

Module 1: Gas Power Cycles

Introduction

An important application of thermodynamics is the analysis of power cycles through which the energy absorbed as heat can be continuously converted into mechanical work. A thermodynamic analysis of the heat engine cycles provides valuable information regarding the design of new cycles or for improving the existing cycles.

The purpose of a thermodynamic cycle is either to produce power, or to produce refrigeration/pumping of heat. Therefore, the cycles are broadly classified as follows:

1. Heat engine or power cycles.
2. Refrigeration/heat pump cycles.

Any thermodynamic cycle is essentially a closed cycle in which, the working substance undergoes a series of processes and is always brought back to the initial state. However, some of the power cycles operate on open cycle. It means that the working substance is taken into the unit from the atmosphere at one end and is discharged into the atmosphere after undergoing a series of processes at the other end.

Analysis of Cycles

In air standard analysis, air is considered as the working medium. The analysis is carried out with the following assumptions.

Assumptions

1. The working substance consists of a fixed mass of air and behaves as a perfect gas. The closed system is considered which under goes a cycle process. Therefore, there are no intake or exhaust process.
2. The combustion process is replaced by an equivalent heat addition process form an external source. Thus there is no change in the chemical equilibrium of the working fluid and also composition.
3. There is no exhaust process; this is replaced by an equivalent heat rejection process.
4. Compression and expansion processes in the cycle are considered as reversible adiabatic process.
5. The specific heats C_p and C_v of air remains constant and does not vary with temperature.

Carnot Cycle

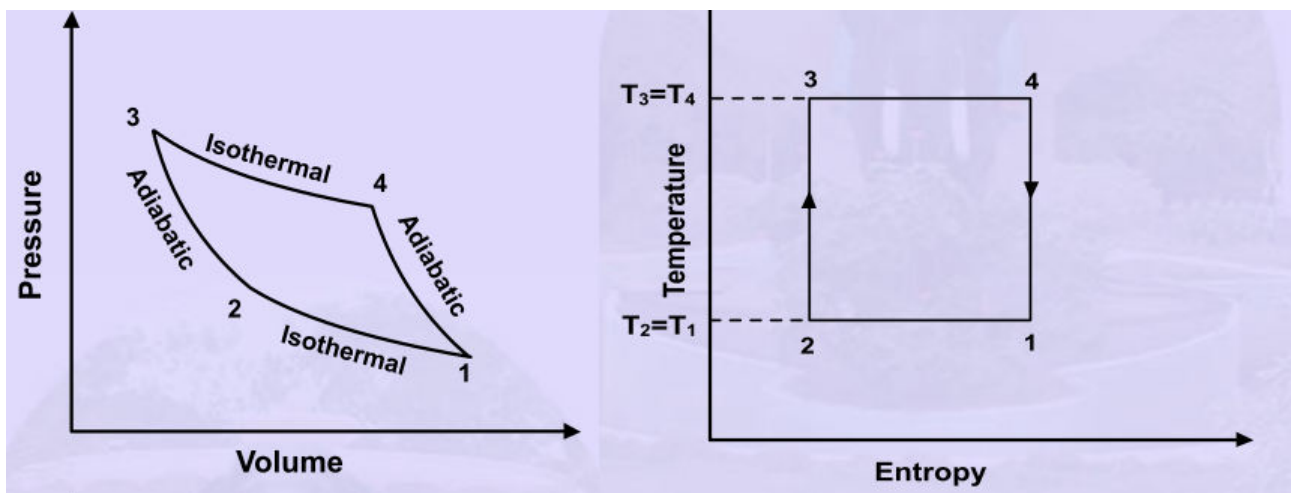


Figure 1: P-V and T-S diagram of Carnot Cycle

The T-s and p-v diagrams for a Carnot power cycle are shown in Fig.1. The cycle consists of two reversible adiabatic and two reversible isothermal processes, The working of the cycle is as follows:

Process 1-2: Reversible isothermal heat rejection of the working substance from state 1 to state 2.

Process 2-3: Isentropic compression of the working substance from state 2 to state 3. During this process work is done on the working substance by the surroundings.

Process 3-4: Reversible isothermal heat addition of the working substance from state 3 to state 4.

Process 4-1: Isentropic expansion of the working substance so that it comes back to its initial state. During this process work is done by the working substance on the surroundings.

Expression for Thermal Efficiency

From the thermodynamics, for unit mass of air

Heat supplied from point 3 to 4

$$p_3 v_3 \ln \frac{v_4}{v_3} \quad (1)$$

Heat rejected from point 1 to 2

$$p_1 v_1 \ln \frac{v_2}{v_1} \quad (2)$$

From characteristic gas equation we can write $p_3 v_3 = RT_{max}$ and $p_1 v_1 = RT_{min}$

Substituting in the above equations, we get

$$RT_{min} \ln \frac{v_2}{v_1}$$

$$RT_{max} \ln \frac{v_4}{v_3}$$

So the Work-done during the cycle is given by $W = \text{Heat supplied} - \text{Heat rejected}$

$$W = RT_{max} \ln \frac{v_4}{v_3} - RT_{min} \ln \frac{v_1}{v_2} \quad (3)$$

As the compression ratio

$$r_c = \frac{v_4}{v_3} = \frac{v_1}{v_2}$$

Substituting the above in the Work-done equation, we get

$$W = RT_{max} \ln r - RT_{min} \ln r$$

Carnot Thermal Efficiency is given by = $\frac{\text{Workdone}}{\text{Heat Supplied}}$

So, Mathematically it can written as,

$$\eta_{th} = \frac{RT_{max} \ln r - RT_{min} \ln r}{RT_{max} \ln r} \rightarrow \frac{T_{max} - T_{min}}{T_{max}} \quad (4)$$

Carnot cycle can be executed in a closed system (a piston and cylinder device or in a steady flow device. It can be seen that the thermal efficiency depends only on two temperatures T_{max} and T_{min} and is independent of working substance. The Carnot cycle is the most efficient cycle that can be executed between a heat source at temperature T_{max} and a heat sink at temperature T_{min} . But reversible isothermal heat transfer process is difficult to achieve in practice, because, it would require very large heat exchangers and it would take a very long time (a power cycle in a typical engine has to be completed in a fraction of a second). Therefore it is not practical to build an engine that would operate on a cycle that closely approximates a Carnot cycle.

The real value of the Carnot cycle comes from the fact that it is used as a standard against which the actual or other ideal power cycles are compared. It can be seen from the equation that the thermal efficiency the

Carnot power cycle increases with increase in T_{max} and with decrease in T_{min} . Hence in actual or other ideal cycles attempts are made in increasing the average temperature at which heat is supplied or by decreasing the average temperature at which heat is rejected. It should also be noted that the source and sink temperatures that can be used in practice have their limitations. The highest temperature in the cycle is limited by the maximum temperature the components of the engine can withstand and the lowest temperature is limited by the temperature of the cooling medium used in the cycle such as the atmospheric air, ocean, lake or a river.

Air Standard Otto Cycle

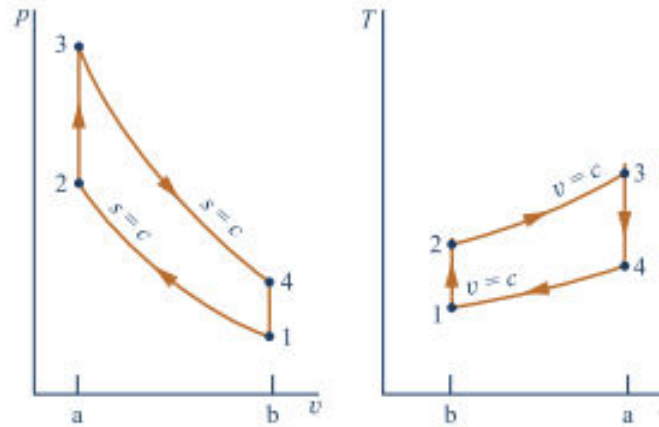


Figure 2: P-V and T-S diagram of Otto Cycle

Otto cycle is the ideal cycle for spark ignition engines. The cycle is named after Nikolaus A Otto, a German who built a four – stroke engine in 1876 in Germany using the cycle proposed by Frenchman Beau de Rochas in 1862. The p–V and T–s diagrams for an Otto cycle are shown in Fig.2. The cycle consists of the following processes:

Process 1-2: Isentropic compression of air from state 1 to state 2. During this process. Work is done on air by the surroundings.

Process 2-3: Constant volume of heating of air from state 2 till the maximum permissible temperature is reached.

Process 3–4: Isentropic expansion of air from state 3 to state 4. During this process work is done by air on the surroundings.

Process 4–1: Constant volume cooling of air till the air comes back to its original state.

Expression for Thermal Efficiency and Work output

No heat is added during the isentropic compression processes 1-2 and 3-4.

Heat added during the constant volume heating process 2-3 is given by,

$$Q_H = c_v * (T_3 - T_2) \quad (5)$$

Heat rejected during the constant volume heating process 4-1 is given by,

$$Q_L = c_v * (T_1 - T_4) \quad (6)$$

Work done during the cycle is given by,

$$W = Q_H - Q_L = c_v(T_3 - T_2) - c_v(T_4 - T_1)$$

Thermal Efficiency is given by, $\eta_{th} = \frac{W}{Q_H}$

$$\eta_{th} = \frac{c_v(T_3 - T_2) - c_v(T_4 - T_1)}{c_v(T_3 - T_2)} \quad (7)$$

Further simplified as,

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$\eta_{th} = 1 - \frac{1}{\left(\frac{T_3 - T_2}{T_4 - T_1}\right)} \quad (8)$$

The equation 8, gives the expression for thermal efficiency of the Otto cycle in terms of the temperatures at the salient points of the cycles. It is possible to express the net work output and thermal efficiency of the Otto cycle in terms of compression ratio. So, for the isentropic processes 1-2 and 3-4, we can write,

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{(\gamma-1)} \quad \text{and} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{(\gamma-1)}$$

But $v_2 = v_3$ and $v_1 = v_4$ Therefore we can write,

$$\frac{(T_3 - T_2)}{(T_4 - T_1)} = \frac{(T_4 \left(\frac{v_1}{v_2}\right)^{(\gamma-1)} - T_1 \left(\frac{v_1}{v_2}\right)^{(\gamma-1)})}{(T_4 - T_1)} = \frac{(T_4 - T_1)}{(T_4 - T_1)} \left(\frac{v_1}{v_2}\right)^{(\gamma-1)}$$

$$\frac{(T_3 - T_2)}{(T_4 - T_1)} = \left(\frac{v_1}{v_2}\right)^{(\gamma-1)}$$

So the equation 8 for thermal efficiency can be written as

$$\eta_{th} = 1 - \frac{1}{\left(\frac{v_1}{v_2}\right)^{(\gamma-1)}} = 1 - \frac{1}{r_c^{(\gamma-1)}} \quad (9)$$

From the above equation, it can be observed that the efficiency of the Otto cycle is mainly the function of compression ratio for the given ratio of C_p and C_v . If we plot the variations of the thermal efficiency with increase in compression ratio for different gases, the curves are obtained as shown in Fig.3, beyond certain values of compression ratios, the increase in the thermal efficiency is very small, because the curve tends to be asymptotic. However, practically the compression ratio of petrol engines is restricted to maximum of 9 or 10 due to the phenomenon of knocking at high compression ratios.

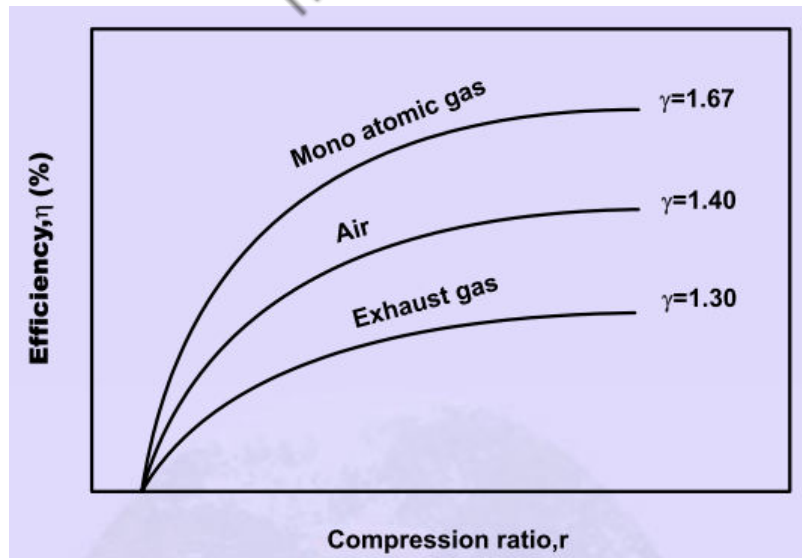


Figure 3: Variation of thermal efficiency with compression ratio

Expression for Mean effective pressure

Generally it is defined as the ratio of new workdone to the displacement volume of the piston. Let us consider unit kg of working substance, Work done is given by,

$$W = c_v(T_3 - T_2) - c_v(T_4 - T_1) \quad (10)$$

Displacement volume is given by,

$$\begin{aligned}
 v_s &= (v_1 - v_2) \\
 v_s &= v_1 \left(1 - \frac{1}{r}\right), \quad \text{But, } v_1 = \frac{RT_1}{p_1} \\
 v_s &= \frac{RT_1}{p_1} \left(1 - \frac{1}{r}\right) \\
 v_s &= \frac{c_v(\gamma - 1)T_1}{p_1} \left(\frac{r - 1}{r}\right) \quad \text{Since, } R = c_v(\gamma - 1)
 \end{aligned} \tag{11}$$

Mean effective pressure is given by;

$$\begin{aligned}
 p_m &= \frac{(T_3 - T_2) - (T_4 - T_1)}{\frac{c_v(\gamma - 1)T_1}{p_1} \left(\frac{r - 1}{r}\right)} \\
 p_m &= \frac{1}{c_v(\gamma - 1)} \frac{p_1}{T_1} \frac{r}{r - 1} [(T_3 - T_2) - (T_4 - T_1)]
 \end{aligned} \tag{12}$$

As $T_2 = T_1(r)^{\gamma-1}$. Let Pressure ratio is $r_p = \frac{p_3}{p_2} = \frac{T_3}{T_2}$ and $T_3 = \frac{p_3}{p_2}T_2 = r_p T_2 = r_p T_1(r)^{\gamma-1}$

$$\begin{aligned}
 \frac{T_4}{T_3} &= \left(\frac{v_3}{v_4}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} \\
 T_4 &= r_p T_1(r)^{\gamma-1} * \left(\frac{1}{r}\right)^{\gamma-1} \\
 T_4 &= r_p T_1
 \end{aligned} \tag{13}$$

So the mean effective pressure is given by

$$\begin{aligned}
 p_m &= \frac{p_1 r}{c_v(r - 1)(\gamma - 1)} [(r_p r^{\gamma-1} - r^{\gamma-1}) - (r_p - 1)] \\
 p_m &= p_1 r \left[\frac{(r^{\gamma-1}(r_p - 1) - (r_p - 1))}{c_v(r - 1)(\gamma - 1)} \right] \\
 p_m &= p_1 r \left[\frac{(r^{\gamma-1} - 1)(r_p - 1)}{c_v(r - 1)(\gamma - 1)} \right]
 \end{aligned} \tag{14}$$

The above expression is the equation for Mean effective pressure in terms of compression ratio r , pressure ratio r_p and specific heat ratio γ

Air Standard Diesel Cycle

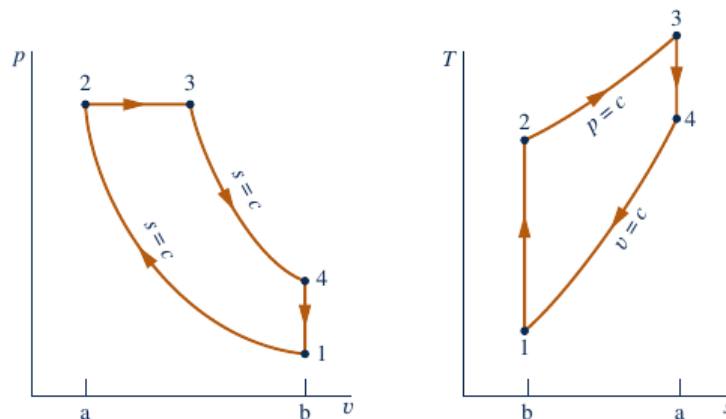


Figure 4: P-V and T-S diagram of Diesel Cycle

The diesel cycle is the ideal cycle for compression ignition engines (CI engines). CI engine was first proposed by Rudolph Diesel in 1890. The diesel engine works on the principle of compression ignition. In such an engine, only air is compressed and at the end of the compression process, the fuel is sprayed into the engine cylinder containing high pressure air, so that the fuel ignites spontaneously and combustion occurs. Since only air is compressed during the compression stroke, the possibility of auto ignition is completely eliminated in diesel engines. Hence diesel engines can be designed to operate at much higher compression ratios (between 12 and 24). Also another benefit of not having to deal with auto ignition is that fuels used in this engine can be less refined (thus less expensive).

Air standard diesel cycle is a idealized cycle for diesel engines. It is as shown on P-v and T-s diagrams. The processes in the cycle are as follows:

Process 1-2: Reversible adiabatic Compression. During this process the work is done on the air by the surroundings.

Process 2-3: Constant pressure heat addition till the maximum permissible temperature is reached.

Process 3-5: Reversible adiabatic Expansion. During this process work is done by the air on the surroundings.

Process 4-1: Constant volume heat rejection so that air comes back to its original state to complete the cycle.

Expression for Work output and Thermal Efficiency

Consider unit kg of working fluid. Since the compression and expansion processes are reversible adiabatic processes, we can write,

$$\begin{aligned} \text{Heat supplied} \quad Q_H &= c_p(T_3 - T_2) \\ \text{Heat Rejected} \quad Q_L &= c_v(T_4 - T_1) \end{aligned}$$

Work done during the cycle is given by,

$$\begin{aligned} W &= Q_H - Q_L \\ W &= c_p(T_3 - T_2) - c_v(T_4 - T_1) \end{aligned}$$

Now, we can write Thermal efficiency as,

$$\begin{aligned} \eta_{th} &= \frac{W}{Q_H} \\ \eta_{th} &= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)} \\ \eta_{th} &= 1 - \frac{1}{\gamma} \left(\frac{T_4 - T_1}{T_3 - T_2} \right) \end{aligned} \quad (15)$$

The above expression gives the equation for thermal efficiency in terms of temperatures and ratio of specific heat γ . We can get the expression for thermal efficiency in terms of compression ratio r_c , cut-off ratio ρ . Now expressing all the Temperatures in terms of T_1, γ and r_c , we get.,

$$\begin{aligned} T_2 &= T_1 r_c^{\gamma-1} \\ \text{such that, } r &= \frac{v_1}{v_2} \text{ also cut-off ratio } \rho = \frac{v_3}{v_2} = \frac{T_3}{T_2} \\ T_3 &= \rho T_2 = \rho T_1 r_c^{\gamma-1} \\ \frac{T_4}{T_3} &= \left(\frac{v_3}{v_4} \right)^{\gamma-1} \\ T_4 &= \rho T_1 r_c^{\gamma-1} \left(\frac{v_3 v_2}{v_2 v_4} \right)^{\gamma-1} \\ T_4 &= \rho T_1 r_c^{\gamma-1} \left(\frac{\rho}{r_c} \right)^{\gamma-1} = \rho^\gamma T_1 \end{aligned}$$

Substituting the above temperature expressions in equation 23, we get

$$\eta_{th} = 1 - \frac{1}{\gamma} \frac{\rho^\gamma T_1 - T_1}{\rho r_c^{\gamma-1} T_1 - r_c^{\gamma-1} T_1}$$

$$\eta_{th} = 1 - \frac{\rho^\gamma - 1}{\gamma r_c^{\gamma-1} (\rho - 1)} \quad (16)$$

From the above equation, it is observed that the thermal efficiency of the diesel engine can be increased by increasing the compression ratio r_c , by decreasing the cut-off ratio ρ and by using a large value of γ . Since the quantity, $\frac{\rho^\gamma - 1}{\gamma(\rho - 1)}$ is always greater than unity, the efficiency of the diesel cycle is always lower than that of an Otto cycle having the same compression ratio. However, practical Diesel engines uses higher compression ratios compared to petrol engines.

Expression for Mean effective pressure

As already discussed during Otto cycle, it is defined as the ratio of new workdone to the displacement volume of the piston. Let us consider unit kg of working substance, Heat supplied during the diesel cycle is given,

$$Q_H = c_p(T_3 - T_2)$$

Heat rejected during the diesel cycle is given,

$$Q_L = c_v(T_1 - T_4)$$

Work done is given by,

$$W = Q_H - Q_L = c_p(T_3 - T_2) - c_v(T_4 - T_1) \quad (17)$$

Displacement volume is given by,

$$v_s = (v_1 - v_2)$$

$$v_s = v_1 \left(1 - \frac{1}{r_c}\right), \quad \text{But, } v_1 = \frac{RT_1}{p_1}$$

$$v_s = \frac{RT_1}{p_1} \left(1 - \frac{1}{r_c}\right)$$

$$v_s = \frac{c_v(\gamma - 1)T_1}{p_1} \left(\frac{r_c - 1}{r_c}\right) \quad \text{Since, } R = c_v(\gamma - 1) \quad (18)$$

Mean effective pressure is given by;

$$p_m = \frac{\gamma(T_3 - T_2) - (T_4 - T_1)}{\frac{c_v(\gamma - 1)T_1}{p_1} \left(\frac{r_c - 1}{r_c}\right)}$$

$$p_m = \frac{1}{c_v(\gamma - 1)} \frac{p_1}{T_1} \frac{r_c}{r_c - 1} [\gamma(T_3 - T_2) - (T_4 - T_1)] \quad (19)$$

As $T_2 = T_1(r_c)^{\gamma-1}$, if Cut-off ratio $\rho = \frac{v_3}{v_2}$, r_e is the expansion ratio $= \frac{v_4}{v_3} = \frac{v_4 v_2}{v_2 v_3}$ so expansion ratio $r_e = \frac{r_c}{\rho}$
Then, $T_3 = \frac{v_3}{v_2} T_2 = \rho T_2 = \rho T_1 (r_c)^{\gamma-1}$

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = \frac{1}{r_e^{\gamma-1}} = \left(\frac{\rho}{r_c}\right)^{\gamma-1}$$

$$T_4 = \rho T_1 (r_c)^{\gamma-1} * \left(\frac{\rho}{r_c}\right)^{\gamma-1}$$

$$T_4 = \rho^\gamma T_1 \quad (20)$$

So the mean effective pressure is given by

$$p_m = \frac{p_1 r_c}{c_v (r_c - 1) (\gamma - 1)} [\gamma (\rho r_c^{\gamma-1} - r_c^{\gamma-1}) - (\rho^\gamma - 1)]$$

$$p_m = p_1 r_c \left[\frac{\gamma r_c^{\gamma-1} (\rho - 1) - (\rho^\gamma - 1)}{c_v (r_c - 1) (\gamma - 1)} \right] \quad (21)$$

The above expression is the equation for Mean effective pressure in terms of compression ration r, cut-off ratio ρ and specific heat ratio γ

Difference between Actual Diesel and Otto cycles

Otto Cycle

1. Homogeneous mixture of fuel and air formed in the Carburettor is supplied to the engine cylinder.
2. Ignition is initiated by means of electric spark plug.
3. Power output is varied by means of throttle-valve near Carburettor, which controls the fuel-air mixture supply to the engine

Diesel Cycle

1. No carburetor is used. Air alone is supplied to the engine cylinder. Fuel is injected directly into the engine cylinder at the end of compression stroke by means of a fuel injector. Fuel-air mixture is heterogeneous.
2. No spark plug is used. Compression ratio is high and the high temperature of air ignites fuel.
3. No throttle valve is used. Power output is controlled only by means of the mass of fuel injected by the fuel injector.

