MODULE - 1 AMPLITUDE MODULATION

INTRODUCTION :-

- Message signals are incompatible for direct transmission. For such a signal, to travel longer distances, its strength has to be increased by modulating with a high frequency carrier wave.
- Modulation is the process of altering any one parameter (amplitude / frequency / phase) of the carrier signal, in accordance with the instantaneous values of the message signal by keeping other parameters of carrier constant.
- Figure 1, shows the general schematic representation of modulation process. It consists of 3-types of signals as follows.

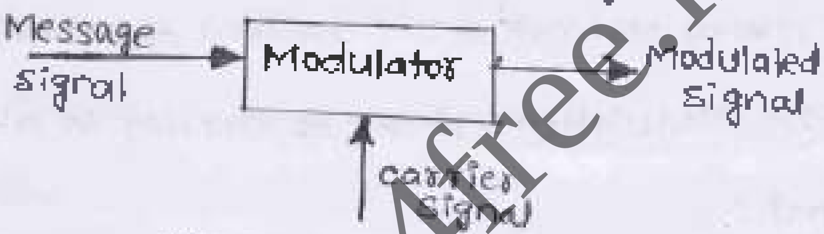


Figure 1: Modulation process

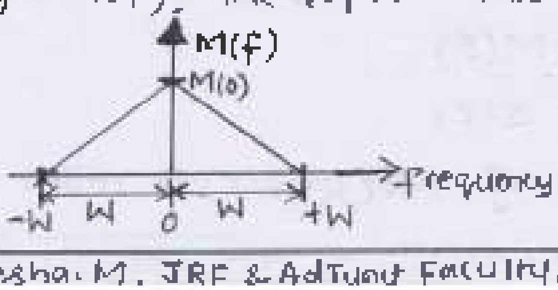
1. Message signal :- The signal which contains a message to be transmitted, is called as message signal. It is also known as base-band signal, which has to undergo the process of modulation. Hence it is also named as "Modulating signal". Mathematically, it is denoted by $m(t)$.

Ex: $m(t) = A_m \cos 2\pi f_m t$

where, A_m = Amplitude of message signal in Volts.

f_m = frequency of message signal in Hz.

** In frequency domain the spectrum of $m(t)$ is denoted by $M(f)$. The spectrum is bandlimited to W Hz.



- Where $\boxed{W = f_m}$
- $M(0)$ = Amplitude of $M(f)$ at frequency $\boxed{f = 0}$.

2. Carrier signal: It is a high frequency signal, used to carry the signal to the receiver after modulation.
Mathematically it is denoted by $c(t)$.

Ex: $c(t) = A_c \cos(2\pi f_c t + \phi_c)$, its 3-parameters are

A_c = Amplitude of carrier signal in Volts

f_c = frequency of carrier signal in Hz

ϕ_c = phase of carrier signal in degrees.

↳ Depending on the altering parameter of carrier signal, there are 3-types of Modulation techniques namely.

i) Amplitude Modulation : Amplitude of carrier is altered.

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ii) Frequency Modulation : Frequency of carrier is altered.
iii) Phase Modulation : phase of carrier is altered.

3. Modulated Signal :-
The resultant signal after the process of modulation is called as "Modulated signal". This signal consists of modulating signal and carrier signal.
Mathematically it is denoted by $S(t)$.

Example: The Amplitude modulated signal for any $m(t)$ is given by,

$$S(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

where, K_a = Amplitude Sensitivity parameter (Discussed in Next Section).

Note: • In this module the time domain and Frequency domain analysis is discussed

• Fourier transformation: It is a technique used to convert time domain signal to frequency domain signal.

Ex: i.e., $m(t) \xrightarrow{F.T} M(f)$

$S(t) \xrightarrow{F.T} S(f)$

* Graphical representation of $M(f)$ ⓐ $S(f)$ is called as "Spectrum".

* Frequency Spectrum gives the details of frequency components.

* Table 1 gives the Fourier transformation of few standard time domain signals, which are repeatedly used in this module.

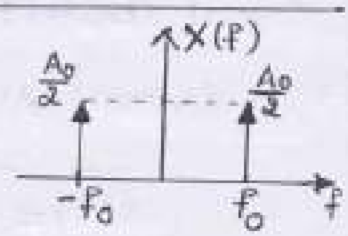
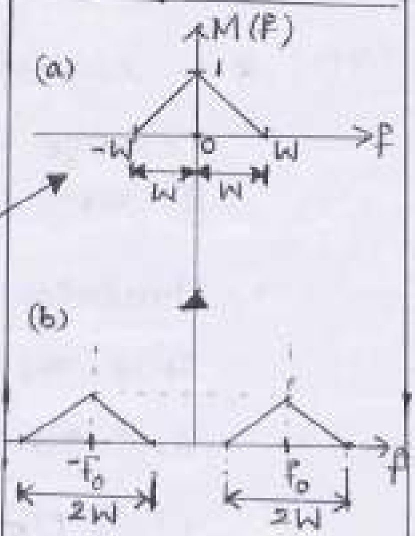
Time Domain Signal	Fourier Transformation (F.T)	Frequency Spectrum
$x(t) = A_0 \cos 2\pi f_0 t$ Any General Cosine Signal with amplitude A_0 and frequency 'f_0'.	$X(f) = \frac{A_0}{2}\left[\underbrace{\delta(f-f_0)}_{\text{Impulse at } f=f_0} + \underbrace{\delta(f+f_0)}_{\text{Impulse at } f=-f_0}\right]$ * $\delta(f-f_0)$ and $\delta(f+f_0)$ are "Impulse" signals.	 ↳ Spectrum of $x(t)$ consists of two impulse signals at frequencies $f = f_0$ and $f = -f_c$ with equal amplitudes $\frac{A_0}{2}$.
$\underbrace{m(t) \cdot \cos 2\pi f_0 t}_{\text{Product}}$ of Any $m(t)$ with Cosine signal having frequency 'f_0'.	$\frac{1}{2}\left[M(f-f_0) + M(f+f_0)\right]$ * $M(f) \rightarrow$ Spectrum of $m(t)$ shown in fig (a) centered at $f=0$ * $M(f-f_0) \rightarrow M(f)$ at center frequency $f = f_0$ * $M(f+f_0)$ - ↳ $M(f)$ at Center frequency $f = -f_0$	

Table 1

* **Advantages of Modulation:**

Advantages of using modulation process in Communication systems are as follows.

1. Reduces the height of Antenna
2. Avoids mixing of signals
3. Allows Multiplexing of signals
4. Allows Adjustments of Bandwidth
5. Increases the range of Communication
6. Improves Quality of Reception.

1.2. Amplitude Modulation: (A.M)

Definition: It is a process of altering the amplitude of carrier signal in accordance with the instantaneous values of message signal by keeping frequency and phase of carrier signal constant.

↳ In General AM signal, standard equation is given by

$$\boxed{s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t} \quad \text{---} *$$

K_a = Amplitude sensitivity parameter

$m(t)$ = Message signal

A_c = Amplitude of carrier signal

f_c = frequency of carrier signal.

Note: * In Amplitude modulation, information present in the message signal $m(t)$, resides only in the amplitude of $s(t)$.

→ **Modulation Index:** - (μ)

It is the product of amplitude sensitivity parameter and the maximum value of message signal.

i.e., $\boxed{\mu = K_a |m(t)|_{max}}$ No units (or) $\boxed{\mu = K_a A_m}$

K_a = Amplitude sensitivity

$|m(t)|_{max}$ = Maximum value (or) Amplitude of $m(t)$.

↳ The maximum value of modulation index, $\mu = 1$.

↳ If $\mu > 1$, the carrier wave becomes over modulated.

↳ If $\mu < 1$, the carrier wave becomes under modulated

↳ If $\mu = 1$, the carrier wave becomes critically modulated.

* In AM, the carrier frequency, $\boxed{f_c \gg f_m}$; f_m = frequency of $m(t)$

** Time and Frequency domain description of AM-signal:

Q) Define Amplitude Modulation. Obtain the expression for AM by both time domain and frequency domain representation with necessary waveforms.

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↳ **Amplitude Modulation:-**

Defn:- It is a process of altering the amplitude of carrier signal in accordance with the instantaneous values of message signal by keeping frequency and phase of carrier signal constant.

Expression for AM signal:-

- The instantaneous value of message signal is given by,

$$m(t) = A_m \cos(2\pi f_m t) \quad \text{——(1)}$$

where, $A_m \Rightarrow$ Amplitude of message signal.

$f_m \Rightarrow$ frequency (or) Bandwidth of message signal.

- The instantaneous value of carrier signal is given by,

$$c(t) = A_c \cos(2\pi f_c t) \quad \text{——(2)}$$

where, $A_c \Rightarrow$ Amplitude of carrier signal.

$f_c \Rightarrow$ frequency of carrier signal.

- We know that the standard equation of AM signal is given by,

$$S(t) = A_c\left[1 + K_a m(t)\right]\cos(2\pi f_c t) \quad \text{——(3)}$$

where, K_a = Amplitude sensitivity parameter.

Substitute $m(t) = A_m \cos 2\pi f_m t$ in equation (3)

$$S(t) = A_c\left[1 + \underbrace{K_a \cdot A_m}\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$\therefore S(t) = A_c\left[1 + \mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t) \quad \text{——(4)}$$

Where $\mu = K_a A_m \Rightarrow$ Modulation Index for AM-signal

$$S(t) = [A_c + \mu A_c \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$S(t) = A_c \cos 2\pi f_c t + \mu A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t$$

We know that, $\cos A \cdot \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$$\therefore S(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos 2\pi (f_c - f_m)t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m)t$$

$$\therefore \boxed{S(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{carrier}} + \frac{\mu A_c}{2} \underbrace{\cos 2\pi (f_c - f_m)t}_{\text{LSB}} + \frac{\mu A_c}{2} \underbrace{\cos 2\pi (f_c + f_m)t}_{\text{USB}}} \longrightarrow (5)$$

Equation (5) gives the simplified expression of AM-signal. It consists of three frequency components

- $f_c \longrightarrow$ carrier frequency with amplitude 'A_c', which does not contains any message signal
- $f_c - f_m \rightarrow$ Lower side band (LSB) with amplitude $\frac{\mu A_c}{2}$
- $f_c + f_m \rightarrow$ upper side band (USB) with amplitude $\frac{\mu A_c}{2}$

Taking Fourier transformation on both sides of equation (5), we get

$$\boxed{\begin{aligned} S(f) = {} & \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \\ & \frac{\mu A_c}{4}[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))] + \\ & \frac{\mu A_c}{4}[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))] \end{aligned}} \longrightarrow (6)$$

Equation (6) gives the fourier transform of $S(t)$.

Figure 1(a) shows the spectrum of message signal m(t). and Figure 1(b) shows the spectrum of AM wave S(t).

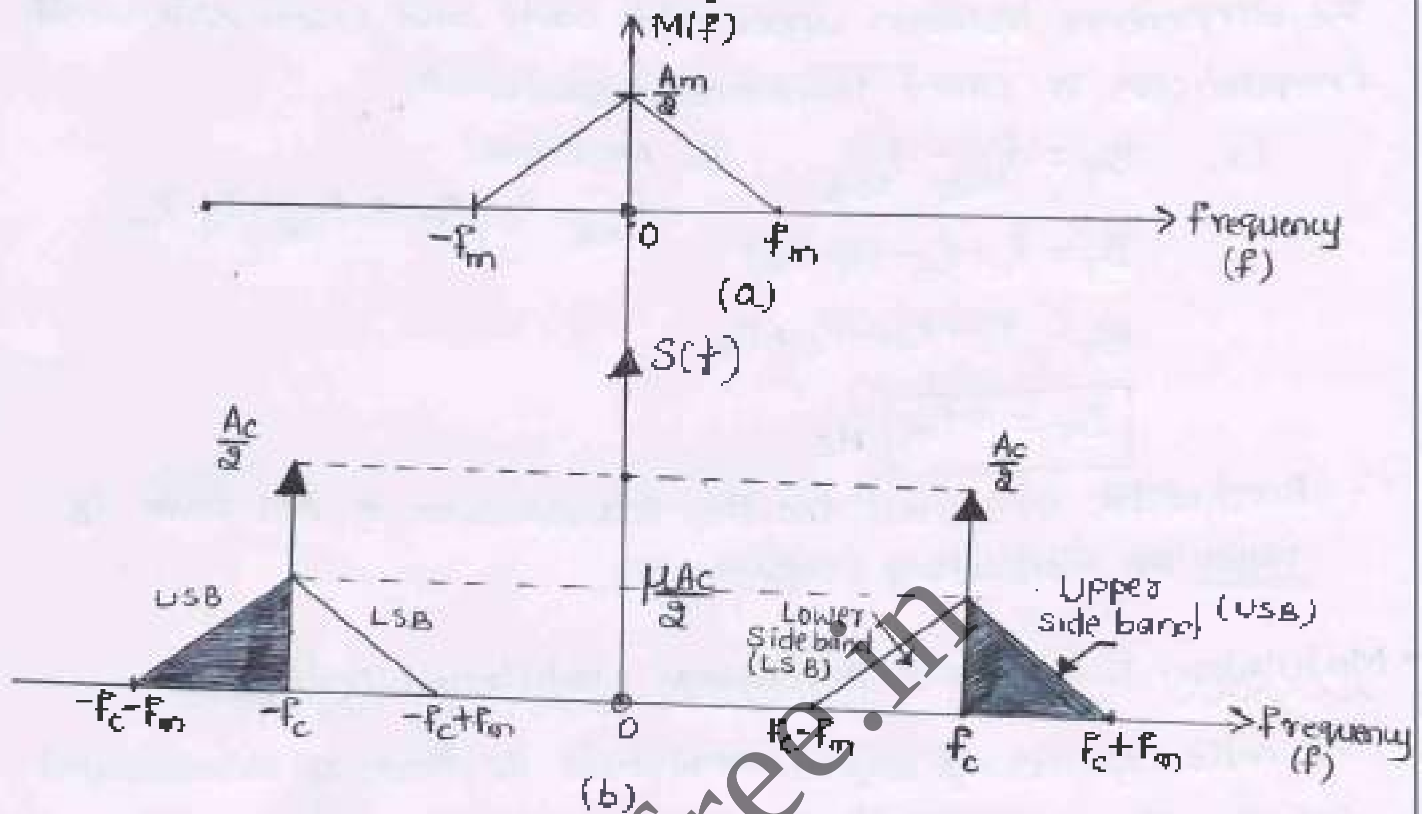

Figure 1. (a) Spectrum of m(t) (b) Spectrum of AM signal.

Figure 2, shows the time domain signal waveforms.

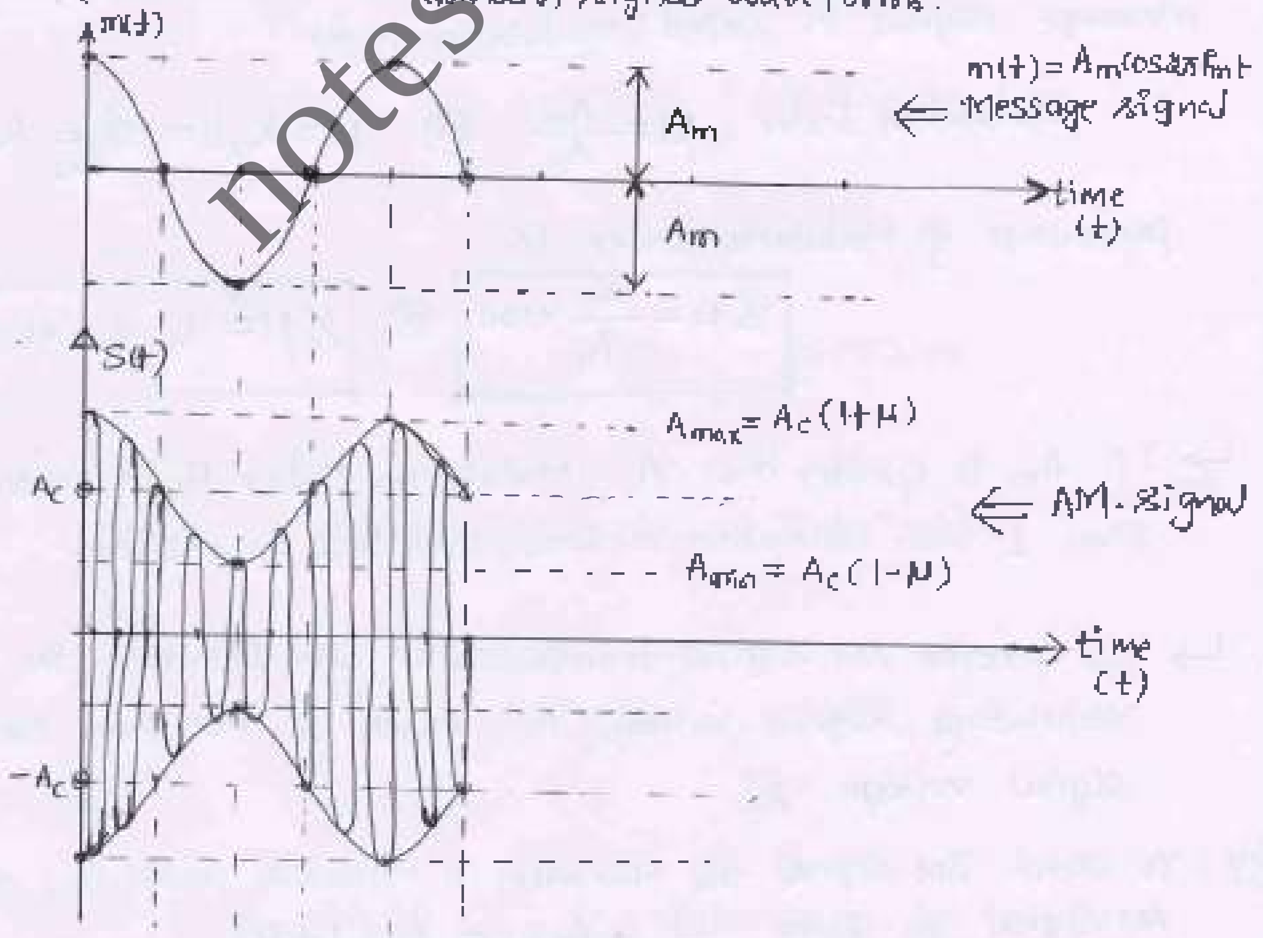

Figure 2: (a) Message signal m(t) (b) AM-Signal S(t) for $\mu<1$

- Transmission Bandwidth of AM signal :- (B_T)

The difference between upper side band and lower side band frequencies is called transmission Bandwidth

i.e., $B_T = f_{USB} - f_{LSB}$ $\because$ for AM signal $f_{USB} = f_c + f_m$ & $f_{LSB} = f_c - f_m$

$\therefore$ $B_T = f_c + f_m - (f_c - f_m)$

$B_T = f_c + f_m - f_c + f_m$

$\boxed{B_T = 2f_m}$ Hz

$\therefore$ Bandwidth required for the transmission of AM-wave is TWICE the modulating frequency.

- Modulation Index and percentage Modulation Index :-

The ratio of message signal amplitude to that of unmodulated carrier signal amplitude is called "Modulation Index".

(or)

The product of amplitude sensitivity parameter and amplitude of message signal is called 'Modulation Index'

i.e., Modulation Index, $\mu = \frac{A_m}{A_c}$ (or) $\mu = K_a |m(t)|_{max} = K_a A_m$

percentage of Modulation Index is

$\boxed{\% \mu = \frac{A_m}{A_c} \times 100}$ (or) $\boxed{\% \mu = K_a . A_m \times 100}$

↳ If A_m is greater than A_c, Modulation index μ becomes greater than 1 then Distortion is Introduced into the system.

↳ For proper AM-signal transmission and Detection, the Modulating signal voltage 'A_m' must be less than carrier signal voltage 'A_c'.

Note: To sketch AM signal the Maximum & Minimum amplitudes of AM signal is given by
- $A_{max} = A_c[1+\mu]$
- $A_{min} = A_c[1-\mu]$

• Expression for AM-signal Modulation Index in terms of its Maximum and Minimum amplitudes:-

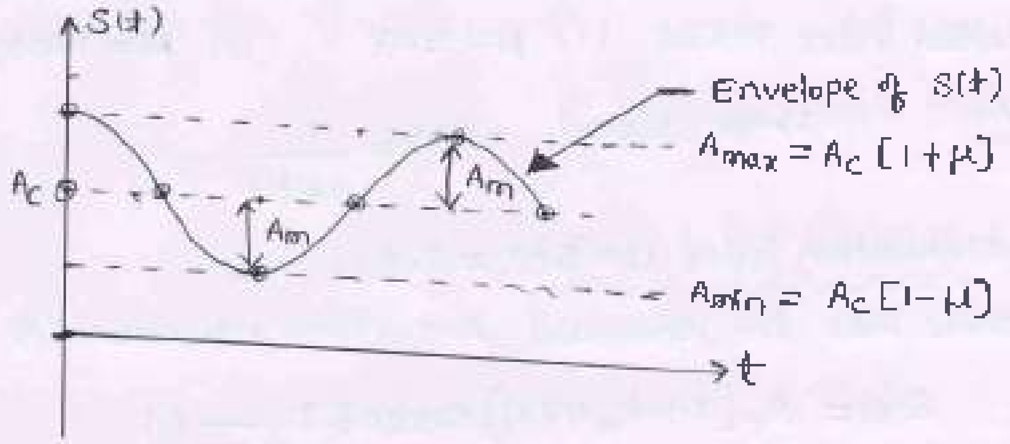

Figure 3: Envelope of AM signal s(t) [(positive Envelope)]

We know that the standard AM-signal is given by

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t \quad \text{—— (1)}$$

Let $A_{max} \Rightarrow$ Maximum amplitude of s(t)

$A_{min} \Rightarrow$ Minimum amplitude of s(t)

Figure 3 shows the Envelope of s(t) [i.e., $A_c[1 + k_a m(t)]$] sketch.

AM signal reaches its maximum value when $m(t) = A_m$ ∴

$$\therefore A_{max} = A_c(1 + k_a \cdot A_m) = A_c(1+\mu) \quad \text{—— (1)} \quad \because \mu = k_a A_m$$

Similarly, AM signal reaches minimum value when $m(t) = -A_m$

$$\therefore A_{min} = A_c(1 - k_a A_m) = A_c(1-\mu) \quad \text{—— (2)}$$

Dividing equation (1) with equation (2) we get

$$\frac{A_{max}}{A_{min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)} = \frac{1+\mu}{1-\mu}$$

$$A_{max}(1-\mu) = A_{min}(1+\mu)$$

$$A_{max} - \mu \cdot A_{max} = A_{min} + \mu A_{min}$$

$$A_{max} - A_{min} = \mu A_{max} + \mu A_{min} = \mu(A_{max} + A_{min})$$

∴ Modulation Index : $\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$

Example 1.1 : Using the message signal $m(t) = \frac{1}{1+t^2}$. Determine and sketch the amplitude modulated wave for the following modulation index values (i) $\mu = 50\%$ (ii) $\mu = 100\%$

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↳ Given : Message signal $m(t) = \frac{1}{1+t^2}$

(i) Modulation index $\mu = 50\% = 0.5$:-

We know that the standard AM signal equation is

$$S(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t \quad \text{--- (1)}$$

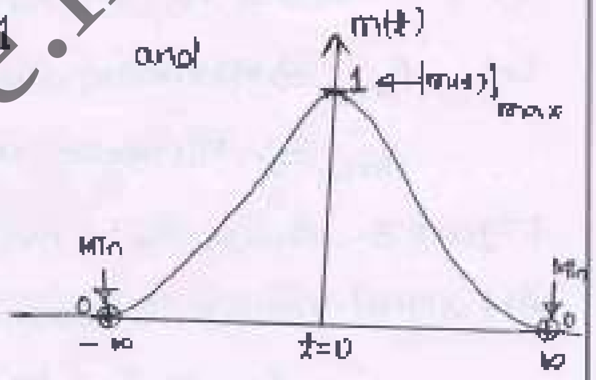
for $\mu = 0.5$: We know that, $\mu = k_a |m(t)|_{max}$

- The given signal $m(t)$ is maximum at $t = 0$.

$\therefore \quad |m(t)|_{max} = \left.\frac{1}{1+t^2}\right|_{t=0} = \frac{1}{1+0} = 1$ and

- $m(t)$ is Minimum at $t = \pm\infty$.

$$|m(t)|_{min} = \left.\frac{1}{1+t^2}\right|_{t=\pm\infty} = 0.$$



— ! Sketch of m(t) :-

$\therefore \quad \mu = k_a |m(t)|_{max} = k_a \times 1$

$\therefore \boxed{\mu = k_a = 0.5}$

$\therefore \quad S(t) = A_c \left[1 + \frac{0.5}{1+t^2}\right] \cos 2\pi f_c t$

To sketch $S(t)$:- $A_{max} = A_c [1 + k_a |m(t)|_{max}] = A_c [1 + 0.5(1)] = 1.5 A_c$

$A_{min} = A_c [1 + k_a |m(t)|_{min}] = A_c [1 + 0] = A_c$

$\therefore$ The Envelope of $m(t)$ appears between $A_{min} = A_c$ and $A_{max} = 1.5 A_c$ in $S(t)$ signal as shown in figure 1.

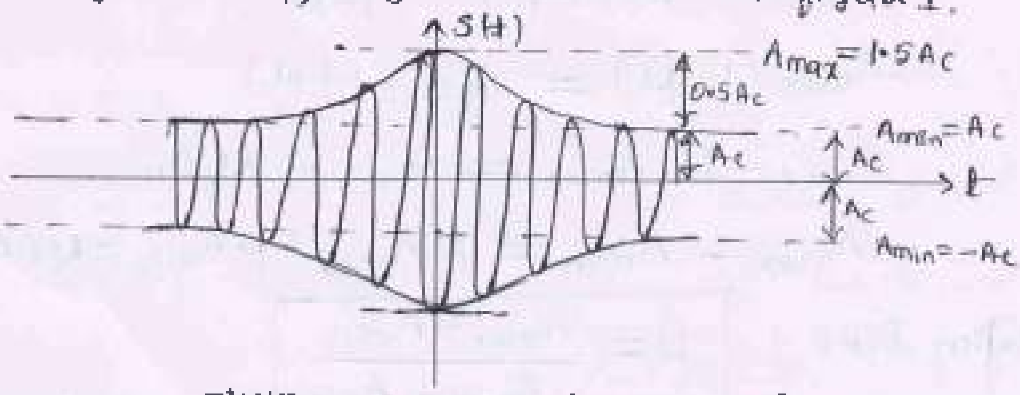


Figure 1: AM signal for $\mu = 0.5$

ii) When $\mu = 100\% = 1$:- $K_a = \frac{\mu}{|m(t)|_{max}} = \frac{1}{1} = 1$. $\left(\because \mu = K_a |m(t)|_{max}\right)$

$\therefore$ AM equation is,

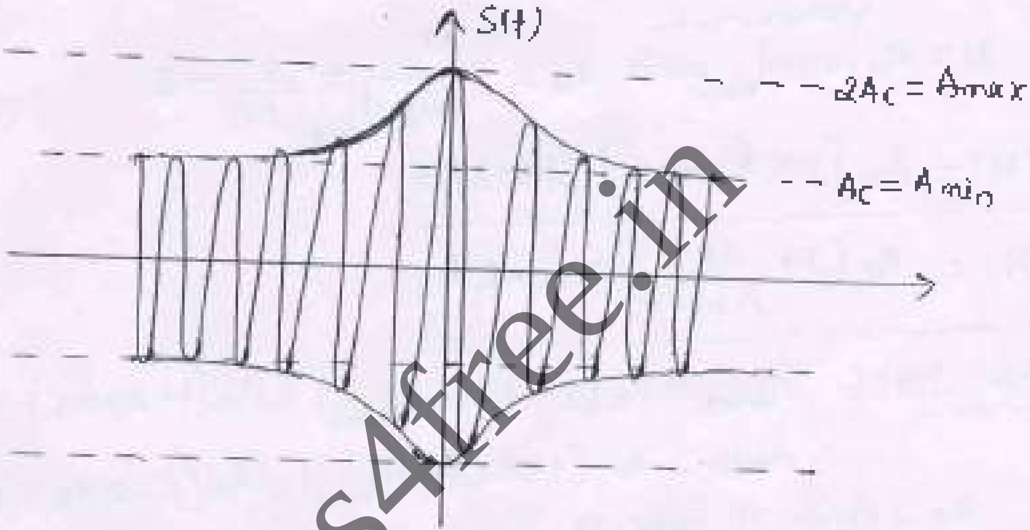
$$S(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t) \quad , \quad m(t) = \frac{1}{1+t^2}$$

$$\Rightarrow S(t) = A_c\left[1 + \frac{1}{(1+t^2)}\right]\cos(2\pi f_c t)$$

To Sketch S(t) :-

$$A_{max} = A_c\left[1 + K_a \overset{1}{|m(t)|_{max}}\right] = A_c\left[1+1\right] = 2A_c$$

$$A_{min} = A_c\left[1 + K_a \overset{0}{|m(t)|_{min}}\right] = A_c\left[1+0\right] = A_c$$



Sketch of AM wave S(t) for $\mu = 1$

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Example 1.2: Using the Message signal,

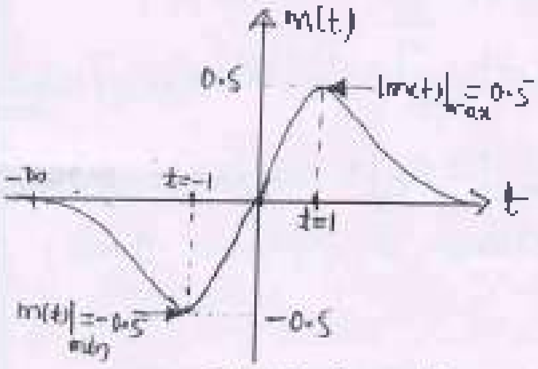
$m(t) = \frac{t}{1+t^2}$. Determine and Sketch AM signal for (i) $\mu = 50\%$ (ii) $\mu = 100\%$ (iii) $\mu = 125\%$

$\mapsto$ Given data: Message signal $m(t) = \frac{t}{1+t^2}$

t	$-\infty$	...	-3	-2	-1	0.5	0	0.5	1	2	3	...	∞
$m(t)$	0	...	-0.3	-0.4	-0.5	-0.4	0	0.4	0.5	0.4		...	0

$|m(t)|_{min} = -0.5$ at $t = -1$

$|m(t)|_{max} = 0.5$ at $t = 1$



Sketch of m(t)

$\therefore$ For this message signal, AM signal S(t) becomes Maximum at $t = 1$ and reaches Minimum at $t = -1$.

Case(i) $\mu = 50\% = 0.5$:- W.K.T. $K_a =$

The standard equation of S(t), AM signal is

$$S(t) = A_c[1 + K_a m(t)]\cos 2\pi f_c t$$

W.K.T, $K_a = \frac{\mu}{|m(t)|_{max}} = \frac{0.5}{0.5} = 1$ & $m(t) = \frac{t}{1+t^2}$

$$\therefore \boxed{S(t) = A_c\left[1 + \left(\frac{t}{1+t^2}\right)\right]\cos 2\pi f_c t}$$ and is shown in figure 1.2(a).

To Sketch S(t):-

$A_{max} = A_c[1 + K_a |m(t)|_{max}]$

$A_{max} = A_c[1 + 1\times 0.5]$

$\boxed{A_{max} = 1.5A_c}$

$A_{min} = A_c[1 - 0.5]$

$\boxed{A_{min} = 0.5A_c}$

Case(ii):- for $\mu = 100\% = 1$:-

$$\mu = K_a|m(t)|_{max} \Longrightarrow K_a = \frac{\mu}{|m(t)|_{max}} = \frac{1}{0.5} = 2$$

$$\therefore S(t) = A_c[1 + K_a m(t)]\cos 2\pi f_c t$$

$$\boxed{S(t) = A_c\left[1 + \frac{2t}{(1+t^2)}\right]\cos 2\pi f_c t}$$

To Sketch S(t):- $A_{max} = A_c[1 + K_a |m(t)|_{max}] = A_c[1 + 2\times 0.5] = 2A_c$.

$A_{min} = A_c[1 + K_a [m(t)]_{min}] = A_c[1 - 2\times 0.5] = 0$

The sketch of S(t) is shown in figure 1.2.(b). (over Modulated)

Case(iii):- for $\mu = 125\% = 1.25$:-

$$\mu = K_a|m(t)|_{max} \Longrightarrow K_a = \frac{\mu}{|m(t)|_{max}} = \frac{1.25}{0.5} = 2.5$$

$$\therefore S(t) = A_c[1 + K_a m(t)]\cos 2\pi f_c t$$

$$S(t) = A_c\left[1 + \frac{2.5\times t}{1+t^2}\right]\cos 2\pi f_c t.$$

$$\therefore S(t) = A_c\left[1 + \frac{2.5t}{(1+t^2)}\right]\cos 2\pi f_c t$$

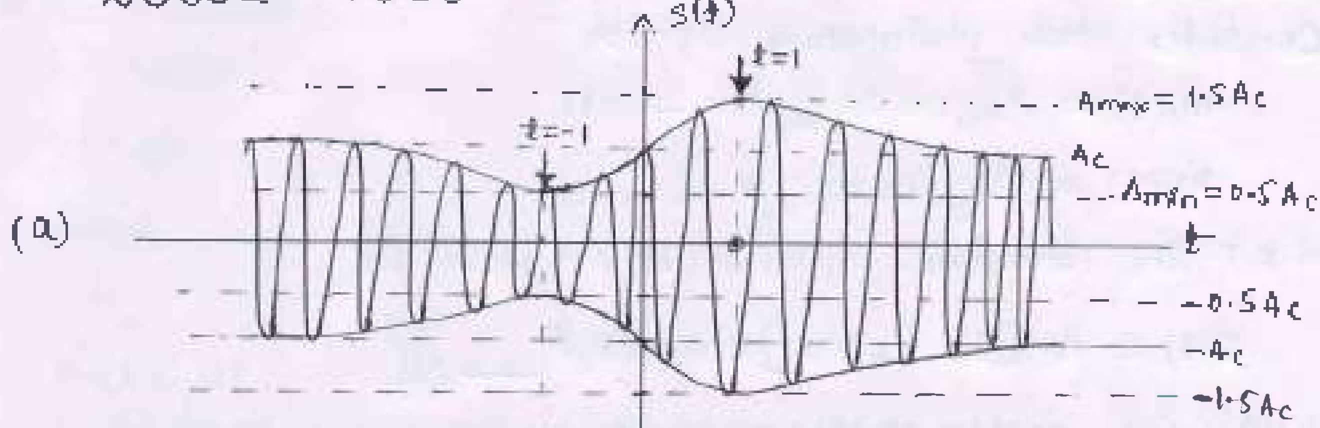
To Sketch S(t):-

$$A_{max} = A_c[1 + K_a|m(t)|_{max}] = A_c[1 + 2.5\times 0.5] = 2.25A_c$$

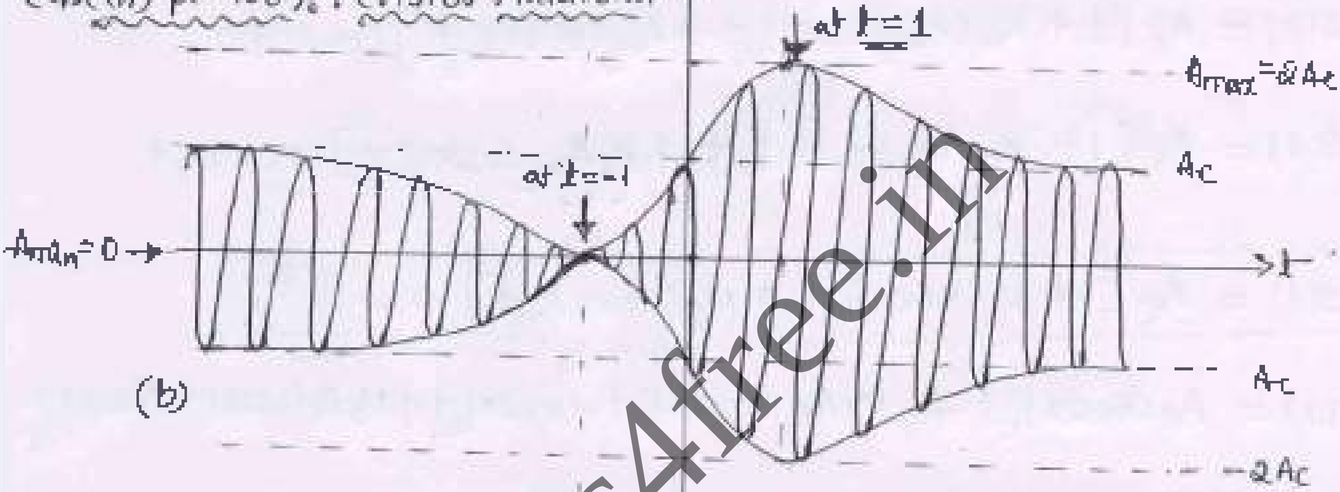
$$A_{min} = A_c[1 + K_a|m(t)|_{min}] = A_c[1 - 2.25\times 0.5] = -0.25A_c$$

The Sketch of S(t) for $\mu = 1.25$ is shown in figure 1.2(c) and is over modulated.

Case(i) : $\mu = 50\%$: Under Modulation :-



Case(ii) $\mu = 100\%$: Critical Modulation



Case(iii) : $\mu = 125\%$: Over Modulation :-

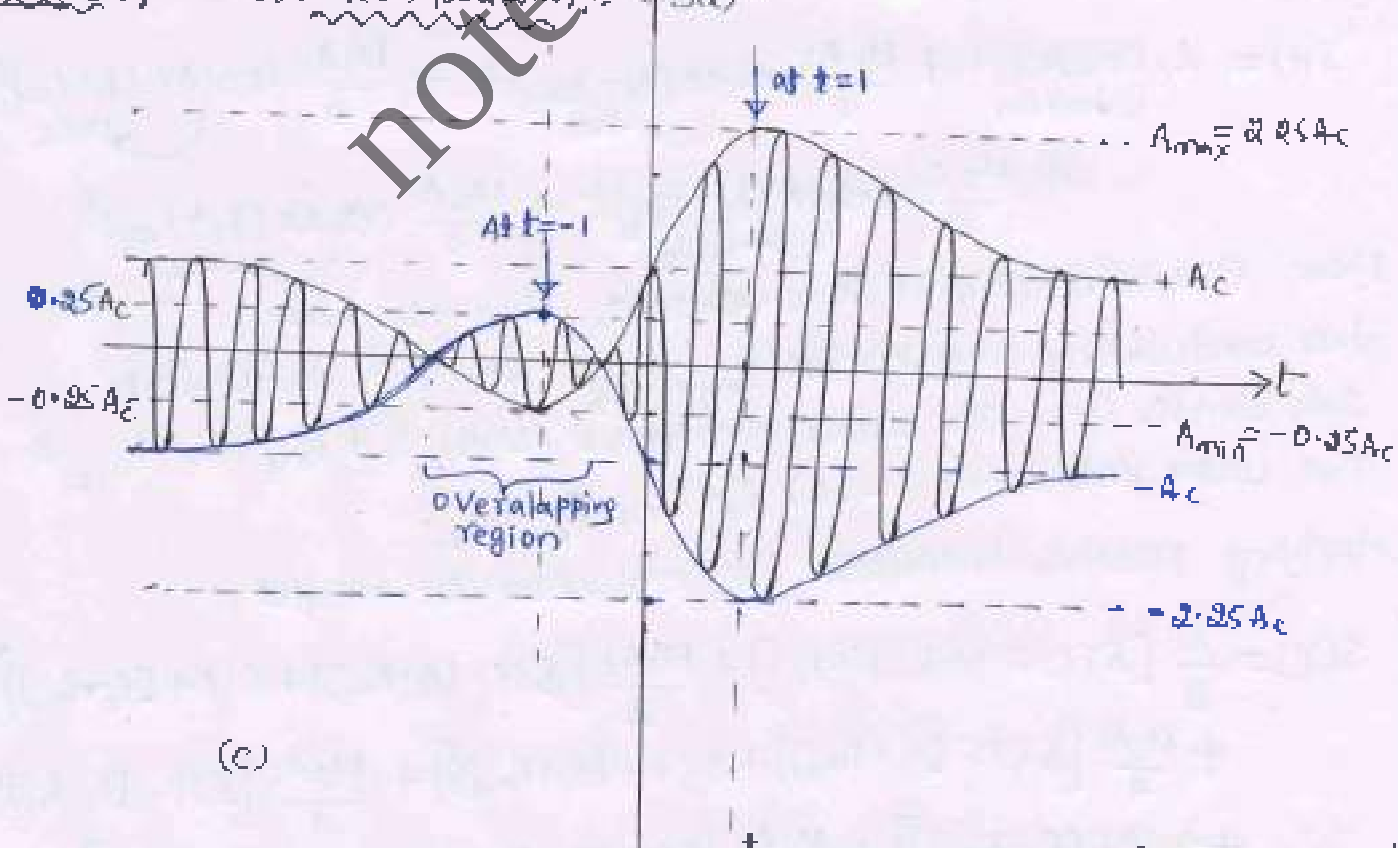


Figure 1.2 : AM signal for $m(t) = \frac{t}{1+t^2}$ for (a) $\mu = 50\%$ (b) $\mu = 100\%$ and (c) $\mu = 125\%$

* Expression for Multitone Amplitude Modulation:-

Consider two modulating signals

$$m_1(t) = A_{m_1} \cos 2\pi f_{m_1} t \quad \text{--- (1)}$$

$$m_2(t) = A_{m_2} \cos 2\pi f_{m_2} t \quad \text{--- (2)}$$

W.K.T the standard equation of AM-wave is

$$S(t) = A_c \left[1 + K_a m(t)\right] \cos 2\pi f_c t \quad \text{--- (3)}$$

In this case $m(t) = m_1(t) + m_2(t) = A_{m_1} \cos 2\pi f_{m_1} t + A_{m_2} \cos 2\pi f_{m_2} t$

$$\therefore \quad S(t) = A_c \left[1 + K_a \left(A_{m_1} \cos 2\pi f_{m_1} t + A_{m_2} \cos 2\pi f_{m_2} t\right)\right] \cos 2\pi f_c t$$

$$S(t) = A_c \left[1 + \underbrace{K_a A_{m_1}}_{\mu_1} \cos 2\pi f_{m_1} t + \underbrace{K_a A_{m_2}}_{\mu_2} \cos 2\pi f_{m_2} t\right] \cos 2\pi f_c t$$

$$\therefore \boxed{S(t) = A_c \left[1 + \mu_1 \cos 2\pi f_{m_1} t + \mu_2 \cos 2\pi f_{m_2} t\right] \cos 2\pi f_c t} \quad \text{--- (4)}$$

$$S(t) = A_c \cos 2\pi f_c t + \mu_1 A_c \cos 2\pi f_c t \cdot \cos 2\pi f_{m_1} t + \mu_2 A_c \cos 2\pi f_c t \cdot \cos 2\pi f_{m_2} t$$

W.K.T. $\cos A \cdot \cos B = \frac{1}{2}\left\{\cos(A-B) + \cos(A+B)\right\}$

$$\therefore \quad S(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{carrier}} + \frac{\mu_1 A_c}{2} \underbrace{\cos 2\pi (f_c - f_{m_1}) t}_{\text{LSB1}} + \frac{\mu_1 A_c}{2} \underbrace{\cos (2\pi (f_c + f_{m_1}) t)}_{\text{USB1}}$$

$$+ \frac{\mu_2 A_c}{2} \underbrace{\cos 2\pi (f_c - f_{m_2}) t}_{\text{LSB}_2} + \frac{\mu_2 A_c}{2} \underbrace{\cos 2\pi (f_c + f_{m_2}) t}_{\text{USB}_2} \quad \text{--- (5)}$$

From equation (5), it is clear that, when we have two modulating frequencies (f_{m_1}, f_{m_2}) we get total four side bands. i.e., Two upper sidebands (USB) $f_c + f_{m_1}$, $f_c + f_{m_2}$ & Two Lower sidebands (LSB) $f_c - f_{m_1}$, $f_c - f_{m_2}$.

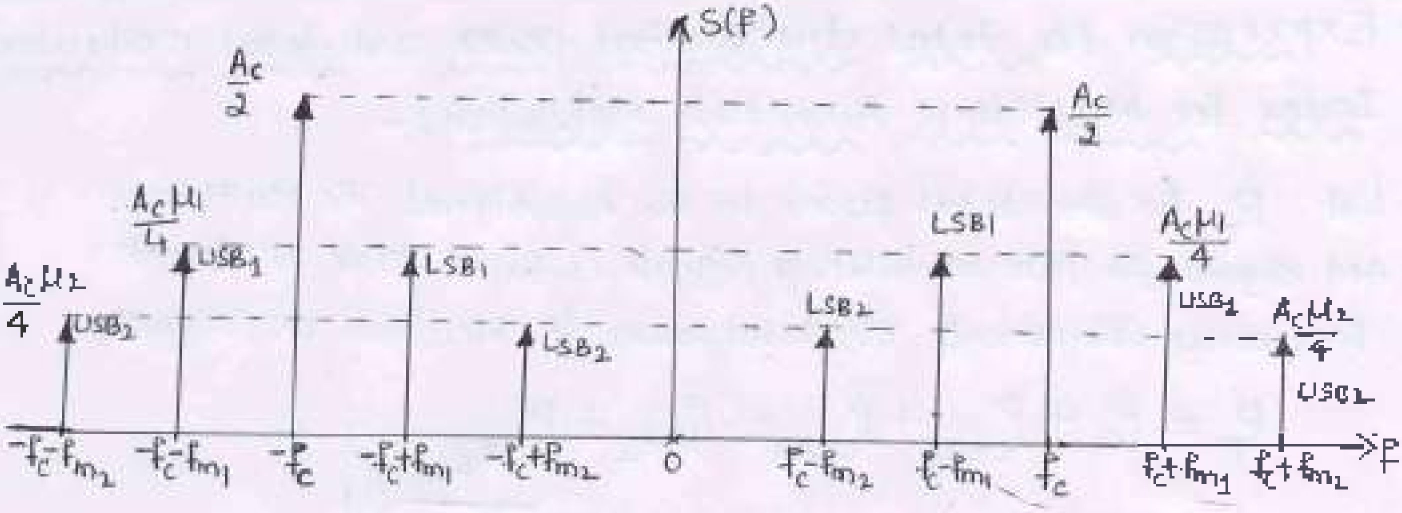
Applying Fourier transform to equation (5) we get,

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{\mu_1 A_c}{4}\left[\delta(f - [f_c - f_{m_1}]) + \delta(f + [f_c - f_{m_1}])\right]$$

$$+ \frac{\mu_1 A_c}{4}\left[\delta(f - [f_c + f_{m_1}]) + \delta(f + [f_c + f_{m_1}])\right] + \frac{\mu_2 A_c}{4}\left[\delta(f - [f_c - f_{m_2}])\right.$$

$$\left. + \delta(f + [f_c - f_{m_2}])\right] + \frac{\mu_2 A_c}{4}\left[\delta(f - [f_c + f_{m_2}]) + \delta(f + [f_c + f_{m_2}])\right]$$

$$\text{--- (6)}$$



— : Spectrum for Multitone Amplitude Modulation :—

Total Bandwidth :- The total transmission Bandwidth for Multitone Amplitude modulated wave is

$$\boxed{BW_T = 2W} \text{ Hz.}$$

where $W = \text{Maximum}\,(f_{m_1}, f_{m_2} \ldots, f_{m_N})$

Note: * In Multitone Amplitude Modulated Wave shown in equation (5) has five different frequencies as follows

- $f_c \longrightarrow$ Carrier Signal with amplitude 'A_c'.
- $f_c - f_{m_1} \longrightarrow$ LSB$_1$ with amplitude $\Big\}\ \frac{\mu_1 A_c}{2}$
- $f_c + f_{m_1} \longrightarrow$ USB1 with amplitude
- $f_c - f_{m_2} \longrightarrow$ LSB$_2$ with amplitude $\Big\}\ \frac{\mu_2 A_c}{2}$
- $f_c + f_{m_2} \longrightarrow$ USB$_2$ with amplitude

* power present in any alternating signal (voltage signal) is given by

$$P = \frac{(V_{RMS})^2}{R} = \frac{(V_M/\sqrt{2})^2}{R} = \frac{V_m^2}{2R} = \frac{(\text{Amplitude})^2}{2R}$$

where R = Load Resistance in Ω.

*** Expression for total transmitted power and total modulation Index for Multitone Amplitude Modulation:-**

Let P_T, be the total power in the AM signal. The Multitone AM signal for two modulating signals contains Five different frequency components. Its total power is calculated as follows,

$$P_T = P_C + P_{LSB_1} + P_{USB_1} + P_{LSB_2} + P_{USB_2} \quad \text{——— (1)}$$

$$P_C = \frac{A_C^2}{2R}$$

$$P_{LSB_1} = P_{USB_1} = \frac{\left(\frac{\mu_1 A_C}{2}\right)^2}{2R} = \frac{\mu_1^2 A_C^2}{8R} = \frac{A_C^2}{2R}\left(\frac{\mu_1^2}{4}\right) = P_C \frac{\mu_1^2}{4}$$

$$P_{LSB_2} = P_{USB_2} = \frac{\left(\frac{\mu_2 A_C}{2}\right)^2}{2R} = \frac{\mu_2^2 A_C^2}{8R} = \frac{A_C^2}{2R}\left(\frac{\mu_2^2}{4}\right) = P_C \frac{\mu_2^2}{4}$$

$$\therefore P_T = P_C + P_C\frac{\mu_1^2}{4} + P_C\frac{\mu_1^2}{4} + P_C\frac{\mu_2^2}{4} + P_C\frac{\mu_2^2}{4}$$

$$P_T = P_C\left[1 + \frac{\mu_1^2}{4} + \frac{\mu_1^2}{4} + \frac{\mu_2^2}{4} + \frac{\mu_2^2}{4}\right]$$

$$P_T = P_C\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right] = P_C\left[1 + \frac{(\mu_1^2 + \mu_2^2)}{2}\right]$$

$$\boxed{P_T = P_C\left[1 + \frac{\mu_t^2}{2}\right]} \Longrightarrow \text{Total power in AM signal}$$

Where $\mu_t^2 = \mu_1^2 + \mu_2^2 = N$

$$\therefore \boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2}} \Rightarrow \text{Net Modulation Index}$$

In General, Net Modulation index for N-message signals is given by

$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + \cdots + \mu_N^2}}$$

Note: For Single Tone AM Signal,

- Total power, $P_T = P_c\left(1+\frac{\mu^2}{2}\right)$

- Total side band power : $P_{SB} = P_{LSB} + P_{USB} = \frac{P_c\mu^2}{4} + \frac{P_c\mu^2}{4}$

$$\therefore \boxed{P_{SB} = P_c\frac{\mu^2}{2}} \leftarrow \text{Total power in side bands}$$

GATE

* Example 1.3 : Determine the ratio of Maximum average total power to unmodulated carrier power in AM signal.

↳ W.k.T. total power in AM signal is given by

$$P_T = P_c\left[1+\frac{\mu^2}{2}\right] \quad \text{--- (1)}$$

Where, P_c = unmodulated carrier power

μ = Modulation Index varies from 0 to 1

$\therefore$ P_T becomes Maximum at $\mu = 1$

$$\therefore \quad P_{T_{max}} = P_c\left[1+\frac{\mu^2}{2}\right]\Bigg|_{\mu=1}$$

$$\therefore \boxed{\frac{P_{T_{max}}}{P_c} = \left(1+\frac{1^2}{2}\right) = 1+\frac{1}{2} = \frac{3}{2} = 1.5}$$

$\therefore$ The ratio of Maximum transmitted power to that of unmodulated carrier power in AM signal is 1.5.

Note: • The Maximum power radiated from an AM-broadcasting station is $\boxed{P_{t_{max}} = 1.5P_c}$

- In AM-signal

↳ carrier power does not contains any message signal.

$\therefore$ The presence of carrier power 'P_c' in total power P_t, reduces the Efficiency of AM-signal.

↳ LSB & USB carries equal power "$P_c\mu^2/4$".

* Efficiency of AM signal: (η)

It is the ratio of total sideband power to that of total transmitted power.

$$\text{i.e., } \eta = \frac{P_{SB}}{P_t} = \frac{P_{LSB}+P_{USB}}{P_t}$$

W.K.T. $\quad P_t = P_c\left[1+\frac{\mu^2}{2}\right]$

$$P_{SB} = P_{LSB}+P_{USB} = P_c\frac{\mu^2}{2}$$

$$\therefore \eta = \frac{P_{SB}}{P_t} = \frac{P_c\frac{\mu^2}{2}}{P_c\left(1+\frac{\mu^2}{2}\right)} = \frac{\frac{\mu^2}{2}}{\frac{2+\mu^2}{2}} = \frac{\mu^2}{2+\mu^2}$$

$$\therefore \% \boxed{\eta = \frac{\mu^2}{2+\mu^2}\times 100}$$

Note: * the Maximum Efficiency of AM signal is

$$\eta_{max} = \left.\frac{\mu^2}{2+\mu^2}\times 100\right|_{\mu=1} = \frac{1^2}{2+1}\times 100 = \frac{1}{3}\times 100$$

$$\therefore \boxed{\eta_{max} = 33.33\%}$$

***

1.3: SWITCHING MODULATOR :-

It is a Diode circuit used to generate AM-Signal.

Q) Explain the operation of switching modulator with circuit diagram and waveforms. Dec/Jan 2016/2017

↳ Switching modulator is used to generate AM signal.

Circuit diagram :-

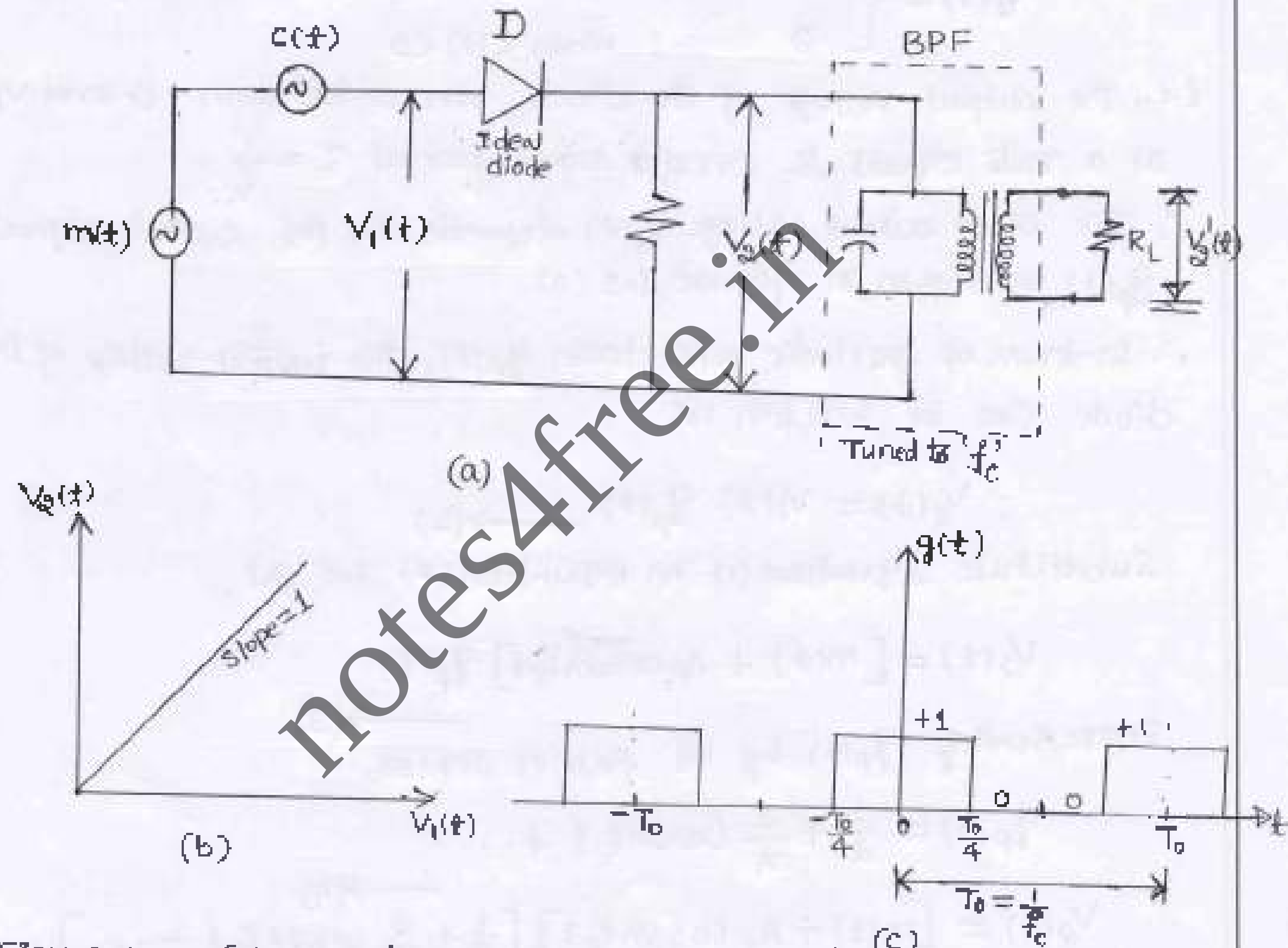


Figure 1.3 : (a) Switching Modulator Circuit diagram

(b) Idealized input ($v_1(t)$) and output $V_2(t)$ relation of Diode

(c) periodic pulse-train of c(t).

Explanation :- Switching modulator consists of an Ideal diode which is used as a switch, followed by Band pass Filter (BPF) tuned to frequency 'f_c' as shown in figure 1.3 (a).

↳ Message signal m(t) and carrier signal c(t) are simultaneously applied as input signal for ideal diode 'D', as shown in figure 1.3(a).

∴ The total input '$v_1(t)$' to the diode is given by

$$v_1(t) = m(t) + c(t)$$

$$\therefore \boxed{v_1(t) = m(t) + A_c \cos 2\pi f_c t} \longrightarrow (1)$$

It is assumed that $|m(t)| \ll A_c$. Therefore ON & OFF of Diode 'D' is controlled by $c(t)$.

∴ the output voltage of Diode 'D' is,

$$v_2(t) = \begin{cases} v_1(t) & ; \text{ when } c(t) > 0 \Rightarrow \text{shown in figure 1.3(b)} \\ 0 & ; \text{ when } c(t) < 0 \end{cases}$$

i.e., the output voltage of the diode varies between 0 and $v_1(t)$ at a rate equal to carrier signal period $T_0 = \frac{1}{f_c}$.

∴ The Diode output voltage $v_2(t)$ depends on the control signal $g_p(t)$ as shown in figure 1.3 (c).

