Advanced Surveying [15CV46]

Module - 1 10/02/17

CURVE SURVEYING

Curves are defined as arcs with finite radius provided between intersecting straights [Tangents] to gradually negotiate a change in direction.

classification of curves :-

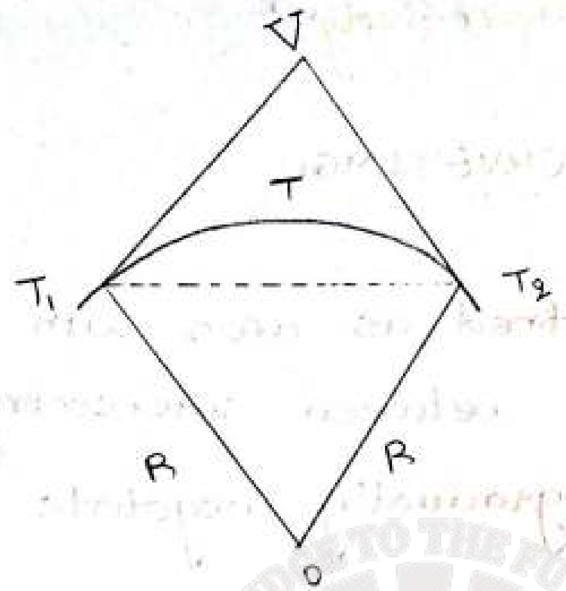
1. Vertical curves → Vertical planes
2. Horizontal curves → Horizontal planes

Circular curves are classified into.

1. SIMPLE CURVES
2. COMPOUND CURVES
3. REVERSE CURVES

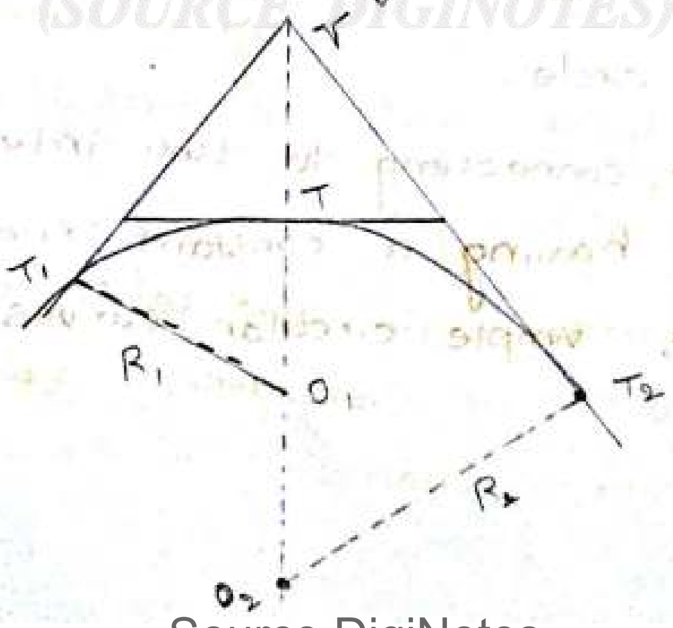
1. SIMPLE CURVES :- It consists of single arc of a circle.

* A curve connecting to two intersecting straights having a constant radius is known as simple circular curves.



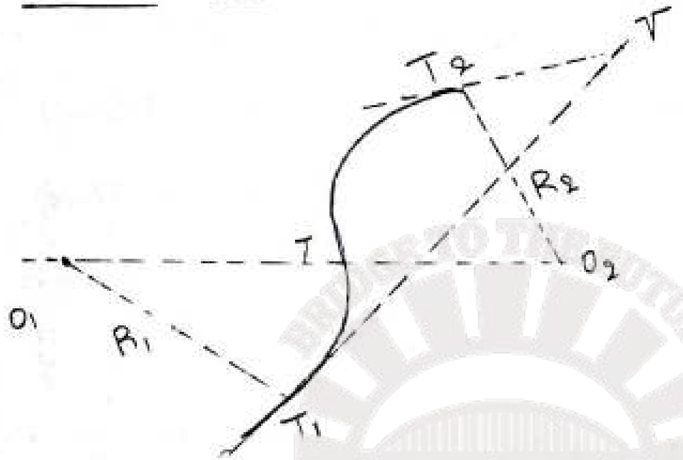
$T_1, T, T_2 \rightarrow$ Simple circular curve of radius of R come joining the 2 Straights [tangents] T_1 and T_2 intersecting at a point V

② COMPOUND CURVES :- It consists of two or more Simple arcs or simple circular curves of different radii turning in the Same directions and join at common tangent points.



The above figure is a compound curve with 2 simple circular curves T_1, T, T_2 of radii R_1 and R_2 respectively.

③ <u>REVERSE CURVE</u> :-



when two simple circular curves of equal or different radii having opposite direction of curvature joined together, the resultant curve is known as reverse curve.

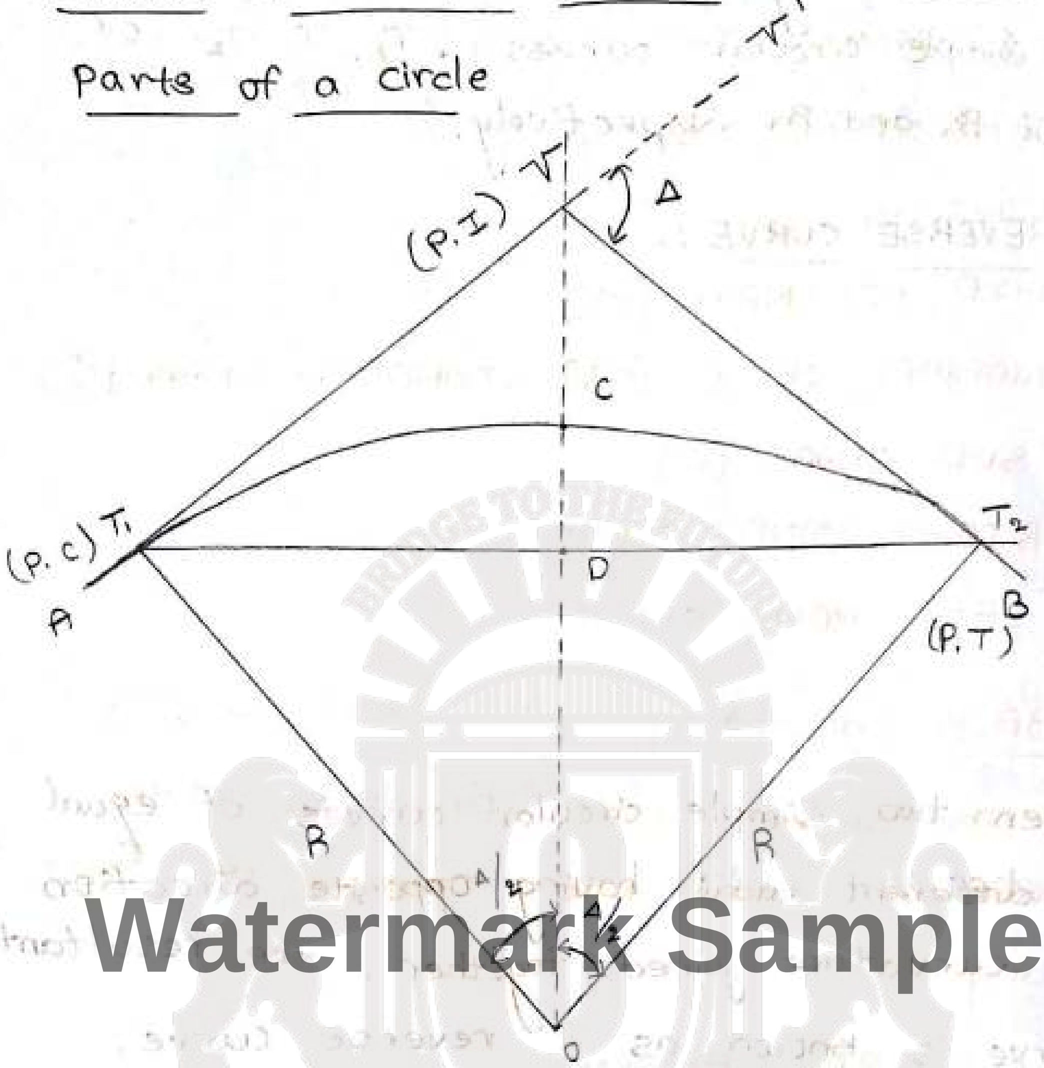
In the above figure T_1, T, T_2 is the reverse curve formed by the curves T_1T and TT_2 of radii R_1 and R_2 respectively.

* Reverse curves are quite common in railway yards but are unsuitable for modern highways.

* These curves are also called as S- curves or serpentine because of their shapes.

Elements of simple circular curve or

Parts of a circle



Following are the elements of the simple circular curves or parts of a circular curve.

① BACK TANGENT

② FORWARD TANGENT

③ POINT OF INTERSECTION [VERTEX]

④ POINT OF CURVE [P.C]

⑤ POINT OF TANGENCY [P.T]

⑥ INTERSECTION ANGLE

⑦ DEFLECTION ANGLE TO ANY POINT

⑧ TANGENT DISTANCE [T]

⑨ EXTERNAL DISTANCE [E]

⑩ LENGTH OF CURVE [L]

⑪ LONG CHORD

⑫ MID ORDINATE [M]

⑬ NORMAL CHORD [OR] STANDARD CHORD [C]

⑭ SUB CHORD [c]

⑮ RIGHT HAND CURVE

⑯ LEFT HAND CURVE

① BACK TANGENT :-

* The tangent in AT_1 previous to the first curve is called back tangent. of first tangent

② FORWARD TANGENT :-

* The tangent T_2B following the curves is called Forward Tangent or Second tangent

③ POINT OF INTERSECTION :- [P.I]

Its the two tangents AT_1 and BT_2 are produced, they will meet in a point called point of intersection [P.I] or Vertex [V]

④ POINT OF CURVE [P.C] :-

It is the begining of the curves

where the allignment changes from a tangent to a curve

⑤ POINT OF TANGENCY [P.T]

It is the end of the curve where the allignment changes from a curve to a tangent

⑥ INTERSECTION ANGLE :-

The angle 'V' 'VB' between the tangent 'AB' produced and 'VB' is called the intersection angle (Δ)

⑦ DEFLECTION ANGLE TO ANY POINT

The deflection angle to any point on the curve is the angle at point of curve [P.C] between the back tangent and the chord from point of curve [P.c] to the point on the curves

⑧ TANGENT DISTANCE [T]:-

It is the distance between point of curve [P.C] to the point of intersection [P.I]; also it is the ditance from point of intersection [P.I] to the point of tangency [P.T]

(9) EXTERNAL DISTANCE (E)

It is the distance from the mid point of the curve to the point of intersection [P.I]. It is also known as apex distance

(10) LENGTH OF THE CURVE (L)

It is the total length of the curve from point of Curve [P.C] to point of tangency [P.T]

(11) LONG CHORD :-

It is the chord joining point of curve [P.C] to point of Tangency [P.T].

(12) MID ORDINATE :- [M]

It is the ordinate from the mid point of the long chord to the mid point of the curve.

(13) NORMAL CHORD [OR] STANDARD CHORD [C]:-

It is a chord between the two successive regular stations on a chord.

(14) SUB CHORD [c]:-

A Sub chord is any chord shorter than the long normal chord.

(15) RIGHT HAND CURVE :-

If the curve deflets to the right of the deflecti direction of the progress of survery, it is called right hand curve.

Relation between degree of curve (D) and radius of curve (R)

ARC DEFINATION:-

If 'R' is the radius of the curve and 'D' diameter of the curve for 100 feet arc then from the familiar proportion we have

$$100 : 2\pi R = D : 360$$

$$\frac{100}{2\pi R} = \frac{D}{360}$$

$$R = \frac{100}{2\pi} \times \frac{360}{D}$$

$$R = \frac{5729.578}{D}$$

$$\approx$$

$$\boxed{R = \frac{5730}{D}}$$

If 'D' is the ~~diameter~~ degree of curve for 30 m arc

$$30 : 2\pi R = D : 360$$

$$\frac{30}{2\pi R} = \frac{D}{360}$$

$$R = \frac{30}{2\pi} \times \frac{360}{D}$$

$$R = \frac{1718.87}{D} \approx$$

$$\boxed{R = \frac{1719}{D} \text{ m}}$$

If 'D' is the degree of curve for 20m arc

$$20 : 2\pi R = D : 360$$

$$\frac{20}{2\pi R} = \frac{D}{360}.$$

$$R = \frac{20}{2\pi} \times \frac{360}{D}$$

$$R = \frac{1145.92}{D}$$

$$\simeq$$

$$\boxed{R = \frac{1146}{D} m}$$

CHORD DEFINATION:

From a Δ^{le} POM

$$\sin\left(\frac{D}{2}\right) = \frac{50}{R}$$

$$R = \frac{50}{\sin\left(\frac{D}{2}\right)}$$

when 'D' is small, $\sin\left(\frac{D}{2}\right)$ may be taken approximately equal to $D/2$

$$\sin\left(\frac{D}{2}\right) \approx \frac{D}{2}$$

$$R = \frac{50}{D/2}$$

$$R = \frac{50}{\frac{D}{2} \times \frac{\pi}{180^\circ}}$$ in degrees

$$R = \frac{5729.578}{D}$$

$$\boxed{R \approx \frac{5730}{D}}$$

From the Δ^{le} pom

$$\sin\left(\frac{D}{2}\right) = \frac{15}{R}$$

$$R = \frac{15}{\sin\left(\frac{D}{2}\right)}$$

$$R = \frac{15}{\frac{D}{2} \times \frac{\pi}{180^\circ}} \text{ in degrees}$$

$$R = \frac{15 \times 2 \times 180}{D\pi}$$

$$R = \frac{1718.873}{D}$$

$$\approx$$

$$\boxed{R = \frac{1719}{D}}$$

For the 20 m

From the Δ^{le} pom

$$\sin\left(\frac{D}{2}\right) = \frac{10}{R}$$

$$R = \frac{10}{\sin\left(\frac{D}{2}\right)}$$

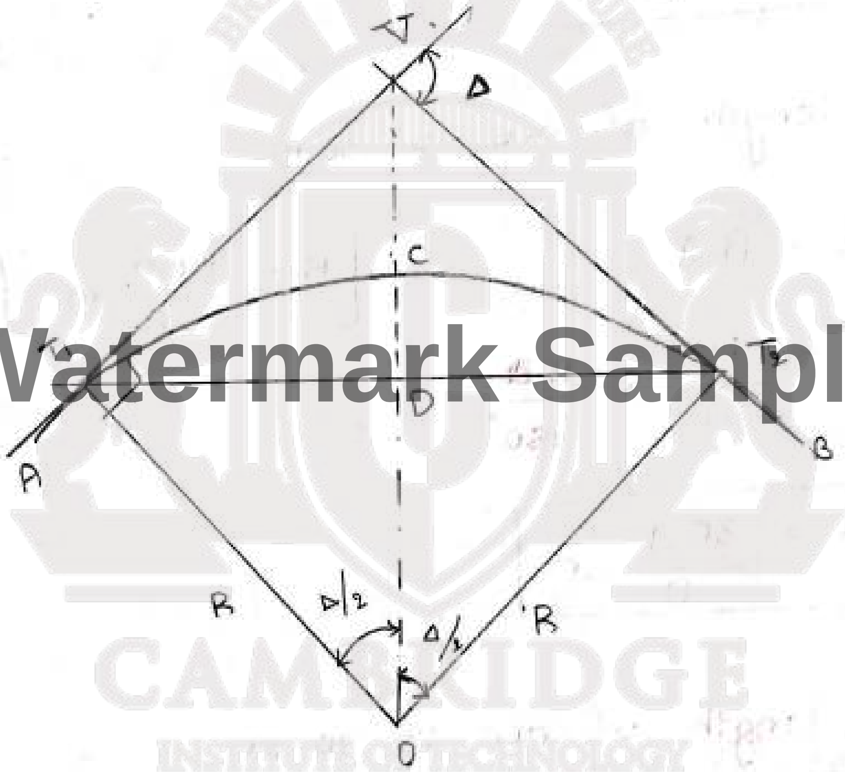
$$R = \frac{10}{\frac{D}{2} \times \frac{\pi}{180}} \text{ in degrees}$$

$$R = \frac{10 \times 2 \times 180}{D\,\pi}$$

$$R = \frac{1145.915}{D}$$

$$\approx$$

$$\boxed{R \simeq \frac{1146}{D}}$$



LENGTH OF THE CURVE (l)

$l = T_1 C T_2$

$l = R\Delta$ (where Δ is in radians)

$l = \dfrac{\pi R \Delta}{180^\circ}$ (where Δ is in degrees)

ARC DEFINATION / CHORD DEFINATION

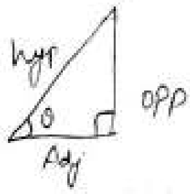
If length of arc is 100 feet

$l = R\Delta$

$\left[R = \dfrac{5730}{D} \text{ feet}\right]$

$l = \dfrac{5730}{D} \times \dfrac{\pi\Delta}{180^\circ}$

$\boxed{l = \dfrac{100\,\Delta}{D} \text{ feet}}$



$\sin\theta = \dfrac{opp}{hyp}$

$\cos\theta = \dfrac{opp}{Adj}$

If length of arc is 30 m

$l = R\Delta$

$\left[R = \dfrac{1719}{D} \text{ m}\right]$

$= \dfrac{1719}{D} \times \dfrac{\pi\Delta}{180^\circ}$

$\boxed{l = \dfrac{30\,\Delta}{D} \text{ m}}$

If length of arc is 20 m

$l = R\Delta$

$\left[R = \dfrac{1146}{D} \text{ m}\right]$

$= \dfrac{1146}{D} \times \dfrac{\pi\Delta}{180^\circ}$

$\boxed{l = \dfrac{20\,\Delta}{D} \text{ m}}$

TANGENT LENGTH (T)

$$T = T_1V = VT_2$$

$$\tan\left(\frac{\Delta}{2}\right) = \frac{T_1V}{OT_1} = \frac{T}{R}$$

$$\tan\left(\frac{\Delta}{2}\right) = \frac{T}{R}$$

$$\boxed{T = R\tan\left(\frac{\Delta}{2}\right)}$$

LENGTH OF LONG CHORD (L)

$$L = T_1 0 T_2$$

$$\sin\left(\frac{\Delta}{2}\right) = \frac{T_1D}{OT_1}$$

$$\sin\left(\frac{\Delta}{2}\right) = \frac{L/2}{R}$$

$$\boxed{L = 2R\sin\left(\frac{\Delta}{2}\right)}$$

APEX DISTANCE OR EXTERNAL DISTANCE [E]

$$E = CV$$

$$= VO - CO$$

$$E = R\sec\left(\frac{\Delta}{2}\right) - R$$

$$E = R\left[\sec\left(\frac{\Delta}{2}\right) - 1\right]$$

MID ORDINATE [M or O_o]

$$M = CD$$

$$= CO - DO$$

$$= R - R\cos\left(\frac{\Delta}{2}\right)$$

$$\boxed{M \text{ or } O_o = R\left[1 - \cos\left(\frac{\Delta}{2}\right)\right]}$$

$$\boxed{M \text{ or } O_o = R \text{ ver Sine}\left(\frac{\Delta}{2}\right)}$$

☆ The mid ordinate of the curve is also known as Versed Sign of the curve.

Problem :-

① A circular curve has 200 m radius and at a deflection angle of 65°. Calculate (a) Length of the curve (b) tangent length (c) Length of long chord (d) Apex distance (e) mid ordinate. also calculate the degree of the curve by arc defination and chord defination.

Sol^n :-

$$R = 200 \text{ m}$$

$$\Delta = 65°$$

(a) Length of the curve (l)

$$l = \frac{\pi R \Delta}{180^\circ}$$

$$= \frac{\pi \times 200 \times 65^\circ}{180^\circ}$$

$$\boxed{l = 226.89 \text{ m}}$$

(b) Tangent length (T)

$$T = R \tan\left(\frac{\Delta}{2}\right)$$

$$= 200 \times \tan\left(\frac{65^\circ}{2}\right)$$

$$\boxed{T = 127.41 \text{ m}}$$

(c) Length of long chord (L)

$$L = 2R \sin\left(\frac{\Delta}{2}\right)$$

$$= 2 \times 200 \times \sin\left(\frac{65}{2}\right)$$

$$\boxed{L = 214.91 \text{ m}}$$

(d) Apex distance (E)

$$E = R\left[\sec\left(\frac{\Delta}{2}\right) - 1\right]$$

$$= 200\left[\sec\left(\frac{65}{2}\right) - 1\right]$$

$$\boxed{E = 37.1378 \text{ m}}$$

(e) mid ordinate (m or O_0)

$$m = R\left[1 - \cos\left(\frac{\Delta}{2}\right)\right]$$

$$= 200\left[1 - \cos\left(\frac{65^\circ}{2}\right)\right]$$

$$\boxed{m = 31.3217\ m}$$

By arc defination by using assuming a arc of 30 m length

$$R = \frac{1719}{D}$$

$$= \frac{1719}{200}$$

$$\boxed{D = 8.595}$$

chord defination

$$R = \frac{15}{\frac{\pi}{180} \times \sin\left(\frac{D}{2}\right)}$$

$$D = \frac{15 \times 2 \times 180}{(\pi \times 180)\,200}$$

$$\boxed{D = 8.5943}$$

By chord defination by assuming a 30 m chord length

$$R = \frac{1719}{D}$$

$$= \frac{1719}{200}$$

$$\boxed{D = 8.595}$$

arc defination

$$\pi \times R \times \frac{D}{180} = 30$$

$$D = \frac{30 \times 180}{200 \times \pi}$$

$$\boxed{D = 8.594}$$

(2) Determine the radius of the curve given the degree of curve is 5°.

