DESIGN OF MACHINE ELEMENTS – 1 (17ME54)

E- Notes for Module - 1

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Module 1

- Mechanical Engineering Design
- Phases of design Process
- Design Considerations
- Engineering Materials and their mechanical properties
- Standards and codes
- Factor of safety
- Material selection
- Static Stresses
- Theories of Failure
- Stress concentration
- Impact Stresses

Outcomes

At the end of this Module, the students should have the knowledge of

- Basic concept of design in general.
- Concept of machine design and their types.
- Factors to be considered in machine design.
- Engineering materials
- Mechanical properties
- Standards and codes used in Machine design in general.
- Concept of Factor of safety
- Factors considered in selection of engineering materials
- Static stresses on machine elements.
- Different theories of failure.
- Stress concentration
- Methods to reduce stress concentration
- Stress concentration factor
- Determination of Stress concentration factor
- Impact loads
- Impact factor
- Impact stresses on machine elements
- Evaluate Impact stress

Introduction

To design is to make <u>decisions</u> (Judgment, assessment)

Machine Design

Investigation of the various decisions which determine the mechanical arrangement of parts in a machine and which influences the size, shape or material of a finished part.

Design by evolution, Design by innovation

Design Process

Among the fundamental elements of the design process are the establishment of objectives and criteria, synthesis, analysis, Construction, testing and evaluation.

Synthesis – Concerned with assembling the elements into a workable model

Analysis – Simplification of real world through models

Design Considerations

Traditional Considerations

- **For bulk or body of the equipment**
  - Strength
  - Deflection
  - Weight
  - Size and shape
- **For the surface of component**
  - Wear
  - Lubrication
  - Corrosion
  - Frictional forces
  - Frictional heat generator
- **Cost**

Modern Considerations

- **Safety**
- **Ecology**

(Land, air, water, thermal pollution, resource conservation)

Quality of life (LQI Life Quality Index)

(Physical health, Material well being, Environment, Cultural – Educational, Equality of opportunity, Personal freedom)

Reliability and maintainability

Aesthetics

Engineering Materials

Engineering materials are normally classified as **metals** and **nonmetals**.

Metals may be divided into **ferrous** and **non-ferrous metals**.

Important ferrous metals are:

(i) cast iron (ii) wrought iron (iii) steel.

Some of the important non-ferrous metals used in engineering design are:

(a) Light metal group such as aluminium and its alloys, magnesium and manganese alloys.

(b) Copper based alloys such as brass (Cu-Zn), bronze (Cu-Sn).

(c) White metal group such as nickel, silver, white bearing metals eg. SnSb7Cu3, Sn60Sb11Pb, zinc etc.

Ferrous materials

1. **Cast iron:** it is an alloy of iron, carbon and silicon, it is hard and brittle carbon content may be within 1.7% – 3% and carbon may be present as free carbon or iron carbide (Fe_3C)
    a) Grey cast iron
    b) White cast iron
    c) Malleable cast iron
    d) Spheroidal or nodular graphite cast iron
    e) Austenitic cast iron
    f) Abrasion resistant cast iron

Grey cast iron

Carbon here is mainly in the form of graphite.

- inexpensive and has high compressive strength.
- Graphite is an excellent solid lubricant and this makes it easily machinable but brittle.
- Examples of this type of cast iron are FG20, FG35 or FG35Si15.
- The numbers indicate ultimate tensile strength in MPa and 15 indicates 0.15% silicon.

Applications: Cylinder block heads, housing, fly wheels, Machine tool beds, Columns etc.

White cast iron

- It is formed when casting is rapidly cooled.
- Has Carbon in combined state (Cementite and Pearlite form)
- The presence of iron carbide increases hardness and makes it difficult to machine but requires grinding as a shaping process.
- These cast irons are abrasion resistant.
- Seldom used as full castings but is formed on wearing surfaces of chilled mould.

Applications: Break shoes, Rollers for rolling wheels, Car wheels, Cam and followers etc.

Malleable cast iron

- These are white cast irons rendered malleable by annealing.
- These are tougher than grey cast iron and they can be twisted or bent without fracture.
- They have excellent machining properties and are inexpensive.
- Malleable cast iron are used for making parts where forging is expensive
- Depending on the method of processing they may be designated as black heart BM32, BM30 or white heart WM42, WM35 etc.

Applications: Complex castings which often need machining, Agricultural equipments, Locomotives, Rail road cars, Pipe fittings, flanges, Valves Chains etc.

Spheroidal or nodular graphite cast iron

- Also known as Ductile cast iron.
- In these cast irons graphite is present in the form of spheres or nodules.
- They have high tensile strength and good elongation properties.
- Stronger, more ductile, tougher and less porous than Gray cast iron.
- They are designated as, for example, SG50/7, SG80/2 etc. where the first number gives the tensile strength in MPa and the second number indicates percentage elongation.

Applications: Crank shafts, Piston, Pulleys, Cylinder heads etc.

Austenitic cast iron or Alloy Cast iron

- Austenitic flake graphite iron designated, ex: AFGNi16Cu7Cr2
- Austenitic spheroidal or nodular graphite iron designated, ex: ASGNi20Cr2.
- Contain small percentages of silicon, manganese, sulphur, phosphorus etc.
- They may be produced by adding alloying elements viz. nickel, chromium, molybdenum, copper and manganese in sufficient quantities.
- These elements give more strength and improved properties.

Applications: Automobile parts such as cylinders, pistons, piston rings, brake drums etc.

Abrasion resistant cast iron

- These are alloy cast iron and the alloying elements render abrasion resistance.
- A typical designation is ABR33 Ni4 Cr2 which indicates a tensile strength in kg/mm^2 with 4% nickel and 2% chromium.

Wrought iron

- This is a very pure iron where the iron content is of the order of 99.5%.
- It is produced by re-melting pig iron and some small amount of silicon, sulphur, or phosphorus may be present.
- It is tough, malleable and ductile and can easily be forged or welded.
- It cannot however take sudden shock.

Applications: Chains, crane hooks, railway couplings etc.

Steel

This is by far the most important engineering material and there is an enormous variety of steel to meet the wide variety of engineering requirements.

Steel is basically an alloy of iron and carbon in which the carbon content can be **less than 1.7%** and carbon is present in the form of iron carbide to impart hardness and strength.

Two main categories of steel are

(a) Plain carbon steel

Alloy steel

Plain carbon steel

The properties of plain carbon steel depend mainly on the carbon percentages and other alloying elements are not usually present in more than 0.5 to 1% such as 0.5% Si or 1% Mn etc. There is a large variety of plane carbon steel and they are designated as C01, C14, C45, C70 and so on where the number indicates the carbon percentage.

Dead mild steel- upto 0.15% C, Low carbon steel or mild steel- 0.15 to 0.46% C

Medium carbon steel- 0.45 to 0.8% C. High carbon steel- 0.8 to 1.5% C

Alloy steel

These are steels in which elements other than carbon are added in sufficient quantities to impart desired properties. Chief alloying elements added are usually

- nickel for strength and toughness,
- chromium for hardness and strength,
- tungsten for hardness at elevated temperature,
- vanadium for tensile strength,
- manganese for high strength in hot rolled and heat treated condition,
- silicon for high elastic limit, cobalt for hardness and
- molybdenum for extra tensile strength.

Examples of alloy steels are 35Ni1Cr60, 30Ni4Cr1, 40Cr1Mo28, 37Mn2. Stainless steel is one such alloy steel that gives good corrosion resistance. (18/8 steel where chromium and nickel percentages are 18 and 8 respectively). A typical designation of a stainless steel is 15Si2Mn2Cr18Ni8 where carbon percentage is 0.15.

Applications of Steel

Crank shafts, Connecting rods, Piston rods, Keys, Pins, Rivets, Bolts, Ball and Roller bearings, Springs, Shafts, Gears, Valves, Frames of heavy stationary and transportation equipments, tubes, dyes, Rolls, Levers etc.

Non-ferrous metals

Metals containing elements other than iron as their chief constituents are usually referred to as non-ferrous metals. There is a wide variety of non-metals in practice. However, only a few exemplary ones are discussed below:

Aluminium

This is the white metal produced from Alumina. In its pure state it is weak and soft but addition of small amounts of Cu, Mn, Si and Magnesium makes it hard and strong. It is also corrosion resistant, low weight and non-toxic.