∴ In terms of periodic pulse train $g_p(t)$, the output voltage of the diode can be written as

$$v_2(t) = v_1(t) \cdot g_p(t) \longrightarrow (2)$$

Substitute equation (1) in equation (2) we get,

$$v_2(t) = [m(t) + A_c \cos 2\pi f_c t]\, g_p(t) \longrightarrow (3)$$

Representing $g_p(t)$ by its Fourier Series,

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t + \cdots \longrightarrow (4)$$

$$\therefore v_2(t) = [m(t) + A_c \cos 2\pi f_c t]\left[\frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t + \cdots\right]$$

$$v_2(t) = \frac{1}{2} m(t) + \frac{2}{\pi} m(t) \cdot \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t + \frac{2A_c}{4} \underbrace{[\cos^2 2\pi f_c t]}_{\left(\frac{1+\cos 2\theta}{2}\right)} + \cdots$$

$$v_2(t) = \frac{1}{2} m(t) + \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t + \underbrace{\frac{A_c}{4}}_{(DC)} + \frac{A_c}{4} \cos 4\pi f_c t + \cdots \longrightarrow (5)$$

The required AM wave centered at 'f_c' is obtained by passing $v_2(t)$ through an ideal BPF having center frequency 'f_c' and $BW = 2f_m$ Hz

∴ The output of the BPF is

$$V_2'(t) = \frac{2}{\pi} m(t) \cos(2\pi f_c t) + \frac{A_c}{2} \cos 2\pi f_c t$$

$$V_2'(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} \cdot m(t)\right] \cos 2\pi f_c t$$

$$V_2'(t) = \frac{A_c}{2}\left[1 + k_a m(t)\right] \cos 2\pi f_c t \quad \Leftarrow \text{AM-Wave.} \longrightarrow (6)$$

where $k_a = \frac{4}{\pi A_c}$ = Amplitude Sensitivity parameter

Equation (6) is the standard AM signal produced by the switching modulator with carrier amplitude scaled down to '$\frac{A_c}{2}$'

* * * * *

*** 1.4. ENVELOPE DETECTOR ***

Q) Explain the operation of envelope detector with neat diagrams and waveforms. Also mention the significance of RC-time constant. June-July 2017

→ Demodulation (or) Detection is the process of recovering the original message signal from the modulated wave at the receiver.

Envelope Detector: It is a simple and highly effective diode circuit which is commonly used for demodulation of AM-signal.

Circuit diagram:-

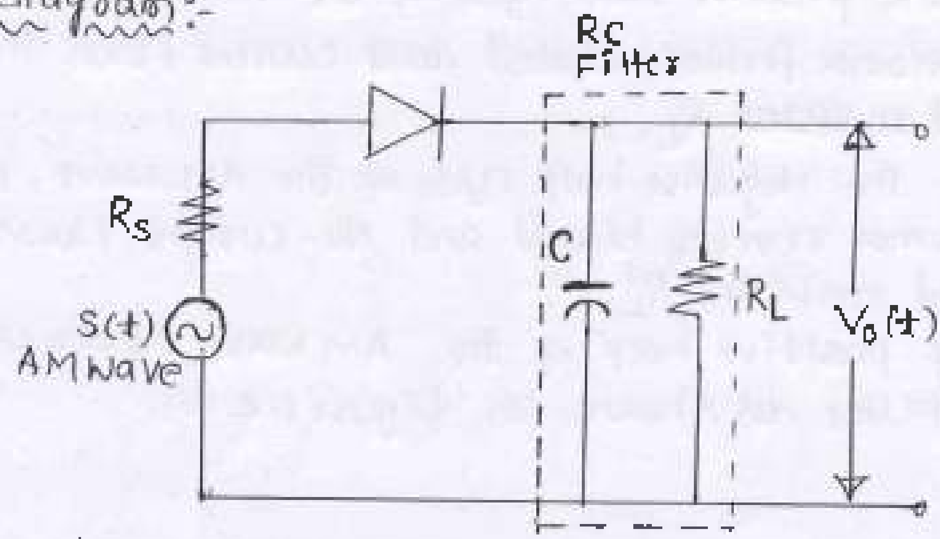


Figure 1.4(a): circuit diagram of Envelope Detector:-

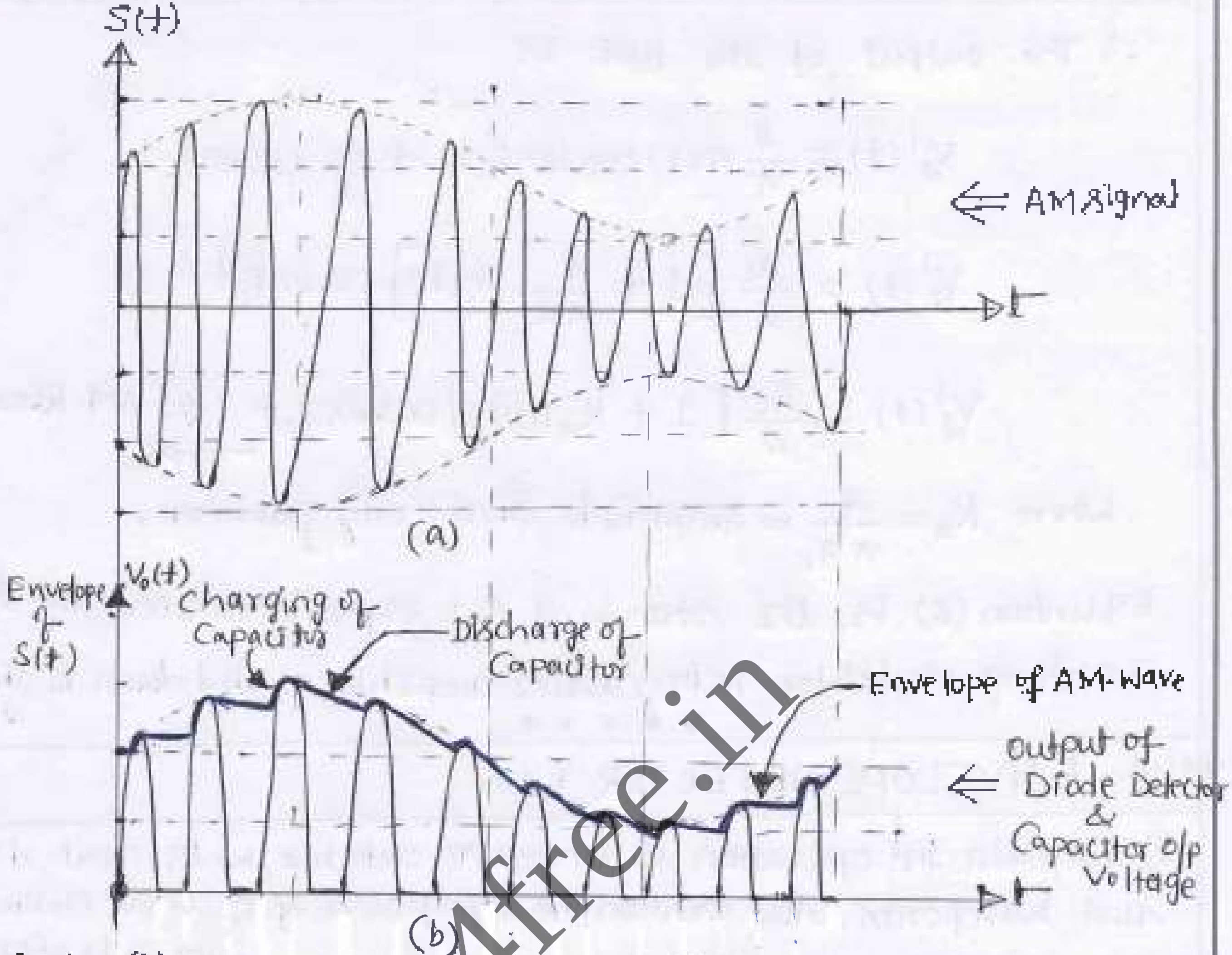


Figure 1.4(b): (a) AM signal S(t), input to the Envelope Detector
(b) Envelope of AM signal and Voltage across Capacitor

Figure 4(a) shows the envelope detector circuit. It consists of a diode and a RC-filter. This circuit is also known as "Diode Detector".

Circuit operation:-

→ In the positive half cycle of the AM-signal, Diode 'D' becomes forward biased and current flows through load resistor 'R_L'.

→ In the Negative half cycle of the AM-wave, Diode 'D' becomes reverse biased and No-current flows through load resistor 'R_L'.

∴ Only positive half of the AM wave appears across RC-Filter as shown in figure 1.4(b).

Working of RC-Filter:-

↳ During the +ve half cycle of AM wave, the capacitor 'C' charges up rapidly towards the peak value of the input signal. When the input signal falls below this value, the Diode becomes Reverse biased and the Capacitor 'C' discharges slowly through the load resistor 'R_L'.

↳ The Discharging process Continues until the next positive half cycle of AM-Wave. When the input signal becomes greater than the voltage across capacitor, the diode starts conducting again and the process is repeated

↳ this Continuous process of charging and Discharging of Capacitor, gives the Envelope of AM signal as shown in figure 1.4(b). which is in same shape as that of message signal.

Selection of RC Constant:-

* The charging time Constant '$R_S C$' must be very much less than the Carrier period '$1/f_c$'.

$$\therefore R_S C \ll \frac{1}{f_c}$$;⟹ To ensure Capacitor Charges up rapidly.

* The Discharging Time Constant '$R_L C$' should be long enough to ensure that the capacitor discharges slowly through the load resistor 'R_L' between positive peaks of the Carrier Wave.

i.e.,
$$\boxed{\frac{1}{f_c} \ll R_L C \ll \frac{1}{f_m}}$$ ⟹ To ensure Slow Discharge of Capacitor

—— * ** —— *** ——

* Advantages of Amplitude Modulation :-

1. Low Bandwidth
2. AM-Waves can travel longer distance.
   i.e., AM waves covers large area
3. AM transmitters are less complex
4. AM receivers are simple, Detection is easy and cost efficient.

** Disadvantages (or) Limitations of AM:

1. power is wasted in the transmitted signal.
2. AM needs Larger Bandwidth ($BW_T = 2W$)
3. AM-signal gets affected due to Noise.

* Applications of AM :

The Major applications of AM are

↳ Radio broadcasting.

↳ picture Transmission in a TV-system. (Television Broadcasting).

DOUBLE SIDE BAND SUPPRESSED CARRIER (DSBSC) MODULATION

↳ To overcome the drawback of power wastage in AM wave DSBSC-Modulation is used.

↳ DSBSC is a method of transmission of message signal, where only two side bands are transmitted without the carrier signal.

↳ Amplitude Modulated wave in which the carrier is suppressed is called "DSBSC-Modulation"

1.5. Time and Frequency domain description of DSBSC-Signal:

Let $m(t) = A_m \cos 2\pi f_m t$ —— (1) : Modulating signal and

$c(t) = A_c \cos 2\pi f_c t$ —— (2) : Carrier signal.

Then the Time domain expression for DSBSC-signal is given by,

$$S(t) = m(t) \cdot c(t)$$

$$\boxed{S(t) = A_c \cos 2\pi f_c t \cdot m(t)} \quad —— (3)$$

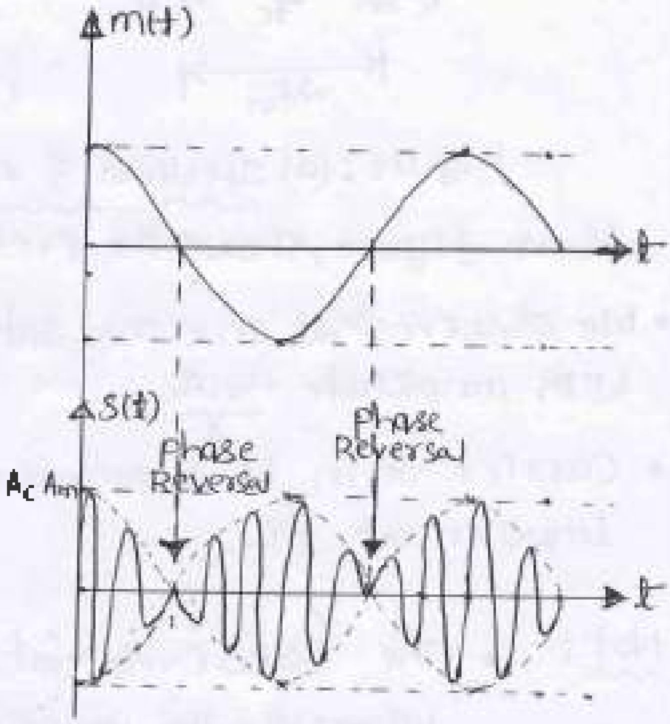


Figure 1.5: DSBSC Modulated Signal

Equation (3) is the general expression of DSBSC-signal for any message signal m(t).

for $m(t) = A_m \cos 2\pi f_m t$

$$S(t) = A_c \cos 2\pi f_c t \times A_m \cos 2\pi f_m t$$

$$S(t) = A_m A_c \cos 2\pi f_c t \cdot \cos 2\pi f_m t$$

W.K.T. $\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$$\therefore S(t) = \frac{A_m A_c}{2}\left[\cos 2\pi (f_c - f_m)t + \cos 2\pi (f_c + f_m)t\right] \longrightarrow (4)$$

Take Fourier Transform on both sides of equation (4), we get

$$S(f) = \frac{A_m A_c}{4}\left[\delta(f-[f_c-f_m]) + \delta(f+[f_c-f_m])\right] + \frac{A_m A_c}{4}\left[\delta(f-[f_c+f_m]) + \delta(f+[f_c+f_m])\right] \longrightarrow (5)$$

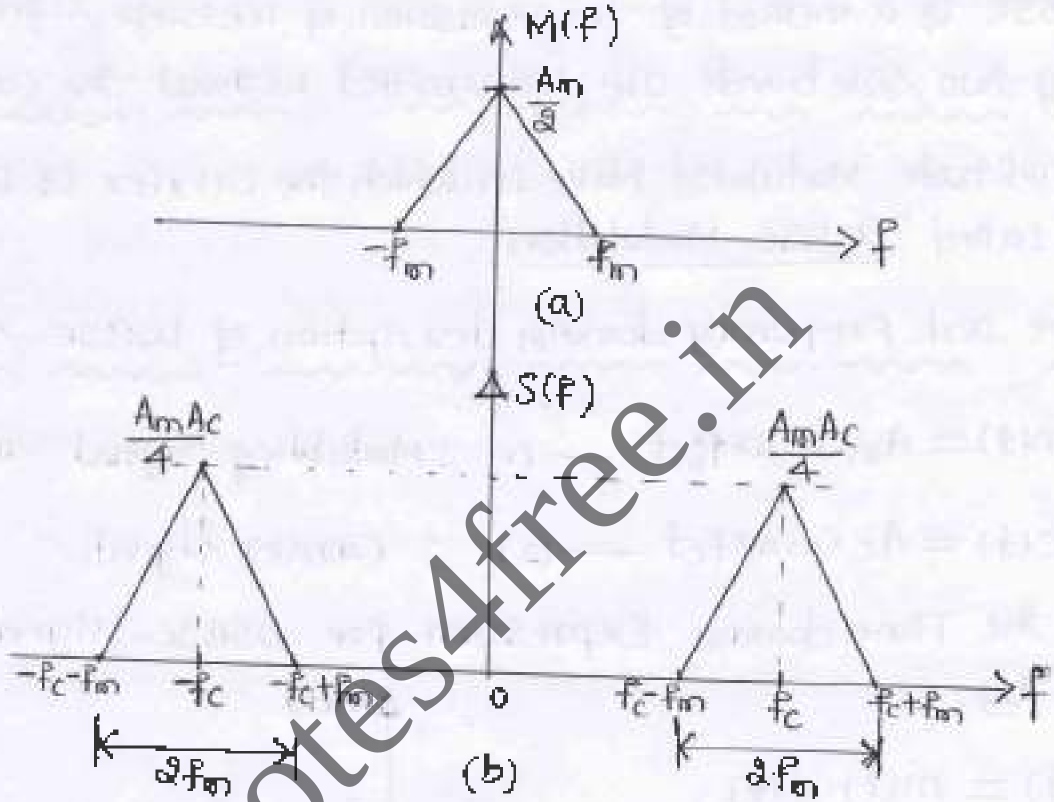


Figure: (a) Spectrum of m(t) (b) Spectrum of DSBSC signal

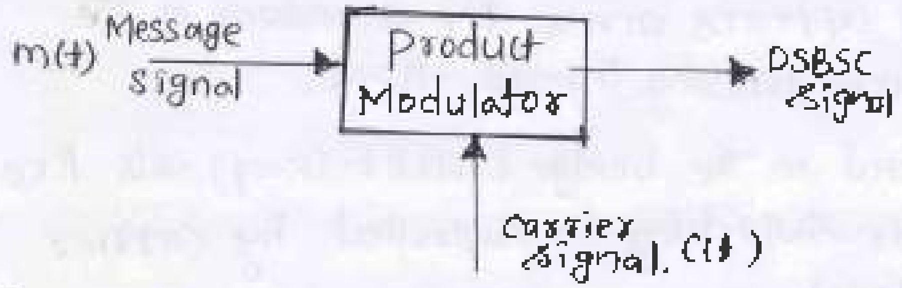
Above figure shows the frequency spectrum of a DSBSC signal.

- We observe that on either side of $\pm f_c$, we have LSB and USB with amplitude $\frac{A_m A_c}{4}$.
- Carrier term is suppressed in the spectrum as there is no Impulse at $\pm f_c$.

Note:-
- The DSBSC-signal $s(t)$, undergoes a phase Reversal whenever the message signal crosses zero, as shown in figure 5.1.
- A DSBSC signal can be generated by a Multiplier (or also called product Modulator)

Generation of DSB-SC Wave:-

The devices used to generate DSBSC waves are known as the product Modulators



$s(t) = m(t) \cdot c(t)$

The most commonly used product Modulator to generate DSBSC signal is "Ring Modulator."

1.6. RING MODULATOR ***

Q) Explain the generation of DSBSC wave using Ring Modulator and also sketch the necessary waveforms.

↳ Ring Modulator is a product Modulator used for Generating DSBSC-Modulated signal.

Circuit diagram:

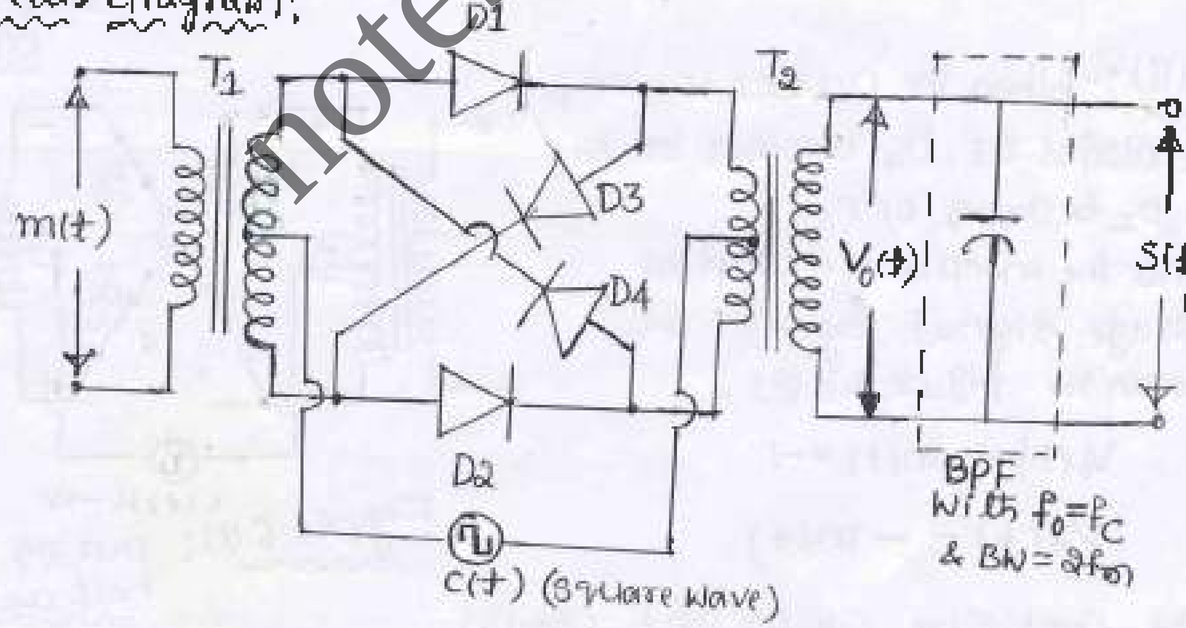


Figure 1.6(a): circuit diagram of Ring Modulator

↳ The circuit diagram of Ring modulator is shown in figure 1.6(a) consists of two Center tapped transforms T1, T2 and Four diodes D1, D2, D3 and D4 Connected in bridge circuit and a BPF with Center frequency 'f_c', $BW = 2f_m$.

↳ The carrier signal is applied to the center taps of the input (T_1) and output (T_2) transformers. Modulating signal is applied to the input transformer T_1.

↳ The output voltage appears across the secondary of the transformer, T_2 (After passing through BPF).

↳ The Diodes connected in the bridge circuit (Ring) acts like switches and their switching is controlled by carrier signal (square wave).

Circuit operation :-

Case(i): When the carrier is +ve, the Diodes D1, D2 becomes ON & Diodes D3, D_4 becomes OFF.
Hence the Modulator Multiplies message signal m(t) by +1.

i.e., $V_0(t) = m(t) \times (+1) = m(t)$

Equivalent circuit is shown in Figure 1.6(b)

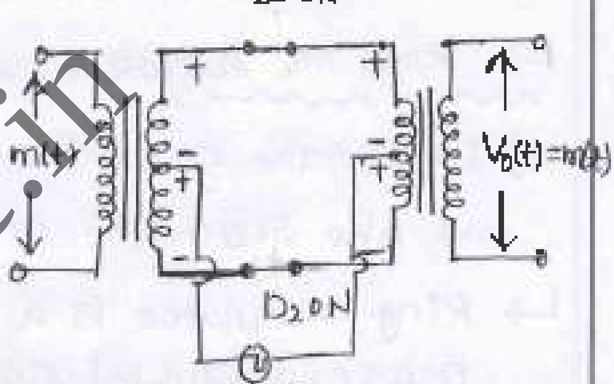


Figure 1.6(b): During +ve half cycle of C(t)

Case(ii):- When the carrier is −ve, the Diodes D3, D_4 becomes ON & D1, D_2 becomes OFF.
Hence the modulator multiplies message signal by "−1" as shown in figure 1.6 (c).

i.e., $V_0(t) = m(t) \times -1$

$V_0(t) = -m(t)$

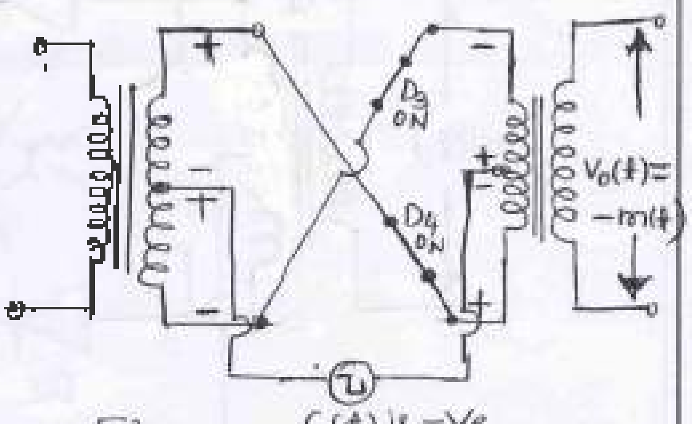


Figure 1.6 (c): During −ve half cycle of C(t)

∴ By combining Case(i) and Case(ii)
The Ring Modulator output at the secondary of transformer T_2 is given by

$$V_0(t) = m(t) \times c(t) \quad \text{——(1)}$$

The square wave carrier $c(t)$ can be represented by a Fourier series as:

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)]$$

$$\therefore c(t) = \frac{4}{\pi}\left[\cos 2\pi f_c t - \frac{1}{3}\cos 6\pi f_c t + \cdots\right] \quad \text{——(2)}$$

$\therefore$ Substitute equation (2) in $V_0(t)$ equation (1) we get

$$V_0(t) = m(t) \times \frac{4}{\pi}\left[\cos 2\pi f_c t - \frac{1}{3}\cos 6\pi f_c t + \cdots\right] \quad \text{——(3)}$$

When $V_0(t)$ is passed through BPF having center frequency 'f_c' and bandwidth '$2f_m$' we get DSBSC signal,

$$\boxed{S(t) = \frac{4}{\pi}\, m(t) \cos 2\pi f_c t} \leftarrow \text{DSBSC Wave generated from RING Modulator}$$

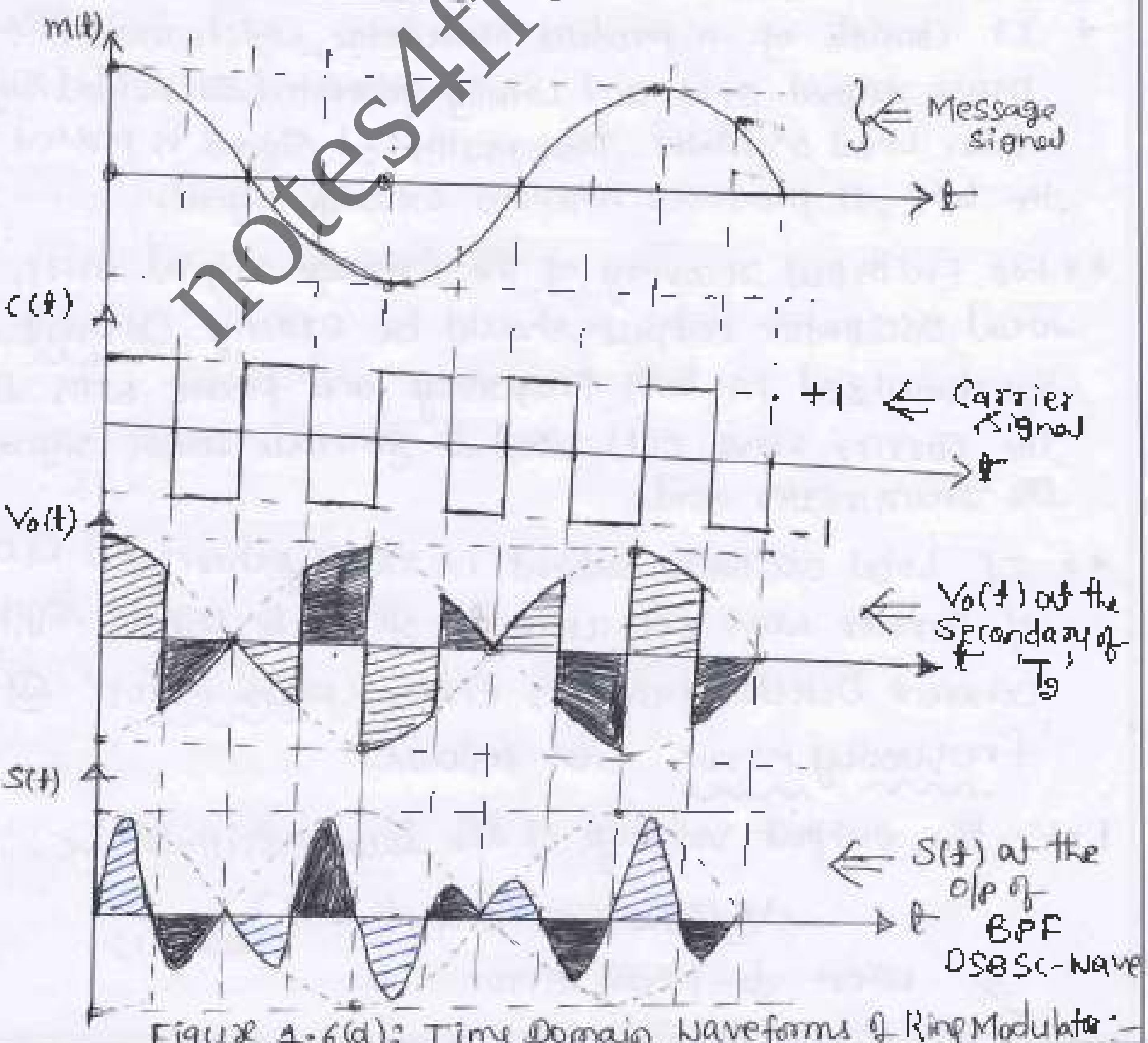


Figure 1.6(d): Time Domain Waveforms of Ring Modulator:-

1.7. Coherent Detection:-

Q) With relevant diagram Explain the operation of the coherent detection of DSBSC Modulated Waves. Also Explain phase error and frequency error.

↳

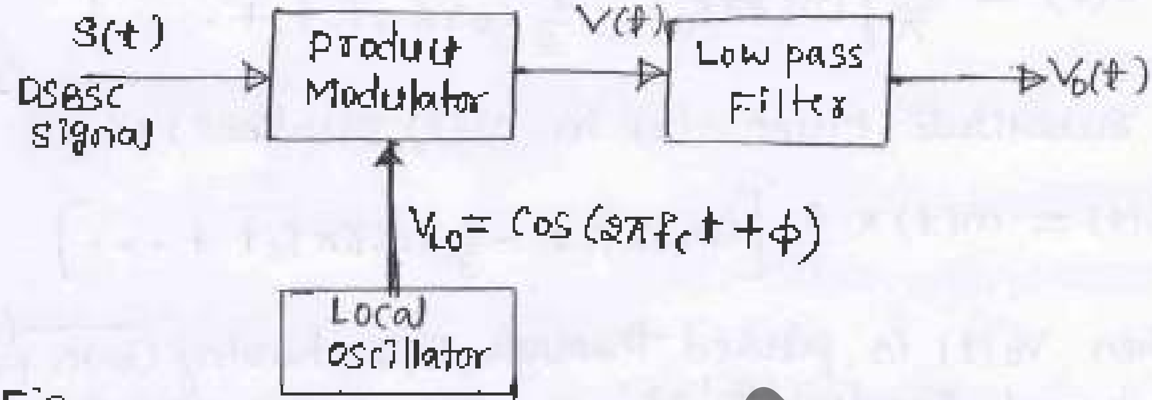


Figure 1.7: Coherent Detector for DSBSC Signal

* The Modulating signal $m(t)$ is recovered from a DSBSC Wave $S(t)$ by Using Coherent Detector shown in figure 1.7.

* It consists of a product Modulator, which multiplies DSBSC signal $S(t)$ and Locally generated sinusoidal signal from Local oscillator. Then Multiplied signal is passed through the LPF, it produces required message signal.

** For Faithful recovery of the message signal $m(t)$, the local oscillator output should be exactly Coherent (or) Synchronized in both frequency and phase with that of the Carrier Wave $c(t)$ used to generate DSBSC-signal at the transmitter End.

** If Local oscillator output is not synchronized with that of Carrier wave $c(t)$ used to generate DSBSC-signal, Coherent Detector produces either 'phase error' (or) 'frequency error' as follows.

Let the output voltage of the local oscillator is

$$V_L(t) = \cos(2\pi f_c t + \phi) \quad \text{—— (1)}$$

where ϕ = phase error

↳ Then the product Modulator output,

$$V(t) = s(t) \times V_L(t) \longrightarrow (2)$$

W.K.T, DSBSC signal $s(t) = A_c\, m(t) \cos 2\pi f_c t$

$$\therefore V(t) = A_c\, m(t) \underbrace{\cos 2\pi f_c t}_{B} \times \underbrace{\cos(2\pi f_c t + \phi)}_{A}$$

$$\therefore V(t) = \frac{A_c\, m(t)}{2}\left[\cos(2\pi f_c t + \phi - 2\pi f_c t) + \cos(2\pi f_c t + \phi + 2\pi f_c t)\right]$$

$$\therefore V(t) = \frac{A_c}{2} \cdot m(t)\left[\cos\phi + \cos(4\pi f_c t + \phi)\right] \longrightarrow (3)$$

When $V(t)$ is passed through LPF having Bandwidth "$\pm f_m$" we get

$$\boxed{V_0(t) = \frac{A_c}{2} \cdot \cos\phi \cdot m(t)}$$ output of LPF

$\therefore$ The Demodulated signal $V_0(t)$ is proportional to $m(t)$.

and $\cos\phi \Rightarrow$ phase error.

* When $\phi = 0°$: $V_0(t) = \frac{A_c}{2} \cdot m(t) \Rightarrow$ output Voltage is Maximum

* When $\phi = 90°$: $V_0(t) = 0$ ($\because \cos 90° = 0$). Then the output Voltage $V_0(t)$ is minimum (zero). This effect is called Quadrature Null Effect of the Coherent Detector.

** **

Note:-

↳ Similarly if frequencies are not synchronized, i.e., Local oscillator frequency $f_c' \neq$ carrier frequency 'f_c'. then the frequency difference is $\boxed{(f_c' - f_c) = \Delta f}$

↳ The presence of frequency difference $\boxed{\Delta f = f_c' - f_c}$, Results in shift in the modulating signal frequency.

i.e., frequency of the output voltage signal of Coherent Detector is : $f_0 = f_m + \Delta f \rightarrow$ upward shift ($f_c' < f_c$) if Δf is Negative

$f_0 = f_m - \Delta f \rightarrow$ Down ward shift ($f_c' > f_c$) (if Δf is positive).

Problems on Coherent Detector:-

** Example 1.4: Consider a Message signal $m(t)$ containing frequency components at 100, 200 and 400 Hz. This signal is applied to an SSB Modulator together with a carrier at 100 KHz, with only the upper side band retained. In the coherent detector used to recover $m(t)$, The Local oscillator supplies a sine wave of frequency 100.02 KHz. ** June/July-2017

(i) Determine the frequency components of the detector output

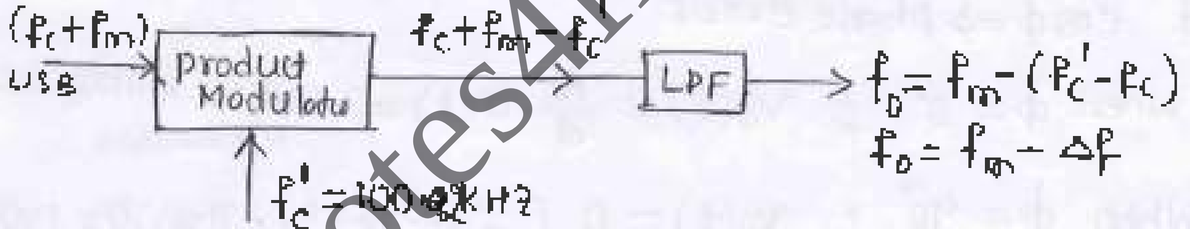
(ii) Repeat the analysis assuming that only the lower side band is transmitted.

(4 Marks)
July 2016 (8 Marks)

↳ Given data: Carrier frequency used at SSB Modulator $\Big\} \Rightarrow f_c = 100\,KHz$

frequency components of $m(t) \Rightarrow 100\,HZ,\ 200\,Hz\ \&\ 400\,Hz$

Case(i): When the upper side band is retained:-



∴ output frequencies of Coherent Detector are down shifted by

$\Delta f = f_c' - f_c = 100.02\,KHz - 100\,KHz = 20\,Hz.$

∴ output frequencies are $(\underset{f_{m_1}}{100} - \underset{\Delta f}{20}) = \underline{80\,Hz}$, $\underset{f_{m_2}}{200} - \underset{\Delta f}{20} = \underline{180\,Hz}$ & $\underset{f_{m_3}}{400} - \underset{\Delta f}{20} = \underline{380\,Hz}$

Case(ii):- When lower side band is transmitted, output frequencies of Coherent detector is $\boxed{f_0 = f_m + \Delta f}$. i.e., all the message signal frequencies are shifted upward by $\Delta f = 20\,Hz$.

∴ Frequency components of the detector output are $\underline{120\,Hz}$, $\underline{220\,Hz}$ and $\underline{420\,Hz}$.

1.8. Costas Receiver :-

Q) Explain how Costas receiver can be used for demodulating DSB-SC signal. — June/July – 2017 (6M)

↳ The Costas receiver is a practical synchronous receiver system, suitable for demodulating DSBSC-Waves. It is also named as Costas loop (or) practical synchronous receiving system.

Block diagram:-

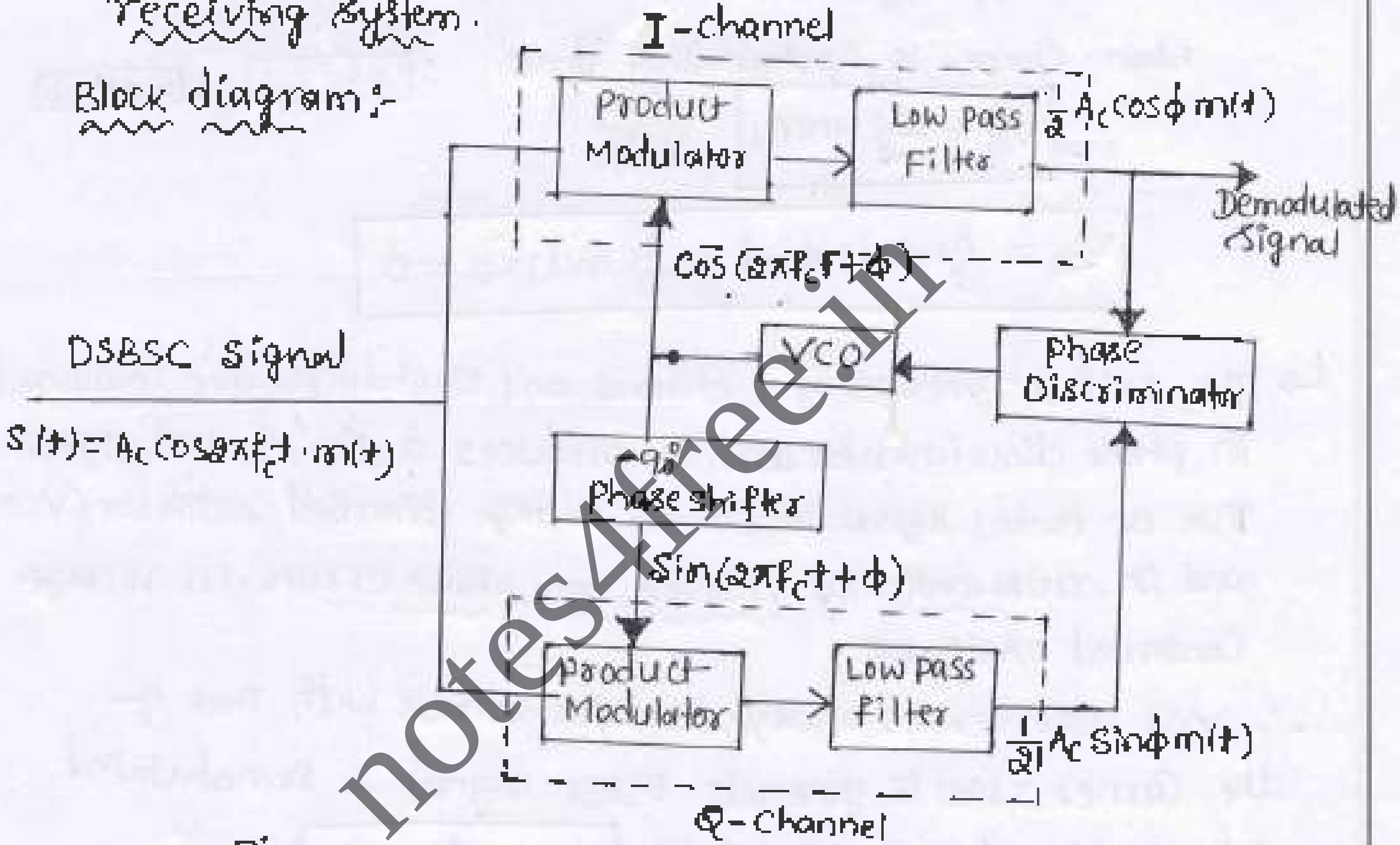


Figure 1.8: Costas receiver (or) Costas loop

↳ The Costas receiver consists of two coherent detectors supplied with same input signal (DSBSC-Wave $s(t)$) but with individual local oscillator signals that are phase quadrature with respect to each other. (i.e., the local oscillator signals supplied to the product modulators are 90° out of phase).

↳ The coherent detector in the upper path is referred to as the In-phase detector [(or) I-channel] and that in the lower path is referred to as Quadrature-phase Detector [(or) Q-channel] as shown in figure 1.8.

Operation:

↳ when local oscillator signal is of the same phase and frequency as that of carrier wave $A_c \cos 2\pi f_c t$ used to generate the incoming DSBSC wave. Then the I-channel output contains the desired demodulating signal $m(t)$ and Q-channel output is zero.

W.K.T, $V_{O_I} = \frac{A_c}{2} \cdot m(t) \cos\phi$

when carrier is synchronized $\phi = 0°$ $\because \boxed{\cos 0° = 1}$ & $\boxed{\sin 0° = 0}$

$$\boxed{V_{O_I} = \frac{A_c}{2} \cdot m(t)}$$

and

$$\boxed{V_{OQ} = \frac{A_c}{2} m(t) \sin\phi = \frac{A_c}{2} m(t) \times 0 = 0}$$

↳ The output voltages of I-channel and Q-channels are combined in phase discriminator and it produces a DC-control signal. This DC control signal is fed to voltage controlled oscillator (VCO) and it automatically corrects any phase errors in voltage controlled oscillator.

∴ VCO carrier is always in synchronous with that of the carrier used to generate DSBSC-signal. ∴ Demodulated output is always equal to $\boxed{V_{O_I} = \frac{A_c}{2} m(t)}$

1.9. Quadrature Carrier Multiplexing:

(Q) With relevant diagrams, explain the operation of the quadrature carrier multiplexing transmitter scheme and receiver scheme. Dec 2016/Jan 2017 8M.

→ Quadrature Carrier Multiplexing is a technique in which we can transmit more number of signals (DSBSC-wave) within the same channel Bandwidth. This technique is also named as Quadrature Amplitude Modulation (QAM).

QAM transmitter:

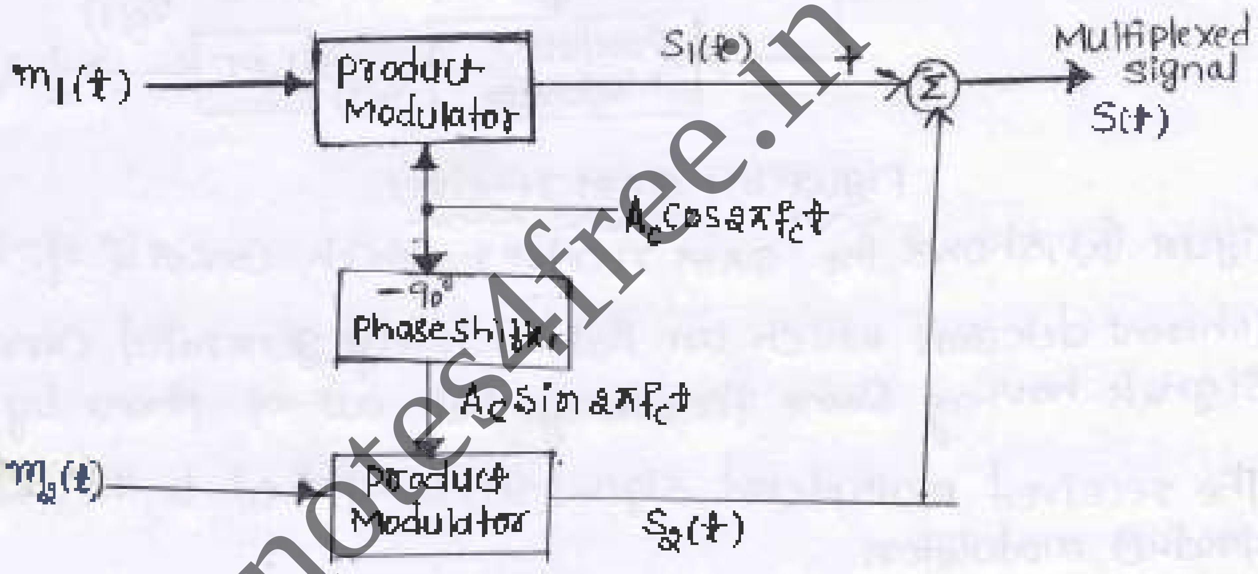


Fig (a) Quadrature Carrier Multiplexing (or) QAM Transmitter

- Figure (a) shows QAM transmitter. It consists of two product modulators that are supplied with carriers which differ in phase by 90° (phase quadrature)
- The output of the two product modulators are summed to produce multiplexed signal. $S(t)$.

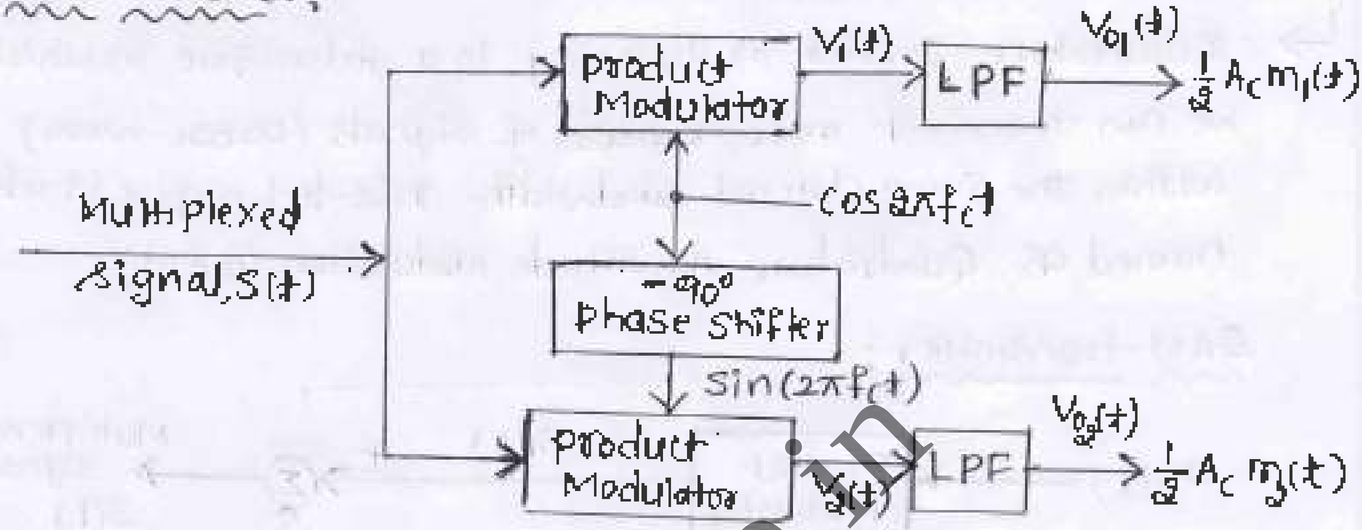
i.e., $S(t) = S_1(t) + S_2(t)$.

$$S(t) = A_c m_1(t) \cos 2\pi f_c t + A_c m_2(t) \sin 2\pi f_c t$$

∴ QAM transmitter allows two modulated (DSBSC) waves to occupy the same transmission channel Bandwidth.

∴ The Multiplexed signal $s(t)$, occupies a channel Bandwidth of $BW = 2W$; $W = \text{Maximum}(f_{m_1}, f_{m_2})$ Centered at the Carrier frequency 'f_c'.

QAM-Receiver:-



Figure(b): QAM receiver

- Figure (b) shows the QAM receiver, which Consists of two Coherent detectors which are fed by locally generated carrier signals having same frequency but out-of-phase by 90°.
- The received multiplexed signal $s(t)$ is applied to the two product modulators.

↳ The output of top product Modulator is

$$V_1(t) = s(t) \times \cos 2\pi f_c t.$$

↳ The top LPF removes the high frequency terms and allows only $\frac{A_c}{2} m_1(t)$.

$$\therefore V_{01}(t) = \frac{A_c}{2} m_1(t)$$

Similarly the output of bottom product modulator is

$$V_2(t) = s(t) \times \sin 2\pi f_c t$$

↳ The bottom LPF removes the high frequency terms and allows only $\frac{A_c}{2} m_2(t)$

$$\therefore V_{02}(t) = \frac{A_c}{2} m_2(t)$$

Application: used in color TV

- It is BandWidth - Conservation scheme.

* Advantages of DSBSC-Modulation :-

1. Carrier signal is supressed
2. Low power Consumption.
3. Efficiency is more than AM.
4. The Modulation system is simple.
5. Linear Modulation.

* Disadvantages (or) Limitations of DSBSC-Modulation :-

1. Design of receiver is complex.
2. Bandwidth required is same as that of AM

   i.e., $BW_{AM} = BW_{DSBSC} = 2f_m$

* Applications of DSBSC :-

↳ point to point Communication.

↳ Analog TV systems to transmit Color Information.

1.10: SINGLE SIDE BAND SUPRESSED CARRIER (SSBSC) MODULATION:

↳ Standard AM and DSBSC Modulation requires a transmission bandwidth of $BW_T = 2f_m$.

↳ ∴ In both AM & DSBSC-Modulation, half of the transmission bandwidth is occupied by the upper side band of the modulated wave and other half of the transmission BW is occupied by LSB.

↳ The USB and LSB are uniquely related to each other by virtue of their symmetry about the carrier frequency "f_c".

∴ only one sideband is necessary for transmission of message signal.

↳ When only one side band is transmitted, the modulation is referred to as "Single side band Modulation"

SSB-Modulation :-

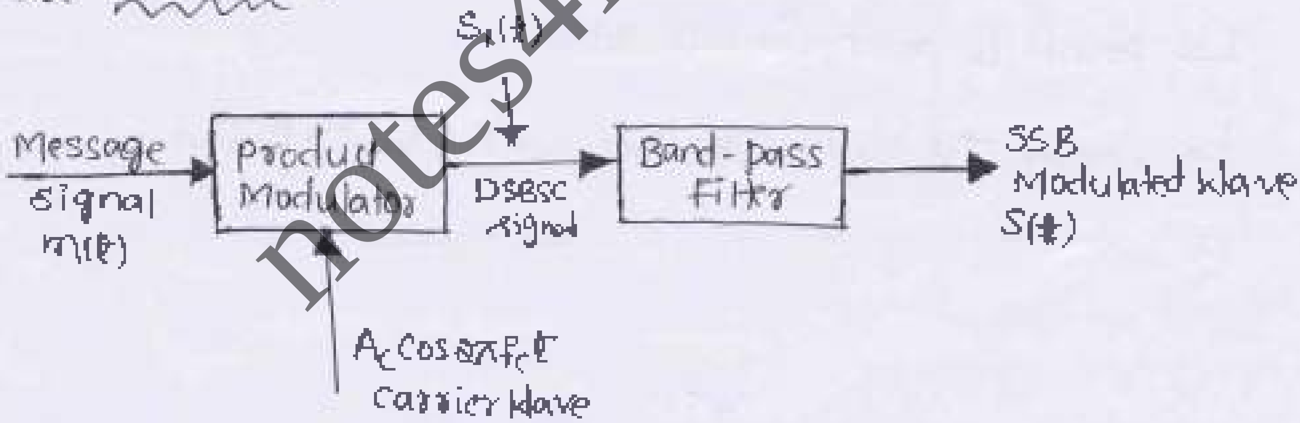


Figure : Frequency descrimination scheme for the generation of SSB Modulated wave.

- Frequency descrimination scheme for the generation of SSB Modulated wave is shown in above figure.
- It Consists of product Modulator followed by BPF.
- The output of the product Modulator is DSBSC signal

$$S(t) = m(t) \times A_c \cos 2\pi f_c t$$

$\therefore \quad s_1(t) = A_c\, m(t) \cdot \cos 2\pi f_c t$ ——(1)

Taking Fourier transform on both sides of equation (1) we get

$$S_1(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

$$S_1(f) = \frac{A_c}{2}\underbrace{M(f-f_c)}_{LSB} + \frac{A_c}{2}\underbrace{M(f+f_c)}_{USB} \quad \text{——(2)}$$

- Where M(f) is frequency spectrum of message signal m(t) as shown in figure 2(a).
- The spectrum of DSBSC signal $S_1(f)$, shown in figure 2(b) consists of both the side bands (LSB & USB) centered with respect to carrier frequency.
- When DSBSC signal is passed through BPF with its center frequency 'f_c' and Bandwidth 'W' Hz. If it transmits USB then we get spectrum of SSB modulated signal.

$$S(f) = \frac{A_c}{2} M(f+f_c) \quad \text{——(3)}$$; SSB Modulated signal with only USB.

- If BPF allows only LSB, then

$$S(f) = \frac{A_c}{2}\left(M(f-f_c)\right) \quad \text{——(4)}$$; SSB Modulated signal with only LSB

$\therefore$ The single side band suppressed carrier modulated signal S(t) produced at the output of BPF contains only one side band and carrier signal, other side band are eliminated (or) Suppressed

The Frequency domain description of SSB-Modulated signal is clearly represented in Figure 2.

Figure 2(c) shows the spectrum of SSB-Signal with USB only.

Figure 2(d) shows the spectrum of SSB-Signal with LSB only.

$\therefore$ Transmission Bandwidth of SSB Modulated signal is $\} = B_T = W \Leftarrow$ Half of that of AM & DSBSC-signals.

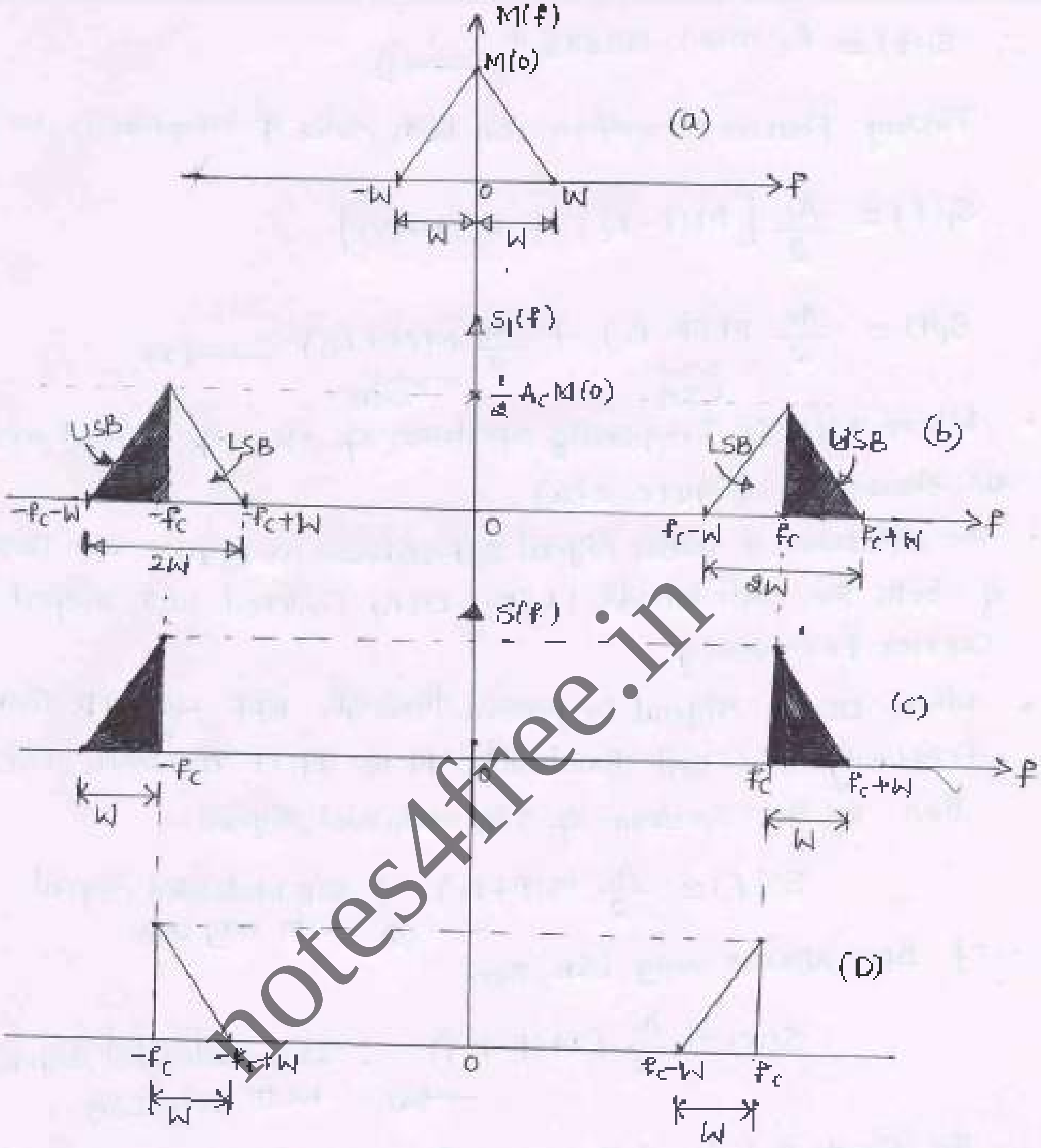


Figure 2: (a) Spectrum of m(t) (b) Spectrum of DSBSC-signal $S_1(t)$. (c) Spectrum of SSB Mod. signal having only USB S(t) (d) Spectrum of SSB Modulated signal having only LSB. S(t)

** Advantages of SSB-Modulation :-

1. SSB-Modulation requires half of the Bandwidth required for AM and DSBSC-signal.
2. Due to suppression of carrier and one side band, power is saved
3. Reduced interference of Noise.
4. Fading does not occur in SSB-transmission.

* Disadvantages of SSB:-

1. The Generation and reception of SSB signal is a Complex process
2. SSB-Modulation system is Expensive and highly Complex to implement.
3. Since the carrier and one side band is suppressed, the SSB transmitter and receiver need to have Excellent frequency stability.

* Applications of SSB:-

1. Mobile Communication system : (Since SSB is power saving modulation)
2. SSB is also used in applications in which bandwidth requirements are Low.

   Ex: • point to point Communication
   - Telemetry
   - Military applications
   - Radio Navigation

1.11
* VESTIGIAL SIDEBAND MODULATION (VSB) :-

Necessity (or) Need for VSB-Modulation:

↳ The SSB modulation is not appropriate way of modulation. Because the upper side band and lower side band meet at the carrier frequency 'f_c' and it is very difficult to isolate one side band. Therefore generating SSB-signal is challenging.

↳ To overcome this difficulty, the modulation technique known as "Vestigial side Band (VSB) - Modulation" is used.