Air Standard Dual-Combustion or Limited Pressure or Semi-Diesel cycle

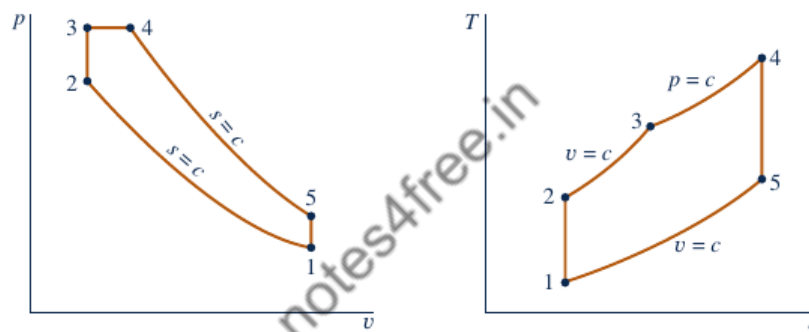


Figure 5: P-V and T-S diagram of Dual Cycle

The cycle is also called as the dual cycle, which is shown in Figure 5. Here the heat addition occurs partly at constant volume and partly at constant pressure. This cycle is a closer approximation to the behaviour of the actual Otto and Diesel engines because in the actual engines, the combustion process doesn't exactly occur at constant volume or at constant pressure but rather as in the dual cycle. Hence for most oil engines the ideal cycle is taken as the dual cycle. The P-V and T-S diagram for the dual cycle is shown above and it consists of the following processes,

Process 1-2: Reversible adiabatic compression.

Process 2-3: Constant volume heat addition.

Process 3-4: Constant pressure heat addition.

Process 4-5: Reversible adiabatic expansion.

Process 5-1: Constant volume heat rejection.

Consider unit kg of mass during the cyclic process,

Heat supplied during the process 2-3 and 3-4 is given by,

$$Q_H = c_v(T_3 - T_2) + c_p(T_4 - T_3)$$

Heat rejected during the process 5-1 is given by,

$$Q_L = c_v(T_1 - T_5)$$

Thermal efficiency of the cycle can be obtained by the equation

$$\eta_{th} = \frac{\text{Work done}}{\text{Heat Supplied}}$$

$$\begin{aligned}\eta_{th} &= \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \\ \eta_{th} &= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)} \\ \eta_{th} &= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}\end{aligned}\quad (22)$$

The efficiency of the cycle can be expressed in terms of the following ratios, compression ratio $r_c = \frac{v_1}{v_2}$, Expansion ratio $r_e = \frac{v_5}{v_4}$, Cut-off ratio $\rho = \frac{v_4}{v_3}$ and pressure ratio $r_p = \frac{p_3}{p_2}$

We can express, $r_c = \rho * r_e$ such that $r_e = \frac{r_c}{\rho}$

For the process 3-4

$$\begin{aligned}\rho &= \frac{v_4}{v_3} = \frac{T_4 p_3}{p_4 T_3} = \frac{T_4}{T_3} \\ T_3 &= \frac{T_4}{\rho}\end{aligned}\quad (23)$$

For the process 2-3

$$\begin{aligned}\frac{p_2 v_2}{T_2} &= \frac{p_3 v_3}{T_3} \\ T_2 &= T_3 \frac{p_2}{p_3} = \frac{T_4}{r_p \rho}\end{aligned}\quad (24)$$

For the process 1-2

$$\begin{aligned}\frac{T_2}{T_1} &= \left(\frac{v_1}{v_2}\right)^{\gamma-1} \\ T_2 &= T_1 (r_c)^{\gamma-1} \\ T_1 &= \frac{T_4}{r_p \cdot \rho \cdot r_c^{\gamma-1}}\end{aligned}\quad (25)$$

For the process 4-5

$$\begin{aligned}\frac{T_5}{T_4} &= \left(\frac{v_4}{v_5}\right)^{\gamma-1} = \frac{1}{r_e^{\gamma-1}} \\ T_5 &= T_4 \frac{\rho^{\gamma-1}}{r_c^{\gamma-1}}\end{aligned}\quad (26)$$

Substituting the values of T_1, T_2, T_3 and T_5 in the expression for the efficiency equation, we get

$$\begin{aligned}\eta_{th} &= 1 - \frac{T_4 \frac{\rho^{\gamma-1}}{r_c^{\gamma-1}} - \frac{T_4}{r_p \cdot \rho \cdot r_c^{\gamma-1}}}{\left(\frac{T_4}{\rho} - \frac{T_4}{r_p \cdot \rho} + \gamma\left(T_4 - \frac{T_4}{\rho}\right)\right)} \\ \eta_{th} &= 1 - \frac{1}{r_c^{\gamma-1}} \cdot \frac{r_p \cdot \rho - 1}{(r_p - 1) + \gamma \cdot r_p (\rho - 1)}\end{aligned}\quad (27)$$

Comparison on Otto, Diesel and Dual cycles

The important variable factors which are used as the basis for comparison of the cycles are compression ratio, peak pressure, heat rejection and the net work. In order to compare the performance of the Otto, Diesel and Dual combustion cycles, some of the variable factors must be fixed. In this section, a comparison of these three cycles is made for the same compression ratio, constant maximum pressure and temperature, same heat rejection and net work output. This analysis will show which cycle is more efficient for a given set of operating conditions.

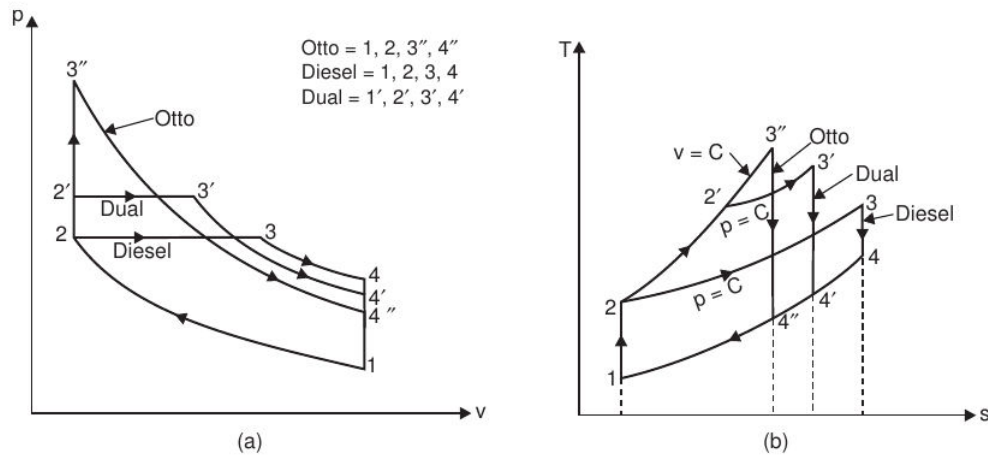


Figure 6: P-V and T-S diagram on same compression ratio and heat input

Case:1 Same compression ratio and heat input

A comparison of the cycles (Otto, Diesel and Dual) on the p-v and T-s diagrams for the same compression ratio and heat supplied is shown in the Fig.6.

We know that, $\eta = 1 - \frac{\text{Heat Supplied}}{\text{Heat Rejected}}$

Since all the cycles reject their heat at the same specific volume, process line from state 4 to 1, the quantity of heat rejected from each cycle is represented by the appropriate area under the line 4 to 1 on the T-s diagram. As is evident from the efficiency equation, the cycle which has the least heat rejected will have the highest efficiency. Thus, Otto cycle is the most efficient and Diesel cycle is the least efficient of the three cycles.

i.e., $\eta_{\text{otto}} > \eta_{\text{dual}} > \eta_{\text{diesel}}$.

Case-2: Same Maximum Temperature, pressure and Heat rejection

The air-standard Otto, Dual and Diesel cycles are drawn on common p-v and T-s diagrams for the same maximum pressure and maximum temperature, for the purpose of comparison.

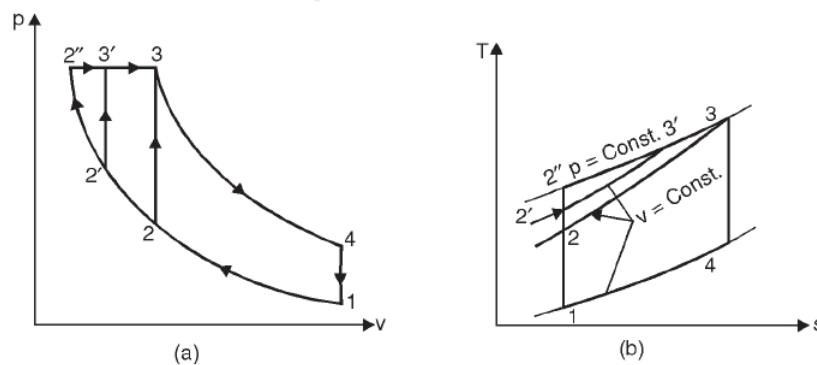


Figure 7: P-V and T-S diagram for comparison on same Maximum Temp, pressure and Heat rejection

Otto 1-2-3-4-1, Dual 1-2'-3'-4-1, Diesel 1-2''-3-4-1 as shown in Fig.7. Slope of constant volume lines on T-s diagram is higher than that of constant pressure lines.

Here the otto cycle must be limited to a low compression ratio (r_c) to fulfill the condition that point 3 (same maximum pressure and temperature) is to be a common state for all the three cycles. The construction of cycles on T-s diagram proves that for the given conditions the heat rejected is same for all the three cycles (area under process line 4-1).

Since, by definition, $\eta = 1 - \frac{\text{Heat Supplied}}{\text{Heat Rejected}} = 1 - \frac{\text{Constant}}{Q_H}$

the cycle with greater heat addition will be more efficient. From the T-S diagram

$Q_{H-Diesel} = \text{Area under } 2''-3$, $Q_{H-Dual} = \text{Area under } 2'-3'-3$, $Q_{H-Otto} = \text{Area under } 2-3$

It can be seen that $Q_{H-Diesel} > Q_{H-Dual} > Q_{H-Otto}$

and thus efficiencies are given by, $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$.

Brayton Cycle

Gas turbine engines operate on either open or closed basis. The Fig. 8a is the open mode of operation which is commonly used. In this type the atmospheric air is continuously drawn into the compressor and compressed to a high

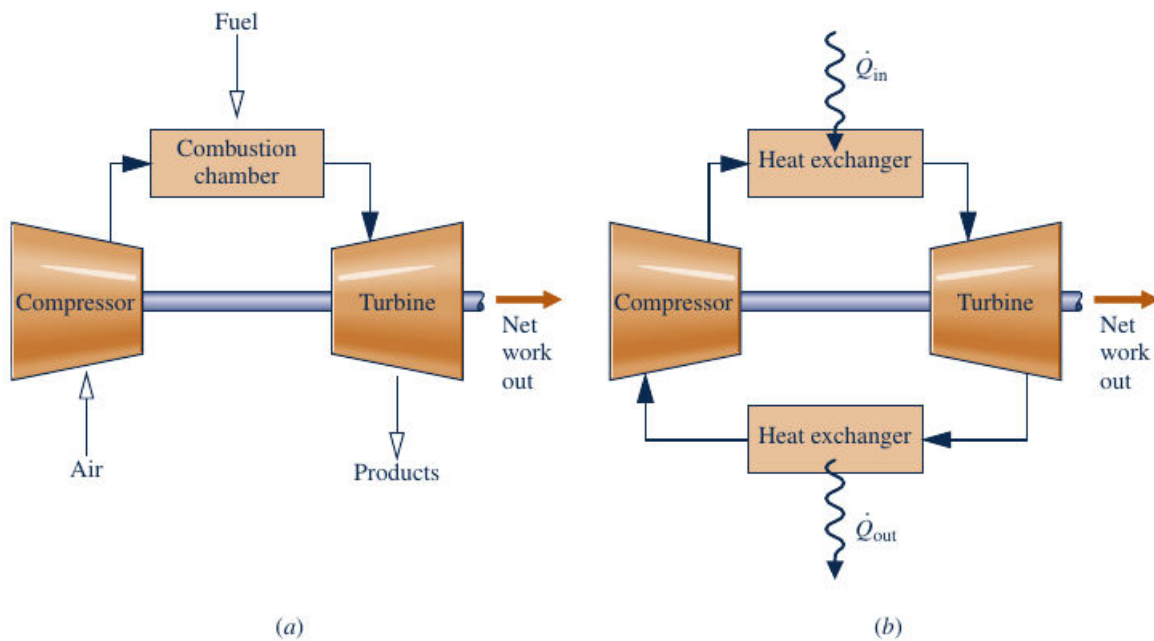


Figure 8: Simple Gasturbine (a) Open cycle (b) Close cycle

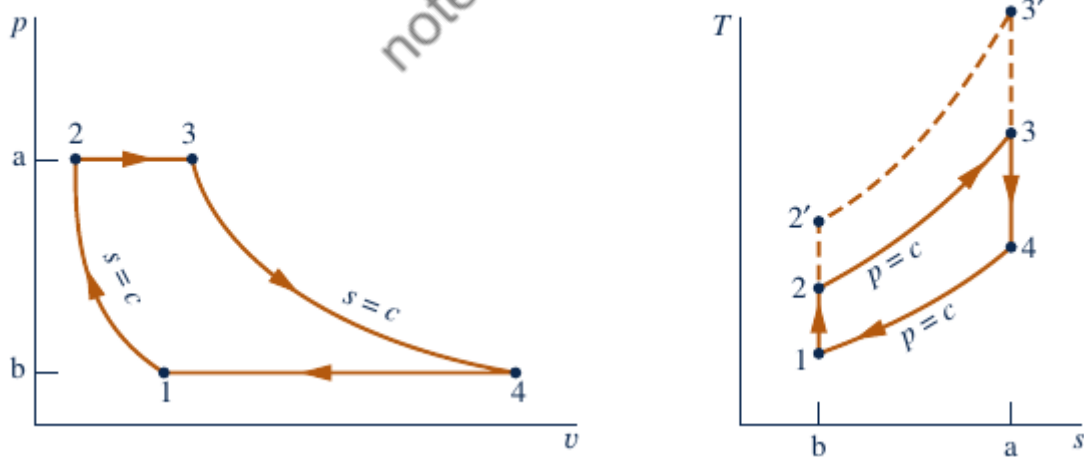
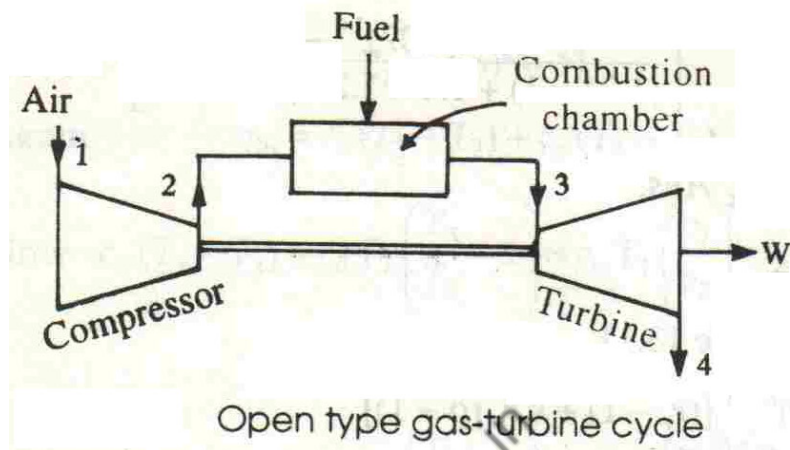


Figure 9: P-V and T-S diagram for Brayton cycle

GAS TURBINES

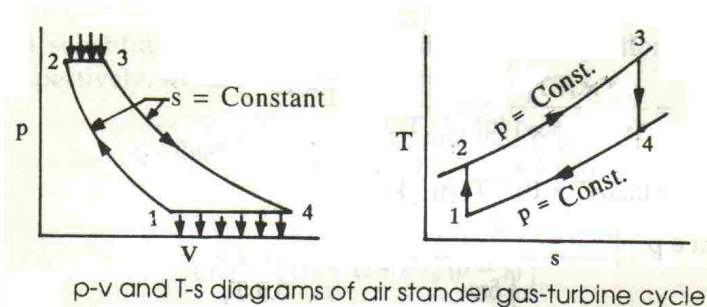
Gas Turbines and Jet Propulsion: Classification of Gas Turbines, Analysis of open cycle gas turbine cycle. Advantages and Disadvantages of closed cycle. Methods to improve thermal efficiency. Jet propulsion and Rocket propulsion.

Simple Gas Turbine Cycle



A schematic diagram of a simple gas turbine power plant is shown in figure. Air is drawn from the atmosphere into the compressor, where it is compressed reversibly and adiabatically. The relatively high pressure is then used in burning the fuel in the combustion chamber. The air fuel ratio is quite high (about 60:1) to limit the temperature of the burnt gases entering the turbine. The gases then expand isentropically in the turbine. A portion of the work obtained from the turbine is utilized to drive the compressor and the auxiliary drive, and rest of the power output is the net power of the gas turbine plant.

A gas turbine plant works using a Brayton or joule cycle. This cycle was originated by joule, a British engineer for use in a hot air reciprocating engine and later in about 1870 an American engineer George Brayton tried this cycle in a gas turbine. This cycle consists of two constant pressures and two adiabatic processes. The P-V and T-S diagrams of the cycle are as shown in figure.



Process 1 – 2: isentropic compression in the compressor

Process 2 – 3: constant pressure heat addition in the combustion chamber

Process 3 – 4: isentropic expansion in the turbine

Process 4 -1: constant pressure heat rejection in the atmosphere or cooling of air in the intercooler (closed cycle).

Expression of net work output:

We have net work output, $W_N = W_T - W_C$

Turbine work, $W_T = h_3 - h_4$
 $= C_P (T_3 - T_4)$ since the working fluid is a perfect gas

Compressor work, $W_C = h_2 - h_1$
 $= C_P (T_2 - T_1)$

$$\therefore W_N = C_P (T_3 - T_4) - C_P (T_2 - T_1)$$

Let $R = \frac{P_2}{P_1}$ = pressure ratio for compression

$t = T_3/T_1$ = Temperature ratio

$$W_N = C_P T_1 \left[\frac{T_3}{T_1} - \frac{T_4}{T_1} - \frac{T_2}{T_1} + 1 \right]$$

$$\text{We have } \frac{T_1}{P_1^{\frac{r-1}{r}}} = \frac{T_2}{P_2^{\frac{r-1}{r}}} \quad \therefore \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}}$$

$$\frac{T_4}{T_1} = \frac{T_4}{T_3} \frac{T_3}{T_1}$$

$$= \left(\frac{P_4}{P_3} \right)^{\frac{r-1}{r}} t = \left(\frac{1}{R} \right)^{\frac{r-1}{r}} t \quad \because P_1 = P_4 \quad \& \quad P_2 = P_3$$

$$\therefore W_N = C_P T_1 \left[t - \frac{t}{R^{\frac{r-1}{r}}} - R^{\frac{r-1}{r}} + 1 \right]$$

Expression for Thermal Efficiency:

$$\text{We have thermal efficiency, } \eta_{th} = \frac{W_N}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Heat added, $Q_H = h_3 - h_2 = C_P (T_3 - T_2)$
 Heat rejected, $Q_L = h_4 - h_1 = C_P (T_4 - T_1)$

$$\therefore \eta_{th} = 1 - \frac{C_P (T_4 - T_1)}{C_P (T_3 - T_2)} = 1 - \frac{T_1 \left[\frac{T_4}{T_1} - 1 \right]}{T_2 \left[\frac{T_3}{T_2} - 1 \right]}$$

$$\text{Now, } \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}} \quad \& \quad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4} \right)^{\frac{r-1}{r}} = \left(\frac{1}{R} \right)^{\frac{r-1}{r}}$$

But as $P_2 = P_3$ & $P_1 = P_4$, it follows that $\frac{T_2}{T_1} = \frac{T_3}{T_4}$ or $\frac{T_4}{T_1} = \frac{T_3}{T_1}$

$$\therefore \eta_{th} = 1 - \frac{T_1}{T_2} \quad \text{i.e.,} \quad \eta_{th} = 1 - \frac{1}{\left(\frac{T_2}{T_1} \right)} \quad \text{or} \quad \eta_{th} = 1 - \frac{1}{R^{\frac{r-1}{r}}}$$

From the above equation, it is seen that the efficiency of the air standard gas turbine cycle increases with increase in pressure ratio (R) and the type of working fluid.