Duralumin

This is an alloy of 4% Cu, 0.5% Mn, 0.5% Mg and aluminium. It is widely used in automobile and aircraft components.

Y-alloy- This is an alloy of 4% Cu, 1.5% Mn, 2% Ni, 6% Si, Mg, Fe and the rest is Al. It gives large strength at high temperature. It is used for aircraft engine parts such as cylinder heads, piston etc.

Magnalium- This is an aluminium alloy with 2 to 10 % magnesium. It also contains 1.75% Cu. Due to its light weight and good strength it is used for aircraft and automobile components.

Copper alloys

Copper is one of the most widely used non-ferrous metals in industry. It is soft, malleable and ductile and is a good conductor of heat and electricity. The following two important copper alloys are widely used:

Brass (Cu-Zn alloy)- It is fundamentally a binary alloy with Zn upto 50% . As Zn percentage increases, ductility increases upto ~37% of Zn beyond which the ductility falls. Small amount of other elements viz. lead or tin imparts other properties to brass. Lead gives good machining quality and tin imparts strength. Brass is highly corrosion resistant, easily machinable and therefore a good bearing material.

Applications:Springs, Radiator tubes, Valve stems, Propeller tubes, Condensor tubes etc.

Bronze (Cu-Sn alloy)

This is mainly a copper-tin alloy where tin percentage may vary between 5 to 25. It provides hardness but tin content also oxidizes resulting in brittleness. Deoxidizers such as Zn may be added. Gun metal is one such alloy where 2% Zn is added as deoxidizing agent and typical compositions are 88% Cu, 10% Sn, 2% Zn. This is suitable for working in cold state. It was originally made for casting guns but used now for boiler fittings, bushes, glands and other such uses.

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Non-metallic materials are also used in engineering practice due to principally their low cost, flexibility and resistance to heat and electricity. Though there are many suitable non-metals, the following are important few from design point of view:

Timber

This is a relatively low cost material and a bad conductor of heat and electricity. It has also good elastic and frictional properties and is widely used in foundry patterns and as water lubricated bearings.

Leather

This is widely used in engineering for its flexibility and wear resistance. It is widely used for belt drives, washers and such other applications.

Rubber

It has high bulk modulus and is used for drive elements, sealing, vibration isolation and similar applications.

Plastics

These are synthetic materials which can be moulded into desired shapes under pressure with or without application of heat. These are now extensively used in various industrial applications for their corrosion resistance, dimensional stability and relatively low cost.

There are two main types of plastics:

a. Thermosetting plastics

b. Thermoplastics

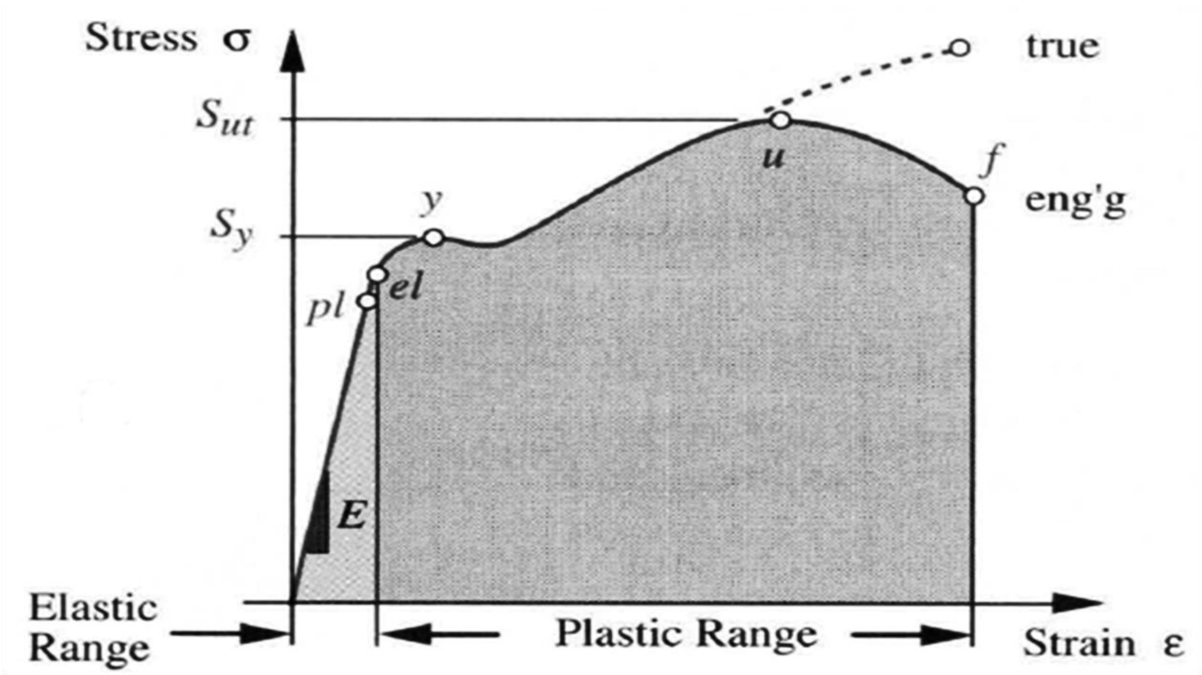
Thermosetting plastics

Thermosetting plastics are formed under heat and pressure. It initially softens and with increasing heat and pressure, polymerisation takes place. This results in hardening of the material. These plastics cannot be deformed or remoulded again under heat and pressure. Some examples of thermosetting plastics are phenol formaldehyde (Bakelite), phenol-furfural (Durite), epoxy resins, phenolic resins etc.

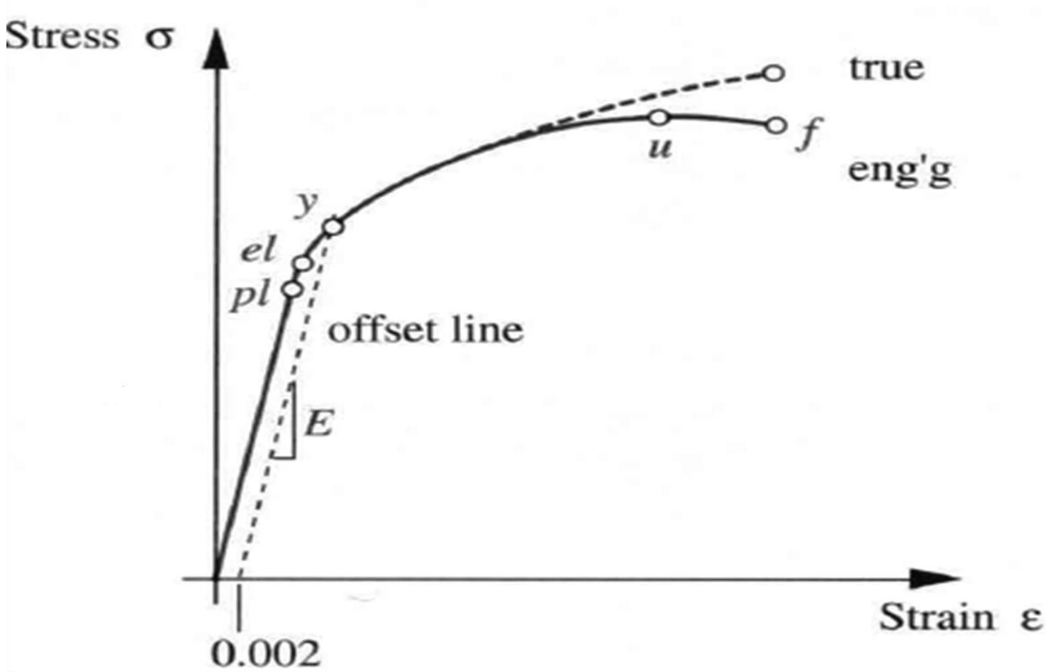
Thermoplastics

Thermoplastics do not become hard with the application of heat and pressure and no chemical change takes place. They remain soft at elevated temperatures until they are hardened by cooling. These can be re-melted and remoulded by application of heat and pressure. Some examples of thermoplastics are cellulose nitrate (celluloid), polythene, polyvinyl acetate, polyvinyl chloride (PVC) etc.