↳ Vestigial sideband modulated signal (VSB-signal) consists of

- Almost one complete side band and
- Vestige (or Trace (or) Part) of the other side band

* Generation of VSB Modulated Wave:

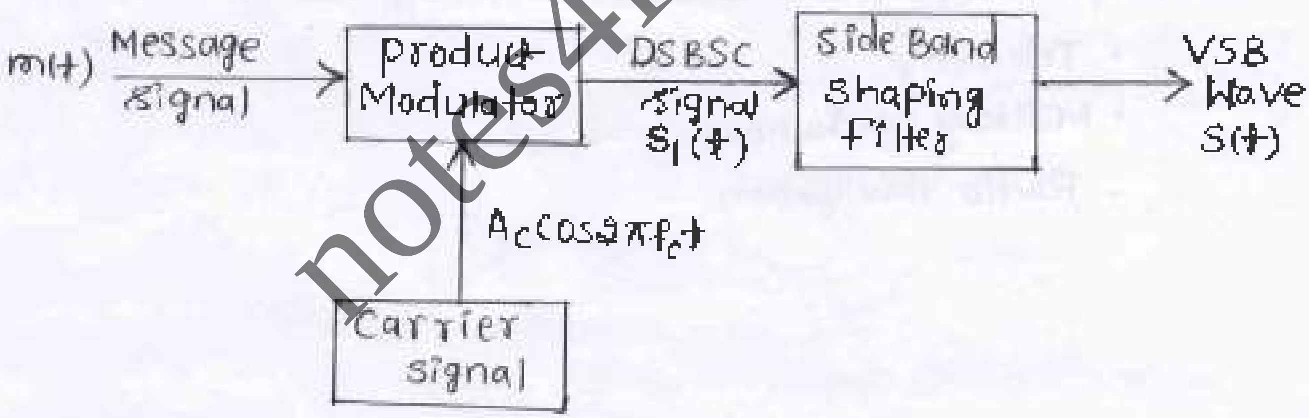


Figure 1: VSB Generator

- VSB signal generator consists of a product modulator and a sideband shaping filter as shown in figure 1.
- Product modulator generates a DSBSC signal and then pass it through a side band shaping filter.
- Let $H(f)$ be the transfer function of side band shaping filter. This filter will pass one complete sideband along with a vestige (or) trace (or) a part of unwanted (other) side band.

↳ The relation between the transfer function $H(f)$ of the filter and the spectrum $S(f)$ of the VSB-Modulated wave $s(t)$ is defined by, $S(f) = S_1(f) \times H(f)$.

$$\therefore \boxed{S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f)}$$

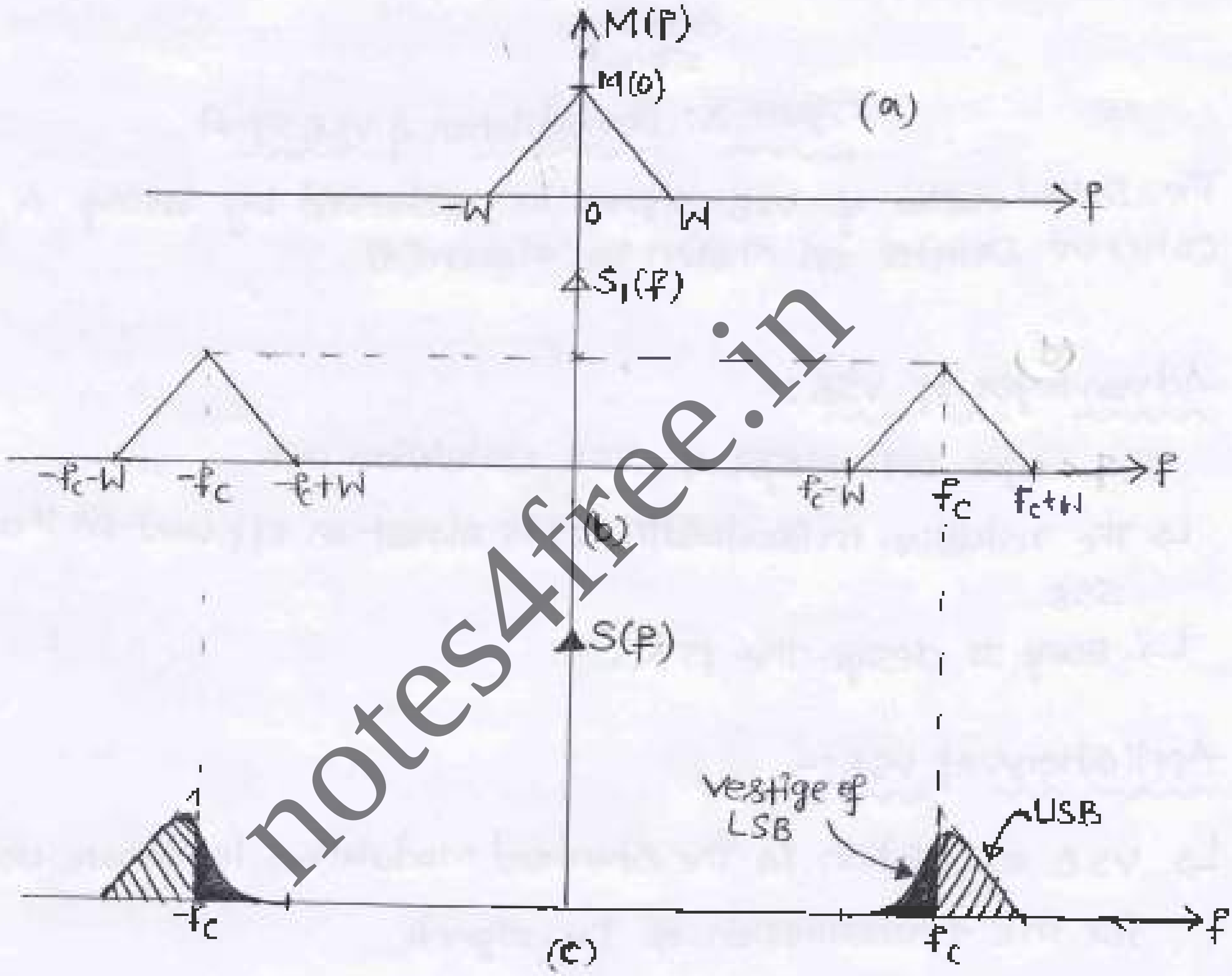


Figure 2: (a) Spectrum of $m(t)$ (b) Spectrum of DSBSC signal $S_1(t)$
(c) Spectrum of VSB-Modulated signal $S(t)$

Frequency domain description of VSB modulated wave is shown in figure 2. Figure 2(b) is the spectrum of DSBSC signal produced at the output of product modulator. Figure 2(c) shows the spectrum of VSB-modulated signal $s(t)$.

From figure 2(c) it is evident that the Total transmission Bandwidth of VSB-Modulated signal is higher than that of SSB and lower than that of DSBSC-signal.

i.e., $W < BW_{T(VSB)} < 2W$

* Demodulation of VSB-Modulated Wave :-

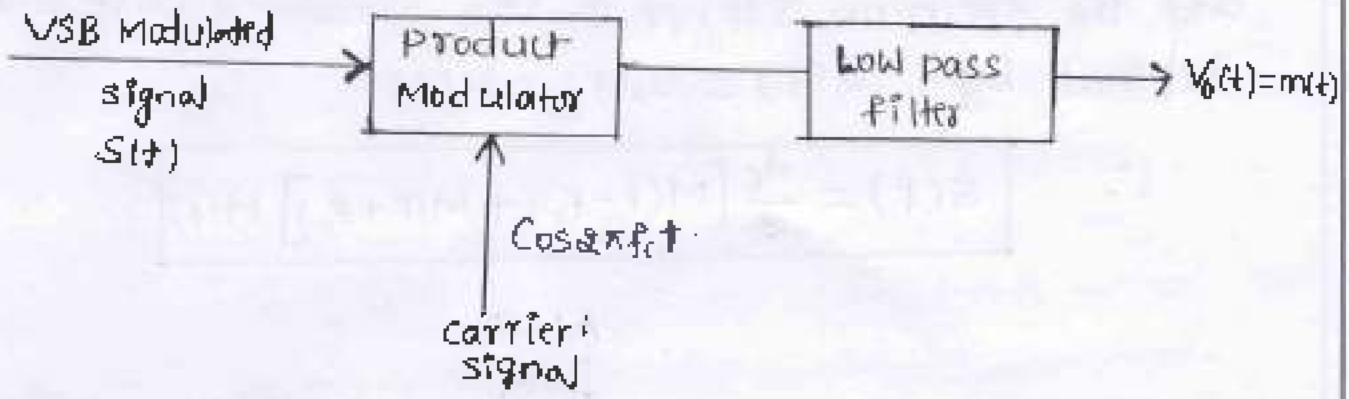


Figure 3: Demodulation of VSB signal

The Demodulation of VSB-signal is achieved by using a coherent Detector as shown in figure(3).

Advantages of VSB :-

The Major advantages of VSB modulation are

↳ The reduction in Bandwidth. It is almost as efficient as that of SSB.

↳ Easy to design the filter.

Applications of VSB :-

↳ VSB modulation is the standard Modulation technique used for the transmission of TV-signals.

1.12: Frequency Translation: [Mixing (or) Heterodyning]

↳ In the communication systems, it is necessary to translate modulated wave $s(t)$, frequency upward (or) Downward in frequency so that it occupies a new frequency band.

↳ i.e., If 'f_c' is the frequency of Modulated wave $s(t)$ then frequency-translator changes modulated wave frequency 'f_c' to either $f_c + f_L \Rightarrow$ upward translation (or)
$f_c - f_L \Rightarrow$ Downward translation.

*↳ Frequency-translation is achieved by using a Frequency-Multiplier (product Modulator) followed by a Band pass filter as shown in Figure 1.

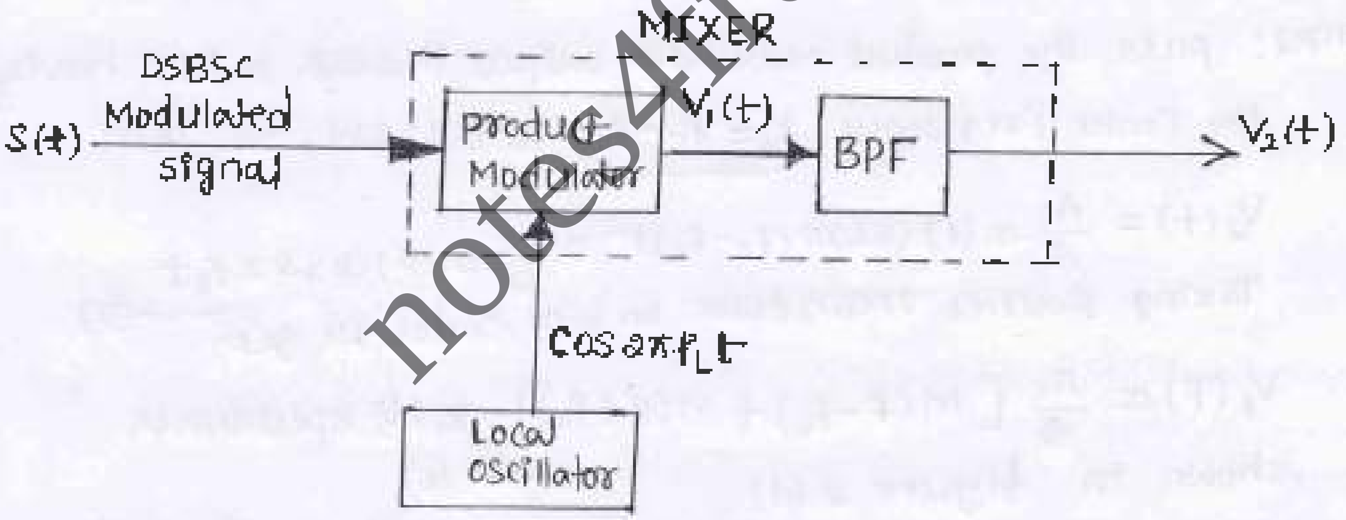


Figure 1: Frequency Translation @ Mixing (or) Heterodyning

Consider a DSBSC Modulated signal,

$$s(t) = A_c\, m(t) \cdot \cos 2\pi f_c t \quad —(1)$$

- Where $m(t)$ is limited to the frequency band $-W \leq f \leq W$ shown in Figure(a)

The fourier-transform of equation(1) is

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] \quad —(2)$$

- The spectrum of $s(t)$ occupies the band of frequencies $(f_c - W)$ to $(f_c + W)$ and $-(f_c - W)$ to $-(f_c + W)$ as shown in figure(b)

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Case(i): Frequency translation to Lower frequency (or) (Downward frequency)

i.e, $(f_c \longrightarrow f_0 = f_c - f_L)$

step1: The output of the product modulator is given by

$$V_1(t) = s(t) \times \cos 2\pi f_L t$$

$$V_1(t) = A_c m(t) \cos 2\pi f_c t \times \cos 2\pi f_L t$$

$$\therefore V_1(t) = \frac{A_c m(t)}{2} \cos 2\pi (f_c - f_L) t + \frac{A_c m(t)}{2} \cos 2\pi (f_c + f_L) t \longrightarrow (3)$$

Taking Fourier transformation on both sides we get

$$V_1(f) = \frac{A_c}{4}\left[M(f-(f_c-f_L)) + M(f+(f_c-f_L))\right] + \frac{A_c}{4}\left[M(f-(f_c+f_L)) + M(f+(f_c+f_L))\right] \longrightarrow (4)$$

The spectrum of $V_1(f)$ is shown in figure 2.(c)

step2: pass the product modulator output through a BPF having the center frequency $f_0 = f_c - f_L$ & $BW = 2W$, we get

$$V_2(t) = \frac{A_c}{2} m(t) \cos 2\pi (f_c - f_L) t = \frac{A_c}{2} m(t) \cos 2\pi f_0 t \longrightarrow (5)$$

Taking Fourier transform on both sides we get

$$V_2(f) = \frac{A_c}{4}\left[M(f-f_0) + M(f+f_0)\right] \longrightarrow (6)$$

& its spectrum is shown in figure 2.(d).

$\therefore$ By comparing equation (2) $[S(f)]$ and output of BPF $V_2(f)$ it is clear that the center frequency 'f_c' of DSBSC modulated signal $s(t)$ is translated to a new frequency band $\boxed{f_0 = f_c - f_L}$

Case(ii): Frequency translation to Higher frequency (or) (Upward frequency)

i.e, $f_c \longrightarrow f_0 = f_c + f_L$

↳ pass the product modulator output $V_1(t)$ shown in eqn (3) through a BPF having center frequency $f_0 = f_c + f_L$ & $BW = 2W$, we get, the output of BPF

$$V_2(t) = \frac{A_c}{2} m(t) \cos 2\pi (f_c + f_L) t$$

$$V_2(t) = \frac{A_c}{2} m(t) \cos 2\pi f_0 t \quad \because f_0 = f_c + f_L \qquad \text{---(7)}$$

Taking Fourier transformation on both sides we get

$$V_2(f) = \frac{A_c}{4} \left[M(f - f_0) + M(f + f_0) \right] \qquad \text{---(8)}$$

$\therefore$ The center frequency of DSBSC Modulated signal is (f_c) translated to upward frequency $\boxed{f_0 = f_c + f_L}$

The spectrum of $V_2(f)$ is shown in figure 2(e).

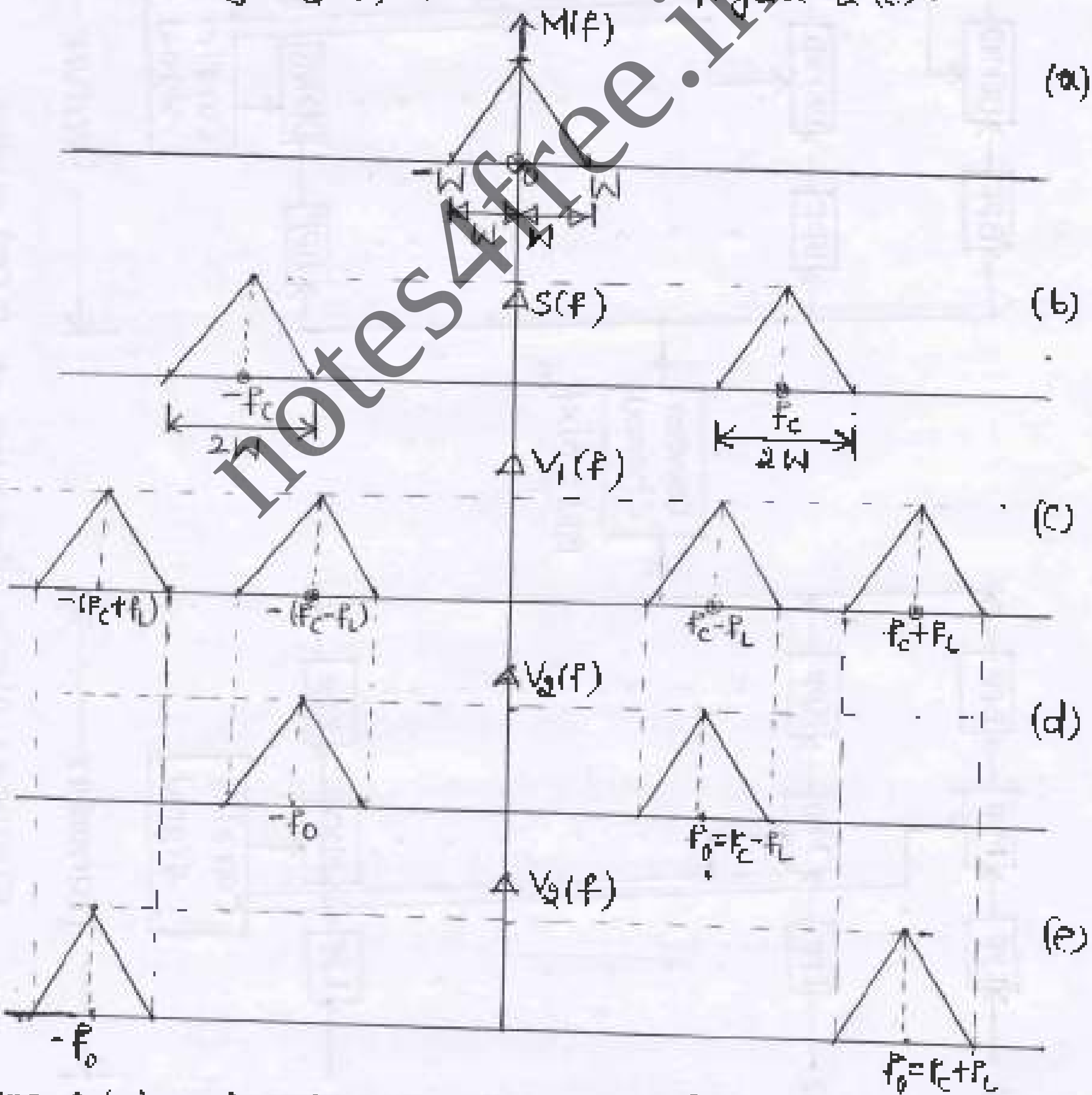


Figure 2: (a) Spectrum of $m(t)$ (b) spectrum of DSBSC signal $s(t)$
(c) Spectrum of product modulator output $V_1(t)$
(d) Spectrum of downward translated signal $V_2(t)$
(e) Spectrum of upward translated signal $V_2(t)$

1.13 * Frequency Division Multiplexing <FDM>

- Multiplexing is a process of combining N-independent message signals into a composite signal suitable for transmission over a common channel.
- Multiplexing is accomplished by seperating the signals either in frequency (or) time.
- The technique of separating the signals in frequency domain is referred to as "Frequency Division Multiplexing".

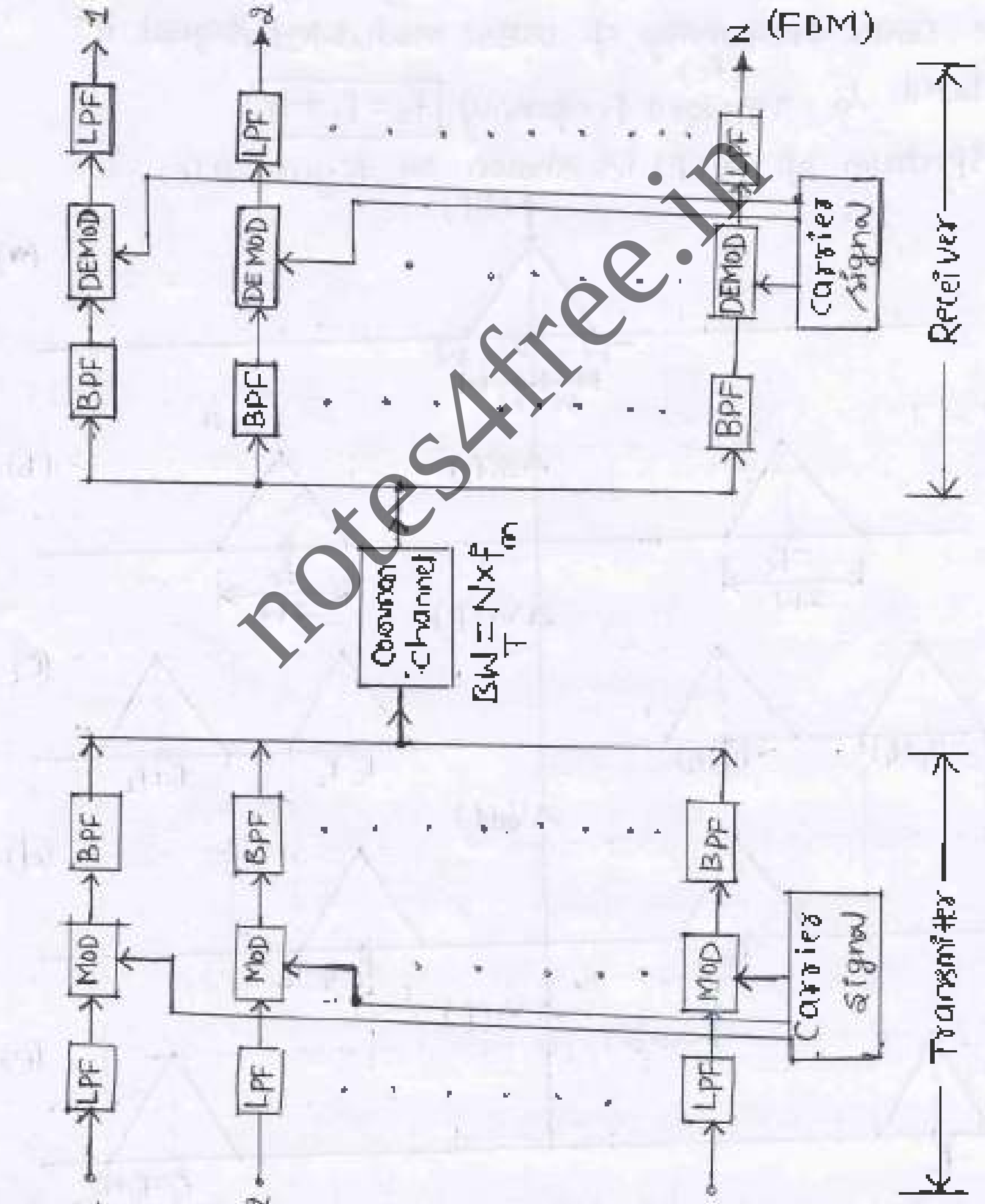


Figure 1: Block diagram of FDM system

The block diagram of FDM system is shown in figure 1.

↳ N-Incoming independent message signals are modulated by mutually exclusive carriers supplied from carrier source at each modulator. The modulated signals are passed through the BPF to select any one side band. Therefore BPF's produces SSB-signals and are separated in frequency and combined into a composite signal and this process is called Frequency division multiplexing.

↳ Multiplexed signal is transmitted over the communication channel.

↳ Total Bandwidth required to N-SSB modulated signals without any guard band is

$$BW_T = N \times f_m \quad ; \quad N = \text{number of input signals}$$

↳ At the receiver side N-independent message signals are recovered by passing the composite signal through the BPF followed by Demodulator and LPF.

Advantages of FDM:-

1. A Large Number of signals can be transmitted simultaneously.
2. FDM does not requires synchronization between Transmitter & receiver.
3. Demodulation of FDM is easy

Disadvantages of FDM:-

1. Communication channel must have Large Bandwidth i.e., $BW_T = N \times f_m$
2. Large Numbers of Modulators & Filters are required.
3. Cross talk occurs in FDM

Theme Example : VSB Transmission of Analog and Digital TV :
1.14 :

-6M-

↳ Vestigial side band modulation plays a key role in commercial television.

↳ The exact details of modulation format used to transmit the video signal characterizing a TV system are influenced by two factors :

i) The video signal exhibits a large bandwidth and significant low frequency information, which requires the use of vestigial side band modulation.

ii) The circuitry used for demodulation in the receiver must be simple and inexpensive. This suggests the use of envelope detection, which requires the addition of a carrier wave to the VSB modulated wave.

Figure (a) shows the ideal spectrum of a transmitted TV-Signal. It consists of the upper side band, 25% of the LSB and the picture carrier are transmitted.

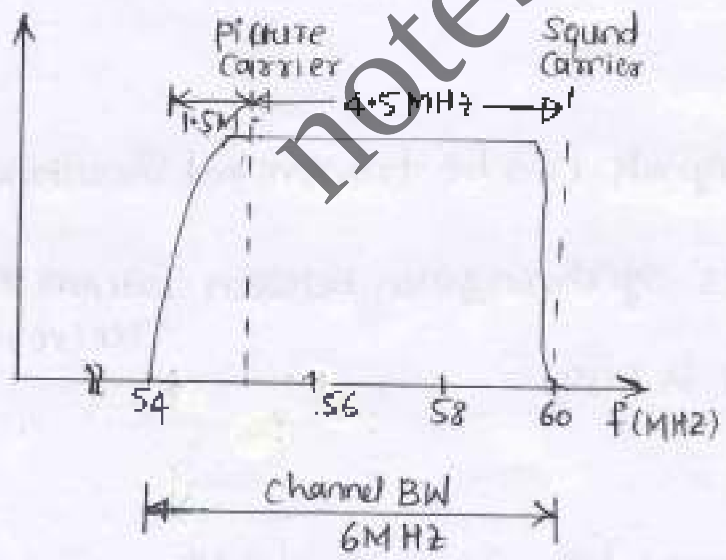


Fig (a): Ideal Transmission spectrum of TV signal

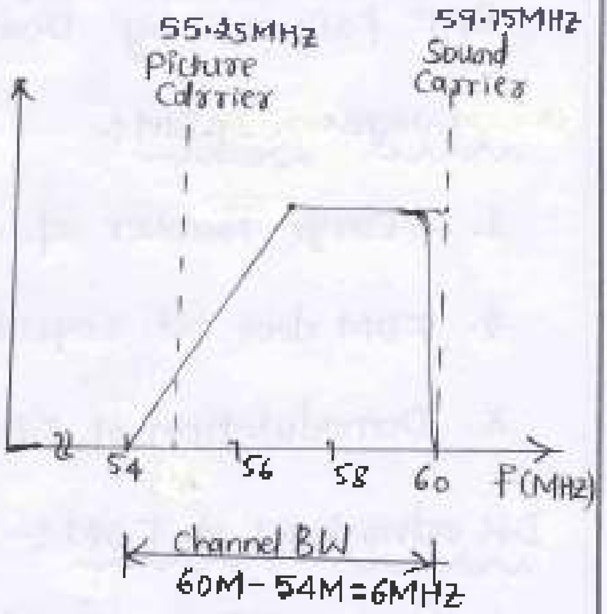


Fig (b): Amplitude response of a VSB-shaping filter at the receiver

- Figure (b) shows the amplitude response of a VSB shaping filter used at the receiver
- The channel bandwidth used for TV broadcasting in North America is 6MHz as shown in fig (a) & fig (b).

Module-1 :- Amplitude Modulation

(Numerical problems) V.T.U Q-Papers

List of Formulae :-

1. Amplitude Modulation Index @ Depth of Modulation :

$\mu = K_a A_m$, where A_m = Amplitude of message signal in Volts

K_a = Amplitude Sensitivity parameter

@ $\boxed{\mu = K_a A_m}$

Note :• The Maximum Value of Modulation index is "1"

• When, $\mu < 1 \Rightarrow$ Results in under modulation
• When, $\mu = 1 \Rightarrow$ Results in Critical Modulation
• when, $\mu > 1 \Rightarrow$ Results in over Modulation.

2. AM Wave equation :

↳ Single-tone AM : (considering single message signal)

• $s(t) = A_c\left[1 + \mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$

↳ Multitone AM : (considering multiple message signals)

• $s(t) = A_c\left[1 + \mu_1\cos(2\pi f_{m_1} t) + \mu_2\cos(2\pi f_{m_2} t) + \cdots\right]\cos(2\pi f_c t)$

where, $\mu_1 = K_a A_{m_1}$; $\mu_2 = K_a A_{m_2}$

3. Net Modulation Index :-

For Multitone Modulation, Net Modulation Index μ_t is

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + \cdots + \mu_n^2}$$

4. The Maximum and Minimum amplitudes of AM :-

• $A_{max} = A_c(1+\mu)$; • $A_{min} = A_c(1-\mu)$

A_{max} & A_{min} are required to sketch resulting AM signal for any given Modulation index 'μ'.

5. Total power in AM-wave :-

↳ In terms of carrier power 'P_c' and 'μ'

$$P_t = P_c\left(1+\frac{\mu^2}{2}\right) \; ; \; P_c = \frac{A_c^2}{2R} \; ; \; R = \text{Load Resistance}$$

↳ power in sidebands : $P_{LSB} = P_{USB} = P_c \frac{\mu^2}{4}$ Watts

↳ Total power in side bands : $P_{SB} = P_{LSB} + P_{USB} = P_c \frac{\mu^2}{2}$

$$P_t = P_c + P_{LSB} + P_{USB} = P_c + P_{SB} \implies P_{SB} = P_t - P_c$$

↳ In terms of Antenna RMS currents : $P_t = I_t^2 R$ and $P_c = I_c^2 R$

Note :- when $\mu = 1$: $P_t\Big|_{max} = P_c\left(1+\frac{\mu^2}{2}\right)\Big|_{\mu=1} = \frac{3}{2} P_c = 1.5 P_c$.

∴ The Max. transmitted power is 1.5 times Carrier power

6. Efficiency of AM :-

$$\eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{\mu^2}{2+\mu^2}$$

Note : Max. efficiency of AM is $\eta_{max} = \frac{\mu^2}{2+\mu^2}\Big|_{\mu=1} = \frac{1}{3} = \underline{\underline{0.3333}}$

7. Total transmission Band Width of AM :-

$$BW_T = 2W = f_{USB} - f_{LSB}$$

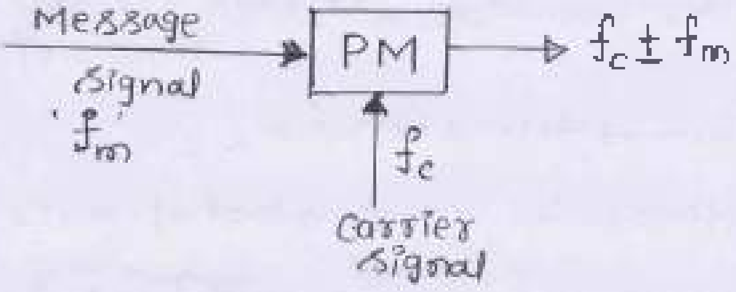
where $W = \text{Maximum}(f_{m_1}, f_{m_2}, \ldots, f_{m_n})$; for Multitone AM.

$W = f_m$, for single tone AM

f_{USB} = frequency of upper side Band $= f_c + f_m$

f_{LSB} = frequency of lower side Band $= f_c - f_m$

8. Product Modulator :- <DSBSC - Generator>



↳ product modulator produces two output frequencies

(I) $f_c + f_m = f_{USB}$

(ii) $f_c - f_m = f_{LSB}$

9. Band pass Filter :- It is used to select any one side band frequency.

- $f_c \pm f_m \longrightarrow \boxed{BPF} \longrightarrow f_c + f_m$: if center frequency of BPF is $(f_c + f_m)$
- $f_c \pm f_m \longrightarrow \boxed{BPF} \longrightarrow f_c - f_m$: if center frequency of BPF is '$f_c - f_m$'

10. Amplitude of each side band in AM $= \frac{\mu A_c}{2}$

11. General formulas :

↳ $\cos A \cdot \cos B = \frac{1}{2}\{\cos(A-B) + \cos(A+B)\}$.

↳ $\cos(2\pi f_0 t) \xrightleftharpoons{F \cdot T} \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$

↳ $A_c\, m(t) \cdot \cos(2\pi f_c t) \xrightleftharpoons{F \cdot T} \frac{A_c}{2}[M(f - f_0) + M(f + f_0)]$

Where, • $\delta(f - f_c) = \begin{cases} 1 & \text{: only at } f = f_c \\ 0 & \text{: ew} \end{cases}$ ← Impulse signal @ Delta function

• $\delta(f + f_c) = \begin{cases} 1 & \text{: only at } f = -f_c \\ 0 & \text{: ew} \end{cases}$

• $M(f)$ is spectrum of $m(t)$.

12. Bandwidth of SSBSC is : $BW_{SSB} = W = f_m$

1. Consider a message signal $m(t) = 20\cos(2\pi t)$ Volts and a carrier signal $c(t) = 50\cos(100\pi t)$ Volts

(i) Find and sketch the resulting AM wave for 75% modulation. (VTU Q.P)

(ii) Sketch the spectrum of this AM wave.

(iii) Find the power dissipated across a load of $100\,\Omega$.

↳ Given data : $m(t) = 20\cos(2\pi t)$ ∴ $A_m = 20$; $f_m = 1$ Hz

$c(t) = 50\cos(100\pi t)$ ∴ $A_c = 50$; $f_c = 50$ Hz

(i) Resulting AM wave for 75% Modulation :- (i.e., for $\mu = 0.75$)

W.K.T. for Single Tone AM,

$$s(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

∴ The Resulting AM wave for $\mu = 0.75$; $A_c = 50V$; $f_m = 1$ and $f_c = 50$ is

$$\boxed{s(t) = 50[1 + 0.75\cos(2\pi t)]\cos(100\pi t)}$$

- To Sketch AM Signal :-
  - $A_{max} = A_c(1+\mu) = 50(1+0.75) = 87.5V$
  - $A_{min} = A_c(1-\mu) = 50(1-0.75) = 12.5V$

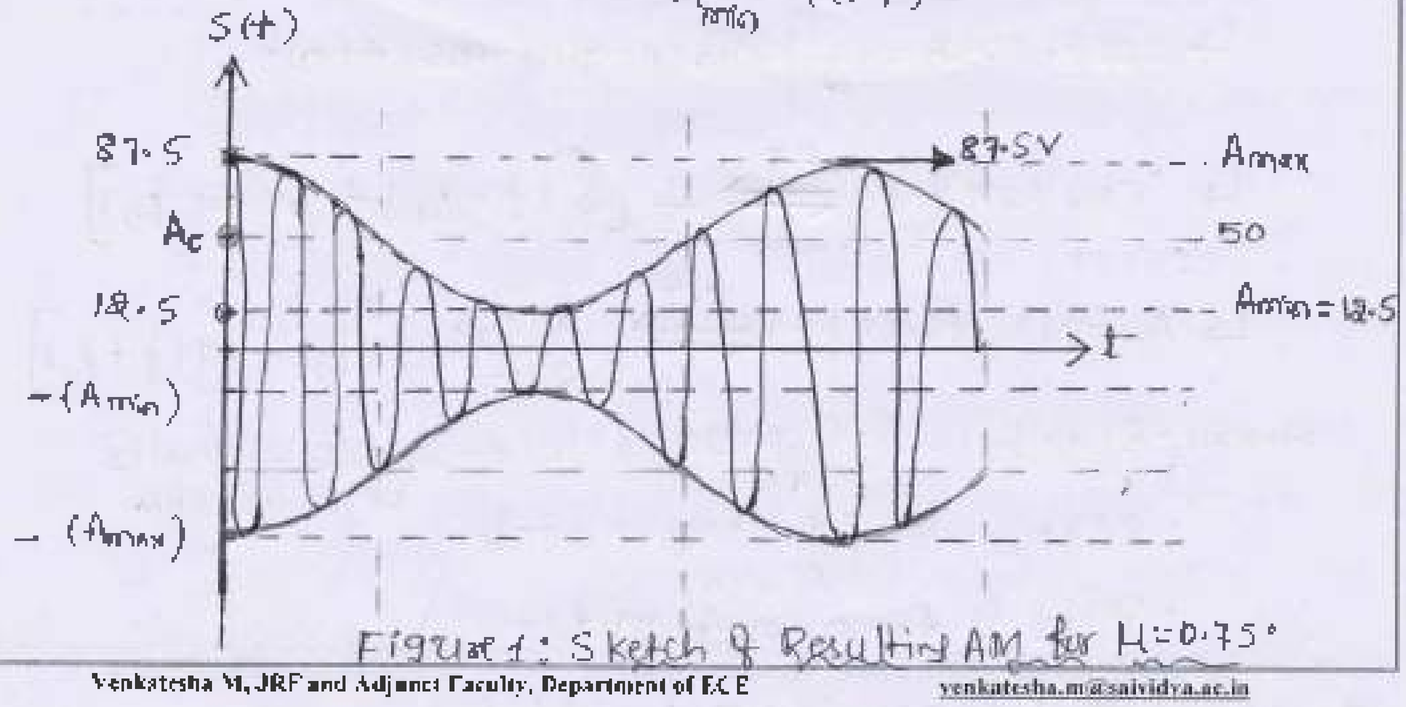


Figure 1 : Sketch of Resulting AM for $\mu = 0.75$.

(ii) Spectrum of AM wave :-

W.K.T the resulting AM wave is

$$s(t) = A_c\left[1 + \mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$s(t) = 50\left[1 + 0.75\cos(2\pi(1)t)\right]\cos(2\pi(50)t)$$

$$\therefore\ s(t) = 50\cos 2\pi(50)t + 37.5\cos 2\pi(50)t\cdot\cos 2\pi(1)t$$

W.K.T $\cos A\cdot\cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$

$$\therefore\ s(t) = 50\cos 2\pi(50)t + \frac{37.5}{2}\left[\cos 2\pi(50-1)t + \cos 2\pi(50+1)t\right]$$

$$\boxed{\therefore\ s(t) = 50\cos 2\pi(50)t + 18.75\cos 2\pi(49)t + 18.75\cos 2\pi(51)t} \quad \text{— (1)}$$

Taking Fourier Transform of Equation (1) we get

$$S(f) = \frac{50}{2}\left[\delta(f-50) + \delta(f+50)\right] + \frac{18.75}{2}\left[\delta(f-49) + \delta(f+49)\right] + \frac{18.75}{2}\left[\delta(f+51) + \delta(f-51)\right]$$

$$\boxed{\therefore\ S(f) = 25\left[\delta(f-50) + \delta(f+50)\right] + 9.375\left[\delta(f-49) + \delta(f+49)\right] + 9.375\left[\delta(f+51) + \delta(f-51)\right]}$$

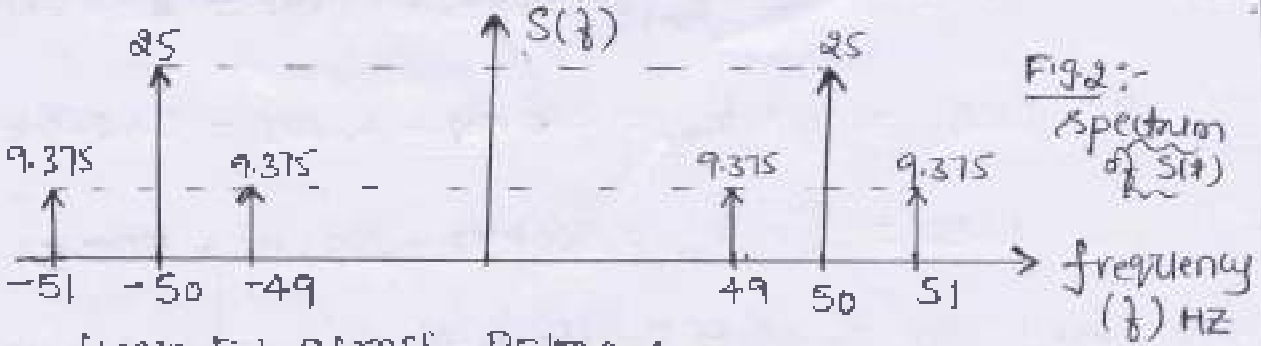


Fig 2:- Spectrum of S(f)

(iii) Power dissipated across $R = 100\Omega$:-

W.K.T. $P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$; $P_c = \frac{A_c^2}{2R} = \frac{(50)^2}{2\times 100} = 12.5\,W$

$$\therefore\ P_t = 12.5\left(1 + \frac{(0.75)^2}{2}\right) = \underline{16.016\,W}$$

2. A carrier wave $4\sin(2\pi \times 500 \times 10^3 t)$ Volts is amplitude modulated by an audio wave $[0.2\sin 3(2\pi \times 500t) + 0.1\sin 5(2\pi \times 500t)]$ Volts. Determine the upper and lower sidebands and sketch the complete spectrum of the modulated wave. Estimate the total power in the side band. (VTU Q.P)

↳ Given: $c(t) = 4\sin(2\pi \times 500 \times 10^3 t)$.

$\therefore A_c = 4V$ & $f_c = 500 \times 10^3 = 500KHZ$.

The Message signal (Audio wave),

$$m(t) = 0.2\sin 2\pi \times 1500 \times t + 0.1\sin 2\pi \times 2500 t$$

$\therefore A_{m_1} = 0.2$; $f_{m_1} = 1500 = 1.5\,kHz$

$A_{m_2} = 0.1$; $f_{m_2} = 2500 = 2.5\,kHz$.

$\therefore \mu_1 = \frac{A_{m1}}{A_c} = \frac{0.2}{4} = 0.05$ and $\mu_2 = \frac{A_{m2}}{A_c} = \frac{0.1}{4} = 0.025$

Net Modulation index : $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{0.05^2 + 0.025^2} = 0.056$

(*) Upper and lower side bands (USB and LSB):-

i) $USB1 = f_c + f_{m_1} = 500kHZ + 1.5\,kHZ = 501.5\,kHZ$

$LSB1 = f_c - f_{m_1} = 500kHZ - 1.5\,kHZ = 498.5\,kHZ$.

ii) $USB_2 = f_c + f_{m_2} = 500\,kHZ - 2.5\,kHZ = 502.5\,kHZ$

$LSB2 = f_c - f_{m_2} = 500\,KHZ - 2.5\,kHZ = 497.5\,KHZ$

• To Sketch the Complete Spectrum of the modulated wave:-

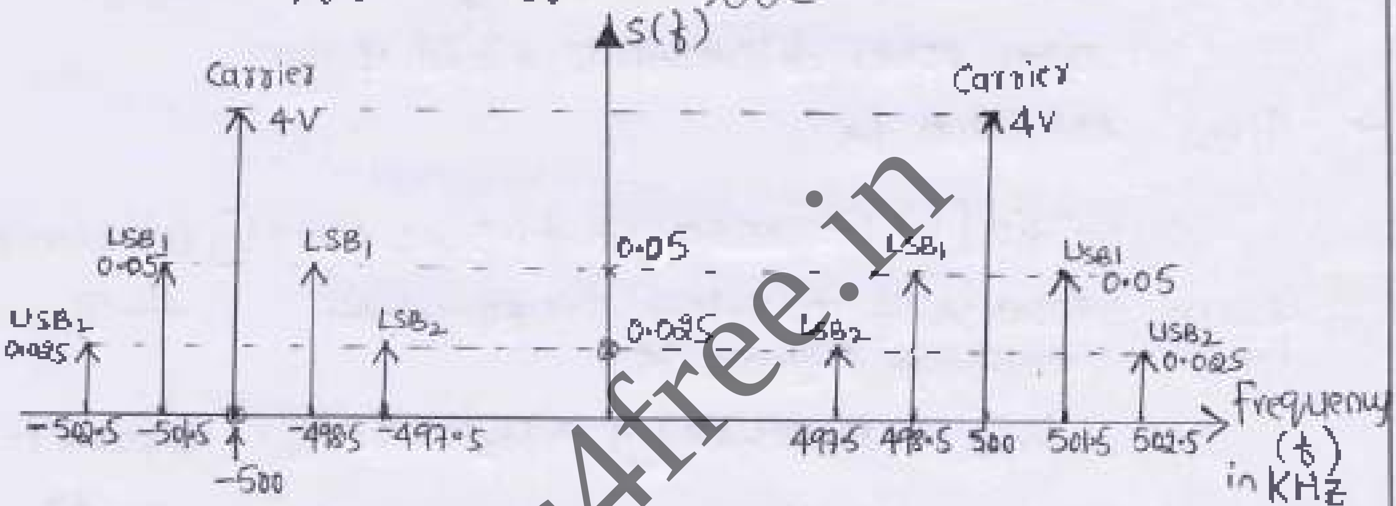
Amplitudes of upper side band and lower side band for frequency spectrum is $\frac{\mu A_c}{4}$ & that of Carrier is '$\frac{A_c}{2}$'.

i) Amplitude of Carrier frequency 'f_c' : $\frac{A_c}{2} = \frac{4}{2} = 2V.$

ii) Amplitudes of USB_1 and LSB_1 is : $\frac{\mu_1 A_c}{4} = \frac{0.05 \times 4}{4} = 0.05$

iii) Amplitudes of USB_2 & LSB_2 is : $\frac{\mu_2 A_c}{4} = \frac{0.025 \times 4}{4} = 0.025$

∴ Complete Spectrum of AM signal is



* Total power in sidebands:-

$$P_{SB} = P_{LSB} + P_{USB} = P_c \frac{\mu_t^2}{2}$$

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = 0.056$$

$$\therefore P_{SB} = P_c \times \frac{0.056^2}{2} \quad ; P_c = \frac{A_c^2}{2R} = \frac{4^2}{2R} = \frac{8}{R} \text{ Watts}$$

$$P_{SB} = \frac{8}{R} \times 0.0125$$

$$\boxed{P_{SB} = \frac{0.0125}{R}} \text{ Watts} \quad ; \text{ Where } R = \text{load resistance}$$

if $\boxed{R = 1\Omega}$

$$\boxed{P_{SB} = 0.0125 \text{ Watts}}$$

<3> An AM wave has the form,

$$s(t) = 20\left[1 + 1.5\cos 2000\pi t + 1.5\cos 4000\pi t\right]\cos 40000\pi t$$

Determine,

(i) Net Modulation Index

(ii) The carrier power and side band power

(iii) $S(f)$ and Draw its frequency spectrum.

(iv) Total power delivered to a load of 100Ω.

↳ Given AM wave is

$$s(t) = 20\left[1 + 1.5\cos 2000\pi t + 1.5\cos 4000\pi t\right]\cos 40000\pi t \quad \text{—— ①}$$

General equation of multitone AM equation for two message signals is

$$s(t) = A_c\left[1 + \mu_1\cos 2\pi f_{m_1} t + \mu_2 \cos 2\pi f_{m_2} t\right]\cos 2\pi f_c t \quad \text{—— ②}$$

∴ By comparing equations ① & ② we get—

$A_c = 20V$; $\mu_1 = 1.5 = \mu_2$; $f_{m_1} = 1kHz$; $f_{m_2} = 2kHz$; $f_c = 20kHz$ and Load resistance $R = 100\Omega$

(i) Net Modulation Index :-

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{1.5^2 + 1.5^2} = 2.12$$

(Note: $s(t)$ is over Modulated) ∵ $\mu_t > 1$

(ii) carrier power & side band power :-

↳ carrier power : $P_c = \frac{A_c^2}{2R} = \frac{20^2}{2 \times 100} = 2$ Watts.

↳ Side band power : $P_{SB} = P_c \frac{\mu_t^2}{2}$

$$P_{SB} = \frac{2 \times 2.12^2}{2} = 4.4944 \text{ Watts}.$$

where, P_{SB} = Total power in all side bands.

(iii) $S(f)$ and Frequency Spectrum :-

from given data

$$S(t) = 20\left[1 + 1.5\cos 2000\pi t + 1.5\cos 4000\pi t\right]\cos 40{,}000\pi t$$

i.e. $$S(t) = \underset{A_c}{20}\left[1 + \underset{\mu_1}{1.5}\cos(2\pi \times \underset{\substack{f_{m1}\\1\times10^3}}{1000} \times t) + \underset{\mu_2}{1.5}\cos 2\times \underset{\substack{f_{m2}\\2\times10^3}}{2000}\pi t\right]\cos 2\pi \times \underset{\substack{f_c\\20\times10^3}}{20{,}000}t$$

$$\therefore S(t) = 20\cos 2\pi\times20\times10^3 t + 30\underbrace{\cos 2\pi\times20\times10^3 t \cdot \cos 2\pi\times1\times10^3 t}$$

$$+ 30\underbrace{\cos 2\pi\times20\times10^3 t \cdot \cos 2\pi\times2\times10^3 t}.$$

W.K.T $\cos A\cdot\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$$\therefore S(t) = 20\cos 2\pi\times20\times10^3 t + \frac{30}{2}\left[\cos 2\pi\times(20-1)\times10^3 t + \cos 2\pi\times(20+1)\times10^3 t\right]$$

$$+ \frac{30}{2}\left[\cos 2\pi\times(20-2)\times10^3 t + \cos 2\pi(20+2)\times10^3 t\right]$$

$$S(t) = 20\cos 2\pi\times20\times10^3 t + 15\left[\cos 2\pi\times19\times10^3 t + \cos 2\pi\times21\times10^3 t\right]$$

$$+ 15\left[\cos 2\pi\times18\times10^3 t + \cos 2\pi\times22\times10^3 t\right]$$

Take Fourier Transformation for Equation ① ——— ①

$$S(f) = \frac{20}{2}\left[\delta(f-20K) + \delta(f+20K)\right] + \frac{15}{2}\left[\delta(f+19K) + \delta(f-19K)\right]$$

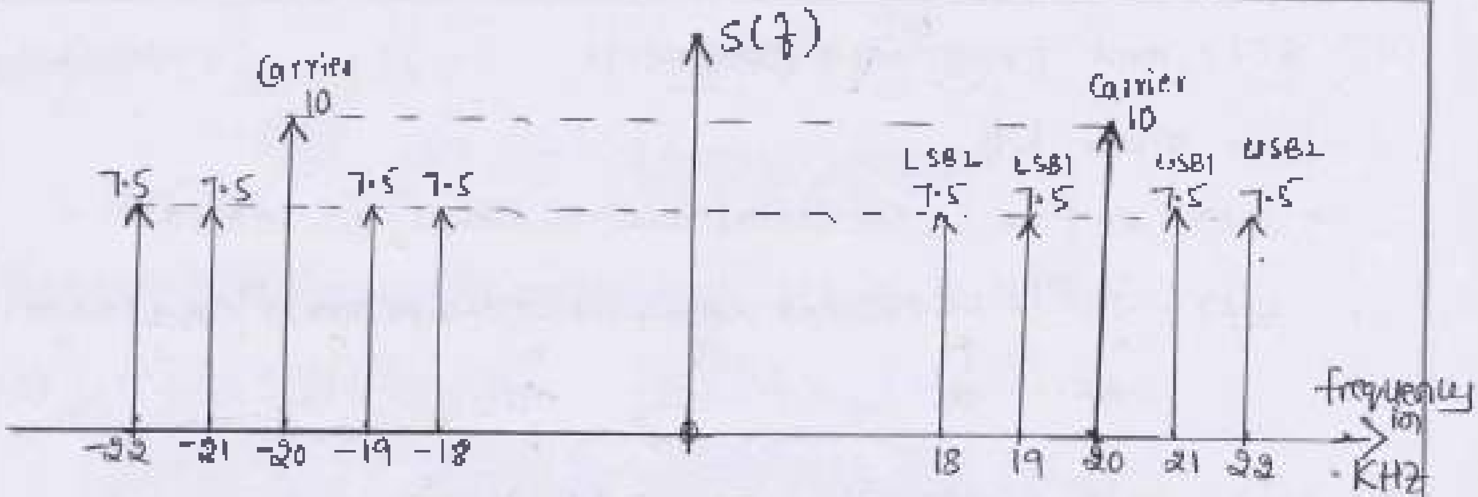
$$+ \frac{15}{2}\left[\delta(f-21K) + \delta(f+21K)\right] + \frac{15}{2}\left[\delta(f+18K) + \delta(f-18K)\right]$$

$$+ \frac{15}{2}\left[\delta(f-22K) + \delta(f+22K)\right]$$

$$\boxed{\begin{aligned}\therefore S(f) = 10\left[\delta(f-20K) + \delta(f+20K)\right] + 7.5\,[&\delta(f+19K) + \delta(f-19K)\\ &+ \delta(f-21K) + \delta(f+21K) + \delta(f-22K) + \delta(f+22K)\\ &+ \delta(f+18K) + \delta(f-18K)]\end{aligned}} \quad ②$$

Equation ② gives the equation of $S(f)$ with ——— ②

$\boxed{\begin{matrix} f_c = 20KHZ \\ \text{Amplitude } 10V \end{matrix}}$; $f_{LSB_1} = 19KHZ$, $f_{USB_1} = 21KHZ$; $f_{LSB_2} = 18KHZ$, $f_{USB_2} = 22KHZ$ $\Big\}$ Amplitude 7.5V



-: complete spectrum of s(t) :- (Plot of S(f))

(iv) Total power delivered to a load of 100Ω :-

Method 1 :

W.K.T.
$$P_t = P_c\left[1+\frac{\mu_t^2}{2}\right]$$

$P_c = 2W$; $\mu_t = 2.12$

$$\therefore P_t = 2\left[1+\frac{2.12^2}{2}\right]$$

$$\boxed{P_t = 6.4944 \text{ Watts}}$$

Method 2 :-

Since we already calculated P_c & Total side bands power P_{SB}. Total power is

$$P_t = P_c + P_{SB}$$
$$P_t = 2 + 4.4944$$
$$\boxed{P_t = 6.4944} \text{ Watts}$$

<4> An AM-Broadcasting transmitter radiates 50KW of carrier power. What will be the radiated power at 85% Modulation?

↳ Given data : carrier power, $P_c = 50KW$; $\mu = 0.85$

$\therefore$ radiated power, $P_t = P_c\left[1+\frac{\mu^2}{2}\right] = 50\times10^3\left[1+\frac{0.85^2}{2}\right]$

$$\boxed{P_t = 68.0625 \text{ KW}}$$

<5> An audio frequency signal $10 \sin 2\pi(500)t$ is used to amplitude modulate a carrier of $50 \sin 2\pi(10^5)t$. Assume modulation index $= 0.2$. Determine.

i) side band frequencies

ii) Amplitude of each side band.

iii) Bandwidth required.

iv) Efficiency of AM wave.

↳ Given: $m(t) = 10 \sin 2\pi(500)t \Rightarrow A_m = 10V$; $f_m = 500\text{ Hz}$

$f_m = 0.5\text{ KHZ}$

$c(t) = 50 \sin 2\pi(10^5)t \Rightarrow A_c = 50V$; $f_c = 10^5\text{ Hz}$

and $\mu = 0.2$ (given) $f_c = 100\text{ KHZ}$

i) Side band frequencies :-

- $f_{USB} = f_c + f_m = 100k + 0.5k = 100.5\text{ KHZ}$
- $f_{LSB} = f_c - f_m = 100k - 0.5k = 99.5\text{ KHZ}$

ii) Amplitude of each side band $= \frac{\mu A_c}{2} = \frac{0.2 \times 50}{2} = 5V$

iii) Bandwidth required : $BW = 2f_m = 2(0.5\text{ KHz})$

$\boxed{BW = 1\text{ KHZ}}$

iv) Efficiency of AM wave :-

$$\% \eta = \frac{\mu^2}{2+\mu^2} \times 100 = \frac{0.2^2}{2+0.2^2} \times 100$$

$$\boxed{\% \eta = 1.96}$$

<V.T.U> <6> An Amplitude modulated signal is given by

$s(t) = [10\cos 2\pi \times 10^6 t + 5\cos 2\pi \times 10^6 t \cdot \cos 2\pi \times 10^3 t + 2\cos 2\pi \times 10^6 t \cdot \cos 4\pi \times 10^3 t]$. Determine

i) Net Modulation index

ii) side band power

iii) Total modulated power. Assume $R = 100\Omega$

↳ Given AM signal is

$$s(t) = [10\cos 2\pi \times 10^6 t + 5\cos 2\pi \times 10^6 t \cdot \cos 2\pi \times 10^3 t + 2\cos 2\pi \times 10^6 t \cdot \cos 4\pi \times 10^3 t]$$

$$s(t) = 10\cos 2\pi \times 10^6 t \left[1 + \frac{5}{10}\cos 2\pi \times 10^3 t + \frac{2}{10}\cos 4\pi \times 10^3 t\right]$$

$$s(t) = 10[1 + 0.5\cos 2\pi \times 10^3 t + 0.2\cos 4\pi \times 10^3 t]\cos 2\pi \times 10^6 t \quad \text{—— (1)}$$

The standard AM equation for two message signals is

$$s(t) = A_c[1 + \mu_1 \cos 2\pi f_{m_1} t + \mu_2 \cos 2\pi f_{m_2} t]\cos 2\pi f_c t \quad \text{—— (2)}$$

$\therefore A_c = 10V$; $\mu_1 = 0.5$; $\mu_2 = 0.2$; $f_{m_1} = 1 \times 10^3 = 1KHZ$, $f_{m_2} = 2 \times 10^3 = 2KHZ$; $f_c = 10^6 Hz$, $f_c = 1000KHZ$

<i> Net Modulation Index : $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{0.5^2 + 0.2^2} = 0.538$

<ii> Sideband power :- $P_{SB} = P_{LSB} + P_{USB} = P_c \cdot \frac{\mu_t^2}{2}$

$$P_c = \frac{A_c^2}{2R} = \frac{10^2}{2 \times 100} = 0.5W.$$

$$\therefore P_{SB} = 0.5 \times \frac{0.538^2}{2} = 0.072\ W$$

<iii> Total modulated power :- $P_t = P_c\left[1 + \frac{\mu_t^2}{2}\right] = P_c + P_{SB} = 0.572W.$

7.) An AM signal has the form,

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$$S(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2800\pi t).$$

Determine the ratio of power in message signal to that of power in unmodulated Carrier signal.

↳ Given AM-equation is

$$S(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2800\pi t)$$

$$\therefore S(t) = \cos(2\pi \times \underset{\uparrow f_{LSB}}{1000} \times t) + 4\cos(2\pi \times \underset{\uparrow f_c}{1200} t) + \cos(2\pi \times \underset{\uparrow f_{USB}}{1400} t) \quad \text{——(1)}$$

It has 3-components, carrier signal, Lower side band (LSB) and upper side band (USB).

∴ the ~~Amplitude~~ of Carrier, $A_c = 4V$.

Amplitude of LSB & USB is $\frac{\mu A_c}{2} = 1 \Rightarrow \boxed{\mu = \frac{2}{A_c} = \frac{2}{4} = \frac{1}{2}}$

∴ Amplitude of message signal, $A_m = A_c \times \mu = 4 \times \frac{1}{2} = 2V \quad (\because \mu = \frac{A_m}{A_c})$

∴ The Ratio of power in Message signal to that of carrier power

$$= \frac{\left(\frac{A_m^2}{2R}\right)}{\left(\frac{A_c^2}{2R}\right)}$$

← power in message signal

← power in Carrier signal

$$\frac{P_m}{P_c} = \frac{A_m^2}{A_c^2} = \left(\frac{A_m}{A_c}\right)^2$$

$$\frac{P_m}{P_c} = \left(\frac{2}{4}\right)^2 = (0.5)^2$$

$$\boxed{\frac{P_m}{P_c} = 0.25}$$

*Note:- In General for Any AM signal the Ratio of power present in Message signal to that of Carrier signal is equal to "μ^2"

• For problem 10, $\boxed{\frac{P_m}{P_c} = \mu^2 = (0.5)^2 = \underline{0.25}}$

8) Consider a 2-stage SSB-Modulator as shown in figure-1. The input signal consists of a voice signal in a frequency range of 300Hz to 3.4KHz. The two oscillators frequencies have values $f_{c_1} = 100KHz$ and $f_{c_2} = 10MHz$. Determine

(i) Sidebands of DSBSC modulated waves appearing at the outputs of the product modulators.