Sol^n

Given:-

$D = 5^\circ$

By arc defination by assuming a arc of 30 m length

$$\frac{\pi}{180} \times R \times \frac{D}{180} = 30$$

$$R = \frac{30 \times 180}{\pi \times 5}$$

$$\boxed{R = 343.77 \text{ m}}$$

By chord defination by assuming a chord length of 30 m

$$R = \frac{15}{\frac{\pi}{180} \times \frac{D}{2}}$$

$$R = \frac{15 \times 180 \times 2}{\pi \times 5}$$

$$\boxed{R = 343.77 \text{ m}}$$

SETTING OUT SIMPLE CIRCULAR CURVES

Setting out a curve means locating various points at equal or convinient distances along the length of the curve.

* The distance between two any Successive points is called Peg interval.
* The method of setting out Simple curves are broadly classified as
  i) Linear methods
  ii) Instrumental methods or Angular methods
  iii)

(i) Linear methods :-

In the linear methods, only a chain or tape is used

* Linear methods are used when high degree of accuracy is not required or when the Curve is Short.

Following are the methods of the linear method for Setting out Simple Circular curves.

(i) By offsets or ordinates from the long chord
(ii) By perpendicular offsets from tangents

(iii) Radial offsets from the tangents

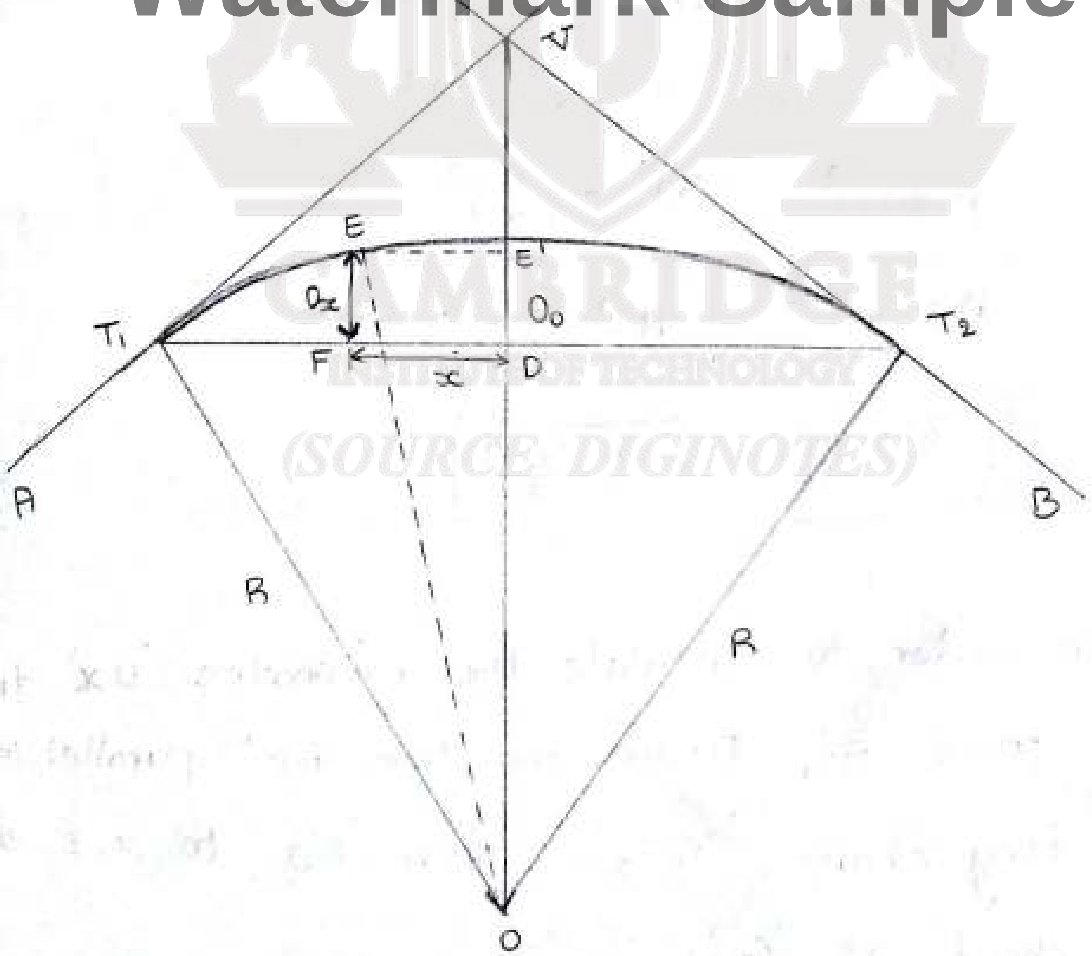
(iv) By successive bisection of arcs.

(v) By offsets from chords produced or deflection distance method

(ii) Instrumental methods or Angular methods

* In this method, an instrument such as the theodolite is used with or without a chain or tape.

ORDINATES FROM LONG CHORD OR OFFSETS FROM LONG CHORD



Let R = Radius of the curve

O_0 = mid ordinate

O_x = ordinate at distance x from the midpoint of the chord

T_1 and T_2 = Tangent points

L = Length of Long chord actually measured on the ground

Bisect the long chord at point 'P'. from the triangle OT_1D

$$(OT_1)^2 = (T_1D)^2 + (OD)^2$$

$$R^2 = \left(\frac{L}{2}\right)^2 + (CO - CD)^2$$

$$R^2 = \left(\frac{L}{2}\right)^2 + (R - O_0)^2$$

$$(R - O_0)^2 = (R)^2 - \left(\frac{L}{2}\right)^2$$

$$R - O_0 = \sqrt{(R)^2 - \left(\frac{L}{2}\right)^2}$$

$$\boxed{O_0 = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}}$$

* In order to calculate the ordinates 'Ox' to any point 'E', Draw the line EE' parallel to the long chord 'T_1T_2'. Join 'EO' to cut the long chord at 'G'.

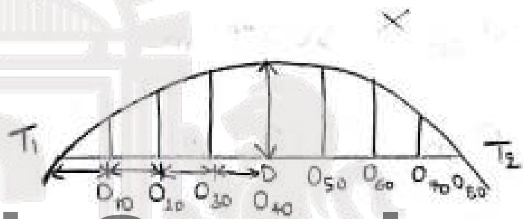
* Then O_x = EF = E'D

$$O_x = E'O - DO$$

$$= \sqrt{(EO)^2 - (EE')^2} - (CO - CD)$$

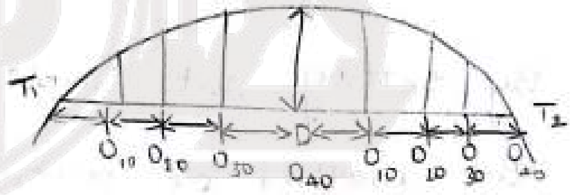
$$\boxed{O_x = \sqrt{(R)^2 - (x)^2} - (R - O_0)}$$

① Set out the simple curve by ordinates from the long chord. Take the length of the long chord has 80 m, ordinates at peg intervals of 10 m and versed sine of 4 m.



$$O_0 = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

$$4 = R - \sqrt{R^2 - \left(\frac{80}{2}\right)^2}$$



$$(R - 4) = \sqrt{(R)^2 - \left(\frac{80}{2}\right)^2}$$

$$(R-4)^2 = R^2 - 40^2$$

$$R^2 + 4^2 - 2(R)(4) = R^2 - 40^2$$

$$R^2 + 16 - 8R = R^2 - 1600$$

$$8R = 1600 + 16$$

$$8R = 1616$$

$$\boxed{R = 202 \text{ m}}$$

$$O_x = \sqrt{(R)^2 - (x)^2} - (R - O_0)$$

$$= \sqrt{(202)^2 - (10)^2} - (202 - 4)$$

$$= \sqrt{40804 - 100} - 198$$

$= 201.752 - 198$

$O_x = 3.752$ m

$O_{20} = 201.00 - 198$

$= 3.007$ m

$O_{30} = 1.759$ m

$O_{40} = 0$ m

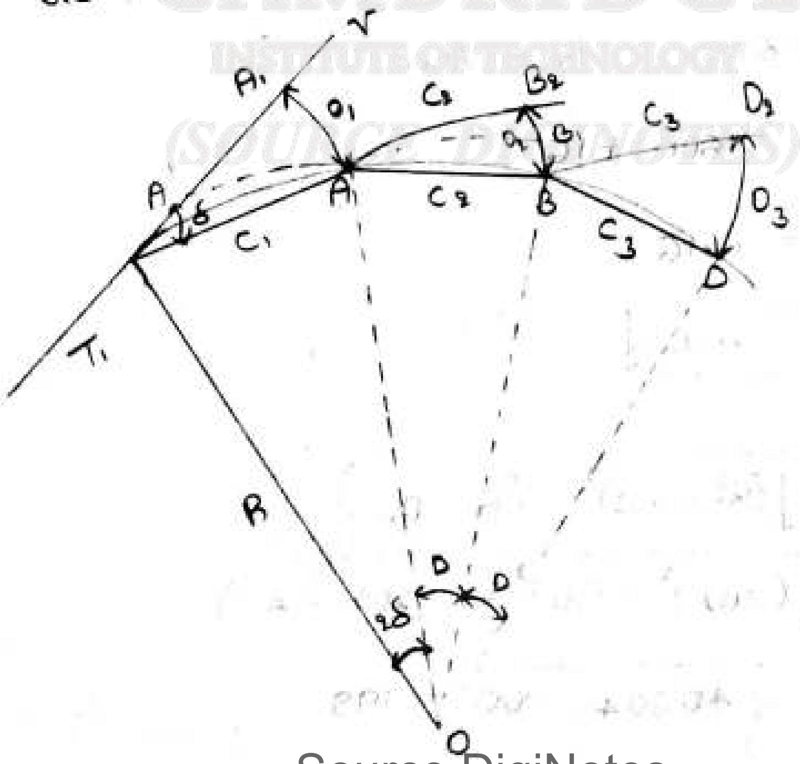
$O_{50} = 3.752$ m

$O_{60} = 3.007$ m

$O_{70} = 1.759$ m

$O_{80} = 0$ m

By setting out simple circular curve by the method of offsets from chords produced OR deflection distance method



This method is very much useful for long curves and is generally used on highway curves then the theodolite is not available.

* Let $T_1A_1 = T_1A = C_1$ initial sub chord $= C_1$
  A, B, D are the points on the curve.
* Then AB = Second chord $= C_2$, $BD = C_3$
* T_1V = Rear tangent.
* $\angle A_1T_1A = \delta$ = deflection angle of first chord
* $A_1A = O_1$ = First offset
  $B_2B = O_2$ = Second offset
  $D_2D = O_3$ = Third offset

* Now arc $A_1A = O_1 = T_1A\delta \longrightarrow (1)$

Since T_1V is the tangent at the circle at T_1

$\angle T_1OA = 2\angle A_1T_1A = 2\delta$

$T_1A = R2\delta$

$$\boxed{\delta = \frac{T_1A}{2R}} \longrightarrow (2)$$

Substituting the value of 'δ' in eqn (1) we get

Arc $A_1A = O_1 = T_1A\delta$

$$= T_1A \times \frac{T_1A}{2R}$$

$$= \frac{(T_1A)^2}{2R}$$

$$= \frac{C_1^2}{2R}$$

$$\boxed{O_1 = \frac{C_1^2}{2R}} \longrightarrow (3)$$

In order to obtain the value of second offset O_2 for getting the point B on the curve, draw a tangent AB_1 to the curve at A to cut the rear tangent at A'. Join T_1A and prolong it to a point B_2 such that $AB_2 = AB = C_2$. Then $O_2 = B_2B$

As from the above equation $\left(O_1 = \frac{C_1^2}{2R}\right)$, the offset B_1B from the tangent AB_1 is given by

$$\boxed{B_1B = \frac{C_2^2}{2R}} \longrightarrow (4)$$

Again $\angle B_2AB_1 = \angle A'AT_1$ [opposite angles]

Since T_1A' and $A'A$ are both tangents, they are equal in length.

$\therefore \angle A'T_1A = \delta$

$\angle B_2AB_1 = \angle A'AT_1 = \delta$

Arc $B_2B_1 = AB_2 \times \delta$

$= C_2\delta$

Substituting the value of δ from eq^n (2) we get

$$B_2B_1 = C_2\delta$$

$$= C_2 \frac{T_1A}{2R}$$

$$B_2B_1 = \frac{C_2C_1}{2R}$$

$$\text{arc } B_2B_0 = B_2B_1 + B_1B_0$$

$$= \frac{C_2C_1}{2R} + \frac{C_2^2}{2R}$$

$$O_2 = \frac{C_2(C_1+C_2)}{2R}$$

$$\boxed{O_2 = \frac{C_2}{2R}\left[C_1+C_2\right]} \rightarrow (5)$$

Similarly the third offset $O_3 = D_2D$ is given by

$$\boxed{O_3 = \frac{C_3}{2R}\left[C_2+C_3\right]}$$

* The last or n^{th} offset is given by

$$\boxed{O_n = \frac{C_n}{2R}\left(C_{n-1}+C_n\right)}$$

* Generally the 1^{st} chord is a sub chord, say of length "c" and the intermediate chords are normal chords, say of length "C". In that case the above formula is reduced to $\boxed{O_1 = \frac{c^2}{R}}$

and $\cancel{O_2} = \frac{c^{\bullet}}{2R}(\cancel{c}+C)$

$O_3 = \frac{C'}{2R}(\cancel{C'}+C)$

$$O_2 = \frac{C}{2R}(c+C)$$

$$O_n = \frac{c'}{2R}(C+c')$$

① Two tangents intersect at a chainage 59 + 60, the deflection angle being 50°30'. Calculate the necessary data for setting out a curve of 15 chains radius to connect the two tangents if it is intended to set out the curve by offsets from the chords produced. Take peg interval = 100 links, length of the chain being 20 m = 100 links.

Sol^n

Given :-

Chainage of P.I = 59 + 60

Deflection angle (Δ) = 50°30'

R = 15 chains = 15 × 20 = 300 m

Peg Intervals = 100 links = 20 m

Length of the chain = 20m = 100 links

Tangent Length (T)

$= R \tan\left(\frac{\Delta}{2}\right)$

$= 15\times20 \tan\left(\frac{50°30'}{2}\right)$ 7.074 chains x20

$\boxed{T = 141.48 \text{ m}}$

Length of the curve $(l) = \frac{\pi R \Delta}{180}$

$\frac{3.14 \times 15 \times 50°30'}{180}$

$\boxed{l = 264.42 \text{ m}}$

chainage of PI = 59 + 60

$= (59 \times 20) + (60 \times 0.2)$

$= 1192$ m

Deduct the Tangent length (T)

1192 − 141.48

= 1050.52

chainage of P.C = 1050.52

Add length of curve (l) = 264.42 m

chainage of P.T = 1050.52 + 264.42

1314.94 m

The chainage of each peg will be multiple of 20 m.

56574

The length of the first Sub chord

$$c = 1060 - 1050.52$$

$$\boxed{c = 9.48 \text{ m}}$$

Length of last subchord (c')

$$c' = 1314.93 - 1300$$

$$\boxed{c' = 14.93 \text{ m}}$$

No of full chords $= \dfrac{1300 - 1060}{20}$

$$= 12$$

Total number of chords $= c + 12 + c'$

$$= 1 + 12 + 1$$

$$= 14$$

Length of first offset

$$O_1 = \frac{c^2}{2R} = \frac{(9.48)^2}{2 \times 300} = 0.149$$

$$\boxed{O_1 = 0.15 \text{ m}}$$

Length of Second offset

$$O_2 = \frac{C}{2R}\left[c + C\right]$$

$$= \frac{20}{2 \times 300}\left[9.48 + 20\right] = 0.982 \text{ m}$$

$$\boxed{O_2 = 0.98 \text{ m}}$$

$$O_3 = \frac{C^2}{2R} = \frac{20^2}{2 \times 300}$$

$$\boxed{O_3 = 0.67 \text{ m}}$$

$$O_3, O_4 \ldots O_{12} = \frac{C^2}{R}$$

$$= \frac{(20)^2}{300}$$

$$= 1.33 \text{ m}$$

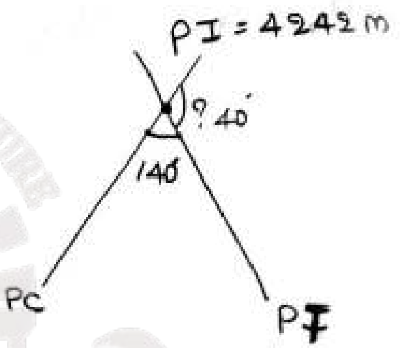
$$O_{14} = \frac{C'}{2R}(c + C)$$

$$O_{14} = \frac{14.93}{2 \times 300}(20 + 14.93)$$

$$\boxed{O_{14} = 0.87 \text{ m}}$$

② Two straights AB and BC intersects at a chainage of 4242 m. The angle of intersection is 140° it is required to set out 5° curve to connect the straights. calculate all the data necessary to set out the curve by the method of offsets from the chord produced. Given the peg interval of 30 m.

<u>Note</u> :- By the method of deflection distance, on offsets of chords produced, all the intermediate offsets will be equal to $\frac{c^2}{R}$, while the last offset will be equal to $\boxed{\frac{c'}{2R}(C+c')}$



chainage of PI = 4242 m

Intersection angle = 140°

Deflection angle =

$180° - 140° = 40°$

Degree of curve 5°

peg interval = 30 m

Radius of curve (R) = $\frac{1719}{D} = \frac{1719}{5} = 343.8$ m

Tangent length (T) = $R \tan\left(\frac{\Delta}{2}\right)$

$= 343.8 \tan\left(\frac{40°}{2}\right)$

$\boxed{T = 125.13 \text{ m}}$

Length of the curve (l) = $\frac{\pi R \Delta}{180}$

$= \frac{3.14 \times 343.8 \times 40}{180}$

$\boxed{l = 240.01 \text{ m}}$

$PC = 4242 - 125.13$

$= 4116.86 \text{ m}$

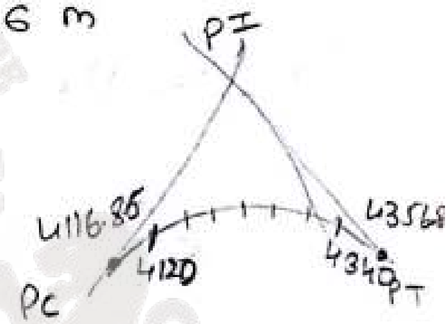
chainage of PT $= 4116.86 + 240.01$

$= 4356.88 \text{ m}$

Length of first sub chord

$C = 4120 - 4116.86 \text{ m}$

$\boxed{C = 3.14 \text{ m}}$



Length of last sub chord

$4356.878 - 4340$

$C' = 16.878$

No. of full chords $= \frac{4340 - 4120}{30}$

$= 7.33 \approx$

$= 7$

Total number of chords $= C + 7 + C'$

$= 1 + 7 + 1$

$= 9$

Length of first offset

$O_1 = \frac{C^2}{2R} = \frac{(3.14)^2}{2 \times 343.8}$

$\boxed{O_1 = 0.0143 \text{ m}}$

Length of second offset

$$O_2 = \frac{c}{2R}\left[f + c \right]$$

$$= \frac{30}{2 \times 343.8}\left[3.14 + 30 \right]$$

$$\boxed{O_2 = 1.445\ m}$$

$$O_3, O_4 \ldots O_7 = \frac{c^2}{R}$$

$$= \frac{(30)^2}{343.8}$$

$$= 2.6178\ m$$

$$O_9 = \frac{c'}{2R}\left[c + c \right]$$

$$= \frac{16.878}{2 \times 343.8}\left[30 + 16.878 \right]$$

$$\boxed{O_9 = 1.1506\ m}$$

RANKINE'S METHOD OF DEFLECTION ANGLE

or Rankine's method of tangential angle

Deflection angle at any point on the curve is angle at PC between the back tangent and the chord Pc to that point.

* Rankine's method is based on the principle that the deflection angle to any point on a circular curve is measured by one half the angle subtended by the arc from PC to that point

* It is assumed that the length of the arc is approximately equal to its chord.

Two tangents intersect at a chainage 49 + 70, the deflection angle being 42° 40' calculate the necessary data for setting out a curve of 15 chains radius to connect the two tangents by the method offset by chords produced. Take peg interval = 100 links, Length of the chain being 20 m = 100 links.

Sol^n

chainage of P.I = 49+70

Deflection angle (Δ) = 42° 40'

R = 15 chains

peg interval = 100 links = 20 m

Length of the chain = 20 m = 100 links

Tangent length (T)

$$R \tan\left(\frac{\Delta}{2}\right)$$

$$= 15 \times 20 \tan\left(\frac{42^\circ 40'}{2}\right)$$

$$\boxed{T = 117.16 \text{ m}}$$

Length of the curve $(l) = \frac{\pi R \Delta}{180}$

$$\frac{3.14 \times 20 \times 15 \times 42^\circ 40'}{180}$$

$$\boxed{l = 223.28 \text{ m}}$$

chainage of PI = 49 + 70

$= (49 \times 20) + (70 \times 0.2)$

$= 994$ m

Deduct the tangent length (T)

994 − 117.16

$$\boxed{T = 876.84 \text{ m}}$$

chainage of PC = 876.84

Add length of curve (1) = 223.28 m

chainage of P.T = 876.84 + 223.28

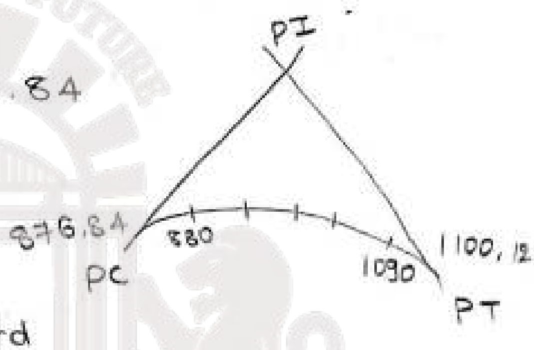
= 1100.12 m

The chainage of each peg will be multiple of 20 m.