Stress – Strain Curves



Low Carbon Steel



Annealed High-Carbon Steel

Mechanical properties

Mechanical parameters	**Definition**	**Measurable parameter**
Stiffness	***Resistance to elastic deformation***	***Young's modulus***
Strength	**Resistance to plastic deformation**	**Yield stress**
Toughness	**Resistance to fracture**	**Energy to fracture**
Ductility	**Ability to plastically deform**	**Strain to fracture**

Elasticity:

It is the property of material to regain its original shape after deformation when the external forces are removed.

Plasticity:

The permanent deformation of the material when the stress level exceeds the yield point under plastic conditions materials ideally deform without any increase in stress.

Elastic deformation until strains of about 0.005 (or 0.5%).Beyond this permanent or non-recoverable strain occurs i.e. Plastic deformation

Hardness:

Property of material that enables it to resist permanent deformation, penetration, indentation etc. Size of indentations by various types of indenters are the measure of hardness e.g. Brinnel hardness test, Rockwell hardness test, Vickers hardness (diamond pyramid) test. These tests give hardness numbers which are related to yield pressure (MPa).

Ductility:

This is the property of the material that enables it to be drawn out or elongated to an appreciate extent before rupture occurs. Normally if percentage elongation exceeds 15% the material is ductile and if it is less than 5% the material is brittle. Lead, copper, aluminium, mild steel are typical ductile materials.

Malleability:

It is a special case of ductility where it can be rolled into thin sheets but it is not necessary to be so strong. Lead, soft steel, wrought iron, copper and aluminium are some materials in order of diminishing malleability.

Brittleness:

This is opposite to ductility. Brittle materials show little deformation before fracture and failure occurs suddenly without any warning.

Creep:

when a member is subjected to a constant load over a long period of time it undergoes a slow permanent deformation. This is dependent on temperature. Usually at elevated temperatures creep is high.

Toughness: The ability of a material to absorb energy per unit volume without fracture is called toughness (also called modulus of toughness)

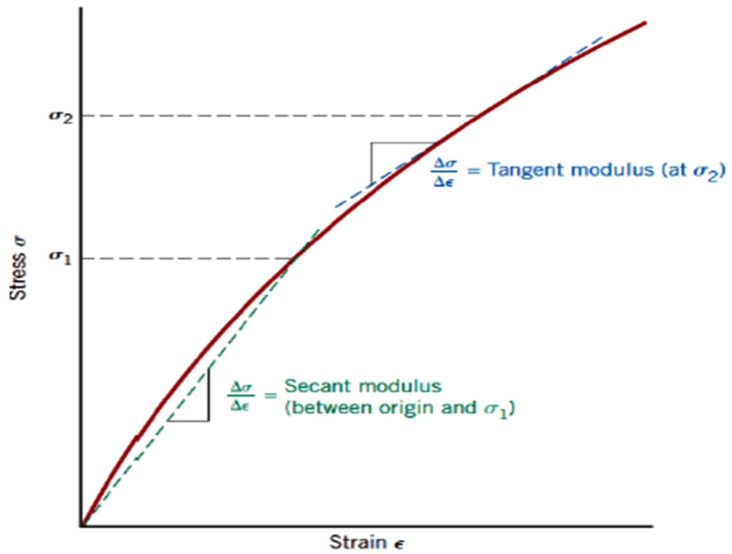
Resilience: The ability of a material to absorb energy per unit volume without permanent deformation is called its resilience (also called modulus of resilience).

$$U_T = \int_0^{\varepsilon_f} \sigma d\varepsilon \cong \left(\frac{S_y + S_{ut}}{2}\right)\varepsilon_f$$

$$U_R = \frac{1}{2} S_{el} \frac{S_{el}}{E} = \frac{1}{2}\frac{S_{el}^2}{E}$$

$$U_R = \frac{1}{2}\frac{S_{el}^2}{E}$$

Materials like gray cast iron have non-linear elastic curve Either tangent or secant modulus used. Tangent modulus slope of curve at specified stress (σ_2).. Secant modulus slope of line from origin to some (σ - ϵ) point on curve.



Standards and codes

What is Standard?

Standard can be defined as a set of technical definitions and guidelines – or simply a "how to" instructions for designers and manufacturers. It gives all the necessary requirements for the product, service, and operation.

A designer will use the standard to design the product, and a manufacturer will use the standard for the manufacturing of the product.

Example:

ASTM international | American Society of Mechanical Engineers | International Organization of Standard….

American National Standard…. | IEEE Standards Association | Deutshes Instiut für Normung

What is Code?

When governmental bodies adopt the standard and become legally enforceable, or when it has been incorporated into a business contract, the standard will become a code.

ASME codes are legally enforceable in many US state. Whereas, in the other part of the world they are not legally enforceable but such countries have their own similar codes. Example : BIS codes, Indian Boiler code etc.

- Code Provides a set of rules that specify the minimum acceptable level of safety & Quality for manufactured, fabricated or constructed goods.
- Codes also refer out to standards or specifications for the specific details on additional requirements that are not specified in the Code

Example: IS 1570 (Part II Section I) 1979 Reaffirmed 1993, IS 1865 – SG Grey cast Iron, IS 1762 – Low Medium alloy steels, IS 919 Part – 1 1993 Fundamental tolerances, IS 2100 – 1962 Rivet steel.

Specification

What is Specification?

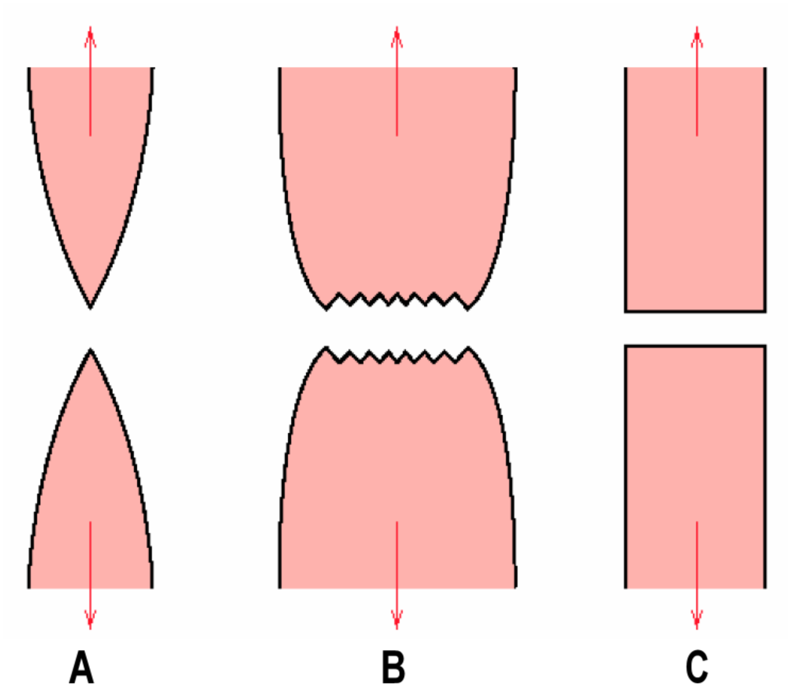
Specifications provide specific/additional requirements for the materials, components or services that are beyond the code or standard requirements.

For Example, if you want A106 Gr B pipe with Maximum carbon of 0.23% against standard requirements of 0.3% Max, you have to specify your requirement in your specification or Purchase Order. Specification is generated by private companies to address additional requirements applicable to a specific product or application.

Why Specification is required?

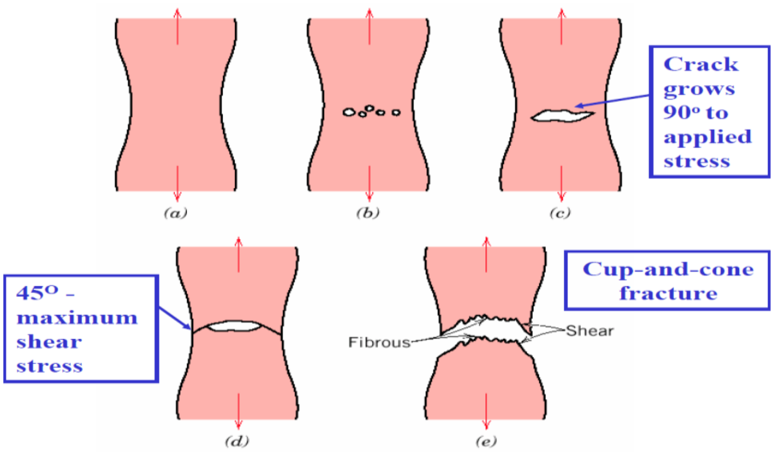
It allows purchaser to include special requirements as per design and service conditions. It allows customizing your product. Please note requirement in specification are must meet requirements. Examples- Product specification, Shell DEP & EIL Specification

Types of static failure



A. Very ductile, soft metals (e.g. Pb, Au) at room temperature, other metals, polymers, glasses at high temperature. **B. Moderately ductile fracture,** typical for ductile metals. **C. Brittle fracture,** cold metals, ceramics.