(ii) Sidebands of SSB modulated wave appearing at two BPF's output.

(iii) passband and Guardband of two BPF's.

(iv) Sketch the spectrum at each stage of the .

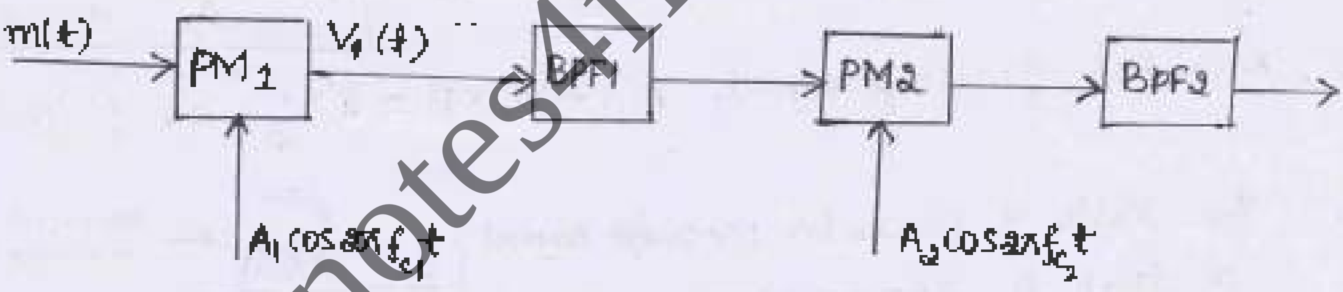


Figure1: Two Stage SSB Modulator

↳ Given :- Frequency of m(t) : $f_m = 300Hz$ to $3.4KHz$
$f_m = 0.3KHz$ to $3.4KHz$

frequency of Carrier1 : $f_{c_1} = 100KHz$
used for PM1

frequency of Carrier 2 : $f_{c_2} = 10MHz$
used for PM2

1) The PM1 output $V_1(t)$ consists of two side bands as follows : $LSB = f_{c_1} - f_m = 100KHz - (0.3KHz$ to $3.4KHz)$

$$\boxed{LSB = 99.7KHz \text{ to } 96.6KHz}$$

and, $USB = f_{c_1} + f_m$

$USB = 100KHz + (0.3KHz \text{ to } 3.4KHz)$

$$\boxed{USB = 100.3KHz \text{ to } 103.4 KHz}$$

ii> **output of BPF1**: Assume that BPF1 is designed to allow (pass) only USB. Then, BPF1 output $S_1(t)$ frequency is.

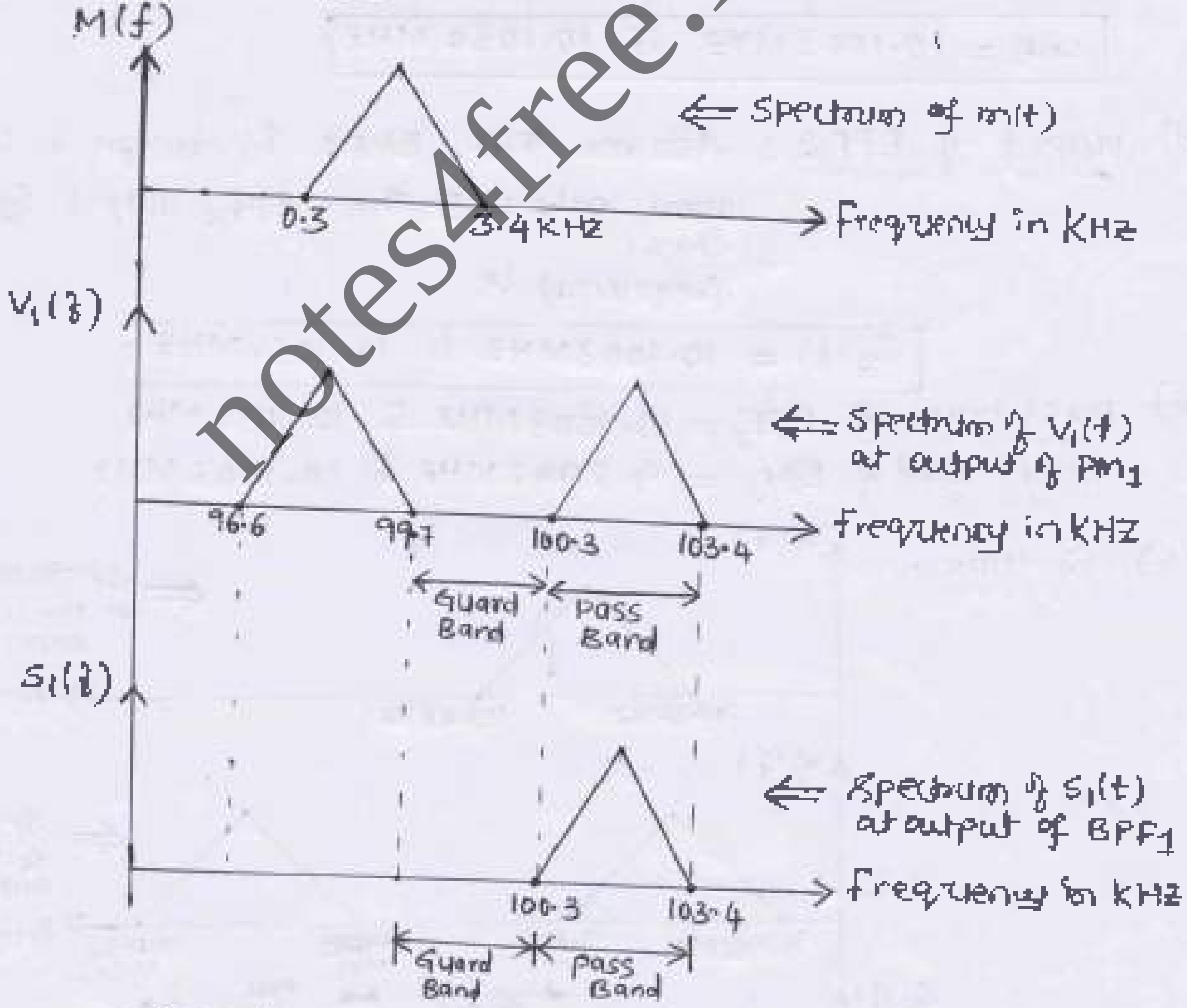
$$\boxed{S_1(t) = 100.3KHz \text{ to } 103.4KHz}$$

iii> ∴ Pass Band of BPF1 = 100.3 KHz to 103.4KHZ.

Guard Band of BPF_1 = 99.7 KHz to 100.3 KHZ

} shown in Spectrum

iv)



—: Spectrum at each stage :—

* Similarly, the PM2 output consists of two side bands as follows. (Its input is $s_1(t)$ as message signal with frequency $f_m = f_{USB} = \underbrace{100.3\text{KHz to } 103.4\text{KHz}}_{m(t)\text{ frequency range for PM2}}$ & carrier frequency $f_{c_2} = 10\text{MHz}$)

(i) $LSB = f_{c_2} - f_m$

$= 10\text{MHz} - (100.3\text{KHz to } 103.4\text{KHz})$

$\boxed{LSB = 9.8997\text{MHz to } 9.8966\text{MHz}}$ and

$USB = f_{c_2} + f_m$

$= 10\text{MHz} + (100.3\text{KHz to } 103.4\text{KHz})$

$\boxed{USB = 10.1003\text{MHz to } 10.1034\text{MHz}}$

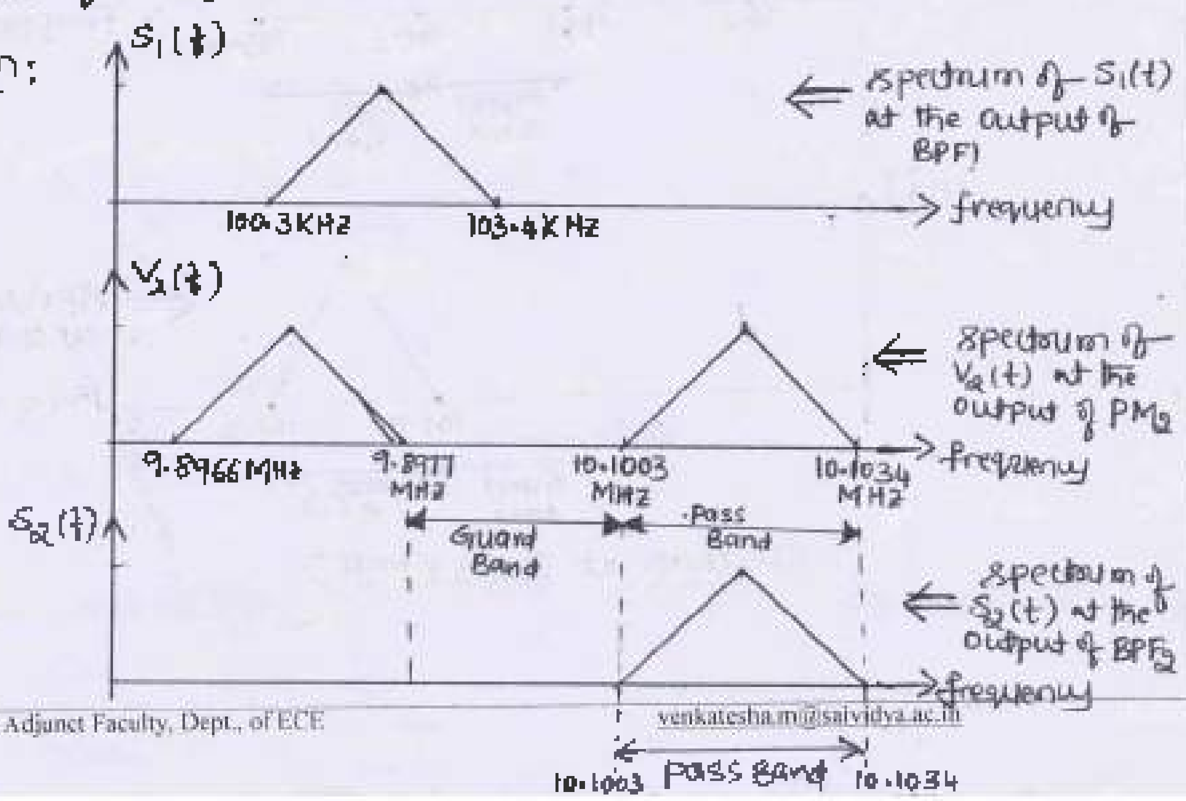
ii) output of BPF2: Assume that BPF2 is designed to allow (pass) only LSB. then BPF_2 output $S_2(t)$ frequency is

$\boxed{S_2(t) = 10.1003\text{MHz to } 10.1034\text{MHz}}$

iii) pass band of BPF_2 = 10.1003 MHz to 10.1034 MHz

Guard Band of BPF_2 = 9.8997 MHz to 10.1003 MHz.

iv) Spectrum:



9) A 250W carrier of 1000KHZ is simultaneously modulated by sinusoidal signals of 2KHZ, 6KHZ and 8KHZ with modulation indices of 35%, 55% and 75% respectively. What are the frequencies present in the modulated wave and what is the radiated power.

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5 Marks

↳ Given Data:- $P_c = 250W$; $f_c = 1000\,KHZ$

$f_{m1} = 2KHZ$; $f_{m2} = 6KHZ$; $f_{m3} = 8KHZ$

$\mu_1 = 0.35$; $\mu_2 = 0.55$; $\mu_3 = 0.75$

i) Frequencies present in the Modulated Wave:-

↳ carrier $f_c = 1000\,KHZ$

↳ $LSB_1 = f_c - f_{m1} = 1000k - 2k = 998\,KHZ$

↳ $USB_1 = f_c + f_{m1} = 1000k + 2k = 1002\,KHZ$

↳ $LSB_2 = f_c - f_{m2} = 1000k - 6k = 994\,KHZ$

↳ $USB_2 = f_c + f_{m2} = 1000k + 6k = 1006\,KHZ$

↳ $LSB_3 = f_c - f_{m3} = 1000k - 8k = 992\,KHZ$

↳ $USB_3 = f_c + f_{m3} = 1000k + 8k = 1008\,KHZ$

(ii) Radiated power:-

W.K.T $P_t = P_c\left(1 + \frac{\mu_t^2}{2}\right)$

$P_c = 250W$

$\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2} = \sqrt{0.35^2 + 0.55^2 + 0.75^2}$

$\mu_t = 0.9937$

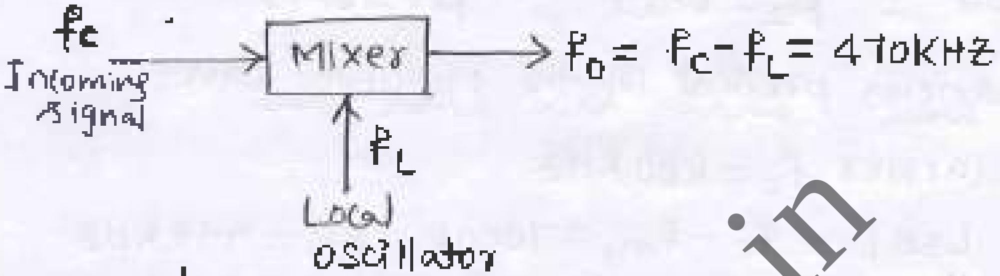
∴ Radiated power, $P_t = 250\left(1 + \frac{0.9937^2}{2}\right)$

$\boxed{P_t = 373.4}$ Watts

10) The Incoming signal has a midband frequency that may lie in the range 530KHZ to 1650KHZ. The associated bandwidth is 10KHZ. this signal is to be translated to a fixed frequency band Centered at 470KHZ. Determine the tuning range that must be provided by the local oscillator.

Given: $f_c = 530KHZ$ to $1650KHZ$; $BW = 10KHZ$

$f_0 = 470KHZ$; $f_L = ?$ $\therefore$ It is down frequency Translator (or) Mixer.



From given data Translator (mixer) output $f_0 = 470KHZ = f_c - f_L$

$\therefore$ Local oscillator frequency, f_L is given by;

$$f_L = f_c - f_0$$

When, $f_c = 530KHZ$; $f_L = 530K - 470K = 60KHZ$

$f_c = 1650KHZ$; $f_L = 1650K - 470K = 1180KHZ$

$\therefore$ Tuning range of Local oscillator frequency f_L is 60KHZ to 1180KHZ

11) Determine the Bandwidth of FDM-system which uses SSB modulation at the transmitter for 24 voice signals having a Bandwidth of 4KHZ each.

↳ Given $N = 24$, Voice signals with SSB Modulation.

$W = f_m = 4KHZ$

$\therefore$ Total Band Width of FDM system is

$$BW = N \times f_m = 24 \times 4KHZ$$

$$\boxed{BW = 96KHZ}$$

* * * * * * * *

MODULE 2

ANGLE MODULATION

Introduction:-

Angle Modulation, is a process of altering either frequency (or) phase of carrier signal in accordance with the instantaneous values of message signal, by keeping amplitude of carrier constant.

↳ General equation of Angle Modulated wave is given by

$$S(t) = A_c \cos \theta_i(t) \quad \text{--- (1)}$$

where, A_c = Amplitude of carrier signal

$\theta_i(t)$ = Angle of the modulated signal.

Angle Modulation techniques are further divided into two types

- → Frequency Modulation (FM)
- → Phase Modulation (PM)

• **Frequency Modulation:-** It is a process of altering frequency of carrier signal in accordance with the instantaneous values of message signal by keeping amplitude, phase of carrier constant.

↳ The General equation of FM-signal is given by

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)\,dt\right] \quad \text{--- (2)}$$

where, K_f = Frequency sensitivity parameter in Hz/volt

$m(t)$ = Message signal

• **Phase Modulation:**

It is a process of altering phase of carrier signal in accordance with the instantaneous values of message signal.

↳ The General equation of PM signal is

$$S(t) = A_c \cos\left[2\pi f_c t + K_p m(t)\right] \quad \text{--- (3)}$$

where K_p = Phase sensitivity parameter.

1.1 Basic Definitions :-

The most commonly used angle modulation technique is "Frequency Modulation".

Some of the basic definitions with respect to frequency Modulation are as follows.

(a) Instantaneous frequency $[f_i(t)]$:-

The Instantaneous frequency of FM-signal is mathematically defined as, $\boxed{f_i(t) = f_c + K_f m(t)}$ as shown in figure 1.

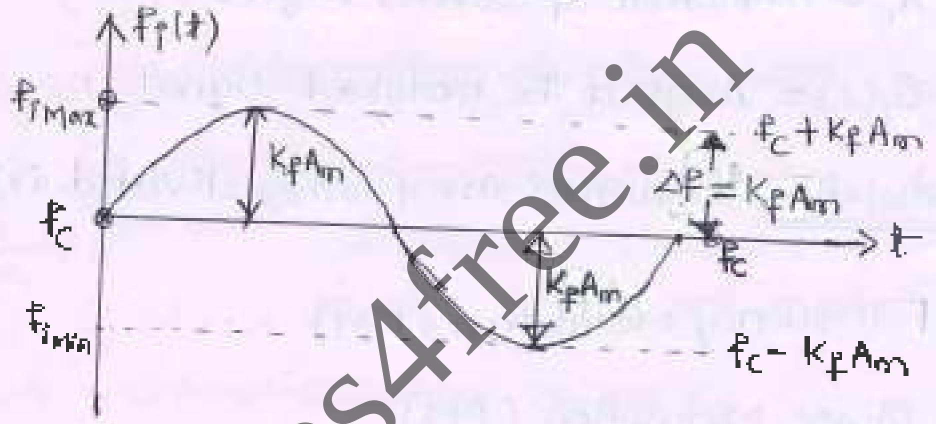


Figure 1: Instantaneous frequency $f_i(t)$ of FM signal for sinusoidal message signal $m(t)$

From figure 1, following observations can be made

↳ when $m(t) = 0$: $f_i(t) = f_c$ → Same as that of unmodulated carrier

↳ At peak value of $m(t)$ i.e., $|m(t)| = A_m$ } $\Rightarrow f_i(t)_{Max} = f_c + K_f A_m$

∴ Maximum frequency of FM signal is $\boxed{f_i(t)_{Max} = f_c + K_f A_m}$

(b) Angle of FM signal :- $[\theta_i(t)]$

Instantaneous value, $\theta_i(t)$ of FM signal is related to its instaneous frequency $f_i(t)$ as follows

$$\boxed{f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t)}$$

Since $W_i(t) = \frac{d}{dt}\theta_i(t)$ ⇓ $2\pi f_i(t) = \frac{d}{dt}\theta_i(t)$

(c) Maximum Frequency deviation $[\Delta f_{max}]$:-

It is the difference between maximum frequency of FM signal to that of unmodulated carrier frequency.

It is denoted by Δf_{max}.

i.e, $\boxed{\Delta f_{max} = k_f A_m}$

Proof : From the definition of frequency deviation,

$$\Delta f_{max} = \text{Max. frequency of FM-signal} - \text{frequency of carrier signal}$$

$$\Delta f_{max} = f_i(t)_{max} - f_c$$

from figure 1, $f_i(t)_{max} = f_c + k_f A_m$

$$\therefore \Delta f_{max} = f_c + k_f A_m - f_c$$

$\boxed{\Delta f_{max} = k_f A_m \text{ Hz}}$ ⇐ Indicated in figure 1.

(d) Carrier Swing :-

It is the difference between maximum and minimum frequencies of FM signal as shown in figure 2.

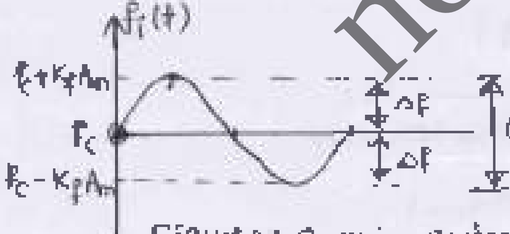


Figure 2: Carrier Swing

∴ Carrier swing or swing in carrier frequency is,

$$= f_c + k_f A_m - (f_c - k_f A_m)$$
$$= f_c + k_f A_m - f_c + k_f A_m$$
$$= 2 k_f A_m$$
$$= 2\Delta f_{max}$$

$$\therefore \boxed{\text{The Carrier Swing} = f_i(t)_{max} - f_i(t)_{min} = 2 \times \Delta f_{max}}$$

(e) Modulation Index : (β)

It is the ratio of maximum frequency deviation to that of frequency of message signal. It is denoted by symbol 'β'.

i.e, Modulation Index in FM $\Rightarrow \boxed{\beta = \dfrac{\Delta f_{max}}{f_m}}$ No units

1.2. Frequency Modulation:-

Q) Define Frequency Modulation. Derive the time domain expression for Frequency modulated wave & also sketch necessary waveforms.

→ Frequency Modulation is a process of altering the frequency of carrier signal in accordance with the instantaneous values of message signal by keeping amplitude & phase of carrier constant.

Time domain expression:-

- Let the instantaneous value of carrier signal is

$$c(t) = A_c \cos 2\pi f_c t \longrightarrow (1)$$

- Let the instantaneous value of message signal is

$$m(t) = A_m \cos 2\pi f_c t \longrightarrow (2)$$

- We know that the standard equation of Angle modulated wave is given by, $S(t) = A_c \cos \theta_i(t) \longrightarrow (3)$

where $\theta_i(t)$ = Angle of FM wave (modulated wave)

- We know that the instantaneous frequency $f_i(t)$ of FM signal is given by $f_i(t) = f_c + K_f m(t)$ —— (4)

where, K_f = frequency sensitivity

$m(t)$ = message signal

- We know that the Angular frequency,

$$\omega_i(t) = \frac{d}{dt}\theta_i(t)$$

$$\Downarrow$$

$$2\pi f_i(t) = \frac{d}{dt}\theta_i(t)$$

$$\therefore f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t) \quad \text{—— (5)}$$

Substitute $f_i(t) = f_c + K_f m(t)$ in equation (5) we get.

$$\therefore \quad f_c + k_f m(t) = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t)$$

$$\therefore \quad \frac{d\,\theta_i(t)}{dt} = 2\pi f_c + 2\pi k_f m(t) \quad \text{--- (6)}$$

→ Apply Integral on both sides of equation (6) we get

$$\int \frac{d}{dt}\theta_i(t) = \int [2\pi f_c + 2\pi k_f m(t)]\,dt$$

$$\Downarrow$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int m(t)\,dt \quad \text{--- (7)}$$

∴ The General equation of FM signal is

$$S(t) = A_c \cos\theta_i(t) \quad \text{using equation (7)}$$

$$\boxed{S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right]} \quad \text{--- (8)}$$

Equation (8) is the general equation of FM signal for any message signal m(t).

for, $m(t) = A_m \cos 2\pi f_m t$

$$\int m(t)\,dt = \int A_m \cos 2\pi f_m t\,dt \qquad \left(\because \int \cos mx\,dx = \frac{\sin mx}{m}\right)$$

$$= \frac{A_m}{2\pi f_m} \cdot \sin 2\pi f_m(t) \quad \text{--- (9)}$$

∴

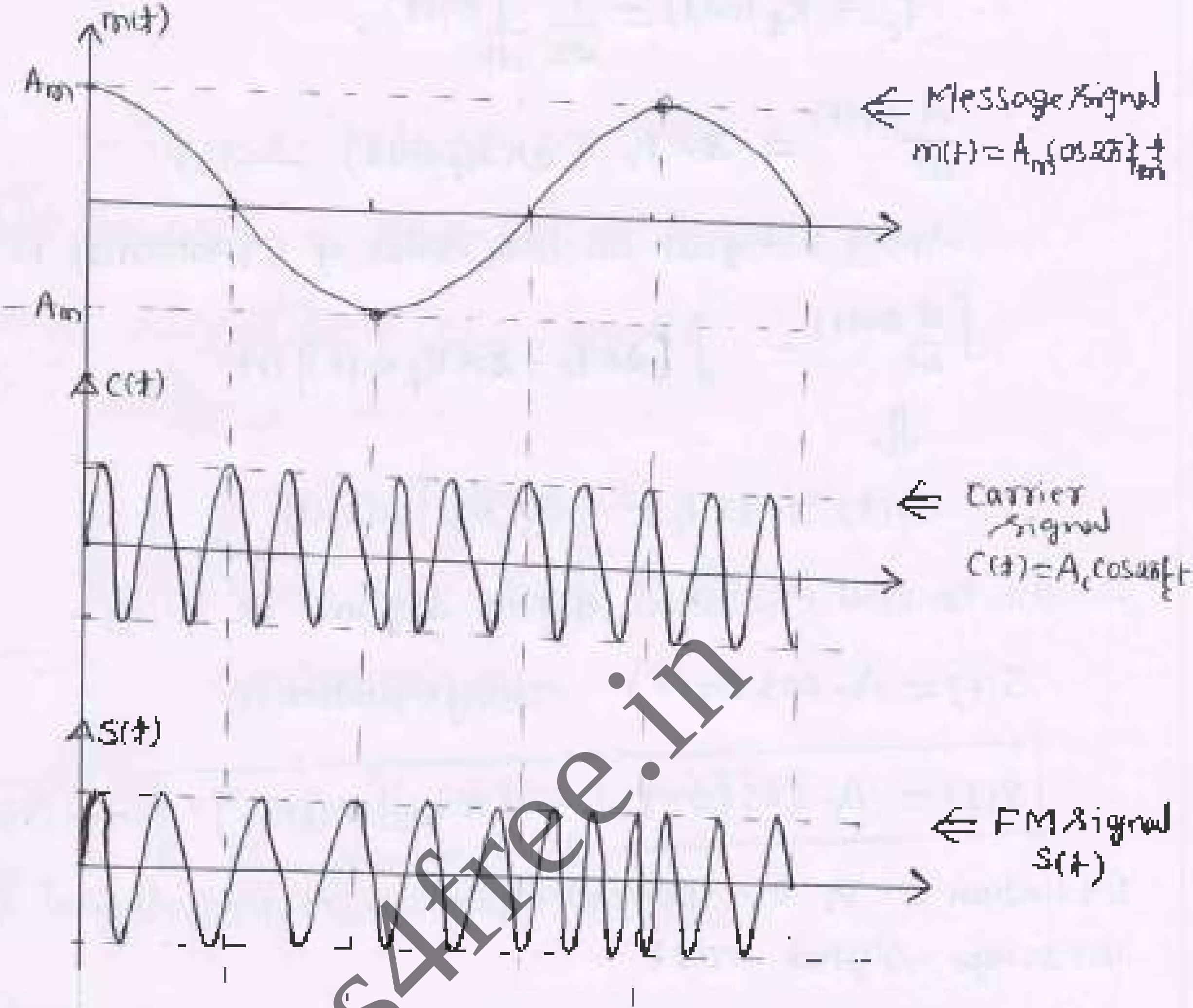
$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \times \frac{A_m}{2\pi f_m} \cdot \sin(2\pi f_m t)\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

$$\boxed{S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]} \quad \text{--- (10)}$$

Equation (10) is the standard equation of FM signal for $m(t) = A_m \cos 2\pi f_m t$ ∴ where $\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f_{max}}{f_m}$ ← Modulation Index of FM Signal.

The required waveforms of m(t), c(t) & s(t) are shown in figure (2).



Figure(2): (a) Message signal m(t) (b) Carrier signal c(t)
(c) Frequency Modulated-(FM) signal.

Figure 2(c) shows the time domain representation of FM-signal S(t) shown in equation (10). The frequency of S(t) linearly varies with respect to message signal m(t). i.e. $[f_i(t) = f_c + k_f m(t)]$.

1.2.1 phase Modulation :-

- It is a process of altering phase of carrier signal in accordance with the instantaneous values of message signal m(t) by keeping amplitude & frequency of carrier constant.

Time Domain Expression of PM-signal is given by

$$S(t) = A_c \cos \theta_i(t) \quad ; \quad \theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$\therefore \boxed{S(t) = A_c \cos[2\pi f_c t + k_p m(t)]}$$ ⇐ phase Modulated signal (PM-signal)

where k_p = phase sensitivity parameter.

1.2.2 : Implementation of phase Modulator using Frequency Modulator and Frequency Modulator using phase Modulator

We know that the standard equation of FM signal for any message signal m(t) is given by

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right] \quad \text{— (1)}$$

Similarly the standard equation of PM-signal for any message signal m(t) is given by,

$$S(t) = A_c \cos\left[2\pi f_c t + k_p\, m(t)\right] \quad \text{— (2)}$$

By comparing equations (1) & (2), it is clear that there is a definite possibility of implementing FM-signal using phase Modulator and vice-versa.

Case (i) : Implementation of FM using PM :-

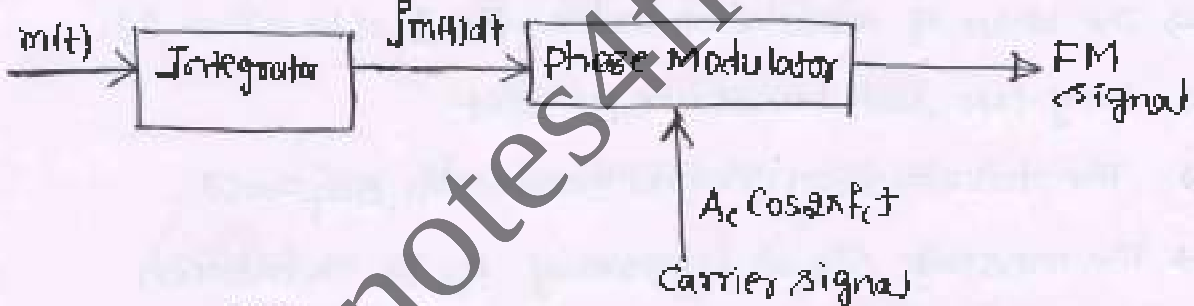


Figure 1: Generation of FM-signal using phase Modulator

↳ By changing phase Modulator input signal to $\int m(t)\,dt$ we can generate FM signal. as shown in figure 1.

Case (ii) : Implementation of PM using FM :-

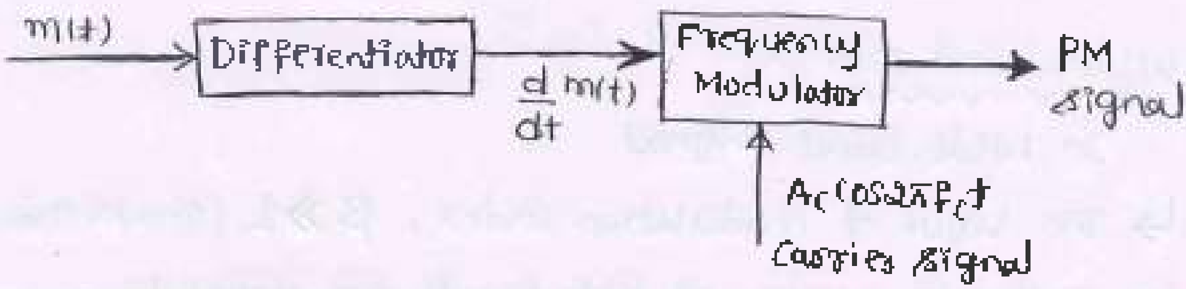


Figure 2 : Generation of PM-signal using FM :-

↳ By changing Frequency modulator input signal to $\frac{d}{dt} m(t)$, we can generate PM-signal. as shown in figure (2).

∴ From Figure(1) & Figure(2) it is clear that both PM & FM signals are correlated to each other.

∴ In this module, only Frequency modulation is discussed in detail. If required PM-Signal is generated by using Indirect method as shown in figure (2).

1.3. Classification of Frequency modulated Signals:-

Depending on the value of modulation index 'β' and channel Bandwidth FM-signals are classified into two two types

i) Narrow Band FM

ii) Wide Band FM

i) Narrow Band FM:

In Narrow band FM signal

- The value of modulation index, $\beta < 1$ (Less than 1)
- only two side bands are present
- The transmission channel Bandwidth, $BW_T = 2W$
- The message signal frequency, f_m is in between 30HZ to 3KHZ.
- Maximum frequency deviation is 15KHZ

Application:- Narrow band FM-technique is mainly used for Speech signal transmission

Example: Mobile communication

ii) Wide-band FM:-

In Wide-band signal

- The value of modulation index, $\beta \gg 1$. (Greater than 1)
- Infinite number of side bands are present.
- ** The message signal frequency, f_m is in between 30Hz to 15KHz.

*** The Bandwidth of Wide band FM signal can be calculated from Carson's Rule shown in equation (1)

$$\boxed{BW_T = 2f_m + 2\Delta f_{max}} \Leftarrow$$ CARSON's Rule to find Bandwidth of Wide band FM-Signal.

↳ The Maximum frequency deviation is 75KHZ.

Applications of Wide band FM:-

* Wide-band FM technique is mainly used in High Quality Music signal transmission.

Example: FM-channels

** Comparison between Narrow band FM and Wide band FM:

Parameter	Narrow Band FM	Wide Band FM
1. Modulation Index β	Less than 1	Greater than 1
2. Band width B_T	$2f_m$	$2f_m + 2\Delta f_{max}$
3. Number of Side bands	Two	Infinity
4. frequency of Message s/l f_m	30HZ to 3KHZ	30HZ to 15KHZ
5. Maximum Frequency Deviation	15KHZ	75KHZ
6 Application	Speech signal Txn Ex: Mobile Communication	Music signal Txn Ex: FM-Stations (or) channels.

3.4 Narrow band Frequency Modulation (In detail)

↳ Narrow band FM signals are characterized by modulation index, β less than 1.

↳ Narrow band FM signal equation can be derived from general FM equation for $m(t) = A_m \cos 2\pi f_m(t)$, as follows

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad \text{—— (1)}$$

equation (1) is general FM equation for $m(t) = A_m \cos(2\pi f_m t)$ obtained in section 1.2.

W.K.T $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$$(1) \overset{\therefore}{\Longrightarrow} \quad S(t) = A_c \cos 2\pi f_c t \cos(\beta \sin 2\pi f_m t) - A_c \sin(2\pi f_c t) \times \sin(\beta \sin 2\pi f_m t) \quad \text{—— (2)}$$

For Narrow band FM-signals, $\beta < 1$

∴ The value of $\beta \sin 2\pi f_m t$ becomes less than 1-degree and it approaches almost $0°$. Therefore

$$\cos(\beta \sin 2\pi f_m t) \simeq 1 \quad \text{—— (3)} \qquad \left(\because \lim_{\theta \to 0°} \cos\theta \simeq 1\right)$$

$$\sin(\beta \sin 2\pi f_m t) \simeq \beta \sin 2\pi f_m t \quad \text{—— (4)} \qquad \left(\because \lim_{\theta \to 0°} \sin\theta \simeq \theta\right)$$

By substituting equations (3) & (4) in equation (2) we get Narrow band FM signal

$$\boxed{S(t) \simeq A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \cdot \sin 2\pi f_m t} \quad \text{—— (5)}$$

∴ Narrow band FM signal consists of 3-frequency components

↳ $f_c \Rightarrow$ Carrier signal
$f_c - f_m \Rightarrow$ Lower side band
$f_c + f_m \Rightarrow$ Upper side band

} Same as that of Standard AM signal.

∴ Total transmission Bandwidth of Narrow band FM $\Rightarrow \boxed{BW_T = 2f_m}$

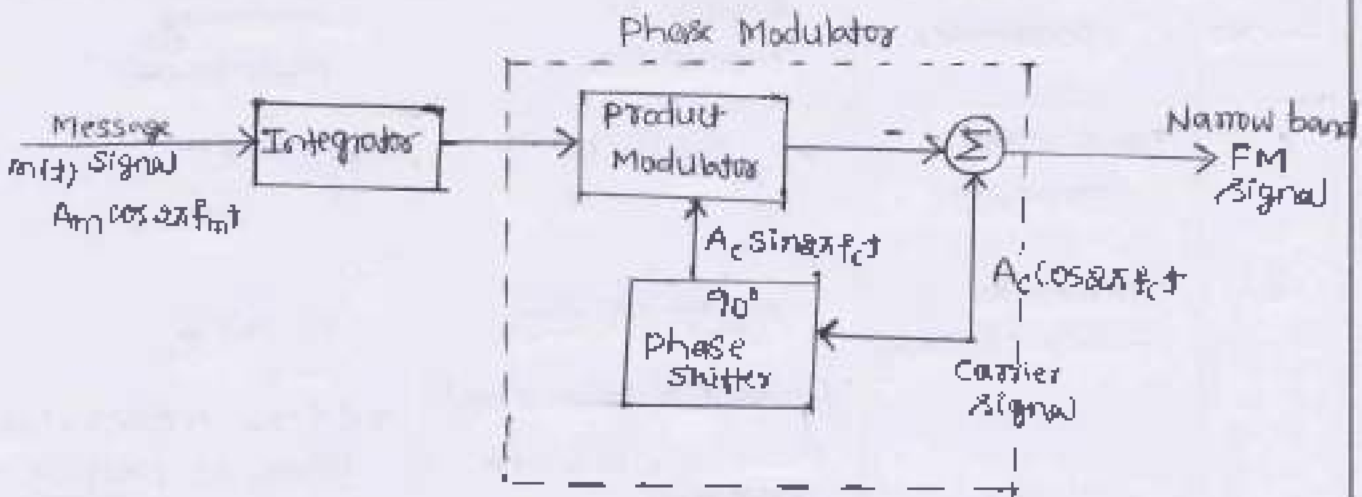


Figure 1: Indirect Method of Generating FM signal (Narrow band) Using phase modulator :-

Figure 1, shows the indirect method of generating Narrow band FM signal shown in figure ⑤, using phase modulator.

Note: The Band width required to transmit Narrow band FM signal is same as that of AM signal - transmission channel bandwidth "$2f_m$".

* Comparison between AM and FM :-

Sl. No.	Parameter	Amplitude Modulation	Frequency Modulation
1.	Altering parameter of Carrier	Amplitude	Frequency.
2.	Constant parameter of Carrier	Frequency and Phase	Amplitude, phase
3.	Modulation Index	$\mu = K_a A_m < 1$	$\beta = \frac{\Delta f_{max}}{f_m}$ $\beta < 1$ (Narrow band) $\beta > 1$ (Wide band)

Sl. No	parameter	Amplitude Modulation	Frequency Modulation
4.	Transmitted power	$P_t = P_c\left[1+\frac{\mu_t^2}{2}\right]$	$P_t = \frac{A_c^2}{2R}$
5.	Maximum power efficiency	$\eta_{max} = 33.33\%$ (one side power and Carrier power are wasted)	$\eta_{max} = 100\%$ i.e. All the transmitted power is useful power
6.	Bandwidth	• $BW = 2f_m$ • Band width is Independent of modulation Index	$BW = 2f_m$ (Narrowband) or $BW = 2f_m + 2\Delta f_{max}$ (wide band) • Bandwidth depends on Modulation Index (wide band FM)
7.	Range of Communication	Covers Large Area EX: Radio	Covers limited Area EX: FM-channels
8	Complexity	Less Complex	More Complex
9.	Cost	Inexpensive	Expensive
10.	Noise Immunity	Affected by Noise	Immune to Noise.
11.	Types	• DSBSC • SSBSC	• Narrow band • Wide band
12.	Applications	Long distance Communication EX: Radi	Short distance Communication EX: FM-stations.

MODULE-2

—: Angle Modulation :—

Numerical problems (VTU Q.P's & Additional problems)

List of Formulae:

1. General Equation of Angle Modulated wave:

$$S(t) = A_c \cos[\theta_i(t)] \quad \therefore \boxed{P_t = \frac{A_c^2}{2R}}$$ ⇐ Total power of FM signal (or) PM signal

2. General equation of FM wave.

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_f \int m(t)\,dt\right]$$

for $m(t) = A_m \cos 2\pi f_m t$

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

3. Instantaneous frequency of FM signal:-

$$f_i(t) = \frac{1}{2\pi}\frac{d[\theta_i(t)]}{dt} \quad ; \quad \theta_i(t) = \text{Angle of FM signal.}$$

4. Maximum Frequency Deviation:-

$$\Delta f_{max} = K_f A_m = \beta \times f_m \quad ; \quad K_f = \text{Frequency Sensitivity parameter.}$$

5. Modulation Index [β]:-

$$\beta = \frac{\Delta f_{max}}{f_m}$$

; $\beta < 1 \Rightarrow$ Narrowband FM signal

$\beta > 1 \Rightarrow$ Wideband FM signal.

6. Carrier frequency swing:-

$$\text{Carrier swing} = 2\Delta f_{max}.$$

7. Band width:-
   - $BW = 2f_m + 2\Delta f_{max} \Rightarrow$ Wide band FM signal CARSON'S Rule.
   - $BW = 2f_m \Rightarrow$ Narrow band FM signal.

1. The equation for a FM wave is,

$$S(t) = 10 \sin[5.7\times10^8 t + 5 \sin 12\times10^3 t]$$. calculate:

(i) carrier frequency — June/July - 2017

(ii) Modulating frequency

(iii) Modulation Index

(iv) Frequency deviation

(v) power dissipated in 100Ω.

Given data :-

Equation of FM signal,

$$S(t) = 10 \sin[5.7\times10^8 t + 5 \sin 12\times10^3 t] \quad \text{——(1)}$$

W.K.T. Standard equation of FM signal,

$$S(t) = A_c \sin[2\pi f_c t + \beta \cdot \sin 2\pi f_m t] \quad \text{——(2)}$$

By comparing (1) and (2) we get

$2\pi f_c = 5.7\times10^8$; $\beta = 5$; $2\pi f_m = 12\times10^3$; $A_c = 10V$.

$\therefore$

(i) Carrier frequency : $f_c = \dfrac{5.7\times10^8}{2\pi} = 90.718\times10^6$ HZ.

(ii) Modulating frequency : $f_m = \dfrac{12\times10^3}{2\pi} = 1.91\times10^3$ HZ.

(iii) Modulation Index : $\beta = 5$ $\therefore$ S(t) is Wide band FM s/l since $\beta > 1$.

(iv) Frequency deviation : $[\Delta f_{max}]$

W.K.T. $\beta = \dfrac{\Delta f_{max}}{f_m}$

$\therefore \Delta f_{max} = \beta \times f_m = 5 \times 1.91\times10^3 = 9.55\times10^3$ HZ

(v) power dissipated in 100Ω :

$$P_t = \frac{A_c^2}{2R} = \frac{10^2}{2\times100} = \frac{100}{2\times100} = 0.5\ W$$

2. When a 50.4 MHZ Carrier is frequency modulated by a Sinusoidal AF modulating signal, the highest frequency reached is 50.405 MHZ. Calculate

Dec 2016 | Jan 2017

(i) The Frequency deviation produced

(ii) Carrier Swing of the wave.

(iii) Lowest frequency reached

↳ Given data: $f_c = 50.4$ MHZ.

highest frequency of FM: $f_i(t)_{max} = 50.405$ MHZ

(i) The Frequency deviation produced:

W.K.T $f_i(t)_{Max} = f_c + \Delta f_{max}$

$\therefore \Delta f_{max} = f_i(t)_{Max} - f_c = 50.405 \times 10^6 - 50.4 \times 10^6$

$\Delta f_{max} = 0.005 \times 10^6 = 5000 = 5$ KHZ

(ii) Carrier Swing: W.K.T Carrier Swing $= 2 \times \Delta f_{max}$

$= 2 \times 5$ KHZ

$= 10$ KHZ

(iii) Lowest frequency reached:

$f_i(t)_{min} = f_c - \Delta f_{max}$

$= 50.4 \times 10^6 - 5 \times 10^3$

$f_i(t)_{Min} = \underline{\underline{50.395 \text{ MHZ}}}$

Note:

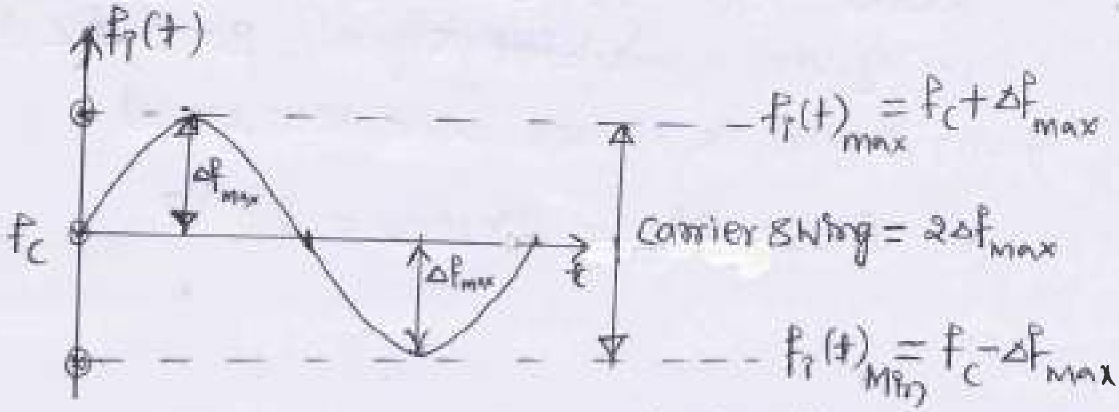


Figure: $f_i(t)$ of FM wave showing its Maximum, Minimum frequency & Carrier Swing

3) An angle modulated signal is defined by,

$s(t) = 10\cos[2\pi \times 10^6 t + 0.2 \sin 200\pi t]$ volts find the following:

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i) power in modulated signal

ii) the Frequency deviation, Δf

iii) The Approximate transmission bandwidth

↳ Given data: The Angle modulated signal (FM signal)

$$S(t) = 10\cos[2\pi \times 10^6 t + 0.2 \sin 2000\pi t] \quad \text{---} \; 1$$

W.K.T the standard equation of FM signal is

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \quad \text{---} \; 2$$

by Comparing (1) & (2) we get

$A_c = 10V$; $2\pi f_c = 2\pi \times 10^6$; $\beta = 0.2$; $2\pi f_m = 2000\pi$

$\Rightarrow f_c = 1 \times 10^6$ Hz $\qquad \Rightarrow f_m = 1000$ Hz

i) Power in $S(t)$: $P_t = \frac{A_c^2}{2R}$; Take $R = 1\Omega$ ($\because$ Not given default value is $R = 1\Omega$)

$$P_t = \frac{10^2}{2 \times 1}$$

$$P_t = 50\,W$$

ii) Frequency deviation (Δf_{max}):-

$$\Delta f_{max} = \beta \times f_m = 0.2 \times 1000 = 200\,Hz.$$

iii) Transmission Bandwidth:-

for given modulated signal, $\beta = 0.2 < 1$ $\therefore$ the given signal is Narrow band FM signal.

$$\therefore BW_T = 2f_m = 2 \times 1000 = 2\,KHz$$

4. An FM wave is defined by $s(t) = 10\cos[2 + \sin 6\pi t]$. Find the instantaneous frequency of $s(t)$

Dec-2014/ Jan-2015

↳ Given FM signal is

$$s(t) = 10\cos[2 + \sin 6\pi t] \quad \text{--- (1)}$$

The instantaneous frequency of $s(t)$ is given by

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}[\theta_i(t)]$$

for given signal Angle, $\theta_i(t) = 2 + \sin 6\pi t$

$$\therefore f_i(t) = \frac{1}{2\pi} \cdot \frac{d}{dt}[2 + \sin 6\pi t]$$

$$= \frac{1}{2\pi}[0 + 6\pi\cos 6\pi t] \quad \left(\because \frac{d}{dx}\sin mx = m\cdot\cos mx\right)$$

$$= \frac{6\pi}{2\pi}\cos 6\pi t$$

$$\boxed{f_i(t) = 3\cos 6\pi t}$$

5. A sinusoidal modulating wave of amplitude 5V and frequency 1KHz is applied to a frequency modulator. The frequency sensitivity of the modulator is 50Hz/volt. The carrier frequency is 100KHz. Calculate
(i) The frequency deviation (ii) Modulation Index.

↳ Given data: $A_m = 5V$; $f_m = 1KHz$; $k_f = 50\ Hz/volt$.
$f_c = 100KHz$.

(i) Frequency deviation: $\Delta f_{max} = k_f \times A_m = 50 \times 5 = 250Hz$.

(ii) Modulation Index: $\beta = \frac{\Delta f_{max}}{f_m} = \frac{250}{1\times10^3} = 0.25$

6) In an FM system, when the audio frequency is 500 HZ and modulating Voltage 2.5V, the deviation produced is 5 KHZ. If the modulating Voltage is increased to 7.5V, calculate the new Value of frequency deviation. Calculate the modulation index in Each Case.

↳ Given data: $f_m = 500\,HZ$; $A_m = 2.5\,V$; $\Delta f = 5\,KHZ$

$$\therefore \Delta f = k_f A_m$$

$$\Longrightarrow k_f = \frac{\Delta f}{A_m} = \frac{5\times10^3}{2.5} = 2\times10^3 = 2\,KHZ$$

$$\Longrightarrow \beta = \frac{\Delta f}{f_m} = \frac{5\times10^3}{500} = 10$$

Case(ii): If $A_m = 7.5\,V$; $\Delta f = ?$; $\beta = ?$

$$\Delta f = k_f A_m = 2\times10^3 \times 7.5 = 15\times10^3 = 15\,KHZ.$$

$$\beta = \frac{\Delta f}{f_m} = \frac{15\times10^3}{500} = 30$$

7) The carrier swing of a frequency modulated signal is 70 KHZ and the modulating signal is 7 KHZ Sine wave. Determine the modulation index of FM signal and Bandwidth.

↳ Given data: Carrier swing $= 70\,KHZ$; $f_m = 7\,KHZ$; $\beta = ?$

W.K.T the Modulation Index,

$$\beta = \frac{\Delta f_{max}}{f_m}$$

Carrier Swing $\Longrightarrow 2\Delta f_{max} = 70\,KHZ$

$$\therefore \Delta f_{max} = \frac{70\,KHZ}{2} = 35\,KHZ$$

$$\therefore \boxed{\beta = \frac{35\,KHZ}{7\,KHZ} = 5} > 1$$ ∴ It is wide Band FM signal

∴ Using Carson's rule, $BW_T = 2f_m + 2\Delta f_{max} = 2\times 7\,KHZ + 2\times 35\,KHZ$

$$= 14\,KHZ + 70\,KHZ$$

$$= 84\,KHZ$$

* Transmission Bandwidth of FM-Signals:- [Carson's Rule]

We know that there are two-types of FM-signals,

⟨i⟩ Narrow-Band FM-signal ($\beta < 1$)

⟨ii⟩ Wide-band FM-signal ($\beta \gg 1$)

The Approximate formula for finding transmission bandwidth $\langle B_T \rangle$ of FM-signals is given by "Carson's Rule"

According to Carson's formula (or rule) the B_T of FM-signal is given by

⟨i⟩ $$B_T = 2f_m + 2\Delta f = 2f_m\left(1 + \frac{\Delta f}{f_m}\right) = 2f_m(1+D)$$

where D = deviation ratio $= \frac{\Delta f}{f_m}$

i.e., $B_T = 2f_m + 2\Delta f = 2f_m(1+D)$ for WBFM-signals ($\beta \gg 1$)

⟨ii⟩ if $\beta < 1$ for NBFM,

$$\boxed{B_T = 2f_m}$$

i.e., According to Carson's rule, BW of FM-signal is

$$BW_T = \begin{cases} 2f_m + 2\Delta f = 2f_m(1+D) \ ; & \beta \gg 1 \text{ (WBFM signals)} \\ 2f_m \ ; & \beta < 1 \text{ (NBFM signals)} \end{cases}$$

Note: The Deviation ratio is same as that of modulation index of FM

i.e., $\beta = \frac{\Delta f}{f_m} = D \rightarrow$ Deviation ratio

36. Generation of FM-Waves :-

There are two basic methods of generating FM-waves.

<i> Direct Method.

<ii> Indirect Method <Armstrong Modulator>

*** Imp**

<i> Generation of frequency modulated signal using DIRECT-METHOD :-

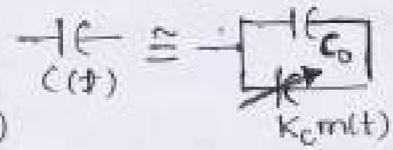
<Q> Explain generation of frequency modulated signal using direct method. V.TU June/July-2017 (5M)

↳ The Direct method uses a sinusoidal oscillator, with one of the reactive elements (example: capacitive element) in the tank circuit of the oscillator being directly controlled by the message signal, m(t).

↳ In direct method of FM-signal generation, the instantaneous frequency of the carrier wave is varied directly in accordance with the message signal.

↳ Fig.1, shows a Hartly oscillator in which the capacitive component of the tank circuit is,

$$C(t) = C_0 + K_c m(t) \quad \text{——} \quad (*)$$



where, C_0 = Total capacitance in the absence of modulation.

K_c = Variable capacitor sensitivity to voltage change.

$m(t)$ = message signal = $A_m \cos(2\pi f_m t)$.

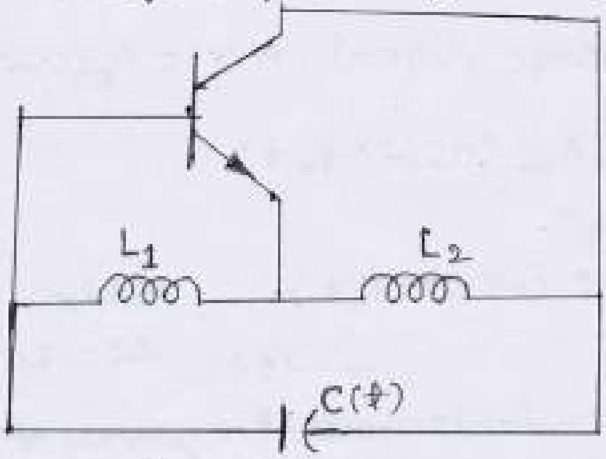


Fig.1: Hartley oscillator

The frequency of the Hartley oscillator is given by

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,c(t)}} \quad : \text{where } c(t) = C_0 + K_c m(t)$$

$$\therefore \quad f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)[C_0 + K_c m(t)]}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C_0\left[1+\frac{K_c}{C_0}m(t)\right]}}$$

$$f_i(t) = \frac{f_0}{\sqrt{1+\frac{K_c}{C_0}m(t)}} \quad : \text{where } f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}}$$

$$f_i(t) = f_0\left(1+\frac{K_c}{C_0}\cdot m(t)\right)^{-1/2} \quad \text{--- (1)}$$

Using Binomial theorem, $(1+x)^{-1/2} = \left(1-\frac{x}{2}\right)$

$$\therefore \quad \left(1+\frac{K_c}{C_0}m(t)\right)^{-1/2} = \left(1-\frac{K_c\, m(t)}{2C_0}\right) \quad \text{--- (2)}$$

Using equation (2) in (1) we get

$$f_i(t) = f_0\left(1-\frac{K_c\, m(t)}{2C_0}\right) \quad \text{--- (3)}$$

Let us assume $-\frac{K_c}{2C_0} = \frac{K_f}{f_0}$, where K_f = frequency sensitivity parameter.

$$\therefore \quad f_i(t) = f_0\left(1+\frac{K_f}{f_0}\cdot m(t)\right)$$

$$f_i(t) = f_0 + K_f m(t) \quad \text{--- (4)}$$

for sinusoidal message signal, $m(t) = A_m \cos 2\pi f_m t$

$$\therefore \quad f_i(t) = f_0 + \underbrace{K_f A_m}_{\Downarrow} \cdot \cos(2\pi f_m t)$$

$$\therefore \quad f_i(t) = f_0 + \Delta f \cos(2\pi f_m t) \quad \text{--- (5)}$$

: where $\Delta f = K_f A_m$ = Maximum frequency deviation.

Equation (5) gives, the instantaneous frequency of FM-wave generated by using direct method.

↳ Therefore, the direct method is stright forward to implement and is capable of providing large frequency deviation (Δf).

↳ One of the major limitation of the direct method is," the carrier frequency is not obtained from a highly stable oscillator.

↳ To overcome, this limitation a closed loop feedback system for the carrier frequency stabilization is used to provide frequency stabilized FM wave. This arrangement is shown in Fig. 2.

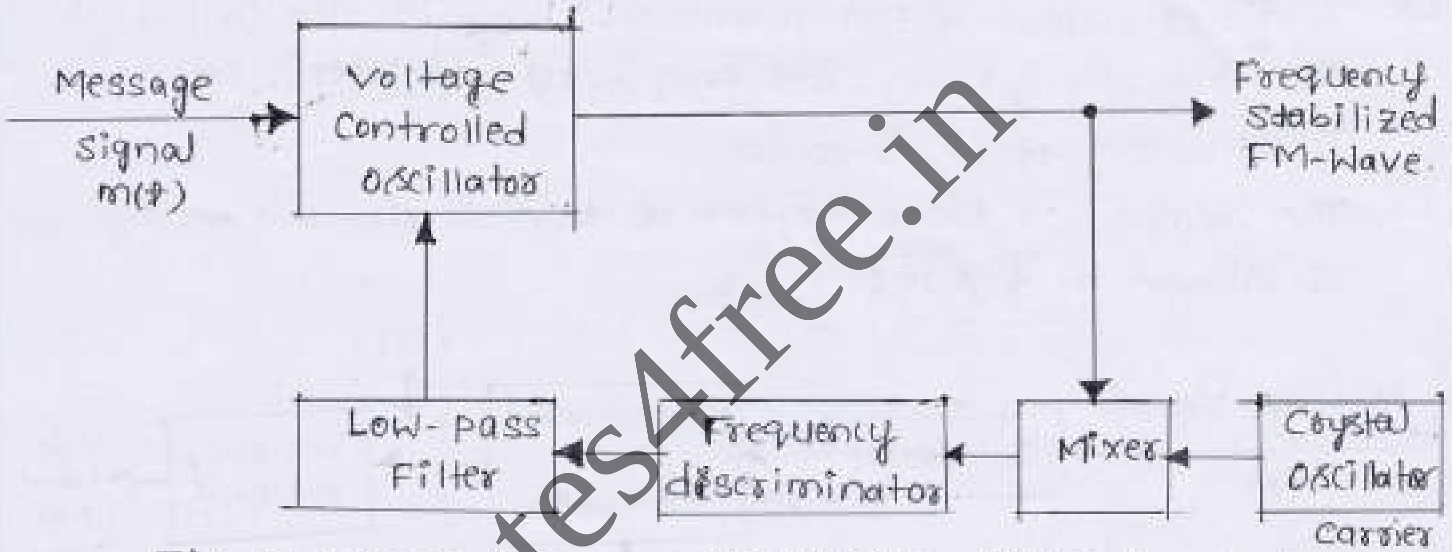


Fig. 2: Carrier frequency stabilization of Direct method FM-generation

↳ Fig.2 consists of crystal oscillator, Mixer, Frequency descriminator, Low-pass filter and voltage controlled oscillator (VCO).