Optimum Pressure Ratio for Specific Power Output

In a gas turbine cycle, T_1 is the temperature of the atmosphere and T_3 is the temperature of the burnt gases entering the turbine. Temperature T_3 is fixed by the metallurgical consideration of the turbine and temperature T_1 is fixed by the atmospheric condition. Between these two extreme values of temperature, there exists an optimum pressure ratio for which the work output of the turbine is maximum.

We have, the net work output of the turbine is,

$$W_N = C_P T_1 \left[t - \frac{t}{R^{\frac{r-1}{r}}} - R^{\frac{r-1}{r}} - 1 \right] \quad \text{--- (1)}$$

The optimum pressure ratio is obtained by differentiating the net work output w.r.t. the pressure ratio and putting the derivative equal to zero i.e., $\frac{dW_N}{dR} = 0$

$$\text{Or } \frac{d}{dR} \left[C_P T_1 \left\{ t - \frac{t}{R^{\frac{\gamma-1}{\gamma}}} - R^{\frac{\gamma-1}{\gamma}} - 1 \right\} \right] = 0$$

Differentiating with respect to R we get,

$$-t \frac{1-\gamma}{\gamma} R^{\frac{1-\gamma-\gamma}{\gamma}} - \frac{\gamma-1}{\gamma} R^{\frac{\gamma-1-\gamma}{\gamma}} = 0$$

$$\text{i.e., } -t \left(\frac{1-\gamma}{\gamma} \right) R^{\frac{1-2\gamma}{\gamma}} = \frac{\gamma-1}{\gamma} R^{-1/\gamma}$$

$$t \left(\frac{\gamma-1}{\gamma} \right) R^{\frac{1-2\gamma}{\gamma}} = \frac{\gamma-1}{\gamma} R^{-1/\gamma}$$

$$\text{or } \frac{R^{-1/\gamma}}{R^{\frac{1-2\gamma}{\gamma}}} = t \quad \text{or } R^{\frac{1-1+2\gamma}{\gamma}} = t$$

$$\text{or } R^{\frac{-1-1+2\gamma}{\gamma}} = t \quad \text{or } R^{\frac{2(\gamma-1)}{\gamma}} = t$$

$$\text{or } (R)_{opt} = t^{\frac{\gamma}{2(\gamma-1)}} \quad \text{i.e., } R_{opt} = \left[\frac{T_3}{T_1} \right]^{\frac{\gamma}{2(\gamma-1)}}$$

Substituting this value of R in the expression for W_N , we get

$$(W_N)_{opt} = C_p T_1 \left[t - \frac{t}{\left[\frac{r}{t^{2(r-1)}} \right]^{\frac{r-1}{r}}} - \left[t^{\frac{r}{2(r-1)}} \right]^{\frac{r-1}{r}} + 1 \right]$$

$$= C_p T_1 \left[t - \frac{t}{t^{\frac{1}{2}}} - t^{\frac{1}{2}} + 1 \right]$$

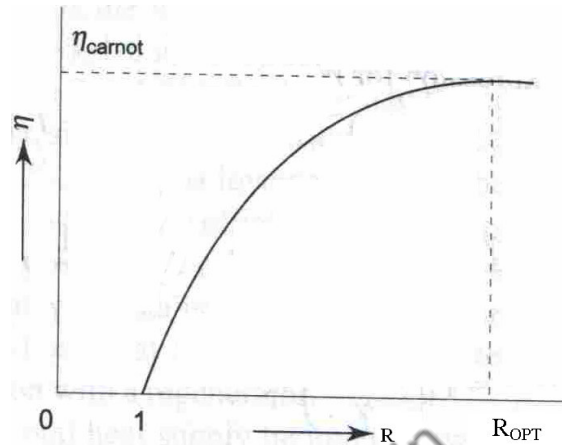
$$= C_p T_1 \left[t - t^{\frac{1}{2}} - t^{\frac{1}{2}} + 1 \right]$$

$$= C_p T_1 \left[t - 2t^{\frac{1}{2}} + 1 \right]$$

$$(W_N)_{opt} = C_p T_1 \left[t^{\frac{1}{2}} - 1 \right]$$

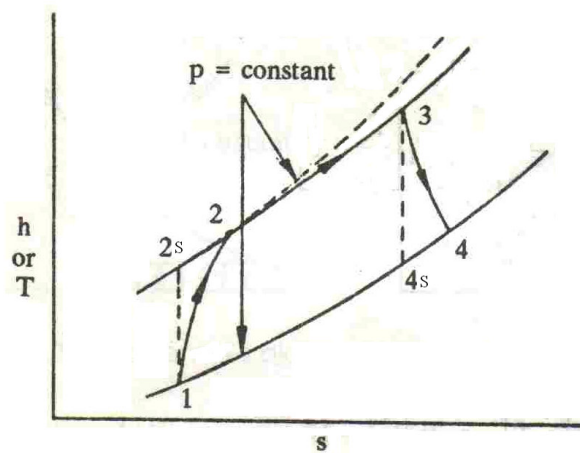
$$\eta_{th} = 1 - \frac{1}{R^{\frac{r-1}{r}}} = 1 - \frac{1}{\left[t^{\frac{r}{2(r-1)}} \right]^{\frac{r-1}{r}}}$$

$$\therefore (\eta_{th})_{opt} = 1 - \frac{1}{t^{\frac{1}{2}}}$$



Effect of pressure ratio on Brayton cycle efficiency

In an ideal gas turbine plant, the compression and expansion processes are isentropic and there is no pressure-drop in the combustion chamber. But because of irreversibilities associated in the compressor and the turbine, and the pressure-drop in the actual flow passages and combustion chamber, an actual gas turbine plant differs from ideal one. The T-S diagram of actual plant is shown in figure.



T-s diagram of the actual cycle

\therefore Compressor efficiency, $\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$

and the turbine efficiency, $\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}}$

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Classification: Gas turbine are mainly divided into two group

I Constant pressure combustion gas turbine

i) Open cycle, ii) Closed cycle

II Constant volume combustion gas turbine

In almost all the field open cycle gas turbine plants are used. Closed cycle plants were introduced at one stage because of their ability to burn cheap fuel.

Advantages and disadvantages of closed cycle over open cycle

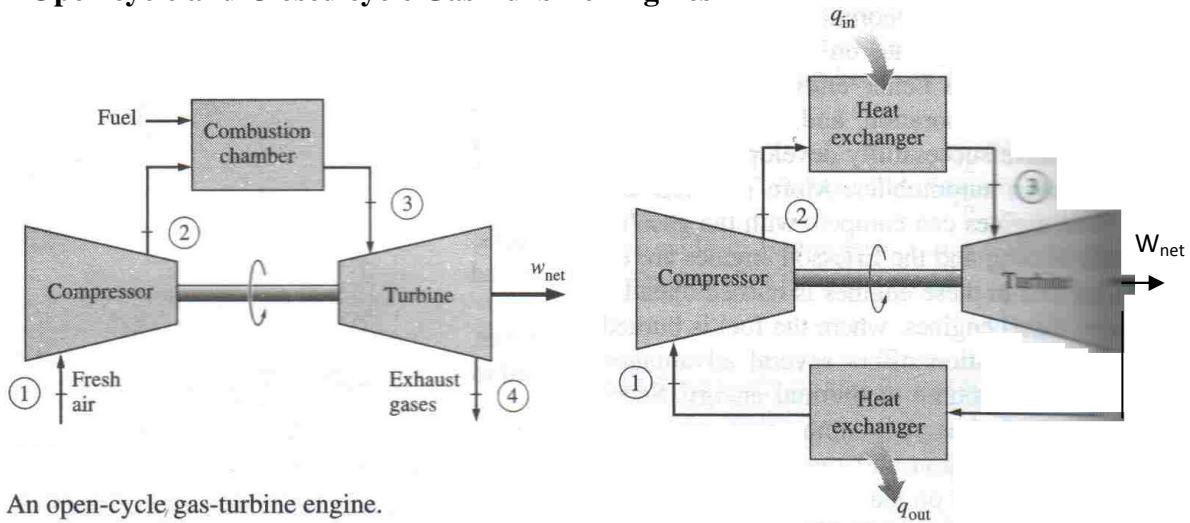
Advantages of closed cycle:

- i) Higher thermal efficiency
- ii) Reduced size
- iii) No contamination
- iv) Improved heat transmission
- v) Improved part load η
- vi) Lesser fluid friction
- vii) No loss of working medium
- viii) Greater output and
- ix) Inexpensive fuel.

Disadvantages of closed cycle:

- i) Complexity
- ii) Large amount of cooling water is required. This limits its use of stationary installation or marine use
- iii) Dependent system
- iv) The wt of the system pre kW developed is high comparatively, \therefore not economical for moving vehicles
- v) Requires the use of a very large air heater.

An Open cycle and Closed cycle Gas Turbine Engines



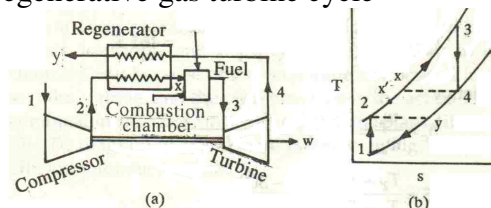
An open-cycle gas-turbine engine.

A closed-cycle gas-turbine engine

Methods to improve the performance of simple gas turbine plants

I Regenerative Gas Turbine Cycle: The temperature of the exhaust gases in a simple gas turbine is higher than the temperature of the air after compression process. The η of the Brayton cycle can be increased by utilizing part of the energy of the exhaust gas from the turbine in heating up the air leaving the compression in a heat exchanger called a regenerator, thereby reducing the amount of heat supplied from an external source and also the amount of heat rejected.

Figure shows a single stage regenerative gas turbine cycle



An ideal air standard regenerative gas turbine cycle

Air is drawn from the atmosphere into the compressor and is compressed isentropically to state 2. It is then heated at constant pressure in the regenerator to state x by the hot burnt gases from the gas turbine. Since the temperature of the air increases before it reaches the combustion chamber, less amount of fuel will be required to attain the designed turbine inlet temperature of the products of combustion. After combustion at constant pressure in the combustion chamber, the gas enters the turbine at state 3 and expands isentropically to state 4 in the turbine. It then enters the counter-flow regenerator as stated earlier, where it gives up a portion of its heat energy to the compressed air from the compressor and leaves the regenerator at state y.

In an ideal cycle, the temperature of the air leaving the regenerator is equal to the temperature of the burnt gases leaving the turbine, i.e., $T_x = T_4$. But in practice, the temperature of the air leaving the regenerator is less than T_x . In T-S diagram, T_x^1 is the temperature of the air leaving the regenerator in an actual plant.

∴ Effectiveness of a regenerator is $\varepsilon = \eta_r = \frac{T_{x^1} - T_2}{T_x - T_2}$ when C_p is constant.

In an ideal regenerator, heat loss by the burnt gases is equal to the heat gained by the air in the regenerator, i.e., $T_4 - T_y = T_x - T_2$,

Where $T_x = T_4$ and $T_y = T_2$ and hence $\eta_r = 1$

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

For an ideal regenerative gas turbine cycle,

$$Q_L = C_p (T_y - T_1) = C_p (T_2 - T_1) \text{ and}$$

$$Q_H = C_p (T_3 - T_x) = C_p (T_3 - T_4)$$

$$\therefore \eta_{th} = 1 - \frac{(T_2 - T_1)}{(T_3 - T_4)} = 1 - \frac{T_1 \left[\frac{T_2}{T_1} - 1 \right]}{T_3 \left[1 - \frac{T_4}{T_3} \right]}$$

$$\text{Since } \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}} \text{ and } \frac{T_4}{T_3} = \left(\frac{P_4}{P_3} \right)^{\frac{r-1}{r}} = \left(\frac{P_1}{P_2} \right)^{\frac{r-1}{r}} = \left(\frac{1}{R} \right)^{\frac{r-1}{r}}$$

$$\therefore \eta_{th} = 1 - \frac{1 - \left(R^{\frac{r-1}{r}} - 1 \right)}{1 - \left(1 - \frac{1}{R^{\frac{r-1}{r}}} \right)} = 1 - \frac{1}{R^{\frac{r-1}{r}}}$$

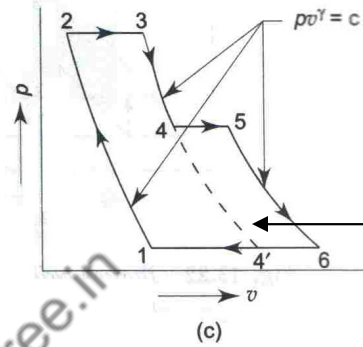
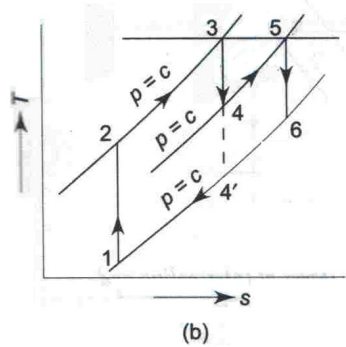
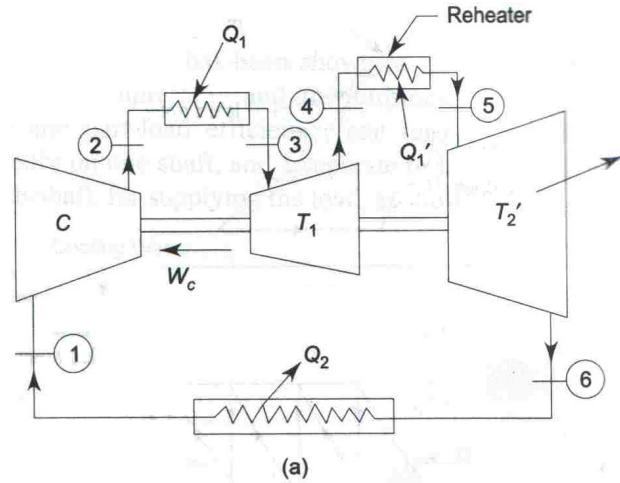
$$\text{i.e., } \eta_{th} = 1 - \frac{1}{R^{\frac{r-1}{r}}}$$

It is evident that the η_{th} of an ideal regenerative gas turbine cycle depends not only on the pressure ratio but also on the ratio of the two extreme temperatures. For a fixed ratio of T_3/T_1 , the cycle η drops with increasing pressure ratio.

In practice the regenerator is costly, heavy and bulky, and causes pressure losses which brings about a decrease in cycle η .

II Ideal regenerative cycle with inter cooling and reheat:

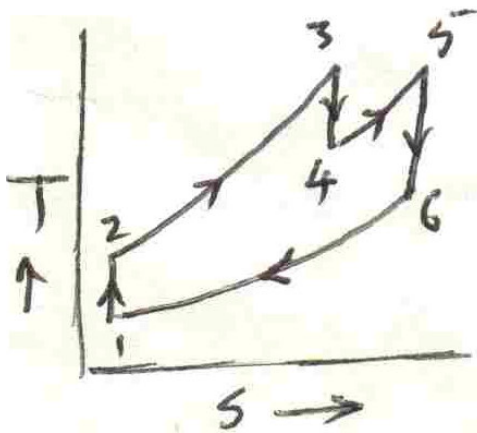
a) Gas turbine cycle with reheat and multistage expansion:



Effect of reheat on Brayton cycle

Work output of the turbine is increased by multistage expansion with reheating. In the above illustration, two-stage expansion is done in low pressure (T_1) and high pressure (T_2^1) turbines with reheating the air in between stages.

Optimum work output for a two-stage reheat cycle:



Assumptions: The air after one stage of expansion is reheated back to its original temperature i.e., $T_3 = T_5$

Let $R = \frac{P_2}{P_1}$, $R_1 = \frac{P_3}{P_4}$ and $R_2 = \frac{P_5}{P_6}$, $t = \frac{T_3}{T_1} = \text{maximum cycle temperature ratio}$

$$\begin{aligned} \therefore R_1 R_2 &= \frac{P_3}{P_4} \frac{P_5}{P_6} \quad \because P_5 = P_4 \\ &= \frac{P_2}{P_1} = R \end{aligned}$$

Net work output is $W_N = C_P (T_3 - T_4) + C_P (T_5 - T_6) - C_P (T_2 - T_1)$

$$= C_P T_1 \left[\frac{T_3}{T_1} - \frac{T_4}{T_1} + \frac{T_3}{T_1} - \frac{T_6}{T_1} - \frac{T_2}{T_1} + 1 \right]$$

we have $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} = R^{\frac{r-1}{r}}$

$$\frac{T_4}{T_1} = \frac{T_4}{T_3} \frac{T_3}{T_1} = \left(\frac{P_4}{P_3} \right)^{\frac{r-1}{r}} \frac{T_3}{T_1} = \frac{t}{R_1^{\frac{r-1}{r}}}$$

$$\frac{T_6}{T_1} = \frac{T_6}{T_5} \frac{T_5}{T_1} = \left(\frac{P_6}{P_5} \right)^{\frac{r-1}{r}} \frac{T_3}{T_1} = \frac{t}{R_2^{\frac{r-1}{r}}}$$

$$\therefore W_N = C_P T_1 \left[t - \frac{t}{R_1^{\frac{r-1}{r}}} + t - \frac{t}{R_2^{\frac{r-1}{r}}} - R^{\frac{r-1}{r}} + 1 \right]$$

but $R_2 = R/R_1$

$$\therefore W_N = C_P T_1 \left[2t - \frac{t}{R_1^{\frac{r-1}{r}}} - t \left(\frac{R_1}{R} \right)^{\frac{r-1}{r}} - R^{\frac{r-1}{r}} + 1 \right]$$

For given values of R , t and T_1 , W_N is maximum if $\frac{dW_N}{dR_1} = 0$

$$\therefore \frac{dW_N}{dR_1} = C_P T_1 \left[0 - t \cdot \frac{1-\gamma}{\gamma} \cdot R_1^{\frac{\gamma-1}{\gamma}} - \frac{t}{R^{\frac{\gamma-1}{\gamma}}} \cdot \frac{\gamma-1}{\gamma} \cdot R_1^{\frac{\gamma-1}{\gamma}} - 0 + 0 \right] = 0$$

i.e., $\frac{\gamma-1}{\gamma} t R_1^{\frac{1-2\gamma}{\gamma}} - \frac{\gamma-1}{\gamma} \frac{t}{R^{\frac{\gamma-1}{\gamma}}} R_1^{-1/\gamma} = 0$

$$R_1^{\frac{1-2r}{r}} = \frac{R_1^{-\frac{1}{\gamma}}}{R^{\frac{r-1}{r}}} \quad \text{or} \quad R^{\frac{r-1}{r}} = \frac{R_1^{-\frac{1}{\gamma}}}{R_1^{\frac{1-2r}{r}}}$$

$$R^{\frac{r-1}{r}} = R_1^{2\frac{(r-1)}{r}}$$

Or $R = R_1^2$ i.e., $R_1 = \sqrt{R}$

$$\therefore R_2 = \frac{R}{R_1} = \frac{R}{\sqrt{R}} = \sqrt{R}$$

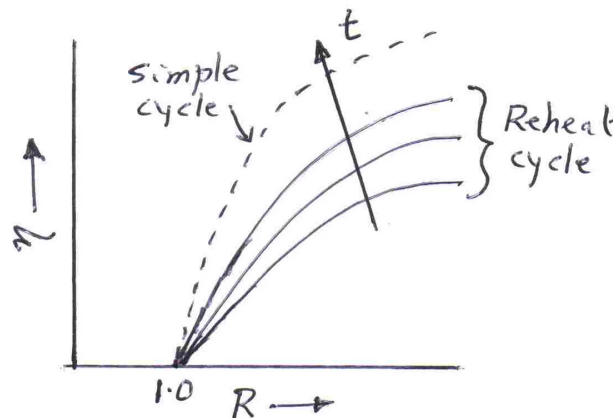
\therefore For maximum work output, $R_1 = R_2 = \sqrt{R}$

Similarly if the cycle has 'N' stages of expansion, with reheating, then for maximum work output, pressure ratio for each = (compression pressure ratio)^{1/N}

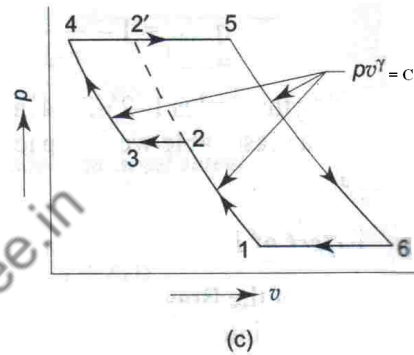
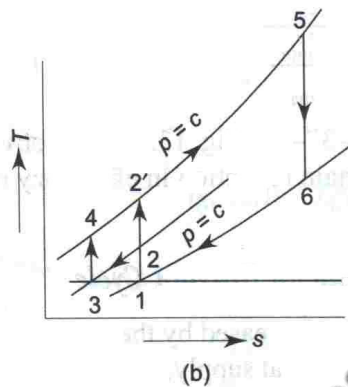
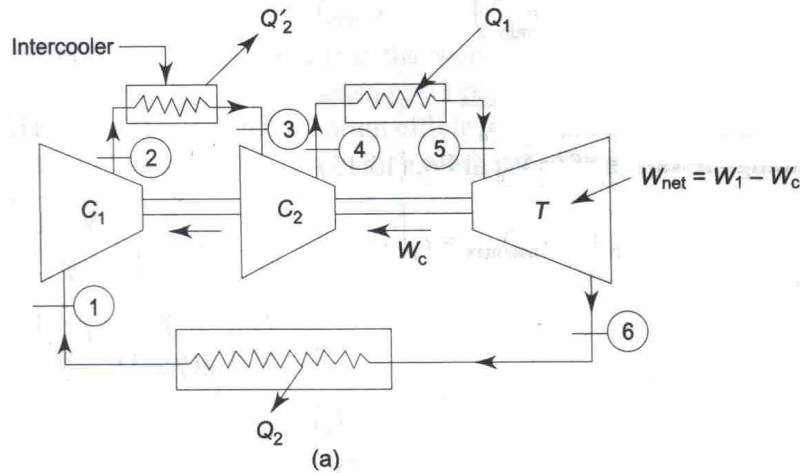
Advantages: By employing multistage expansion with reheating in between the stages, the net work output from the gas turbine cycle can be increased. This is illustrated on the T-S diagram shown for a 2-stage expansion with reheating in between the stages.