The length of the first sub chord

$C = 880 - 876.84$

$\boxed{C = 3.16 \text{ m}}$



Length of last sub chord

1100.12 - 1090

$\boxed{c' = 10.12 \text{m}}$

No of full chords = $\dfrac{1090 - 880}{20}$

= 10.5

Total number of chords = C + 7 + C'

= 1 + 10.5 + 1

= 12

Length of first offset

$$O_1 = \frac{c^2}{2R} = \frac{(3.16)^2}{2 \times 300}$$

$\boxed{O_1 = 0.0166 \text{ m}}$

Length of Second offset

$$O_2 = \frac{C}{2R}\left(c + C\right]$$

$$= \frac{20}{2 \times 300}\left(3.16 + 20\right)$$

$$\boxed{O_2 = 0.772 \text{ m}}$$

$$O_3, O_4 -- O_{10} = \frac{C^2}{R}$$

$$= \frac{(20)^2}{300}$$

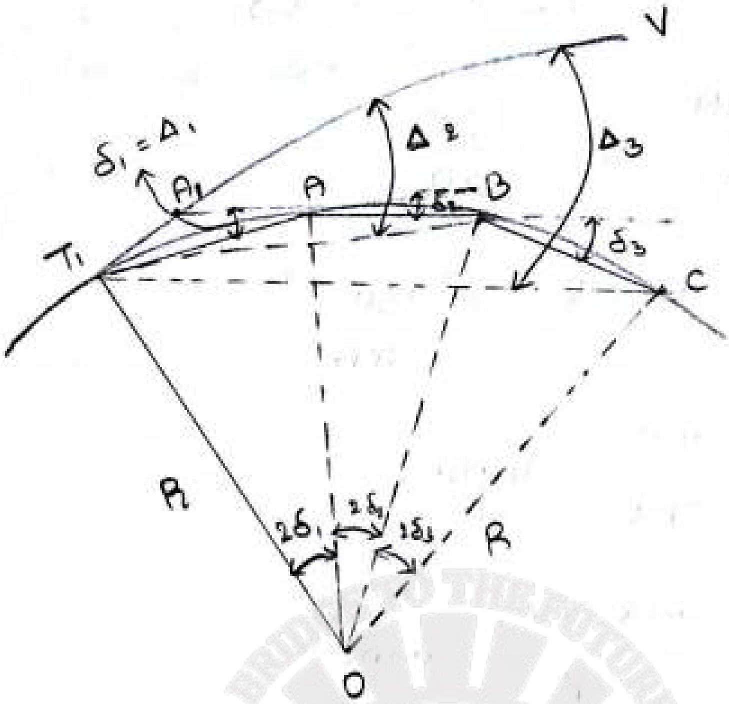
$$= 1.33 \text{ m}$$

$$O_{12} = \frac{C'}{2R}\left[c + C\right]$$

$$= \frac{10.12}{2 \times 300}\left[20 + 10.12\right]$$

$$\boxed{O_{12} = 0.5080 \text{ m}}$$

RANKINES METHOD OF TANGENTIAL ANGLES OR DEFLECTION ANGLES



Let,

The T_1V = Rear tangent

T_1 = Point of curve (P.C)

$\delta_1, \delta_2, \delta_3 \rightarrow$ Tangential angle or angle with each of successive chords

T_1A, AB, BC makes the respected tangents to the curve at T_1, A, B ---

$\Delta_1, \Delta_2, \Delta_3$-- Total tangential angles or the deflection angles to the points A, B, C ---

C_1, C_2, C_3 = Lengths of chords T_1A, AB, BC

A_1A = Tangent to the curve at A.

From the property of circle,

$$\angle VT_1A = \frac{1}{2} \angle T_1OA$$

or

$$\angle T_1OA = 2\angle VT_1A = 2\delta_1$$

Now,
$$\frac{\angle T_1OA}{C_1} = \frac{180^\circ}{\pi R}$$

$$\angle T_1OA = 2\delta_1 = \frac{180^\circ}{\pi R}$$

$$\delta_1 = \frac{90C_1}{\pi R} \text{ degrees}$$

$$\delta_1 = \frac{90 \times 60C_1}{\pi R} \text{ minutes}$$

$$= 1718.9 \frac{C_1}{R} \text{ minutes}$$

Similarly,
$$\delta_2 = 1718.9 \frac{C_2}{R} \text{ minutes}$$

$$\delta_3 = 1718.9 \frac{C_3}{R} \text{ minutes}$$

In general

$$\boxed{\delta = 1718.9 \frac{C}{R} \text{ minutes}}$$

where,

C is the length of the chord. For the First chord T_1A, the deflection angle = Its tangential angle

$$\Delta_1 = \delta_1$$

For the second point B, Let the deflection angle be Δ_2

Since δ_2 = Tangential angle for the chord AB

$$\angle AOB = 2\delta_2$$

$\angle AT_1B$ = Half the angle subtended by AB at the centre $= \delta_2 \left[\frac{2\delta_2}{2} = \delta_2\right]$

Now, $\Delta_2 = \angle VT_1B$

$$= \angle A_1T_1A + \angle AT_1B$$

$$\Delta_2 = \delta_1 + \delta_2$$

$$\Delta_2 = \Delta_1 + \delta_2$$

Similarly $\Delta_3 = \delta_1 + \delta_2 + \delta_3$

$$\Delta_3 = \Delta_2 + \delta_3$$

$$\Delta_n = \delta_1 + \delta_2 + \dots \delta_n$$

$$\boxed{\Delta_n = \Delta_{n-1} + \delta_n}$$

Hence the deflection angle for any chord is equal to the deflection angle for the previous chord + tangential angle for that chord.

Check :- The deflection angle of the long chord $\angle VT_1T_2 = \Delta_n = \frac{\Delta}{2}$

where δ is intersection angle or the external deflection angle for the curve.

Sum of all the individual deflection angle is equal to half the deflection angle of the curve.

$$\angle VT_1T_2 = \Delta_n = \frac{\Delta}{2}$$

Field notes :-

The record of deflection angles for various points is shown in the below format.

Point	Chainage	Chord length (c)	Tangential angle (δ)			Deflection angle (Δ)			Actual Theodolite reading			Remarks
			°	'	"	°	'	"	°	'	"	

Two tangents intersect at a chainage 1000 m, the deflection angle being 18°. Calculate the necessary data to set out the Simple Curve of radius 250 m by Rankines method and tabulate the results.

Peg interval = 20 m

Least count of theodolite = 20"

chainage of PI = 1000 m

$\Delta = 28^\circ$

R = 250 m

peg interval = 20 m

Least count of Theodolite = 20"

$T = R \tan\left(\frac{\Delta}{2}\right)$

$= 250 \tan\left(\frac{28^\circ}{2}\right)$

$\boxed{T = 62.33 \text{ m}}$

Length of curve (l) $= \frac{\pi R \Delta}{180}$

$= \frac{3.14 \times 250 \times 28}{180}$

$\boxed{l = 122.17 \text{ m}}$

chainage of $P.T_1$ (P.C)

$= PI - T$

$= 1000 - 62.33$

$\boxed{T_1 = 937.67 \text{ m}}$

chainage of T_2 (P.T)

$= P.C + l$

$= 937.67 + 122.17$

$\boxed{T_2 = 1059.84 \text{ m}}$

Length of first sub chord (c_1)

$940 - 937.67$

$= 2.33$ m

Length of last sub chord (c_2)

$1059.84 - 1040$

$= 19.8$ m

No of normal chords $= \dfrac{1040 - 940}{20} = 5$

Total number of chords

$c_1 + c + c_2$

$\Rightarrow 1 + 5 + 1$

$\Rightarrow 7$

$\delta_1 = 1718.9 \dfrac{c_1}{R}$

$\delta_1 = 1718.9 \times \dfrac{2.33}{250}$ minutes

$\boxed{\delta_1 = 0° 16' 1''}$

(or)

$\delta_1 = \dfrac{180 \times c_1}{2\pi R}$ degrees

$= \dfrac{180 \times 2.33}{2 \times \pi \times 250}$

$\boxed{\delta_1 = 0° 16' 1''}$

$$\delta_2, \delta_3 .. \delta_7 = 1718.9 \frac{C_2}{R}$$

$$= 1718.9 \left(\frac{\cancel{180}}{250 \times 60} \right)$$

$$= 2^\circ 17' 30''$$

$$\delta_7 = 1718.9 \times \frac{19.8}{250}$$

$$\boxed{\delta_7 = 2^\circ 16' 24''}$$

point	chainaige	chord length	Tangential Angle (δ)			Deflection Agle Δ			Actual Theodolite reading		
			°	'	"	°	'	"	°	'	"
A	940	2.33	0	16	1	0	16	1	0	16	0
B	960	20	2	17	30	2	33	31	2	33	40
C	980	20	2	17	30	4	51	1	4	51	0
D	1000	20	2	17	30	7	8	31	7	8	40
E	1020	20	2	17	30	9	26	1	9	26	0
F	1040	20	2	17	30	11	43	31	11	43	40
G	1059.84	19.84	2	16	24	13	59	55	14	0	0

$$\delta_2, \delta_3 ---\ \delta_6 = 1718.9 \frac{C}{R}$$

$$= \frac{90C}{\pi R} = \frac{90 \times 20}{\pi \times 250}$$

$$\delta_2, \delta_3, ---\ \delta_6 = 2^\circ 17' 30''$$

check $\Delta_7 = \frac{\Delta}{2} = \frac{28^\circ}{2} = 14^\circ 0' 0''$

$$\Delta_1 = \delta_1$$

$$\Delta_2 = \Delta_1 + \delta$$

$$\Delta_3 = \Delta_2 + \delta$$

Calculate the necessary data for setting out a curve by Rankien's method of tangential angles, the chainage of point of intersection is 1192 m. the deflection is 50°30', radius of the curve is 300 m. Take the peg interval of 20 m. If the theodolite has the least count of 20''. Tabulate the actual readings of deflection angles to be set out.

PI = 1192 m

$\Delta = 50^\circ 30'$

R = 300 m

peg interval 20 m

Least count of Theodolite 20''.

$$T = R \tan\left(\frac{\Delta}{2}\right)$$

$$= 300 \tan\left(\frac{50^\circ 30'}{2}\right)$$

$$\boxed{T = 141.489 \text{ m}}$$

Length of curve $(l) = \frac{\pi R \Delta}{180}$

$$= \frac{\pi \times 300 \times 50^\circ 30'}{180}$$

$$\boxed{l = 264.417 \text{ m}}$$

chainage of T_1 (P.C) = P.I − T

$$= 1192 - 141.489$$

$$\boxed{T_1 = 1050.511 \text{ m}}$$

Chainage of T_2 (P.T) = P.C + l

$$= 1050.511 + 264.417$$

$$\boxed{T_2 = 1314.928 \text{ m}}$$

Length of first sub chord (C_1)

1060 − 1050.511

$(C_1) = 9.489$ m

Length of last sub chord (C_2)

1314.928 − 1300

(C_2) 14.928

No of normal chords $= \frac{1310 - 1060}{20}$

$= 12$

Total number of chords

$C_1 + C + C_2$

$1 + 12 + 1$

14

$$\delta_1 = \frac{90 C_1}{\pi R}$$

$$= \frac{90 \times 9.48}{\pi \times 300}$$

$$\boxed{\delta_1 = 0^\circ 54' 19''}$$

$$\delta_2, \delta_3 \text{ --- } \delta_{13} = \frac{90 C}{\pi R}$$

$$= \frac{90 \times 20}{\pi \times 300}$$

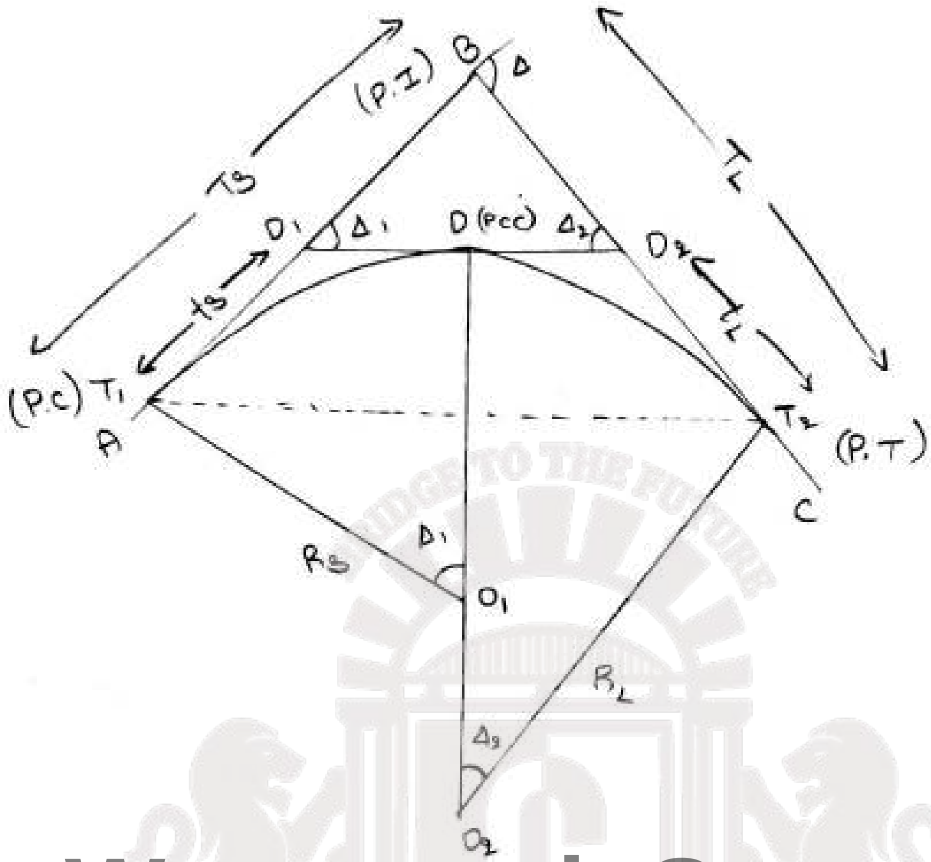
$$= 1^\circ 54' 35''$$

$$\delta_{14} = \frac{90 \times C}{\pi R}$$

$$= \frac{90 \times 14.94}{\pi \times 300}$$

$$\boxed{\delta_{14} = 1^\circ 25' 36''}$$

COMPOUND CURVES



A compound curve is a combination of two or more simple curves with different radii.

* The two centered compound curve has two circular arcs of different radii that deviate in the same direction and join at the common tangent point also known as point of common curvature.

* There are 7 elements for the compound curves.

* From the figure T_1DT_2 is the two centered compound curves having two circular arcs

T_1D and DT_2 meeting at a common point 'D' known as point of compound curvature. [PCC].

* T_1 is the point of curve and T_2 is the point of tangency [P.T]. O_1 and O_2 are the centres of two arcs.
* R_S is the smaller radius $(O_1 T_1)$ and R_L is the larger radius $(O_2 T_2)$
* $D_1 D_2$ is the common tangent
* Δ_1 is the deflection angle between the rear tangent and common tangent
* Δ_2 is the deflection angle between the common tangent and forward tangent
* Δ is the total deflection angle
* t_S is the length of the tangent to the arc T_1D having the smaller radius
* t_L is the length of the tangent to the arc DT_2 having the longer radius
* T_S is the tangent distance.
* T_1B corresponding to the shorter radius.
* T_L is the tangent distance BT_2 corresponding to the longer radius.

From the figure we have

$$t_s = T_1D_1 = D_1D$$

$$\boxed{t_s = R_s \tan\left(\frac{\Delta_1}{2}\right)} \longleftarrow (1)$$

$$t_L = T_2D_2 = D_2D$$

$$\boxed{t_L = R_L \tan\frac{\Delta_2}{2}} \longrightarrow (2)$$

$$\Delta = \Delta_1 + \Delta_2$$

From triangle BD_1D_2 we have

$$\frac{D_1B}{\sin\Delta_2} = \frac{BD_2}{\sin\Delta_1} = \frac{D_1D_2}{\boxed{\sin(180-\Delta)}}$$

$$\frac{D_1B}{\sin\Delta_2} = \frac{BD_2}{\sin\Delta_1} = \frac{D_1D_2}{\sin\Delta}$$

$$D_1B = \frac{D_1D_2}{\sin\Delta}\sin\Delta_2$$

$$BD_2 = \frac{D_1D_2}{\sin\Delta}\sin\Delta_1$$

$$D_1B = (t_s + t_L)\frac{\sin\Delta_2}{\sin\Delta}$$

$$BD_2 = (t_s + t_L)\frac{\sin\Delta_1}{\sin\Delta}$$

$$T_s = T_1D_1 + D_1B$$

$$\boxed{T_s = t_s + (t_s + t_L)\frac{\sin\Delta_2}{\sin\Delta}} \longrightarrow (3)$$

$T_L = T_2D_2 + D_2B$

$$\boxed{T_L = t_L + (t_S + t_L)\frac{\sin\Delta_1}{\sin\Delta}} \longrightarrow (4)$$

A compound curve consisting of two simple circular curves of radii 350 m and 500 m is to be laid out between two straights. The angle of intersection between the two straights are 55° and 25°. Calculate the various elements of the compound curve.

Sol^n:

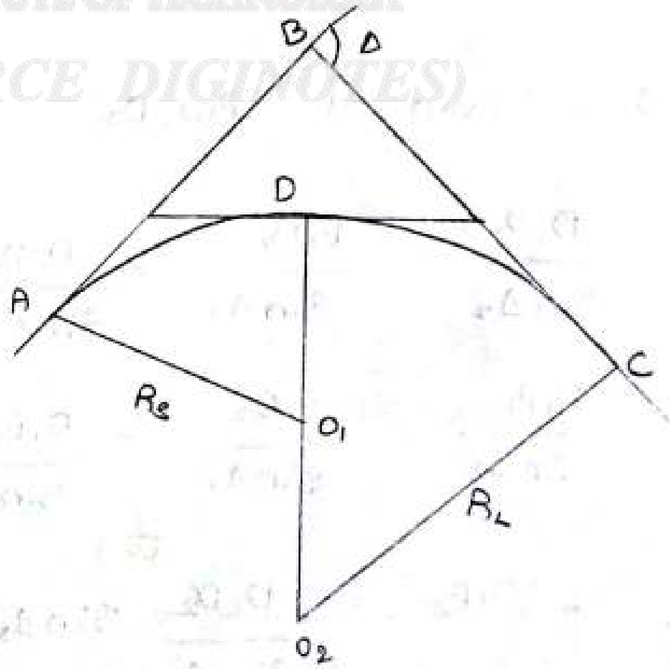
** [Note :- If the radius is small the deflection angle increases and the radius is large the deflection angle decreases]

$R_S = 350$ m

$R_L = 500$ m

$\Delta_1 = 55°$

$\Delta_2 = 25°$



$$t_S = R_S \tan\left(\frac{\Delta_1}{2}\right)$$

$$= 350 \tan\left(\frac{55^\circ}{2}\right)$$

$$\boxed{t_S = 182.198 \text{ m}}$$

$$t_L = R_L \tan\left(\frac{\Delta}{2}\right)$$

$$= 500 \tan\left(\frac{25^\circ}{2}\right)$$

$$\boxed{t_L = 110.84 \text{ m}}$$

$$T_S = t_S + (t_S + t_L)\frac{\sin \Delta_2}{\sin \Delta}$$

~~From Δ^{le} $BD_1 D_2$~~

$$\cancel{= 182.198 + (182.198 + 110.84)\frac{\sin 25^\circ}{\sin 80^\circ}}$$

$\emptyset$, From Δ^{le} BD_1D_2

$D_1D_2 = t_S + t_L$

$= 182.198 + 110.847$

$$\boxed{D_1D_2 = 293.045 \text{ m}}$$

$$\frac{D_1B}{\sin \Delta_2} = \frac{BD_2}{\sin \Delta_1} = \frac{D_1D_2}{\sin(180-\Delta)}$$

$$\frac{D_1B}{\sin \Delta_2} = \frac{BD_2}{\sin \Delta_1} = \frac{D_1D_2}{\sin \Delta}$$

$$D_1B = \frac{D_1D_2}{\sin \Delta}\sin \Delta_2$$

$$D_1B = \frac{D_1D_2}{\sin(100)} \times \sin 25$$

$$= \frac{293.038}{\sin(100)} \times \sin 25$$

$$\boxed{D_1B = 125.75 \text{ m}}$$

$$BD_2 = \frac{D_1D_2}{\sin\Delta} \sin\Delta_1$$

$$= \frac{293.038}{\sin 100} \times \sin 55$$

$$\boxed{BD_2 = 243.74 \text{ m}}$$

$$T_S = T_1D_1 + D_1B$$

Watermark Sample

$$T_S = t_S + (t_S + t_L)\frac{\sin\Delta_2}{\sin\Delta}$$

$$= 182.198 + (182.198 + 110.84)\frac{\sin 25}{\sin(55^\circ + 25^\circ)}$$

$$\boxed{T_S = 307.955 \text{ m}}$$

$$T_L = T_L + (t_S + t_L)\frac{\sin\Delta_1}{\sin\Delta}$$

$$= 110.847 + 293.045 \frac{\sin 55^\circ}{0.9848}$$

$$\boxed{T_L = 354.6 \text{ m}}$$

Setting out compound curve

* The compound curve can be set by the method of deflection angle.
* The first branch is set out by setting the theodolite T_1 [P.C]
  the second branch is set out by setting the theodolite at D [PCC].
* The procedure is as follows
  ① After having known any four parts, calculate the points of rest three parts by the formulae given.
  ② Knowing T_s and T_L, locate the points T_1 and T_2 by linear measurements by the point of intersection
  ③ calculate the length of curves L_s, calculate the chainage of T_1, D and T_2 asusual.
  ④ For the first curve, calculate the tangential angles for setting out the curve by Rankine's method.
  ⑤ Set the theodolite at T_1 and set out the first branch of a curve.
  ⑥ After locating the point D [Pcc], shift

the theodolite to 'D' and set it there. with the vernier set to $\left(360 - \frac{\Delta_1}{2}\right)$ reading take the back side at T_1 and plunge the telescope. The line of side is thus oriented along $T_1 D$ produced and if the theodolite is through $\Delta_1/2$, the line of side will be directed along the common tangent BD_2 DT_2. Thus the theodolite is correctly oriented at 'D'.