↳ This configuration provides

- Good frequency stability
- Required frequency deviation to generate WBFM.
- Constant proportionality between output frequency change to input voltage change.

∴ the required WBFM-Wave is obtained.

(ii) Indirect Method (or) Armstrong Modulator :-

(Q) Explain the generation of FM-wave using Indirect (or) Armstrong method.

↳ In the indirect method, the message signal is first used to produce Narrow-band FM, which is followed by frequency multiplier to increase the frequency deviation to the desired level.

↳ The frequency multiplier produces wide-band FM wave.

↳ Indirect method of FM-generation scheme is also called as the "Armstrong wide-band-frequency modulator", in recognition of its inventor.

↳ The simplified block diagram of this Indirect FM-System is shown in figure 1.

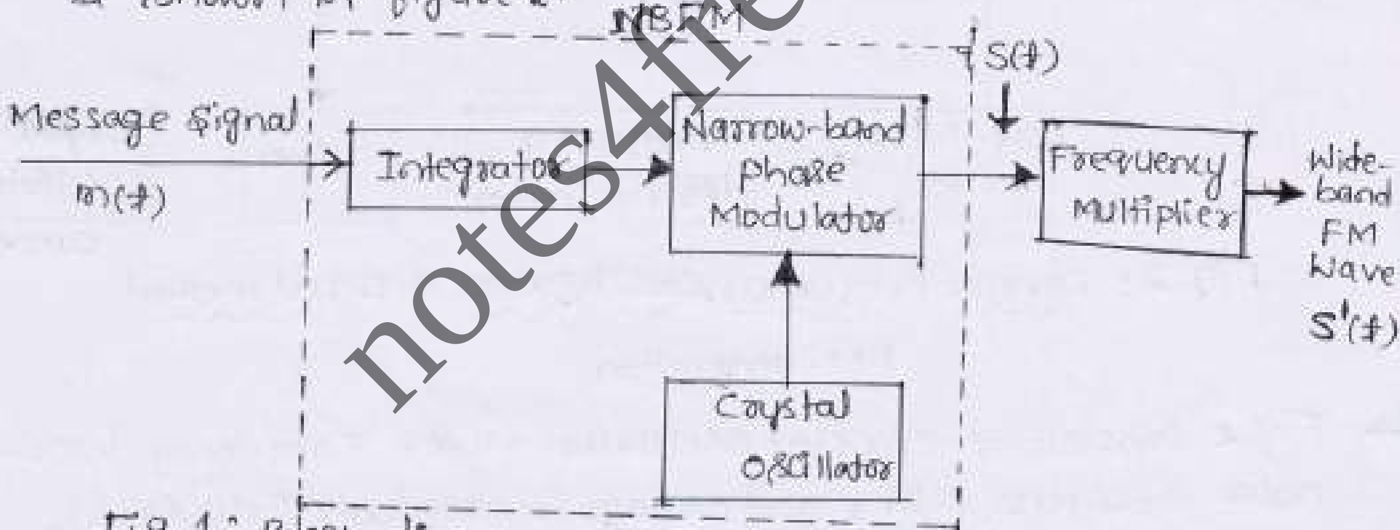


Fig.1: Block diagram of the indirect method of generating a wide-band FM-wave

↳ In Fig.1, the message signal m(t) is first integrated and then used to phase-modulate a carrier wave generated by crystal oscillator. which results in a NBFM (Narrow band FM-wave) S(t) with carrier frequency 'f_c' and modulation index '$\beta<1$', as shown in equation 1.

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi K_p \int m(t)\,dt\right] \quad (1)$$

↳ The use of crystal oscillator provides frequency stability.

↳ The Narrowband FM-wave is next multiplied in frequency by using frequency multiplier.

↳ Frequency multiplier produces the required wide band FM wave, $s'(t)$ as shown in Fig.2.

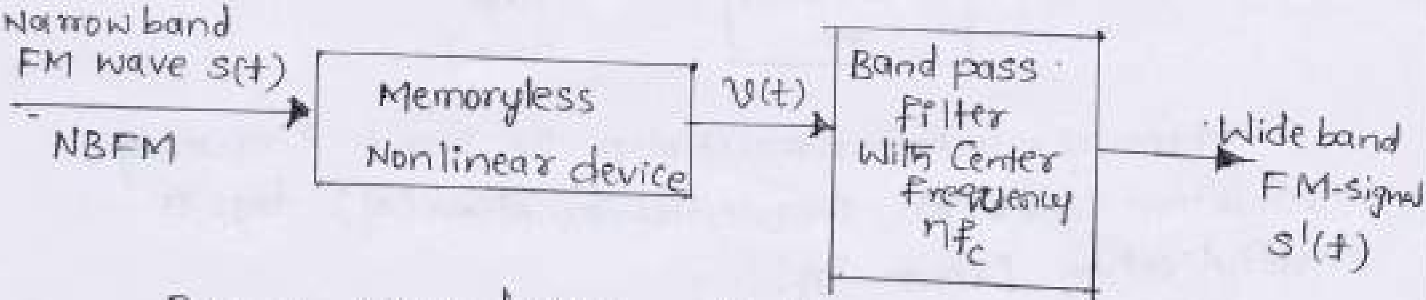


Fig.2: Block diagram of frequency multiplier

↳ A frequency multiplier consists of a memoryless non-linear device followed by a band pass filter having center frequency "$n \times f_c$".

↳ The output voltage of memoryless non-linear device is

$$v(t) = a_1 s(t) + a_2 s^2(t) + a_3 s^3(t) + \dots + a_n s^n(t) \quad \text{--- (2)}$$

where, $s(t)$ = Narrowband FM-signal with carrier frequency f_c and modulation index 'β'.

i.e., $s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right]$ and its instantaneous frequency is,

$$f_i(t) = f_c + k_f\, m(t)$$

↳ When $v(t)$, the output of non-linear device is passed through a BPF having center frequency "$n \times f_c$", we get required wide band FM-signal $s'(t)$.

$$s'(t) = A_c \cos\left[2\pi f_c' t + 2\pi k_f' \int m(t)\,dt\right] \Rightarrow \text{required WBFM signal} \quad \text{--- (3)}$$

where $f_c' = nf_c$ and $k_f' = n \times k_f$ & $\beta' = n \times \beta$

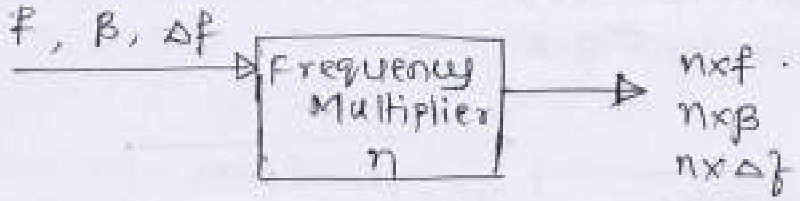
its instantaneous frequency is

$$f_i'(t) = f_c' + k_f' m(t).$$

∴ Required WBFM is obtained.

Problems on NBFM & WBFM with Frequency multipliers :- 24

Note :- Frequency multiplier,



∴ Frequency multiplier, multiplies the input frequency, modulation index (β), frequency deviation (Δf) by a multiplication factor "n".

1. A block diagram of FM transmitter is shown in Fig. 1 Compute the maximum frequency deviation Δf of the output of the FM transmitter and the carrier frequency f_c. If $f_1 = 200$ KHZ, $f_{Lo} = 10.8$ MHZ, $\Delta f_1 = 25$ HZ, $n_1 = 64$ and $n_2 = 48$.

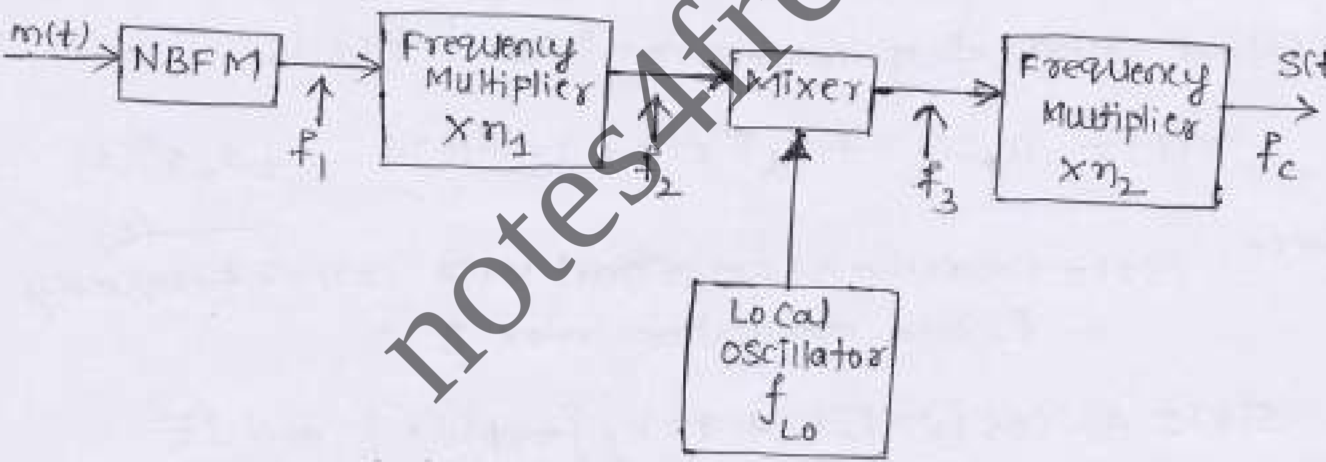


Fig 1 : FM transmitter :-

↳ **Solution :-**

Given :-

$f_1 = 200$ KHZ
$\Delta f_1 = 25$ HZ
$f_{Lo} = 10.8$ MHZ
$n_1 = 64$
$n_2 = 48$
$f_c = ?$
$\Delta f = ?$

↳ The Maximum frequency deviation 'Δf' at the output :-

$$\Delta f' = n_1 \times n_2 \times \Delta f_1 = 64 \times 48 \times 25 = 76.8 \text{ KHZ}$$

↳ The Carrier frequency "f_c" at the output :-

from the block diagram, the carrier frequency

$$f_c = n_2 \times f_3 \quad \text{---(1)}$$

$$f_3 = f_2 \pm f_{Lo} = n_1 \times f_1 \pm f_{Lo} = (64 \times 200 \times 10^3 \pm 10.8 \times 10^6)$$

$$f_3 = 23.6 \text{ MHZ to } 2 \text{ MHZ}$$

$$\therefore f_c = n_2 \times f_3 = 48 (23.6 \text{ MHZ to } 2 \text{ MHZ}) = 1132.8 \text{ MHZ to } 47.2 \text{ MHZ}$$

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2. For a Wideband Frequency modulator, if a narrowband carrier, $f_1 = 0.1$ MHZ, Second carrier $f_2 = 9.5$ MHZ, output carrier frequency = 100 MHZ and $\Delta f = 75$ KHZ. Calculate the multiplying factors n_1 and n_2 if NBFM frequency deviation is 20HZ. Draw the suitable block diagram of the modulator.

(6-Marks) VTU Q.P.

Given:-

output carrier, $f_C = 100$ MHZ

overall frequency deviation

$\Delta f = 75$ KHZ

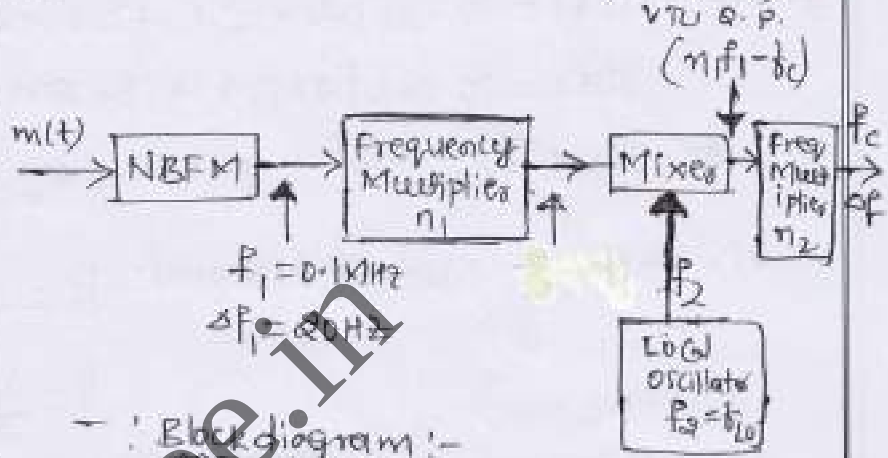
$f_1 = 0.1$ MHZ (carrier1)

$f_2 = 9.5$ MHZ ← Local oscillator frequency for Mixer

$n_1 = ?$

$n_2 = ?$

$\Delta f_1 = 20$ Hz ← NBFM freq. deviation



— : Block diagram :-

from given data the output frequency deviation is

$$\Delta f = n_1 \times n_2 \times \Delta f_1$$

$$\therefore n_1 \times n_2 = \frac{\Delta f}{\Delta f_1} = \frac{75\text{ KHZ}}{20\text{ HZ}} = 3.75 \times 10^3$$

$$\therefore \boxed{n_1 n_2 = 3750}$$

The o/p frequency of frequency Multiplier1 $\} = n_1 \times f_1$

The o/p frequency of Mixer is $\} = (\underline{n_1 f_1}) - \underline{f_2}$ [Assuming Lower frequency $f_{IF} = f_{RF} - f_{LO}$]

Frequency Multiplier2 o/p frequency $\} = f_C = n_2$ (output frequency of Mixer)

$$\therefore f_C = n_2 (n_1 f_1 - f_2)$$

$$f_C = n_1 n_2 f_1 - n_2 f_2 \quad \left(\begin{array}{l}\text{W.K.T}\\ n_1 n_2 = 3750\end{array}\right)$$

$$(100 \times 10^6) = 3750 \times 0.1 \times 10^6 - n_2 \times 9.5 \times 10^6$$

$$\therefore n_2 = \frac{3750 \times 0.1 \times 10^6 - 100 \times 10^6}{9.5 \times 10^6} = \underline{28.95}$$

$$\therefore n_1 = \frac{3750}{n_2} = \frac{3750}{28.95} = 129.5\underline{4}$$

(3) An angle modulated signal is represented by 26 .

$S(t) = 10\cos[2\pi\times10^6 t + 5\sin 2000\pi t + 10\sin 3000\pi t]$ Volts

Find the following :

(i) Power in the modulated signal (ii) Frequency deviation

<iii> Deviation Ratio <iv> phase deviation <v> Transmission Bandwidth

Given : $S(t) = 10\cos[2\pi\times10^6 t + 5\sin 2000\pi t + 10\sin 3000\pi t]$

$S(t) = 10\cos[2\pi f_c t + \beta_1 \sin 2\pi f_{m_1} t + \beta_2 \sin 2\pi f_{m_2} t]$

$\therefore A_c = 10V ;\ f_c = 1\times10^6 ;\ \beta_1 = 5 ;\ f_{m_1} = 1000Hz ;\ \beta_2 = 10 ;\ f_{m_2} = 1500Hz$

<i> power in Modulated signal: $P_t = \frac{A_c^2}{2R}$ Let $\boxed{R = 1\Omega}$ ← Normalized Value.

$P_t = \frac{A_c^2}{2} = \frac{10^2}{2} = \frac{100}{2} = 50W$ //

<ii> Frequency deviation: "Δf"

W.K.T $\Delta f = \beta\times f_m$ ← for single message signal.

given modulated signal S(t) consists of 2-message signals with $\beta_1 = 5 ;\ f_{m_1} = 1000Hz$ & $\beta_2 = 10 ;\ f_{m_2} = 1500Hz$.

$\therefore \Delta f = \beta_1 f_{m_1} + \beta_2 f_{m_2} = 5\times1000 + 10\times1500$

$\Delta f = 5000 + 15,000 = 20,000\ Hz$

$\boxed{\Delta f = 20KHz}$

<iii> Deviation Ratio <Modulation Index> :-

$\beta = D = \frac{\Delta f}{f_m} = \frac{\Delta f}{W}$; where $W = \max(f_{m_1}, f_{m_2})$

$W = \max(1000, 1500)$

$W = 1500$

$\therefore \beta = D = \frac{20K}{1.5K} = 13.333$

<iv> phase Deviation :-

$\Delta\theta = |\theta_i(t) - \theta_c|_{max}$

$\Delta\theta = |5\sin 2000\pi t + 10\sin 3000\pi t|_{max}$

$\Delta\theta = 5 + 10 = 15$ radians

<v> Transmission Bandwidth :-

$BW = 2\Delta f + 2f_m = 2\Delta f + 2W = 2\times20K + 2(1.5K)$

$BW = 43KHz$ //

3.6 FM-STEREO MULTIPLEXING:

Stereo Multiplexing is a form of frequency division multi-plexing <FDM> designed to transmit two separate signals $[m_R(t)$ and $m_L(t)]$ via the same carrier.

FM-stereo system consists of

a) FM-Stereo transmitter
b) FM-Stero Receiver.

FM-stereo transmitter:-

- Let $m_L(t)$ and $m_R(t)$ denote the two message signals picked up by Left hand and Right hand microphones at the transmitting end of the system, as shown in Fig 1.
- It uses a pilot carrier frequency $f_c = 19\,KHZ$. Frequency doubler produces, sub carrier, $\cos(4\pi f_c t)$.

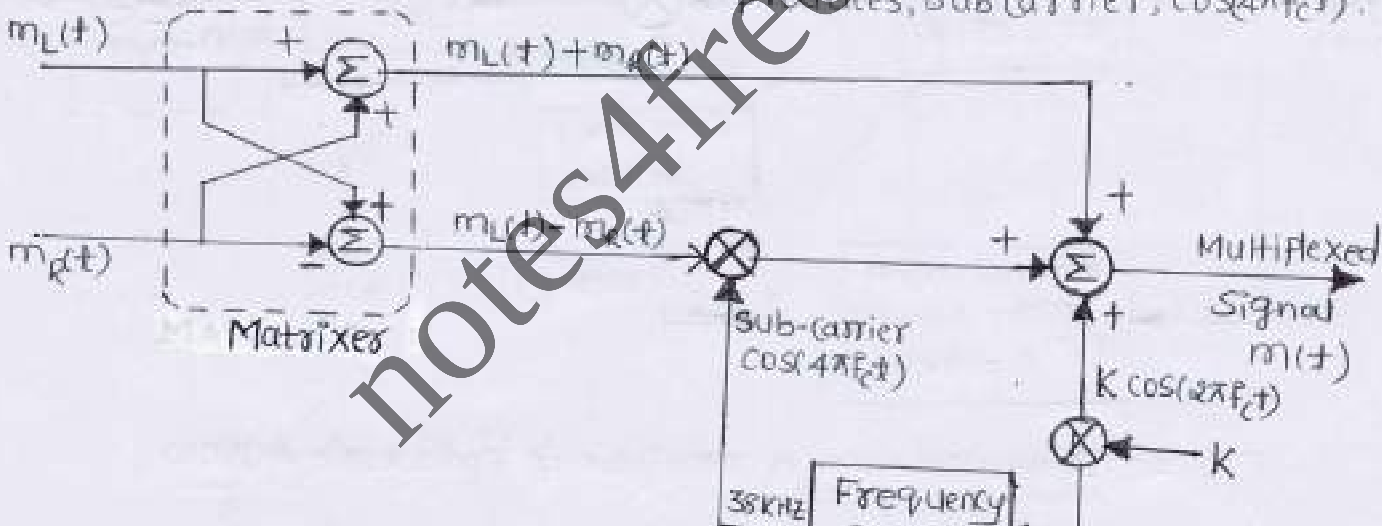


Fig 1: Multiplexer in transmitter of FM-Stereo System

- The Multiplexed signal $m(t)$, produced at the output of multiplexer in transmitter of FM stereo system is,

$$m(t) = \underbrace{[m_L(t)+m_R(t)]}_{\text{Baseband signal ①}} + \underbrace{[m_L(t)-m_R(t)]\cos 4\pi f_c t}_{\text{DSBSC ② signal } 2f_c = 38KHZ} + \underbrace{K\cos(2\pi f_c t)}_{\text{Pilot ③ Carrier signal } f_c = 19KHZ} \quad \text{--- (1)}$$

- Multiplexed signal, m(t) consists of three different signals. (28)

1. $[m_L(t)+m_R(t)] \Rightarrow$ sum of $m_L(t)$, $m_R(t)$ generated by the simple matrixer. It is baseband signal.
2. $[m_L(t)-m_R(t)]\cos(4\pi f_c t) \Rightarrow$ DSBSC-signal produced by the product modulator.
3. $K.\cos(2\pi f_c t) \Rightarrow$ pilot carrier signal multiplied by a constant 'K'.

FM-stereo Receiver :-

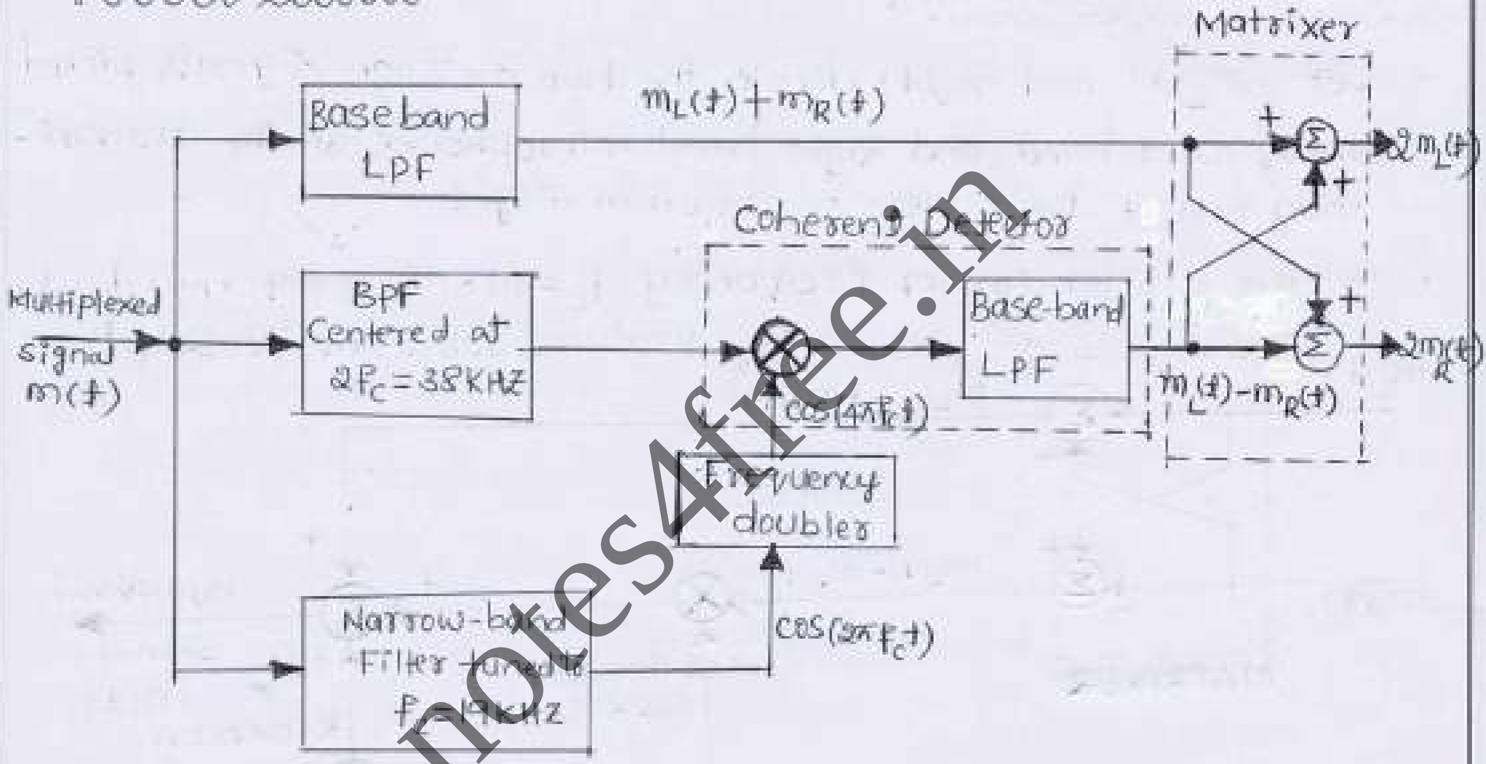


Fig. 2 : Demultiplexer in receiver of FM-stereo system

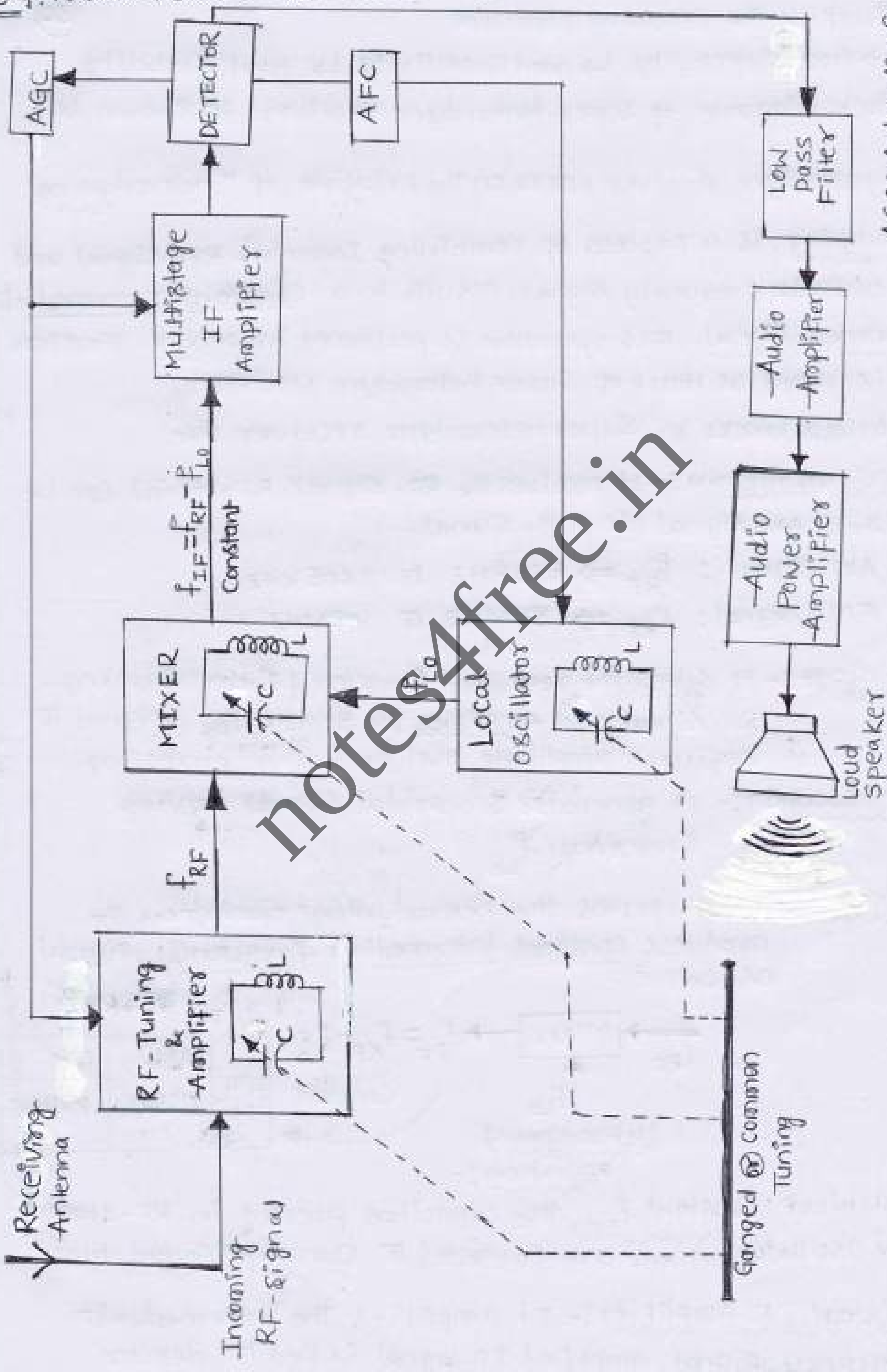
- Fig. 2 shows the demultiplexer in receiver of FM-stereo system. It is used to recover the two message signals $m_L(t)$ and $m_R(t)$.
- FM-stereo demultiplexer consists of 3-filters,

1. Baseband LPF : It selects the base-band component $[m_L(t)+m_R(t)]$ present in multiplexed signal m(t).
2. BPF : (Bandpass filter) :- It selects the DSBSC-signal.
3. Narrowband filter :- It selects the pilot carrier signal, $\cos(2\pi f_c t)$.

- Frequency doubler produces the required subcarrier signal, $\cos(4\pi f_c t)$ for coherent detection of DSBSC-signal.
- Coherent Detector, recovers the difference signal $[m_L(t)-m_R(t)]$.
- Finally the Matrixer, produces the required signals $2m_L(t)$ and $2m_R(t)$.

3.7: ** SUPER HETERODYNE RECEIVER:

(29)



AGC: Automatic Gain Controller

AFC: Automatic Frequency control

Fig 1: Block diagram of Super-Heterodyne Receiver:

(30)

- The superheterodyne receiver, is a special type of receiver that fullfills the following features
  - ↳ Good Selectivity ↳ Good Sensitivity ↳ Good stability.
- The block diagram of Super-heterodyne receiver is shown in Fig.1.
- Super-heterodyne receiver works on the principle of "heterodyning".
- Heterodyning, is a process of combining Incoming RF-signal and Local oscillator frequency signal, results in a Constant Intermediate frequency signal. This operation is performed in Mixer. Therefore Mixer is called as Heart of Super-heterodyne receiver.
- The various blocks in Superheterodyne receiver are.

↳ Receiving Antenna:- It receives the RF-signal. RF-signal can be either AM-signal (or) FM-signal.

for AM-signal : $f_{RF} \Rightarrow$ 535 kHz to 1605 kHz

for FM-signal : $f_{RF} \Rightarrow$ 88 MHz to 108 MHz

↳ RF-stage:- It selects the required frequency from incoming RF-signal and Amplifies the selected f_{RF}, signal to required amplitude level for further processing.

↳ Local oscillator:- It generates sinusoidal signal having frequency 'f_{LO}'.

↳* Mixer*: It performs the heterodyning operation & produces constant intermediate frequency signal as output.

$$f_{IF} = \underbrace{f_{RF} - f_{LO}}_{\text{Constant}}$$

	AM	FM
f_{IF}	455 kHz	10.7 MHz

:- Heterodyning operation :-

* To achieve constant f_{IF}, the capacitors present in RF-stage, Local oscillator, Mixer are connected to Common (or) Ganged tuner.

↳ Multistage IF Amplifier:- It amplifies the intermediate frequency signal. Amplified IF-signal is fed to detector.

↳ Detector :- It detects the message signal present in amplified IF-signal. (31)

↳ Low pass filter :- It eliminates any higher order harmonics present in detector output. It produces the required base-band signal at audio-frequency (AF)

↳ Audio Amplifier :- It amplifies the AF-signal to the required amplitude level.

↳ Audio power Amplifier :- It boosts the power level of amplified AF-signal to a power level suitable to drive the Loud-speaker.

↳ Loud-Speaker :- It converts electrical signal to physical sound signal.

Note: • In AM-superheterodyne receiver, $f_{IF} = f_{RF} - f_{LO} = 455\,KHZ$.
• In FM-superheterodyne receiver, $f_{IF} = f_{RF} - f_{LO} = 10.7\,MHZ$.

Numerical problems :-

1) prove that the number of sidebands in wide band FM-signal is Infinite.

↳ We know that the general expression of WBFM-signal is,

$$S(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \quad \text{——(1)}$$

where β = Modulation Index = $\frac{\Delta f}{f_m} \gg 1$ for WBFM.

Equation (1) can be expressed as,

$$S(t) = Re\left[A_c e^{j[2\pi f_c t + \beta \sin 2\pi f_m (t)]}\right] \quad \left[\because \cos\theta = Re(e^{j\theta})\right] \quad \text{——(2)}$$

$$S(t) = Re\left[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))\right] \quad \text{——(3)}$$

By using n^{th} order Bessel function, $J_n(\beta)$ equation (3) can be re-arranged as 32

$$s(t) = \text{Re}\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp[j2\pi(f_c + nf_m)t]\right] \quad (4)$$

The carrier amplitude A_c, is constant. Therefore it can be taken outside the real-time operator Re[·]. Take Re[·] inside the summation we get

$$s(t) = A_c \sum_{n=-\infty}^{\infty} \text{Re}\left[J_n(\beta)\exp[j2\pi(f_c+nf_m)t]\right]$$

$$\text{Re}\left[J_n(\beta)\exp[j2\pi(f_c+nf_m)t]\right] = J_n(\beta)\cos[2\pi(f_c+nf_m)t]$$

$$\therefore s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c+nf_m)t] \quad (5)$$

By taking Fourier transform on both sides we get

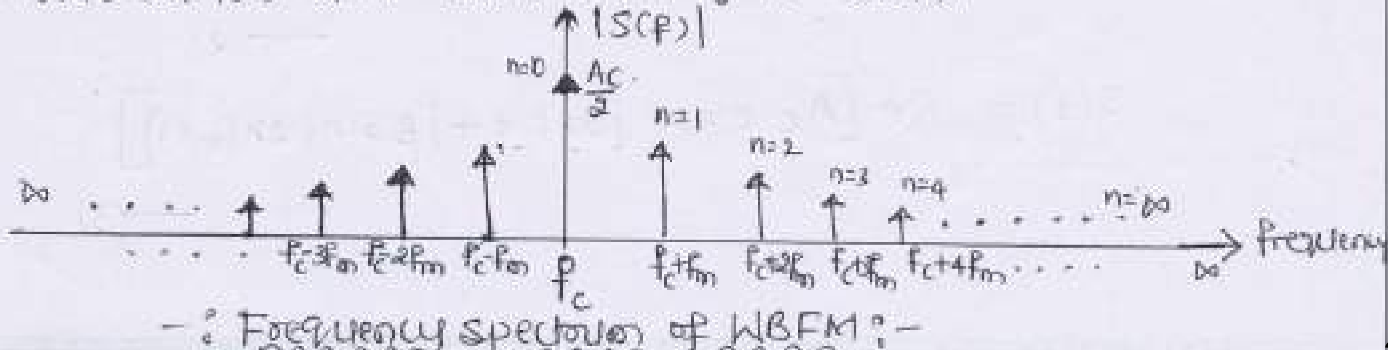
$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right] \quad (6)$$

∴ from equation (6) It is clear that the spectrum of wide band FM-signal consists of an infinite number of delta functions (side bands) spaced at $f = f_c \pm nf_m$ for $n = 0, \pm1, \pm2 \ldots \pm\infty$

In WBFM, $J_n(\beta)$ is finite for all values of 'n'. and

- $J_n(\beta) = J_{-n}(\beta)$ for n-even
- $J_n(\beta) = -J_{-n}(\beta)$ for n-odd
- As n, Increases $|J_n(\beta)|$ decreases and $\boxed{J_0(\beta) \simeq 1}$

∴ The spectrum of WBFM-signal consists of Infinite number of side bands as shown in figure below.



-: Frequency spectrum of WBFM :-

3.8 Demodulation of FM-Waves :-

Frequency Demodulation is the process of recovering original message signal from an incoming FM-wave.

There are two methods in frequency demodulation

<i> Frequency descriminator (OR) Balanced slope detector

<ii> phase-Locked Loop.

<i> Frequency descriminator (OR) Balanced slope Detector :-

- The Balanced slope detector Consists of two slope detector circuits. The Block diagram of Frequency descriminator (or) balanced slope detector is shown in Fig.1 and its equivalent circuit diagram is shown in Fig.2.

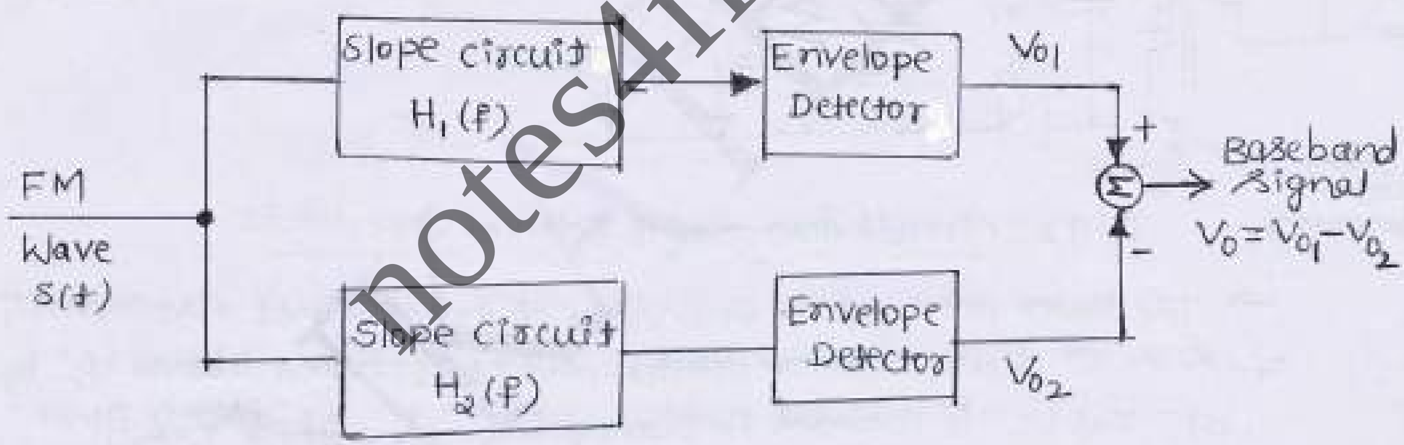


Fig1: Block diagram

The output voltage of frequency descriminator is, (or)(Balanced slope detector)

$$V_0 = V_{01} - V_{02} = \begin{cases} 0 & ; f_{in} = f_c \\ +ve & ; f_{in} > f_c \\ -ve & ; f_{in} < f_c \end{cases} \quad \text{——— (1)}$$

where, f_{in} = Frequency of FM-wave

f_c = Carrier frequency (unmodulated carrier).

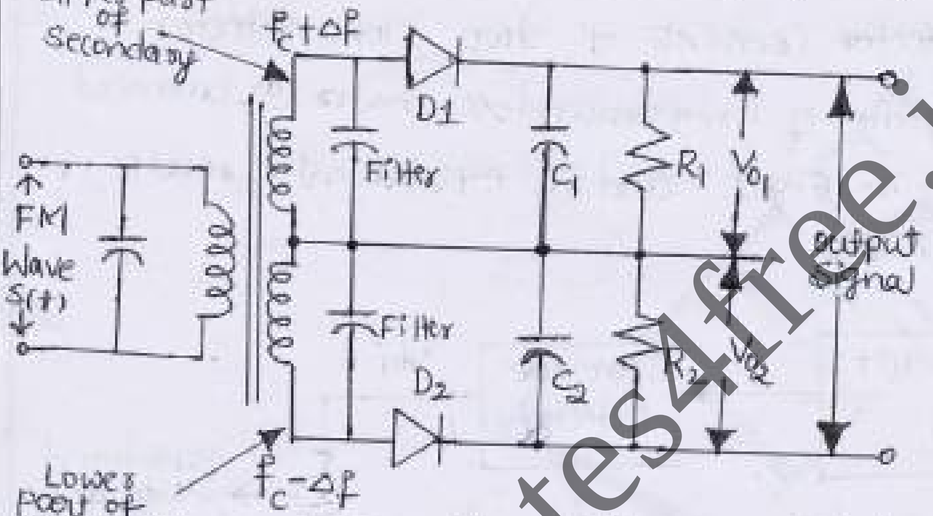
- This method is popular Known as Balanced slope detector.

- The equivalent circuit diagram is shown in Fig. 2. It Consists of,

(34)

↳ Center tapped transformer : • Its primary is tuned to frequency of FM-signal. "$f_c \pm \Delta f$". (Intermediate frequency)
• It produces 180° out-of-phase Voltages at Secondary windings

↳ The upper part of the secondary of transformer, Consists of Diode-Envelope detector and it is tuned above 'f_c' by Δf. That is its resonant frequency is "$f_c + \Delta f$" [upper tuned filter]



$$V_0 = V_{01} - V_{02} = \begin{cases} 0 & : f_{in} = f_c \\ +ve & : f_{in} > f_c \\ -ve & ; f_{in} < f_c \end{cases}$$

Fig 2: Circuit diagram of Balanced slope detector

↳ The lower part of the secondary of transformer also consists of similar diode envelope detector and it is tuned below 'f_c' by Δf. That is its resonant frequency is "$f_c - \Delta f$". [Lower Tuned filter]

↳ It produces the required output voltage (Baseband message signal) $V_0 = m(t)$ as shown in Equation (1).

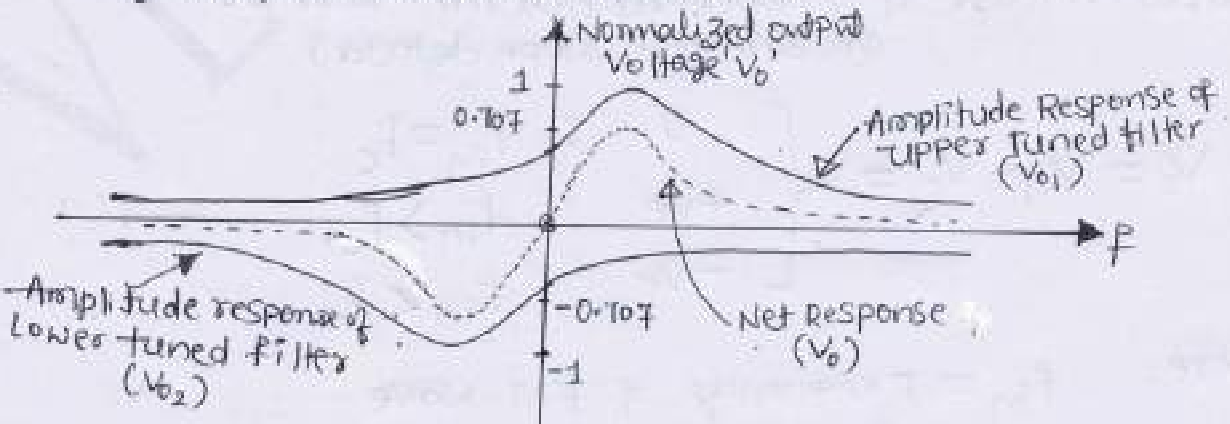


Fig 3: Frequency Response

↳ The frequency response of upper & Lower tuned filters, Net response of the circuit is shown in Fig. 3.

—*—*—

FM-demodulation using phase Locked Loop:- (PLL)

phase Locked Loop (PLL) is a negative feedback system that consists of three major components

<i> A Multiplier used as a phase detector (or) phase comparator.

<ii> A - Voltage Controlled oscillator <VCO>

<iii> A - Loop filter, which is a Low pass filter <LPF>

The Block diagram of PLL is shown in Fig.1.

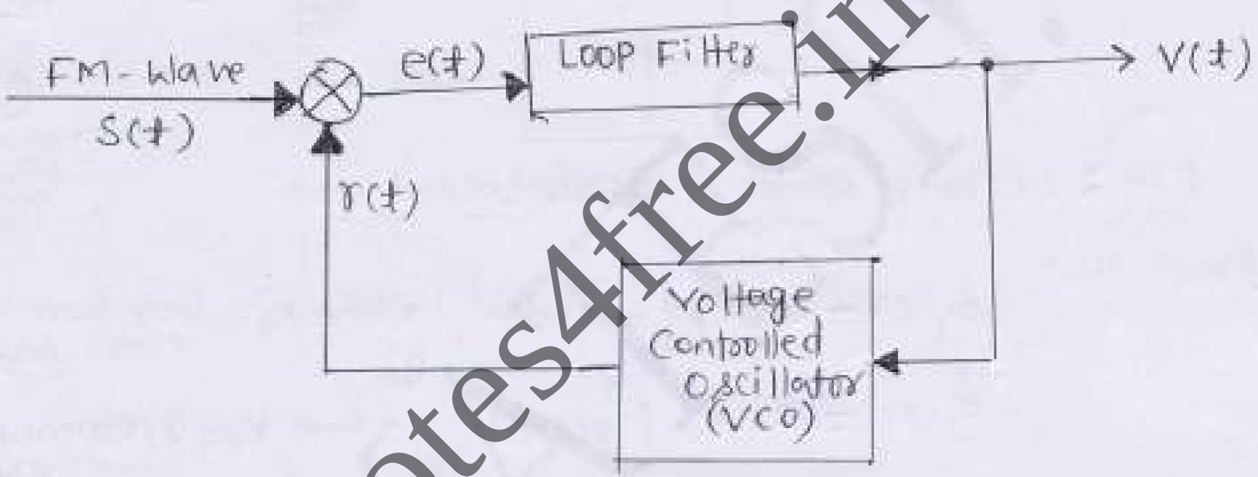


Fig.1: Block diagram of PLL

↳ The VCO output is defined as

$$r(t) = A_v \cos(2\pi f_c t + \phi_2(t)) \quad — (1)$$

where $\phi_2(t) = 2\pi K_v \int_0^t v(t)\, dt$.

↳ Then, the incoming signal (FM) $(S(t))$ and the VCO output $r(t)$ are applied to the multiplier, then it gives error signal,

$$e(t) = r(t) \cdot S(t) \quad — (2)$$

where $S(t) = A_c \sin[2\pi f_c t + \phi_1(t)] \quad — (3)$

where $\phi_1(t) = 2\pi K_f \int_0^t m(t)\, dt$. — (4)

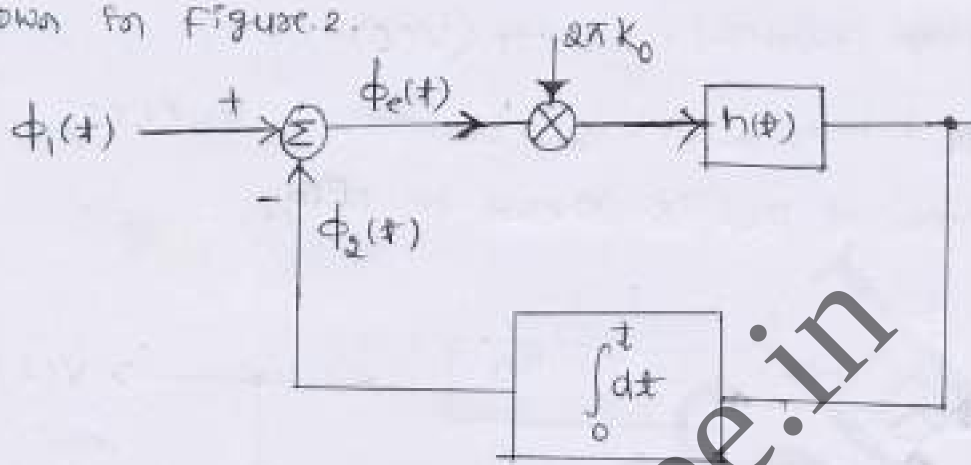
*** 36

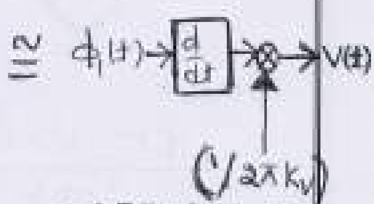
<i> Linear Model of phase-Locked-Loop (Linear-PLL):-

VTU Q.P. 8 Mark

The phase locked loop (PLL) is said to be in phase-Lock, when the phase error $\boxed{\phi_e(t) = 0}$

The Linear model of PLL for the demodulation of FM-signal is shown in figure.2.





-: Equivalent Model :-

Fig.2 : Linear Model of Phase Locked Loop

We know that,

$$\phi_1(t) = 2\pi k_f \int_0^t m(t)\,dt \quad \text{— ①}$$ where k_f = freq. sensitivity of FM-wave.

$$\phi_2(t) = 2\pi k_v \int_0^t v(t)\,dt \quad \text{— ②}$$ where k_v = Frequency Sensitivity Constant of VCO

From fig.2,

$$\phi_e(t) = \phi_1(t) - \phi_2(t) \quad \text{— ③}$$

W.K.T. for phase-lock mode : $\phi_e(t) = 0$ (Assuming Small error ≈ 0)

$\therefore$ Equation (3) $\Rightarrow$

$$0 = \phi_1(t) - \phi_2(t)$$

$$\therefore \phi_1(t) = \phi_2(t)$$

Using equations ① & ② we get

$$2\pi k_f \int_0^t m(t)\,dt = 2\pi k_v \int_0^t v(t)\,dt$$

$$k_f \int_0^t m(t)\,dt = k_v \int_0^t v(t)\,dt \quad \text{— (4)}$$

(37)

Differentiating both sides of equation (4), we get

$$k_f\, m(t) = k_v\, v(t)$$

$$\therefore \boxed{v(t) = \frac{k_f}{k_v} \cdot m(t) = K\, m(t)} \quad ; \text{ where } K = \frac{k_f}{k_v}$$

i.e., $\boxed{v(t) \propto m(t)}$

Thus, the output $v(t)$ of the low pass-loop filter $[h(t)]$ is proportional to the original modulating signal. i.e., The message signal present in FM-modulated wave $s(t)$ is recovered and it is produced at the output of loop filter.

—*—*—*—

3.9: Non-Linear effects in FM-wave :- <VTUQP>

Q) Write a short note on Non-linear effects in FM-system.

↳ Non-linear effects can be of two-types
(i) strong (ii) weak.

* Non-linearity is said to be strong, if it is intentionally introduced into the circuit in a controlled manner.
  Ex: square law devices.
* Non linearity is said to be weak, when it is inherently present in the circuit.

The effect such non-liniarities will limit $m(t)$ levels in the system.

In FM-generation system, weak non-linearity is present. the effect of weak non-linearity in FM-systems can be by considering the input and output relation of the memoryless-non linear device used in the frequency multiplier.

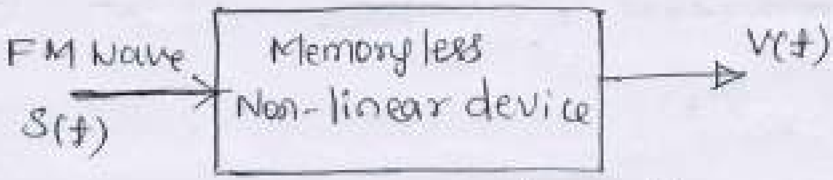


Fig 1: Non-linear device used in FM system:-

Consider a memoryless Non-linear device as shown in fig-1.

W.K.T, the relation between input & output signal is

$$V(t) = a_1 s(t) + a_2 s^2(t) + a_3 s^3(t) + \cdots + a_n s^n(t) \quad \text{---} (1)$$

Let us consider upto 3^{rd} order

i.e., $V(t) = a_1 s(t) + a_2 s^2(t) + a_3 s^3(t)$ ——(2)

W.K.T the expression for FM-Wave is

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)] \quad \text{where } \phi(t) = 2\pi k_f \int_0^t m(t)\,dt$$

$$\therefore V(t) = a_1 A_c \cos[2\pi f_c t + \phi_1(t)] + a_2 A_c^2 \cos^2[2\pi f_c t + \phi_1(t)] + a_3 A_c^3 \cos^3[2\pi f_c t + \phi_1(t)] \quad \text{---} (3)$$

(1) (2) (3)

$\therefore$ The output voltage consists of DC components and three FM-signals with carrier frequencies f_c, $2f_c$ & $3f_c$ having frequency deviations Δf, $2\Delta f$ and $3\Delta f$ respectively.

The Desired FM-signal can be separated using a BPF as shown in fig-2.

We get the FM-system o/p after passing through BPF is

$$V_o'(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] \quad \text{---} (4)$$

equation (4) is same as that of FM-input signal

$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$ except for change in amplitude.

$\therefore$ Amplitude Non-linearities of the FM-S/m does not affect FM-signal

Module-3

-: 3.1 : Random Variables and process :-

Introduction : Random ⟶ unpredictable

Randomness, is the essence of communication. In communication system, the signal at any stage belongs to either Deterministic (or) Random signals.

The signals are deterministic, if their behaviour is known and can be represented by formula (or) graph.

The signals are said to be Random <or stochastic> if their behaviour is not known/unpredictable.

This module, gives the overview of probability theory, random variables and various statistical parameters such as mean, variance, moments, standard deviation, Autocorrelation, Auto-variance and cross correlation functions.

Basics of probability:-

- Random Experiment :- Experiments whose outcomes are not predictable are known as random experiments.

  Example :
  - ↳ Tossing a coin
  - ↳ Throwing a die
  - ↳ Measuring the Noise voltage at the terminals of Resistor.

- Sample space (S):-

  The sample space of a random experiment is a mathematical abstraction used to represent all possible outcomes of the experiment. It is denoted by 'S'.

- Sample point :(s_k)

  It represents each outcome of the random experiment.

  i.e., sample space 'S', consists of any number of sample points & each point represents an outcome of the random experiment.

i.e., $S = \{s_1, s_2, s_3 \ldots\ldots, s_n\}$ $s_1, s_2 .. s_n \Rightarrow$ Sample points in Sample space 'S'

2

- Random Event :-

A Random event is an outcome (or) set of outcomes (or) set of sample points that share a common attribute.

Events are denoted by upper case letters such as A, B, etc.

- Mutually exclusive events :- (or) (Independent events)

Two events A and B are said to be mutually exclusive if they are completely independent (or) cannot occur together.

- Occurance of an Event : An event 'A' of an random experiment is said to have occurred if the experiment gives an outcome that belongs to event 'A'.

- Complement of an event :- The complement of an event A, is the event containing all sample points in "S" but not in A.

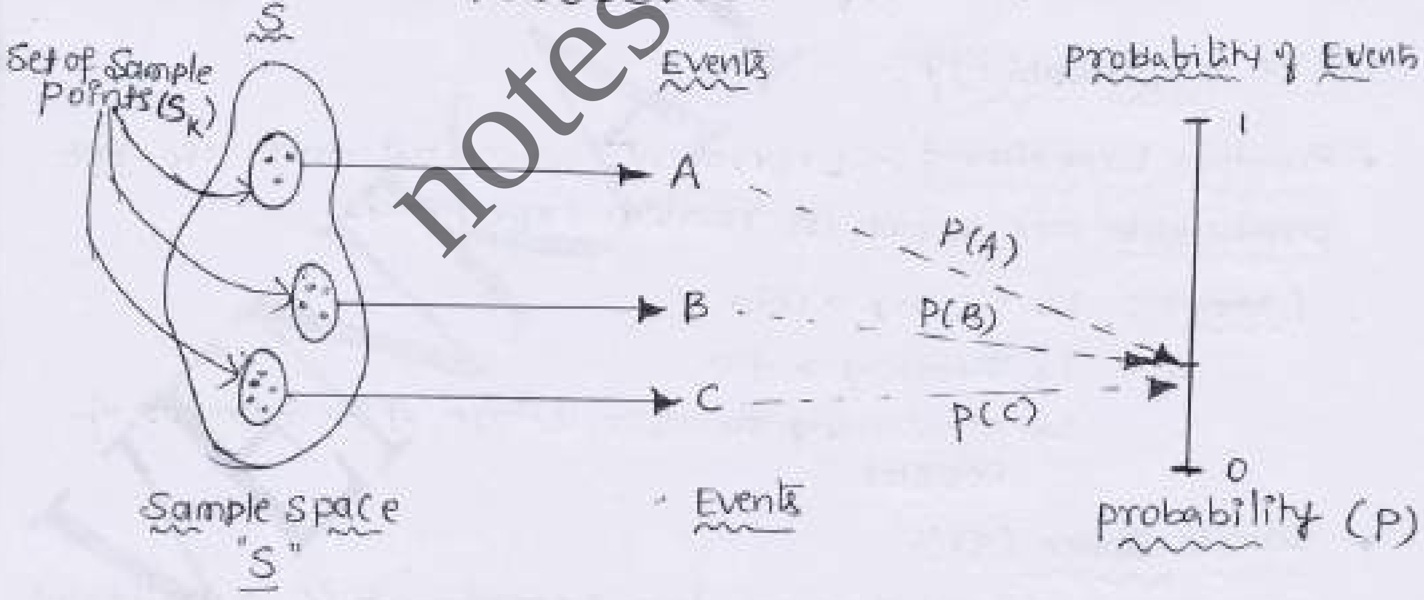


Fig.1: Illustration of Relationship between sample space (S), Events and probability

Figure 1, illustrates the relationship between sample space (S), random events (A, B, C) and their probabilities [P(A), P(B), P(C)].

- Dependent Events :- Events A, B are said to be dependent if they share common sample points in "S".

3.1: Probability :-

Probability of event A, is denoted by P(A).

Let a random Experiment is repeated n-times. If the event 'A' occurs n_A-times, then probability of event A, i.e, P(A) is defined as,

$$P(A) = \lim_{n \to \infty} \left(\frac{n_A}{n} \right)$$

- The ratio, $\frac{n_A}{n}$ represents the fraction of occurance of event 'A'.

Axioms (Laws) of probability :-

1. $P(A) \geq 0$
2. $P(S) = 1$ [Total probability of all events in Sample Space is 1]
3. If A and B are mutually Exclusive events then,

   $P(A+B) = P(A) + P(B)$
4. If A and B are dependent events then,

   $P(A+B) = P(A) + P(B) - P(AB)$

   where $P(AB) \Rightarrow$ Joint probability of events A, B.

- Joint probability :- [P(A,B) (or) P(AB)]

  Joint probability of two events A and B, is defined as

$$P(A,B) = P(AB) = \lim_{n \to \infty} \left(\frac{n_{AB}}{n} \right)$$

↳ The ratio $\frac{n_{AB}}{n}$, represents the fraction of occurance of both the events A, B.

**** 3.2: Conditional probability:- 4

Let us consider two-events X and Y of a Random Experiment.

- The conditional probability, $P[Y/x]$ refers to the probability of Y-occuring, given that X-has occured. It is defined as

$$P[Y/x] = \frac{P[X,y]}{P[x]} \quad \text{— (1)}$$

where $P[x,y]$ = Joint probability of events X, Y
$P[x]$ = probability of event X. [known event]

i.e., Conditional probability is defined as the "ratio of Joint-probability to that of probability of known event."

Similarly, the Conditional probability $P[X/y]$ refers to the probability of X-occuring, given that Y-has occured. It is defined as.

$$P[X/y] = \frac{P[X,y]}{P[Y]} \quad \text{— (2)}$$

***

Baye's theorem for Conditional probability:-

It gives the relationship between conditional probabilities $\{P[Y/x], P[X/y]\}$ and probabilities of individual events P(A) & P(B).