Failure in a **ductile material** is usually considered to have occurred when yielding i.e plastic deformation becomes sufficiently large to destroy engineering usefulness even though no fracture has occurred. **Brittle materials** fail by fracture with little or no notice.



(a) Necking (b) Formation of microvoids (c) Coalescence of microvoids to form a crack (d) Crack propagation by shear deformation (e) Fracture

Factor of safety

In designing a component it is necessary to ensure sufficient reserved strength in case of an accident. This is ensured by a suitable factor of safety which is defined as "the ratio of failure stress to allowable or design stress".

$$FOS = \frac{Failure\ stress}{Design\ stress} = \frac{Failure\ load}{Allowable\ load}$$

Allowable Stress or Design stress

The value of stress used in design to determine the dimensions of the component or stress which the designer expects not to exceed under normal operating conditions.

for ductile materials

$$\sigma_d = \frac{\sigma_{yield}}{FOS}$$

for brittle materials

$$\sigma_d = \frac{\sigma_{ultimate}}{FOS}$$

Factors which are difficult to evaluate accurately in the design analysis

- Uncertainty in loading
- Inhomogeneity of materials
- Various material behaviours (corrosion, plastic flow, creep)
- Residual stress due to different manufacturing processes
- Safety and reliability

In addition to these factors, the numbers of assumptions made in design analysis, in order to simplify the calculations, may not be exactly valid in working conditions. FOS ensures against these uncertainties and unknown conditions.

Magnitude of factor safety depends upon the following factors

- Effect of failure
- Type of load
- Degree of accuracy in force analysis
- Material of component
- Reliability of component
- Cost of component
- Testing of machine element
- Service conditions
- Quality of manufacture

Range of FoS Ex.:

- For Cast iron components - 3 to 6 on Ultimate strength
- For ductile materials made of steel 1.5 to 2 on Yield strength
- For components made of ductile materials and those subjected to fluctuation of loads - 1.3 to 1.5 on Endurance strength.
- The design of components such as Cam and follower, Gears, Rolling contact bearings – 1.8 to 2.5 on Surface endurance strength due to contact stresses.

Refer Data handbook for range of FoS

Material selection

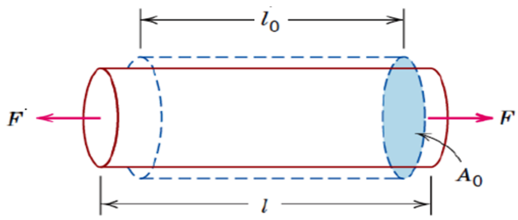
Factors to be considered in the selection of materials :

1. **Availability**
2. **Cost**
3. **Mechanical properties**
    a. Strength (Static load) – S_{ut} or S_y
    b. Strength (Fluctuating load) - S_{en}
    c. Rigidity - Young's Modulus
    d. Ductility - Percent elongation
    e. Hardness - Brinell (HB) or Rockwell (HRC)
    f. Toughness - Izod or Charpy value
    g. Frictional property – Coefficient of friction
4. **Manufacturing considerations**

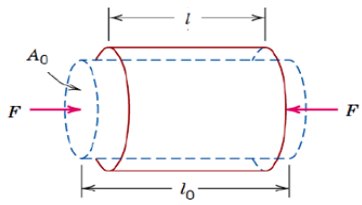
- Depending on service conditions and fundamental requirements materials are selected.
- Machinability is an important factor
- Machining governs the selection of materials

Manufacturing processes like Casting, Rolling, Forging, Extrusion, Welding etc.

Normal Stresses



Tensile



Compressive

A prismatic bar is a straight structural member having the same cross section throughout its length, and **an axial force is a force directed along the axis** of the member, resulting in either tension or compression in the bar.

The intensity of the force (i.e. force per unit area) is called **stress**. The axial force **F** acting at the cross section is the resultant of the continuously distributed stresses.

The stresses act in a direction perpendicular to the cut surface, they are called **Normal stresses.**

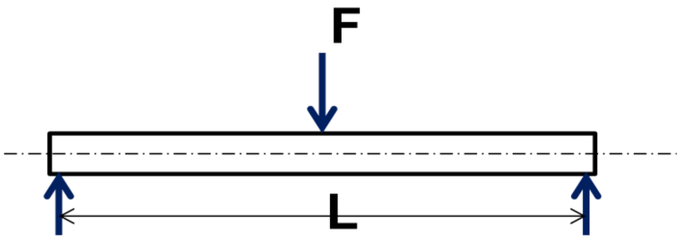
Limitation: The equation for normal stress is valid only if the stress is uniformly distributed over the cross section. This condition is realized **only if the axial force F acts through the centroid of the cross sectional area**.

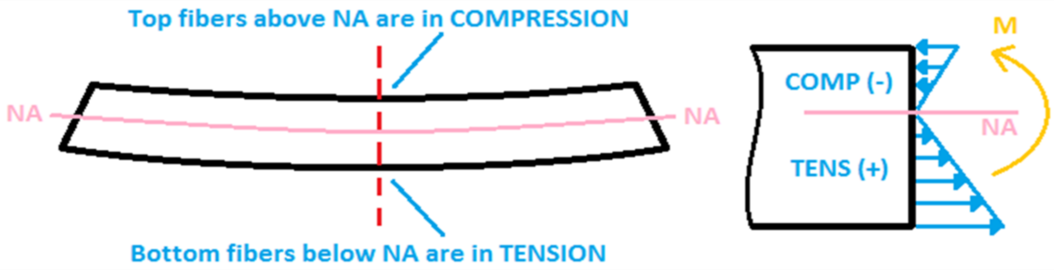
Bending stress

M = Bending moment N-mm,

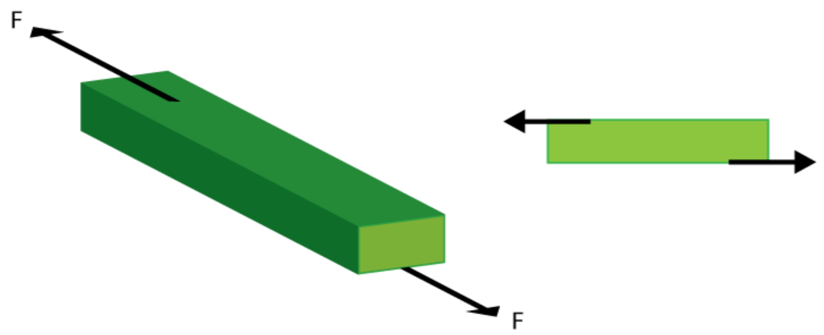
Z = Section Modulus (I/c)

$$\sigma_b = \frac{M}{Z}$$

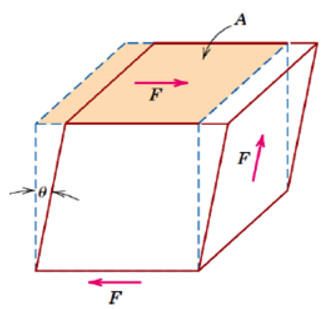




Shear stress



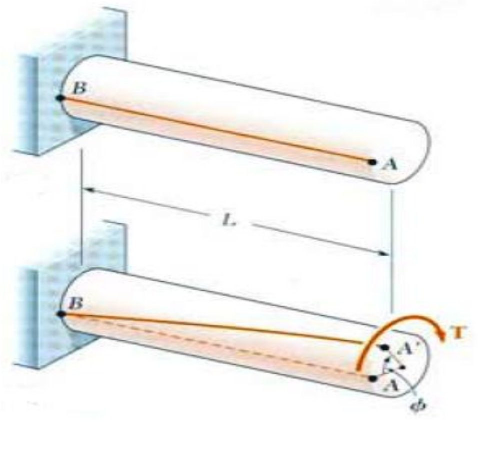
These forces are causing shear stress because they are acting within the same plane of a surface of the prismatic bar.