Disadvantages: But disadvantage of reheating is due to the fact that additional heat has to be supplied in order to reheat the air after each stage of expansion. This may result in a decrease in the thermal efficiency of the cycle. This is shown in figure below.

It can be seen that for a given value of t , the thermal η of the reheat cycle increases with increase in R and for a given value of R , the thermal η increases with increase in t . However, the thermal η of a reheat cycle will be less than that of a simple cycle for a given value of R .



b) Gas turbine cycle with multistage compression with inter cooling in between the stages:



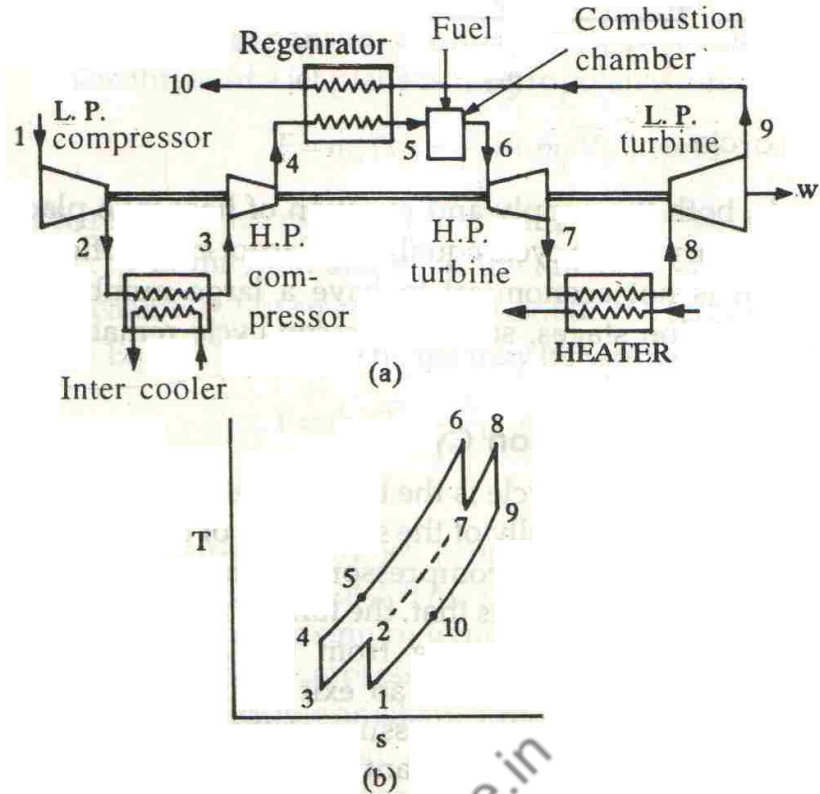
Effect of intercooling on Brayton cycle

$$\text{Let } \frac{P_2}{P_1} = R_1 \quad \frac{P_4}{P_3} = R_2 \quad \frac{P_5}{P_6} = R$$

$$\text{For maximum work output, } R_1 = R_2 = \sqrt{R}$$

The work output from a simple gas turbine cycle can be increased also by having multistage compression with inter cooling in between the stages. The effect of having two stage compression with inter cooling in between the stages is illustrated on the T-S diagram. It can be seen that, a higher work output has been achieved than that of simple cycle by an amount shown by the shaded area. The disadvantage of it is that more heat has to be supplied to heat the air than that is required for simple cycle. This may reduce the thermal η of the cycle.

c) Gas turbine cycle with two stage compression two-stage expansion and regenerator.



The thermal η of a gas turbine cycle may be improved by incorporating multistage compression with intercooling between the stages and multistage expansion with reheating between the turbines and also providing a regenerator. There is a definite saving of work due to multistage compression with intercooling arrangement between the stages. Similarly, the work output of the turbine is increased by multistage expansion with reheating. As a result, the net work of the plant increases.

The thermal efficiency of the cycle is given by $\eta_{th} = 1 - \frac{Q_L}{Q_H}$

$$= 1 - \frac{(h_{10} - h_1) + (h_2 - h_3)}{(h_6 - h_5) + (h_8 - h_7)}$$

Deviation of Practical gas turbine cycle from ideal cycle:

1) The working substance will not be air through out the cycle. Air is compressed in compressor where as the products of combustion coming out of the combustion chamber is expanded in the turbine. The value of C_p and γ will be different for expansion and heating as compared to compression process.

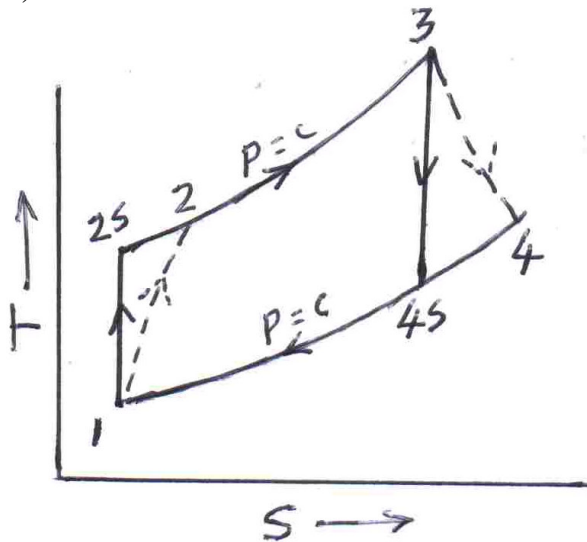
For compression, $C_p = 1.005 \text{ kJ/kg}^\circ\text{K}$, $\gamma = 1.4$

For expansion, $C_p = 1.135 \text{ kJ/kg}^\circ\text{K}$, $\gamma = 1.33$

2) There will be pressure loss in the piping connecting the various components of the plant. \therefore the pressure with which the products of combustion enters the turbine will be less than the

pressure with which air is coming out of the compressor i.e., the pressure ratio for expansion will be less than pressure ratio for compression.

3)



- 1-2s \Rightarrow isentropic work of compression
- 1-2 \Rightarrow actual work of compression
- 3-4s \Rightarrow isentropic turbine work
- 3-4 \Rightarrow Actual turbine work

In a practical gas turbine cycle the compression and expansion processes are not isentropic but adiabatic with certain amount of frictional losses. The friction losses are accounted for by defining a parameter called isentropic efficiency.

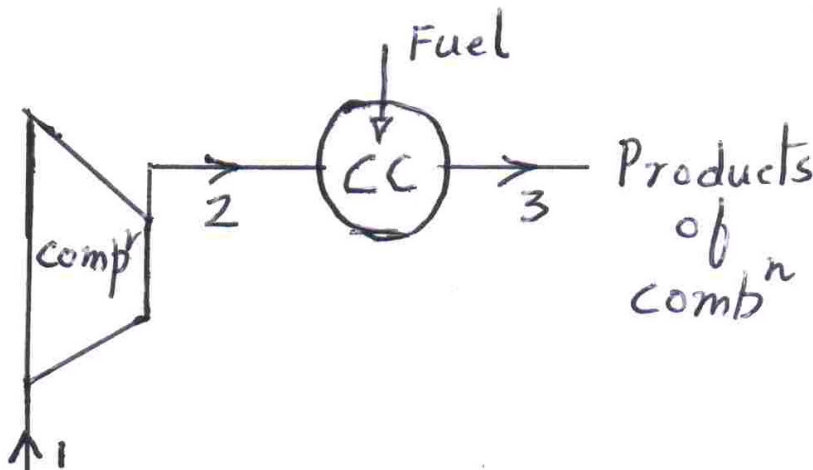
a) Isentropic η of compression (η_c)

$$\eta_c = \frac{\text{Isentropic work of compression}}{\text{Actual work of compression}} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

b) Isentropic η of turbine (η_T)

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

To determination the air-fuel ratio:



Let m_a = mass of air entering the combustion chamber
 m_f = mass of fuel entering the combustion chamber
 CV = Calorific value of fuel

Applying SFEE to combustion chamber, we get,

$$m_a h_2 + m_f CV = (m_a + m_f) h_3$$

$$\div \text{by } m_f \quad \frac{m_a}{m_f} h_2 + CV = \left(\frac{m_a}{m_f} + 1 \right) h_3$$

$$\cong \frac{m_a}{m_f} h_3 \quad \text{as } \frac{m_a}{m_f} \gg \gg \gg 1$$

$$\therefore \frac{m_a}{m_f} (h_3 - h_2) = CV$$

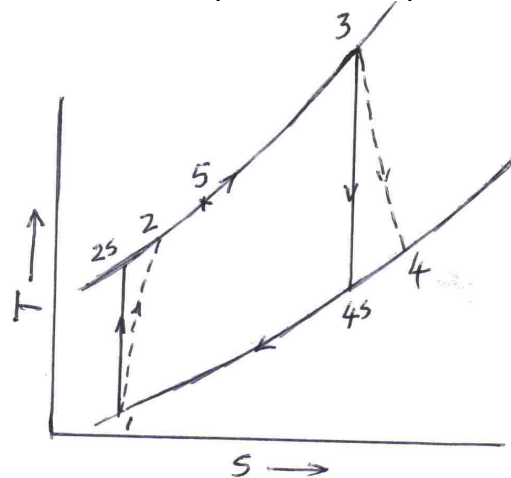
$$\text{or } \frac{m_a}{m_f} = \frac{CV}{(h_3 - h_2)} \quad \text{i.e., } \frac{m_a}{m_f} = \frac{CV}{C_p(T_3 - T_2)}$$

Problems:

- In a G.T. installation, the air is taken in at 1 bar and 15°C and compressed to 4 bar. The isentropic η of turbine and the compressor are 82% and 85% respectively. Determine (i) compression work, (ii) Turbine work, (iii) work ratio, (iv) Th. η .

What would be the improvement in the th. η if a regenerator with 75% effectiveness is incorporated in the cycle. Assume the maximum cycle temperature to be 825°K .

Solution: $P_1 = 1 \text{ bar}$ $T_1 = 288^\circ\text{K}$ $P_2 = 4 \text{ bar}$ $T_3 = 825^\circ\text{K}$
 $\eta_c = 0.85$ $\eta_t = 0.82$



Process 1-2 is isentropic i.e., $\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{r-1}{r}}$

$$\therefore T_{2s} = 288 (4)^{\frac{0.4}{1.4}} = 428.14^0 K$$

But $\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$ i.e., $0.85 = \frac{428.14 - 288}{T_2 - 288}$ $\therefore T_2 = 452.87^0 K$

Process 3-4 is isentropic i.e., $\frac{T_{4s}}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{r-1}{r}}$ $\therefore T_{4s} = 825 \left(\frac{1}{4}\right)^{\frac{0.4}{1.4}} = 554.96$

But $\eta_t = \frac{T_3 - T_4}{T_3 - T_{4s}}$ i.e., $0.82 = \frac{825 - T_4}{825 - 554.96}$ $\therefore T_4 = 603.57^0 K$

(i) Compressor work, $W_C = C_p (T_2 - T_1)$
 $= 1.005 (452.87 - 288) = 165.69 \text{ kJ/kg}$

(ii) Turbine work, $W_t = C_p (T_3 - T_4)$
 $= 1.005 (825 - 603.57) = 222.54 \text{ kJ/kg}$

(iii) Work ratio = $W_R = \frac{\text{Net work output}}{\text{Turbine work}} = \frac{W_T - W_C}{W_T} = 0.255$

(iv) Th. η , $\eta_{th} = \frac{W_{net}}{Q_H} = \frac{222.54 - 165.69}{C_p (825 - 452.87)} = \frac{56.85}{373.99}$
 $= 15.2\%$

we have effectiveness = $\frac{T_5 - T_2}{T_4 - T_2} = 0.75 = \frac{T_5 - 452.87}{603.57 - 452.87}$
 $\therefore T_5 = 565.89^0 K$

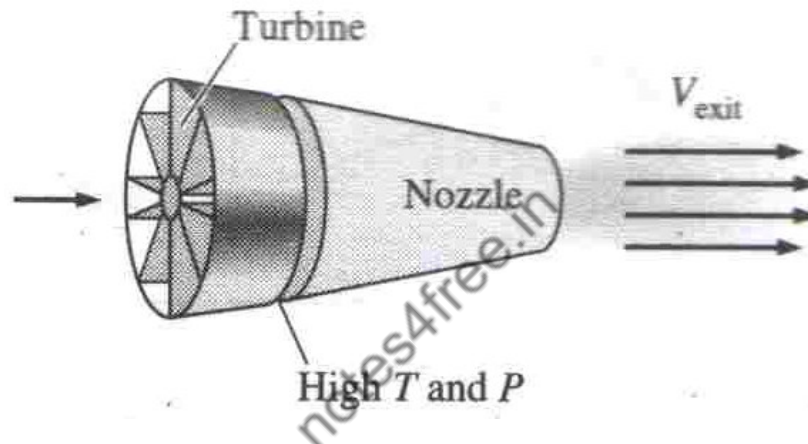
\therefore Heat supplied, $Q_H^1 = Q_{5-3} = C_p (T_3 - T_5)$
 $= 1.005 (825 - 565.89)$
 $= 260.4 \text{ kJ/kg}$

$\therefore \eta_{th} = \frac{W_T - W_C}{Q_H^1} = \frac{56.85}{260.4} = 0.218$

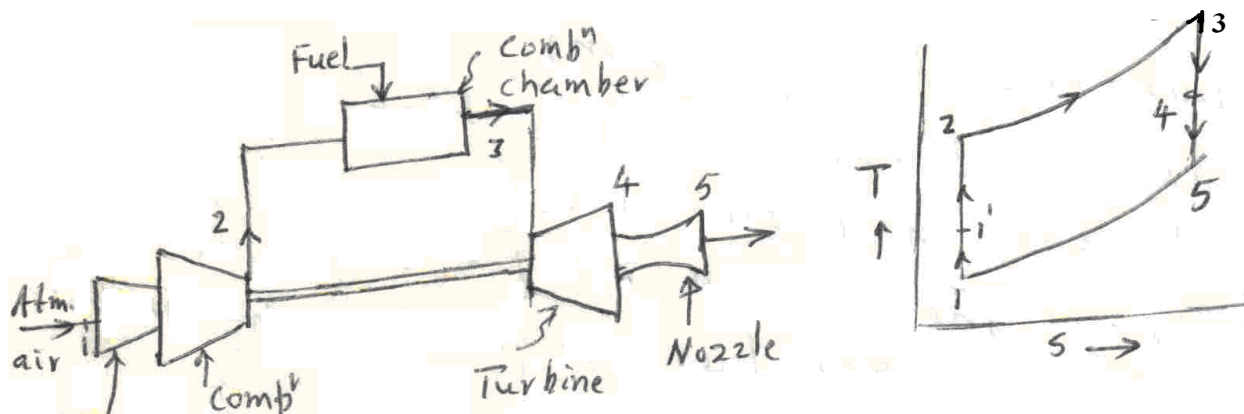
\therefore Improvement in η_{th} due to regenerator = $\frac{0.218 - 0.152}{0.152}$
 $= 0.436$ i.e., 43.6%

Ideal Jet-Propulsion Cycles:

Gas turbine engines are widely used to power air-crafts because they are light and compact and have a high power-to-weight ratio. Air craft gas turbine operates on an open cycle called a jet-propulsion cycle. The ideal jet propulsion cycle differs from the simple ideal Brayton cycle in that the gases are not expanded to the ambient pressure in the turbine. Instead, they are expanded to a pressure such that the power produced by the turbine is just sufficient to drive the compressor and the auxiliary equipment, such as a small generator and hydraulic pumps. That is, the net work output of a jet propulsion cycle is zero. The gases that exit the turbine at relatively high pressure are subsequently accelerated in a nozzle to provide the thrust to propel the air craft. Also, air craft gas turbine operate at higher pressure ratios (typically between 10-25), and the fluid passes through a diffuser first, where it is decelerated and its pressure is increased before it enters the compressor.



Air standard Jet Propulsion cycle:



An air standard Brayton or Joule cycle is the basic cycle for jet propulsion. The jet engine consists practically of the same type of components as in gas turbine plants, namely a compressor, a combustion chamber and a turbine. The only difference is that, the former also consists of an inlet diffuser, where the air entering from the atmosphere is decelerated and

slightly compressed, and an exit nozzle, where the products of combustion expand to the pressure of the surroundings with increase in relative velocity. Such a plant and the corresponding T-S diagram are shown in Fig. In this plant, the work output of the turbine is just sufficient to drive the compressor.

Air enters the diffuser from the atmosphere, is slightly compressed from state 1 to state 1'. It then enters the compressor, where it is further compressed to state 2. The compressed air then flows into the combustion chamber where it burns the fuel at constant pressure from state 2 to state 3 and the products of combustion then expand in the turbine from state 3 to state 4 developing power which is just sufficient to drive the compressor. Further expansion of the burnt gases takes place in the exit nozzle at state 5, after which the gases make an exit into the atmosphere with a very high velocity. The momentum of the exhaust gases flowing at high velocity from the nozzle result in a thrust upon the aircraft on which the engine is installed. In an actual jet propulsion plant, there is a slight pressure drop in the combustion chamber and the processes of combustion and expansion are not strictly reversible adiabatic. The thrust of a jet plane is the propulsive force i.e.,

$$F = \dot{m}_a (\bar{v}_5 - \bar{v}_1) + \dot{m}_f \bar{v}_5$$

where \dot{m}_a = mass flow rate of air,
 \bar{v}_1 & \bar{v}_5 = inlet and exit velocity of the fluid
 \dot{m}_f = mass flow rate of fuel in the combustion chamber

but since \dot{m}_f is very small, $\dot{m}_f \bar{v}_5$ term is neglected.

$$\therefore F = \dot{m}_a (\bar{v}_5 - \bar{v}_1)$$

Turbo jet:

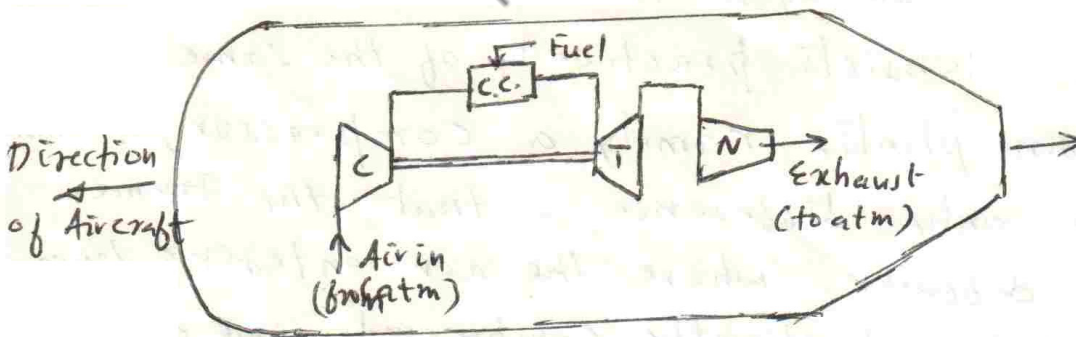


Figure shows a turbo jet unit. Power produced by the turbine is just sufficient to drive the compressor. The exhaust gases from the turbine which are at a higher pressure than atmosphere are expanded in the nozzle and a very high velocity jet is produced which provides a forward motion to the air-craft by the jet reaction (Newton's third law of motion).

At higher speeds the turbo jet gives higher propulsion efficiency. The turbo-jets are most suited to the air-crafts traveling above 800 km/hr. The overall η of a turbo jet is the product of the thermal η of the gas turbine plant and the propulsive η of the jet (nozzle).

Turbo-Propellers:

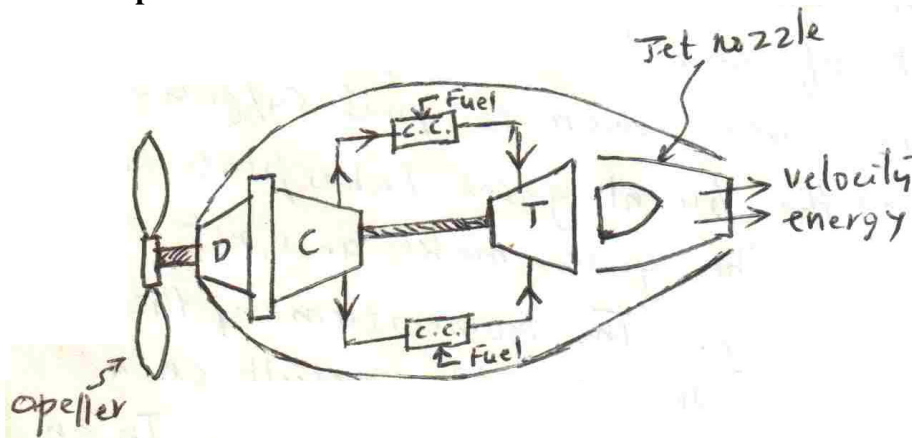


Figure shows a turbo-propeller system used in air-crafts. Here expansion takes place partly in turbine (80%) and partly (20%) in the nozzle. The power developed by the turbine is consumed in running the compressor and the propeller. The propeller and jet produced by the nozzle give forward motion to the aircraft.

It is having an added advantage over turbojet i.e., low specific weight and simplicity in design and propeller i.e., high power for take-off and high propulsion η at speeds below 600 km/hr.

Its overall η is improved by providing the diffuser before compressor, which increases the pressure rise. This pressure rise takes place due to conversion of kinetic energy of the incoming air (equal to air-craft velocity) into pressure energy by the diffuser. This type of compression is called 'ram effect'.

Ram jet: Ram jet engines have the capacity to fly at supersonic speeds. A diffuser increases the pressure of incoming air which is adequate to self ignite the fuel. In a ram jet engine the temperature of the rammed air is always above the self ignition temperature of the fuel employed. The ram-jet engine consists of a diffuser, combustion chamber and nozzle. The air enters the ram-jet plant with supersonic speed and is slowed down to sonic velocity in the supersonic diffuser, consequently the pressure suddenly increases to the formation of shock wave. The pressure of the air is further increased in the subsonic diffuser, increasing the temperature of air above the ignition temperature. The burning of the fuel takes place in the combustion chamber with the help of flame stabilization. The high pressure and high temperature gases pass through the nozzle where the pressure energy is converted to kinetic energy. The high velocity gases leaving the nozzle is a source of forward thrust to the ram-jet.