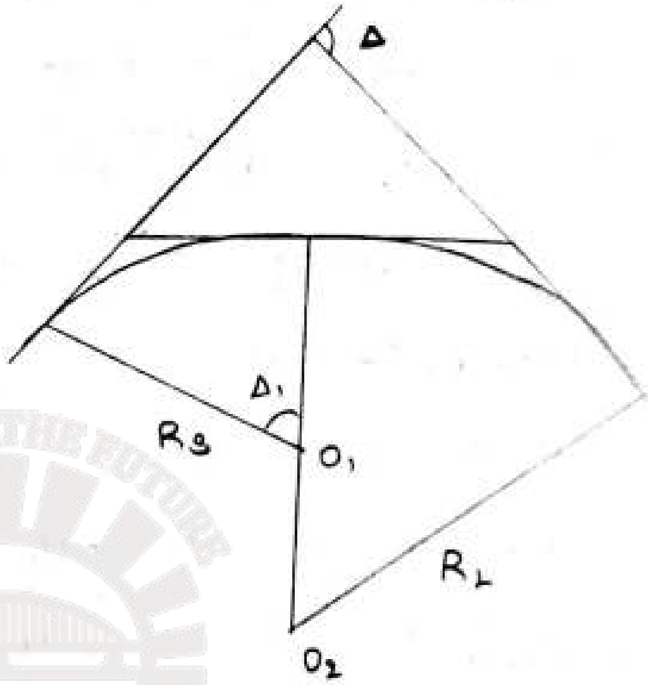
(6) calculate the tangential angles for the second branch and set out the curve by observations from D till T_2 is reached.

(7) check the observations by measuring the angle $T_1 B T_2$ which should be equal to

$$180 - \frac{\Delta_1 + \Delta_2}{2} \quad \text{or} \quad 180 - \frac{\Delta}{2}$$

(1) Two straights AB and BC are intersected by a line $D_1 D_2$. The angles BD_1D_2 and BD_2D_1 are $40^\circ 30'$ and $36^\circ 24'$ respectively. The radius of the first arc is 600 m and that of the second arc is 800 m if the chainage of intersection point B is 8248.1 m. find out the chainage of tangent point

and the point of compound curve.



$\Delta_1 = 40^\circ 30'$

$\Delta_2 = 36^\circ 24'$

$R_s = 600$ m

$R_L = 800$ m

$t_s = R_s \tan\left(\frac{\Delta_1}{2}\right)$

$= 600 \tan\left(\frac{40^\circ 30'}{2}\right)$

$\boxed{t_s = 221.35 \text{ m}}$

$t_L = R_L \tan\left(\frac{\Delta_2}{2}\right)$

$= 800 \tan\left(\frac{36^\circ 24'}{2}\right)$

$\boxed{t_L = 263.02 \text{ m}}$

$\Delta = \Delta_1 + \Delta_2$

$= 40^\circ 30' + 36^\circ 24'$

$\Delta = 76^\circ 54'$ $76^\circ 54'$

$D_1 D_2 = t_s + t_L$

$= 484.37$ m

$$D_1B = \frac{D_1D_2}{\sin\Delta}\sin\Delta_2$$

$$= \frac{484.37}{\sin(76°54')}\sin(36°24')$$

$$\boxed{D_1B = 295.118\ m}$$

$$BD_2 = \frac{D_1D_2}{\sin\Delta}\sin\Delta_1$$

$$= \frac{484.37}{\sin(76°54')}\sin(40°30')$$

$$\boxed{BD_2 = 322.98\ m}$$

$$T_S = t_S + (t_S + t_L)\frac{\sin\Delta_2}{\sin\Delta}$$

$$= 221.35 + 484.376\ \frac{\sin 36°24'}{\sin 76°54'}$$

$$\boxed{T_S = 516.906}$$

$$T_L = t_L + (t_S + t_L)\frac{\sin\Delta_1}{\sin 76°54'}$$

$$= 263.026 + 322.98$$

$$\boxed{T_L = 586.006}$$

$$L_S = \frac{\pi \times 600 \times 40°30'}{180°} \qquad L_L = \frac{\pi \times 800 \times 36°24'}{180°}$$

$$\boxed{L_S = 424.115\ m} \qquad \boxed{L_L = 508.23}$$

REVERSE CURVES:-

A reverse curve consists of two simple curves of opposite directions that join at a common tangent point called POINT OF REVERSE CURVATURE [PRC].

* They are used when the straights are parallel or include a very small angle of intersection and are frequently encountered in mountaineous countries.

* The use of reverse curve should be avoided on highways, on main railway lines where the speeds are high for the following reasons

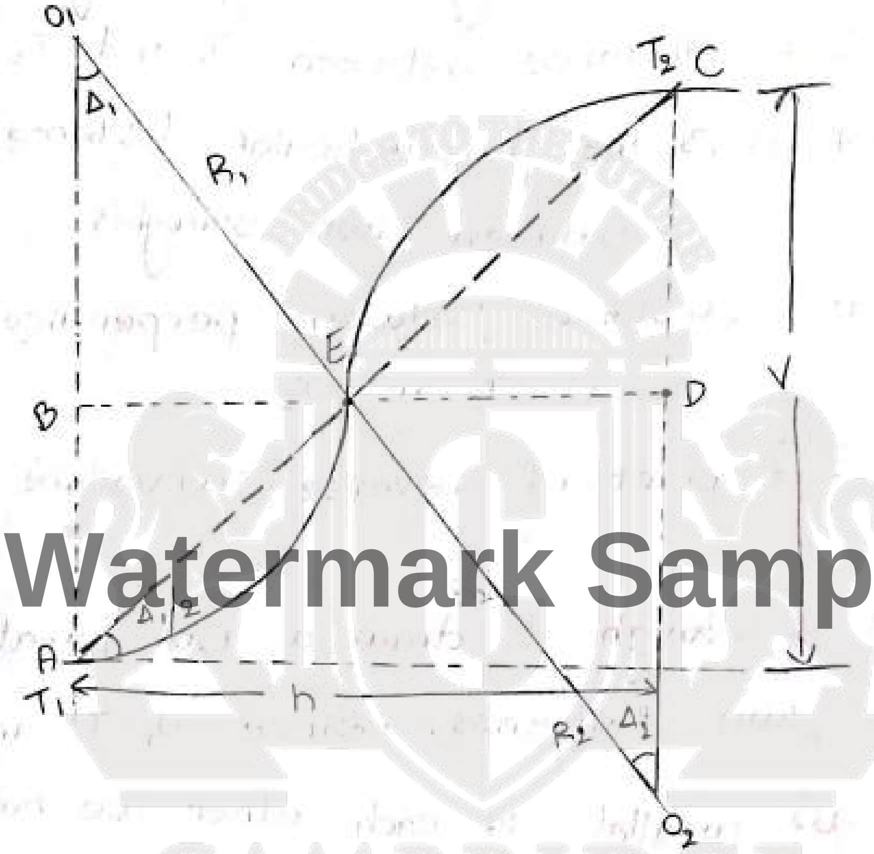
① Sudden change of cant is required from one side of point of reverse curvature to the other.

② There is no oppurtunity to elevate the outer bank at point of reverse curvature

③ The sudden change of direction is uncomfortable to passangers and is objectionable.

(4) Steering is dangerous in the case of highways and the driver as to be very coausious.

Reverse curve between two parallel straights



Given the two radii R_1 and R_2 and the central angles.

we are required to calculate the various elements of the reverse curve.

* condition repuired $\Delta_1 = \Delta_2$

Step (1) In the figure Let AT_1 and T_2C be the two straights parallel to each other so that there is no point of intersection.

Let R_1 = Smaller radius

R_2 = Larger radius

Δ_1 = central angle corresponding to R_1

Δ_2 = central angle corresponding to R_2

L = Distance between T_1 and T_2.

V = is the perpendicular distance between two straights

h = distance between perpendiculars at T_1 and T_2

E = point of reverse curvature.

Step 2 :- Through E draw a BD parallel to the two tangents. Since O_1T_1 and O_2T_2 are parallel to each other we have

$$\Delta_1 = \Delta_2$$

$$T_1B = O_1T_1 = O_1B$$

$$= R_1 - R_1 \cos \Delta_1$$

$$= R_1 (1 - \cos \Delta_1)$$

$$\boxed{T_1B = R_1 \text{ ver sin } \Delta_1}$$

$$T_2D = O_2T_2 - O_2D$$

$$T_2D = R_2 - R_2 \cos \Delta_2$$

$$= R_2 - R_2 \cos \Delta_1$$

$$= R_2 (1 - \cos \Delta_1)$$

$$\boxed{T_2D = R_2 \text{ ver sin } \Delta_1}$$

$$V = T_1B + DT_2$$

$$= R_1 \text{ ver sin } \Delta_1 + R_2 \text{ ver sin } \Delta_1$$

$$= (R_1 + R_2) \text{ ver sin } \Delta_1$$

$$\boxed{V = (R_1 + R_2)(1 - \cos \Delta_1)} \longrightarrow (1)$$

Again $T_1E = 2R_1 \sin \frac{\Delta_1}{2}$

$$T_2E = 2R_2 \sin \frac{\Delta_2}{2}$$

$$= 2R_2 \sin \frac{\Delta_1}{2}$$

$$T_1T_2 = L = T_1E + ET_2$$

$$= 2R_1 \sin \frac{\Delta_1}{2} + 2R_2 \sin \frac{\Delta_1}{2}$$

$$\boxed{T_1T_2 = 2(R_1 + R_2) \sin \frac{\Delta_1}{2}} \longrightarrow (2)$$

But $\sin \frac{\Delta_1}{2} = \frac{V}{L}$

$$L = 2(R_1 + R_2) \frac{V}{L}$$

$$L = \sqrt{2V(R_1 + R_2)}$$

$BE = R_1 \sin \Delta_1$; $ED = R_2 \sin \Delta_2$

$= R_2 \sin \Delta_1$

$BD = h = (R_1 \sin \Delta_1 + R_2 \sin \Delta_1)$

$$\boxed{h = (R_1 + R_2) \sin \Delta_1} \rightarrow (3)$$

Special Case :-

If $R_1 = R_2 = R$

$V = 2R(1 - \cos \Delta_1)$

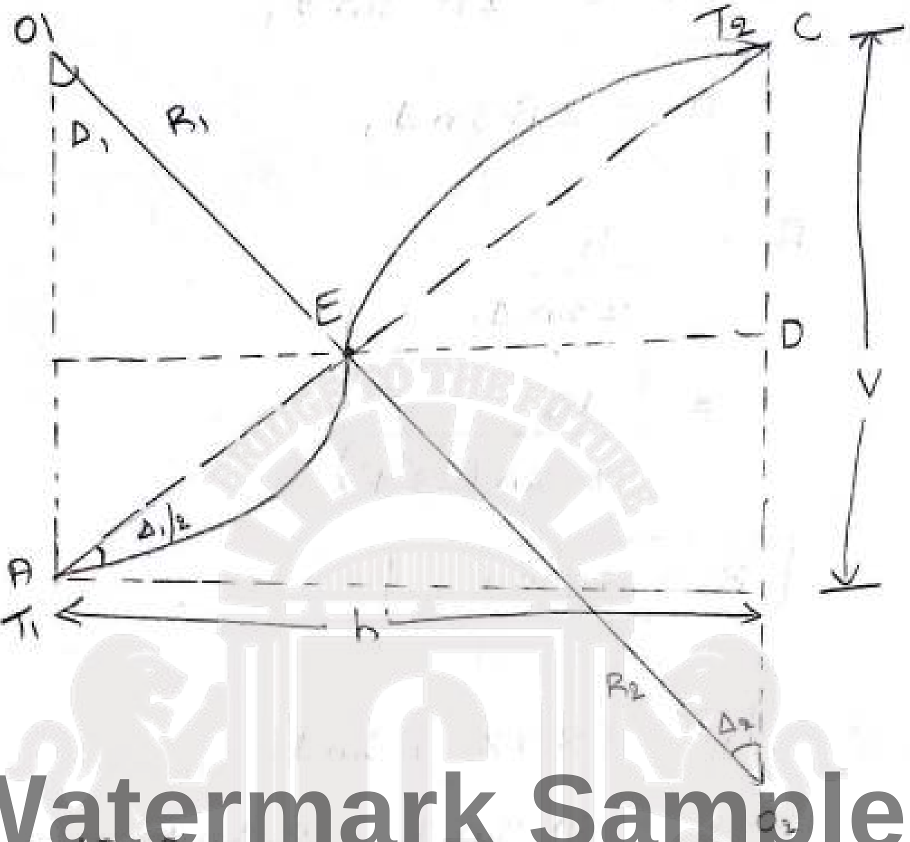
$L = 4R \sin \frac{\Delta_1}{2}$

$L = \sqrt{4RV}$

$h = 2R \sin \Delta_1$

(1) Two parallel railway lines are to be connected by a reverse curve each sections having the same radius. If the lines are 12 m apart and the maximum distance between tangents points measured parallel to the straight is 48 m (h). Find the maximum allowable radius.

If however both the radii are to be different. calculate the radius of 2nd branch is that 1st of the branch 60 m also calculate the length of both the branches.



Sol^n:

$h = 48$ m

$V = 12$ m

$$\tan\left(\frac{\Delta_1}{2}\right) = \frac{V}{h}$$

$$= \frac{12}{48}$$

$$= 0.25$$

$$\Delta_1 = \tan^{-1}(0.25) \times 2$$

$$\boxed{\Delta_1 = 28^\circ\, 4'}$$

$$\sin \Delta_1 = 0.4705$$

$$BE = R_1 \sin \Delta_1$$

$$ED = R_2 \sin \Delta_2$$

Here $R_1 = R_2 = R$

$$BE + ED = 2R \sin \Delta_1$$

$$h = 2R \sin \Delta_1$$

$$R = \frac{h}{2 \sin \Delta_1}$$

$$= \frac{48}{2 \sin (28^\circ 4')}$$

$$\boxed{R = 51.009 \text{ m}}$$

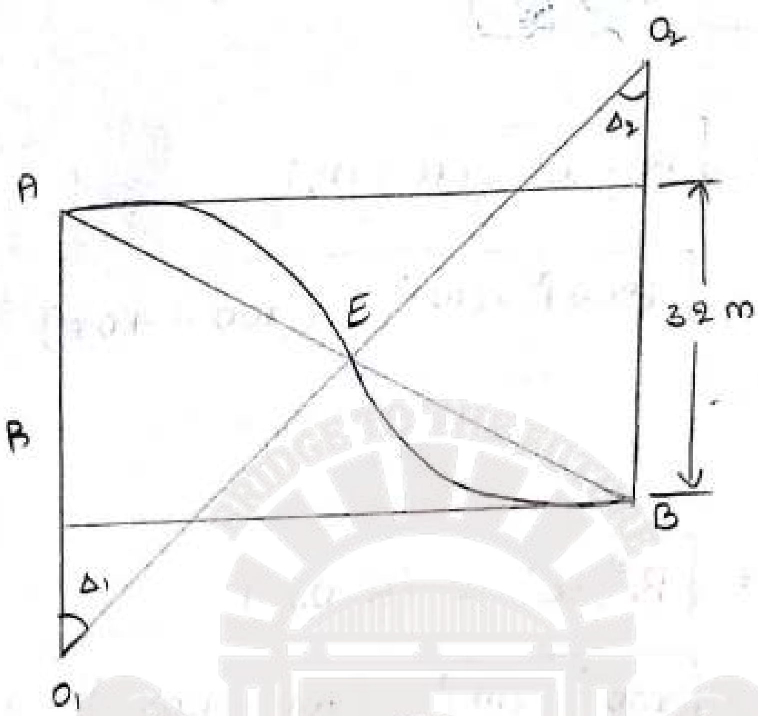
$$h = (R_1 + R_2) \sin \Delta_1$$

$$48 = 60 \sin \Delta_1 + R_2 \sin \Delta_1$$

$$19.77 = R_2 (0.4705)$$

$$\boxed{R_2 = 42.019 \text{ m}}$$

② A reverse curve AB is to be set out between two parallel railway tangents 32 m apart. If the two arcs of the curve are to have the same radius and the distance between tangents A and B is 60 m . calculate the radius. The curve is to be set out from AB at 10 m intervals along that line. calculate the length of offsets.



$L = 160$ m

$L = \sqrt{4 \times RV}$

$L = \sqrt{4 \times R \times 32}$

$160 = \sqrt{4 \times R \times 32}$

$R^2 = \frac{160}{11.31}$

$R = 200$ m

<u>Note:-</u>

The maximum ordinates at a midpoint AE and BE is given by

$$O_0 = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

$$= 200 - \sqrt{200^2 - \left(\frac{180}{2}\right)^2}$$

$O_0 = 200 - 195.95$

$$\boxed{O_0 = 4.05 \text{ m}}$$

$\cancel{O_x = \sqrt{R^2 - x^2} - (R - O_0)}$

$\cancel{O_{10} = \sqrt{(200)^2 - (10)^2} - (200 - 4.05)}$

$\cancel{O_{10} =}$

$$O_x = \sqrt{R^2 - x^2} - (R - O_0)$$

$$O_{10} = \sqrt{(200)^2 - (10)^2} - (200 - 4.05)$$

$$O_{10} = 3.799 \text{ m}$$

$$O_{20} = \sqrt{(200)^2 - (20)^2} - (200 - 4.05) = 3.04 \text{ m}$$

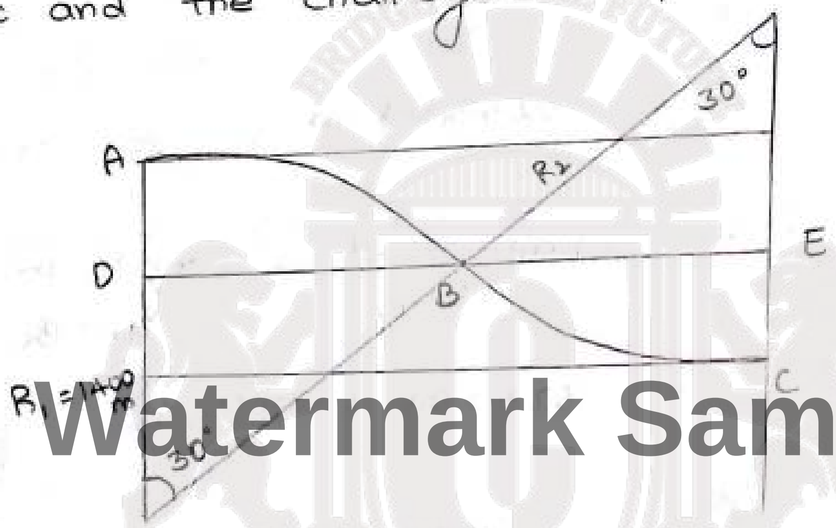
$$O_{30} = \sqrt{(200)^2 - (30)^2} - (200 - 4.05)$$

$$O_{30} = \cancel{3.02}\ 1.787 \text{ m}$$

$$O_{40} = 0 \text{ m}$$

(2) The ordinates of the other arc are the Same as above .. For the each curve the offsets are 0.00, 1.777, 3.03, 3.789, 3.03, 1.777, 0.00.

(3) Two parallel lines which are 469 m are joined by a reverse curve ABC which deflects to the right by an angle of 30° from the 1st straight, if the radius of the 1st arc is 1400 m and the chainage of PI is 2500 m, calculate the radius of 2nd arc and the chainages of points B and C



Let A and C be the points of tangencies and B point of reverse curvature [PRC]

The distance between the lines, V = 469 m

$$V = (R_1 + R_2) \text{ ver } \sin \Delta_1$$

$$V = (R_1 + R_2)(1 - \cos\Delta)$$

$$469 = (1400 + R_2)(1 - \cos 30)$$

$$469 = (1400 + R_2)\ 0.1339$$

$$1400 + R_2 = \overset{3515.74}{3502.61} = 1400 + R_2$$

$$\boxed{R_2 = 2100.66 \text{ m}}$$

The chainage of B = chainage of A + Length of arc AB

$$= 2500 + \frac{\pi R_1 \Delta_1}{180}$$

$$= 2500 + \frac{\pi \times 1400 \times 30}{180}$$

$$= 3233.038$$

$$\simeq$$

$$= 3233.04 \text{ m}$$

chainage of C = chainage of B + Length of ar BC

$$= 3233.04 + \frac{\pi R_2 \Delta_2}{180}$$

$$= 3233.04 + \frac{\pi \times 2100.66 \times 30}{180}$$

$$= 4332.95 \text{ m}$$

A TRANSITION CURVES

Transistion curve is a curve of varying radius introduce between a straight and a circular curve or between two branches of a compound curve or reverse curve.