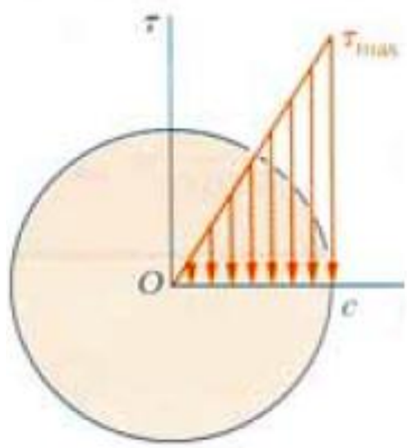


Direct Shear: When an external force acting on a component tends to slide the adjacent planes with respect to each other, the resulting stress on these planes are called shear stress.

Torsional Shear: The internal stress which are induced to resist the action of twist are called torsional shear stress.

T = Torque, r = radius, J = Polar Moment of Inertia, τ = Shear stress

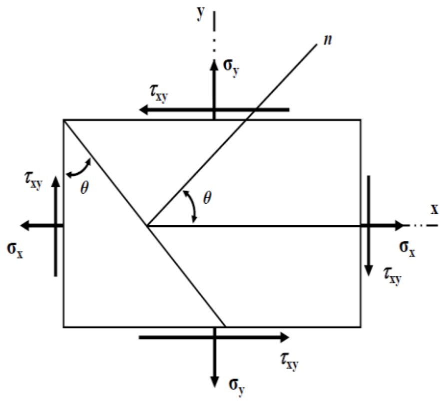


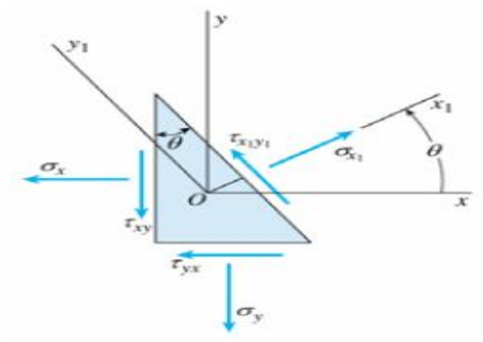


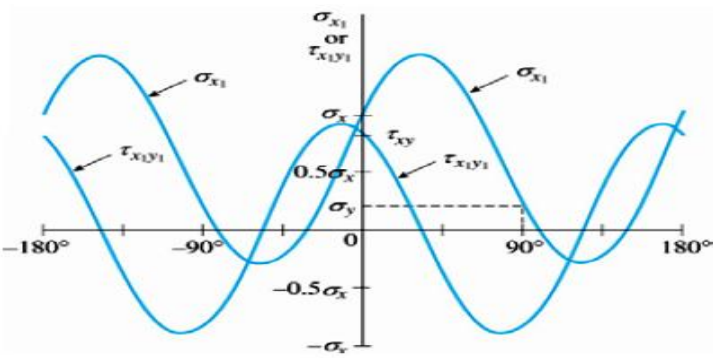
$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

$$\tau_{Max} = \frac{Tr}{J}$$

Biaxial Stress System







$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x1y1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\,\tau_{xy}}$$

the principal stresses can be written as

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm [(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2]^{½}$$

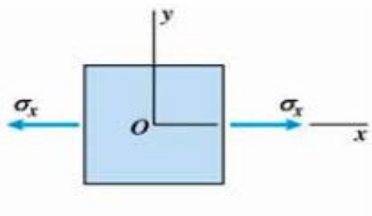
and the corresponding maximum shear stress is

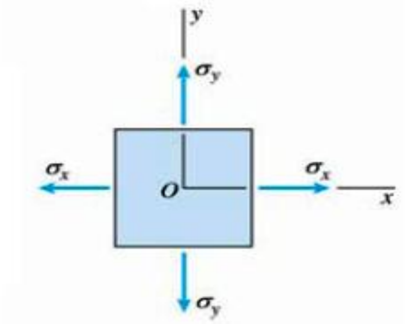
$$\tau_{max} = [(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2]^{½}$$

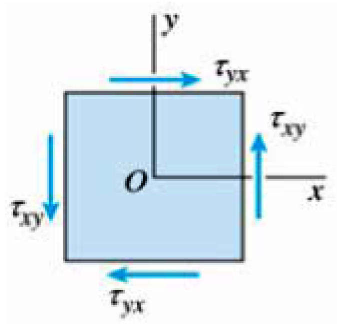
note that there are no shear stresses on the principal plane

for uniaxial and biaxial stress states, $\tau_{xy} = 0$

$\tan 2\theta_p = 0 \qquad \theta_p = 0^o$ and 90^o





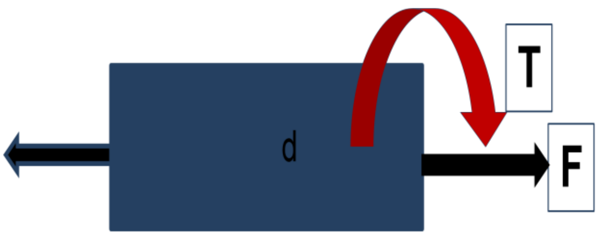


for pure shear stress, $\sigma_x = \sigma_y = 0$

$$\tan 2\theta_p = \infty$$

$$\theta_p = 45^o \text{ and } 135^o$$

Combined axial and torsion



Normal stress $\sigma = \frac{F}{A}$

Shear stress $\tau = \frac{T}{Z_t}$

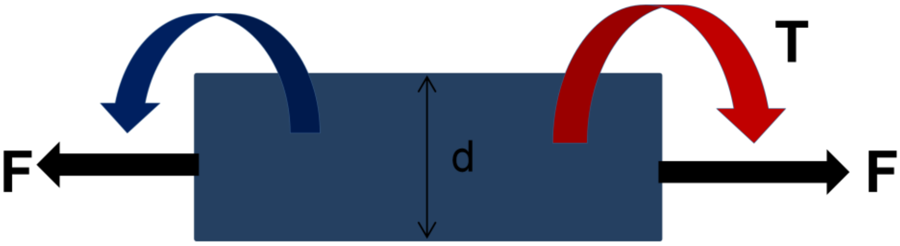
Maximum or Equivalent normal stress

$$\sigma_{max} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Maximum or Equivalent shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Combined axial, bending moment and torsion



Axial stress

$$\sigma_a = \frac{F}{A}$$

Bending s

$$\sigma_b = \frac{M}{Z_b}$$

$$\sigma = \sigma_a + \sigma_b$$

$$\tau = \frac{T}{Z_t}$$

Shear stress

Maximum or Equivalent normal stress

$$\sigma_{max} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Maximum or Equivalent shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Theories of failure

The relationship between the strength of mechanical component subjected to complex state and mechanical properties in simple tension test is obtained by theories of failure. With the help of these theories the data obtained in simple tension test is used to determine the dimensions of components due to complex loads.

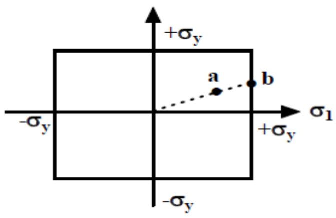
1. Maximum normal stress theory (Rankines theory)
2. Maximum shear stress theory (Guest's theory)
3. Distortion energy theory (Von-Mises theory)
4. Maximum strain theory (St. Venant's theory)

The Maximum-Normal-Stress Theory (Rankine theory)

The theory states that failure of mechanical components subjected to biaxial or triaxial stresses occurs when maximum normal stress reaches the yield or ultimate stress of material as in simple tension test.

$$\sigma_1 = \pm\, \sigma_y$$

$$\sigma_2 = \pm \sigma_y$$

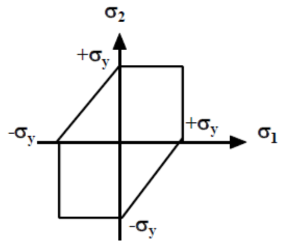


Yield surface corresponding to Maximum Principal stress theory

The Maximum Shear Stress Theory (Guest's theory)

It states that failure of mechanical components subjected to biaxial or triaxial stresses occurs when maximum shear stress at any point in component becomes equal to maximum shear stress in the specimen as in simple tension test.

$$\sigma_1 - \sigma_2 = \pm\sigma_y$$



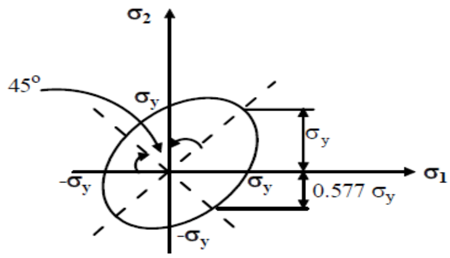
Yield surface corresponding to maximum Shear stress theory

Maximum Distortion energy theory (Hencky Von Mise's theory)

The theory states that the failure of mechanical component subjected to biaxial or triaxial stresses occur when strain energy of distortion for unit volume at any point in the component becomes equal to the strain energy of distortion per unit volume as in a simple tension test when the yielding starts.