Advantages:

- i) No moving parts
- ii) Wide variety of fuels may be used
- iii) Light in weight

The major short coming of ram-jet engine is that it cannot be started of its own. It has to be accelerated to certain flight velocity by some launching device. It is always equipped with a small turbo-jet which starts the ram-jet.

Rocket Propulsion: Similar to jet propulsion, the thrust required for rocket propulsion is produced by the high velocity jet of gases passing through the nozzle. But the main difference is that in case of jet propulsion the oxygen required for combustion is taken from the atmosphere and fuel is stored whereas for rocket engine, the fuel and oxidizer both are contained in a propelling body and as such it can function in vacuum also.

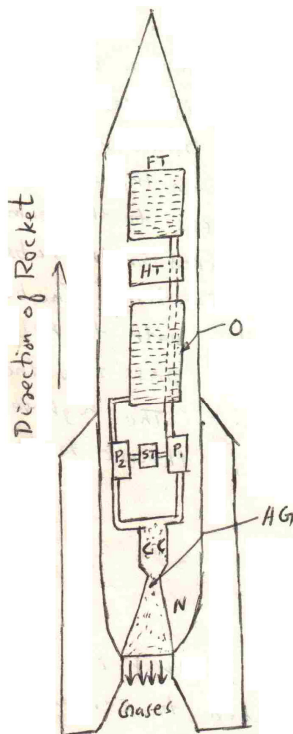
Rockets are classified as follows:

I According to the type of propellants

- i) Solid propellant rocket
- ii) Liquid propellant rocket

II According to the number of motors

- i) Single-stage rocket (consists of one rocket motor)
- ii) Multi-stage rocket (consists more than one rocket motor)



- FT = Fuel tank
- HT = Hydrogen peroxide tank
- O = Oxidizer tank
- ST = Steam turbine
- P₁, P₂ = Pumps
- C.C = Combustion chamber
- HG = Hot gases
- N = Nozzle

Fig :- Rocket

Figure shows a simple type single stage liquid propellant (the fuel and the oxidizer are commonly known as propellants) rocket. It consists of a fuel tank FT, an oxidizer tank O, two pumps P₁, P₂, a steam turbine ST and a combustion chamber C.C. The fuel tank contains alcohol and oxidizer tank contains liquid oxygen. The fuel and the oxidizer are supplied by the pumps to the combustion chamber where the fuel is ignited by electrical means. The pumps are driven with the help of steam turbine. Here the steam is produced by mixing a very concentrated hydrogen-peroxide with potassium permanganate. The products of combustion are discharged from the combustion chamber through the nozzle N. So the rocket moves in the opposite direction.

In some modified forms, this type of rockets may be used in missiles.

Uses:

1. Long range artillery
2. Signaling and fire work display
3. Jet assisted take-off
4. For satellites
5. For space ships
6. Research

Effect of Boiler Pressure (Using Mollier Diagram i.e., h-s diagram)

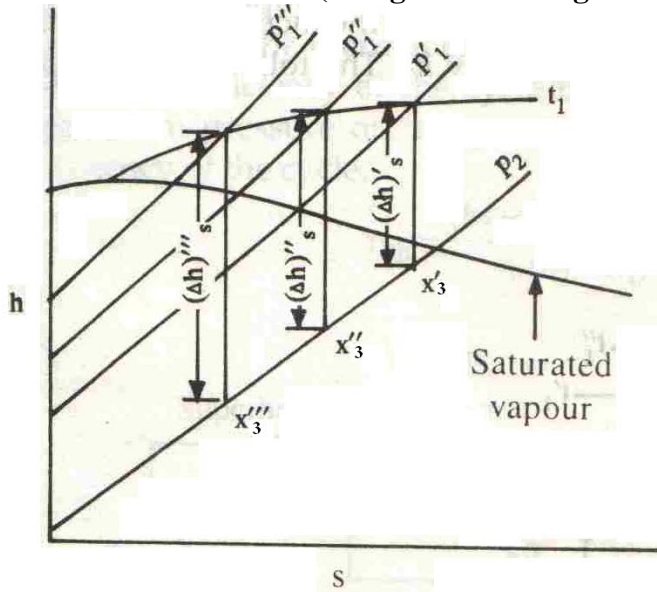


Fig. 13.5 Effect of boiler pressure

We have,

$$\eta_{th} = \frac{(h_2 - h_3) - (h_1 - h_4)}{h_2 - h_1} \text{ but } W_P \ll W_T$$

$$\therefore \eta_{th} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{(\Delta h)_s}{(h_2 - h_1)}$$

i.e., Rankine cycle η depends on h_2 , h_1 and Δh_s . From figure as $P_1''' > P_1'' > P_1'$ for the fixed maximum temperature of the steam t_1 and condenser pressure P_2 , Isentropic heat drops increases with boiler pressure i.e., from the figure therefore it is evident that as boiler pressure increases, the isentropic heat drop $(\Delta h)_s$ increases, but the enthalpy of the steam entering the turbine decreases, with the result that the Rankine η increases. But quality of the steam at the exit of the turbine suffers i.e., $x_3''' < x_3'' < x_3'$, which leads to serious wear of the turbine blades.

Effect of Super Heating (Using Mollier Diagram i.e., h-s diagram)

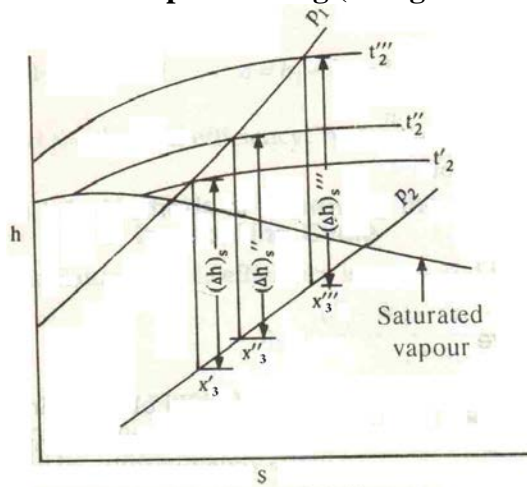
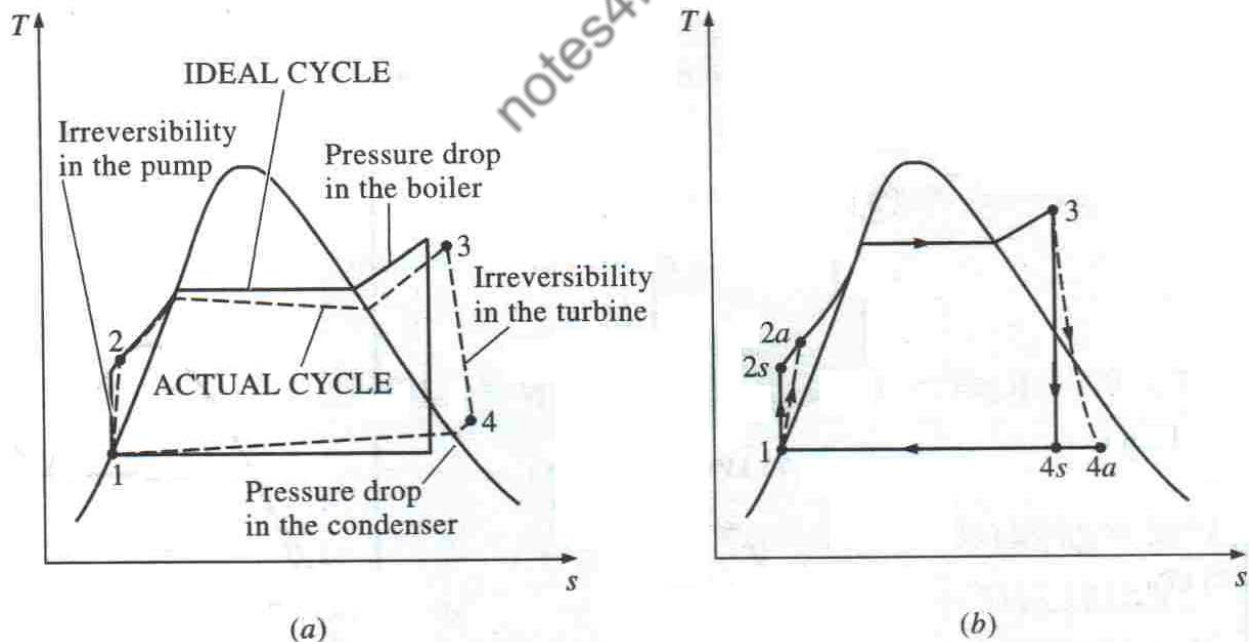


Fig. Effect of superheating

The moisture in the steam at the end of the expansion may be reduced by increasing the super heated temperature of steam t_1 . This can be seen in figure where $t_1''' > t_1'' > t_1'$, but $x_3' < x_3'' < x_3'''$. It is, therefore, natural that to avoid erosion of the turbine blades, an increase in the boiler pressure must be accompanied by super heating at a higher temperature and since this raises the mean average temperature at which heat is transferred to the steam, the Rankine η increases.

Deviation of Actual Vapour Power cycles from Ideal cycle



The actual Vapour power cycle differs from the ideal Rankine cycle, as shown in figure, as a result of irreversibilities in various components mainly because of fluid friction and heat loss to the surroundings.

Fluid friction causes pressure drops in the boiler, the condenser, and the piping between various components. As a result, steam leaves the boiler at a lower pressure. Also the pressure at the turbine inlet is lower than that at the boiler exit due to pressure drop in the connecting pipes. The pressure drop in the condenser is usually very small. To compensate these pressure drops, the water must be pumped to sufficiently higher pressure which requires the larger pump and larger work input to the pump.

The other major source of irreversibility is the heat loss from the steam to the surroundings as the steam flows through various components. To maintain the same level of net work output, more heat needs to be transferred to the steam in the boiler to compensate for these undesired heat losses. As a result, cycle efficiency decreases.

As a result of irreversibilities, a pump requires a greater work input, and a turbine produces a smaller work output. Under the ideal conditions, the flow through these devices are isentropic. The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as

$$\eta_P = \frac{W_s}{W_a} = \frac{h_{1s} - h_4}{h_1 - h_4}$$

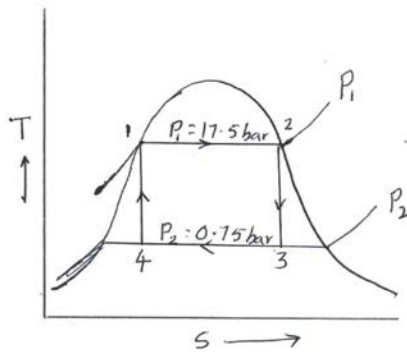
$$\text{And } \eta_t = \frac{W_a}{W_s} = \frac{h_2 - h_3}{h_2 - h_{3s}}$$

Numerical Problems:

- Dry saturated steam at 17.5 bar enters the turbine of a steam power plant and expands to the condenser pressure of 0.75 bar. Determine the Carnot and Rankine cycle efficiencies. Also find the work ratio of the Rankine cycle.**

Solution: $P_1 = 17.5 \text{ bar}$ $P_2 = 0.75 \text{ bar}$ $\eta_{\text{Carnot}} = ?$ $\eta_{\text{Rankine}} = ?$

a) Carnot cycle: At pressure 17.5 bar from steam tables,



P	t_s	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
17	204.3	871.8	1921.6	2793.4	2.3712	4.0246	6.3958
18	207.11	884.5	1910.3	2794.8	2.3976	3.9776	6.3751

For $P = 17.5 \text{ bar}$, using linear interpolation

$$\text{For } t_s, 204.3 + \frac{207.11 - 204.3}{1} \times 0.5 = 205.71^\circ\text{C}$$

$$= 478.71 \text{ K}$$

$$\text{Similarly, } h_f = 878.15 \text{ kJ/kg} \quad h_{fg} = 1915.95 \text{ kJ/kg} \quad h_g = 2794.1 \text{ kJ/kg}$$

$$S_f = 2.3844 \text{ kJ/kg}^\circ\text{K} \quad S_{fg} = 4.0011 \text{ kJ/kg}^\circ\text{K} \quad S_g = 6.3855 \text{ kJ/kg}^\circ\text{K}$$

Also at pressure 0.75 bar from steam tables

P	t_s	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
0.8	93.51	391.7	2274.0	2665.8	1.233	6.2022	7.4352
0.7	89.96	376.8	2283.3	2660.1	1.1921	6.2883	7.4804

∴ For 0.75 bar, using linear interpolation,

$$t_s = 91.74^\circ\text{C} \quad h_f = 384.25 \quad h_{fg} = 2278.65 \quad h_g = 2662.95$$

$$S_f = 1.2126 \quad S_{fg} = 6.2453 \quad S_g = 7.4578$$

$$\text{The Carnot cycle } \eta, \eta_C = \frac{T_1 - T_2}{T_1} = \frac{478.71 - 364.74}{478.71} = 0.2381$$

$$\text{Steam rate or SSC} = \frac{1}{\oint \delta W} = \frac{1}{W_T - W_P}$$

Since the expansion work is isentropic, $S_2 = S_3$

$$\text{But } S_2 = S_g = 6.3855 \text{ and } S_3 = S_{f3} + x_3 S_{fg3}$$

$$\text{i.e., } 6.3855 = 1.2126 + x_3 (6.2453) \quad \therefore x_3 = 0.828$$

$$\therefore \text{Enthalpy at state 3, } h_3 = h_{f3} + x_3 h_{fg3}$$

$$= 384.25 + 0.828 (2278.65) = 2271.63 \text{ kJ/kg}$$

$$\therefore \text{Turbine work or expansion work or positive work} = h_2 - h_3$$

$$= 2794.1 - 2271.63 = 522.47 \text{ kJ/kg}$$

Again since the compression process is isentropic

$$\text{i.e., } S_4 = S_1 = S_{f1} = 2.3844$$

$$\text{Hence } 2.3844 = S_{f4} + x_4 S_{fg4}$$

$$= 1.2126 + x_4 (6.2453) \quad \therefore x_4 = 0.188$$

$$\therefore \text{Enthalpy at state 4 is } h_4 = h_{f4} + x_4 h_{fg4}$$

$$= 384.25 + 0.188 (2278.65)$$

$$= 811.79 \text{ kJ/kg}$$

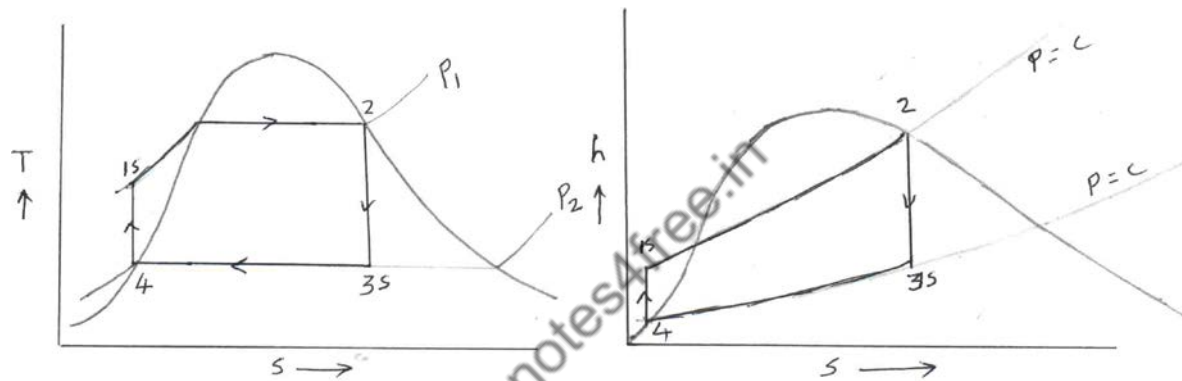
$$\therefore \text{Compression work, } = h_1 - h_4 = 878.15 - 811.79$$

$$W_P = 66.36 \text{ kJ/kg}$$

$$\therefore SSC = \frac{1}{522.47 - 66.36} = 2.192 \times 10^{-3} \text{ kg / kJ}$$

$$\text{work ratio} = r_w = \frac{\oint \delta w}{+ve \text{ work}} = \frac{W_T - W_P}{W_T} = \frac{456.11}{522.47} = 0.873$$

b) Rankine cycle:



$$\eta_R = \frac{W_T - W_P}{Q_H} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

Since the change in volume of the saturated liquid water during compression from state 4 to state 1 is very small, v_4 may be taken as constant. In a steady flow process, work $W = -v \int dp$

$$\therefore W_P = h_{1S} - h_4 = v_{fP2} (P_1 - P_2)$$

$$= 0.001037 (17.5 - 0.75) \times 10^5 \times (1/1000)$$

$$= 1.737 \text{ kJ/kg}$$

$$\therefore h_{1S} = 1.737 + 384.25 = 385.99 \text{ kJ/kg}$$

Hence, turbine work = $W_T = h_2 - h_3 = 522.47 \text{ kJ/kg}$

Heat supplied = $Q_H = h_2 - h_{1S} = 2.794.1 - 385.99 = 2408.11 \text{ kJ/kg}$

$$\therefore \eta_R = \frac{522.47 - 1.737}{2408.11} = 0.2162$$

$$\therefore SSC = \frac{1}{522.47 - 1.737} = 19204 \times 10^{-3} \text{ kg / kJ}$$

$$\text{Work ratio, } r_w = \frac{522.47 - 1737}{522.47} = 0.9967$$

2. If in problem (1), the turbine and the pump have each 85% efficiency, find the % reduction in the net work and cycle efficiency for Rankine cycle.

Solution: If $\eta_P = 0.85$, $\eta_T = 0.85$

$$W_P = \frac{W_P}{0.85} = \frac{1.737}{0.85} = 2.0435 \text{ kJ/kg}$$

$$W_T = \eta_T W_T = 0.85 (522.47) = 444.09 \text{ kJ/kg}$$

$$\therefore W_{\text{net}} = W_T - W_P = 442.06 \text{ kJ/kg}$$

$$\therefore \% \text{ reduction in work output} = \frac{520.73 - 442.06}{520.73} = 15.11\%$$

$$W_P = h_{1S} - h_4 \therefore h_{1S} = 2.0435 + 384.25 = 386.29 \text{ kJ/kg}$$

$$\therefore Q_H - h_2 - h_{1S} = 2794.1 - 386.29 = 2407.81 \text{ kJ/kg}$$

$$\therefore \eta_{\text{cycle}} = \frac{442.06}{2407.81} = 0.1836$$

$$\therefore \% \text{ reduction in cycle efficiency} = \frac{0.2162 - 0.1836}{0.2162} = 15.08\%$$

Note: Alternative method for problem 1 using h-s diagram (Mollier diagram) though the result may not be as accurate as the analytical solution. The method is as follows

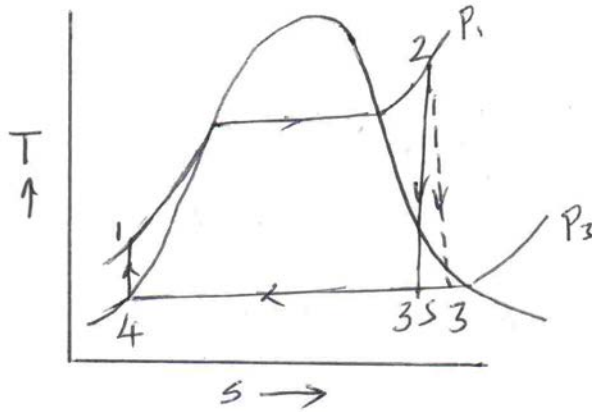
Since steam is dry saturated at state 2, locate this state at the pressure $P_2 = 17.5$ bar on the saturation line and read the enthalpy at this state. This will give the value of h_2 .

As the expansion process 2-3 is isentropic, draw a vertical line through the state 2 to meet the pressure line, $P = 0.75$ bar. The intersection of the vertical line with the pressure line will fix state 3. From the chart, find the value of h_3 .

The value of h_4 can be found from the steam tables at pressure, $P = 0.75$ bar, as $h_4 = h_{f4}$. After finding the values of h_2 , h_3 and h_4 , apply the equation used in the analytical solution for determining the Rankine cycle η and SSC.

3. Steam enters the turbine of a steam power plant, operating on Rankine cycle, at 10 bar, 300°C. The condenser pressure is 0.1 bar. Steam leaving the turbine is 90% dry. Calculate the adiabatic efficiency of the turbine and also the cycle η , neglecting pump work.