Statement: Bayes theorem states that, the Conditional probability.

$$P[Y/x] = \frac{P[X/y] \cdot P[Y]}{P[X]}.$$

Proof:- We know that the Conditional probability,

$$P[Y/x] = \frac{P[X,Y]}{P[Y]} \quad \text{— (1)}$$

$$\therefore \quad P[X,Y] = P[Y/X] \cdot P[Y] \quad \text{--- (2)}$$

Similarly $$P[X/Y] = \frac{P[X,Y]}{P[Y]} \quad \text{--- (3)}$$

$$\therefore \quad P[X,Y] = P[X/Y] \cdot P[Y] \quad \text{--- (4)}$$

By comparing equations (3) and (4) we get

$$P[Y/X] \cdot P[X] = P[X/Y] \cdot P[Y]$$

$$\therefore \quad \boxed{P[Y/X] = \frac{P[X/Y] \cdot P[Y]}{P[X]}} \Leftarrow \text{Bayes theorem}$$

Note: • If X and Y are independent (Mutually Exclusive) events

then ↳ $P[Y/X] = P[Y]$ ← Event 'Y' is independent of occurance of Event X

↳ $P[X/Y] = P[X]$

• Therefore Conditional probability is applicable for dependent variables/events.

• for Independs events A, B, Joint probability is

$$P(A,B) = P(AB) = P(A) \cdot P(B)$$

$$P(X,Y) = P(XY) = P(A) \cdot P(B)$$

3.3. Random Variables :-

Definition:- Random Variable is a real Valued function, which can take any value from the sample space and its range is set of real numbers.

Random variable may be classified as

(a) Discrete Random Variable (DRV)

(b) Continuous Random Variable (CRV)

- probability mass function ($P[X \leq x]$)

Let X, be a discrete random variable and $x_1, x_2, x_3 \ldots, x_n$ be the values of 'X' can take, then $P[X \leq x]$ is called probability mass function.

***

- probability Distribution function:- For any random variable 'X' probability distribution function is denoted by $F_X(x)$: It is related to probability mass function as

$$F_X(x) = P[X \leq x] \quad : \quad X \Rightarrow \text{Random Variable}, \; x = \{x_1, x_2, x_3 \ldots x_n\}$$

- The $F_X(x)$ is a function of 'x' and not a function of random variable 'X'

↳ properties of distribution function

Distribution function, $F_X(x)$ satisfies the following properties

i> $F_X(x) \geq 0 \quad : -\infty \leq x \leq \infty$ (Non-Negative)

** ii> $F_X(-\infty) = 0$ and $F_X(\infty) = 1$ [Bounded between 0 & 1]

iii> $P[X \leq a] = F_X(a) \quad : \; a \Rightarrow$ Any real number

iv> $P[b < X \leq a] = F_X(a) - F_X(b) \quad : \; a > b$

Note: $F_X(x)$ is also known as cumulative distribution function.

** probability density function :-

<Q> What do you mean by probability density function? Prove that the total area under the surface of a probability density function (pdf) is always 1.

VTU June/July-2017 (5 Marks)

↳ Definition: The Derivative of probability distribution function is called as probability density function (pdf). It is denoted by, $f_X(x)$.

Mathematically :

$$f_X(x) = \frac{d\,F_X(x)}{dx}$$

Where, $F_X(x) \Rightarrow$ probability Distribution function.

$x \Rightarrow$ Random variable that can take any real value 'x'.

* The total area under the probability density curve is always equal to 1 (unity).

i.e.,
$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

Proof :- By definition,

$$f_X(x) = \frac{d\,F_X(x)}{dx} \quad \text{—— (1)}$$

Apply Integral on both sides between the limits $-\infty$ to ∞.

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_{-\infty}^{\infty} \frac{d\,F_X(x)}{dx}\,dx$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \Big[F_X(x)\Big]_{x=-\infty}^{\infty} = F_X(+\infty) - F_X(-\infty)$$

$$\therefore \int_{-\infty}^{\infty} f_X(x)\,dx = F_X(\infty) - F_X(-\infty) = 1 - 0 = 1$$

(because, $F_X(\infty) = 1$ and $F_X(-\infty) = 0$ (properties of distribution function))

3.4 Several Random Variables :- [More than 1 Random Variable] (8)

Consider two random Variables X and Y.

- Joint distribution function :-

↳ Joint distribution function for two random Variables X, Y is denoted by $F_{X,Y}(x,y)$.

↳ It is defined as the probability that the random Variable X is less than or equal to 'x' and that the random Variable Y is less than or equal to 'y'.

i.e., $F_{X,Y}(x,y) = P[X \le x \, ; \, Y \le y]$

***

- Joint probability density function :-

Q) Explain the term Joint probability density function of random Variables X and Y. Show that the volume under the surface of pdf curve is unity?

↳ Joint probability density function of two random Variables X and Y is given by the partial derivative of the joint distribution function. It is denoted by $f_{X,Y}(x,y)$.

i.e., $$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \cdot \partial y}$$

where $F_{X,Y}(x,y)$ = Joint probability distribution function of random Variables X, Y.

- : Total Volume under the surface of a probability density function is always 1 (unity).

i.e., $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx \cdot dy = 1$$

Proof :- By, definition of Joint probability density function,

$$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \cdot \partial y} \quad \text{---(1)}$$

-Apply double integral on both sides between limits $-\infty$ to ∞.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx.dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \cdot \partial y} \cdot dx \cdot dy$$

$$= \left[F_{X,Y}(x,y)\right]_{x=y=-\infty}^{x=y=\infty}$$

$$= \left[F_{X,Y}(\infty,\infty) - F_{X,Y}(-\infty,-\infty)\right] \quad \text{---(2)}$$

W.K.T using the properties of distribution function,

$F_{X,Y}(\infty,\infty) = 1$ and

$F_{X,Y}(-\infty,\infty) = 0$

$$\therefore \int_{-\infty}^{\infty}\int_{\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy = \underbrace{F_{X,Y}(\infty,\infty)}_{1} - \underbrace{F_{X,Y}(-\infty,-\infty)}_{0}$$

$$= 1 - 0$$

$$= 1$$

$\therefore$ Total volume under the surface of a Joint probability density function is equal to 1. (unity).

10

Additional problems on Density function & Distribution function:-

Formulae :- Distribution function: $F_X(x) = P[X \le x]$

Density function: $f_X(x) = \frac{d}{dx} F_X(x)$

Properties of distribution function:-

(i) $F_X(\infty) = 1$

(ii) $F_X(-\infty) = 0$

(iii) $P[X \le a] = F_X(a)$

(iv) $P[b < X \le a] = F_X(a) - F_X(b)$: $a > b$.

<1> A Random Variable X has distribution function

$$F_X(x) = \begin{cases} 0 & : x < 0 \\ kx^2 & : 0 \le x \le 10 \\ 100k & : x > 10 \end{cases}$$

(a) Determine the constant k

(b) Evaluate $P(X \le a)$ and $P(5 < X \le 7)$

(c) Determine probability density function.

→ From the given data, probability distribution function

$$F_X(x) = \begin{cases} 0 & : x < 0 \\ kx^2 & : 0 \le x \le 10 \\ 100k & : x > 10 \end{cases}$$

(a) To find Constant 'k' :-

We know that $F_X(\infty) = 1$

from given data $F_X(\infty) = 100k$

$\therefore$ $100k = 1 \Rightarrow \boxed{k = \frac{1}{100}}$

(b) • $P[X \le 5]$:-

W.K.T $P[X \le a] = F_X(a)$

$\therefore P[X \le 5] = F_X(5)$

from given data $F_X(5) = Kx^2 = \left(\frac{1}{100}\right)(5)^2 = \frac{25}{100} = 0.25$

$x=5$; K-value

$\therefore P[X \le 5] = F_X(5) = 0.25$

• $P[5 < X \le 7]$:-

W.K.T $P[b < X \le a] = F_X(a) - F_X(b)$

$\therefore P[5 < X \le 7] = F_X(7) - F_X(5)$

$F_X(7) = Kx^2 = \frac{1}{100} \times 49 = \frac{49}{100}$; $F_X(5) = 0.25$

$\therefore P[5 < X \le 7] = \frac{49}{100} - 0.25 = \underline{0.24}$

(c) probability density function :- $\{f_X(x)\}$

W.K.T $f_X(x) = \frac{d}{dx} F_X(x)$

$$F_X(x) = \begin{cases} 0 & ; x < 0 \\ \frac{x^2}{100} & ; 0 < x \le 10 \\ 1 & ; x > 10 \end{cases}$$

$\left(\because K = \frac{1}{100}\right)$

$\left(\because \frac{d}{dx}\left(\frac{x^2}{100}\right) = \frac{2x}{100} = \frac{x}{50}\right)$

$$\therefore f_X(x) = \frac{dF_X(x)}{dx} = \begin{cases} 0 & ; x < 0 \\ \frac{x}{50} & ; 0 < x \le 10 \\ 0 & ; x > 10 \end{cases}$$

3.5. Statistical Averages:-

Q) With an example, explain what is meant by statistical averages. V.T.U. June/July 2017 (6-Marks)

→ Statistical average or Mean of any random variable is expressed by the summation of the values of random variable 'x' weighted by their probabilities.

Statistical average is also called mean or Expectation value of random variable. It is denoted by $E[X]$ or m_X.

i.e.,
$$E[x] = m_X = \sum_{\text{all } 'x'} x\, P[X=x] = \sum_{\text{all } 'x'} x\left(\frac{n_x}{N}\right)$$

Example: Let 'x' be the random variable that is defined on the die experiment, which can take $x = \{1,2,3,4,5,6\}$ real values.

Suppose that the die is rolled N-times and the number x appears n_x times. Then, the statistical average is calculated by summing all the numbers and dividing by the total number of times that the die is rolled.

i.e., Statistical average,
$$E[x] = \sum_{x=1}^{6} x.\frac{n_x}{N} \qquad \left(\because P[X=x] = \frac{n_x}{N}\right)$$

$$E[x] = \frac{1.n_1 + 2n_2 + 3n_3 + 4n_4 + 5n_5 + 6n_6}{N}$$

i.e., $$E[X] = \frac{n_1 + 2n_2 + 3n_3 + 4n_4 + 5n_5 + 6n_6}{N}$$

∴ for continuous random variable, X statistical average is also calculated by,
$$E[x] = \int x f_X(x)\, dx$$

Note: Statistical averages includes Mean, Median and Mode.

3.6 : Moments :-

Moments are an application of Random Variable 'X'. There are two types of moments

(a) Moments about the origin

(b) Central moments

(a) Moments about the origin :-

Let 'X' be a continuous random variable, then n^{th}-order moment of random variable 'X' is defined as,

$$E[X^n] = \int_{-\infty}^{\infty} x^n \cdot f_X(x)\,dx$$

Most widely used moments are,

↳ First moment : for $\boxed{n=1}$; $E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx \Leftarrow$ Results in Mean

↳ Second moment : for $\boxed{n=2}$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx \Leftarrow$$ Results in Mean square Value of Random Variable 'X'.

(b) Central moments :-

moments about the mean value of random Variable X are called "Central moments".

n^{th}- order Central moment is defined as

$$E[(X-m_x)^n] = \int_{-\infty}^{\infty} (x-m_x)^n f_X(x)\,dx$$

* The most widely used Central moment is "Second order Central moment. i.e., for $n=2$

$$E[(X-m_x)^2] = \int_{-\infty}^{\infty} (x-m_x)^2 \cdot f_X(x)\,dx$$

* $E[(X-m_x)^2] \Rightarrow$ Results in Variance of random Variable 'X'.

3.7: Mean, Variance and Standard deviation :-

a) **Mean :** Mean Value is also known as Expected Value of random Variable 'X'. It is the first order moment about the Origin.

It is denoted by $E[X]$ (or) m_X.

Where $E \rightarrow$ Expectation operator.

Mathematically, mean of Continuous random Variable 'X' is given by,

$$m_X = E[X] = \int_{-\infty}^{\infty} x f_X(x)dx$$

Note: If Random Variable 'X' represents Voltage ('v'), then mean $\longrightarrow$ represents Average DC-value (V_{dc})

b) **Variance :-**

The Second order Central moment of random Variable 'X' is known as Variance.

Variance of random Variable is denoted by σ_X^2.

Mathematically it is given by,

$$\sigma_X^2 = E\left[(X-m_x)^2\right]$$

Simplification: W.K.T, $(X-m_x)^2 = X^2 + m_x^2 - 2Xm_x$

Where m_x = Mean of R.V 'X' = Constant

$$\therefore \sigma_X^2 = E[(X-m_x)^2] = E[X^2 + m_x^2 - 2Xm_x]$$

$$\sigma_X^2 = E[X^2] + E[m_x^2] - 2E[X.m_x]$$

Since m_x is Constant $E[m_x^2] = m_x^2$; $2E[Xm_x] = 2m_xE[X]$

$$\therefore \sigma_X^2 = E[X^2] + m_x^2 - 2E[X] \cdot m_x$$

W.K.T $E[X] = m_x \Leftarrow$ mean of random variable 'X'.

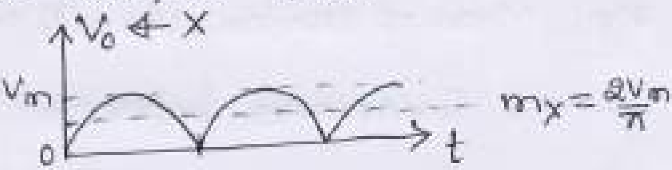
$$\therefore \sigma_X^2 = E[X^2] + m_x^2 - 2m_x \cdot m_x$$

$$\therefore \quad \sigma_X^2 = E[X^2] + m_x^2 - 2m_x^2$$

$$\therefore \quad \sigma_X^2 = E[X^2] - m_x^2$$

i.e., Var(X) = Second order moment about origin − square of Mean

Note: If Random Variable 'X' represents output voltage in full-wave rectifier then



Mean $\longrightarrow$ Average DC-voltage $= E[X] = m_x$

$E[X^2] \Longrightarrow$ Mean square value $\Rightarrow$ It gives total power (Assuming $R = 1\Omega$)

$\sigma_X^2 \Longrightarrow$ Variance $\Rightarrow$ It gives AC power.

$m_x^2 \Longrightarrow$ square of average DC-voltage $\Rightarrow$ It gives DC-power

$\therefore \quad \sigma_X^2 = E[X^2] - m_x^2$ can be read as

AC-power = Total power − DC-power. (Assuming $R = 1\Omega$)

Standard deviation:

Square root value of Variance is called as "Standard deviation".

It is denoted by the symbol 'σ_X'

i.e., $\sigma_X = +\sqrt{Var[X]}$

Note: If 'X' represents the output voltage of rectifier circuit then standard deviation gives root mean square value. (RMS-value).

3.8: Covariance and Correlation Co-efficient :-

- Covariance :- It gives joint expectation of two-random-variables X and Y. It is denoted by symbol 'λ_{XY}'

i.e., $\text{cov}(X,Y) = \lambda_{XY} = E[(X-m_x)(Y-m_y)]$

where m_x = Mean of random Variable 'X'
m_y = Mean of random Variable 'Y' } Constants

↳ Simplification :- W.K.T Covariance between X, Y is

$$\text{cov}(X,Y) = \lambda_{XY} = E[(X-m_x)(Y-m_y)]$$
$$= E[XY - Xm_y - m_xY + m_xm_y]$$
$$= E[XY] - E[X\underset{\uparrow \text{Constant}}{m_y}] - E[\underset{\uparrow \text{Constant}}{m_x}Y] + E[\underbrace{m_xm_y}_{\text{Constant}}]$$
$$= E[XY] - m_yE[X] - m_xE[Y] + m_xm_y$$
$$= E[XY] - m_ym_x - m_xm_y + m_xm_y$$
$$= E[XY] - 2m_xm_y + m_xm_y$$

$$\boxed{\text{cov}(X,Y) = \lambda_{XY} = E[XY] - m_xm_y} \quad — (1)$$

Note: • If $E[XY] = 0$, then the random Variables X, Y are said to be Orthogonal.

- If two random Variables X and Y are Independent then they are said to be "Uncorrelated".
- For Independent Random Variables, X and Y $E[XY] = E[X] \cdot E[Y] = m_x \cdot m_y$ and Covariance is zero (using Equation (1))

- Correlation co-efficient :-

Correlation Co-efficient is a measure of dependancy between random variables X and Y.

It is defined as the ratio of Covariance (λ_{XY} or $Cov(X,Y)$) to the product ($\sigma_X \cdot \sigma_Y$).

It is denoted by symbol ρ_{XY}

i.e., Correlation Co-efficient is given by

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y}\ ;$$

where
σ_X = Standard deviation of X
σ_Y = Standard deviation of Y

$$\therefore \quad \rho_{XY} = \frac{E[XY] - m_x m_y}{\sigma_X \cdot \sigma_Y}$$

It gives dependancy between random variables X and Y.

i.e., (i) If $\rho_{XY} = 0 \Rightarrow$ Random Variables X and Y are said to be Completely Independent

(ii) If $\rho_{XY} = 1 \Rightarrow$ X and Y are strongly dependent on each other.

(iii) If $0 < \rho_{XY} < 1 \Rightarrow$ X and Y are said to neither independent nor totally dependent on each other.

***

3.9: Auto correlation function :- <VTU Q.P>

Q) Define auto correlation function. Mention its important properties.

V.T.U Dec 2017/Jan 2018 (6 Marks)

→ **Definition**: The auto correlation function of the random variable 'X', is defined as the Expectation of the product of two random variables, $X(k)$ and $X(l)$, obtained by observing the random variable 'X' at times 'k' and 'l' respectively.

It is denoted by $\gamma_X(k,l)$ (or) $\gamma_X(\tau)$

i.e., Auto Correlation function is given by

$$\boxed{\gamma_X(k,l) = E[X(k) \cdot X(l)] = \gamma_X(k-l)}$$

∴ Auto correlation function gives correlation of a signal with itself but delayed in time.

- Auto correlation function $\gamma_X(k,l)$ is a function of time difference $(k-l)$.

Properties :-

Let $\gamma_X(\tau)$ be the auto correlation function of Random process 'X' (Variable) then

⟨i⟩ $\gamma_X(\tau) = \gamma_X(k-l)$: It is a function of time difference $(k-l)$

⟨ii⟩ $\gamma_X(0) = E[X^2]$; when $k=l$.

⟨iii⟩ $\gamma_X(\tau)$ is maximum value at $\tau = 0$.

i.e., $\gamma_X(0) \geq \gamma_X(\tau)$ for any value of 'τ'

(iv) $\gamma_X(\tau)$ is a even function of (τ)

i.e, $\gamma_X(\tau) = \gamma_X(-\tau)$

* Auto Covariance :- (VTU. Q.P)

Auto Covariance function of random process 'X' is defined as,

$$\boxed{\begin{aligned} C_X(K,l) &= E\left[(X(K)-m_x)\cdot(X(l)-m_x)\right] \\ C_X(K,l) &= r_X(K-l)-m_X^2 \end{aligned}}$$

where $C_X(K,l)$ = Auto Covariance
$r_X(K-l)$ = Auto Correlation between $X(K)$ and $X(l)$
m_X = Mean of random variable 'X'.

3.10 :- Cross Correlation function :-

(Q) Define Cross Correlation function. Mention its properties. (8 Marks) VTU Q.P.

→ Definition:- Consider two random variables X and Y, observed at time instants 'K' and 'l' respectively, then the cross correlation function between random variables X and Y is given by

$$r_{XY}(K,l) = E\left[X(K)\cdot Y(l)\right]$$

(or)

$$r_{YX}(l,K) = E\left[Y(l)\cdot X(K)\right]$$

∴ cross correlation function gives the correlation between two different random processes.

→ Properties :-

1. It is symmetric function
   i.e, $r_{XY}(\tau) = r_{XY}(-\tau)$
2. It does not have maximum at origin
3. $|r_{XY}(\tau)| \leq \frac{1}{2}\left[r_X(0)+r_Y(0)\right]$
   where $r_X(0)$ & $r_Y(0)$ are auto-correlation functions of X and Y respectively at $\boxed{\tau=0}$

Numerical problem on Mean, Variance & Standard deviation :- 20

Formulae : • $m_x = E[x] = \int_{-\infty}^{\infty} x \, f_x(x) \, dx \Rightarrow$ Mean

• $Var(x) = E[x^2] - m_x^2 \Rightarrow$ Variance (σ_x^2)

• Standard deviation : $\sigma_x = +\sqrt{Var(x)}$

① A random variable 'x', has distribution function

$$F_X(x) = \begin{cases} 0 & ; x<0 \\ \frac{x}{8} & ; 0 \le x \le 2 \\ \frac{x^2}{16} & ; 2 \le x \le 4 \\ 1 & ; 4 \le x \end{cases}$$

Determine (a) Mean $(\bar{x})$ (b) Variance (c) Standard deviation.

Ans :- from given data, probability distribution function,

$$F_X(x) = \begin{cases} 0 & ; x<0 \\ x/8 & ; 0 \le x \le 2 \\ x^2/16 & ; 2 \le x \le 4 \\ 1 & ; x \ge 4 \end{cases}$$

∴ probability density function $f_X(x)$ is,

$$f_X(x) = \frac{d F_X(x)}{dx} = \begin{cases} 0 & ; x<0 \\ 1/8 & ; 0 \le x \le 2 \\ x/8 & ; 2 \le x \le 4 \\ 0 & ; x \ge 4 \end{cases}$$

We can find Mean, Variance & Standard deviation using density function $f_X(x)$.

(a) Mean :

$$E[X] = \int_{-\infty}^{\infty} x \, f_X(x) \, dx = \int_0^2 x \cdot \left(\tfrac{1}{8}\right) dx + \int_2^4 x \cdot \left(\tfrac{x}{8}\right) \cdot dx$$

$$\therefore E[X] = 2.583$$

(b) Variance : W.K.T $Var(x) = E[x^2] - m_x^2$

$$\therefore E[x^2] = \int_{-\infty}^{\infty} x^2 f_X(x) \, dx = \int_0^2 x^2 \cdot \left(\tfrac{1}{8}\right) dx + \int_2^4 x^2 \left(\tfrac{x}{8}\right) dx = 7.833$$

$$\therefore Var(x) = \sigma_x^2 = E[x^2] - m_x^2 = 7.833 - (2.583)^2 = 1.161$$

(c) Standard deviation :- $\sigma_x = +\sqrt{Var(x)} = 1.107$

Module 3: (b); Noise :-

Q) Define Noise. Mention its various types. (VTU Q.P 6 Marks)

→ Noise is a disturbance (or) unwanted frequency signal which appears within the required frequency/signal.

The classification of Noise-sources are shown in Fig.1.

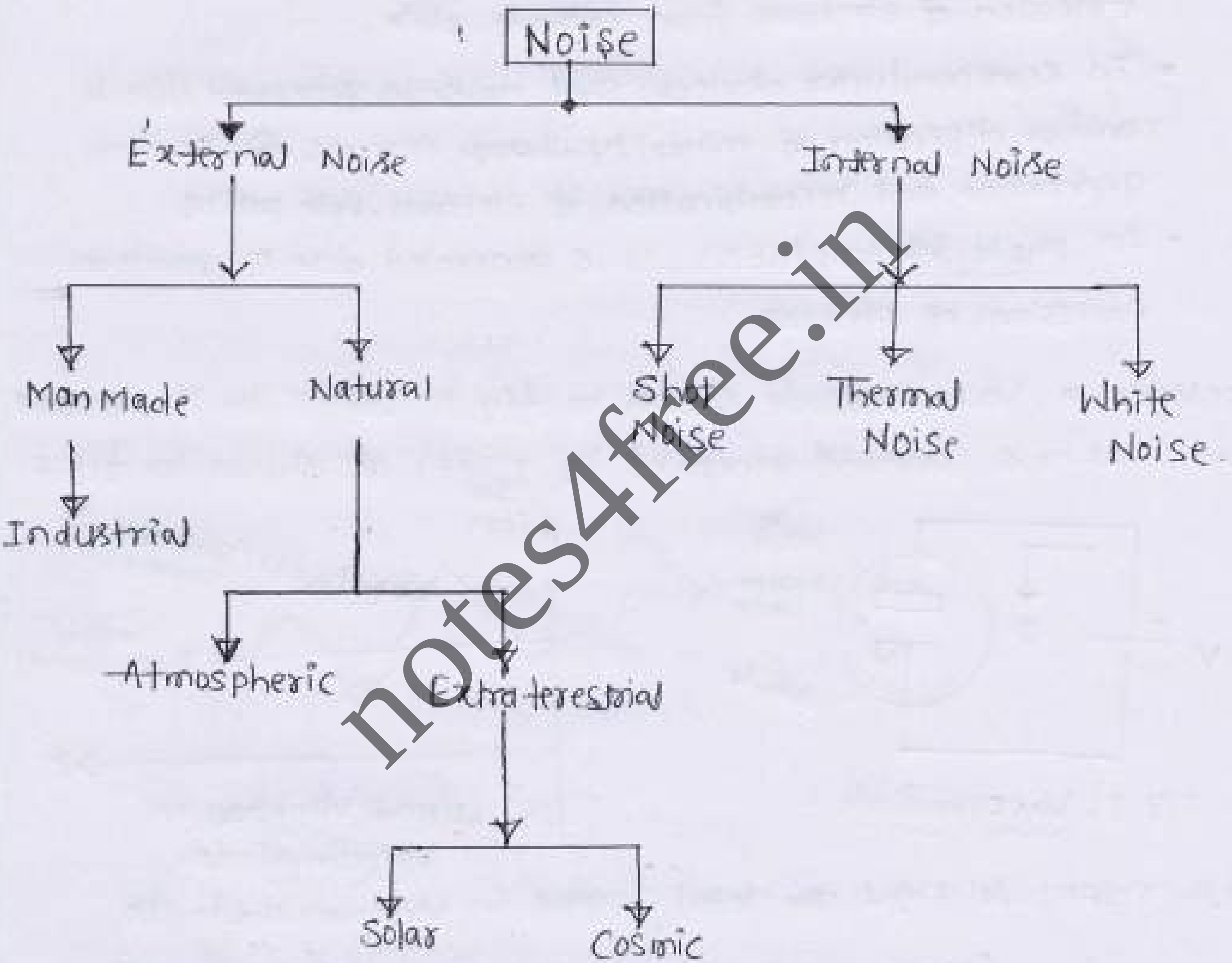


Fig 1: Classification of Noise-sources/signals

Shot Noise: Shot Noise appears in active devices due to random behaviour of charge carriers (electrons and holes).

Thermal Noise: Thermal noise is generated to random movement of thermally electrons.

White Noise: The Noise which is gaussian distribution and has flat (or) constant power spectral density over wide range of frequencies.

*** VTUQP

3.11: Shot Noise :-

(Q) Write a short note on shot Noise. 4-Marks.

↳ Shot Noise appears in active devices due to random behaviour of charge carriers (electrons and holes).

Example:

- In Vaccum tubes, shot noise is generated due to random emission of electrons from Cathode-plate.
- In semiconductor devices, shot noise is generated due to random diffusion of minority charge carriers (or) random generation and recombination of electron-hole pairs.
- In photodetectors/LED's, it is generated due to random emission of photons.

Consider a vaccum diode shown in fig.1. Let 'I' be the current and shot noise component generated is $I_{SN}(t)$ as shown in fig.2.

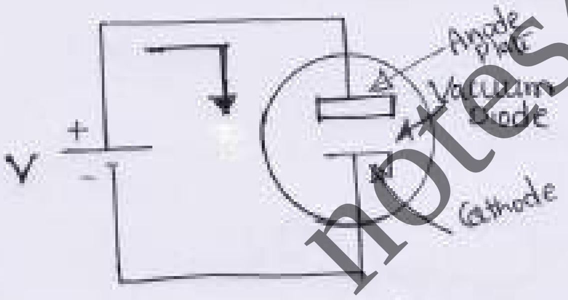


Fig.1: Vaccumdiode

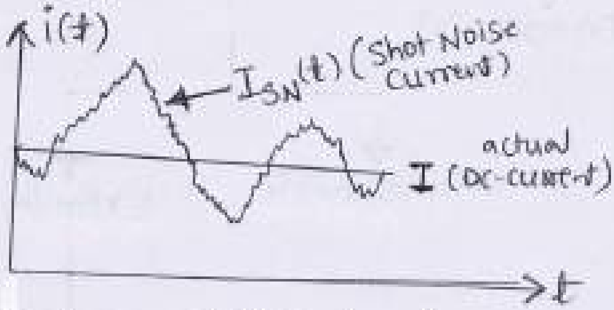


Fig2: Current Variation in Vaccumdiode

With respect to Fig.2 the total current in vaccum diode is,

$$i(t) = I + I_{SN}(t)$$; $I_{SN}(t) \Rightarrow$ shot noise current generated due to random emission of electrons from Cathode.

↳ ∴ The Mean square value of fluctuating shot noise current in vaccum diode is given by

$$E\left[I_{SN}^2\right] = 2qIB_N$$

; Where $q = 1.6 \times 10^{-19}$ Coulumbs

B_N = Noise Equivalent Bandwidth

I = required DC current in Vaccum diode.

↳ for PN-Junction diodes,

$$E\left[I_{SN}^2\right] = 2q(I+I_s)B_N$$

I_s = Reverse Leakage current ≈ 0A

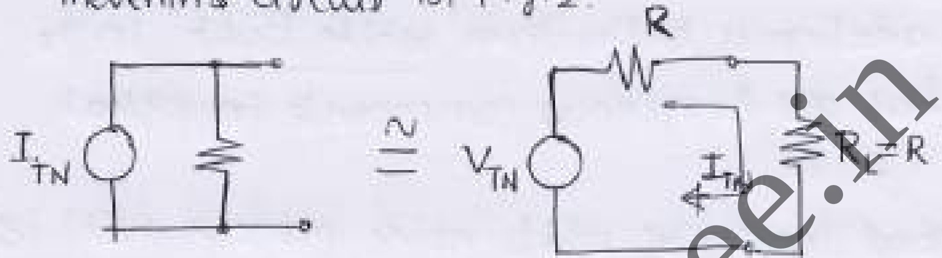
3.12: Thermal Noise :- (VTU Q.P)

a) Write a short note on Thermal Noise.

↳ Thermal Noise is generated due to random moment of thermally induced carriers (electrons) in a conductor.

↳ The random motion of thermally induced electrons produces electric current which is random in nature. This random current is called "thermal noise" @ "Johnson Noise"

↳ Figure 1 shows noise-Model using resistor and its equivalent thevenin's circuit in fig. 2.



I_{TN} = Thermal Noise Current

$V_{TN} = I_{TN} \times R$ ↳ Thermal Noise voltage.

Fig 1: Noise model

Fig 2: Equivalent circuit (To find Max Noise Power) $R_L = R$

↳ The Mean value of thermal noise current is always zero.

↳ Mean square value of the thermal noise voltage is

$$E[V_{TN}^2] = 4KB_NTR$$

where K = Boltzmann Constant $= 1.38 \times 10^{-28}$ J/s

T = temperature at which resistor is operating $= 290\,K$ (Standard)

B_N = Noise equivalent Bandwidth in Hz.

R = Resistor in Ω

↳ The Maximum noise power, produced across noisy resistor model shown in fig. 2 is

$$P_N = \frac{E[V_{TN}^2]}{4R} = \frac{4KB_NTR}{4R} = KTB_N \text{ Watts.}$$

Note: Max. power delivered to load when $R_L = R$ &

$$P_{max} = \frac{V^2}{4R} \ //.$$

3.13 : White Noise:- (VTU Q.P)

Q) Define white Noise. plot power spectral density and auto correlation function (ACF) of white noise

VTU June/July 2017 - 5 Marks -

↳ **White Noise:-**

The Noise which has Gaussian distribution and have flat power spectral density over a wide range of frequencies is called white Noise.

- It is denoted by $w(t)$.
- White Noise is analogous to the term "white light" in the Sense that it has all frequency components in equal amounts.

↳ The power spectral density of white noise process, $w(t)$ is

$$S_W(f) = \frac{N_0}{2} \quad \text{--- (1)}$$

where
$N_0 = K T_e$
T_e = Noise equivalent temperature

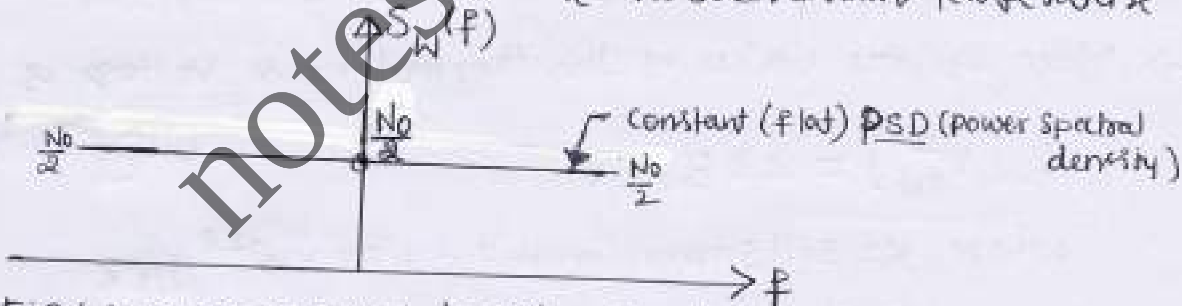


Fig 1: power spectral density of white Noise:-

↳ Auto-correlation function of white noise, $w(t)$ is

$$r_W(\tau) = \frac{N_0}{2}\delta(t) \quad \left(\text{i.e., } S_N(f) \xleftrightarrow{IFT} r_W(\tau)\right)$$

$$\therefore r_W(\tau) = \begin{cases} \frac{N_0}{2} ; & \tau = 0 \\ 0 & \text{elsewhere} \end{cases}$$

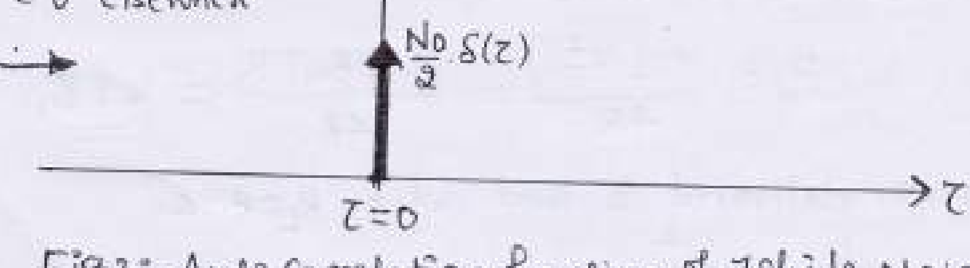


Fig 2: Auto correlation function of white Noise

3.14. Noise Equivalent Bandwidth ***

Q) What is noise equivalent Bandwidth? Derive an expression for the same. VTU June/July - 2017 (5 Marks)

→ Noise equivalent bandwidth represents the frequency selectivity of the filter.

It is denoted by symbol 'B_N' HZ.

Mathematically, it is defined as the ratio of total noise - output power to that of Noise spectral density at input.

i.e., $B_N = \frac{P_{no}}{S_{n_i}}$ —(*)

where, P_{no} = Total noise output power in Watts

S_{n_i} = Input noise spectral density in Watts/Hz.

equation (1) is simplified to, $B_N = \frac{1}{4RC}$ HZ.

Proof :- consider a Low pass Filter (LPF) having voltage ratio transfer function $|H(j\omega)|$, as shown in Fig.1 and its equivalent circuit in Fig.2.

Fig 1: LPF with i/p & o/p spectral density

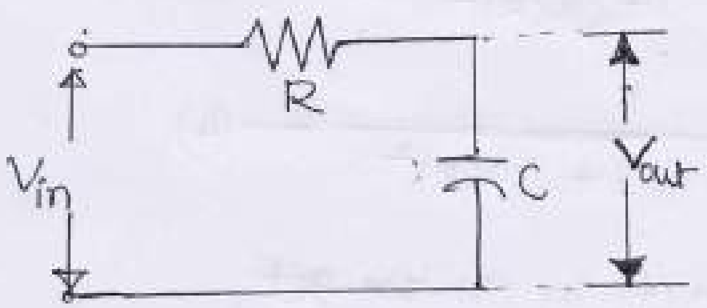


Fig 2: RC-LPF with input and output Voltage

We Know that Noise bandwidth B_N, is given by

$$B_N = \frac{P_{n_0}}{S_{n_i}} \longrightarrow (1)$$

Input noise Spectral density, $S_{n_i} = KT \rightarrow (2)$

K = Boltzmann's Constant

To find total Noise output power: (P_{n_0})

From Fig. 1 the Voltage transfer function, $H(j\omega)$ in terms of input and output noise spectral density S_{n_i} & S_{n_0} is

$$H(j\omega) = \sqrt{\frac{S_{n_0}}{S_{n_i}}} \Rightarrow |H(j\omega)|^2 = \frac{S_{n_0}}{S_{n_i}}$$

$$\therefore \quad S_{n_0} = |H(j\omega)|^2 S_{n_i}$$

$$S_{n_0} = |H(j\omega)|^2 \cdot KT \longrightarrow (3)$$

From Fig. 2, RC-LPF the Voltage transfer function,

$$H(S) = \frac{V_{out}}{V_{in}} = \frac{1/SC}{R + 1/SC} = \frac{1/SC}{\frac{SCR+1}{SC}} = \frac{1}{1+SCR}$$

$$\therefore H(S) = \frac{1}{1+SCR} \; ; \; S = j\omega$$

$$\therefore H(j\omega) = \frac{1}{1+j\omega CR}$$

Take Magnitude on both sides, we get

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega CR)^2}}$$

$$\therefore |H(j\omega)|^2 = \frac{1}{1+(\omega CR)^2} \quad \text{---} (4)$$

Using equation (4) in (3) we get,

$$S_{n_0} = \frac{1}{1+(\omega CR)^2} \cdot KT$$

$$S_{n_0} = \frac{KT}{1+(\omega CR)^2} \quad \text{——(5)}$$

$\therefore$ The ~~total~~ noise output power, P_{n_0} is

$$P_{n_0} = \int_0^\infty S_{n_0} \cdot df$$

$$P_{n_0} = \int_0^\infty \frac{KT}{1+(\omega CR)^2} \cdot df$$

W.K.T $\omega = 2\pi f$

$$\therefore P_{n_0} = \int_0^\infty \frac{KT}{[1+(2\pi f CR)^2]} \cdot df \quad \text{——(6)}$$

Let $2\pi f CR = x$ then $(2\pi f CR)^2 = x^2$

differentiate 'f' with respect to 'x' we get

$$2\pi CR \frac{df}{dx} = 1 \Rightarrow df = \frac{dx}{2\pi CR}$$

$$\therefore P_{n_0} = \int_0^\infty \frac{KT}{(1+x^2)} \cdot \frac{dx}{2\pi CR} = \frac{KT}{2\pi CR} \int_0^\infty \frac{1}{1+x^2} \cdot dx.$$

$$\therefore P_{n_0} = \frac{KT}{2\pi CR} \left[\tan^{-1}(x)\right]_0^\infty \qquad \left(\because \int \frac{1}{1+x^2} = \tan^{-1} x\right)$$

$$P_{n_0} = \frac{KT}{2\pi CR}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right] = \frac{KT}{2\pi CR}\left(\frac{\pi}{2} - 0\right)$$

$$P_{n_0} = \frac{KT}{2\pi CR} \times \frac{\pi}{2} = \frac{KT}{4CR} \quad \text{——(7)}$$

using equations (7) & (2) in equation (1) we get

$$B_N = \frac{P_{n_0}}{S_{n_i}} = \frac{KT}{4CR \times KT} = \frac{1}{4CR} \qquad (\because S_{n_i} = KT)$$

hence the proof.

3.15 : Noise Factor and Noise Figure:-

Noise factor, of an any amplifier (or) network is defined as the ratio of signal to noise power ratio at the input and to that of signal to noise power at the output.

i.e, Noise factor, $F = \dfrac{(S/N) \text{ power ratio at the input}}{(S/N) \text{ power ratio at the output}}$

$$\text{i.e, } F = \frac{(P_{Si}/P_{Ni})}{(P_{So}/P_{No})} = \frac{P_{Si}}{P_{Ni}} \times \frac{P_{No}}{P_{So}} \longrightarrow (1)$$

P_{Si} = signal power at input in Watts.

P_{Ni} = Noise power at input in Watts.

P_{So} = Signal power at output in Watts.

P_{No} = Noise power at output in Watts.

Noise Figure:- (F_{dB})

Noise factor, F expressed in dB, is known as Noise Figure.

$$\text{i.e, } \boxed{\text{Noise Figure} = 10 \log [F]}$$

(or)

$$\text{Noise Figure} = 10 \log \left[\frac{(S/N)_{input}}{(S/N)_{output}} \right]$$

$$\Downarrow$$

$$F_{dB} = \underbrace{10 \log (S/N)_{input}}_{(S/N)_{input} \text{ in dB}} - \underbrace{10 \log (S/N)_{output}}_{(S/N)_{output} \text{ in dB}}$$

Note: The ideal value of Noise figure is 0 dB

* Equivalent Noise temperature ⟨ T_e ⟩ :-

The equivalent Noise temperature is given by,

$$\boxed{T_e = (F-1)\,T}$$

where, F = Noise factor

T = Temperature in kelvin = 290K (T_0) (Standard value)

* Cascade Connection of two port networks :- (OR)

⟨Frii's formula for Amplifiers connected in Cascade⟩ VTU Q.P (6Marks)

Consider cascade connection of Noisy two-port networks (Amplifiers) shown in fig. 1.

Fig. 1: Cascade connection of N-noise networks (Amplifiers)

where G_N = Gain of Network-N

F_N = Noise factor of Network-N

The overall Noise factor F for N-amplifiers (networks) connected in cascade is given by Friis-formula,

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1 G_2} + \cdots\cdots \longrightarrow (i)$$

and its equivalent noise temperature is given by

$$T_e = (F-1)\,T$$

where 'F' is given by equation (i)

Note: For 2-Amplifiers connected in Cascade $\Big\}$ $F = F_1 + \dfrac{F_2 - 1}{G_1}$

Numerical problems:-

List of Formulae:-

1. Noise Figure: $F_{dB} = 10 \log (F)$: where F = Noise factor $= \frac{(S/N)_{Input}}{(S/N)_{output}}$

2. Noise equivalent Temperature : $T_e = (F-1)T$

3. Friis formula for cascaded Networks/Amplifiers:-

(i) For 2-Networks (Amplifiers)

Noise factor: $F = F_1 + \frac{F_2 - 1}{G_1}$

Noise Figure: $F_{dB} = 10 \log (F)$

(ii) For 3-Networks (Amplifiers)

Noise factor: $F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$

Noise Figure: $F_{dB} = 10 \log (F)$

4. Noise equivalent Bandwidth:-

$$B_N = \frac{1}{4RC} \quad Hz.$$

5. Shot Noise:

Mean square value of shot noise current is

$E[I_{SN}^2] = 2qIB_N$: $q = 1.6 \times 10^{-19}$ columbs

6. Thermal Noise:-

- Mean square value of Thermal Noise voltage is

$E[V_{TN}^2] = 4KB_NTR$ and K = Boltzmann's Constant

$K = 1.38 \times 10^{-28}$

- Noise power: $P_N = KTB_N$

1. Suppose amplifier 1 has a noise figure of 9 dB and power gain of 15 dB. It is connected in cascade to the amplifier 2 with noise figure of 20 dB. Calculate the overall noise figure for this cascade connection in decibel units.

VTU Dec-2017/Jan-2018 4-Marks

↳ Given data :-

$F_{1_{dB}} = 9\,dB \;;\; G_{1_{dB}} = 15 \;;\; F_{2_{dB}} = 20 \;;\; F_{dB} = ?$

$\therefore F_1 = 10^{\left(\frac{9}{10}\right)} \;;\; G_1 = 10^{\left(\frac{15}{10}\right)} \;;\; F_2 = 10^{\left(\frac{20}{10}\right)}$

$\boxed{F_1 = 7.943} \;;\; \boxed{G_1 = 31.623} \;;\; \boxed{F_2 = 100}$

Using Frii's formula for 2-amplifiers connected in cascade,

overall noise factor, $F = F_1 + \frac{F_2 - 1}{G_1} = 7.943 + \frac{100-1}{31.623}$

$\therefore F = 11.073$

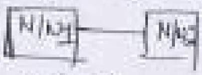
$\therefore$ overall Noise figure, $F_{dB} = 10\log(F) = 10\log(11.073)$

$\boxed{F_{dB} = 10.443}\ \underline{dB}$

2. Two port devices are connected in cascade. For the first stage, the noise figure and available power gain are 5 dB and 12 dB respectively. For the second stage, the noise figure and available power gain are 15 dB and 10 dB respectively. Determine the overall noise figure in dB

June/July-2016 (6 Marks)

↳ Given data :



$F_{1dB} = 5 \Rightarrow F_1 = 10^{(5/10)} = 3.162$

$G_{1dB} = 12 \Rightarrow G_1 = 10^{(12/10)} = 15.85$

$F_{2dB} = 15 \Rightarrow F_2 = 10^{(15/10)} = 31.623$

$G_{2dB} = 10 \Rightarrow G_2 = 10^{\left(\frac{10}{10}\right)} = 10$

$\therefore$ overall Noise factor,

$F = F_1 + \frac{F_2 - 1}{G_1} = 3.162 + \frac{31.623 - 1}{15.85}$

$\therefore \boxed{F = 5.094}$

$\therefore$ overall Noise Figure,

$F_{dB} = 10\log(F) = 7.07\,dB$ //

(3) In a TV-receiver, the antenna is often mounted on a tall mast and a long cable is used to connect the antenna, to the receiver. To overcome the effect of lossy cable, a pre-amplifier is mounted on the antenna as shown in figure 3. (32)

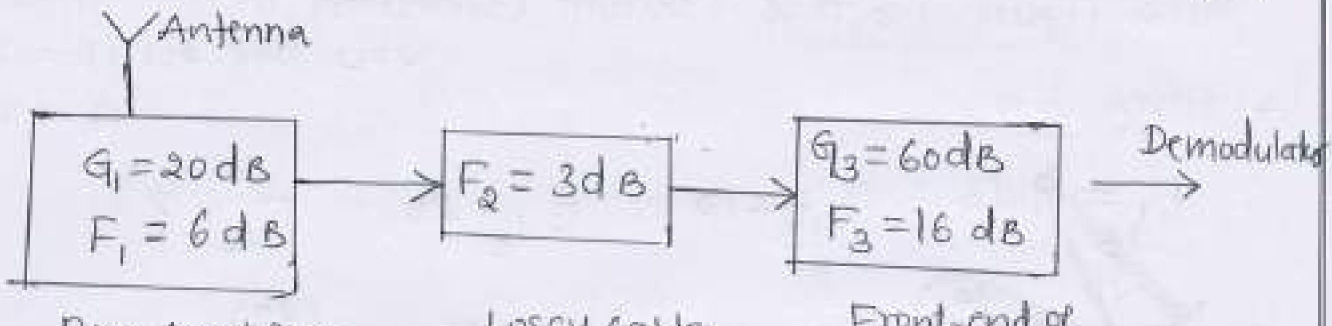


Figure3: TV-receiver

(a) Find the overall noise figure of the system

(b) Find the overall noise figure of the system, if the pre-amplifier is omitted and the gain of front end receiver is increased by 20dB.

→ Given data :-

(i) pre-Amplifier:

$G_{1dB} = 20 \Rightarrow G_1 = 10^{\left(\frac{20}{10}\right)} = 100$

$F_{1dB} = 6 \Rightarrow F_1 = 10^{\left(\frac{6}{10}\right)} = 3.98$

(ii) Lossy cable:-

$F_{2dB} = 3 \Rightarrow F_2 = 10^{\left(\frac{3}{10}\right)} \approx 2$

* $G_2 = \frac{1}{F_2} = \frac{1}{2} = 0.5$

Gain for Lossy cable

(iii) Front-end of the receiver:-

$G_{3dB} = 60 \Rightarrow G_3 = 10^{\left(\frac{60}{10}\right)} = 1000$

$F_{3dB} = 16 \Rightarrow F_3 = 10^{\left(\frac{16}{10}\right)} = 39.81$

(a) overall Noise figure of the system :-

for 3-Networks Friis formula is

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

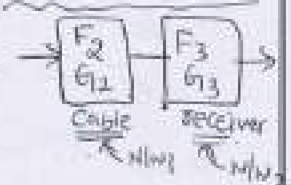
$$F = 3.98 + \frac{(2-1)}{100} + \frac{(39.81-1)}{100 \times 0.5}$$

$F = 4.7662$

$\therefore F_{dB} = 10\log(F) = 6.78\,dB$

(b) If pre-amplifier is omitted:-

$$F = F_2 + \frac{F_3 - 1}{G_2}$$



$F = 79.62$

$\therefore F_{dB} = 10\log F = 19.01\,dB$

4) The signal power and noise power measured at the input of an amplifier are 150μW and 1.5μW respectively. If the signal power at the output is 1.5μW and noise power is 40mW calculate the amplifier noise factor and noise figure.

↳ Given data:

$P_{S_i} = 150\mu W$

$P_{n_i} = 1.5\mu W$

$P_{S_o} = 1.5\mu W$

$P_{n_o} = 40 mW$

$F = ?$

$F_{dB} = ?$

W.K.T the Noise factor in terms of signal, noise power ratio is given by

$$F = \frac{P_{Si}}{P_{n_i}} \times \frac{P_{no}}{P_{So}} = \frac{150\times10^{-6}}{1.5\times10^{-6}} \times \frac{40\times10^{-3}}{1.5\times10^{-6}}$$

$\therefore F = 2.666$

$\therefore$ Noise Figure, $F_{dB} = 10\log(F) = \underline{4.26\,dB}$

5) The SNR at the input of an amplifier is 40dB. If the noise figure of an amplifier is 20dB, calculate the signal to noise ratio in dB at the amplifier output

↳ Given data:-

$(SNR)_{input} = 40\,dB$

$F_{dB} = 20$

$(SNR)_{output\,dB} = ?$

W.K.T

Noise factor, F in-terms of (SNR) ratio is

$$F = \frac{(SNR)_{input}}{(SNR)_{output}} \Rightarrow 10\log F = 10\log\left(\frac{(SNR)_{input}}{(SNR)_{output}}\right)$$

$\therefore F_{dB} = (SNR)_{input\,dB} - (SNR)_{output\,dB}$

$\Downarrow \qquad \Downarrow$

$20 = 40 - (SNR)_{output\,dB}$

$$\therefore \boxed{(SNR)_{output} = 20\,dB}$$

6) A signal circuit is equivalent to a parallel combination of $R = 1k\Omega$ and $C = 0.47\mu F$. Calculate the effective noise bandwidth.

$R = 1k\Omega$

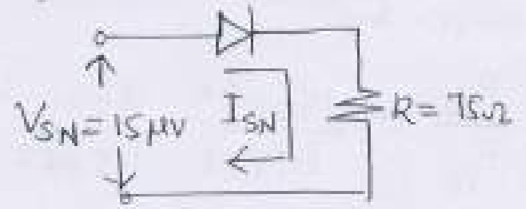
$C = 0.47\mu F$

W.K.T Noise Equivalent bandwidth (Effective Noise Bandwidth)

$$B_N = \frac{1}{4RC} = \frac{1}{4\times1\times10^{3}\times0.47\times10^{-6}} = \underline{531.915\,Hz}$$

<7> A noise generator using diode is required to produce $15\mu V$ noise voltage in a receiver which has an input impedance of 75Ω (purely resistive). The receiver has a noise power bandwidth of 200KHZ. Calculate the current through the diode.

↳ Data:-



and $B_N = 200\,KHZ$

$\therefore$ Shot noise current in diode is: $I_{SN} = \frac{V_{SN}}{R} = \frac{15\times10^{-6}}{75} = 2\times10^{-7}\,A$

the actual current, I, through the diode is given by

$$E[I_{SN}^2] = 2qIB_N$$

$$\Downarrow$$

$$(2\times10^{-7})^2 = 2\times1.6\times10^{-19}\times I\times200\times10^3$$

$$\therefore I = \frac{(2\times10^{-7})^2}{2\times1.6\times10^{-19}\times200\times10^3} = 625\,mA$$

<8> Calculate the rms noise voltage and thermal noise power appearing across a $20k\Omega$ resistor at $25^\circ c$ temperature with an effective noise Bandwidth of 10KHZ

↳ Given: $R = 20K\Omega$; $T = 25^\circ c = 273 + 25 = 298\,K$; $B_N = 10\,KHZ$

(i) rms noise voltage :- $\left(\sqrt{E[V_{TN}^2]}\right)$ (Root Mean Square Value of V_{TN})

W.K.T $E[V_{TN}^2] = 4KTB_NR$

$$E[V_{TN}^2] = 4\times1.38\times10^{-23}\times298\times10\times10^3\times20\times10^3$$

$$E[V_{TN}^2] = 3.2899\times10^{-12}\ \text{Volt}^2$$

$$\sqrt{E[V_{TN}^2]} = 1.8138\,\mu V \Leftarrow \text{RMS noise voltage.}$$

(ii) Thermal noise power:-

$$P_N = KTB_N = 1.38\times10^{-23}\times298\times10\times10^3$$

$$P_N = 4.11\times10^{-17}\ \text{Watts}$$

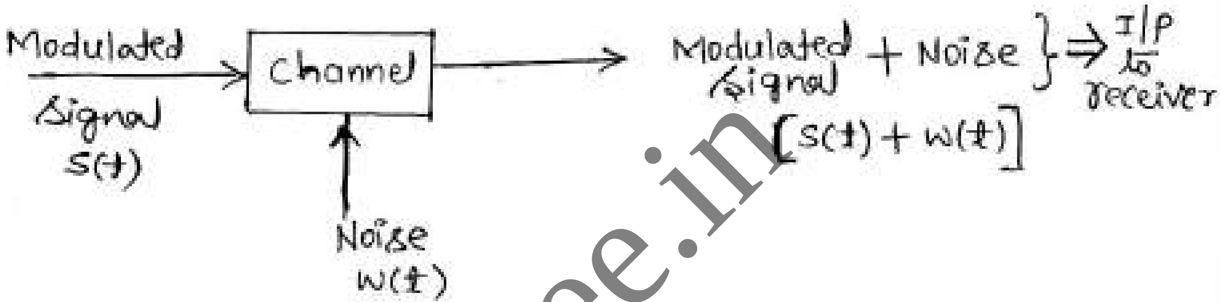
Module- 4

Noise in Analog Modulation

Introduction :-

Any unwanted frequency component which appears within the operating frequency is considered as "Noise".

↳ Noise gets added to the transmitted signal s(t), during the transmission over communication channel.



↳ In this module the effect of Noise on receiver performance parameters such as

(i) pre-SNR (or) channel signal to Noise ratio $(SNR)_{channel}$

(ii) post-SNR (or) output signal to Noise ratio $(SNR)_{output}$

(iii) Figure-of-Merit (FOM), for analog modulation techniques such as DSBSC, AM and FM are discussed in detail.

↳ This module also gives the overview of pre-emphasis and De-emphasis methods used to improve signal to Noise ratio at transmitter and receiver respectively.

↳ Higher the value of Figure-of-Merit <FOM> better the performance of the receiver.

↳ FOM, for various analog Modulation techniques is found to be

- for DSBSC ⟶ FOM = 1
- for AM ⟶ $FOM = \frac{\mu^2}{2+\mu^2}$
- for FM ⟶ $FOM = 1.5\beta^2$

2

4.1 : Receiver Model :-

Q) With relevant equations, explain how noise is produced in a receiver model. June/July - 2017 (8-Marks)

↳

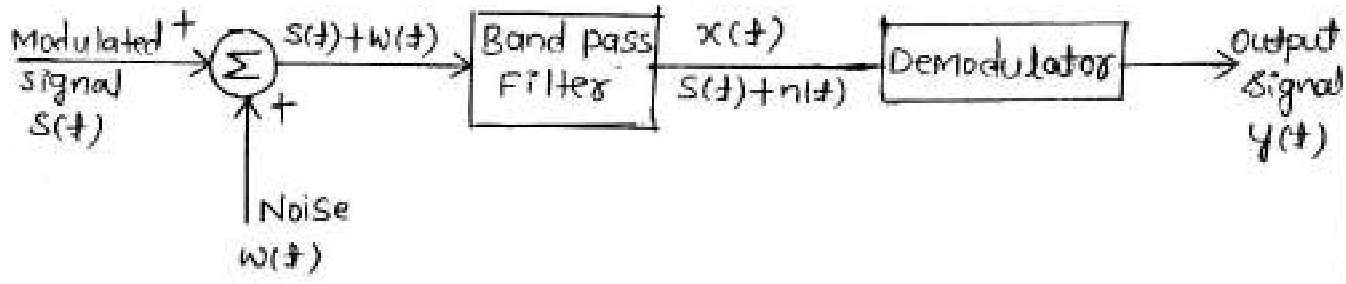


Fig.1 : Receiver Model

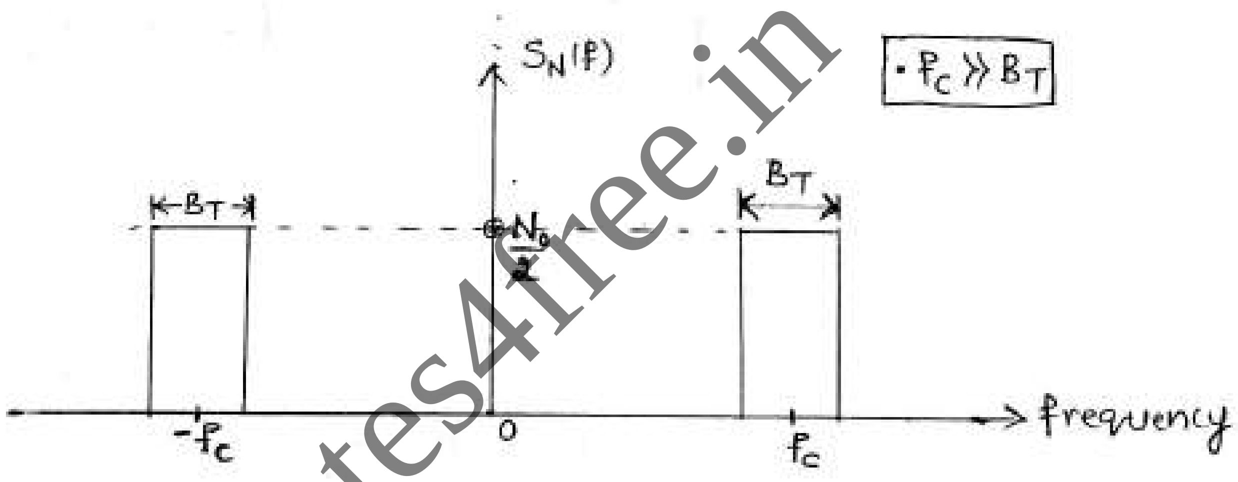


Fig.2 : Idealized characteristics of Band pass filter noise

Figure 1, shows the basic form of receiver model.

Let s(t) ⇒ Modulated signal

w(t) ⇒ Noise signal (wide-band noise)

↳ The receiver input signal is the sum of S(t) and W(t).

↳ The band pass filter in the receiver model represents the combined filtering action of the tuned amplifiers used in the actual receiver.

↳ The Bandwidth of a Band pass Filter <BPF> is kept just wide enough to pass the modulated signal s(t) without distortion.