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2$$

Von Mises Yield Criterion



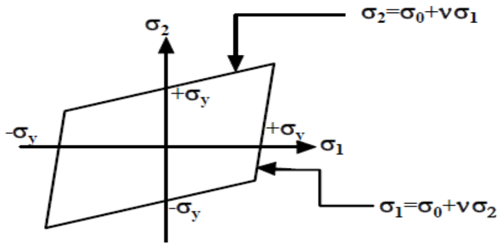
Yield surface corresponding to maximum Distortion Energy theory

Maximum strain theory (Saint Venant's theory)

This theory states that in-elastic action occurs when the maximum unit strain in any direction exceeds the unit strain at the tensile yield point for the material as determined in simple tension test.

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad |\sigma_1| \geq |\sigma_2|$$

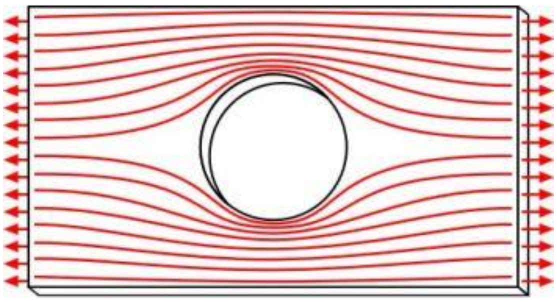
$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad |\sigma_2| \geq |\sigma_1|$$



Yield surface corresponding to maximum Strain theory

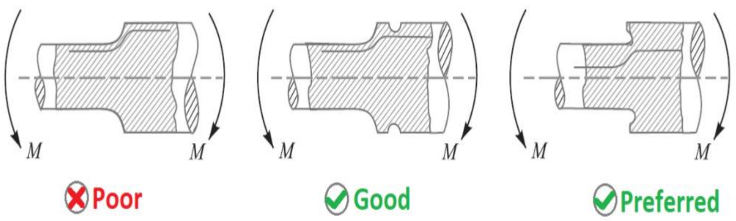
Stress concentration

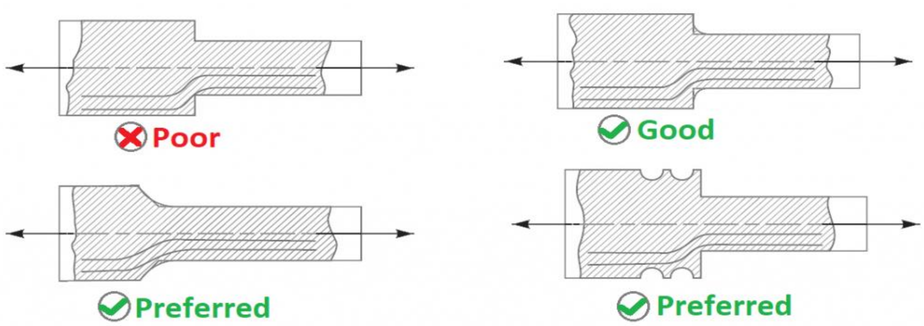
It is defined as the condition which causes the actual stresses in machine member to be higher than the normal values predicted by the elementary, direct and combined stress equations. The irregularities in stress distribution is caused by abrupt changes in the form is called stress concentration. It occurs for all kinds of loads, axial, bending, torsional in presence of fillets, holes, scratches, key ways, splines, tool marks or accidental scratches

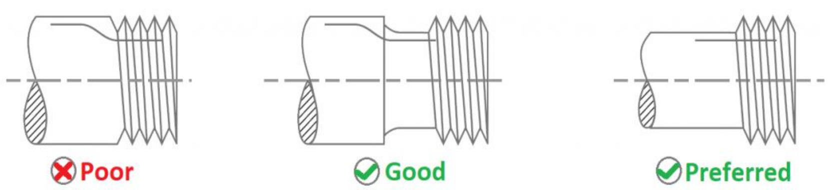


Stress concentration around a hole

Methods to reduce Stress concentration (Courtesy: extrudesign.com)







Stress concentration factor (K_t)

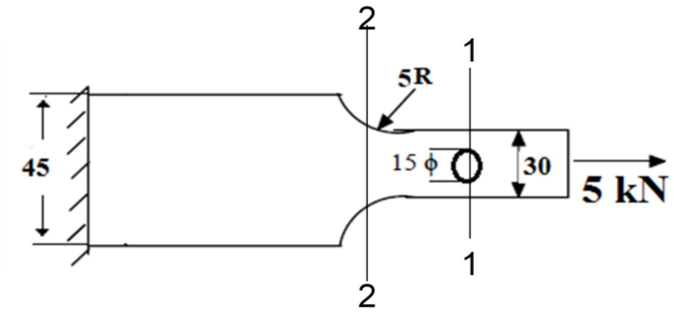
It is defined as the ratio of maximum or significant stress at the discontinuity to the nominal Stress at minimum cross section.

$$K_t = \frac{\textit{Highest value of actual stress at fillet, notch, hole etc.}}{\textit{Nominal stress as given by elementary equation at minium cross section}}$$

Theoretical stress concentration factor is based on the geometrical shape of the discontinuity and the material is assumed to be elastic, isotropic and homogenous. Values of stress concentration factor can be found experimentally by Photo elastic analysis, Lasers, Holography, direct measurement using strain gauges.

Problem 1: A flat plate is subjected to a tensile load of 5 kN as shown in Fig. The plate is made up of grey cast iron FG200 and FoS is 2.5. Determine the thickness of the plate.

All dimensions in mm



$$\sigma_d = \frac{\sigma_{Ut}}{FoS} = \frac{200}{2.5} = 80\ MPa$$

At 1-1 d/w = 0.5, Kt_1 = 2.15

At 2 - 2 r/d = 1/6 = 0.17, D/d = 1.5

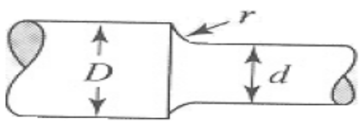
$$\sigma_{Max} = K_t\ \sigma_{Nominal}$$

Kt_2 = 1.8

$$\sigma_{Nominal} = \frac{F}{A}$$

Note: A = Area at minimum cross section (td) t at 1-1 > t at 2-2 hence t= 9 mm

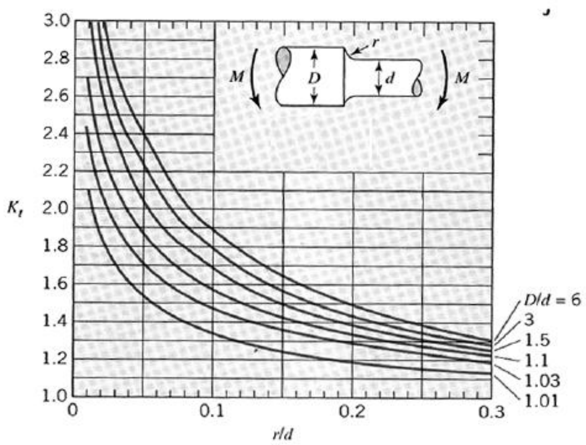
Problem 2: A machine shaft is subjected to a bending moment of 15 Nm. The stress concentration factor at the fillet is 1.5 and ultimate strength of the shaft material is 200 Mpa. Determine the minimum dia., magnitude of stress at fillet and FoS.