Solution:



$$P_1 = 10 \text{ bar} \quad t_2 = 300^\circ\text{C} \quad P_3 = 0.1 \text{ bar}$$

$$x_3 = 0.9 \quad \eta_t = ? \quad \eta_{\text{cycle}} = ? \quad \text{Neglect } W_p$$

From superheated steam tables,

$$\text{For } P_2 = 10 \text{ bar and } t_2 = 300^\circ\text{C}, h_2 = 3052.1 \text{ kJ/kg}, s_2 = 7.1251 \text{ kJ/kg}$$

From table A - 1, For $P_3 = 0.1 \text{ bar}$

$$t_s = 45.83^\circ\text{C} \quad h_f = 191.8 \quad h_{fg} = 2392.9$$

$$S_f = 0.6493 \quad S_{fg} = 7.5018$$

$$\begin{aligned} \text{Since } x_3 = 0.9, \quad h_3 &= h_{f4} + x_3 h_{fg3} \\ &= 191.8 + 0.9 (2392.9) \\ &= 2345.4 \text{ kJ/kg} \end{aligned}$$

Also, since process 2-3s is isentropic, $S_2 = S_{3s}$

$$\begin{aligned} \text{i.e., } 7.1251 &= S_{fg4} + x_{3s} S_{fg3} \\ &= 0.6493 + x_{3s} (7.5018) \end{aligned}$$

$$\therefore x_{3s} = 0.863$$

$$\therefore h_{3s} = 191.8 + 0.863 (2392.9) = 2257.43 \text{ kJ/kg}$$

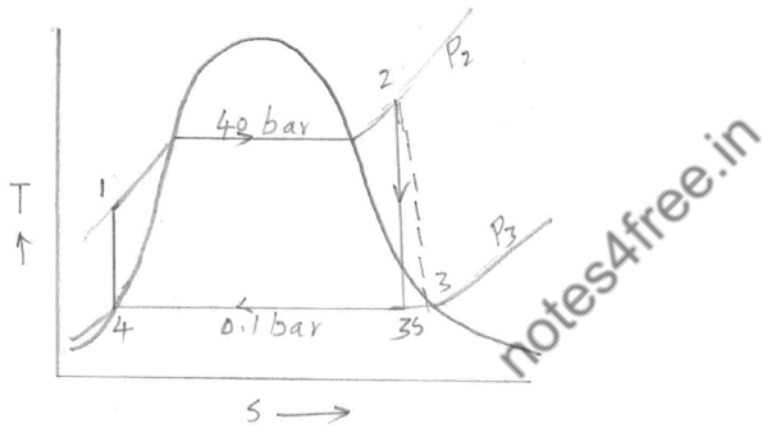
$$\therefore \text{Turbine efficiency, } \eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{3052.1 - 2345.4}{3052.1 - 2257.43} = 0.89$$

$$\eta_{\text{cycle}} = \frac{W_T}{Q_H} = \frac{h_2 - h_3}{h_2 - h_1} \quad \text{but } h_1 = 191.8 \text{ kJ/kg}$$

$$= \frac{3052.1 - 2345.4}{3052.1 - 191.8} = 0.25 \quad \text{i.e., } 25\%$$

4. A 40 mW steam plant working on Rankine cycle operates between boiler pressure of 4 MPa and condenser pressure of 10 KPa. The steam leaves the boiler and enters the steam turbine at 400°C. The isentropic η of the steam turbine is 85%. Determine (i) the cycle η (ii) the quality of steam from the turbine and (iii) the steam flow rate in kg per hour. Consider pump work.

Solution:



$$P_2 = 4 \text{ MPa} = 40 \text{ bar} \quad P_3 = 10 \text{ KPa} = 0.1 \text{ bar}$$

$$P = 40000 \text{ kW} \quad t_2 = 400^\circ\text{C} \quad \eta_t = 0.85 \quad \eta_{\text{cycle}} = ? \quad x_3 = ?$$

$$\dot{m} = ?$$

$$h_2 = h \Big|_{40 \text{ bar}, 400^\circ\text{C}} = 3215.7 \text{ kJ/kg} \quad \text{and } s_2 = 6.7733 \text{ kJ/kg-K}$$

$$h_4 = h_f \Big|_{0.1 \text{ bar}} = 191.8 \text{ kJ/kg}$$

Process 2-3s is isentropic i.e., $S_2 = S_{3s}$

$$6.7733 = 0.6493 + x_{3s} (7.5018)$$

$$\therefore x_{3s} = 0.816$$

$$\begin{aligned}\therefore h_{3S} &= h_{f3} + x_{3S} h_{fg3} = 191.8 + 0.816 (2392.9) \\ &= 2145.2 \text{ kJ/kg}\end{aligned}$$

$$\text{But } \eta_t = \frac{h_2 - h_3}{h_2 - h_{3S}} \quad \text{i.e., } 0.85 = \frac{3215.7 - h_3}{3215.7 - 2145.2}$$

$$\therefore h_3 = 2305.8 \text{ kJ/kg}$$

$$\therefore W_T = h_2 - h_3 = 3215.7 - 2305.8 = 909.9 \text{ kJ/kg}$$

$$W_P = \int v dP = 0.0010102 (40 - 0.1) 10^5 / 10^2$$

$$= 4.031 \text{ kJ/kg}$$

$$= h_1 - h_4 \quad \therefore h_1 = 195.8 \text{ kJ/kg}$$

$$(i) \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{909.9 - 4.031}{(3215.7 - 195.8)} = 29.9\%$$

$$(ii) x_3 = ? \quad \text{we have } 2305.8 = 191.8 + x_3 (2392.9) \quad \therefore x_3 = 0.88$$

$$(iii) P = \dot{m} W_{net} \quad \text{i.e., } 40000 = \dot{m} (905.87)$$

$$\therefore \dot{m} = 44.2 \text{ kg/s}$$

$$= 159120 \text{ kg/hr}$$

Ideal Reheat cycle: We know that, the efficiency of the Rankine cycle could be increased by increasing steam pressure in the boiler and superheating the steam. But this increases the moisture content of the steam in the lower pressure stages in the turbine, which may lead to erosion of the turbine blade. \therefore The reheat cycle has been developed to take advantage of the increased pressure of the boiler, avoiding the excessive moisture of the steam in the low pressure stages. In the reheat cycle, steam after partial expansion in the turbine is brought back to the boiler, reheated by combustion gases and then fed back to the turbine for further expansion.

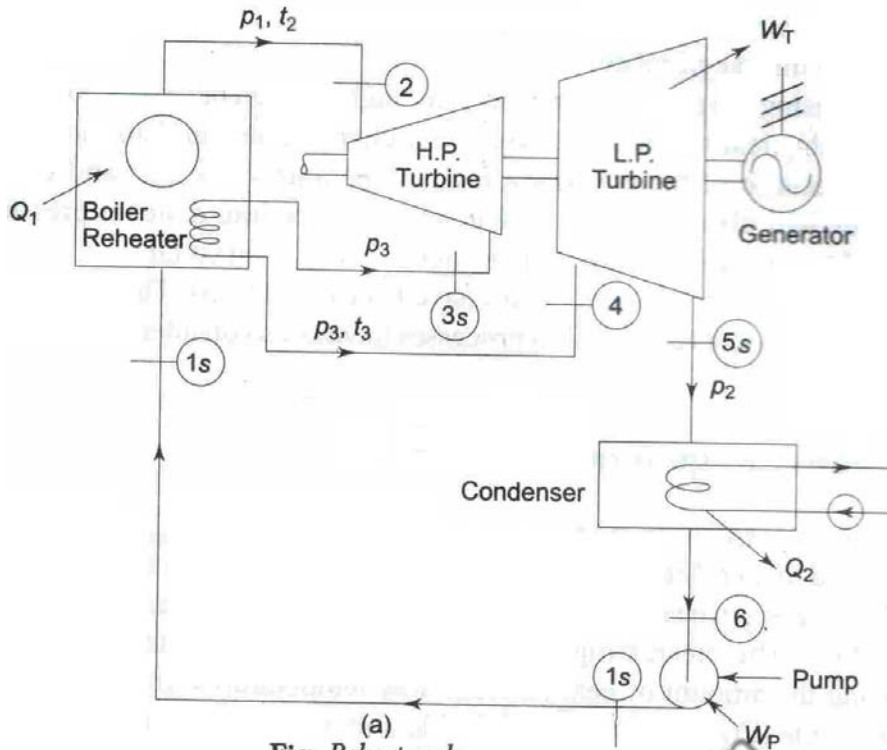
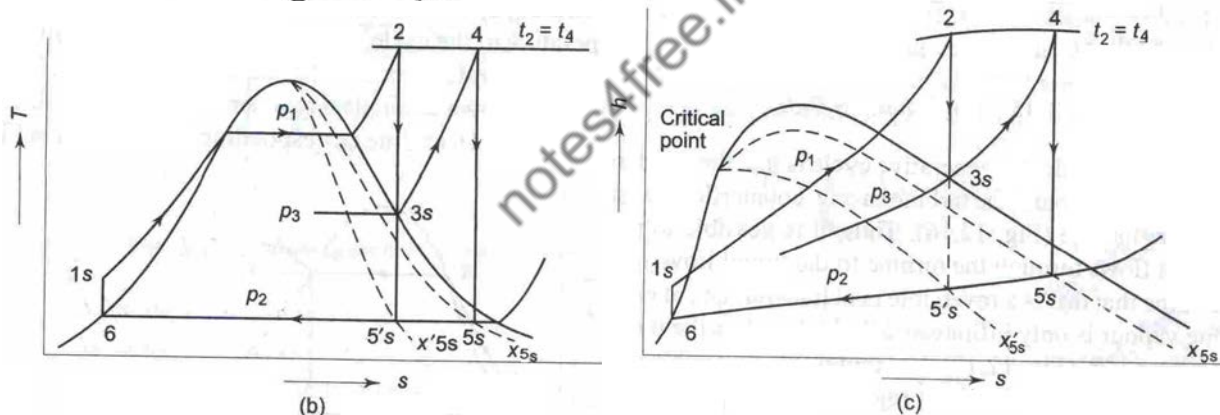


Fig. Reheat cycle



In the reheat cycle the expansion of steam from the initial state (2) to the condenser pressure is carried out in two or more steps, depending upon the number of reheats used.

In the first step, steam expands in HP turbine from state 2 to approximate the saturated vapour line (process 2-3s). The steam is then reheated (or resuperheated) at constant pressure in the boiler (or in a reheater) process 3s-4 and the remaining expansion process 4s-5 is carried out in the LP turbine.

Note: 1) To protect the reheater tubes, steam is not allowed to expand deep into the two-phase region before it is taken for reheating, because in that case the moisture particles in steam while evaporating would leave behind solid deposits in the form of scale which is difficult to remove. Also a low reheat pressure may bring down T_{m1} and hence cycle η . Again a high reheat pressure increases the moisture content at turbine exhaust. Thus reheat pressure is optimized. Optimum reheat pressure is about 0.2 to 0.25 of initial pressure.

We have for 1 kg of steam

$$Q_H = (h_2 - h_{1S}) + (h_4 - h_{3S}); \quad Q_L = h_{5S} - h_6$$
$$W_T = (h_2 - h_{3S}) + (h_4 - h_{5S}); \quad W_P = h_{1S} - h_6$$

$$\therefore \eta_R = \frac{W_T - W_P}{Q_H};$$

$$\text{Steam rate} = \frac{3600}{(W_T - W_P)} \text{ kg / kWh}$$

Since higher reheat pressure is used, W_P work is appreciable.

2) In practice, the use of reheat gives a marginal increase in cycle η , but it increases the net work output by making possible the use of higher pressures, keeping the quality of steam at turbine exhaust within a permissible limit. The quality improves from $x_{5'S}$ to x_{5S} by the use of reheat.

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Ideal Regenerative cycle: The mean temperature of heat addition can also be increased by decreasing the amount of heat added at low temperatures. In a simple Rankine cycle (saturated steam entering the turbine), a considerable part of the total heat supplied is in the liquid phase when heating up water from 1 to 1¹, at a temperature lower than T₂, the maximum temperature of the cycle. For maximum η, all heat should be supplied at T₂, and feed water should enter the boiler at 1¹. This may be accomplished in what is known as an ideal regenerative cycle as shown in figures (a) and (b).

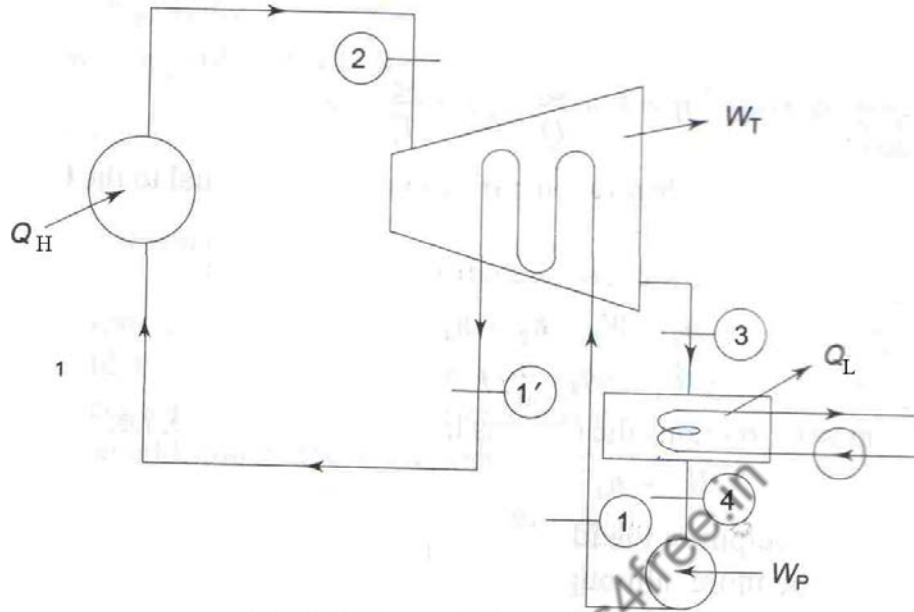


Fig. Ideal regenerative cycle

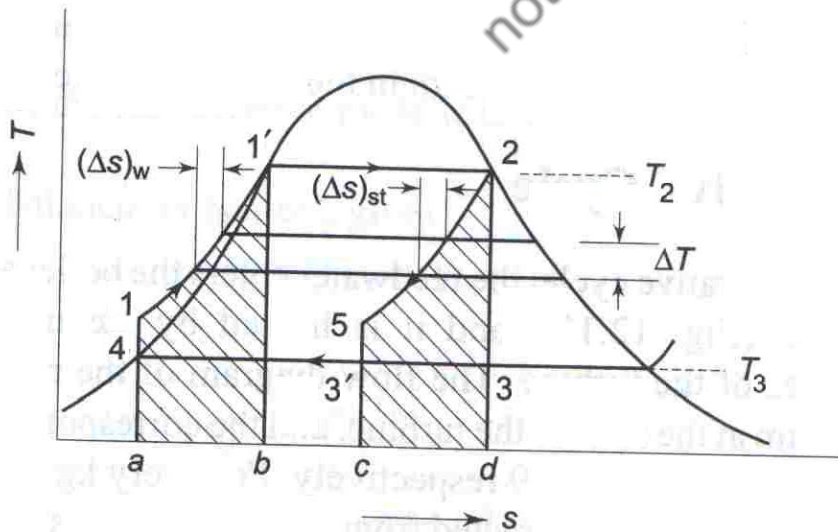


Fig. Ideal regenerative cycle on T-s plot

$$\Delta T(\text{water}) = -\Delta T(\text{steam})$$

The unique feature of the ideal regenerative cycle is that the condensate, after leaving the pump circulates around the turbine casing, counter-flow to the direction of vapour flow in the turbine. Thus it is possible to transfer heat from the vapour as it flows through the turbine to the liquid flowing around the turbine.

Let us assume that this is a reversible heat transfer i.e., at each point, the temperature of the vapour is only infinitesimally higher than the temperature of the liquid. \therefore The process 2-3¹ represents reversible expansion of steam in the turbine with reversible heat rejection. i.e., for any small step in the process of heating the water $\Delta T_{(water)} = -\Delta T_{(steam)}$ and $(\Delta S)_{water} = (\Delta S)_{steam}$. Then the slopes of lines 2-3¹ and 1¹-4 will be identical at every temperature and the lines will be identical in contour. Areas 1-1¹-b-a-1 and 3¹-2-d-c-3¹ are not only equal but congruous. \therefore , all heat added from external source (Q_H) is at constant temperature T_2 and all heat rejected (Q_L) is at constant temperature T_3 , both being reversible.

$$\begin{aligned} \text{Then } Q_H &= h_2 - h_1^1 = T_2 (S_2 - S_1^1) \\ Q_L &= h_3^1 - h_4 = T_3 (S_3^1 - S_4) \\ \text{Since } S_1^1 - S_4 &= S_2 - S_3^1 \quad \text{or} \quad S_2 - S_1^1 = S_3^1 - S_4 \end{aligned}$$

$$\therefore \eta_{Reg} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_3}{T_2} \quad \text{i.e., the } \eta \text{ of ideal regenerative cycle is thus equal to the Carnot cycle } \eta.$$

Writing SFEE to turbine,

$$\begin{aligned} h_2 + h_1 &= W_T + h_1^1 + h_3^1 \\ \text{i.e., } W_T &= (h_2 - h_3^1) - (h_1^1 - h_1) \end{aligned}$$

$$\text{or } W_T = (h_2 - h_3^1) - (h_1^1 - h_1) \quad \text{--- (1)}$$

and the W_P is same as simple rankine cycle i.e., $W_P = (h_1 - h_4)$

\therefore The net work output of the ideal regenerative cycle is less and hence its steam rate will be more. Although it is more efficient when compared to rankine cycle, this cycle is not practicable for the following reasons.

- 1) Reversible heat transfer cannot be obtained in finite time.
- 2) Heat exchanger in the turbine is mechanically impracticable.
- 3) The moisture content of the steam in the turbine is high.

Regenerative cycle:

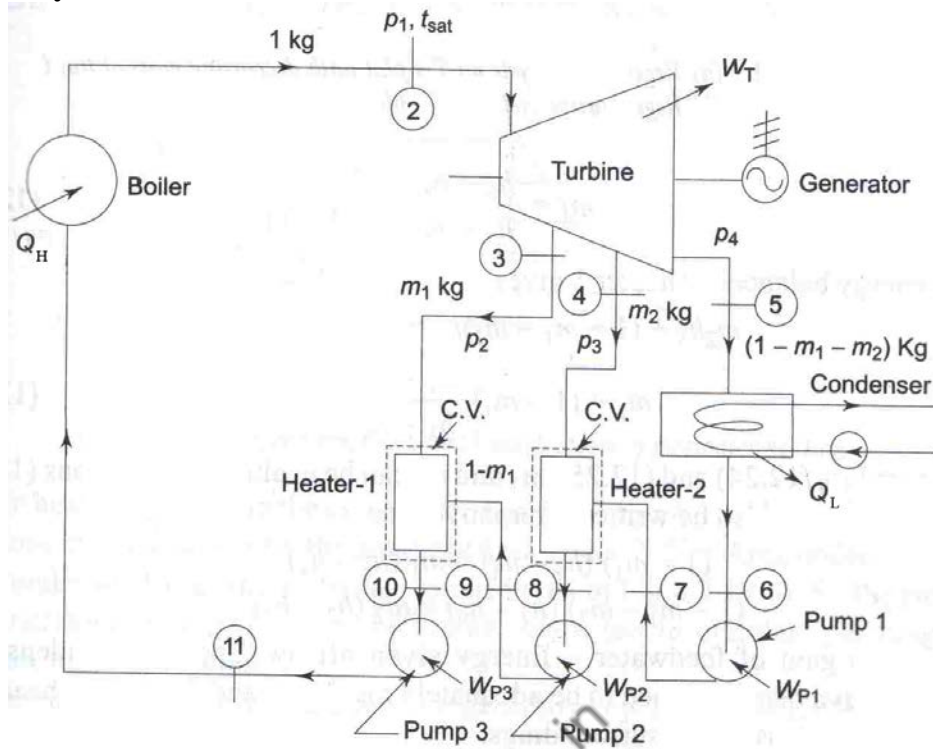
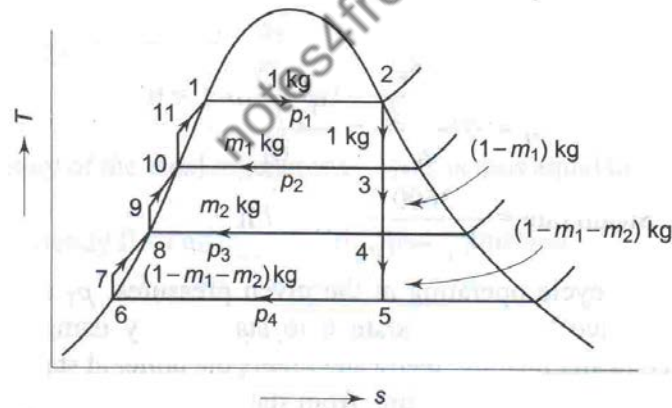
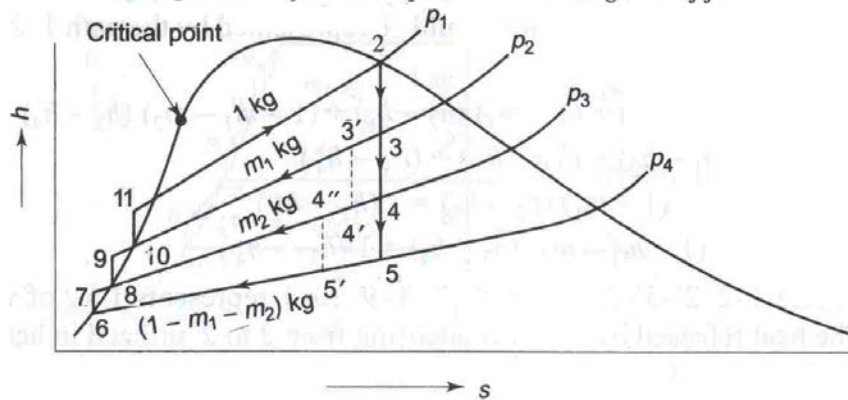


Fig. Regenerative cycle flow diagram with two feedwater heaters



(a) Regenerative cycle on T-s plot with decreasing mass of fluid



In a practical regenerative cycle, the feed water enters the boiler at a temperature between 1 and 1¹ (previous article figure), and it is heated by steam extracted from intermediate stages of the turbine. The flow diagram of the regenerative cycle with saturated steam at the inlet to the turbine and the corresponding T-S diagram are shown in figure.