Necessity of a transition curve

As soon as the mooving vehicle starts negotiating a curve, It is acted upon by the centrifugal force which tends to over turn the moving vehicle.

* sudden change of curvature from zero to a definite value at the point of commencement of the curve causes great discomfort for the vehicle and passangers. The effect of the centrifugal force can be neutralized by raising the outer edge of the highway. The raising of the outer edge of the highway or track [outer rail of the railways] is called super elevation or Cant.

* The amount of superelevation depends on speed of the vehicles and radius of the curve.

* The effect of the centrifugal force may be reduced and the gradual increase of the super elevation may be made by introducing the transition curve between the straight and the curve.

Introduction of following on transition curves of following advantages.

* Int enables the introduce super elevation in proportion to the rate of change of curvature.

* It avoids the danger of derailment [railways at the points of commencement if the full amount of Super elevation is suddenly applied at the point

* It avoids over turning, side slipping of moving vehicles

* It eliminates the discomfort cause the passagers while negotiating the curve

* The main functions of a transition curve being.

① To accomplish gradually the transition from the tangent to the circular curve. So that the curvature is increased gradually from zero to the specified value.

② To provide a medium for the gradual introduction are change of a required

Super elevation.

Types of transition curves :-

1. cubical Spiral
2. Cubic parabola
3. Lemniscate

Cubical Spiral and cubic parabola transition curves are best Suited to railway curves. and the Lemniscate transition curves are best suited to highways.

MODULE - 2

GEODETIC SURVEYING AND THEORY OF ERRORS

The objective of geodetic surverying is to determine very precisely the relative positions or an absolute positions on the earth Surface of a system of widely seperated points

* The relative positions are determined in the terms of lengths and Azimuths of the lines joining them. The absolute positions are determined in terms of lattitude, longitude and elevation above the mean Sea level [MSL)

* Since the area embraced by a geodetic survey form an appreciable portion of earth's Surface, the sphericity or curvature of the earth is taken in to consideration while taking the commputation.

* The geodetic points So determined furnish the most precise control to which a more detailed survey is reffered.

* Geodetic work is usely undertaken by the

government agency. In our country it is done by 'survey of India'. The horizontal control in geodetic survey is established either by triangulation or by precise traverse.

TRIANGULATION –

→ In triangulation the system consists of a number of inter connected triangles in which the length of only one line "BASE LINE", and the angles of the triangles are measured very precisely.

→ Knowing the length of one side and the three angles, the lengths of the other two sides of the each triangle can be computed.

→ The apex of the triangles are known as "TRIANGULATION STATION" and the whole figure is called "TRIANGULATION SYSTEM" or TRIANGULATION FIGURE".

→ The defect of triangulation is that it tends to occumilate errors of length and azimuth, since the length and azimuth of each of line is based on the length and azimuth of the proceeding line.

→ To control the accumilation of errors, Subsidiary bases are also Selected.

* At certain Stations astronomical observations for Azimut and longitude are also made these stations are called laplace stations

The objectives of geodetic stations

* To provide the most accurate system of horizontal control points to less precise triangles may be based, which intern form a framework to which the hydrographical, topographical, engineering and other Survey's may be reffered.

* To assist in the determination of shape and size of the earth by making observations for lattitude, longitude and gravity.

Watermark Sample

classification of Triangulation System

The basis of the triangulation figure or triangulation System is the accuracy with which the length and azimuth of a line of the triangulation are determined.

* Triangulation System of different accuracies depend on the extent and purpose of the Survey.

The triangulation system or triangulation figure are classified as:-

(i) First ORDER OR PRIMARY TRIANGULATION

(ii) SECOND ORDER OR SECONDARY TRIANGULATION

(iii) THIRD ORDER OR TERTIARY TRIANGULATION

① FIRST ORDER OR

The first order triangulation is of the highest order and is employed either to determine the earth's figure and or to furnish the most precise control points to which secondary triangulation may be connected.

* The primary triangulation system embrases the vast area (usually the whole of the country) Every precaution is taken in making linear and angular measurements and in performing the reductions.

General specifications of the primary triangulation

① Average triangle closure - less than 1 sec

② maximum triangle closure - Not more than 3 sec

③ Length of base line - 5 to 15 km

④ Length of size of triangles - 30 to 150 km

⑤ Actual error of base - 1 in 3,00,000

⑥ probable error of base - 1 in 10,00,000

⑦ Discripency between two measures of a Section - 10 mm $\sqrt{\text{Kilometres}}$

⑧ probable error of computate distance - 1 in 60,000 to 1 in 2,50,000.

⑨ Probable error in astronomic azimuth - 0.5 seconds

(ii) SECOND ORDER OR SECONDARY TRIANG-ULATION

The Secondary triangulation consists of number of points fixed within the frame work of primary triangulation.

* The stations are fixed at closed interval so that the sizes of triangles formed are smaller than the primary triangulation.

* The instruments and the methods used are not of the some atmost refinement

General specifications of Secondary triangulation are :-

① Average triangle closure - 3 sec

② maximum triangle closure - 8 sec

③ Length of base line - 1.5 km to 5 km

④ Length of Size of triangles - 8 km to 65 km

⑤ Actual error of base - 1 in 1,50,000

⑥ probable error of base - 1 in 5,00,000

⑦ Discripency between two measures of a Section - 20 mm √Kilometers

⑧ probable error of computate distance - 1 in 20,000 to 1 in 50,000

⑨ probable error in astronomic azimuth - 20 Seconds.

(iii) THIRD ORDER OR TERTIARY TRIANGULATION

* The third order triangulation consists of number of points fixed within the frame work of Secondary triangulation and forms the immediate control for detail engineering and other Surveys

* The sizes of the triangulations are small and the instrument with moderate precision may be used.

General Specification of third order triangulation are

1. Average triangle closure - 6 sec
2. Maximum triangle closure - 12 sec
3. Length of base line - 0.5 km to 3 km
4. Length of size of triangles - 1.5 km to 10 km
5. Actual error of base - 1 in 75,000
6. probable error of base - 1 in 2,50,000
7. Discripency between two measures of a section - 25 mm $\sqrt{\text{kilometres}}$
8. probable error of compute distance 1 in 5,000 to 1 in 20,000
9. probable error in astronomic azimuth - 5 sec

Triangulation figures or triangulation System

Triangulation figure is a group or system of triangles such that any figure has 1 side and only one common to each of the preceeding ~~and~~ or following figures. The Common figures or Systems are

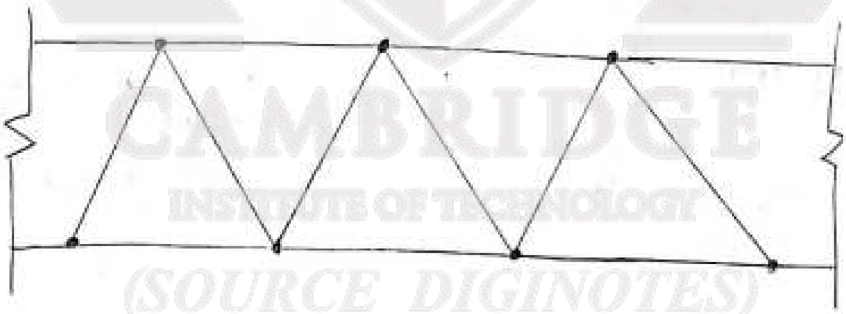
1. Single CHAIN OF TRIANGLES
2. DOUBLE CHAIN OF TRIANGLES
3. CENTRAL POINT FIGURES

④ QUADRILATERALS

① SINGLE CHAIN OF TRIANGLES

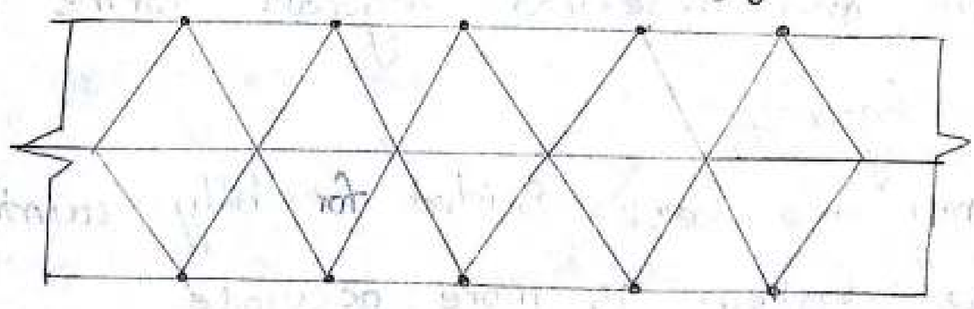
This figure is used where the narrow strip of terrain is to be covered. Though the System is rapid and economical, It is not so accurate for primary work since the number of conditions to be fulfilled in the figure adjustment is relatively small.

* Also it is not possible to carry the Solution of triangles through the figures by two independent routes.



② DOUBLE CHAIN OF TRIAGLES

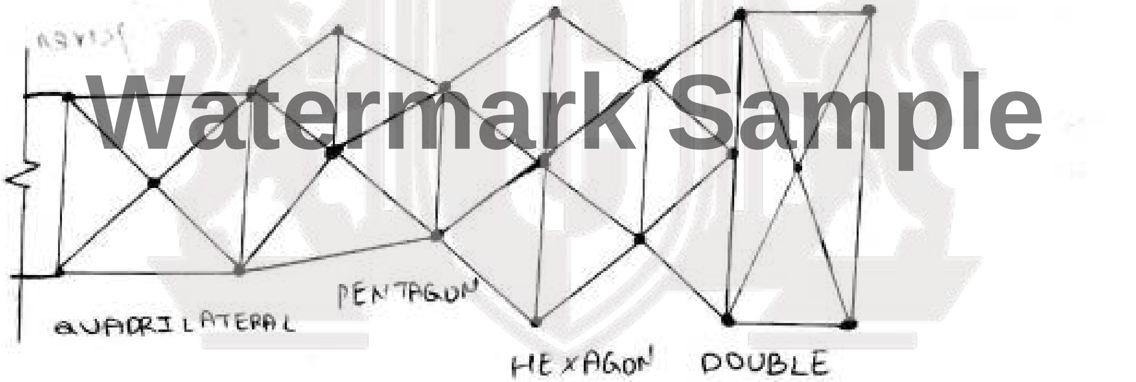
It is used to cover greater area.



③ CENTERED FIGURES :

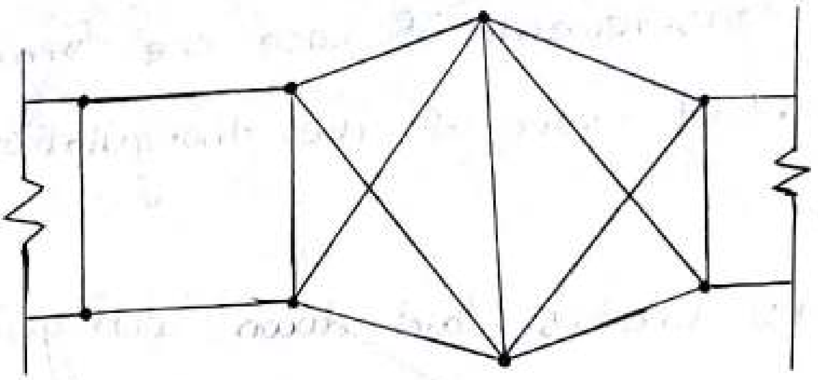
centered figures are used to cover the area and give very satisfactory results in the flat country

* The centered figures may be quadrilaterals, pentagons or hexagons with central stations
* However the progress of the work is slow due to more settings of instrument.



④ QUADRILATERALS :

* The Quadrilateral with four common corner stations and observed diagonals forms a best figures.
* They are best suited for hilly country.
* This system is more accurate.



QUADRILATERALS

CRITERIA FOR SELECTION OF TRIANGULATION FIGURES OR TRIANGULATION SYSTEMS

* The triangulation figure should be such that the computations can be ~~done~~ through two independent ~~roots~~ routes.
* The triangulation figure should be such that atleast one or preferibly both the routes are well conditioned.
* All the lines in the figure should be comparable length. very long lines should be avoided.
* The figure should be such that least work may secure maximum progess.
* complex figures should not involve more than 12 conditons

BASE LINE MEASUREMENT :-

The measurement Of base line forms the most important part of the triangulation operations.

* The base line is laid down with great accuracy of measurement and allignment as it forms the basis for the computations of triangulation System

* The length of the base line depends upon the grades of triangulation.

* Apart from the base line [main] several other check bases are also measured at some suitable intervals.

SELECTION OF SITE FOR SELECTION

Since the accuracy in the measurement of the base line depends upon the site condition, the following points should be taken in to consideration while selecting the site.

① The site should be fairly level. If, However the ground is slopy, the slope should be uniform and gentle. Undulating ground shall be avoided as far as much as possible.

* The site should be free from obstructiom throughout the whole of the length. The line clearing should be economical in both labour and compansation.

* The extremities of the base should be intervisible at ground level.

* The ground should be reasonabily firm and smooth. water gaps should will be fumed and if possible not wider than the length of the long wire or tape.

* The Sight should story site extension to primary triangulation. This is an important factor since the error in extension is likely to exceed the error in measurement.

WELL CONDIONED TRIANGE

There are various triangulation figures and the accuracy attained in each figure depends upon (i) The magnitude of the angles in each individual triangle

(ii) The arrangement of the triangles

Regarding the shape of the triangle should be such that any error in the measurement of the angle shall have the minimum effect upon the lengthe of the calculated side. Such a triangle is called well conditioned triangle.

ROUTIN OF TRIANGOLATION SURVEY

The routin of traingulation surveys generally consists of following operations

1. Reconnaissance
2. Erection of signals and towers
3. measurement of base lines
4. measurement of horizontal base angles
5. Astronomical observation at laplace stations
6. computations

SELECTION OF TRIANGULATION STATION

1. The trangulation stations should be intervisible ; For this purpose the triangulation station must be placed upon the most elevated

ground (hilltops) so that long sights through undisturbed atmosphere may secure.

* They should form well shaped triangles as far as possible the iscol with base angles are of about 56° or equaliteral . In general, no angle should be smaller than 30° and greater than 120°.

* The stations should be easily accessible and should be such that food and water are easily available and the campaign ground are nearest accomodation is available.

* They should be so selected then the length (Station) of site neighter neither too small nor too long. Small length of sight will result in errors due to centering and bisection while large line of sight will make the signal too indistinct for accurate bisection.

⑤ They should be in a comanding situation so as to seärve serve as the control of the subsidiary triangulation and for possible future extension of the principal system. The stations of the subsidiary triangulation should be such that they are useful for detailed surveys.

⑥ In heavily wooden country, the stations ~~the~~ so located that the cost of clearing and cutting and of building towers is minimum

⑦ The stations should be suitiated so that lines of sight do not pass over towns, factories etc nor graze in an obstruction, so that the effects of triangulation are irregular atmos--pheric ~~d~~iffraction is avoided.

MARKING OF TRIANGULATION STATIONS

* The triangulation station should be perman--ently marked with copper or bronze tablets. The name of the station and the year in which it is set should be stamped on the tablet.

* The following are the essentials of good construction of station marks.

  (i) The mark should be distincitive and indestructable. Two marks should be provided, one visible on the surface and the other burried vertically below.

The mark may be set on firm work on a concrete monument.

(ii) Two or three reference marks, Similar in material and shape to the Station mark should be installed. The distance and bearings of these reference marks from the Station mark and from each other should be recorded on them.

(iii) At each Station where at all signal tower is needed, an Azimuth mark should be established at some distance away from the station mark. The Azimuth mark should be of same size and character as that of the reference mark.

SIGNALS AND TOWERS :-

Towers :- A tower is a structure errected over a station for the support of a instrument and observing the bodies partly is provided when the station or signal are both are to be elevated.

* The triangulation tower must be built in duplicate, securly founded. The inner tower supports the instrucment only and the outer tower supports the observer and the signal. The two towers should

be entirely independent ~~to~~ each other.

* Towers may be of masonry, timber or Steel

SIGNALS :

A Signal is a device to define the exact position of an observed station. The Signal can be classified as

(i) Day Light Signal or Non opaque or Non Luminous Signal.

(ii) Sunlight or Sun or Luminous Signal

(iii) Night Signal

A Signal should be capable of fulfilling the following requirements.

(i) The Signal should be clearly ~~visible~~ against any back ground [conspicuous]

(ii) It should be capable of accurately centered over the station mark.

(iii) It should be Suitable for accurate bisection

(iv) It should be free from phase or Should exhibit little phase.

REDUCTION TO CENTRE :-

In order to secure well conditioned triangle or better visibility objects such as flag poles, towers etc are sometimes selected as triangulation stations. * when the observations are to be taken from such station, it is impossible to set up and an instrument over it.

* In such a case, a subsidiary station known as satellite station or eccentric station or false station is selected as near to the main station as possible.

* The observations are taken to the other triangulation station with the same precision as would have been used in the measurement of angles at the true station.

* These angles are later corrected and reduced to what they would have been if the true station was occupied.

* The operation of applying the corrections due to the eccentricity of the station is generally known as the reduction of centre.

* The distance between the true Station and Satellite Station is determined either by trignometric levelling or triangulation method

* Satellite stations should be avoided as far as possible in primary triangulation

Error :- The difference between the observed value and the true value is known as error.

THEORY OF ERRORS

Errors of measurements are of three kinds which occurs while we do the surveying.

1. mistakes
2. SYSTEMATIC ERRORS
3. ACCIDENTAL ERRORS

① MISTAKES :- mistakes are the errors that arise from in experience, carelessness, inattentive tion and poor judgement or confusion in the mind of the observer.

* If the mistake is undetected, it produces a serious effect on the final result.

* Hence every value to be recorded in the field must be checked in some independent field observation

② SYSTEMATIC ERRORS:-

* An systmatic error is an error that under the same conditions will always be of same size and sign

* A systematic error always follows same definite mathematical and physical law and a correction can be determined and applied. Such errors are of constant character and are regarded as positive or negative according to the result as the result too great or too small. Their effect is therefore cumulative

③ ACCIDENTAL ERRORS:-

Accidental errors are those which remain after mistakes and systematic errors have been eliminated and are caused by combination of reasons beyond the ability of the observer to control. * The error may be sometimes in one direction and sometimes in the other, they are likely to make the apparent result too large or too small.

* An accidental error of a single determination is the difference between

(i) the true value of the quantity and

(ii) a determination that is free from mistakes and systematic errors.

* Accidental error obey the laws of chance and therefore must be handled according to mathematical laws of probability

DEFINITATIONS

① INDEPENDENT QUANTITY :- An

independent quantity is one whose value is independent of the values of other quantities. There is no It bears no relation with any other quantity and hence change in other quantities does not affect the value of this quantity.

Example:- Reduced levels of several bench marks.

(2) CONDITION QUANTITY :- A conditioned quantity is one whose value is dependent upon the values of one or more quantities. It is also called a dependent quantity.

Example:- In a triangle ABC, angle A+ $\angle A + \angle B + \angle C = 180°$. In this conditioned equation, any two angles may be regarded as independent and the third angled as dependent or conditioned quantity.

(3) DIRECT OBSERVATION :-

An observation is the numerical value of a measured quantity and may be either direct or and indirect.

(1) direct
(2) Indirect

A direct observation is the one made directly on the quantity being determined.

Example :- The measurement of a base, the Single measurement of an angle etc.

④ INDIRECT OBSERVATION :-

An indirect observation is the one in which the observed value is detected or deduced from the measurement of some relative quantities.

Ex :- The measurement of angle by repitions (a multiple of an angle being measured)

⑤ WEIGHT OF AN OBSERVATION :-

The weight of an observation is one in which a number giving an indication of it's precision and trust worthyless when making the comparison between several quantities of different worth.

Example :- thus, If a certain observation is of weight four. It means that it is 4 times as much realiable as an observation of weight '1'.

④ when two quantities or observation are assumed to be equally reliable, the observed values are said to be of equal weight or of unit weight

→ weights are assigned to the observation or quantities observed in direction proportion to the number of observations.

⑥ OBSERVED VALUE OF THE QUANTITY

An observed value of a quantity is the value obtained when it is corrected for all the known errors.

⑦ TRUE VALUE OF QUANTITY :-

The true value of a quantity is the value which is absolutely free from all quantities of all the errors

* The true value of a quantity is indeterminate since the true error is never known.

⑧ MOST PROBABLE VALUE :-

The most probable value of a quantity is which as more chances of being true than any other value.

⑨ TRUE ERROR :- A true error is a difference between the true value of a quantity and is a observed value.

⑩ MOST PROBABLE ERROR :-

The most probable error is defined as that quantity which is added to, subtracted from, the most probable value fixes the limits within which an even chance the true value of the measured quantity must live.

(ii) **RESIDUAL ERROR** :- A residual error is the difference between the most probable value of a quantity and its observed value.

OBSERVATION EQUATION :- An observation equation is the relation between the observed quantity and it's numerical value.

CONDITIONED EQUATION :- A conditioned equation is the equation expressing the relation existing between the several dependent quantities.

NORMAL EQUATION :- A normal equation is the one which is formed by the multiplying each equation by the co-efficient of the unknown whose normal equation is to be found and by adding the equation, thus formed as the number of normal equations is the same as the number of unknowns. The most probable values of the unknown can be found from this equation.

THE LAWS OF ACCIDENTAL ERRORS

The theory of errors that is discussed in this chapter deal with only accidental error after all the known errors are eliminated.

Investigations of observations of various types show that accidental errors follow a definite law, the law of probability.

This law defines the occurence of errors and can be expressed in the form of equation which is used to compute the probable value or a probable precision of a quantity. The most important features of accidental errors which usually occur are

(i) Small errors tend to be more frequent than the large ones, that is they are the most probable.

(ii) positive and negative errors of same size happen with equal frequency, i.e they are equally probable.