D = 1.5d r = 2 mm

$$\sigma_{Max} = K_t\ \sigma_{Nominal}$$

$$\sigma_{Nominal} = \frac{M}{Z}, Z = \frac{\pi d^3}{32}$$



At fillet From graph r/d = 0.15

Hence d = 2/0.15 = 13.33 mm

$$FoS = \frac{\sigma_{ut}}{\sigma_{max}}$$

Impact Stresses

Examples of impact loading

- Driving a nail with hammer
- Breaking of concrete with an air hammer
- Automobile collision
- Dropping of cartons by freight handlers
- Razing of building with an impact ball
- Automobile wheels dropping into pot holes etc.

When do you call it as impact?

Time period of oscillation is given by $\tau = 2\pi \sqrt{\frac{m}{k}}$ **Sec**

$t_{\text{applied loading}} > 3\tau$ Static loading

$\frac{1}{2}\tau < t_{\text{applied loading}} < 3\tau$ Grey Area

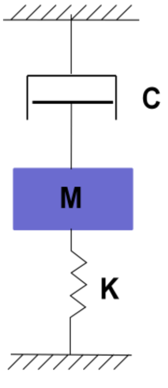
$t_{\text{applied loading}} < \frac{1}{2}\tau$ Dynamic loading

If the time required to apply the load is greater than 3 times, natural period dynamic effects are negligible static loading can be assumed. If the time of loading is < half of the natural period impact is definitely involved.

Models for impact loads

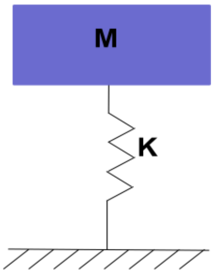
Impact loads are classified into three types in the order of increasing severity

Case (1): Rapidly moving loads are essentially constant magnitude as produced by vehicle crossing a bridge.



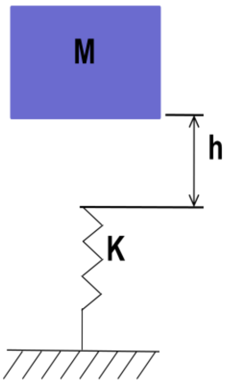
Mass M is held so that it just touches the top of spring and is suddenly released dashpot acts as a frictional supporting force that prevent the full force M×g

Case (2): Suddenly applied load as those resulting from an explosion or combustion in an IC engine cylinder.



Mass M is released and full force M×g is applied on the spring. Since the dashpot is absent it results in the instantaneous application of full force M×g

Case (3): Direct impact loads as produced by a pile driver, drop forge, vehicle collision.

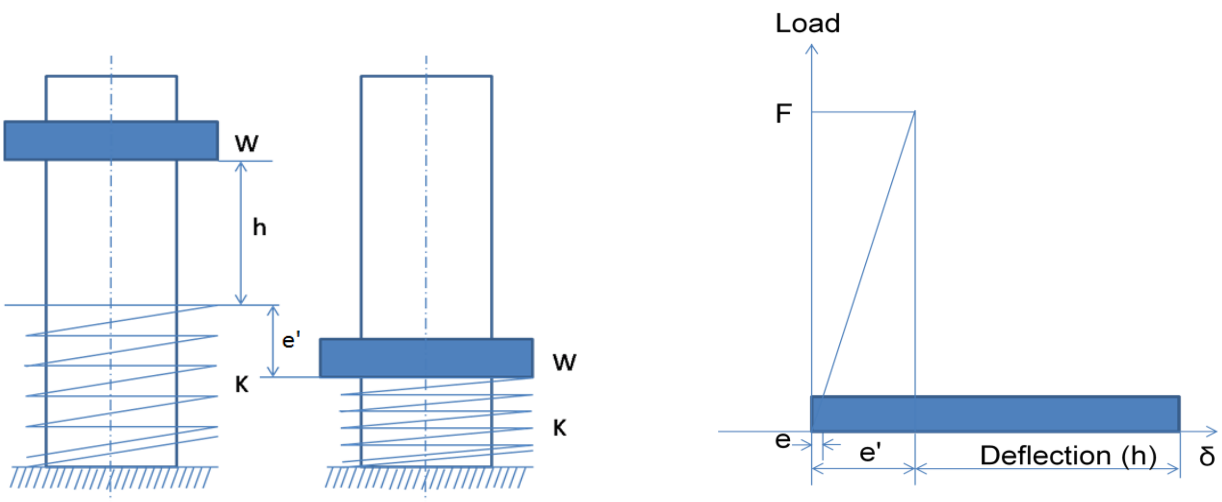


In this case not only a force is applied instantaneously but mass acquires kinetic energy before it strikes the spring.

Impact loads can be compressive, tensile, torsional, bending or combination of these. Static loaded parts must be designed to carry loads, parts subjected to impact must be designed to absorb energy.

Expression for impact stress

e’ = Instantaneous deflection
e = static deflection
F = equivalent force which provides Deflection e’



W= load, K = spring stiffness

For static deflection

$$e = \frac{w}{k} \qquad \text{..................... (1)}$$

For dynamic deflection

$$e' = \frac{F}{k} \qquad \text{..................... (2)}$$

From equation (1) & (2)

$$k = \frac{w}{e} = \frac{F}{e'}$$

$$F = \left(\frac{e'}{e}\right) w \qquad \text{............... (3)}$$

Energy released = Energy absorbed

$$w\,(h + e') = \frac{1}{2} \times W \times \left(\frac{e'}{e}\right) e'$$

$$2\,(h + e') = \frac{(e')^2}{e}$$

$$(2h + 2e') = \frac{(e')^2}{e}$$

$$e'^2 - 2ee' - 2he = 0$$

$$e' = e\left[1 + \sqrt{1 + \frac{2h}{e}}\right] \qquad \text{..................... (4)}$$

$$\frac{e'}{e} = \left[1 + \sqrt{1 + \frac{2h}{e}}\right]$$

$$F = \left(\frac{e'}{e}\right)w$$

$$F = w\left(1 + \sqrt{1 + \frac{2h}{e}}\right)$$

$$\frac{F}{A} = \frac{w}{A}\left(1 + \sqrt{1 + \frac{2h}{e}}\right)$$

$$\sigma' = \sigma\left(1 + \sqrt{1 + \frac{2h}{e}}\right)$$

The quantity $\left(1 + \sqrt{1 + \frac{2h}{e}}\right)$ is called Impact factor.

It is the factor by which the load, stress and deflection caused by dynamically applied load, exceed those caused by slow, Static application of the same weight.

$$\sigma' = \sigma\left(1 + \sqrt{1 + \frac{2h \times AE}{wl}}\right)$$

For suddenly applied load h=0

$$F = w\left(1 + \sqrt{1 + 0}\right)$$

$$F = 2w$$

$$E = \frac{\sigma}{\varepsilon}$$

$$E = \frac{w/A}{e/l}$$

$$E = \frac{wl}{Ae}$$

Expression for energy absorbed

$$e' = e\left[1 + \sqrt{1 + \frac{2h}{e}}\right]$$

e which is the static deflection is very small compared to e'.

$$v^2 = 2gh \qquad h = \frac{v^2}{2g}$$

$$e' = e\sqrt{\frac{v^2}{ge}} = \sqrt{\frac{v^2 e}{g}}$$

$$e' = e\left[1 + \sqrt{1 + \frac{v^2}{ge}}\right]$$

$$e' = \sqrt{\frac{2U}{w} \times \frac{w}{k}}$$

$$\frac{F}{k} = \sqrt{\frac{2U}{k}}$$

$$e' = \sqrt{\frac{2U}{k}}$$

$$F = \sqrt{2Uk}$$

$$F = \sqrt{2U\left(\frac{AE}{L}\right)}$$

$$\mathrm{K} = \frac{w}{e} = \frac{AE}{L}$$

$$\frac{F}{A} = \frac{1}{A}\sqrt{2U\left(\frac{AE}{L}\right)}$$

$$\sigma = \sqrt{\frac{2UE}{AL}} = \sqrt{\frac{2UE}{V}}$$

V = volume

$$\sigma^2 = \frac{2UE}{V}$$

$$U = \frac{\sigma^2 V}{2E}$$

Numerical

Problem 1. A weight of 50 kg is brought on to a platform from a height of 600 mm. The platform is supported by steel bar of cross sectional area 625 mm^2. The bar is 1.25 m long and is supported at the top. Find the maximum stress induced in the bar. What would be the stress if the load were to be applied statically?