For every kg of steam entering the turbine, let m_1 kg steam be extracted from an intermediate stage of the turbine where the pressure is P_2 , and it is used to heat up feed water [(1 - m_1) kg at state 9] by mixing in heater (1). The remaining (1 - m_1) kg of steam then expands in the turbine from pressure P_2 (state 3) to pressure P_3 (state 4) when m_2 kg of steam is extracted for heating feed water in heater (2). So (1 - m_1 - m_2)kg of steam then expands in the remaining stages of the turbine to pressure P_4 , gets condensed into water in the condenser, and then pumped to heater (2), where it mixes with m_2 kg of steam extracted at pressure P_3 . Then (1 - m_1) kg of water is pumped to heater (1) where it mixes with m_1 kg of steam extracted at pressure P_2 . The resulting 1kg of steam is then pumped to the boiler where heat from an external source is supplied. Heaters 1 and 2 thus operate at pressure P_2 and P_3 respectively. The amounts of steam m_1 and m_2 extracted from the turbine are such that at the exit from each of the heaters, the state is saturated liquid at the respective pressures.

$$\therefore \text{Turbine work, } W_T = 1(h_2 - h_3) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5)$$

$$\begin{aligned} \text{Pump work, } W_P &= W_{P1} + W_{P2} + W_{P3} \\ &= (1 - m_1 - m_2)(h_7 - h_6) + (1 - m_1)(h_9 - h_8) + 1(h_{11} - h_{10}) \end{aligned}$$

$$Q_H = (h_2 - h_{11}); \quad Q_L = (1 - m_1 - m_2)(h_5 - h_6)$$

$$\therefore \text{Cycle efficiency, } \eta = \frac{Q_H - Q_L}{Q_H} = \frac{W_T - W_P}{Q_H}$$

$$SSC = \frac{3600}{W_T - W_P} \text{ kg / kWh}$$

In the Rankine cycle operating at the given pressure P_1 and P_4 , the heat addition would have been from state 7 to state 2. By using two stages of regenerative feed water heating, feed water enters the boiler at state 11, instead of state 7, and heat addition is, therefore from state 11 to state 2.

$$\text{Therefore } (T_{m1})_{\text{with regeneration}} = \frac{h_2 - h_{11}}{S_2 - S_{11}}$$

$$\text{And } (T_{m1})_{\text{without regeneration}} = \frac{h_2 - h_7}{S_2 - S_7}$$

Since $(T_{m1})_{\text{with regenerative}} > (T_{m1})_{\text{without regenerative}}$, the η of the regenerative cycle will be higher than that of the Rankine cycle.

The energy balance for heater 1,

$$m_1 h_3 + (1 - m_1) h_9 = 1 h_{10}$$

$$\therefore m_1 = \frac{h_{10} - h_9}{h_3 - h_9} \quad \text{--- (1)}$$

The energy balance for heater 2,

$$m_2 h_4 + (1 - m_1 - m_2) h_7 = (1 - m_1) h_8$$

$$\text{Or } m_2 = (1 - m_1) \frac{(h_8 - h_7)}{(h_4 - h_7)} \quad \text{--- (2)}$$

Above equations (1) and (2) can also be written alternatively as

$$(1 - m_1) (h_{10} - h_9) = m_1 (h_3 - h_{10})$$

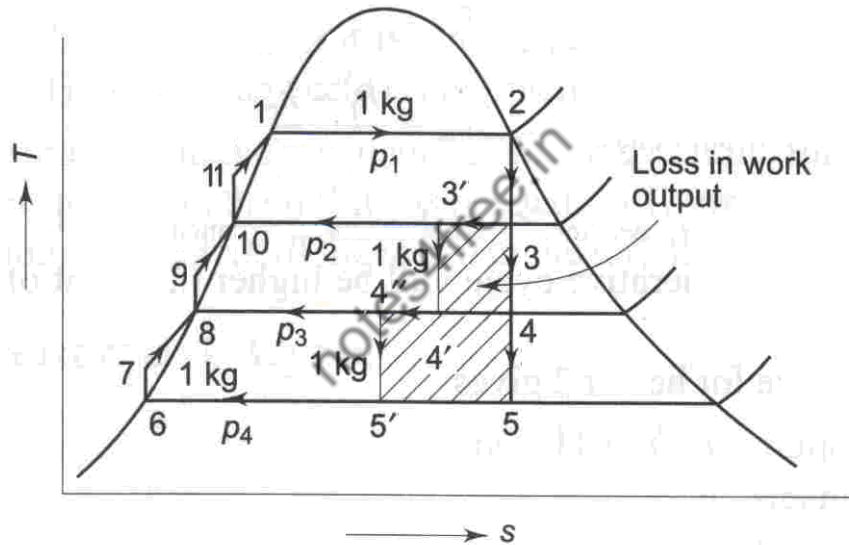
$$\text{and } (1 - m_1 - m_2) (h_8 - h_7) = m_2 (h_4 - h_8)$$

Energy gain of feed water = energy given off by vapour in condensation.

Heaters have been assumed to be adequately insulated and there is no heat gain from, or heat loss to, the surroundings.

In figure (a) path 2-3-4-5 represents the states of a decreasing mass of fluid.

For 1kg of steam, the states would be represented by the path 2-3¹-4¹¹-5¹. [Figure (b)].



(b) Regenerative cycle on T-s plot for unit mass of fluid

$$\text{We have } W_T = (h_2 - h_3) + (1 - m_1) (h_3 - h_4) + (1 - m_1 - m_2) (h_4 - h_5)$$

$$= (h_2 - h_3) + (h_3^1 - h_4^1) + (h_4^{11} - h_5^1) \quad \text{[From Figure b]} \quad \text{--- (3)}$$

The cycle 2 - 3 - 3¹ - 4¹ - 4¹¹ - 5¹ - 6 - 7 - 8 - 9 - 10 - 11 - 2 represents 1kg of working fluid. The heat released by steam condensing from 3 to 3¹ is utilized in heating up the water from 9 to 10.

$$\therefore 1 (h_3 - h_3^1) = 1 (h_{10} - h_9) \quad \text{--- (4)}$$

$$\text{Similarly, } 1 (h_4^1 - h_4^{11}) = 1 (h_8 - h_7) \quad \text{--- (5)}$$

From equation (3), (4) and (5),

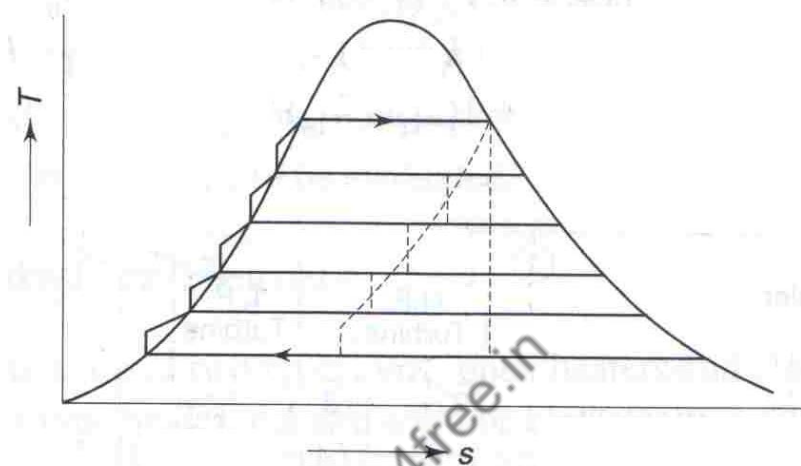
$$W_T = (h_2 - h_5^1) - (h_3 - h_3^1) - (h_4^1 - h_4^{11})$$

$$= (h_2 - h_5^1) - (h_{10} - h_9) - (h_8 - h_7) \quad \text{--- (6)}$$

Also from Ideal regenerative cycle, [Previous article]

$$W_T = (h_2 - h_3^1) - (h_1^1 - h_1) \quad \text{--- (1)}$$

The similarity of equations (6) and equation (1) from previous article is noticed. It is seen that the stepped cycle $2 - 3^1 - 4^1 - 4^{11} - 5^1 - 6 - 7 - 8 - 9 - 10 - 11$ approximates the ideal regenerative cycle in Figure (1) [previous article] and that a greater no. of stages would give a closer approximation. Thus the heating of feed water by steam 'bled' from the turbine, known as regeneration, "Carnotizes" the Rankine cycle.



Regenerative cycle with many stages of feedwater heating

The heat rejected Q_L in the cycle decreases from $(h_5 - h_6)$ to $(h_5^1 - h_6)$. There is also loss in work output by the amount (area under $3 - 3^1 +$ area under $4^1 - 4^{11} -$ area under $5 - 5^1$) as shown by the hatched area in Figure (b). So the steam rate increases by regeneration i.e., more steam has to circulate per hour to produce unit shaft output.

Reheat – Regenerative cycle:

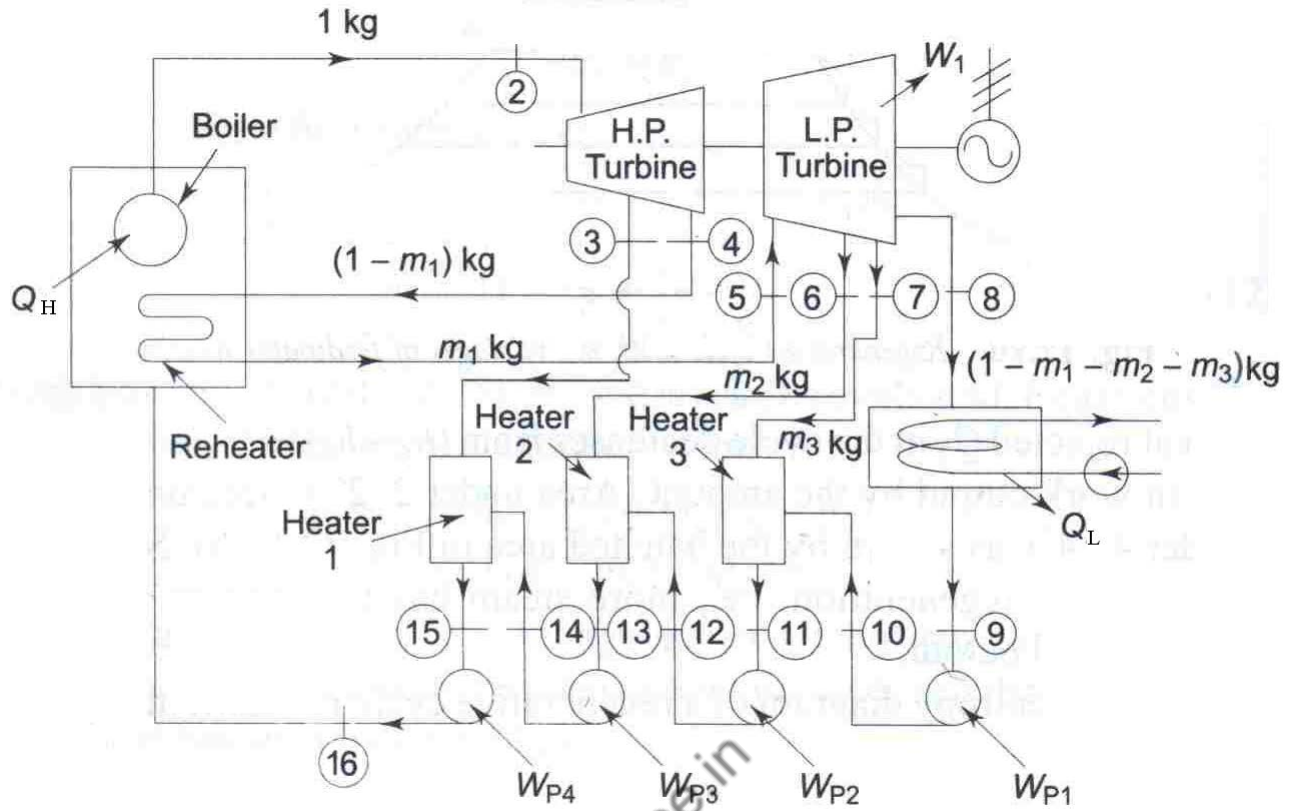


Fig. Reheat-regenerative cycle flow diagram

Reheat – regenerative cycle flow diagram (Three-stages of feed water heating)

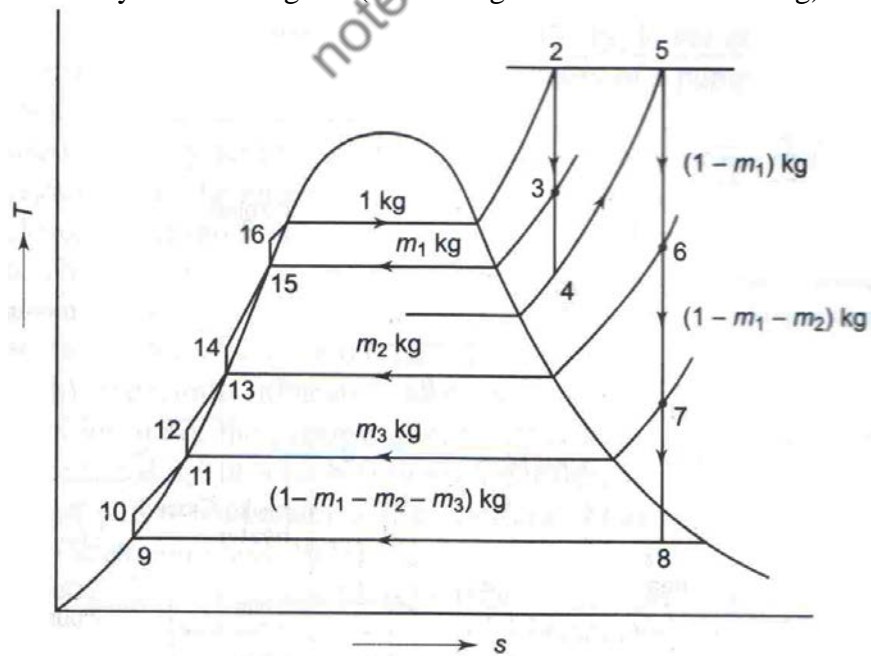


Fig. T-s diagram of reheat-regenerative cycle

The reheating of steam is employed when the vapourization pressure is high reheat alone on the thermal η is very small. \therefore Regeneration or the heating up of feed water by steam extracted from the turbine will effect in more increasing in the η_{th} .

$$\text{Turbine work, } W_T = (h_1 - h_2) + (1 - m_1) (h_2 - h_3) + (1 - m_1) (h_4 - h_5) + (1 - m_1 - m_2) (h_5 - h_6) \\ + (1 - m_1 - m_2 - m_3) (h_6 - h_7) \text{ kJ/kg}$$

$$\text{Pump work, } W_P = (1 - m_1 - m_2 - m_3) (h_9 - h_8) + (1 - m_1 - m_2) (h_{11} - h_{10}) \\ + (1 - m_1) (h_{13} - h_{12}) + 1 (h_{15} - h_{14}) \text{ kJ/kg}$$

$$\text{Heat added, } Q_H = (h_1 - h_{15}) + (1 - m_1) (h_4 - h_3) \text{ kJ/kg}$$

$$\text{Heat rejected, } Q_L = (1 - m_1 - m_2 - m_3) (h_7 - h_8) \text{ kJ/kg}$$

The energy balance of heaters 1, 2 and 3 gives

$$m_1 h_2 + (1 - m_1) h_{13} = 1 \times h_{14}$$

$$m_2 h_5 + (1 - m_1 - m_2) h_{11} = (1 - m_1) h_{12}$$

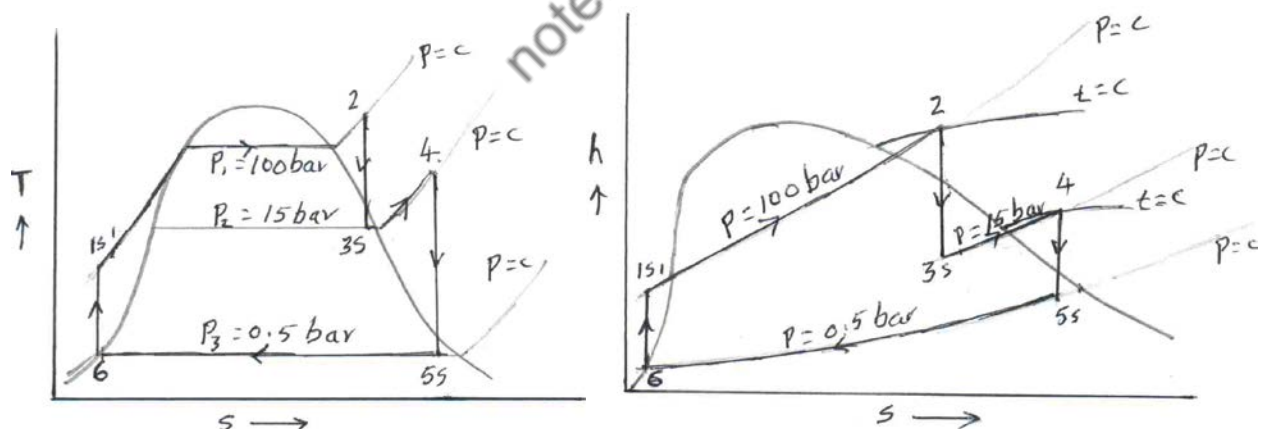
$$m_3 h_6 + (1 - m_1 - m_2 - m_3) h_9 = (1 - m_1 - m_2) h_{10}$$

From which m_1 , m_2 and m_3 can be evaluated

Numerical Problems:

1. An ideal reheat cycle utilizes steam as the working fluid. Steam at 100 bar, 400°C is expanded in the HP turbine to 15 bar. After this, it is reheated to 350°C at 15 bar and is then expanded in the LP turbine to the condenser pressure of 0.5 bar. Determine the thermal η and steam rate.

Solution:



From steam tables

$$P = 100 \text{ bar } t = 400^\circ\text{C} = \quad v = 0.026408 \text{ m}^3/\text{kg} \\ h = 3099.9 \text{ kJ/kg} \\ S = 6.2182 \text{ kJ/kg-K}$$

$$P = 15 \text{ bar} \quad t_s = 192.28^\circ\text{C}, \quad v_f = 0.0011538 \text{ m}^3/\text{kg}, \quad v_g = 0.13167 \text{ m}^3/\text{kg} \\ h_f = 844.6 \text{ kJ/kg}, \quad h_{fg} = 1845.3 \text{ kJ/kg}, \quad h_g = 2789.9 \text{ kJ/kg} \\ s_f = 2.3144 \text{ kJ/kg-K}, \quad s_{fg} = 4.1262 \text{ kJ/kg-K}, \quad s_g = 6.4406 \text{ kJ/kg-K}$$

$$P = 0.5 \text{ bar} \quad t_s = 81.35^\circ\text{C}, \quad v_f = 0.0010301 \text{ m}^3/\text{kg}, \quad v_g = 3.2401 \text{ m}^3/\text{kg}$$

$$h_f = 340.6 \text{ kJ/kg}, \quad h_{fg} = 2305.4 \text{ kJ/kg} \quad h_g = 2646.0 \text{ kJ/kg}$$

$$s_f = 1.0912 \text{ kJ/kg-K} \quad s_{fg} = 6.5035 \text{ kJ/kg-K}, \quad s_g = 7.5947 \text{ kJ/kg-K}$$

$$h_2 = 3099.9 \text{ kJ/kg},$$

Process 2-3s is isentropic, i.e., $S_2 = S_{3s}$

$$6.2182 = 2.3144 + x_{3s} (4.1262)$$

$$\therefore x_{3s} = 0.946$$

$$\therefore h_{3s} = 844.6 + x_{3s} (1845.3)$$

$$= 2590.44 \text{ kJ/kg}$$

$$\therefore \text{Expansion of steam in the HP turbine} = h_2 - h_{3s}$$

$$= 3099.9 - 2590.44$$

$$= 509.46 \text{ kJ/kg}$$

$$P = 15 \text{ bar}, t = 350^\circ\text{C} = \quad v = 0.18653$$

$$\quad \quad \quad h = 3148.7$$

$$\quad \quad \quad s = 7.1044$$

Expansion of steam in the LP cylinder = $h_4 - h_{5s}$

$$h_4 = 3148.7 \text{ kJ/kg}$$

To find h_{5s} :

We have $S_4 = S_{5s}$

$$7.1044 = S_{f5} + x_{5s} S_{fg5}$$

$$= 1.0912 + x_{5s} (6.5035)$$

$$\therefore x_{5s} = 0.925$$

$$\therefore h_{5s} = 340.6 + 0.925 (2305.4) = 2473.09 \text{ kJ/kg}$$

$$\therefore \text{Expansion of steam in the LP turbine} = 3148.7 - 2473.09$$

$$= 675.61 \text{ kJ/kg}$$

$$h_6 = h_f \text{ for } P_3 = 0.5 \text{ bar i.e., } h_6 = 340.6 \text{ kJ/kg}$$

$$\text{Pump work, } W_P = h_{1s} - h_6 = v_{f5} (P_3 - P_1) = 0.0010301 (100 - 0.501 \times 10^5)$$

$$= 10.249 \text{ kJ/kg}$$

$$\therefore h_{1s} = 350.85 \text{ kJ/kg}$$

$$\therefore \text{Heat supplied, } Q_H = (h_2 - h_{1s}) + (h_4 - h_{3s})$$

$$= (3099.9 - 350.85) + (3148.7 - 2590.44)$$

$$= 2749.05 \text{ kJ/kg} + 558.26$$

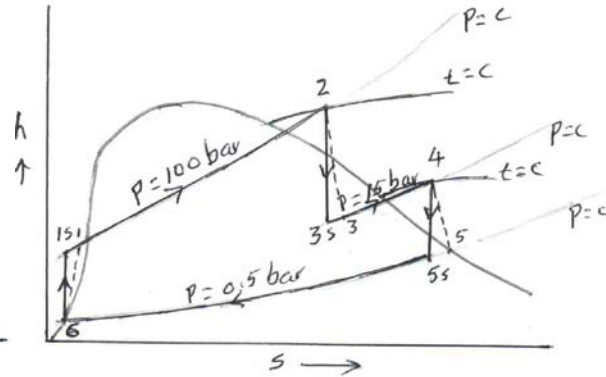
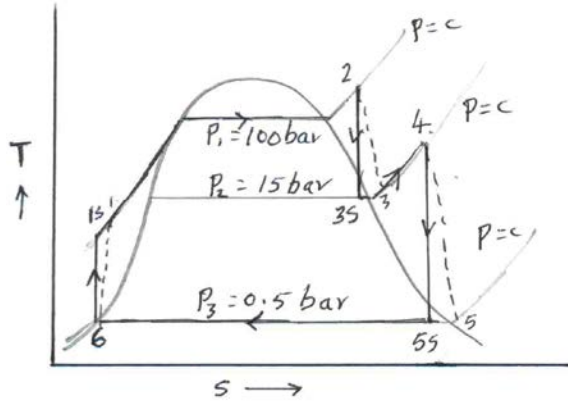
$$= 3307.31 \text{ kJ/kg}$$

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{(W)_{HP} + (W)_{LP} - W_P}{Q_H}$$

$$= \frac{509.46 + 675.61 - 10.25}{3307.31} = 0.355$$

$$\text{Steam rate, } SSC = \frac{3600}{W_{net}} = 3.064 \text{ kg / kWh}$$

1. b) When η of the HP turbine, LP turbine and feed pump are 80%, 85% and 90% respectively.



$$\eta_{iHP} = \frac{h_2 - h_3}{h_2 - h_{3s}} = 0.8 = \frac{3099.9 - h_3}{3099.9 - 2590.44}$$

$$\therefore h_3 = 2692.33 \text{ kJ/kg}$$

$$\eta_{iLP} = \frac{h_4 - h_5}{h_4 - h_{5s}} = 0.85 = \frac{3148.7 - h_5}{3148.7 - 2473.09}$$

$$\therefore h_5 = 2574.43 \text{ kJ/kg}$$

$$\eta_P = \frac{h_{1s} - h_6}{h_1 - h_6} = 0.9 = \frac{350.85 - 340.6}{h_1 - 340.6}$$

$$\therefore h_1 = 351.99 \text{ kJ/kg}$$

$$\therefore \eta_{th} = \frac{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)}{(h_2 - h_1) + (h_4 - h_3)}$$

$$= \frac{(3099.9 - 2692.33) + (3148.7 - 2574.43) - (351.99 - 340.6)}{(3099.9 - 351.99) + (3148.7 - 2692.33)}$$

$$= 0.303 \text{ or } 30.3\%$$

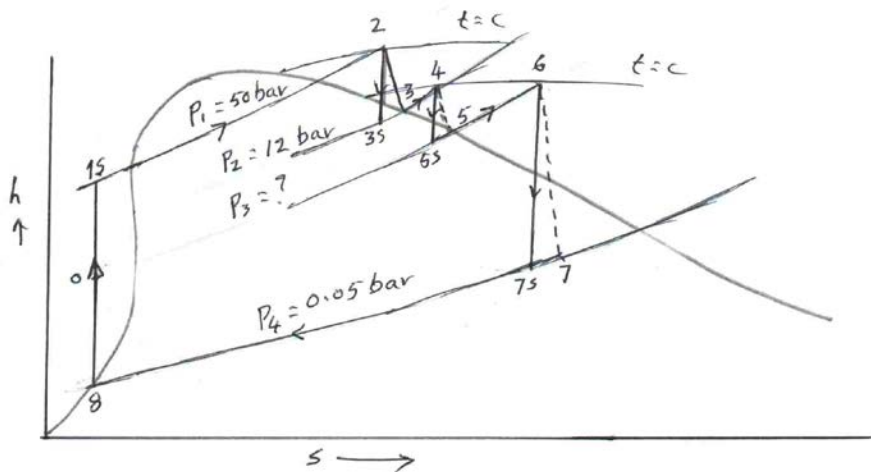
$$\therefore \text{SSC} = 3.71 \text{ kg/kWh}$$

Using Mollier-chart: $h_2 = 3095 \text{ kJ/kg}$, $h_{3s} = 2680 \text{ kJ/kg}$, $h_4 = 3145 \text{ kJ/kg}$
 $h_5 = 2475 \text{ kJ/kg}$, $h_6 = 340.6 \text{ kJ/kg}$ (from steam tables)
 $W_P = 10.249 \text{ kJ/kg}$

2. Steam at 50 bar, 350°C expands to 12 bar in a HP stage, and is dry saturated at the stage exit. This is now reheated to 280°C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280°C . Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit

of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.