(iii) Large errors occur infrequently and are impossible.

LAWS OF WEIGHTS :-

GENERAL PRINCIPLES OF LEAST SQUARES

It is found that from the probability equation that the most probable values of a Series of errors arising from observations of equal weight are those for which the Sum of the Squares is a minimum.

* The fundamental law of least Squares is derived from this.

* According to the principle of least Squares, the most probable value of an observed quantity available from a given Set of observations is the one for which the Sum of the Squares of the residual errors is minimum.

* when a quantity being deducted from a Series of observation, the residual errors will be the difference between the adopted value and several observed values.

LAWS OF WEIGHTS

From the method of least Squares the following laws of weights are established

(1) The weight of the arithematic mean of the measurement of unit weight is equal to the number of observations.

Ex:- Let an angle A measured 6 times and the following being the values

$\angle A$ weight

(1) $30^\circ\ 20'\ 80''$	1
(2) $30^\circ\ 20'\ 10''$	1
(3) $30^\circ\ 20'\ 7''$	1
(4) $30^\circ\ 20'\ 10''$	1
(5) $30^\circ\ 20'\ 9''$	1
(6) $30^\circ\ 20'\ 10''$	1

$\therefore$ Arithematic mean

$$= 30^\circ\ 20' + \frac{1}{6}\left(8'' + 10'' + 7'' \ldots\right)$$

$$= 30^\circ\ 20'\ 9''$$

weight of arithematic mean = Number of observation = 6

(2) The weight of the weighted arithmatic mean is equal to the sum of the individual weight.

Ex:- Let $\angle A$ be measured 6 times the following being the values.

$\angle A$	weight
$30^\circ\ 20'\ 8''$	2
$30^\circ\ 20'\ 10''$	3
$30^\circ\ 20'\ 6''$	2
$30\ 20'\ 10''$	3

$30^\circ \quad 20' \quad 9''$ 4

$30^\circ \quad 20' \quad 10''$ 2

Sum of individual weights

$= 2 + 3 + 2 + 3 + 4 + 2$

$= 16$

weighted Arthimatic mean =

$$30^\circ\, 20' + \frac{1}{16}\left[(8''\times 2) + (10''\times 3) + (6''\times 2) + (10''\times 3) + (9''\times 4) + (10''\times 2)\right]$$

$= 30^\circ\, 20'\, 9''$

$\therefore$ weight of weighted Arithematic mean = 16

(3) The weight of algebraic Sum of two or more quantities is equal to the reciprocal of the Sum of reciprocal of individual weights

Ex:- Let $\alpha = 42^\circ\, 10'\, 20''$, weight - 4

$\beta = 30^\circ\, 40'\, 10''$, weight = 2

Sum of reciprocals of individual weights =

$\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$\therefore$ weight of $\alpha + \beta = 42^\circ 10' 20'' + 30^\circ 40' 10''$

$= 72^\circ\, 50'\, 30''$

$$= \frac{1}{\frac{1}{4}+\frac{1}{2}} = \frac{1}{3/4} = \frac{4}{3}$$

weight of $\alpha - \beta = 11^\circ 30' 10'' = \frac{1}{\frac{1}{4}+\frac{1}{2}} = \frac{1}{3/4} = \frac{4}{3}$

(4) If the quantity of given weight is multiplied by a factor, the weight of the result is obtained by dividing its given weight by the square of the factor

Ex:- Let $\alpha = 42° \; 10' \; 20''$ weight = 6

Then, weight of $3\alpha = 126° \; 31'$

$$= \frac{6}{(3)^2} = 0.66$$

$$= \frac{2}{3}$$

(5) If the quantity of given weight is divided by a factor, the weight of the result is obtained by multiplying its given weight by the square of the factor

Ex:- Let $\alpha = 42° \; 10' \; 30''$, weight = 4

Then, weight of $\frac{\alpha}{3} = \frac{42° \; 10' \; 30''}{3}$

$$= 14° \; 3' \; 30''$$

$$= 4\,(3)^2$$

$$= 36$$

(6) If an equation is multiplied by its own weight, the weight of the resulting equation is equal to the reciprocal of the weight of the equation

Ex :- Let $A + B = 98° 20' 30''$, weight $= \frac{3}{5}$

Then, weight of $\frac{3}{5}(A+B) = 59° 0' 18''$
$= 1$

The reciprocal of the weight of the equation

$$= \frac{1}{3/5} = \frac{5}{3}$$

(7) The weight of the equation remains unchanged, if all the signs of the equations are changed are if the equations are added to or substration from a constant

Ex :. Let $A+B = 80° 20'$, weight $= 3$

Then the weight of $180° - (A+B)$

$180° - (80° 20')$

$(99° 40') = 3$

RULES OF ASSIGNING WEITAGE TO THE FIELD OBSERVATION

(1) The weight of an angles varies directly as the number of observations made for the measurement of that angle.

(2) weights vary inversly as the length of varies roots in the case of lines of levels.

(3) If an angle is measured a large number of times, its weight is inversly proportional

to the Square of probable error.

4 The corrections to be applied to the various observed quantities are in inverse properties proportione to their weights.

DISTRIBUTION OF ERRORS TO THE FIELD OBSERVATION / MEASUREMENTS

whenever the observations are made in the field, it is always necessary for check for closing error. if any.

* The closing error should be distributed to the observed quantities.

For example :- The sum of angles measured at the central angle should be 360°. if the sum is not equal to 30°, the error should be distrubuted to the observed angles after giving proper weightage to the observations × The following rules should be applied for the distrubution of errors.

(1) The correction to be applied to an observation is inversly proportional to the weight of the observation.

(2) The correction to be applied to an observation is directly proportional to square of the probable error.

(3) In case of line of levels, the correction to be applied is proportional to the length.

Example :- An LA ~~wbs~~ was measured by different persons and the following are the values.

Angle	No of measuments
65° 30' 10"	2
65° 29' 50"	3
65° 30' 00"	3
65° 30' 20"	4
65° 30' 10"	3

Find the most probable value of the angle.

Sol:- The most probable value of the angle is equal it's weighted arithematic mean.

Angle				
65° 30' 10"	× 2	=	131° 0' 20"
65° 29' 50"	× 3	=	196° 29' 30"
65° 30' 00"	× 3	=	196° 30' 00"
65° 30' 20"	× 4	=	262° 1' 20"
65° 30' 10"	× 3	=	196° 30' 30"
	sum Σ	=	982° 31' 40"

Σ weight = $2+3+3+4+3 = 15$

$\therefore$ weighed arithematic mean $= \dfrac{982^\circ\ 31'\ 40''}{15}$

$= 65^\circ\ 30'\ 6''.67$

DETERMINATION OF THE MOST PROBABLE VALUES

The most probable value of the quantity is the one which has more changes of being true. It is deduced from several measurements on which it is based. In practice the following cases may arise of which the most probable values may be required to be determined.

CASE (1) :- DIRECT OBSERVATRION OF EQUAL WEIGHTS

CASE (2) :- DIRECT OBSERVATIONS OF UNEQUAL WEIGHTS

CASE (3) :- INDIRECTLY OBSERVED QUANTITES ENVOLVING UNKNOWNS OF EQUAL WEIGHTS

CASE (4) :- INDIRECTLY OBSERVED QUANTITIES INVOLVING UNKNOWNS OF UNEQUAL WEIGHTS

CASE (5) :- OBSERVATION EQUATIONS ACCOMPAINED BY CONDITION EQUATION

(1) DIRECT OBSERVATION OF EQUAL WEIGHTS,

The most probable value of the directly observed quantity of equal weights is equal to the arithematic mean of the observed values.

* If $V_1, V_2, V_3 \ldots V_n$ is the observed value of the quantity of equal weigh, 'm' being the arithematic mean. then

$$\boxed{m = \frac{V_1 + V_2 + V_3 + \ldots\ldots}{n} = \text{most probable value}}$$

CASE (2) :- DIRECT OBSERVATION OF UNEQUAL WEIGHTS

* The most probable value of an observed quantity of unequal weights is equal to the weighed arithematic mean of the observed quantities.

Then,
* If $V_1, V_2, V_3 \ldots V_n$ are the observed quantities with weights $w_1, w_2, w_3 \ldots w_n$ and N is the most probable vale of the quantity

$$\boxed{N = \frac{w_1 V_1 + w_2 V_2 + w_3 V_3 + \ldots w_n V_n}{w_1 + w_2 + w_3 + \ldots w_n} = \text{most probable value}}$$

case 3 and case 4

INDIRECTLY OBSERVED QUANTITIES INVOLVING UNKNOWNS OF EQUAL AND

UNEQUAL WEIGTHS

* when the unknows are independent ~~of~~ each other there the most probable values can be found by forming the normal equations for each of the unknown quantities and treating them as simultaneous equations to get the value of unknowns.

CASE(5):- OBSERVATION EQUATION ACCOMPAINED BY CONDITION EQUATION

when the observation equations are accompained by one or more condition equation, the ladder may be reduced to an observation equation which will eliminate one of the unknowns.

* The normal equation can be formed for the remaining unknows.

① Find the most probable value of ∠A from the following observation equation

$A = 30° \ 28' \ 40''$

$3A = 91° \ 25' \ 55''$

$4A = 121° \ 54' \ 30''$

Sol^n ☆ Note :- There is only one unknown and all the observation are of equal weight.

The co-efficience of A in the three equations are 1, 3, 4

Hence multiply these equations by 1, 3 and 4 respectively

$A = 30° \; 28' \; 40'' \times 1 = 30° \; 28' \; 40''$

$3A = 91° \; 25' \; 55'' \times 3 = 274° \; 17° \; 45''$

$4A = 121° \; 54' \; 30'' \times 4 = 487° \; 38' \; 0'$

Sum $26A = 792° \; 24' \; 25''$

$A = 30° \; 28' \; 37''.9$

② Find the most probable value of $\angle A$ from the following observation equations

$A = 30° \; 28' \; 40''$ weight 2

$3A - 91° \; 25' \; 55''$ weight 3

$A = 30° \; 28' \; 40'' \times 2 = 60° \; 57' \; 20''$

$3A = 91° \; 25' \; 55'' \times 3 = 274° \; 17° \; 45'$

Sum

There is only one unknown but the observation are of unequal weights. The normal equation can be formed by multiplying each of the two observation equations by the corresponding weight and co-efficient of weight and adding them.

* In the first equation, co-efficient of A is 1 and the weight of observation is 2

Hence $1 \times 2 = 2$

* In the second equation, coefficient of A is 3 and the weight of observation is 3

Hence $3 \times 3 = 9$

Thus we have

$$A = 30^\circ\ 28'\ 40'' \times 2 = 60^\circ\ 57'\ 20''$$

$$3A = 91^\circ\ 25'\ 55'' \times 9 = 822^\circ\ 53'\ 15''$$

$$29\,A = 883^\circ\ 50'\ 35''$$

$$\boxed{A = 30^\circ\ 28'\ 38.45''}$$

③ Find the most probable values of the angles A and B from the following observations at a station 'O'.

$A = 9^\circ\ 48'\ 36.6''$, weight 2

$B = 54^\circ\ 37'\ 48.3''$ weight 3

$A+B = 104^\circ\ 26'\ 28.5''$ weight 4

Sol^n Note :- There are two unknowns A and B and both are independent of each other and there will be two normal reactions.

* In the first equation,

Co-efficient of A is $\underline{1}$

weight of observation is $\underline{2}$

$\therefore$ Hence $1 \times 2 = 2$

* In the second equation, coefficient of A is $\underline{0}$ and weight of observation is $\underline{0}$

To find the normal equation of A. multiply by '2', '2' by '0' (since the coefficient of A is 0.

In the 3^{rd} equation, coefficient of A is 1 and weight is 4, multiply eq^n ③ by '4'

$\rightarrow 1 \times 4 = 4$

$$2A = 2 \times 9^\circ 48' 36''.6 = 19^\circ 37' 13.2''$$

$$4A + 4B = 417^\circ 45' 54''$$

$$6A + 4B = 437^\circ 23' 7.2''$$

↳ Normal equation for A —①

To find normal equation for B.

multiply eq^n ① by 0 $\Rightarrow$ $(0 \times 2 = 0)$

multiply eq^n ② by 3 $\Rightarrow$ $(1 \times 3 = 3)$

multiply eq^n ③ by 4 $\Rightarrow$ $(1 \times 4 = 4)$

$$3B = 163^\circ\ 53'\ 24.9''$$

$$4A + 4B = 417^\circ\ 45'\ 54''$$

$$4A + 7B = 581^\circ\ 39'\ 18.9'' \longrightarrow \text{Normal equation for } B$$

Solving both normal of A and B, we get

$$6A + 4B = 437^\circ\ 23'\ 7.2'' = 28.27216$$

$$4A + 7B = 581^\circ\ 39'\ 18.9'' = 66.9380$$

$$A = 28^\circ\ 16'\ 19.84''$$

$$B = 66^\circ\ 56'\ 17.14''$$

UNIT – IV

ASTRONOMICAL SURVEYING

Celestial sphere - Astronomical terms and definitions - Motion of sun and stars - Apparent altitude and corrections - Celestial co-ordinate systems - Different time systems –Use of Nautical almanac - Star constellations - calculations for azimuth of a line.

Celestial Sphere.

The millions of stars that we see in the sky on a clear cloudless night are all at varying distances from us. Since we are concerned with their relative distance rather than their actual distance from the observer. It is exceedingly convenient to picture the stars as distributed over the surface of an imaginary spherical sky having its center at the position of the observer. This imaginary sphere on which the star appear to lie or to be studded is known as the celestial sphere. The radius of the celestial sphere may be of any value – from a few thousand metres to a few thousand kilometers. Since the stars are very distant from us, the center of the earth may be taken as the center of the celestial sphere.

Zenith, Nadir and Celestial Horizon.

The Zenith (Z) is the point on the upper portion of the celestial sphere marked by plumb line above the observer. It is thus the point on the celestial sphere immediately above the observer's station.

The Nadir (Z') is the point on the lower portion of the celestial sphere marked by the plum line below the observer. It is thus the point on the celestial sphere vertically below the observer's station.

Celestial Horizon. (True or Rational horizon or geocentric horizon): It is the great circle traced upon the celestial sphere by that plane which is perpendicular to the Zenith-Nadir line, and which passes through the center of the earth. (Great circle is a section of a sphere when the cutting plane passes through the center of the sphere).

Terrestrial Poles and Equator, Celestial Poles and Equator.

The terrestrial poles are the two points in which the earth's axis of rotation meets the earth's sphere. The terrestrial equator is the great circle of the earth, the plane of which is at right angles to the axis of rotation. The two poles are equidistant from it.

If the earth's axis of rotation is produced indefinitely, it will meet the celestial sphere in two points called the north and south celestial poles (P and P'). The celestial equator is the great circle of the celestial sphere in which it is intersected by the plane of terrestrial equator.

Sensible Horizon and Visible Horizon.

It is a circle in which a plane passing through the point of observation and tangential to the earth's surface (or perpendicular to the Zenith-Nadir line) intersects with celestial sphere. The line of sight of an accurately leveled telescope lies in this plane.

It is the circle of contract, with the earth, of the cone of visual rays passing through the point of observation. The circle of contact is a small circle of the earth and its radius depends on the altitude of the point of observation.

Vertical Circle, Observer's Meridian and Prime Vertical?

A vertical circle of the celestial sphere is great circle passing through the Zenith and Nadir. They all cut the celestial horizon at right angles.

The Meridian of any particular point is that circle which passes through the Zenith and Nadir of the point as well as through the poles. It is thus a vertical circle.

It is that particular vertical circle which is at right angles to the meridian, and which, therefore passes through the east and west points of the horizon.

Latitude (θ) and Co-latitude (c).

Latitude (θ): It is angular distance of any place on the earth's surface north or south of the equator, and is measured on the meridian of the place. It is marked + or – (or N or S) according as the place is north or south of the equator. The latitude may also be defined as the angle between the zenith and the celestial equator.

The Co-latitude of a place is the angular distance from the zenith to the pole. It is the complement of the latitude and equal to (90°-θ).

longitude (ϕ) and altitude (α).

The longitude of a place is the angle between a fixed reference meridian called the prime of first meridian and the meridian of the place. The prime meridian universally adopted is that of Greenwich. Te longitude of any place varies between 0° and 180°, and is reckoned as Φ° east or west of Greenwich.

The altitude of celestial or heavenly body (i.e, the sun or a star) is its angular distance above the horizon, measured on the vertical circle passing through the body.

Co-altitude or Zenith Distance (z) and azimuth (A).

It is the angular distance of heavenly body from the zenith. It is the complement or the altitude, i.e z = (90° - α).

The azimuth of a heavenly body is the angle between the observer's meridian and the vertical circle passing through the body.

Declination (δ) and Co-declination or Polar Distance (p).

The declination of a celestial body is angular distance from the plane of the equator, measured along the star's meridian generally called the declination circle, (i.e., great circle passing through the heavenly body and the celestial pole). Declination varies from 0° to 90°, and is marked + or – according as the body is north or south of the equator.

It is the angular distance of the heavenly body from the near pole. It is the complement of the declination. i.e., p = 90° - δ.

Hour Circle, Hour Angle and Right ascension (R.A).

Hour circles are great circles passing though the north and south celestial poles. The declination circle of a heavenly body is thus its hour circle.

The hour angle of a heavenly body is the angle between the observer's meridian and the declination circle passing through the body. The hour angle is always measured westwards.

Right ascension (R.A): It is the equatorial angular distance measured eastward from the First Point of Aries to the hour circle through the heavenly body.

Equinoctial Points.

The points of the intersection of the ecliptic with the equator are called the equinoctial points. The declination of the sun is zero at the equinoctial points. The Vernal Equinox or the First point of Aries (Y) is the sun's declination changes from south to north, and marks the commencement of spring. It is a fixed point of the celestial sphere. The Autumnal Equinox or the First Point of Libra (Ω) is the point in which sun's declination changes from north to south, and marks the commencement of autumn. Both the equinoctial points are six months apart in time.

ecliptic and Solstices?

Ecliptic is the great circle of the heavens which the sun appears to describe on the celestial sphere with the earth as a centre in the course of a year. The plan of the ecliptic is inclined to the plan of the equator at an angle (called the obliquity) of about 23° 27', but is subjected to a diminution of about 5" in a century.

Solstices are the points at which the north and south declination of the sun is a maximum. The point C at which the north declination of the sun is maximum is called the summer solstice; while the point C at which south declination of the sun is maximum is know as the winter solstice. The case is just the reverse in the southern hemisphere.

North, South, East and West Direction.

The north and south points correspond to the projection of the north and south poles on the horizon. The meridian line is the line in which the observer's meridian plane meets horizon place, and the north and south points are the points on the extremities of it. The direction ZP (in plan on the plane of horizon) is the direction of north, while the direction PZ is the direction of south. The east-west line is the line in which the prime vertical meets the horizon, and east and west points are the extremities of it. Since the meridian place is perpendicular to both the equatorial plan as well as horizontal place, the intersections of the equator and horizon determine the east and west points.

spherical excess and spherical Triangle?

The spherical excess of a spherical triangle is the value by which the sum of three angles of the triangle exceeds 180°.

Thus, spherical excess E = (A + B + C - 180°)

A spherical triangle is that triangle which is formed upon the surface of the sphere by intersection of three arcs of great circles and the angles formed by the arcs at the vertices of the triangle are called the spherical angles of the triangle.

Properties of a spherical triangle.

The following are the properties of a spherical triangle:

1. Any angle is less than two right angles or π.
2. The sum of the three angles is less than six right angles or 3π and greater than two right angles or π.
3. The sum of any two sides is greater than the third.
4. If the sum of any two sides is equal to two right angles or π, the sum of the angles opposite them is equal to two right angles or π.
5. The smaller angle is opposite the smaller side, and vice versa.

formulae involved in Spherical Trigonometry?

The six quantities involved in a spherical triangle are three angles A, B and C and the three sides a, b and c. Out of these, if three quantities are known, the other three can very easily be computed by the use of the following formulae in spherical trigonometry:

1. Sine formula: $\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$

2. Cosine formula: $\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$

Or $\cos a = \cos b \cos c + \sin b \sin c \cos A$

Also, $\cos A = - \cos B \cos C + \sin B \sin C \cos a$

systems used for measuring time?

There are the following systems used for measuring time:

1. Sidereal Time
2. Solar Apparent Time
3. Mean Solar Time
4. Standard Time

terrestrial latitude and longitude.

In order to mark the position of a point on the earth's surface, it is necessary to use a system of co-ordinates. The terrestrial latitudes and longitudes are used for this purpose.

The terrestrial meridian is any great circle whose plane passes through the axis of the earth (i.e., through the north and south poles). Terrestrial equator is great circle whose plane is perpendicular to the earth's axis. The latitude θ of a place is the angle subtended at the centre of the earth north by the are of meridian intercepted between the place and the equator.

The latitude is north or positive when measured above the equator, and is south or negative when measured below the equator. The latitude of a point upon the equator is thus 0°, while at the North and South Poles, it is 90° N and 90° S latitude respectively. The co-latitude is the complement of the latitude, and is the distance between the point and pole measured along the meridian.

The longitude (ϕ) of a place is the angle made by its meridian plane with some fixed meridian plane arbitrarily chosen, and is measured by the arc of equator intercepted between these two meridians. The prime meridian universally adopted is that of Greenwich. The longitude of any place varies between 0° to 180°, and is reckoned as ϕ° east or west of Greenwich. All the points on meridian have the same longitude.

Spherical Triangle? & its properties.

A spherical triangle is that triangle which is formed upon the surface of the sphere by intersection of three arcs of great circles and the angles formed by the arcs at the vertices of the triangle are called the spherical angles of the triangle.