W= 50 kg = 490.5 **N**

h = 600 mm, A = 625 mm^2 , l = 1.25 m

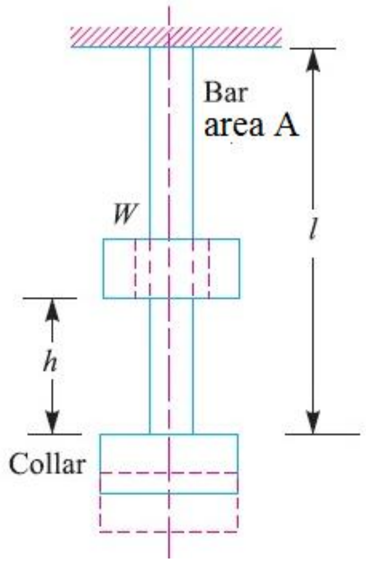
σ' = ?

σ = w/A = 490.5/625 = 0.785 M Pa

$$\sigma' = \sigma\left(1 + \sqrt{1 + \frac{2h}{e}}\right)$$

$$e = \frac{wl}{AE} = \frac{490.5 \times 1250}{625 \times 210 \times 1000}$$

e = 4.67 x 10^{-3} mm



$\sigma' = 0.785(1 + \sqrt{(1+(2\times600)/(4.67 \times 10^{-3}))}$

= 398.5 M Pa

Problem 2. A weight of 1400 N is dropped on to a collar at the lower end of a vertical 3m long 25 mm diameter bar. Calculate the height of drop if the maximum instantaneous stress produce should not exceed 120 MPa.

W = 1400 **N,** E = 200 G Pa, L = 3 m

σ' = 120 M Pa, d = 25 mm, h = ?

$$A = \frac{\pi\, d^2}{4} = 490.87\ mm^2$$

σ = w/A = 1400/490.87 = 2.85 M Pa

$$e = \frac{wl}{AE} = \frac{1400 \times 3000}{490.87 \times 200 \times 1000} = 0.0407 \text{ mm}$$

$$\sigma' = \sigma\left(1 + \sqrt{1 + \frac{2h}{e}}\right)$$

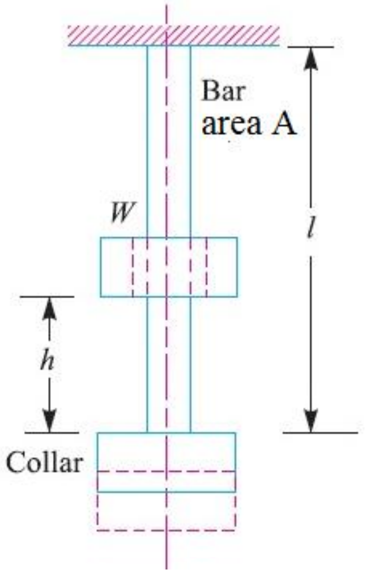
$120 = 2.85(1 + \sqrt{(1+(2\times h)/(0.0407))}$

h = 34.36 mm

Problem 3. An unknown weight fall through 10 mm to the column to a lower end of a vertical bar 3m and 600 mm^2 in section. If maximum instantaneous expansion is known to be 2 mm. What is the corresponding stress and values of unknown weight. E=210 Gpa.

W= ?   h = 10 mm, A = 600 mm^2 , l = 3 m

e' = 2 mm



$$e' = e\left[1 + \sqrt{1 + \frac{2h}{e}}\right]$$

$$2 = e(1 + \sqrt{\frac{2 \times 10}{e}})$$

Square both sides and solve for e, e = 0.167 mm

$$0.167 = \frac{wl}{AE} = \frac{w \times 3000}{600 \times 210 \times 1000}$$

W = 7014 **N**

$$\mathrm{F} = \frac{\mathrm{e}'}{\mathrm{e}} \times \mathrm{w} = \frac{2}{0.167} \times 7014$$

F = 84000 N

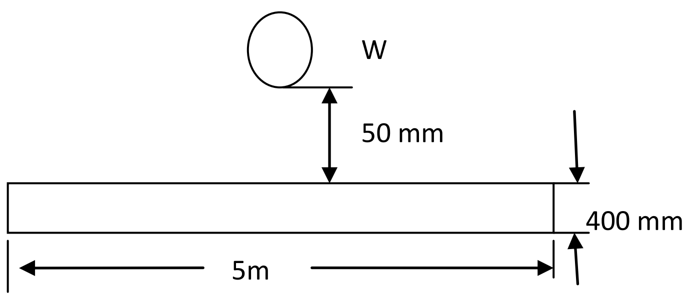
$$\sigma' = \frac{F}{A} = \frac{84000}{600} = 140\ M\ Pa$$

Problem 4. A weight of 6000 N fall from a height of 50 mm on a boxed section beam is simply supported at its ends and is 400 mm deep. The moment of inertia for the box section is 10^8 mm^4, determine the maximum stress induced and compare it with static value.

W= 6000 **N**, h = 50 mm, l = 5 m

H = 400 mm, I = 10^8 mm^4

σ' = ?, σ = ?



$$e = \frac{wl^3}{48EI} = \frac{6000 \times 5000^3}{48 \times 210 \times 10^3 \times 10^8} = 0.74\ mm$$

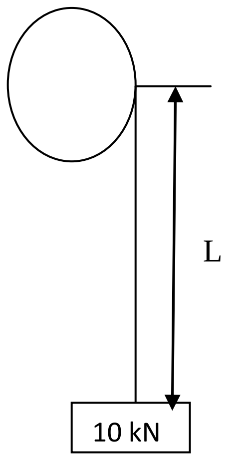
$$\sigma = \frac{M}{Z} = \frac{Wl/4}{I/y} = \frac{6000 \times 5000 \times 200}{4 \times 10^8} = 15\ \mathrm{M\ Pa}$$

$$\sigma' = \sigma\left(1 + \sqrt{1 + \frac{2h}{e}}\right)$$

$$\sigma' = 15\left(1 + \sqrt{1 + \frac{2 \times 50}{0.74}}\right) = 189.6\ M\ Pa$$

$$\frac{\sigma'}{\sigma} = \frac{189.6}{15} = 12.6$$

Problem 5. An elevator carrying lod of 10 kN and is descending by means of steel rope at a speed of a 1 m/sec. The cross section area of the rope is 400 mm^2. The rope is suddenly brought to rest by breaking after 30 sec of descend. Calculate the stress induced in the rope due to sudden stoppage if the Young's Modulus for the rope is 80×10^3 MPa.



W= 10 kN, u = 1 m/s, L = 5 m

t = 30 s, A = 400 mm^2 , E = 80×10^3 MPa,

σ' = ?

v = u + at, v = 0

t = - 1/30 m/s^2

$v^2 = u^2 - 2as$

s = 15 m

$$e = \frac{wl}{AE} = \frac{10000 \times 15000}{400 \times 80 \times 1000} = 4.69 \text{ mm}$$

Σ = W/A = 10000/400 = 25 M Pa

$$\sigma' = \sigma\left(1 + \sqrt{1 + \frac{v^2}{g \times e}}\right)$$

$$\sigma' = 25\left(1 + \sqrt{1 + \frac{1^2}{9.81 \times 4.69\ x\ 10^{-3}}}\right) = 144.3 \text{ M Pa}$$

Exercise problems

Problem 1. A cantilever beam of span 800 mm has a rectangular cross section of depth 200 mm. Free end of the beam is subjected to a transverse load of 1 kN dropped from a height of 40 mm if the material is C50, determine the width of the cross section. Assume FOS 2.5.

Problem 2. A unknown weight falls through 150 mm on a collar rigidly attached to the lower end of vertical bar. 3 m long 500 mm in section, if the maximum instantaneous extension is known to be 2 mm. What is the maximum stress induced.

Problem 3. A machine element in the form of cantilever beam is made of a rod of circular cross section with a span of 800 mm. The material of the rod is C30, determine the safe value for a transverse load that fall on to the free end of the beam from a height of 25 mm. The diameter of the rod is 40 mm.