↳ The Demodulator block represented in Figure 1, depends on the type of modulation used to generate modulated signal, $s(t)$.

↳ The BPF, shown in receiver model is assumed to be ideal with characteristics of band pass filtered noise as shown in figure 2.

↳ For the receiver model shown in figure 1, we can define the following parameters

- We denote $\frac{N_0}{2}$ as the power spectral density of the noise $w(t)$ for both positive and Negative frequencies. where N_0 = Average noise power per unit bandwidth
- Mid-band frequency is equal to the carrier frequency and is denoted by "f_c"
- Typically the carrier frequency, $f_c \gg B_T$ as shown in figure 2.

↳ We consider the filter noise, $n(t)$ as a narrowband noise and is defined in canonical form by

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \longrightarrow (1)$$

where, $n_I(t)$ is the inphase noise component and $n_Q(t)$ is the Quadrature noise component, both components are measured with respect to the carrier wave $A_c\cos(2\pi f_c t)$

↳ The filtered signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t) \longrightarrow (2)$$

The Average Noise power is given by "N_0W" $\left(\because \frac{N_0}{2} \times 2W = N_0W\right)$

$x(t)$ is the output signal obtained from channel and is available for demodulation. Therefore Pre-SNR (SNR- before demodulation) or Channel signal to Noise ratio $(SNR)_C$ is defined as

• $(SNR)_c = \dfrac{\text{Average power of the Modulated signal}}{\text{Average power of the noise in message Bandwidth}}$ $\longrightarrow$ (3)

↳ The Demodulated signal <output signal> of the receiver model shown in figure 1, is given by

$$y(t) = m_d(t) + n_d(t) \longrightarrow (4)$$

where, $m_d(t)$ = demodulated signal Extracted from $s(t)$ and

$n_d(t)$ = demodulated output noise signal.

• Therefore, post-SNR (SNR after demodulation) or output signal-to-noise-ratio $(SNR)_0$ is defined as

• $(SNR)_0 = \dfrac{\text{Average power of the demodulated output signal}}{\text{Average power of output Noise.}}$ $\longrightarrow$ (5)

• Therefore, the Figure-of-Merit <FOM> for the receiver is given by,

$$\text{Figure of Merit} = \frac{(SNR)_0}{(SNR)_c} \longrightarrow (6)$$

Higher the value of Figure of merit, better the performance of the receiver. Its value depends on the type of modulation.

Note: (i) The average power of any continuous time signal is given by its second-order moment.

i.e., • average power of $s(t) = E[(s(t))^2] \longrightarrow$ depends on $s(t)$

• average power of $n(t) = E[(n(t))^2] = \underline{N_0 W} \Leftarrow$ constant for all modulation types

• Average power of "$A_c\cos(2\pi f_c t)$" $= E[(A_c\cos 2\pi f_c t)^2] = \dfrac{A_c^2}{2} \Leftarrow$ constant for sine & cosine functions.

4.2 Noise in DSB-SC Receivers :-

<Q> Show that Figure of merit for DSBSC System is Unity.

June/July-2017 (8-Marks)

↳

Let • $m(t)$ be the message signal and 'P' be the average power in $m(t)$.

• $c(t)$ be the carrier signal, then ~~time~~ domain expression for DSBSC-signal is given by the product of $m(t)$ and $c(t)$.

∴ DSBSC-modulated signal, $S(t)$ is

$$S(t) = m(t) \cdot c(t)$$

$$S(t) = m(t) \cdot [A_c \cos(2\pi f_c t)] \quad \rightarrow (1)$$

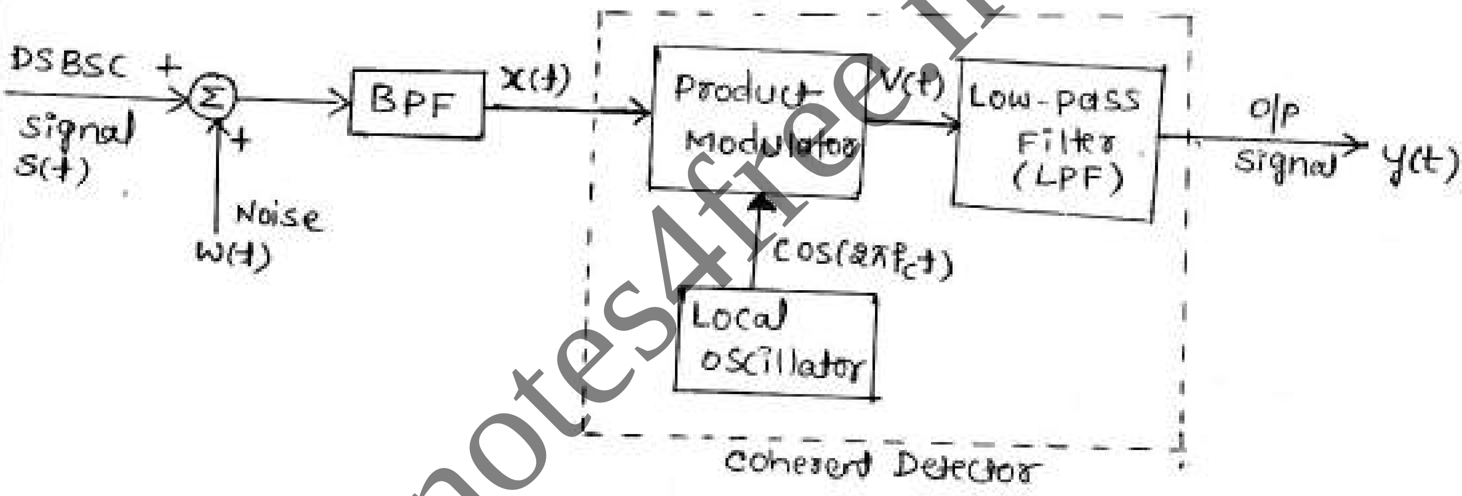


Figure 1 : Model of DSBSC receiver using Coherent Detector

- Figure 1, shows the model of a DSBSC receiver using a Coherent Detector.
- In figure 1, the filtered signal $[x(t) = s(t) + n(t)]$ is applied to product modulator.
- The product modulator multiplies the filtered signal $x(t)$ with locally generated carrier "$\cos 2\pi f_c t$" & produces the product signal $V(t) = x(t) \cdot \cos(2\pi f_c t)$
- $V(t)$ is applied to low-pass-filter it eliminates all higher frequency components & produces output signal $y(t) = m_d(t) + n_d(t)$.

To find channel SNR $(SNR)_c$:-

↳ The DSBSC signal is given by

$$S(t) = m(t) \times A_c \cos(2\pi f_c t) \quad \text{—— } (*)$$

• Therefore, the average power of the modulated signal $s(t)$ is

$$E\left[(s(t))^2\right] = E\left[(m(t))^2 \cdot (A_c \cos 2\pi f_c t)^2\right] = \frac{A_c^2}{2} \cdot P$$

Where P_m = Average power of message signal $= E\left[(m(t))^2\right]$.

• Average power of the noise in message bandwidth is given by "$N_0 W$". where W = Bandwidth of message signal, $m(t)$.

∴ channel signal to noise ratio is

$$(SNR)_C = \frac{\text{Average power of the modulated signal, } s(t)}{\text{Average power of the noise in message bandwidth}}$$

$$(SNR)_C = \frac{A_c^2 P}{2 N_0 W} \quad \text{——} (A)$$

To find output SNR $(SNR)_O$:-

Total signal at the input of Coherent detector is

$$x(t) = s(t) + n(t) \quad \longrightarrow (2)$$

We know that the narrow band noise signal $n(t)$ in its Canonical Form is represented by

$$n(t) = n_I(t) \cdot \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \longrightarrow (3)$$

Where $n_I(t)$ = Inphase noise Component and $n_Q(t)$ = Quadrature phase noise Component, measured with respect to Carrier signal $\cos(2\pi f_c t)$.

Substitute equation (3) in equation (2) we get

$$x(t) = s(t) + n_I(t) \cdot \cos(2\pi f_c t) - n_Q(t) \cdot \sin(2\pi f_c t) \longrightarrow (4)$$

Therefore, the output of product Modulator is given by,

$$V(t) = x(t) \times \cos(2\pi f_c t) \quad \text{——} (5)$$

Substitute $x(t)$ equation (4) in (5) we get

$$V(t) = \left[s(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t)$$

$$V(t) = s(t)\cdot\cos(2\pi f_c t) + n_I(t)\cos^2(2\pi f_c t) - n_Q(t)\cdot\sin(2\pi f_c t)\cdot\cos(2\pi f_c t)$$

DSBSC signal, $s(t) = m(t)\cdot A_c\cos(2\pi f_c t)$, Therefore.

$$V(t) = A_c\cdot m(t)\cdot\cos^2(2\pi f_c t) + n_I(t)\cos^2(2\pi f_c t) - n_Q(t)\cdot\sin(2\pi f_c t)\cdot\cos(2\pi f_c t) \longrightarrow (6)$$

Using Trignometric Identities

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \quad \text{and} \quad \sin\theta\cdot\cos\theta = \frac{\sin 2\theta}{2}$$

$$V(t) = \frac{A_c\cdot m(t)}{2}\left(1+\cos(4\pi f_c t)\right) + \frac{n_I(t)}{2}\left(1+\cos(4\pi f_c t)\right) - \frac{n_Q(t)}{2}\sin(4\pi f_c t)$$

$$V(t) = \frac{A_c m(t)}{2} + \frac{A_c\cdot m(t)}{2}\cdot\cos(4\pi f_c t) + \frac{n_I(t)}{2} + \frac{n_I(t)}{2}\cdot\cos(4\pi f_c t) - \frac{n_Q(t)}{2}\sin(4\pi f_c t) \longrightarrow (7)$$

The output of product Modulator $V(t)$ is applied to low pass filter it allows only $\frac{m(t)\cdot A_c}{2}$ & $\frac{n_I(t)}{2}$ components & eliminates all other higher frequency terms.

$\therefore$ The output signal of Coherent detector is

$$y(t) = \underbrace{\frac{A_c m(t)}{2}}_{\text{demodulated signal}} + \underbrace{\frac{n_I(t)}{2}}_{\text{output noise}}$$

Therefore Average power of demodulated output signal $E\left[\frac{A_c^2}{4}\cdot m^2(t)\right]$ $\Big\} = \frac{A_c^2}{4}P$

Average power of output Noise $E\left[\left(\frac{n_I(t)}{2}\right)^2\right]$ $\Big\} = \frac{N_0 W}{2}$ $\Leftarrow$ Half of input noise power.

$\therefore$ Output signal to Noise ratio is

$$(SNR)_0 = \frac{\text{Average power of the demodulated signal}}{\text{Average power of output noise}}$$

$$(SNR)_0 = \frac{(A_c^2/4)P}{(N_0 W/2)} = \frac{A_c^2 P}{2N_0 W} \longrightarrow (8)$$

∴ Figure-of-Merit for DSBSC-receiver system is

$$\text{Figure of Merit} = \frac{(SNR)_0}{(SNR)_C} \longrightarrow (8)$$

Substitute equation (A) and equation (B) in equation (8) we get

$$FOM = \frac{(SNR)_0}{(SNR)_C} = \frac{\left(\frac{A_c^2 P}{2N_0 W}\right)}{\left(\frac{A_c^2 P}{2N_0 W}\right)} = \underline{\underline{1}}$$

∴ Figure-of-Merit (FOM) for DSBSC receiver is unity.

4.3: Noise in AM receivers:-

(Q) obtain the expression for Figure of Merit of AM receivers using Envelope detector.

↳ Let $m(t)$ be the message signal with average power 'P'

$$P = E[m^2(t)] = E[(A_m \sin(2\pi f_m t))^2] = \frac{A_m^2}{2}$$

$c(t)$ be the carrier signal with $c(t) = A_c \cos 2\pi f_c t$. Then the Amplitude modulated (AM)-signal, $s(t)$ is given by

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t) \longrightarrow (1)$$

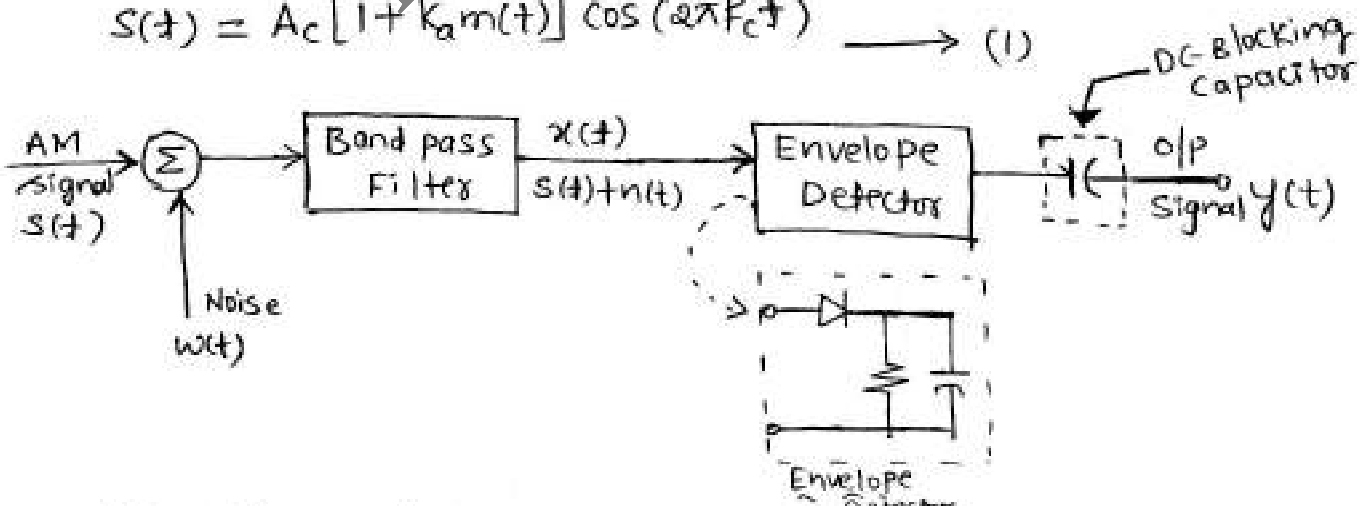


Figure 1: Model of AM receiver using Envelope Detector

Figure 1, shows the model of a AM receiver using a Envelope detector.

Consider noise in AM receivers using Envelope detection.

- To determine channel SNR $(SNR)_C$:-

↳ The AM signal is given by

$$s(t) = A_c[1 + K_a m(t)]\cos(2\pi f_c t) \longrightarrow (1)$$

↳ ∴ The Average power of modulated signal $s(t)$ $= E[\{s(t)\}^2]$

$$= E[\{A_c[1+K_a m(t)]\cos(2\pi f_c t)\}^2]$$

$$= E[\{A_c^2 \cdot (1+K_a m(t))^2 \cdot \cos^2(2\pi f_c t)]$$

$$= E[\{1+K_a m(t)\}^2] \cdot E[(A_c\cos(2\pi f_c t))^2]$$

$$= E[1+K_a^2 m^2(t) + 2K_a m(t)] \cdot \left(\frac{A_c^2}{2}\right)$$

$$= \left(E[1] + E[K_a^2 m^2(t)] + E[\overset{0}{2K_a m(t)}]\right) \cdot \frac{A_c^2}{2}$$

Since m(t) is pure ac-cosine signal

$$= (1 + K_a^2 P) \cdot \frac{A_c^2}{2}$$

∴ Average power of modulated signal $s(t)$ $= \dfrac{(1+K_a^2 P)A_c^2}{2} \longrightarrow (2)$

↳ Average power of the noise in message bandwidth is given by "$N_0 W$". where W = Bandwidth of message signal m(t).

∴ channel signal to Noise ratio is

$$(SNR)_C = \frac{\text{Average power of modulated signal}}{\text{Average power of Noise in message Bandwidth}}$$

$$\therefore (SNR)_C = \frac{(1+K_a^2 P)A_c^2}{2N_0 W} \longrightarrow (A)$$

- To Determine output SNR $(SNR)_0$:-

The total signal at the input of Envelope detector is

$$x(t) = s(t) + n(t) \longrightarrow (3)$$

where $n(t)$ represents narrowband noise in terms of in-phase and Quadrature Components.

$$n(t) = n_I(t) \cdot \cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \longrightarrow (4)$$

Substitute $s(t)$ & $n(t)$ in equation (3) we get-

$$x(t) = A_c[1 + K_a m(t)]\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

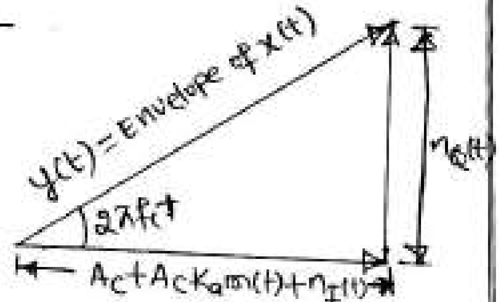
$$= [A_c + A_c K_a m(t)]\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

$$x(t) = \{(A_c + A_c K_a m(t) + n_I(t))\}\cos 2\pi f_c t - n_Q(t)\sin(2\pi f_c t) \longrightarrow (5)$$

∴ The output of the Envelope detector is

$$y(t) = \sqrt{[\underbrace{A_c + A_c K_a m(t) + n_I(t)}_{a}]^2 + \underbrace{n_Q^2(t)}_{b}} \longrightarrow (6)$$

Equation (6) gives the o/p of an ideal envelope detector.



Phasor-diagram of $x(t)$

Let us assume that the signal is much larger than Noise.

Then $\sqrt{a^2+b^2} \approx a$ when $a \gg b$.

$$\therefore y(t) = \underbrace{A_c}_{\text{DC Component}} + \underbrace{A_c K_a m(t)}_{\text{Signal Component}} + \underbrace{n_I(t)}_{\text{Noise Component}} \longrightarrow (7)$$

The DC-component (first term) can be removed with a capacitor (⊣⊢) placed next to Envelope detector as shown in Figure 1.

∴ The resultant demodulated signal is

$$y(t) = \underbrace{A_c K_a m(t)}_{\text{demodulated signal}} + \underbrace{n_I(t)}_{\text{output noise}} \longrightarrow (8)$$

$\therefore$ Average power of demodulated output signal $\{E[A_c^2 K_a^2 m^2(t)]\}$ $= A_c^2 K_a^2 P$

Average power of output Noise $E\{n_I^2(t)\}$ $= N_0 \times B_T$ $\because B_T = 2W$ for Amplitude Modulation

$= 2N_0W$

$\therefore$ The output signal to noise ratio is

$$(SNR)_0 = \frac{A_c^2 K_a^2 P}{2N_0W} \longrightarrow (B)$$

$\therefore$ Figure of Merit for AM receiver is

$$\text{Figure-of-Merit} = \frac{(SNR)_0}{(SNR)_C} \longrightarrow (C)$$

Substituting equation (A) and (B) in equation (C) we get,

$$\text{Figure-of-Merit} = \frac{\left(\frac{A_c^2 K_a^2 P}{2N_0W}\right)}{\left(\frac{A_c^2(1+K_a^2P)}{2N_0W}\right)}$$

$$FOM = \frac{A_c^2 K_a^2 P}{A_c^2(1+K_a^2P)}$$

$$FOM = \frac{K_a^2 P}{(1+K_a^2P)} \longrightarrow (D)$$

The Average power of the modulating wave $m(t)$ is

$$P = \frac{A_m^2}{2}$$

Substituting value of 'P' in equation (D) we get

$$FOM = \frac{K_a^2 \cdot A_m^2/2}{(1+K_a^2 A_m^2/2)}$$ $\because$ W.K.T Modulation Index of AM is $\mu = K_a \cdot A_m$

$$\therefore FOM = \frac{\mu^2/2}{1+\mu^2/2} = \frac{\mu^2/2}{2+\mu^2/2} = \frac{\mu^2}{2+\mu^2}$$

$\therefore$ Figure of Merit for AM receiver is $FOM = \frac{\mu^2}{2+\mu^2}$ //

4.4 : Noise in FM receivers :-

Q) prove that Figure-of merit for single tone Frequency modu-lated signal is $1.5\beta^2$.

↳ The single-tone Frequency modulated wave s(t) is given by,

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right) \longrightarrow (1)$$

where m(t) = Message signal.

Let $\phi(t) = 2\pi k_f \int_0^t m(t)dt$, then

$$s(t) = A_c \cos(2\pi f_c t + \phi(t)) \longrightarrow (2)$$

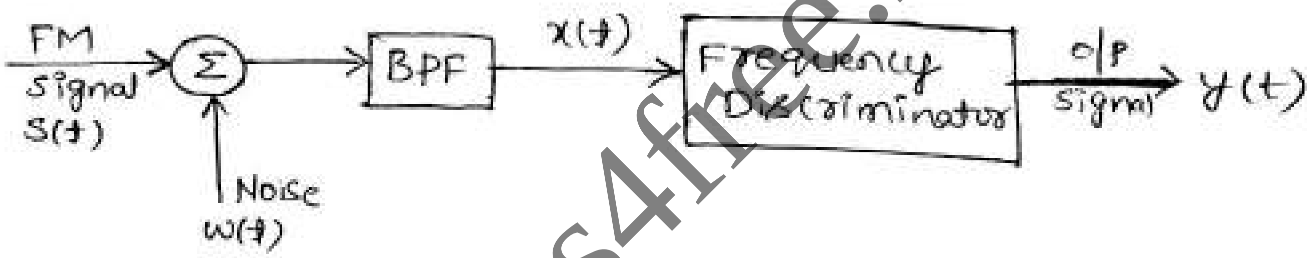


Fig1: Model of FM receiver using Frequency Discriminator

Fig1. shows the model of FM receiver using frequency discriminator.

To determine Channel SNR $(SNR)_c$:-

W.K.T the FM signal is

$$s(t) = A_c \cos(2\pi f_c t + \phi(t)) \text{ --- } (*)$$

∴ Average power of Modulated signal s(t) $\Big\} = \dfrac{A_c^2}{2}$

Average power of noise in message band width is $\Big\} = N_0 \times W$

$$\therefore (SNR)_C = \frac{A_c^2}{2N_0W} \longrightarrow (A)$$

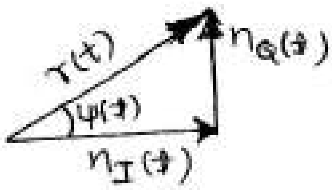
- To Determine output SNR $(SNR)_0$:-

The total signal at the input of frequency discriminator is,

$$x(t) = s(t) + n(t) \longrightarrow (3)$$

For output SNR, analysis let us express $n(t)$ in terms of its magnitude $[r(t)]$ and phase $[\psi(t)]$ given by the equation

$$n(t) = r(t)\cos(2\pi f_c t + \psi(t)) \longrightarrow (4)$$



where $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)} \longrightarrow (5)$

$\psi(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right) \longrightarrow (6)$

∴ Total signal at the input of demodulator is

$x(t) = s(t) + n(t)$ becomes

$$x(t) = A_c\cos(2\pi f_c t + \phi(t)) + r(t)\cos(2\pi f_c t + \psi(t)) \longrightarrow (7)$$

The relative phase $\theta(t)$ can be expressed as

$$\theta(t) \simeq \phi(t) + \frac{n_Q(t)}{A_c} \longrightarrow (8)$$

where $n_Q(t) = r(t)\cdot\sin\psi(t)$

With an ideal phase discriminator, the output $y(t)$ is proportional to the phase deviation $\frac{d\theta(t)}{dt}$.

i.e., the output signal,

$$y(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \longrightarrow (9)$$

Substitute $\theta(t)$ from equation (8) in equation (9) we get

$$y(t) = \frac{1}{2\pi}\frac{d}{dt}\left[\phi(t) + \frac{n_Q(t)}{A_c}\right] \quad \because \phi(t) = 2\pi k_f \int_0^t m(t)\,dt$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f \int_0^t m(t)\,dt + \frac{n_Q(t)}{A_c}\right]$$

$$y(t) = \underbrace{k_f m(t)}_{\text{demodulated signal}} + \underbrace{\frac{1}{2\pi A_c}\frac{d\,n_Q(t)}{dt}}_{\text{o/p Noise}}$$

$\therefore$ Average power of demodulated o/p signal $= k_f^2 P \longrightarrow (10)$

where P = power in message signal $m(t) = \frac{A_m^2}{2}$

Average power of output Noise $= \frac{N_0}{A_c^2}\int_{-W}^{W} f^2\,df = \frac{N_0}{A_c^2}\left(\frac{f^3}{3}\right)_{-W}^{W}$

$$= \frac{2N_0W^3}{3A_c^2}$$

$\therefore$ output signal to noise ratio is

$$(SNR)_o = \frac{k_f^2 P}{\left(\frac{2N_0W^3}{3A_c^2}\right)} = \frac{3A_c^2 k_f^2 P}{2N_0W^3} \longrightarrow (B)$$

$\therefore$ Figure-of-Merit for FM receiver is

$$FOM = \frac{(SNR)_o}{(SNR)_C} \longrightarrow (C)$$

Substitute equation (A) & equation (B) in equation (C) we get,

$$FOM = \frac{\left(\frac{3A_c^2 k_f^2 P}{2N_0W^3}\right)}{\left(\frac{A_c^2}{2N_0W}\right)} = \frac{3k_f^2 P}{W^2} \longrightarrow (D)$$

$\therefore$ substitute $P = \frac{A_m^2}{2}$; Average power of message signal in equation (D) we get

$$FOM = \frac{3\,K_f^2 A_m^2}{2W^2} = \frac{3}{2}\left(\frac{K_f A_m}{W}\right)^2 \longrightarrow (E)$$

We know that the modulation Index of FM-Signal

$$\beta = \frac{\Delta f}{f_m} = \frac{K_f A_m}{W}$$

∴ using the value of 'β' in FOM equation (E) we get Figure-of-Merit of FM receiver

$$\boxed{FOM = \frac{3}{2}\beta^2 = 1.5\,\beta^2}$$ where $\beta = \frac{K_f A_m}{W}$

———*———*———

• Problems on Figure-of-Merit Calculation:-

List of Formulae:-

⟨1⟩ $FOM = \frac{\mu^2}{2+\mu^2}$ for Amplitude Modulation

⟨2⟩ $FOM = 1.5\,\beta^2$ for frequency Modulation

where β = Modulation Index of FM.

⟨3⟩ General definition of FOM is

$$FOM = \frac{(SNR)_o}{(SNR)_c}$$

⟨4⟩ $(SNR)_{o\,dB} = 10 \log (SNR)_o$

⟨5⟩ $(SNR)_{c\,dB} = 10 \log (SNR)_c$

<1> An AM receiver operating with a sinusoidal wave of 80% modulation has an output signal to noise ratio of 30dB. Calculate the corresponding channel s/l-to-noise ratio. prove the formula used.

↳ Given data:

$\mu = 0.8$

$(SNR)_{o\,dB} = 30dB$

$(SNR)_c = ?$

W.K.T. The FOM of AM receiver is

$$FOM = \frac{\mu^2}{2+\mu^2} = \frac{0.8^2}{2+0.8^2} = 0.2424$$

W.K.T., $FOM = \frac{(SNR)_0}{(SNR)_c}$

$$\therefore (SNR)_c = \frac{(SNR)_0}{FOM}$$

$$(SNR)_0 = 10^{\left(\frac{30}{10}\right)} = \underline{1000}$$

$$\therefore (SNR)_c = \frac{(SNR)_0}{FOM} = \frac{1000}{0.2424} = 4125$$

$$(SNR)_{c\,dB} = 10\log(4125)$$

$$(SNR)_{c\,dB} = \underline{\underline{36.15dB}}$$

· For derivation of FOM for AM receiver refer the AM-receiver FOM derivation.

<2> The average noise per/unit BW measured at the front end of AM receiver is 10^{-3} Watt/HZ. The Modulating wave is sinusoidal with a carrier power of 80KW and sideband power of 10KW per side band. The message BW is 4KHZ. Determine the $(SNR)_0$ of the system and FOM. VTU Q.P

↳ Given: $N_0 = 10^{-3}$ Watt/HZ; $P_c = \frac{A_c^2}{2} = 80KW \Rightarrow A_c = 400V$

Side band power $P_S = \frac{A_c^2\mu^2}{8} = 10\times10^3 \Rightarrow \boxed{\mu = 0.707}$

Message bandwidth, W = 4KHZ

$$\therefore (SNR)_0 = \frac{A_c^2\mu^2}{2N_0W} = \frac{(400)^2\times0.707^2}{2\times10^{-3}\times4\times10^3} = \underline{\underline{5000}} \;\&\; FOM = \frac{\mu^2}{2+\mu^2} = 0.2$$

** 3) Find FOM of AM receiver when depth of Modulation is (a) 100 %. (b) 50% (c) 30% VTU Q.P

↳ W.K.T. FOM of AM receiver is given by

$$FOM = \frac{\mu^2}{2+\mu^2} \quad \text{---(1)}$$

(a) when $\mu = 100\% = 1$.

$$FOM = \frac{\mu^2}{2+\mu^2} = \frac{1}{2+1} = \frac{1}{3} = 0.3333$$

(b) when $\mu = 50\% = 0.5$

$$FOM = \frac{\mu^2}{2+\mu^2} = \frac{0.5^2}{2+0.5^2} = 0.1111$$

(c) when $\mu = 30\% = 0.3$

$$FOM = \frac{\mu^2}{2+\mu^2} = \frac{0.3^2}{2+0.3^2} = 0.043$$

*4) An FM signal with a maximum frequency deviation of 75KHZ is applied to an FM demodulator. When the input (channel) SNR is 15dB and the modulating frequency is 10KHZ. Estimate FOM and SNR at demodulator output. VTU.QP

↳

Given data :-

$\Delta f = 75KHZ$

$(SNR)_{c\,dB} = 15dB$

$f_m = W = 10KHZ$

$FOM = ?$

$(SNR)_0 = ?$

W.K.T FOM of FM receiver is

$$FOM = 1.5\beta^2$$

$$\beta = \frac{\Delta f}{f_m} = \frac{75K}{10K} = 7.5$$

$$\therefore FOM = 1.5\beta^2 = 1.5\,(7.5)^2$$

$$FOM = 84.375$$

To find output SNR:-

W.K.T. General definition of Figure-of-Merit

$$FOM = \frac{(SNR)_o}{(SNR)_c} \longrightarrow (1)$$

from given data $(SNR)_{c_{dB}} = 15\,dB = 10\log[(SNR)_c]$

$\therefore (SNR)_c = 10^{\wedge}\left(\frac{15}{10}\right) = 10^{1.5}$

$(SNR)_c = 31.6227$

Also W.K.T FOM = 84.375

$\therefore$ substitute FOM = 84.375 & $(SNR)_c = 31.6227$ in equation (1) we get

$(SNR)_o = FOM \times (SNR)_c$

$= 84.375 \times 31.6227 = 2668.16 = \underline{34.262\,dB}$

5) An FM receiver recieves an FM signal $s(t) = 10\cos[(2\pi \times 10^8 t) + 6\sin(2\pi \times 10^3 t)]$. Calculate the figure-of-Merit for this receiver. VTU Q.P

↳ Given

$s(t) = 10\cos[2\pi \times 10^8 t + 6\sin(2\pi \times 10^8)t] \longleftarrow (1)$

General equation of FM is

$s(t) = A_c\cos[2\pi f_c t + \beta\sin(2\pi f_m t)] \longleftarrow (2)$

Comparing (1) & (2) we get

Modulation index, $\boxed{\beta = 6}$

$\therefore$ Figure-of-Merit $= 1.5\beta^2$

$= 1.5 \times 6^2$

$= \underline{54}$

4.5: Capture effect :-

(Q) Write a short note on Capture effect in FM. (4-Marks)

→ In FM system, the signal can be affected by another frequency modulated signal whose frequency is close to the carrier frequency of the desired FM-signal. Then the receiver may lock such an interference signal and suppresses the desired FM-signal & interference signal becomes more stronger than the desired signal.

→ When the strength of the desired signal and interference signal are nearly equal, the receiver locks interference signal for some time and desired signal for the some time and this goes on randomly and receiver captures the stronger signal. This effect is known as "Capture-effect".

4.6 : FM-Threshold Effect :-

(Q) Explain FM threshold effect in FM-system. VTU (6-Marks)

→ The $(SNR)_0$ of an FM-signal is valid only if the (CNR) mesured at the frequency discriminator input is very much greater than unity.

i.e., $$(SNR)_0 = \frac{3A_c^2 k_f^2 P}{2N_0 W^3} \text{ is Valid iff } CNR \gg 1$$

if CNR < 1 then FM signal is corrupted by noise and FM receiver breaks down. & is called Threshold effect of FM.

i.e, Threshold effect is defined as the minimum carrier to noise ratio (CNR) that gives the $(SNR)_0$ not less than the value predicted by the usual SNR-formula assuming a small noise power.

→ The Threshold effect can be avoided by keeping carrier to noise ratio of FM-system much greater than 1.

↳ When CNR ⩽ 1, FM receiver breaks down due to FM-threshold effect. This point is called Threshold point.

4.7: FM Threshold reduction:-

<Q> Explain about the FM-Threshold effect and its reduction Method (V.T.U. 8M)

↳ FM-Threshold can be reduced by using FM-demodulators with negative feedback (or) by using a phase-Locked loop demodulator. Such devices are referred as Extended threshold demodulators.

↳ FM-demodulator with negative feedback is also known as FMFB-demodulator.

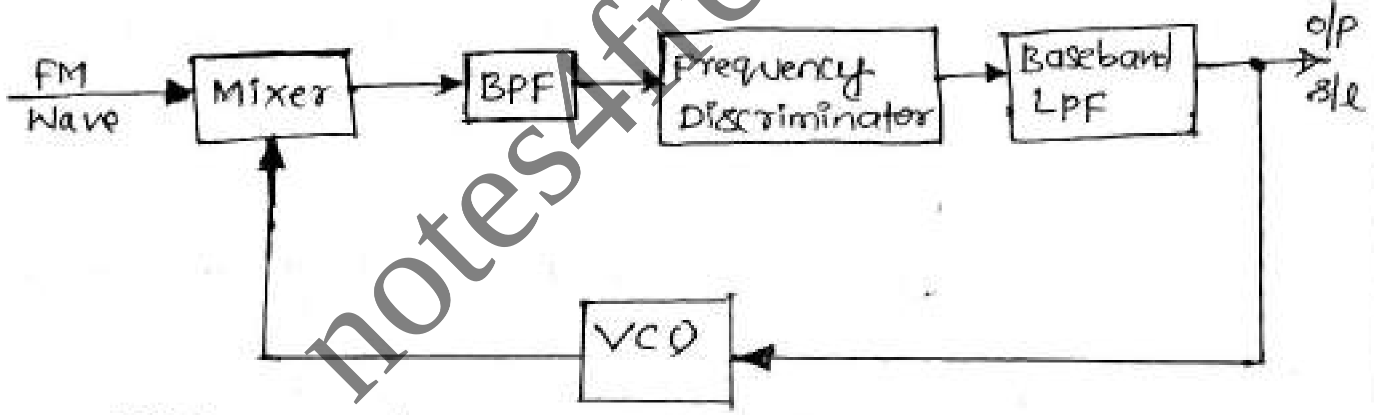


Fig1: Block diagram of FMFB demodulator for threshold reduction

↳ Figure 1, shows the block diagram of FMFB demodulator. In this system local oscillator is replaced by Voltage Controlled Oscillator <VCO>.

↳ The instantaneous output frequency of such VCO is Controlled by demodulated output signal.

↳ The bandwidth of noise to which the FMFB receiver responds is precisely the band of noise that the VCO-tracks, thus the FMFB receiver acts as a tracking filter, that Can track only the slowly varying frequency of a

Wide-band FM signal. 21

↳ Therefore it responds only to a narrow band of noise Centered about frequency "f_c", as a result FMFB-receiver allows a threshold Extension upto 5dB to 7dB as shown in Figure. 2.

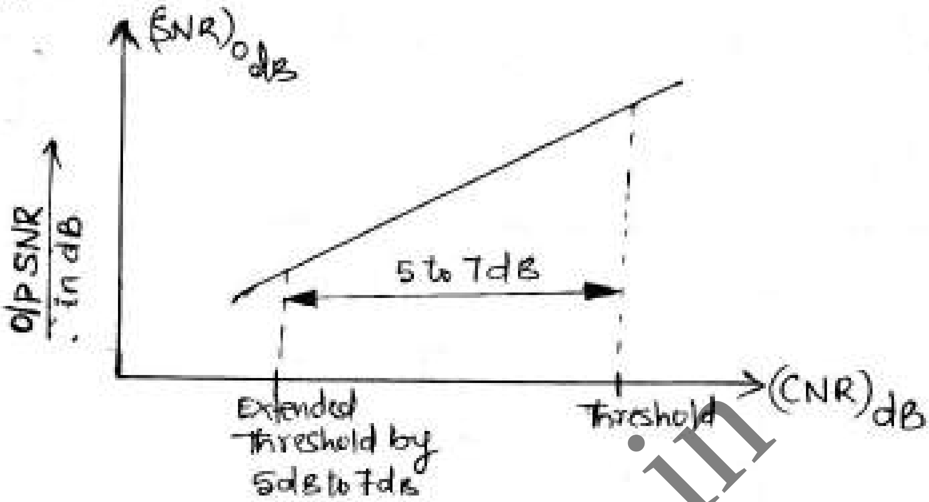


Fig. 2: Graph showing the Extended threshold effect

↳ ∴ FMFB demodulator with negative feedback provides 5dB to 7dB Enhancement in (CNR) & it always maintains CNR≫1 and it avoids FM-threshold effect.

Pre-emphasis and De-emphasis in FM :-

<Q> With circuits and characteristics, explain the importance of pre-emphasis and De-emphasis in FM-systems.

VTU - 8M -

↳ pre-emphasis and De-emphasis methods are commonly used in FM-transmitter and FM-receiver respectively to improve the Threshold.

↳ pre-emphasis and De-emphasis are simple RC networks used to improve threshold upto 13dB to 16dB.

↳ Figure 1 shows the FM transmitter with pre-emphasis filter having transfer function $H_{pre}(f)$.

↳ Figure 1, shows the pre-emphasis filter used before FM-transmitter.

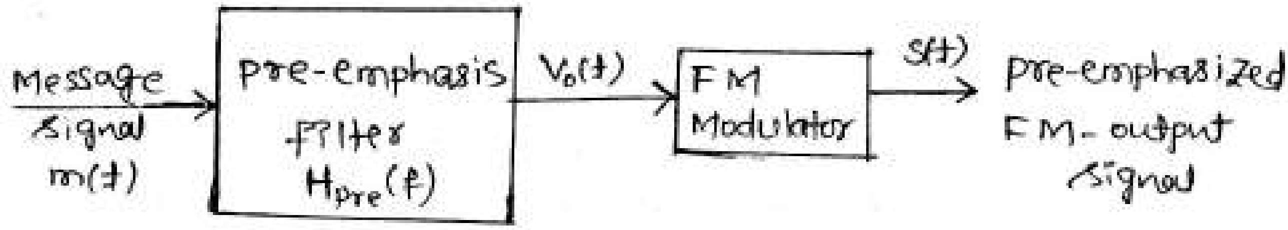


Fig1: Use of pre-emphasis filter in FM transmitter

* pre-emphasis filter (or) circuit :-

↳ pre-emphasis circuit is a High-pass-Filter (HPF) with transfer function shown in figure 2.

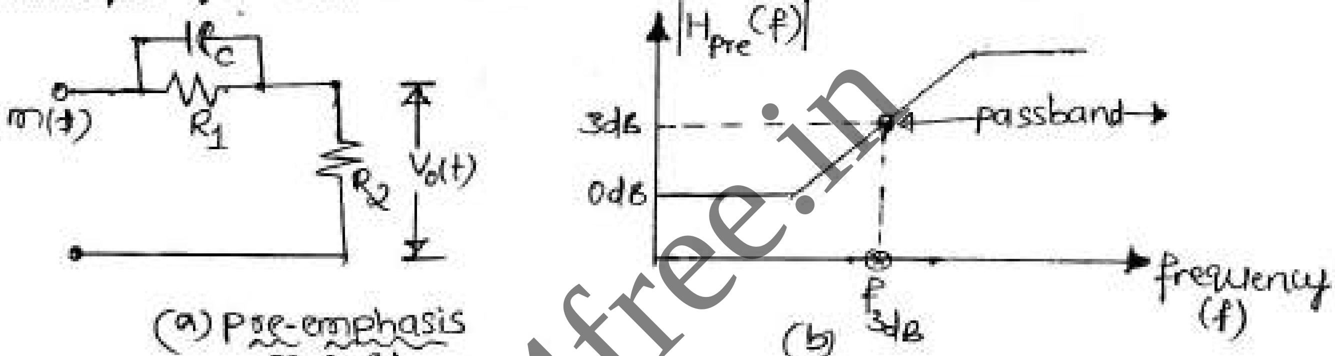


Fig.2: pre-emphasis filter circuit diagram with frequency response

↳ To improve $(SNR)_O$ at the FM-Modulator output, the high freque--ncy components of the message signal m(t) are artificially emphasized at the transmitter, before the Modulation-process & is shown in fig.1.

↳ After pre-emphasis, m(t) occupies entire range of bandwidth assigned. Then at the frequency discriminator (FM-demodulator) of FM receiver, inverse operation of pre-emphasis called De-emphasis is performed.

- Figure 3, shows the FM receiver with de-emphasis filter having frequency response $|H_{de}(f)|$.

- De-emphasis filter/circuit is used after FM-demodulator as shown in figure.3.

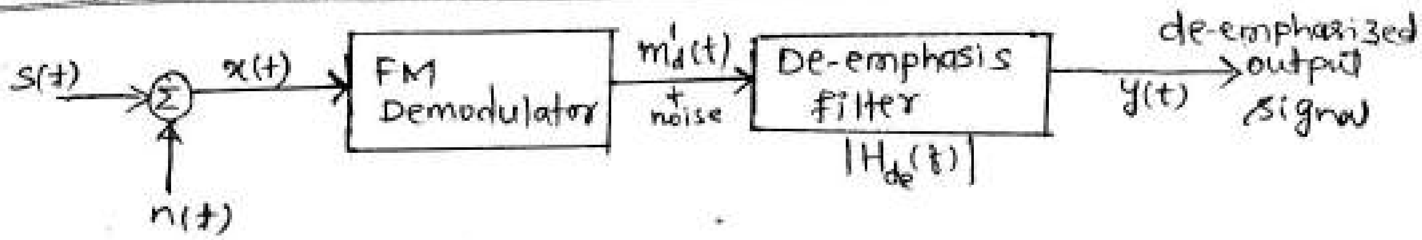
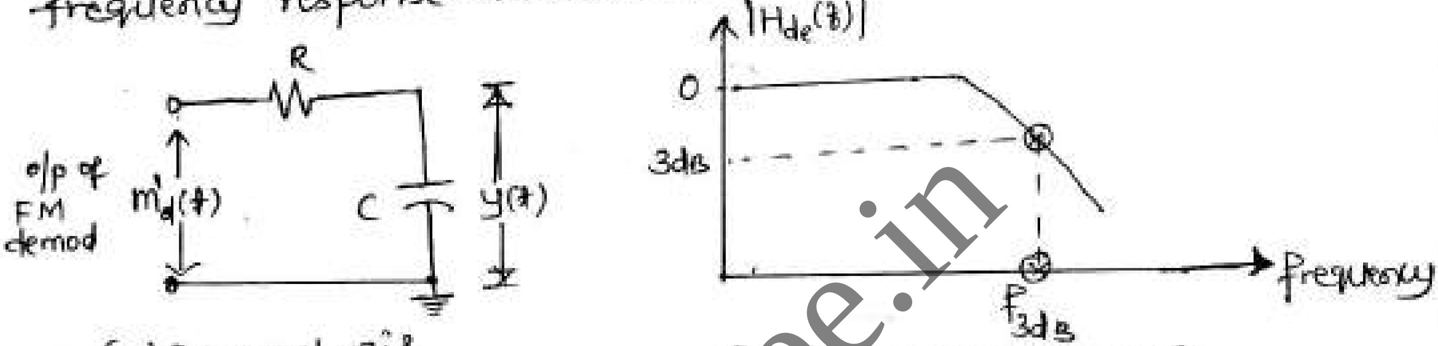


Figure 3: FM receiver with De-emphasis filter

De-emphasis filter (or) Circuit :-

↳ De-emphasis Circuit is a simple RC-Lowpass filter with frequency response as shown in Figure 4.



(a) De-emphasis Circuit

(b) Frequency response

Fig. 4: De-emphasis filter circuit diagram & its frequency response

↳ High-frequency Components present at the output of FM-demodulator o/p signal ($m'_d(t)$) are de-emphasized to restore the original message signal power distribution.

↳ During, de-emphasis process, High-Frequency Components of the noise are also reduced and thereby efficiently increases output signal to noise ratio. $(SNR)_0$.

↳ The frequency response of de-emphasis filter & De-emphasis filter are related by,

$$|H_{pre}(f)| = \frac{1}{|H_{de}(f)|}$$

where $|H_{de}(f)| = \dfrac{1}{1+j\frac{f}{f_{3dB}}}$ and $|H_{pre}(f)| = 1+j\dfrac{f}{f_{3dB}}$

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↳ The effect of de-emphasis filter on output noise spectrum is illustrated in Figure-5.

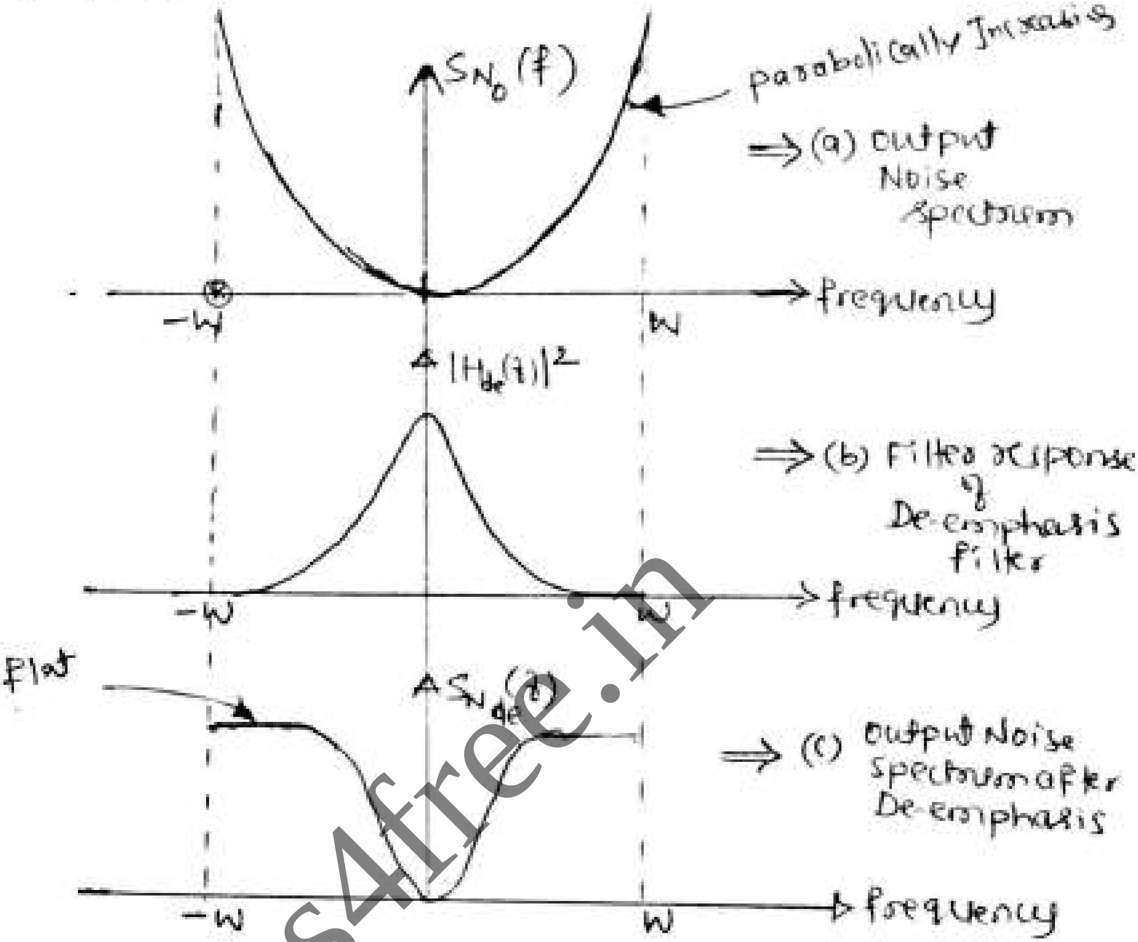


Figure 5: Effect of De-emphasis filter on output Noise

↳ Fig 5(a) shows output noise spectrum which is parabolically increasing towards the edge of $\pm W$. Therefore $(SNR)_o$ reduces before deemphasis.

↳ When Noise is filtered by using de-emphasis filter having frequency response shown in fig 5(b), the output noise spectrum after de-emphasis becomes almost flat at $\pm W$ Hz. As a result $(SNR)_o$ increases and it avoids FM-threshold.

∴ Pre-emphasis and De-emphasis circuits are effectively used to increase (SNR) at the transmitter (before modulation) and receiver (after demodulation) of FM-system respectively.

~ end ~

MODULE 5

DIGITAL REPRESENTATION OF ANALOG SIGNALS

* SYALLABUS :

Introduction, Why Digitize Analog sources ?, The sampling Process, Pulse Amplitude Modulation, Time Division Multiplexing, Pulse position modulation, Generation of PPM waves, Detection of PPM waves, The Quantization Process, Quantization Noise, Pulse code Modulation : sampling, Quantization, Encoding, Regeneration, Decoding, Filtering, multiplexing, Application to vocoder.

* INTRODUCTION :

* The evolution from analog to digital transmission is the conversion of common information sources such as voice and music, which are inherently analog to digital representation.
* In the first step from analog to digital, an analog source is sampled at discrete times. The resulting analog samples are then transmitted by means of analog pulse modulation.
* In the second step from analog to digital, an analog source is not only sampled at discrete times but the samples themselves are also quantized to discrete levels.

* WHY DIGITIZE ANALOG SOURCES :

There are many advantages that the transmission of digital information has over analog.

1) Digital systems are less sensitive to noise than analog.

2) With digital systems, it is easier to integrate different services. For example, video and the accompanying sound track, into the same transmission scheme.

3) The transmission scheme can be relatively independent of the source. For example, a digital transmission scheme that transmits voice at 10kbps could also be used to transmit computer data at 10kbps.

4) Circuitry for handling digital signals is easier to repeat and digital circuits are less sensitive to physical effects such as vibration and temperature.

5) Digital signals are simpler to characterize in terms of bits 1 and 0 and do not have variability as analog signals. This makes the associated hardware easier to design.

6) Various media sharing stratergies known as multiplexing techniques are more easily implemented with digital transmission stratergies.

7) Digital techniques make it easier to specify complex standards that may be shared on a worldwide basis.

8) The techniques such as equalization, especially adaptive versions, are easier to implement with digital transmission techniques.

* COMPARISON BETWEEN ANALOG & DIGITAL COMM SYSTEMS:

S.NO	Parameter	Analog system	Digital System
1)	Bandwidth	Less	More
2)	Error correction and Detection	Not possible	Possible
3)	Immune to Noise	Less	More
4)	System Complexity	Less	More
5)	System Cost	More	Less
6)	Quality of reconstruction	Good	Very Good
7)	Synchronisation	Not required	required
8)	Privacy and security to data	Not possible	possible
9)	Flexibility and Liability	Less	More
10)	Power required	More	Less
11)	Implementation	Difficult	Easy
12)	Programming	Not possible	Possible

* SAMPLING PROCESS :

Statement: Sampling theorem states that any continuous time signal can be completely represented in its samples and recovered back if the sampling frequency is greater than or equal to twice the highest frequency component of base band signal.

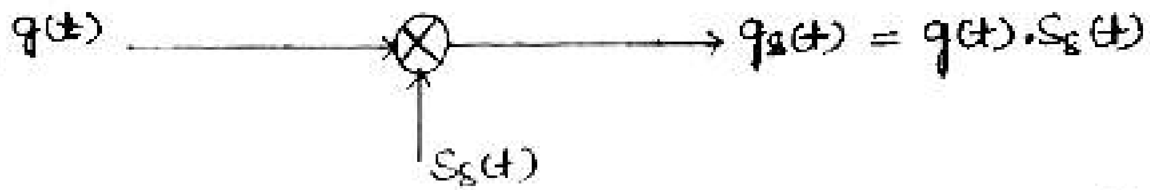
That is Sampling frequency, $f_s \geq 2W$ **.**

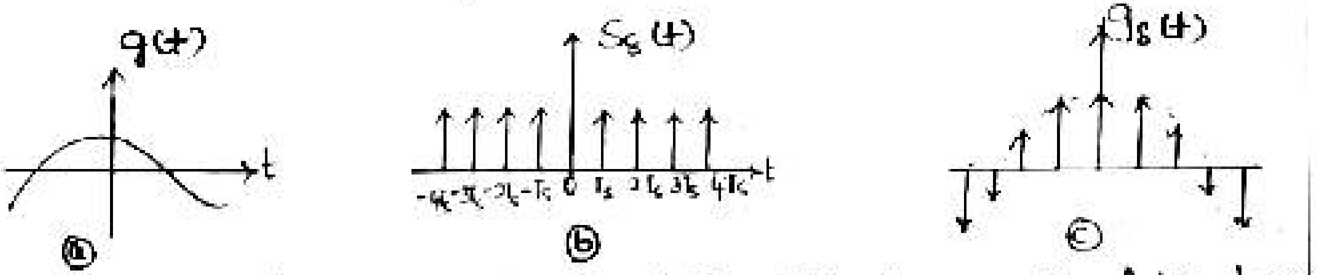
Where W= Highest frequency in base band continuous time signal.

This condition is also called Nyquist condition for sampling process.

<u>Explanation and Proof:</u>

* Consider an arbitrary signal g(t) of finite energy, which is specified for all time. A segment of the signal g(t) is shown in fig(1)(a). Suppose that we sample the signal g(t) instantaneously and at a uniform rate, once every 'Ts' seconds. Consequently, we obtain an infinite sequence of samples spaced 'Ts' seconds apart and denoted by $\{g(nT_s)\}$, where 'n' takes on all possible integer values. We refer to T_s as the sampling period, and to its reciprocal $f_s = 1/T_s$ as the sampling rate. This ideal form of sampling is called instantaneous sampling.





Fig(1): (a) analog signal (b) Periodic Signal ($\delta_s(t)$) (c) Sampled Signal $g_\delta(t)$

* Let $g_\delta(t)$ denote the signal obtained by individually weighting the elements of a periodic sequence spaced T_s seconds. Therefore, sampled output $g_\delta(t)$ is given by,

$$g_\delta(t) = g(t) \cdot \delta_\delta(t) \qquad (1)$$

* Let $\delta_\delta(t)$ denotes the periodic impulse train and is represented as,

$$\delta_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \qquad (2)$$

Substituting Eq^n(2) in Eq^n(1) we get

$$g_\delta(t) = g(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Using shifting property of impulse function

WKT,
$$g(t) \cdot \delta(t - nT_s) = g(nT_s)\,\delta(t - nT_s)$$

$$\therefore \boxed{g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s)} \qquad (3)$$

For frequency domain consider,

$$g_\delta(t) = g(t) \cdot \delta_\delta(t)$$

Taking Fourier Transform on both sides, we get

$$G_\delta(f) = G(f) * \delta_\delta(f) \qquad (4)$$

where,

$$S_\delta(f) = f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \quad \text{——— (5)}$$

substituting Eqⁿ (5) in Eqⁿ (4) we get.

$$G_\delta(f) = G(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

From convolution property of impulse function

WKT, $G(f) * \delta(f - nf_s) = G(f - nf_s)$

$$\therefore \quad G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s) \quad \text{——— (6)}$$

Eqⁿ (6) can be rewritten as,

$$G_\delta(f) = f_s\, G(f) + f_s \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} G(f - nf_s) \quad \text{——— (7)}$$

When the spectrum of $G_\delta(f)$ is passed through an LPF then the 2nd term in RHS of Eqⁿ (7) is eliminated resulting in

$$G_\delta(f) = f_s \cdot G(f)$$

$$\therefore \quad G(f) = \frac{1}{f_s} \cdot G_\delta(f) \quad \text{——— (8)}$$

where $f_s = 2W$

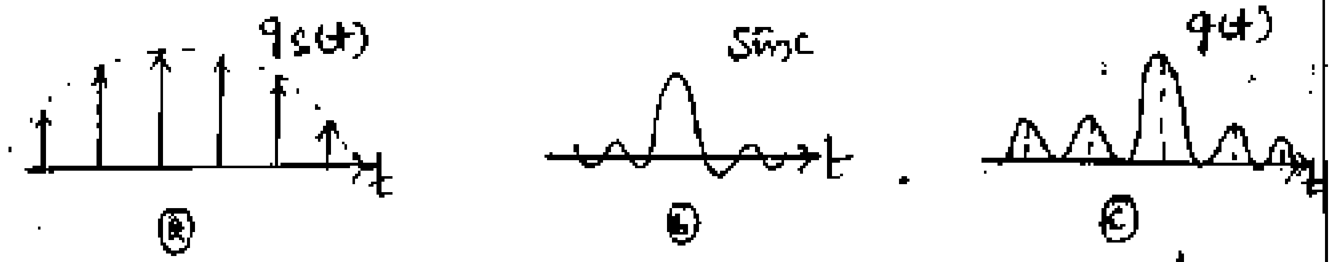


Fig : Recovering g(t) signal from sequence of samples g(nTs).

Now, we may state the sampling theorem for strictly band-limited signals of finite energy into two equivalent parts :

1) A bandlimited signal of finite energy, which only has frequency components less than "W" Hertz, is completely described by specifying the values of the signal at instants of time separated by $\frac{1}{2W}$ seconds.

2) A bandlimited signal of finite energy, which only has frequency components less than "W" Hertz, may be completely recovered from a knowledge of its samples taken at the rate of 2W samples per second.

The sampling rate of 2W samples per second, for a signal bandwidth of 'W' Hertz is called the Nyquist rate; its reciprocal $1/2W$ (measured in seconds) is called the Nyquist interval.