AB, BC and CA are the three arcs of great circles and intersect each other at A, B and C. It is usual to denote the angles by A, B and C and the sides respectively opposite to them, as a, b and c. The sides of spherical triangle are proportional to the angle subtended by them at the centre of the sphere and are, therefore, expressed in angular measure. Thus, by sin b we mean the sine of the angle subtended at the centre by the arc AC. A spherical angle is an angle between two great circles, and is defined by the plane angle between the tangents to the circles at their point of intersection. Thus, the spherical angle at A is measured by the plane angle A1AA2 between the tangents AA1 and AA2 to the great circles AB and AC.

Properties of a spherical triangle

The following are the properties of a spherical triangle:

1. Any angle is less than two right angles or π.
2. The sum of the three angles is less than six right angles or 3π and greater than two right angles or π.
3. The sum of any two sides is greater than the third.
4. If the sum of any two sides is equal to two right angles or π, the sum of the angles opposite them is equal to two right angles or π.
5. The smaller angle is opposite the smaller side, and vice versa.

Formulae in Spherical Trigonometry

The six quantities involved in a spherical triangle are three angles A, B and C and the three sides a, b and c. Out of these, if three quantities are known, the other three can very easily be computed by the use of the following formulae in spherical trigonometry:

1. since formula : $\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$

2. Cosine formula :$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$

or $\cos a = \cos b \cos c + \sin b \sin c \cos A$

Also, $\cos A = -\cos B \cos C + \sin B \sin C \cos a$

The Spherical Excess

The spherical excess of a spherical triangle is the value by which the sum of three angles of the triangle exceeds 180°.

Thus, spherical excess $E = (A + B + C - 180°)$

Also, $$\tan^2 \frac{1}{2} E = \tan \frac{1}{2} s \tan \frac{1}{2}(s-a) \tan \frac{1}{2}(s-b) \tan \frac{1}{2}(s-c)$$

In geodetic work the spherical triangles on the earth's surface are comparatively small and the spherical excess seldom exceeds more than a few seconds of arc. The spherical excess, in such case, can be expressed by the approximate formula

$$E = \frac{\Delta}{R^2 \sin 1''} \text{ seconds}$$

where R is the radius of the earth and Δ is the area of triangle expressed in the same linear units as R.

the relationship between co-ordinates?

1. **The Relation between Altitude of the Pole and Latitude of the Observer.**

In the sketch, H-H is the horizon plane and E-E is the equatorial plane. O is the centre of the earth. ZO is perpendicular to HH while OP is perpendicular to EE.

Now latitude of place = $\theta = \angle EOZ$

And altitude of pole = $\alpha = \angle HOP$

$$\angle EOP = 90° = \angle EOZ + \angle ZOP$$
$$= \theta + \angle ZOP \qquad \text{.... (i)}$$
$$\angle HOZ = 90° = \angle HOP + \angle POZ$$
$$= \alpha + \angle POZ \qquad \text{.... (ii)}$$

Equating the two, we get

$\theta + \angle ZOP = \alpha + \angle POZ$ or $\theta = \alpha$

Hence the altitude of the pole is always equal to the latitude of the observer.

2. The Relation between Latitude of Observer and the Declination and Altitude of a Point on the Meridian.

For star M1, EM1 = δ = declination.

SM1 = α = meridian altitude of star.

M1Z = z = meridian zenith distance of star.

EZ = θ = latitude of the observer.

Evidently, EZ = EM1 + M1Z

Or $\theta = \delta + z$ …. (1)

The above equation covers all cases. If the star is below the equator, negative sign should be given to δ. If the star is to the north of zenith, negative sign should be given to z.

If the star is north of the zenith but above the pole, as at M2, we have

ZP = Z M2 + M2 P

or $(90° - \theta) = (90° - \alpha) + p$, where p = polar distance = M2 P

or $\theta = \alpha - p$ …. (2)

Similarly, if the star is north of the zenith but below the pole, as at M3, we have

ZM3 = ZP + PM3

$(90° - \alpha) = (90° - \theta) + p$, where p = polar distance = M3 P

$\theta = \alpha + p$ …. (3)

The above relations form the basis for the usual observation for latitude.

3. The Relation between Right Ascension and Hour Angle.

Fig 1.22. shows the plan of the stellar sphere on the plane of the equator. M is the position of the star and ∠SPM is its westerly hour angle. HM. Y is the position of the First Point of Aries and angle SPY is its westerly hour angle. ∠YPM is the rit ascension of the star. Evidently, we have

∴ Hour angle of Equinox = Hour angle of star + R.A. of star.

Find the difference of longitude between two places A and B from their following longitudes :]

(1) Longitude of A = 40° W

Longitude of B = 73° W

(2) Long. Of A = 20° E

Long. Of B = 150° E

(3) Longitude of A = 20° W

Longitude of B = 50° W

Solution.

(1) The difference of longitude between A and B = 73° - 40° = **33°**

(2) The difference of longitude between A and B = 150° - 20° = **130°**

(3) The difference of longitude between A and B = 20° - (- 50°) = **70°**

(4) The difference of longitude between A and B = 40° - (- 150°) = **190°**

Since it is greater than 180°, it represents the obtuse angular difference. The acute angular difference of longitude between A and B, therefore, is equal to

360° - 190° = **170°.**

Calculate the distance in kilometers between two points A and B along the parallel of latitude, given that

(1) Lat. Of A, 28° 42' N : longitude of A, 31° 12' W

Lat. Of B, 28° 42' N : longitude of B, 47° 24' W

(2) Lat. Of A, 12° 36' S : longitude of A, 115° 6' W

Lat. Of B, 12° 36' S : longitude of B, 150° 24' E.

Solution.

The distance in nautical miles between A and B along the parallel of latitude = difference of longitude in minutes x cos latitude.

(1) Difference of longitude between A and B = 47° 24' – 31° 12' =

16° 12' = 972 minutes

∴ Distance = 972 cos 28° 42' = 851.72 nautical miles

= 851.72 x 1.852 = **1577.34 km.**

(2) Difference of longitude between A and B

= 360° - { 115° 6' – (- 150° 24') } = 94° 30' = 5670 min.

∴ Distance = 5670 cos 12° 36’ = 5533.45 nautical miles

= 5533.45 x 1.852 = **10,247.2 km.**

Find the shortest distance between two places A and B, given that the longitudes of A and B are 15° 0’ N and 12° 6’ N and their longitudes are 50° 12’ E and 54° 0’ E respectively. Find also the direction of B on the great circle route.

Radius of earth = 6370 km.

Solution.

The positions of A and B have been shown.

In the spherical triangle ABP,

B = AP = 90° - latitude of A

= 90° - 15° 0’ = 75°

A = BP = 90° - latitude of B

= 90° - 12° 6’ = 77° 54’

P = ∠A P B = difference of longitude

= 54° 0’ – 50° 12’ = 3° 48’.

The shortest distance between two points is the distance along the great circle passing through the two points.

Knowing the two sides one angle, the third side AB (=p) can be easily computed by the cosine rule.

Thus cos P = $\dfrac{\cos p - \cos a \cos b}{\sin a \sin b}$

or cos p = cos P sin a sin b + cos a cos b

= cos 3° 48’ sin 77° 54’ sin 75° + cos 77° 54’ cos 75°

= 0.94236 + 0.05425 = 0.99661

∴ p = AB = 4° 40’ = 4°.7

Now, arc ≈ radius x central angle = $\dfrac{6370x4°.7x\pi}{180°} = 522.54km.$

Hence distance AB = **522.54 km.**

Direction of A from B :

The direction of A from B is the angle B, and the direction of B from A is the angle A. Angles A and B can be found by the tangent semi-sum and semi-difference formulae

Thus tan ½ (A + B) = $\dfrac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\frac{1}{2}p$

And tan ½ (A – B) = $\dfrac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\frac{1}{2}p$

Here $\dfrac{(a-b)}{2}=\dfrac{77°54'-75°}{2}=\dfrac{2°54'}{2}=1°27'$

$\dfrac{(a+b)}{2}=\dfrac{77°54'+75°}{2}=\dfrac{152°54'}{2}=76°27';\dfrac{p}{2}=\dfrac{3°48'}{2}=1°54'$

$\therefore$ $\tan\frac{1}{2}(A+B)=\dfrac{\cos 1°27'}{\cos 76°27'}\cot 1°54$

From which, $\dfrac{A+B}{2}=89°35'$ …. (i)

and $\tan\dfrac{1}{2}(A-B)=\dfrac{\sin 1°27'}{\sin 76°27'}\cot 1°54'$

From which, $\dfrac{A-B}{2}=38°6'$ …. (ii)

∴ Direction of B from A = angle A = 89°35’ + 38°6’ = 127°41’ = **S 52°19’ E**

∴ Direction of A from B = angle B = 89°35’ + 38°6’ = 51°29’ = **N 51°29’ W.**

Determine the hour angle and declination of a star from the following data :

(i)	**Altitude of the star**	**= 22° 36'**
(ii)	**Azimuth of the star**	**= 42° W**
(iii)	**Latitude of the place of observation**	**= 40° N.**

Solution.

Since the azimuth of the star is 42° W, the star is in the western hemi-sphere.

In the astronomical Δ PZM, we have

PZ = co-latitude = 90° - 40° = 50° ;

ZM = co-altitude = 90° - 22° 36° = 67 24' ;

angle A = 42°

Knowing the two sides and the included angle, the third side can be calculated from the cosine formula

Thus, cos PM = cos PZ . cos ZM + sin PZ. Sin ZM. cos A

= cos 50°. cos 67° 24' + sin 50°. sin 67° 24'. cos 42°

= 0.24702 + 0.52556 = 0.77258

∴ PM = 39° 25'

∴ Declination of the star = δ = 90° - PM = 90° - 39° 25' = **50° 35' N.**

Similarly, knowing all the three sides, the hour angle H can be calculated from Eq. 1.2

$$\cos H = \frac{\cos ZM - \cos PZ . \cos PM}{\sin PZ . \sin PM} = \frac{\cos 67°24' - \cos 50° . \cos 39°25'}{\sin 50° . \sin 39°25'}$$

$$= \quad = \frac{0.38430 - 0.49659}{0.48640} = -0.23086$$

∴ cos (180° - H) = 0.23086 ∴ 180° - H = 76° 39'

H = 103° 21'.

astronomical parameters of the earth and the sun.

The Earth:

The Earth is considered approximately spherical in shape. But actually it is very approximately an oblate spheroid. Oblate spheroid is the figure formed by revolving an ellipse about its minor axis. The earth is flattened at poles – its diameter along the polar axis being lesser than its diameter at the equator. The equatorial radius a of the earth, according to Hayford's spheroid is 6378.388 km and the polar radius b of the earth is 6356.912 km. The ellipticity is expressed by the ratio $\frac{a-b}{a}$, which reduces to $\frac{1}{297}$. For the Survey of India; Everest's first constants were used as follows:

a = 20,922,932 ft and b = 20,853,642 ft, the ellITicity being $\frac{1}{311.04}$.

The earth revolves about its minor or shorter axis (i.e. polar axis), on an average, once in twenty-four hours, from West to East. If the earth is considered stationary, the whole celestial sphere along with its celestial bodies like the stars, sum, moon etc. appear to revolve round the earth from East to West. The axis of rotation of earth is known as the polar axis, and the points at which it intersects the surface of earth are termed the North and South Geographical or Terrestrial Poles. In addition to the motion of rotation about its own polar axis, the earth has a motion of rotation relative to the sun, in a plane inclined at an angle of 23° 27' to the plane of the equator. The time of a complete revolution round the sun is one year. The apparent path of the sun in the heavens is the result of both the diurnal and annual real motions of the earth.

The earth has been divided into certain zones depending upon the parallels of latitude of certain value above and below the equator. The zone between the parallels of latitude 23° 27 ½ ' N and 23° 27 ½ ' S is known as the torrid zone (see Fig. 1.12). This is the hottest portion of the earth's surface. The belt included between 23° 27 ½ ' N and 66° 32 ½ ' N of equator is called the north temperate zone. Similarly, the belt included between 23° 27 ½ ' S and 66° 32 ½ ' S is called south temperate zone. The belt between 66° 32 ½ ' N and the north pole is called the north frigid zone and the belt between 66° 32 ½ ' S and the south pole is called south frigid zone.

The sun:

The sun is at a distance of 93,005,000 miles from the earth. The distance is only about $\frac{1}{250,000}$ of that of the nearest star. The diameter of the sun is about 109 times the diameter of the earth, and subtends and angle of 31' 59" at the centre of the earth. The mass of the sun is about 332,000 times that of the earth. The temperature at the centre of the sun is computed to be about 20 million degrees.

The sun has twp apparent motions, one with respect to the earth from east to west, and the other with respect to the fixed stars in the celestial sphere. The former apparent path of the sun is in the plane which passes through the centre of the celestial sphere and intersects it in a great circle called the ecliptic. The apparent motion of the sun is along this great circle. The angle between the plane of equator and the ecliptic is called the Obliquity of Ecliptic, its value being 23° 27'. The obliquity of ecliptic changes with a mean annual diminution of 0'.47.

The points of the intersection of the ecliptic with the equator are called the equinoctial points, the declination of the sun being zero at these points. The Vernal Equinox or the First point of Aries (Y) is the point in which the sun's declination changes from south to north. Autumnal Equinox or the First point of Libra (Ω) is the point in which the sun's declination changes from north to south. The points at which sun's declinations are a maximum are called solstices. The point at which the north declination of sun is maximum is called the summer solstice, while the point at which the south declination of the sun is maximum is known as the winter solstice.

The earth moves eastward around the sun once in a year in a pat that is very nearly a huge circle with a radius of about 93 millions of miles. More accurately, the path is described as an ellipse, one focus of the ellipse being occupied by the sun.

Various measurements of time.

Due to the intimate relationship with hour angle, right ascension and longitude, the knowledge of measurement of time is most essential. The measurement of time is based upon the apparent motion of heavenly bodies caused by earth's rotation on its axis. Time is the interval which lapses, between any two instants. In the subsequent pages, we shall use the following abbreviations.

G.M.T.	… Greenwhich Mean Time	G.M.M.	… Greenwich Mean Midnight
G.A.T.	… Greenwich Apparent Time	L.A.N.	… Local Apparent Noon
G.S.T.	… Greenwich Sidereal Time	L.M.M.	… Local Mean Midnight
L.M.T.	… Local Mean Time	L.Std.T.	… Local Standard Time
L.A.T.	… Local Apparent Time	N.A.	… Nautical Almanac
L.S.T.	… Local Sidereal Time	S.A.	… Star Almanac
G.M.N.	… Greenwich Mean Noon		

The units of time.

There are the following systems used for measuring time :

1. Sidereal Time
2. Solar Apparent Time
2. Mean Solar Time
4. Standard Time

Sidereal Time:

Since the earth rotates on its axis from west to east, all heavenly bodies (i.e. the sun and the fixed stars) appear to revolve from east to west (i.e. in clock-wise direction) around the earth. Such motion of the heavenly bodies is known as apparent motion. We may consider the earth to turn on it axis with absolute regular speed. Due to this, the stars appear to complete one revolution round the celestial pole as centre in constant interval of time, and they cross the observer's meridian twice each day. For astronomical purposes the sidereal day is one of the principal units of time. The sidereal day is the interval of time between two successive upper transits of the first point of Aries (Y). It begins at the instant when the first point of Aries records 0h, 0m, 0s. At any other instant, the sidereal time will be the hour angle of Y reckoned westward from 0h to 24h. The sidereal day is divided into 24 hours, each hour subdivided into 60 minutes and each minute into 60 seconds. However, the position of the Vernal Equinox is not fixed. It has slow (and variable) westward motion caused by the precessional movement of the axis, the actual interval between two transits of the equinox differs about 0.01 second of time from the true time of one rotation.

Local Sidereal Time (L.S.T.):

The local sidereal time is the time interval which has elapsed since the transit of the first point of Aries over the meridian of the place. It is, therefore, a measure of the angle through which the earth has rotated since the equinox was on the meridian. The local sidereal time is, thus, equal to the right ascension of the observer's meridian.

Since the sidereal time is the hour angle of the first point of Aries, the hour angle of a star is the sidereal time that has elapsed since its transit. M1 is the position of a star having SPM1 (= H) as its hour angle measured westward and YPM1 is its right ascension (R.A.) measured eastward. SPY is the hour angle of Y and hence the local sidereal time.

Hence, we have SPM1 + M1PY = SPY

or **star's hour angle + star's right ascension = local sidereal time** … (1)

If this sum is greater than 24 hours, deduct 24 hours, while if it is negative add, 24 hours.

The star M2 is in the other position. Y PM2 is its Right Ascension (eastward) and ZPM2 is its hour angle (westward). Evidently,

ZPM2 (exterior) + YPM2 – 24h = SPY = L.S.T.

or **star's hour angle + star's right ascension – 24h = L.S.T**

This supports the preposition proved with reference to Fig. 1.30 (a). The relationship is true for all positions of the star.

When the star is on the meridian, its hour angle is zero. Hence equation 1 reduces to

Star's right ascension = local sidereal time at its transit.

A sidereal clock, therefore, records the right ascension of stars as they make their upper transits.

The hour angle and the right ascension are generally measured in time in preference to angular units. Since one complete rotation of celestial sphere through 360° occupies 24 hours, we have

24 hours = 360° ; 1 hour = 15°

The difference between the local sidereal times of two places is evidently equal to the difference in their longitudes.

Solar Apparent Time:

Since a man regulates his time with the recurrence of light and darkness due to rising and setting of the sun, the sidereal division of time is not suited to the needs of every day life, for the purposes of which the sun is the most convenient time measurer. A solar day is the interval of time that elapes between two successive lower transits of the sun's centers over the meridian of the place. The lower transit is chosen in order that the date may change at midnight. The solar time at any instant is the hour angle of the sun's centre reckoned westward from 0h to 24h. This is called the apparent solar time, and is the time indicated by a sun-dial. Unfortunately, the apparent solar day is not of constant length throughout the year but changes. Hence our modern clocks and chronometers cannot be used to give us the apparent solar time. The non-uniform length of the day is due to two reasons :

(1) The orbit of the earth round the sun is not circular but elliptical with sun at one of its foci. The distance of the earth from the sun is thus variable. In accordance with the law of gravitation, the apparent angular motion of the sun is not uniform – it moves faster when is nearer to the earth and slower when away. Due to this, the sun reaches the meridian sometimes earlier and sometimes later with the result that the days are of different lengths at different seasons.

(2) The apparent diurnal path of the sun lies in the ecliptic. Due to this, even though the eastward progress of the sun in the ecliptic were uniform, the time elapsing between the departure of a meridian from the sun and its return thereto would vary because of the obliquity of the ecliptic.

The sun changes its right ascension from 0h to 24h in one year, advancing eastward among the stars at the rate of about 1° a day. Due to this, the earth will have to turn nearly 361° about its axis to complete one solar day, which will consequently be about minutes longer than a sidereal day. Both the obliquity of the ecliptic and the sun's unequal motion cause a variable rate of increase of the sun's right ascension. If the rate of change of the sun's right ascension were uniform, the solar day would be of constant length throughout the year.

Mean Solar Time :

Since our modern clocks and chronometers cannot record the variable apparent solar time, a fictitious sun called the mean sun is imagined to move at a uniform rate along the equator. The motion of the mean sun is the average of that of the true sun in its right ascension. It is supposed to start from the vernal equinox at the same time as the true sun and to return the vernal equinox with the true sun. The mean solar day may be defied as the interval between successive transit of the mean sun. The mean solar day is the average of all the apparent solar days of the year. The mean sun has the constant rate of increase of right ascension which is the average rate of increase of the true sun's right ascension.

The local mean noon (L.M.N.) is the instant when the mean sun is on the meridian. The mean time at any other instant is given by the hour angle of the mean sun reckoned westward from 0 to 24 hours. For civil purposes, however, it is found more convenient to begin the day at midnight and complete it at the next midnight, dividing it into two periods of 12 hours each. Thus, the zero hour of the mean day is at the local mean midnight (L.M.M.). The local mean time (L.M.T.) is that reckoned from the local mean midnight. The difference between the local mean time between two places is evidently equal to the difference in the longitudes.

From Fig. 1.30 (a) if M1 is the position of the sun, we have

Local sidereal time = R.A. of the sun + hour angle of the sun … (1)

Similarly,

Local sidereal time = R.A. of the mean sun + hour angle of the mean sun … (2)

The hour angle of the sun is zero at its upper transit. Hence

Local sidereal time of apparent noon = R.A. of the sun … (3)

Local sidereal time of mean noon = R.A. of the mean sun … (4)

Again, since the our angle of the sun (true or mean) is zero at its upper transit while the solar time (apparent or mean) is zero as its lower transit, we have

The apparent solar time = the hour angle of the sun + 12h … (5)

The mean solar time = the hour angle of mean sun + 12h … (6)

Thus, if the hour angle of the mean sun is 15° (1 hour) the mean time is 12 + 1 = 13 hours, which is the same thing as 1 o'clock mean time in the afternoon; if the hour angle of the mean sun is 195° (13 hours), the mean time is 12 + 13 =25 hours, which is the same as 1 o'clock mean time after the midnight (i.e., next. Day).