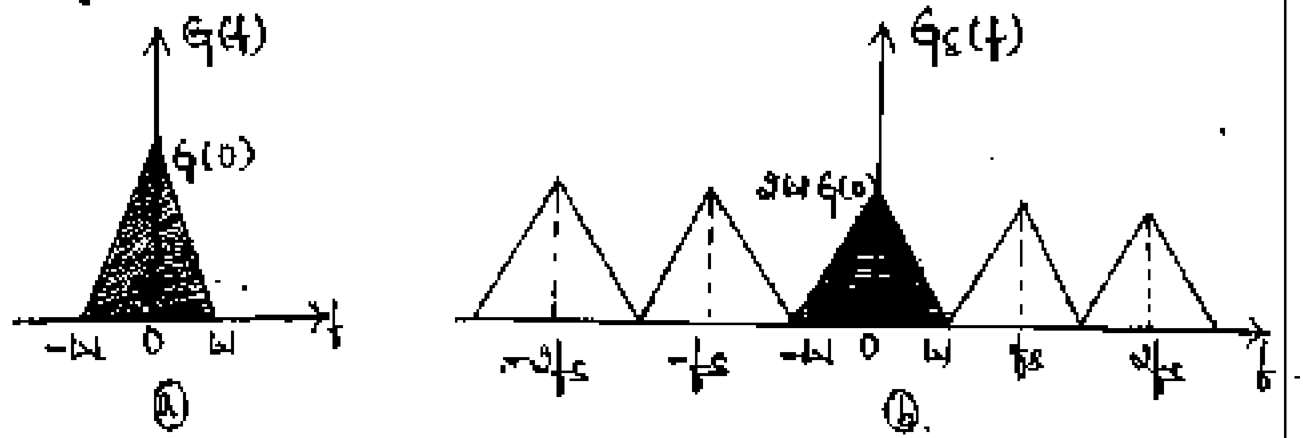
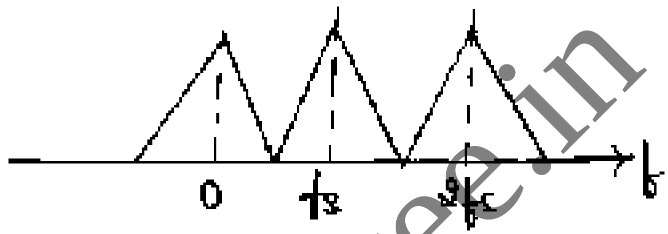


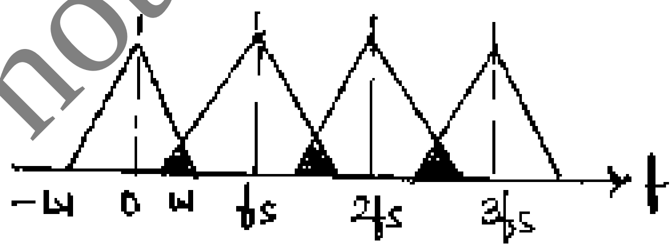
Fig : (a) Spectrum of a strictly band limited signal g(t).
(b) Spectrum of a sampled version of g(t) for $T_s = \frac{1}{2W}$.

NOTE : The concept of undersampling and over sampling is explained below.

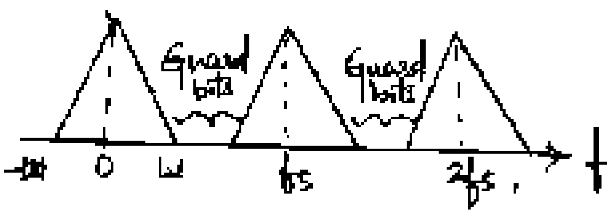
1) When sampling frequency $f_s = 2W$ then this type of sampling is called correct sampling and here there is no aliasing effect seen in this mechanism, i.e when $f_s = 2W$.



2) When $f_s < 2W$ then it is undersampling and there will be aliasing effect induced here.



3) When $f_s > 2W$ then it is over sampling and there will no aliasing effect.



* PROBLEMS :

(1) An analog signal is expressed by the equation $x(t) = 3\cos 50\pi t + 10 \sin 300\pi t + \cos 100\pi t$. Calculate the Nyquist rate and Nyquist Interval for this signal.

Sol^n: Given

$$x(t) = 3\cos 50\pi t + 10\sin 300\pi t + \cos 100\pi t \quad \text{———} (1)$$

Comparing eq^n (1) with std equation

$$x(t) = 3\cos \omega_1 t + 10 \sin \omega_2 t + \cos \omega_3 t \quad \text{———} (2)$$

comparing ω_1, ω_2 and ω_3 then

$\omega_1 = 50\pi$ $\qquad$ $\omega_2 = 300\pi$ $\qquad$ $\omega_3 = 100\pi$

$2\pi f_1 = 50\pi$ $\qquad$ $2\pi f_2 = 300\pi$ $\qquad$ $2\pi f_3 = 100\pi$

$\therefore \boxed{f_1 = 25\text{ Hz}}$ $\qquad$ $\therefore \boxed{f_2 = 150\text{ Hz}}$ $\qquad$ $\boxed{f_3 = 50\text{ Hz}}$

$\therefore f_m = \max(f_1, f_2, f_3)$

$\therefore \boxed{f_m = 150\text{ Hz}}$

$\therefore$ Nyquist rate $f_s = 2f_m = 2 \times 150$

$\therefore \boxed{f_s = 300\text{Hz}}$

Nyquist Interval, $T_s = 1/f_s = 1/300\text{ Hz}$

$\therefore \boxed{T_s = 3.3\text{ ms}}$

(8) An analog signal is expressed by the equation $x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t)$. Calculate the Nyquist rate and Nyquist Interval for this signal.

Sol: Given

$$x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t) \quad \text{——— (1)}$$

WKT $\cos A \cdot \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$\therefore$ $x(t) = \frac{1}{4\pi}[\cos(4000\pi - 1000\pi)t + \cos(4000\pi + 1000\pi)t]$

i.e. $x(t) = \frac{1}{4\pi}[\cos\omega_1 t + \cos\omega_2(t)]$

$\therefore$ $\omega_1 = 3000\pi$ $\qquad$ $\omega_2 = 5000\pi$

$\cancel{2\pi}f_1 = 3000\pi$ $\qquad$ $2\pi f_2 = 5000\pi$

$\boxed{f_1 = 1500\text{ Hz}}$ $\qquad$ $\boxed{f_2 = 2500\text{ Hz}}$

$\therefore$ $f_m = \max(f_1, f_2)$

$= \max(1500\text{Hz}, 2500\text{Hz})$

$\therefore$ $\boxed{f_m = 2500\text{ Hz}}$

$\therefore$ Nyquist Rate $f_s = 2\times f_m = 2\times 2500$

$\therefore$ $\boxed{f_s = 5000\text{ Hz}}$

$\therefore$ Nyquist Interval $T_s = 1/f_s = 1/5000$ $\quad\therefore$ $\boxed{T_s = 0.2\text{ms}}$

* PULSE AMPLITUDE MODULATION :

* It is an analog pulse Modulation scheme in which the amplitudes of a train of rectangular carrier pulses are varied in accordance with the sample values of the modulating signal.

* In PAM, the top of each modulated rectangular pulse is maintained flat. So PAM is same as flat-top sampling.

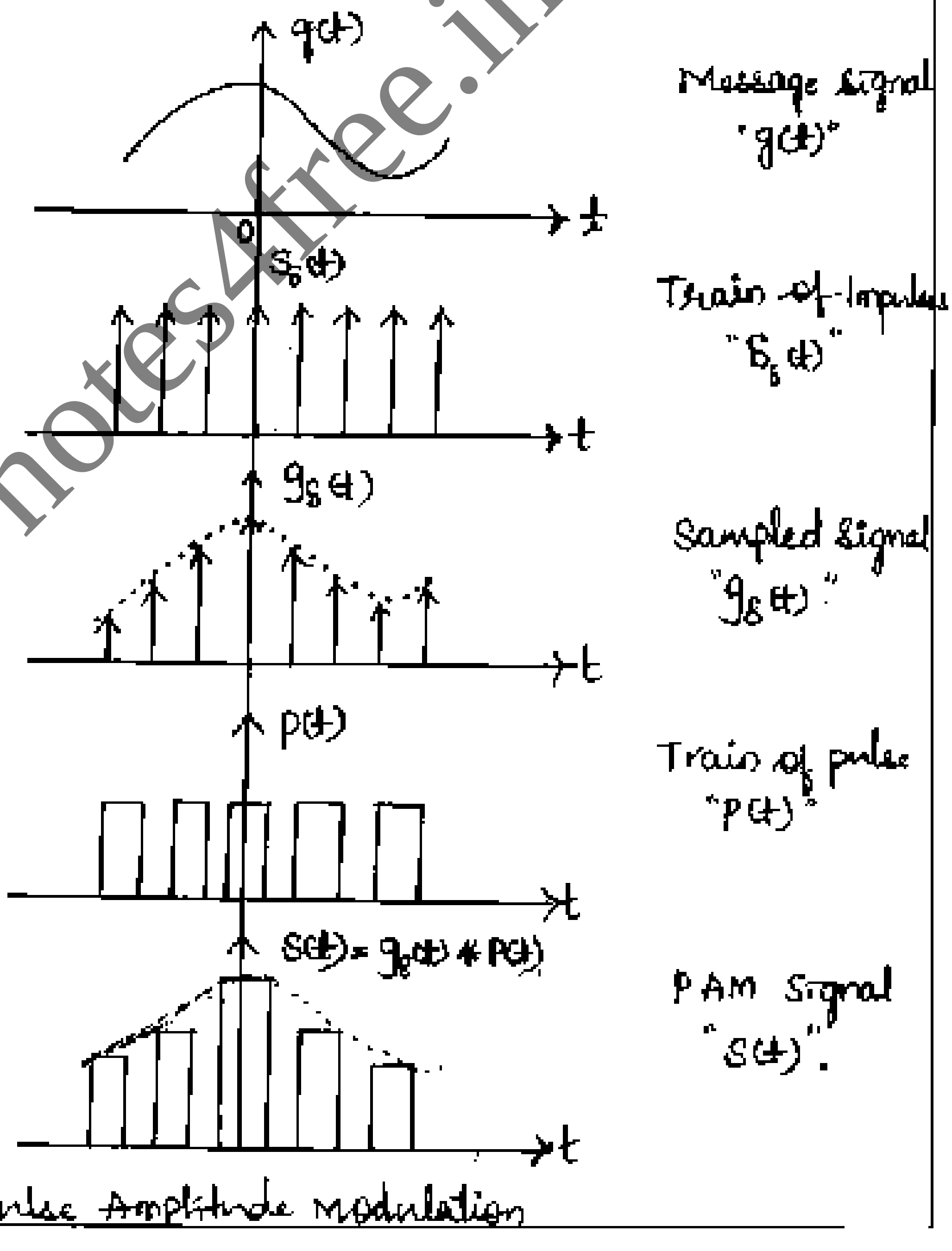


Fig4: Pulse Amplitude Modulation

The waveform of a PAM signal is illustrated in fig(4).

* Let $s(t)$ denote the sequence of flat-top samples or PAM signal, and it is expressed as

$$s(t) = \sum_{n=-\infty}^{\infty} q(nT_s)\, p(T - nT_s) \qquad (1)$$

where,

$q(nT_s)$ is the sample value of $q(t)$ obtained at time $t = nT_s$.

T_s is sampling period.

$p(t)$ is standard rectangular pulse train of duration T.

Advantages of PAM :

It is a base for all the digital modulation technique.

Disadvantages of PAM :

1) Due to Nyquist Criteria, it requires high bandwidth for transmission.

2) Since, amplitude keeps varying, so there is noise associated with it.

* Detection of PAM signal

The original message signal $m(t)$ is obtained by passing PAM signal to the reconstruction filter followed by equilizer.

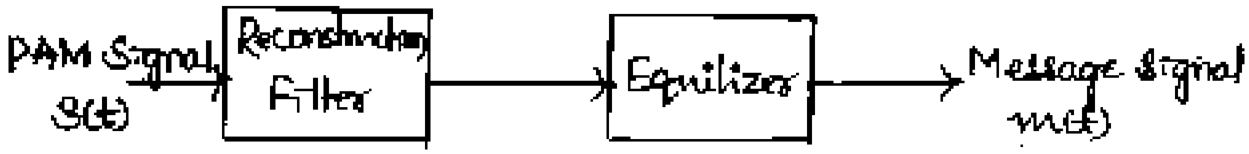


Fig.4(b) : Recovering m(t) from PAM signal

* TIME DIVISION MULTIPLEXING : [TDM]

Time Division Multiplexing is a method of transmitting and receiving independent signals over a common channel by means of synchronised switches at each end of the transmission line so that each signal appears on the line only a fraction of time in an alternating pattern.

* Fig(5) shows the block diagram of TDM system.

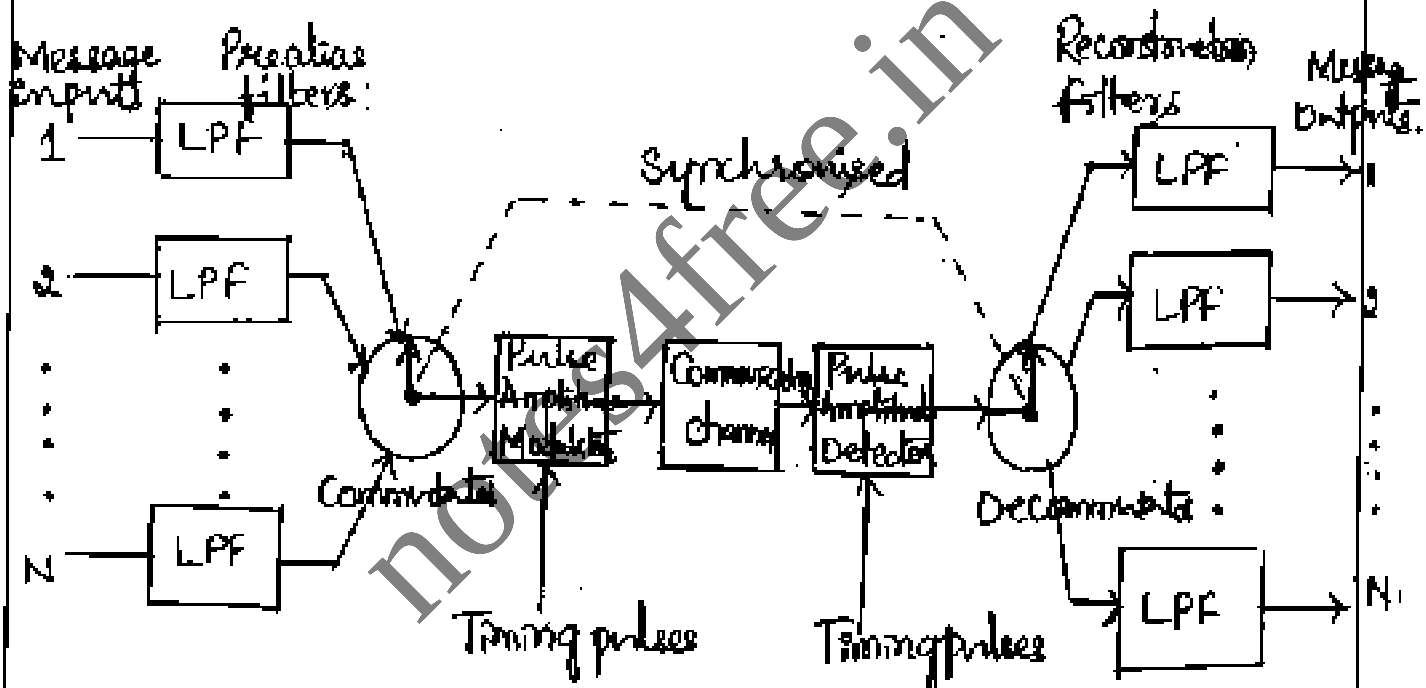


Fig 5 : Block Diagram of TDM system.

* The concept of TDM is illustrated in the fig(5). The Lowpass filters are used to remove high frequency components present in the message signal. The output of the pre-alias filters are then fed to a commutator which is usually implemented using electronic switching circuitry.

* The function of commutator is as follows:

1) To take a narrow sample of each of the 'N' samples of input at a rate of $f_s \geq 2W$.

2) To sequentially interleave (multiplex) these 'N' samples inside a sampling interval $T_s = 1/f_s$.

* The multiplexed signal is then applied to a pulse amplitude modulator whose purpose is to transform the multiplexed signal into a form suitable for transmission over a common channel.

* At the receiving end, the pulse amplitude demodulator performs the reverse operation of PAM and the decommutator distributes the signals to the appropriate low pass reconstruction filters. The decommutator operates in synchronisation with the commutator.

* PULSE-POSITION MODULATION :

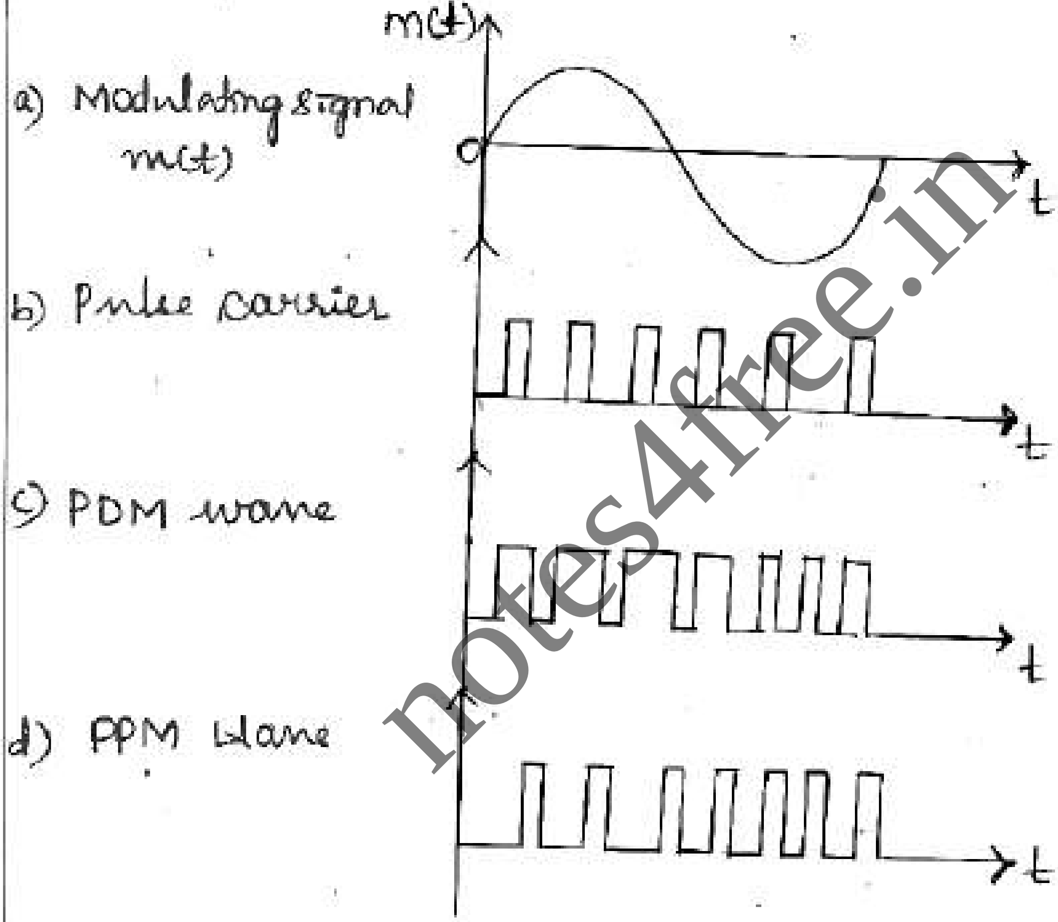
* In pulse-duration modulation (PDM), the samples of the message signal are used to vary the duration of the individual pulses. This form of modulation is also referred to as pulse-width modulation or pulse-length modulation.

* In PPM, the position of a pulse relative to its unmodulated time of occurance is varied in accordance with the message signal as shown in fig(6)(d) for the case of sinusoidal modulation.

Let T_s denote the sample duration. Using the sample $m(nT_s)$ of a message signal $m(t)$ to modulate the position of the n^{th} pulse, we obtain the PPM signal

$$s(t) = \sum_{n=-\infty}^{\infty} g(t - nT_s - k_p m(nT_s)) \quad \text{——— (1)}$$

where k_p is the sensitivity of the pulse-position modulator



Fig(6) : Illustrating two different forms of pulse-time modulation.

* GENERATION OF PPM WAVES :

The PPM signal which is generated is shown in fig(7)(a). The message signal m(t) is first converted into a PAM signal by means of a sample and Hold circuit, generating a staircase waveform u(t), which is shown in 8(b) for the message signal m(t) shown in fig 8(a).

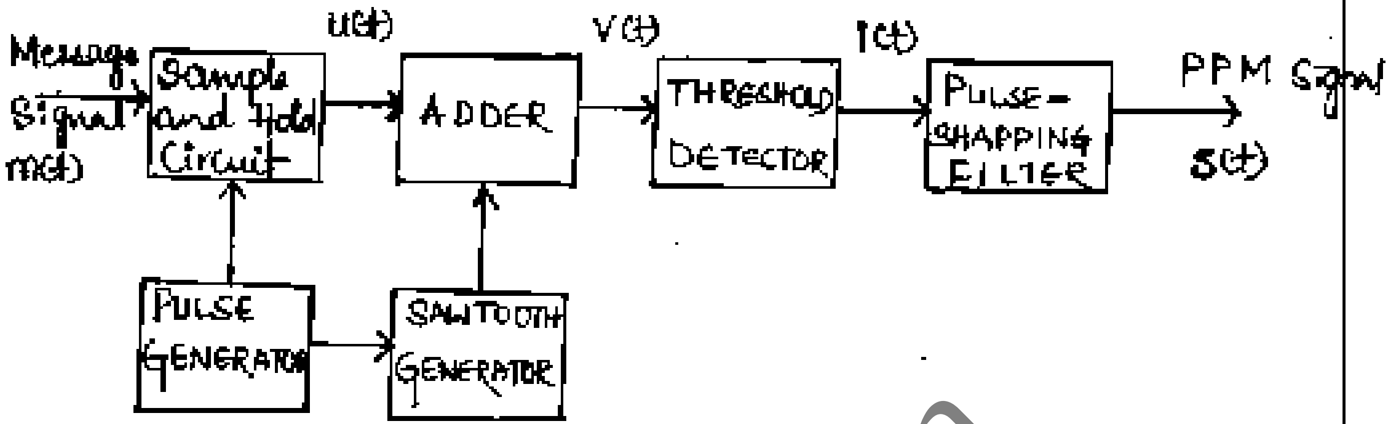
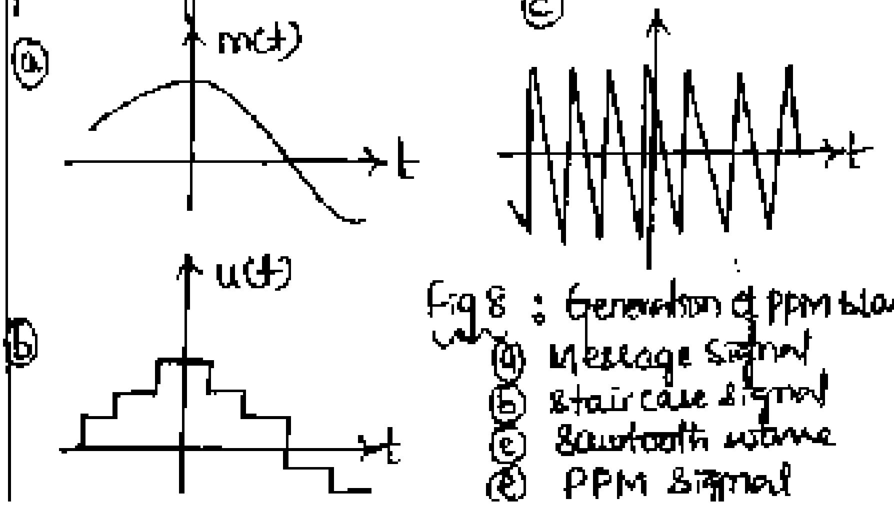


Fig 7(a) : Block diagram of PPM generator.

* Next, the signal u(t) is added to a sawtooth wave, yielding the combined signal v(t). The combined signal v(t) is applied to a threshold detector that produces a very narrow pulse each time v(t) crosses zero in the -ve going direction. The resulting sequence of "impulses" i(t) is shown in fig(8)(e). Finally, the PPM signal s(t) is generated by using this sequence of impulses to excite a filter whose impulse response is defined by the standard pulse g(t).



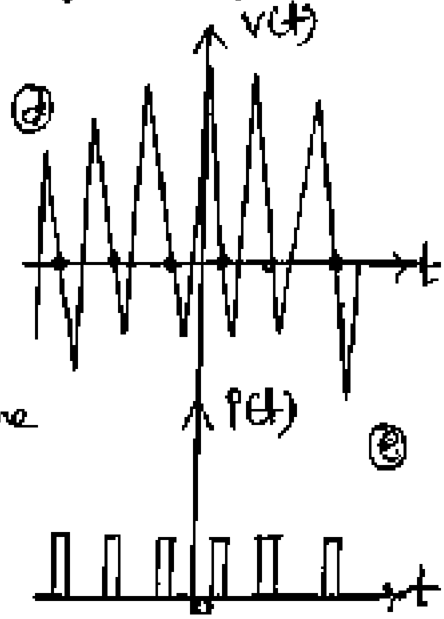


Fig 8 : Generation of PPM wave
(a) Message signal
(b) staircase signal
(c) Sawtooth wave
(e) PPM signal

* DETECTION OF PPM WAVES :

Consider a PPM wave $s(t)$ with uniform sampling, and assume that the message signal $m(t)$ is strictly band limited. The operation of one type of PPM receiver may proceed as follows :

1) Convert the received PPM wave into a PDM wave with the same modulation.

2) Integrate this PDM wave using a device with a finite integration time, thereby computing the area under each pulse of the PDM wave.

3) Sample the output of the integrator at a uniform rate to produce a PAM wave, whose pulse amplitudes are proportional to the signal samples $m(nT_s)$ of the original PPM wave $s(t)$.

4) Finally, demodulate the PAM wave to recover the message signal $m(t)$.

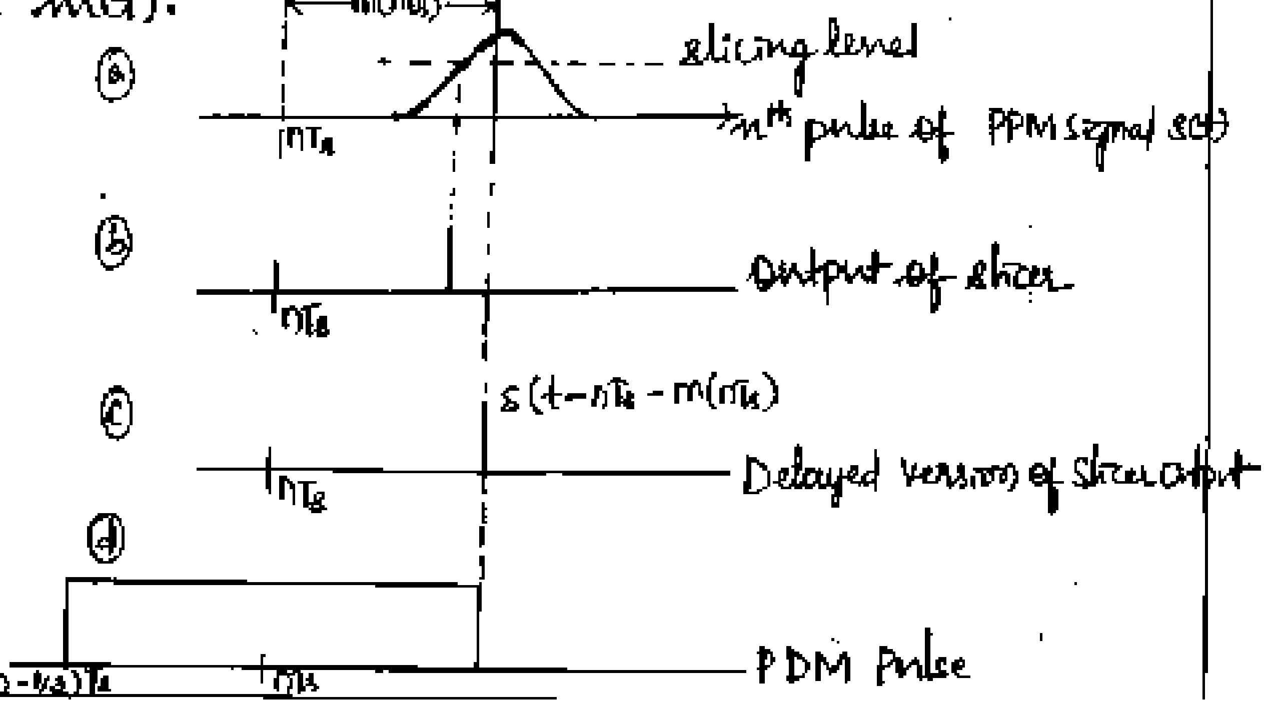


Fig : Detection for waveforms of PPM signal.

* NOISE IN PULSE-POSITION MODULATION :

* In a PPM system, the transmitted information is contained in the relative positions of the modulated pulses. The presence of additive noise affects the performance of such a system.

* The output signal to noise ratio, assuming a full-load sinusoidal modulation, is therefore

$$(SNR)_0 = \frac{\pi^2 B_T T_s^2 A^2}{32 N_0} \quad \text{——— (1)}$$

* The avg noise power in a message bandwidth 'W' is equal to WN_0. The channel signal to noise ratio is therefore,

$$(SNR)_C = \frac{3A^2}{4 T_s B_T W N_0} \quad \text{——— (2)}$$

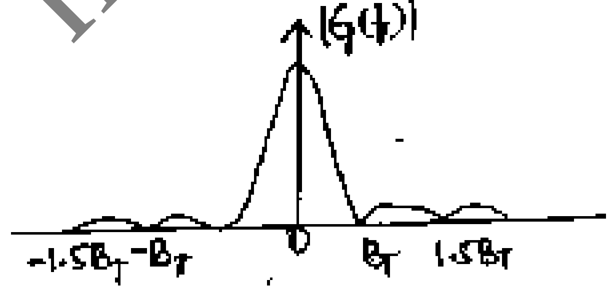


Fig : Amplitude spectrum of a raised Cosine pulse.

* Thus, the figure of merit of a PPM system using a raised cosine pulse is as follows.

$$\text{Figure of Merit} = \frac{(SNR)_0}{(SNR)_C} = \frac{\pi^2 B_T T_s^2 A^2}{32 N_0} \times \frac{4 T_s B_T W N_0}{3A^2}$$

$$\therefore \boxed{\text{Figure of Merit} = \frac{\pi^2}{24} B_T^2 T_s^3 W}$$

* THE QUANTIZATION PROCESS :

The process of transforming sampled amplitude values of a message signal into a discrete amplitude value (levels) is referred to as quantization.

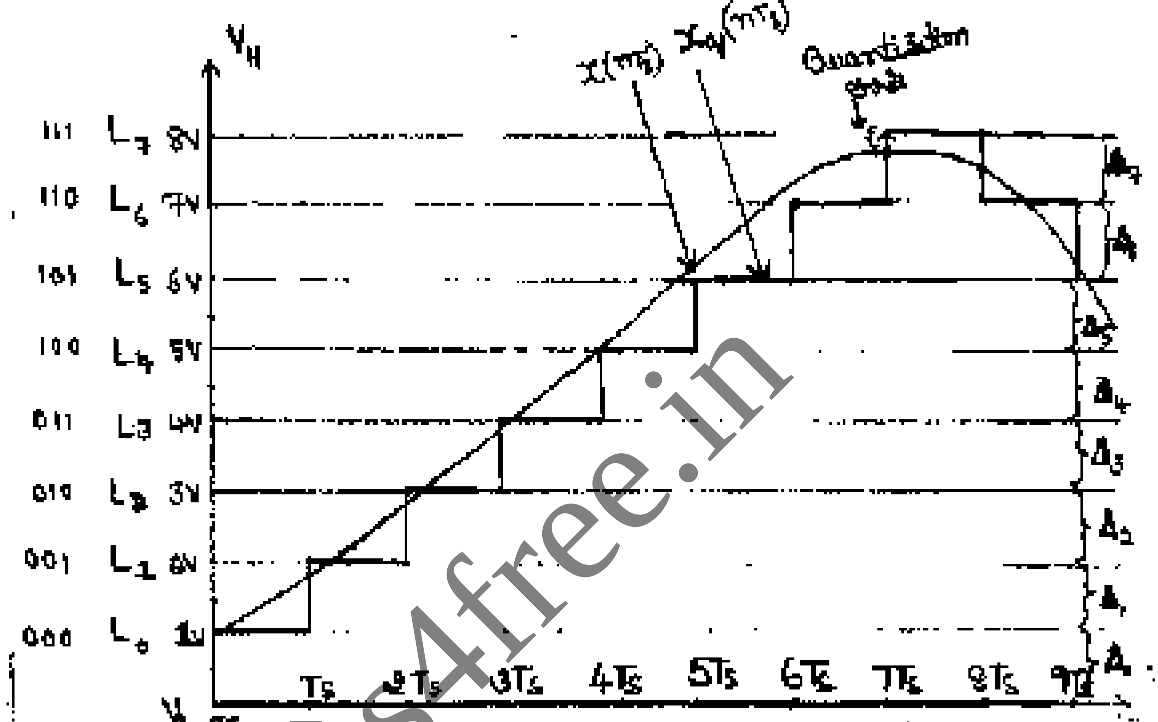


Fig : Quantization process

* The signal $x(t)$ whose excursion is confined to the range from V_L to V_H being divided into 8-equal levels.

* Step size is denoted by 'Δ' and is given by

$$\Delta = \frac{V_H - V_L}{L}$$

where $L = 2^R$ and R is no. of bits.

$$\therefore \quad \Delta = \frac{V_H - V_L}{2^R}$$

If the step size 'Δ' is maintained same through the process of quantization, then it is called "uniform quantization".

* The difference between the continuous amplitude sample level and quantized signal level is known as quantization error.

$$e(t) = x_q(nT_s) - x(nT_s)$$

where quantization error varies from $+\Delta/2$ to $-\Delta/2$.

* The random errors due to quantization process produce a noise at the output of the quantizer and this noise is referred to as Quantization noise.

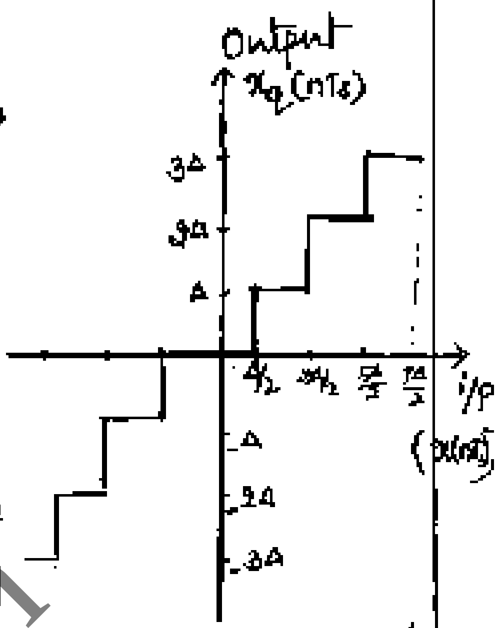
* Consider fig(A), the sampling, Quantizing and coding of an analog signal is as follows.

Sampled values of an analog signal	1.7V	2.7V	3.9V	5V	6.2V	7.2V	7.7V	7.4V
Nearest Quantizer level	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8
Quantizer level voltage	2V	3V	4V	5V	6V	7V	8V	7V
Binary Code	001	010	011	100	101	110	111	110

* There are two types of quantizer they are
  - Mid-tread type quantizer
  - Mid-riser type quantizer

1) Mid-Tread type Quantizer :

* In mid-tread quantizer, the decision threshold of the quantizers are located at $\pm\frac{\Delta}{2}, \pm\frac{3\Delta}{2}, \ldots$, and the representation levels are located at $0, \pm\Delta, \pm 2\Delta$, where Δ is the step size.

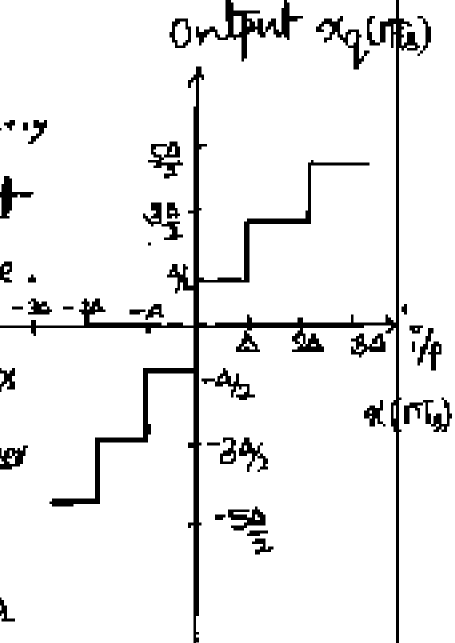


Midtread Quantizer

* A uniform quantizer characterized in this way is referred to as a symmetric quantizer of the midtread type, because the origin lies in the middle of a tread of a staircase graph. Here, quantization levels are odd number.

2) Mid-Riser type quantizer :

* In midriser quantizer, the decision threshold of the quantizers are located at $0, \pm\Delta, \pm 2\Delta, \ldots$, and the representation levels are located at $\pm\frac{\Delta}{2}, \pm\frac{3\Delta}{2}, \pm\frac{5\Delta}{2}, \ldots$, where Δ is the step size.

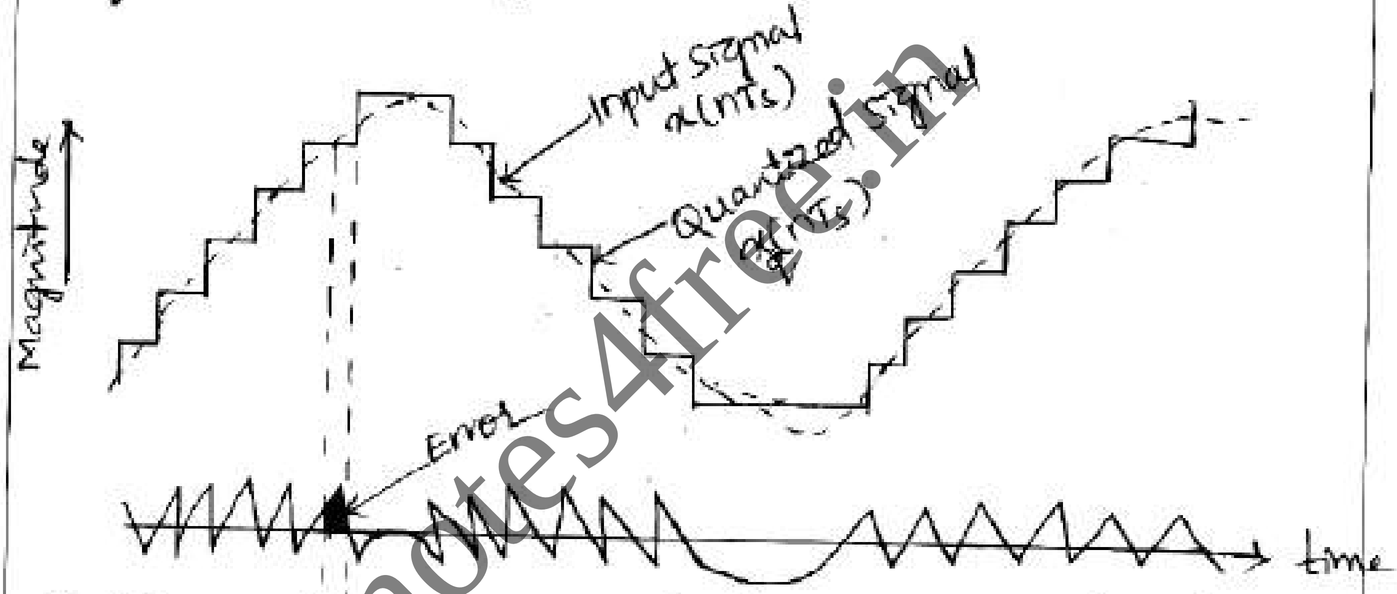


Midrise Quantizer

* A uniform quantizer characterized in this way is referred to as a symmetric quantizer of the midriser type, because the origin lies in the middle of a riser of the staircase graph.
* Here quantization levels are even number.

* QUANTIZATION NOISE

* The use of quantization introduces an error defined as the difference between the input signal 'm' and the output signal 'v'. This error is called Quantization noise. Fig(B), illustrates a typical variation of the Quantization noise as a function of time, assuming the use of a uniform quantizer of the midtread type.



Fig(B) : Illustration of Quantization process and Noise

* Let the random variable 'Q' denotes the quantization error and 'q' its sample value.

$$q = m - v \qquad (1)$$

* Consider then an input 'm' of continuous amplitude in the range $(-m_{max}, m_{max})$ then, the step-size of the quantizer is given by,

$$\Delta = \frac{2m_{max}}{L} \qquad (2)$$

* Now, the probability density function of the quantization error 'Q' as follows,

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} < q \leq \frac{\Delta}{2} \\ 0, & \text{otherwise} \end{cases} \quad \text{——— (3)}$$

* Now, the variance of Quantization error is

$$\sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} q^2 f_Q(q)\, dq$$

$$\sigma_Q^2 = \int_{-\Delta/2}^{\Delta/2} q^2 \cdot \frac{1}{\Delta}\, dq \qquad \because f_Q(q) = \frac{1}{\Delta}$$

$$= \frac{1}{\Delta}\left[\frac{q^3}{3}\right]_{-\Delta/2}^{\Delta/2} = \frac{1}{3\Delta}\left[\left(\frac{\Delta}{2}\right)^3 - \left(\frac{-\Delta}{2}\right)^3\right] = \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} - \frac{(-\Delta)^3}{8}\right]$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right] = \frac{1}{3\cancel{\Delta}} \times \cancel{2}\left(\frac{\Delta^{\cancel{3}}}{\cancel{8}4}\right) = \frac{1}{3} \cdot \frac{\Delta^2}{4}$$

$$\therefore \boxed{\sigma_Q^2 = \frac{\Delta^2}{12}} \quad \text{——— (A)}$$

This is known as "Mean squared quantization error" or Normalized Noise power or Quantization error in terms of power.

* Let us consider "R" which denote the number of bits per sample then the Quantized level is given by,

$$L = 2^R \quad \text{——— (4)}$$

substituting Eq^n (4) in Eq^n (2) we get,

$$\Delta = \frac{2 m_{max}}{2^R} \quad \text{——— (5)}$$

Now substitute Eq^n (5) in Eq^n (A) we get

$$\therefore \sigma_Q^2 = \left[\frac{\frac{2m_{max}}{2^R}}{12}\right]^2 = \frac{4 m_{max}^2}{2^{2R}} \times \frac{1}{12}$$

$$\therefore \sigma_Q^2 = \frac{1}{3} m_{max}^2 2^{-2R} \qquad (6)$$

Let 'P' denote the avg power of message signal m(t). We may express the output signal to noise ratio of a uniform quantizer as,

$$(SNR)_0 = \frac{P}{\sigma_Q^2}$$

$$= \frac{P}{\frac{1}{3} m_{max}^2 2^{-2R}}$$

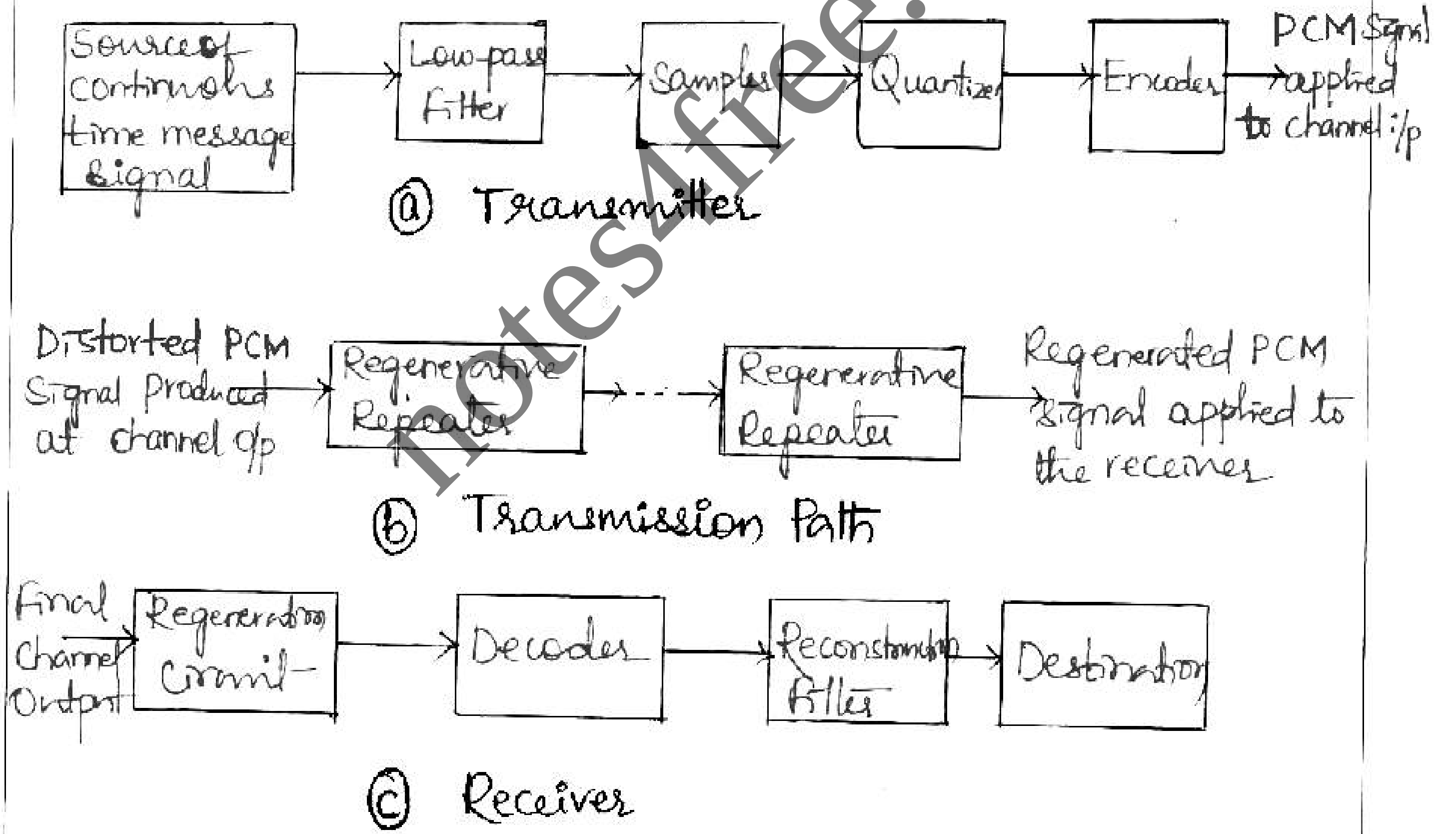
$$\therefore \boxed{(SNR)_0 = \left(\frac{3P}{m_{max}^2}\right) 2^{2R}}$$

* PULSE CODE MODULATION :

* In pulse code Modulation (PCM), a message signal is represented by a sequence of coded pulses, which is accomplished by representing the signal in discrete form in both time and amplitude.
* The basic operations performed in the transmitter of a PCM system are sampling, quantizing and encoding as shown in fig 6(a). The Lowpass filter prior to sampling

is included to prevent aliasing of the message signal. The quantizing and encoding operations are usually performed in the same circuit, which is called an analog to-digital converter.

* The basic operations in the receiver are regeneration of impaired signals, decoding and reconstruction of the train of quantized samples as shown in fig 6(c). Regeneration also occurs at intermediate points along the transmission path as necessary as indicated in fig(6)b.



Fig(6) : The basic elements of a PCM system.

* SAMPLING :

The incoming message signal is sampled with a train of narrow rectangular pulses so as to closely approximate the instantaneous sampling process. In order to ensure perfect reconstruction of the message signal at the receiver, the sampling rate must be greater than or equal to the highest frequency component W of the message signal in accordance with the sampling theorem.

$$f_s \geq 2W.$$

* Quantization :

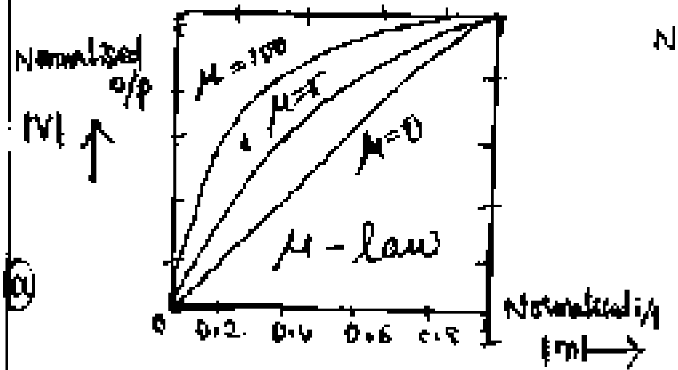
the sampled version of the message signal is then quantized, thereby providing a new representation of the signal that is discrete in both time and amplitude.

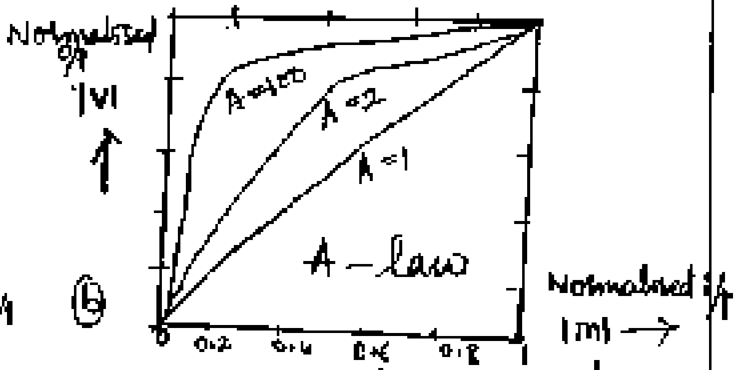
* For uniform quantization, we have mid-tread and mid-rise quantizer and for non-uniform quantization, we have two compression laws μ-law and A law.

* The use of a non-uniform quantizer is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. A particular form of compression law that is used in practice is the so called μ-law, defined by

$$|V| = \frac{\log(1+\mu|m|)}{\log(1+\mu)} \quad \text{——— (1)}$$

where m and v are normalized input & output vlgs. and μ' is positive constant.





* Another compression law that is used in practice is the so called A-law as shown above.

$$|V| = \begin{cases} \dfrac{A|m|}{1+\log A}, & 0 \leq |m| \leq \dfrac{1}{A} \\ \dfrac{1+\log(A|m|)}{1+\log A}, & \dfrac{1}{A} \leq |m| \leq 1 \end{cases}$$

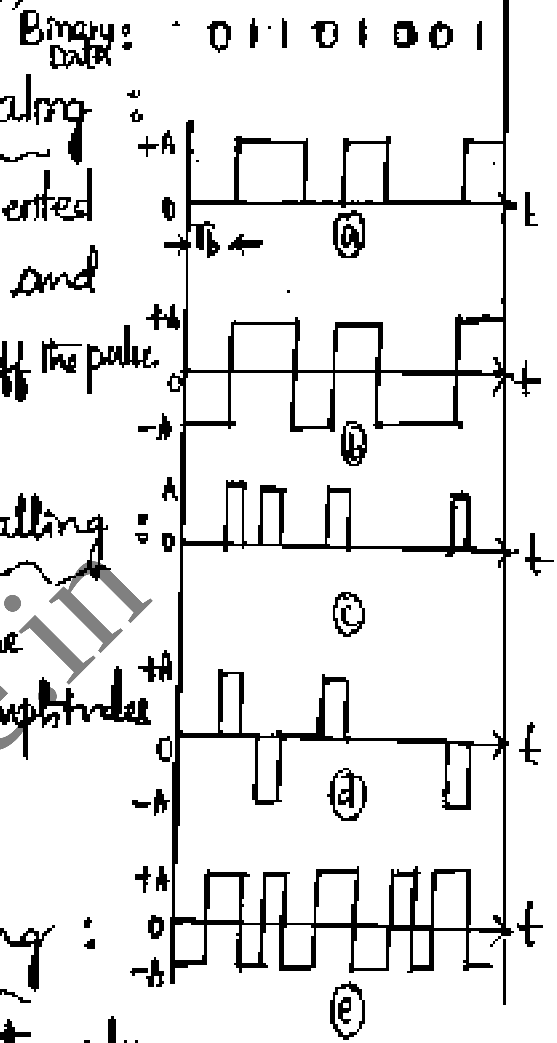
* Encoding :

* In combining the processes of sampling and quantizing the specification of a continuous message (baseband) signal becomes limited to a discrete set of values, but not in the form best suited to transmission over a line or radio path.

* In a binary code each symbol may be either of two distinct values or kinds, such as the presence or absence of a pulse. The two symbols of a binary code are customarily denoted as 0 and 1.

* Line code : It is a line code that a binary stream

of data takes on an electrical representation. The five line codes are illustrated in fig(7).



(a) → Unipolar NRZ

(b) → Polar NRZ

(c) → Unipolar RZ

(d) → Bipolar RZ

(e) → Manchester code

1) **Unipolar Non return to zero (NRZ) signaling :**

In this line code symbol '1' is represented by transmitting a pulse of amplitude 'A' and symbol '0' is represented by switching off the pulse.

2) **Polar Non-return to zero (NRZ) signalling :**

In this line code, symbol 1 and 0 are represented by transmitting pulses of amplitudes +A and -A respectively.

3) **Unipolar Return to zero (RZ) signalling :**

Here, symbol 1 is represented by a rectangular pulse of amplitude A and half-symbol width and symbol 0 is represented by transmitting no pulse.

4) **Bipolar Return to zero (BRZ) signalling :**

This line code uses three amplitude levels as shown in fig. Specifically, +ve & -ve pulses of equal amplitude are used alternatively for symbol 1. 0 for no pulse.

5) **Split-phase (Manchester code)**

Symbol 1 is represented a positive pulse of amplitude 'A' followed by a negative pulse of amplitude -A with both pulses being a half-symbol wide. For symbol '0', the polarities of these two pulses are reversed.

* Differential Encoding :

This method is used to encode information in terms of signal transitions. In particular, a transition is used to designate symbol 0 in the incoming binary data stream, while no transition is used to designate symbol 1 as shown in fig.

ⓐ Original binary data 0 1 1 0 1 0 0 1

ⓑ Differentially encoded data 1 0 0 0 1 1 0 1 1

ⓒ Waveform

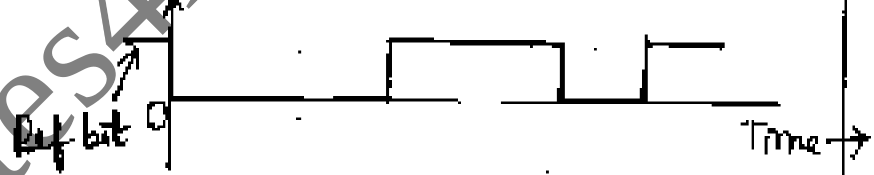


Fig : Differential encoding.

* REGENERATION :

The distorted PCM wave obtained from the transmitter is sent to the amplifier equilizer. The output of equilizer device is passed to the Decision making device to decide the signal interms of 1 or 0 (coded o/p).

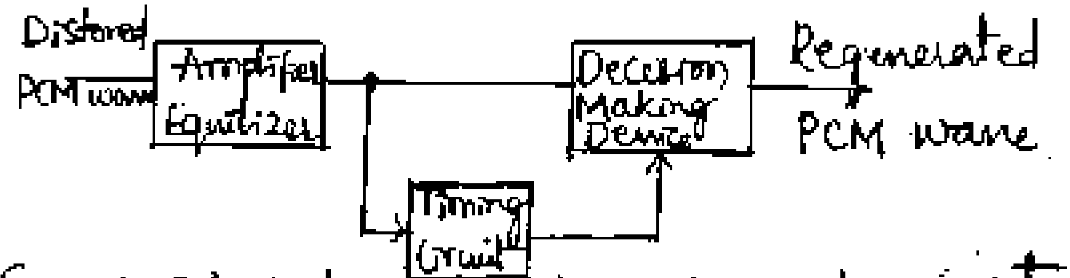


fig : Block diagram of a regenerative repeater.

* **Decoding :**

The decoding process involves generating a pulse the amplitude of which is the linear sum of all the pulses in the codeword, with each pulse being weighted by its place value; $(2^0, 2^1, 2^2 \ldots, 2^{R-1})$ in the code, where 'R' is the number of bits per sample.

* **FILTERING :**

The final operation in the receiver is to recover the message signal wave by passing the decoder output through a lowpass reconstruction filter whose cutoff frequency is equal to the message bandwidth 'W'.

* **MULTIPLEXING :**

In applications using PCM, it is natural to multiplex different message sources by time division whereby each source keeps its individuality throughout the journey from the transmitter to receiver.

* This individuality accounts for the comparative ease with which message sources may be dropped or reinserted in a time division multiplex system.

* **APPLICATION TO VOCODERS :**

* Linear Prediction coding (LPC) vocoders are model-based systems. Based on the human speech model, a

voice encoding approach can be established. Fig8 shows analysis and synthesis of voice signals in an LPC encoder and decoder.

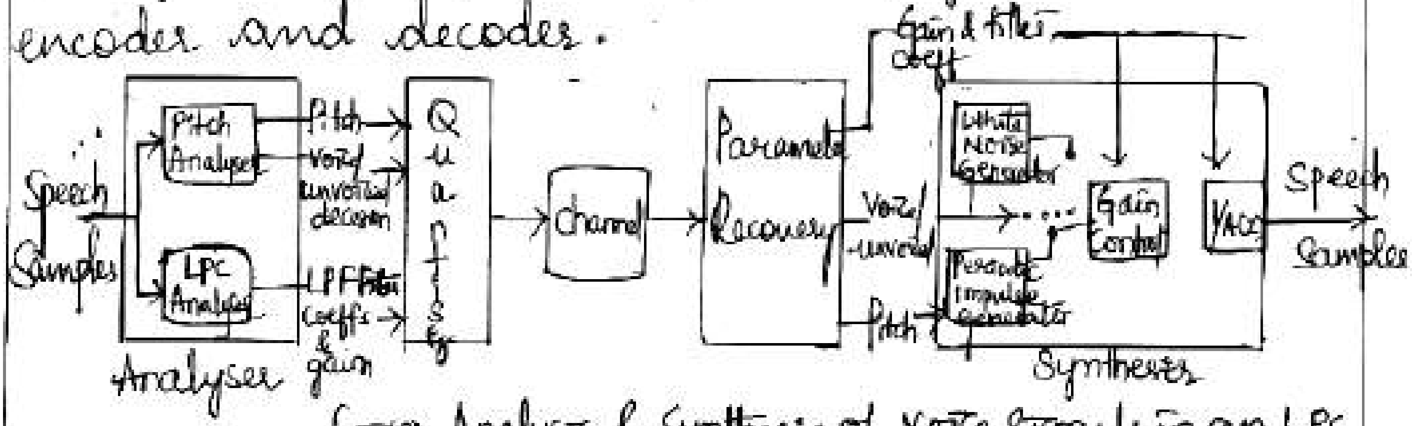


Fig8 Analysis & Synthesis of voice signals in an LPC

* In the analysis of a sampled voice segment, the pitch analysis will first determine whether the speech is a voiced or an unvoiced piece. The LPC analyser will estimates the all-pole filter co-efficients.

* The synthesizer, which produces the speech samples at the destination followed by the Quantizer, channel Parameter recovery and gain control.