The Equation of Time

The difference between the mean and the apparent solar time at any instant is known as the equation of time. Since the mean sun is entirely a fictitious body, there is no means to directly observe its progress. Formerly, the apparent time was determined by solar observations and was reduced to mean time by equation of time. Now-a-days, however, mean time is obtained more easily by first determining the sidereal time by steller observations and then converting it to mean time through the medium of wireless signals. Due to this reason it is more convenient to regard the equation of time as the correction that must be applied to mean time to obtain apparent time. The nautical almanac tabulates the value of the equation of time for every day in the year, in this sense (i.e. apparent – mean). Thus, we have

Equation of time = Apparent solar time – Mean solar time

The equation of time is positive when the apparent solar time is more than the mean solar time ; to get the apparent solar time, the equation of time should then be added to mean solar time. For example, at 0h G.M.T. on 15 October 1949, the equation of the time is + 13m 12s. This means that the apparent time at 0h mean time is 0h 13m 12s. In other words, the true sun is 13m 12s ahead of the mean sun. Similarly, the equation of time is negative when the apparent time is less than the mean time. For example, at 0h G.M.T. on 18 Jan., 1949, the equation of time is – 10m 47s. This means that the apparent time at 0h mean time will be 23h 49m 13s on January 17. In other words, the true sun is behind the mean sun at that time.

The value of the equation of time varies in magnitude throughout the year and its value is given in the Nautical Almanac at the instant of apparent midnight for the places on the meridian of Greenwich for each day of the year. For any other time it must be found by adding or subtracting the amount by which the equation has increased or diminished since midnight.

It is obvious that the equation of time is the value expressed in time, of the difference at any instant between the respective hour angles or right ascensions of the true and mean suns.

The amount of equation of the time and its variations are due to two reasons :

(1) obliquity of the ecliptic, and (2) elasticity of the orbit. We shall discuss both the effects separately and then combine them to get the equation of time.

Explain the conversion of local time to standard time and vice versa.

The difference between the standard time and the local mean time at a place is equal to the difference of longitudes between the place and the standard meridian.

If the meridian of the place is situated east of the standard meridian, the sun, while moving apparently from east to west, will transit the meridian of the place earlier than the standard meridian. Hence the local time will be greater than the standard time. Similarly, if the meridian of the place is to the west of the standard meridian, the sun will transit the standard meridian earlier than the meridian of the place and hence the local time will be lesser than the standard time. Thus, we have

L.M.T = Standard M.T ± Difference in the longitudes $\left(\frac{E}{W}\right)$

L.A.T = Standard A.T ± Difference in the longitudes $\left(\frac{E}{W}\right)$

L.S.T = Standard S.T ± Difference in the longitudes $\left(\frac{E}{W}\right)$

Use (+) sign if the meridian of place is to the east of the standard meridian, and (-) Sign if it to the west of the standard meridian.

If the local time is to be found from the given Greenwich time, we have

L.M.T = Standard M.T ± Difference in the longitudes $\left(\frac{E}{W}\right)$

The standard time meridian in India is 82° 30’ E. If the standard time at any instant is 20 hours 24 minutes 6 seconds, find the local mean time for two places having longtitudes (a) 20° E, (b) 20° W.

Solution:

(a) The longitude of the place = 20° E

Longtitude of the standard meridian = 82° 30'E

∴ Difference in the longitudes = 82° 30' – 20° = 62° 30', E. the place being to the west of the standard meridian.

Now 62° of longitude = $\frac{62}{15}$ h = $4^h\,8^m\,0^s$

Now 30' of longitude = $\frac{30}{15}$ m = $0^h\,2^m\,0^s$

Total = $4^h\,10^m\,0^s$

Now L.M.T = Standard time – Difference in longitude (W)

= $20^h\,24^m\,6^s$ --

module - 4

AERIAL PHOTOGRAMMETRY

photogrammetric survey is the Science and art of obtaining accurate measurements by use of photographs. for various purposes Such as construction of topographic maps, classification of soils, interpretation of geology, Aquisition of military intelligence and the preparation of composite pictures of the ground.

The photographs are either taken from the air or from station on the ground

photogrammetric survey is classified into two types

Terrestrial photogrammetry is that branch of photogrammetry wherein photographs are taken from fixed position on or near the ground.

Aerial photogrammetry is that branch of photogrammetry where in the photographs are taken by a camera mounted in an aircraft flying over the area.

mapping from aerial photographs is the best mapping procedure for large projects and are invliable for military intelligents

The major uses of Aerial mapping method are the civilian and military mapping agencies of government.

Definitions and Nomanclature

* vertical photographs :- It is an aerial photograph made with a camera axis (or optical axis) coinciding with the direction of gravity.

* Tilted photographs :- It is an aerial photograph made with the camera axis unintentionally titted from the vertical by a small amount usually less than 3°. (unintentionally)

* oblique photographs :- It is an aerial photograph taken from the camera axis directed intentionally between the horizontal and vertical.

* If the apparent horizon is shown in photograph it is said to be high oblique.
* If the apparent horizon is not shown in photograph it is said to be low oblique.

* perspective projection is the one produced by straight lines radiating from a common [selected] point and passing through point on the sphere to the plane of projection.

Note
A photograph is a perspective projection.

Diagram

Exposure Station :- is a point in space, in the air, occupy by the camera lens at the instant of exposure

precisely it is the space position of the front noddle point at the instant of excposure

Flying height :- Is the elevation of the exposure Station above sea level or any selected datum

Flight line :- It is a line drawn on a map to represent the track of aircraft.

(line 'OK' in fig)

Focal length :- It is a distance from the front nodal point of the lens to the plane of the photograph. It is also the distance of the image plane from the rear nodle point.

principle point (K) :- It is point where perpendicular is dropped from the front nodle point strikes the photograph. This principle point is considered to coincide with the intersection of x-axis and the y-axis, in figure 'k' is the principal point and 'K' is known as the ground principal point.

Nadir point (n) :- It is a point where the plumb line dropped from the front nodlal point ~~pearses~~ intersect the photograph. This point is also known as photo nadir or photo plumb point. In fig 'n' is the nadir point, which is a point on the photograph vertically beneath the exposure station

(Ground plumb point) N

Ground Nadir point :- Ground Nadir point / ground plumb point is the datum intersection with the plumb line through the front noddle point
[is the point on the ground vertically beneath the exposure station such as point N.]

Tilt :- It is the vertical angle defined by intersection at the exposure station of the optical axis with the plumb line.
(< KON : t = tilt)

principle plane :- (NOK or nok) It is the plane NOK or nok. principle plane is a vertical plane containing the optical axis.

principle Line (nk) :- Is a line of intersection of principle plane with the plane of photograph.
Thus it is the line on a photograph obtained by joining the principal point and the nadir point (nk)

Isocentre (i) :- Is a point in which bisector of angle of the tilt meets the photograph.

GROUND CONTROL FOR PHOTO

* The ground control Survey consists in locating the ground positions of point which can be identified on aerial photograph.

* The ground control is essential for establishing the position and orientation of each photographs in space of each relative to the ground.

* Extent of ground control required is determined by (i) the scale of the map (ii) the navigational control (iii) cartographical process by which the map will be produced.

* The ground survey for establishing control can be divided into 2 parts namely.

(i) BASIC CONTROL (ii) PHOTO CONTROL

The basic control consist in establishing the basic network of triangulation stations, traverse stations, azimuth marks, bench marks etc.

→ The photo control consist of in establishing the horizontal positions or elevations of the images of the identified points on the photograph with respect to the basic control. The

The photo control can be established by two methods

(i) post - marking method

(ii) pre - marking method

In the post - marking method, the photo control points are selected after the aerial photography, the distint advantage of this method is in positive identification and favourable location of points.

In the pre - marking method, the photo control points are selected on the ground first and then included in the photograph the marked points on the ground can be identified on the subsequent photographs. The selected photo control points should be sharp and cleared in the plan.

MOSAICS :- vertical photographs look so much like the ground that the set can be fitted together to form a map like photograph of the ground. Such an assembly of getting of a

Series of overlapping photographs is called a mosaic.

* The mosaic as an overall average scale comparable to the Scale of planymetric map.

* A controlled mosaic is obtained when the photograph are carefully assembled So that the horizontal controlled points agree with their previously plotted positions.

* A mosaic which is assembled without regard to any plotted control is called an uncontrolled mosaic.

* The photographs are laid in such a Sequence as to allow photo number and flight number of each photograph is appear on the finished assembly. This assembly is called index mosaic. Any index mosaic is a form of uncontrolled mosaic.

* A mosaic which is assembled on the Single strip of a photograph is called a Strip mosaic.

<u>OVERLAP</u>:- A when vertical photographs are to be used for the preparation of maps, all the methods of compailation required that the plumb points preeceeding and Succeeding prints are available or visibile in each photograph.

* photographs are taken at proper interval along each strip to give the desired overlap of

photographs in the given strip.

* Each strip is spaced at pre-determined distance to ensure the desired side lap between adjucent strips.

* The overlap of photographs in the direction of flight line is called longitudinal overlap or forward overlap or simply overlap. along

* Along a given flight line photographs are taken at such frequeny has as to cause successive photographs to overlap each other by 55 to 65%.

STEREOSCOPS AND PARALLAX :-

* Stereoscopic vision :- The depth perception is the mental process of determining relative distance of objects from the observer from the impressions received through the eyes.

* Due to binocular vision, the observer is able to perceive the spatial relations that is the three dimensions of the field of view.

* The impression of depth is caused mainly due to three reasons

(i) Relative apparent size of near and far objets

(ii) Effect of light and shade

(iii) Viewing of an object simultaneously by two eyes which are separated in space.

* stereoscope is an instrument used for viewing stereopairs.

* stereoscopes are designed for two purposes.

(1) To assist in ~~purposi~~ presenting to the eyes, the images of a pain of photographs so that the relationship between convergence and accomodiation is the same as would be in natural vision.

(2) To magnify the perciption of depth

* ~~(3)~~ There are two basic types of Stereoscope,

(i) mirror stereoscope

(ii) Lens Stereoscope

(i) <u>mirror Stereoscope</u> :- The mirror stereoscope consists of a pair of small eye piece mirrors ~~Cons~~ and a pair of larger wing mirrors. Each of which is oriented at 45° with the plane of the photographs.

(ii) <u>Lens Stereoscope</u> :- A lens stereoscope consists of a single magnifying lens for each eye and no mirrors. The two magnifying lenses are mounted with a separation equal to the average distances of the human eye, but provision is made for changing this separation to suit the individual user.

MODULE - 5

MODERN SURVEYING INSTRUMENTS

Electro magnetic Distance measurement [EDM]

There are three types or methods of measuring distances between any two given points

(1) Direct distance measurement [DDM]:-

It is chaining or taping

(2) optical distance measurement [ODM]:-

(3) Electromagnetic distance measurement [EDM]

EDM is a general term embrasing the measurement of distance using electronic methods

* In electro magnetic method or electronic method, distances are measured with instruments that relay on propogation, reflection and subsequent reception of either radio waves, visible light waves or infrared waves.

Electromagnetic waves :- The EDM method is based on generation, propagation, reflection and subsequent reception of electromagnetic waves.

* The type of electroelectromagnetic waves generated depends on many factors but principly on the nature of electric signal used to generate the waves.

* The method based on the po progation of modulated light waves using an instrument called geodimeter water was developed.

* Another instrument TELLUROMETER was developed using radio waves.

* modern, short and medium range EDM instrument such as DISTOMATES commonly used in surveying, used modulated infrared rays.

Types of medium instruments depending upon the type of carrier waves employed EDM instruments can be classified into

1. MICRO WAVE INSTRUMENTS
2. VISIBLE LIGHT INSTRUMENTS
3. INFRARED INSTRUMENTS

① MICRO WAVE INSTRUMENTS :-

These instruments come under the category of long range instruments. wherein carrier frequencies of range 3 to 30 GHz enable distance measurements upto 100 Km range.

* Tellurometer comes under this category.

② VISIBLE LIGHT INSTRUMENTS :-

These instruments use as visible light as carrier wave with a higher frequency of a order of 5×10^{14} Hz.

* Since the transmitting power of carrier wave of such high frequency falls off rapidly with the distance, the range of such EDM instruments is lesser than those of EDM microwave instrument.

* Geodimeter comes under this category of instruments

* The EDM instrument in this category has a range of 25 Km.

* The advantage of visible light EDM instruments is that only one instrument is required

③ INFRARED INSTRUMENTS :-

The EDM instruments in this group used infrared radiation band of wavelength about 0.9 μm as a carrier wave which is easily obtained from galleium, arsenide [Ga As] infrared emitting diode. These diodes can be easily, directly amplitude modulated at high frequencies

* Thus, modulated carrier wave is obtained by an inexpensive method. Due to this reason there is predominance of infrared instruments in EDM.

* wild distomates fall under these category of EDM instrument.

* The power output of diode is low hence the range of these instruments is limited to 2-5 Km.

Electromagnetic Spectrum :-

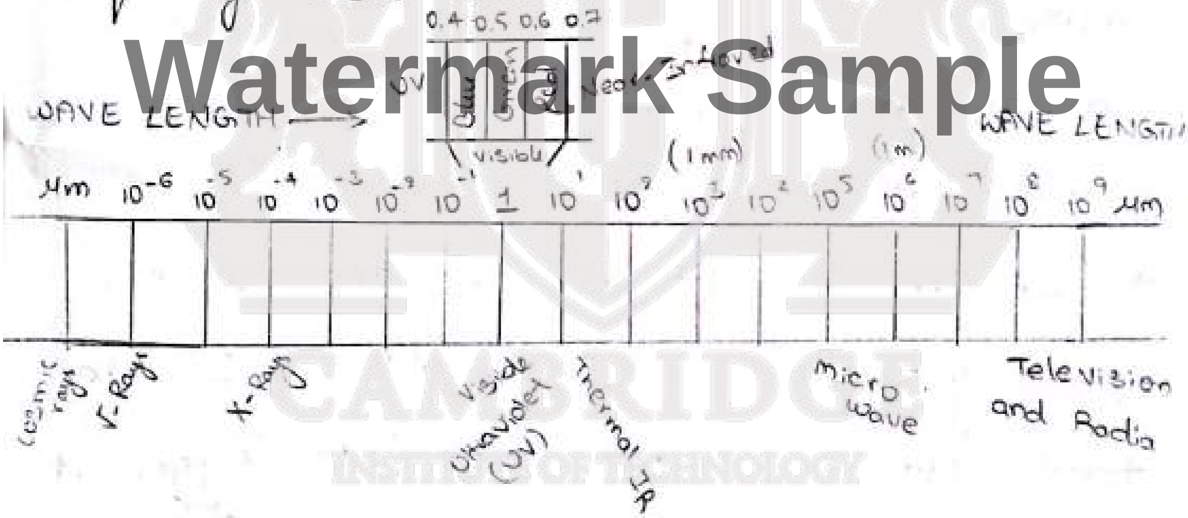
Electromagnetic radiation can be produced at a range of wavelengths and can be categorized according to its position into discrete regions which is generally referred to electromagnetic Spectrum.

* Thus the electromagnetic spectrum is the Continuum of energy that ranges from meters to nano meters in wavelength travels

at a Speed of light. and propogates through a vaccum like the outer space.

* All matter radiates a range of electro magnetic energy with the peak intensity shifting toward progressively shorter wave length at an increasing temperature of the matter.

* In general the wavelengths and frequencies vary from shorter wavelength - high frequency cosmic waves to long wavelength low frequency radio waves.



Total station :-

A total station is a combination of an electronic theodolite and an electromagnetic distance meter [EDM].

* This combination makes it possible to determine the co-ordinates of a reflector by alligning the instruments, cross hairs

on the refractor and simultaneously measuring vertical and horizontal angles and slope distance

* A microprocessor in the instrument takes care of the recordings and readings and the necessary computations.

* The data is easily transferred to computer when it can be used to generate a map

* As a teaching tool, the total stations fulfill several stations, learning how to use properly the total station involves the physics of making measurements, the geometry of calculations, statistics for analysing the result of traverse.

* In the field, it requires team work, planning and careful observations. If the total station is equiped with data logger it also involves the interfacing the data logger with the computer, transforming the data. and working with a data on a computer.

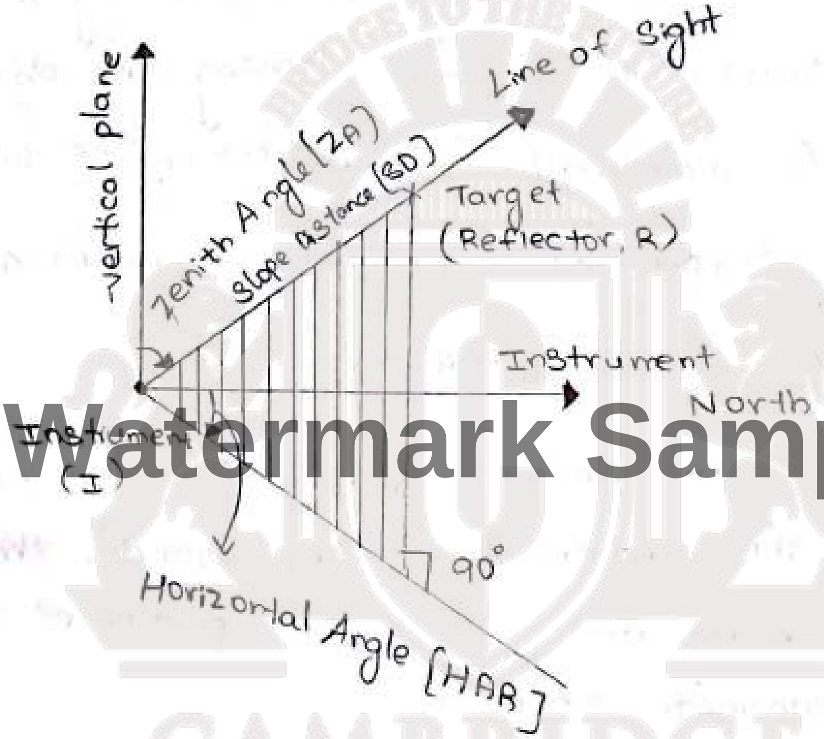
Fundamental measurements when aimed at a appropriate target a total station measures three parameters

(1) HORIZONTAL ANGLE

(2) VERTICAL ANGLE

(3) SLOPE DISTANCE

All the numbers that are provided by the total stations are derived from these three fundamental measurements.



REMOTE SENSING :-

Remote sensing is broadly defined as science and art of collecting information about objects, area or phenomena from distance without being in physical contant with them.

* In the present contest, the defination of remote sensing is restricted to mean the process of acquiring information about any object without physically contanting with it in any ways regardless

whether the observer is immediately adjacent to the object or millions of miles away.

* Aircrafts and satellites are common platform used for remote Sensing

* Remote Sensing data basically consists of wavelength intensity information by collecting the electromagnetic radiation leaving the object at Specific wavelength and measuring its intensity.

* photo interpretation can be considered as primitive form of remote Sensing.

* most of the modern remote Sensing make use of the reflected infrared bands, thermal infrared bands and microwave portion of the electro magnetic Spectrum.

Remote Sensing is broadly classified into two categories

(1) Passive remote Sensing

(2) Active remote Sensing

(1) Passive remote Sensing :- It uses Sun as a source of electro magnetic energy and records the energy that is naturally radiated and are reflected from the objects

(2) Active remote Sensing :- It uses its own source of electro magnetic energy, It which is directed towards the object and the return energy is measured.

Idealized remote Sensing System

An idealized remote Sensing System consists of following stages

stage : 1 → Energy Source

→ propogation of energy through atmosphere

→ Energy interaction with earth's surface features

→ Air borne or space borne sensors receiving the reflected and emitted energy

→ Transmission of data to earth station and generation of data provided

→ multiple data users

Basic principles of remote sensing

Remote Sensing employ electromagnetic energy and to the great extend realise on the interation of electro magnetic with matter (object)

* It refers to the Sensing of electro magnetic

radiation which is reflected, Scattered, or emitted from the object.

2

principles of energy interaction in atmosphere and earth Surface features

① Electromagnetic radiation and the atmosphere

(i) In Remote Sensing, electromagnetic radiation must pass through the atmosphere in order to reach the earth's Surface. Ind to the Sensor after reflection and emission from earth's Surface features.

(ii) The water vapour, oxygen, ozone, carbon dioxide, aerosols etc present in the atmosphere influence the electromagnetic radiation through the mechansim of

(a) Scattering (b) absorption

(a) Scattering :- It is unpredictable diffusion of radiation by molecules of the gases, dust and Smoke in the atmosphere.

* Scattering reduses the image Contrast and changes the Spectral Signatures of ground objects.

* Scattering is basically classified as

(i) Selective and Non Selective

Depending upon the Size of the particle with which the electro magnetic radiation interact.

3

Non Selective Scatter :-

Non Selective Scatter occurs when the diameter of the particles is several times more [app-roximately 10 times] than radiation wavelength

* For visible wavelengths, the main Sources of non selective Scattering are pollen grains, cloud droplets, ice and Snow crystals and rain drops

* It scatters all wavelength of visible light with equal efficiency.

b) Absorption :- In contrast to Scattering, atmospheric absorption results the effective loss of energy as a consequence of the nature of atmospheric constituents like molecules of ozone, carbon dioxide and water vapour.

* oxygen absorbs in ultra violet region and also as an absorption band, Similarly carbon dioxide prevents the number of wavelengths reaching the Surface.

* water Vapour is an extremely important Obserber of electro magnetic radiation within the infrared part of the Spectrum.

Interaction of electromagnetic radiation with earth's surface

4

Electromagnetic energy that strikes or encounters the matter [object] is called incident radiation.

* The electromagnetic radiation striking the surface may be
    (i) Reflected or scatterd
    (ii) Absorbed
    (iii) Transmitted

Interaction with matter can change the following properties of incident radiation.
(a) Intensity (b) Direction (c) wavelength (d) polerization and phase.

The science of remote sensing detects and records the changes.

The energy balance equation for radiation at a given wavelength [λ] can be expressed as follows :-

$$E_{I\lambda} = E_{R\lambda} + E_{A\lambda} + E_{T\lambda}$$

$E_{I\lambda}$ = Incident Energy
$E_{R\lambda}$ = Reflected Energy
$E_{A\lambda}$ = Absorbed energy
$E_{T\lambda}$ = Transmitted energy