Solution:



$$P_1 = 50 \text{ bar} \quad t_2 = 350^\circ\text{C} \quad P_2 = 12 \text{ bar} \quad t_4 = 280^\circ\text{C}, \quad t_6 = 280^\circ\text{C}$$

$$P_3 = ? \quad P_4 = 0.05 \text{ bar}$$

From Mollier diagram

$$h_2 = 3070 \text{ kJ/kg} \quad h_{3s} = 2755 \text{ kJ/kg} \quad h_3 = 2780 \text{ kJ/kg} \quad h_4 = 3008 \text{ kJ/kg}$$

$$(a) \eta_t \text{ for HP stage} = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{3070 - 2780}{3070 - 2755}$$

$$= 0.921$$

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$$h_2 - h_3 = h_4 - h_5$$

$$\text{i.e., } 3070 - 2780 = 3008 - h_5$$

$$\therefore h_5 = 2718 \text{ kJ/kg}$$

Since state 5 is on the saturation line, we find from Mollier chart, $P_3 = 2.6 \text{ bar}$,
Also from Mollier chart, $h_{5s} = 2708 \text{ kJ/kg}$, $h_6 = 3038 \text{ kJ/kg}$, $h_{7s} = 2368 \text{ kJ/kg}$

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93 \text{ kJ/kg}$$

$$\therefore \text{At a pressure } 0.05 \text{ bar, } h_7 = h_{f7} + x_7 h_{fg7}$$

$$2420.93 = 137.8 + x_7 (2423.8)$$

$$\therefore x_7 = 0.941$$

$$\text{Total power output} = (h_2 - h_3) + (h_4 - h_5) + (h_6 - h_7)$$

$$= (3070 - 2780) + (3008 - 2718) + (3038 - 2420.93)$$

$$= 1197.07 \text{ kJ/kg}$$

\therefore Total power output /kg of steam = 1197.07 kW

For $P_4 = 0.05 \text{ bar}$ from steam tables, $h_8 = 137.8 \text{ kJ/kg}$;

$$W_P = 0.0010052 (50 - 0.05) 10^2 = 5.021 \text{ kJ/kg}$$

$$= h_8 - h_{1s}$$

$$\therefore h_{1s} = 142.82 \text{ kJ/kg}$$

$$\begin{aligned} \text{Heat supplied, } Q_H &= (h_2 - h_{1s}) + (h_4 - h_3) + (h_6 - h_5) \\ &= (3070 - 142.82) + (3008 - 2780) + (3038 - 2718) \\ &= 3475.18 \text{ kJ/kg} \end{aligned}$$

$$W_{\text{net}} = W_T - W_P = 1197.07 - 5.021 = 1192.05 \text{ kJ/kg}$$

$$\therefore \eta_{th} = \frac{W_{\text{net}}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m}$$

$$\therefore T_m = 465.6 \text{ K}$$

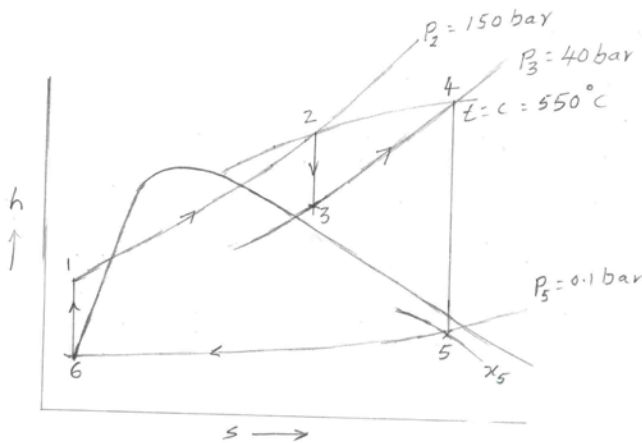
Or

$$T_m = \frac{h_2 - h_{1s}}{s_2 - s_{1s}} = \frac{3070 - 142.82}{6.425 - 0.4763} = 492 \text{ K}$$

$$SSC = \frac{3600}{1192.05} = 3.02 \text{ kg/kWh}$$

3. A steam power station uses the following cycle: Steam at boiler outlet – 150 bar; reheat at 40 bar, 550°C; condenser at 0.1 bar. Using Mollier chart and assuming that all processes are ideal, find (i) quality at turbine exhaust (ii) cycle η (iii) steam rate.

Solution:



$$P_2 = 150 \text{ bar} \quad t_2 = 550^\circ\text{C} \quad P_3 = 40 \text{ bar} \quad t_3 = 550^\circ\text{C}$$

$$P_5 = 0.1 \text{ bar}$$

From Mollier diagram i.e., h-s diagram

$$h_2 = h \Big|_{150 \text{ bar}, 550^\circ \text{C}} = 3450 \text{ kJ/kg}$$

$$h_4 = h \Big|_{40 \text{ bar}, 550^\circ \text{C}} = 3562 \text{ kJ/kg}$$

$$h_3 = 3050 \text{ kJ/kg}$$

$$h_5 = 2290 \text{ kJ/kg}$$

$$x_5 = 0.876 \text{ kJ/kg}$$

h_6 can not be determined from h-s diagram, hence steam tables are used.

$$h_6 = h_f \Big|_{0.1 \text{ bar}} = 191.8 \text{ kJ/kg}$$

Process 6-1 is isentropic pump work i.e., $W_{P1} = \int v dp$

$$= 0.0010102 (40 - 01) 10^5 / 10^3 = 4.031 \text{ kJ/kg}$$

$$= (h_1 - h_6)$$

$$\therefore h_1 = 195.8 \text{ kJ/kg}$$

(i) Quality of steam at turbine exhaust = $x_5 = 0.876$

$$(ii) \eta_{cycle} = \frac{W_T - W_P}{Q_H}$$

$$\begin{aligned} \text{Turbine work, } W_T &= W_{T1} + W_{T2} \\ &= (h_2 - h_3) + (h_4 - h_5) \\ &= (3450 - 3050) + (3562 - 2290) \\ &= 1672 \text{ kJ/kg} \end{aligned}$$

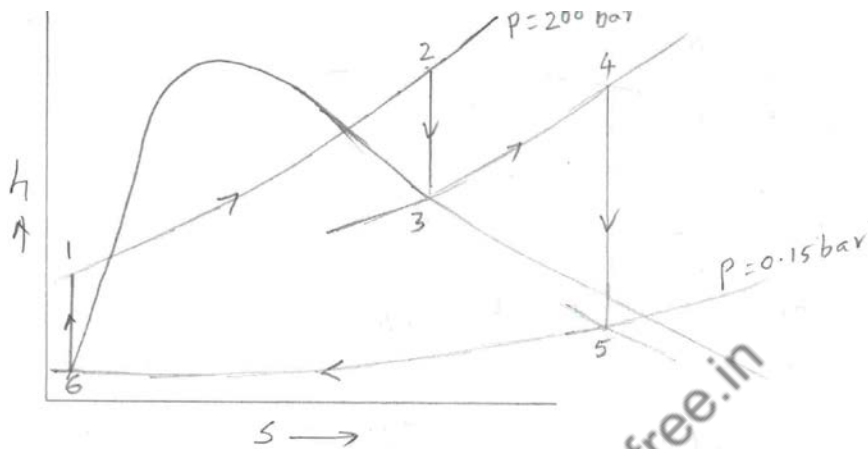
$$\begin{aligned} Q_H = Q_1 + Q_R &= (h_2 - h_1) + (h_4 - h_3) \\ &= (3450 - 195.8) + (3562 - 3050) \\ &= 3766.2 \text{ kJ/kg} \end{aligned}$$

$$\therefore \eta_{cycle} = \frac{1672 - 4.031}{3766.2} = \frac{1667.97}{3766.2} = 0.443$$

$$(iii) \text{ Steam rate} = \frac{3600}{1667.97} = 2.16 \text{ kg / kWh}$$

4. An ideal Rankine cycle with reheat is designed to operate according to the following specification. Pressure of steam at high pressure turbine = 20 MPa, Temperature of steam at high pressure turbine inlet = 550°C, Temperature of steam at the end of reheat = 550°C, Pressure of steam at the turbine exhaust = 15 KPa. Quality of steam at turbine exhaust = 90%. Determine (i) the pressure of steam in the reheater (ii) ratio of pump work to turbine work, (iii) ratio of heat rejection to heat addition, (iv) cycle η .

Solution:



$$P_2 = 200 \text{ bar} \quad t_2 = 550^\circ\text{C} \quad t_4 = 550^\circ\text{C} \quad P_5 = 0.15 \text{ bar} \quad x_5 = 0.9$$

From Mollier diagram,

$$h_2 = 3370 \text{ kJ/kg}$$

$$h_3 = 2800 \text{ kJ/kg}$$

$$h_4 = 3580 \text{ kJ/kg}$$

$$h_5 = 2410 \text{ kJ/kg}$$

$$x_5 = 0.915$$

$$P_3 = P_4 = 28 \text{ bar}$$

But given in the data $x_5 = 0.9$

From steam tables $h_6 = 226 \text{ kJ/kg}$

$$\text{Pump work } W_P = \int v dP$$

$$= 0.001014 (200 - 0.15) 10^5 / 10^3$$

$$= 20.26 \text{ kJ/kg}$$

$$\text{But } W_P = h_1 - h_6 \quad \therefore h_1 = 246.26 \text{ kJ/kg}$$

(i) Pressure of steam in the reheater = 28 bar

$$\begin{aligned} \text{(ii) Turbine work } W_T &= (h_2 - h_3) + (h_4 - h_5) \\ &= (3370 - 2800) + (3580 - 2410) \\ &= 1740 \text{ kJ/kg} \end{aligned}$$

$$\therefore \text{Ratio of } \frac{W_P}{W_T} = 0.0116 \quad \text{i.e., } 1.2\%$$

$$\begin{aligned} \text{(iii) } Q_L &= (h_5 - h_6) = (2410 - 226) = 2184 \text{ kJ/kg} \\ Q_H &= (h_2 - h_1) + (h_4 - h_3) \\ &= (3370 - 226) + (3580 - 2800) \\ &= 3924 \text{ kJ/kg} \end{aligned}$$

$$\therefore \frac{Q_L}{Q_H} = 0.5565 \quad \text{i.e., } 55.65\%$$

$$\text{(iv) } \eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_{\text{Total}}} = \frac{(1740 - 20.26)}{3924} = 0.4383 \quad \text{i.e., } 43.8\%$$

Feedwater Heaters (FWH)

A practical Regeneration process in steam power plants is accomplished by extracting or bleeding, steam from the turbine at various points. This steam, which could have produced more work by expanding further in the turbine, is used to heat the feed water instead. The device where the feedwater heated by regeneration is called a Regenerator or a Feedwater Heater (FWH).

A feedwater heater is basically a heat exchanger where heat is transferred from the steam to the feedwater either by mixing the two streams (open feedwater heaters) or without mixing them (closed feedwater heaters).

Open Feedwater Heaters

An open (or direct-contact) feedwater heater is basically a mixing chamber, where the steam extracted from the turbine mixes with the feedwater exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure.

The advantages of open heater are simplicity, lower cost, and high heat transfer capacity. The disadvantage is the necessity of a pump at each heater to handle the large feedwater stream.

Closed Feedwater Heaters

In closed feedwater heater, the heat is transferred from the extracted steam to the feedwater without mixing taking place. The feedwater flows through the tubes in the heater and extracted steam condenses on the outside of the tubes in the shell. The heat released from the condensation is transferred to the feedwater through the walls of the tubes. The condensate (saturated water at the steam extraction pressure), some times called the heater-drip, then passes through a trap into the next lower pressure heater. This, to some extent, reduces the steam required by that heater. The trap passes only liquid and no vapour. The drip from the lowest pressure heater could similarly be trapped to the condenser, but this would be throwing away energy to the condenser cooling water. To avoid this waste, the drip pump feeds the drip directly into the feedwater stream.

A closed heaters system requires only a single pump for the main feedwater stream regardless of the number of heaters. The drip pump, if used is relatively small. Closed heaters are costly and may not give as high a feedwater temperature as do open heaters.

In most steam power plants, closed heaters are favoured, but at least one open heater is used, primarily for the purpose of feedwater deaeration. The open heater in such a system is called deaerator.

Note: The higher the number of heater used, the higher will be the cycle efficiency. The number of heater is fixed up by the energy balance of the whole plant when it is found that the cost of adding another does not justify the saving in Q_H or the marginal increase in cycle efficiency. An

increase in feedwater temperature may, in some cases, cause a reduction in boiler efficiency. So the number of heaters get optimized. Five feedwater heaters are often used in practice.

Characteristics of an Ideal working fluid

The maximum temperature that can be used in steam cycles consistent with the best available material is about 600°C , while the critical temperature of steam is 375°C , which necessitates large superheating and permits the addition of only an infinitesimal amount of heat at the highest temperature.

The desirable characteristics of the working fluid in a vapour power cycle to obtain best thermal η are as follows:

- The fluid should have a high critical temperature so that the saturation pressure at the maximum permissible temperature (metallurgical limit) is relatively low. It should have a large enthalpy of evaporation at that pressure.
- The saturation pressure at the temperature of heat rejection should be above atmosphere pressure so as to avoid the necessity of maintaining vacuum in the condenser.
- The specific heat of liquid should be small so that little heat transfer is required to raise the liquid to the boiling point.
- The saturation vapour line of the T-S diagram should be steep, very close to the turbine expansion process so that excessive moisture does not appear during expansion.
- The freezing point of the fluid should be below room temperature, so that it does not get solidified while flowing through the pipe lines.
- The fluid should be chemically stable and should not contaminate the materials of construction at any temperature.
- The fluid should be nontoxic, non corrosive, not excessively viscous, and low in cost.

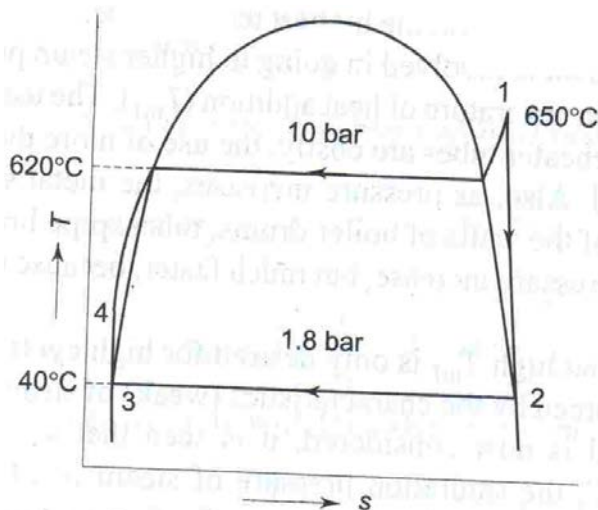
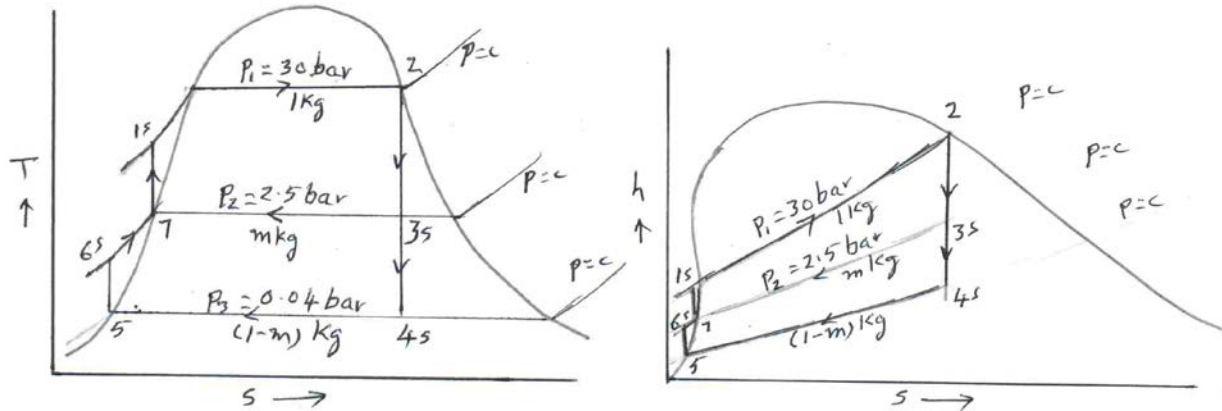


Fig. T-S diagram for an Ideal Working Fluid for a Vapour Power Cycle

Numerical Problems:

1. An ideal regenerative cycle operates with dry saturated steam, the maximum and minimum pressures being 30 bar and 0.04 bar respectively. The plant is installed with a single mixing type feed water heater. The bled steam pressure is 2.5 bar. Determine (a) the mass of the bled steam, (b) the thermal η of the cycle, and (c) SSC in kg/kWh.

Solution:



$$P_1 = 30 \text{ bar} \quad P_2 = 2.5 \text{ bar} \quad P_3 = 0.04 \text{ bar}$$

$$\text{From steam tables, For } P_1 = 30 \text{ bar, } h_2 = 2802.3 \text{ kJ/kg, } S_2 = 6.1838 \text{ kJ/kg-}^0\text{k}$$

$$\text{But } S_2 = S_{3s} \quad \text{i.e., } 6.1838 = 1.6072 + x_3 (5.4448) \\ \therefore x_3 = 0.841$$

$$\therefore h_3 = 535.4 + 0.841 (2281.0) \\ = 2452.68 \text{ kJ/kg}$$

$$\text{Also } S_2 = S_{4s} \quad \text{i.e., } 6.1838 = 0.4225 + x_4 (8.053) \\ \therefore x_4 = 0.715$$

$$\therefore h_4 = 121.4 + 0.715 (2433.1) \\ = 1862.1 \text{ kJ/kg}$$

$$\text{At } P_3 = 0.04 \text{ bar, } h_5 = 121.4 \text{ kJ/kg, } v_5 = 0.001004 \text{ m}^3/\text{kg} \\ \therefore \text{Condensate pump work} = (h_6 - h_5) = v_5 (P_2 - P_3) \\ = 0.001004 (2.5 - 0.04) (10^5/10^3) \\ = 0.247 \text{ kJ/kg}$$

$$\therefore h_6 = 0.247 + 121.4 = 121.65 \text{ kJ/kg}$$

$$\text{Similarly, } h_1 = h_7 + v_7 (P_1 - P_2) (10^5/10^3) \\ = 535.4 + 0.0010676 (30 - 2.5) 10^2 \\ = 538.34 \text{ kJ/kg}$$

a) **Mass of the bled steam:**

Applying the energy balance to the feed water heater

$$mh_3 + (1 - m) h_6 = 1 (h_7)$$

$$\therefore m = \frac{(h_7 - h_6)}{(h_3 - h_6)} = \frac{(535.4 - 121.65)}{(2452.68 - 121.65)} = 0.177 \text{ kg / kg of steam}$$

b) **Thermal η :**

$$\begin{aligned} \text{Turbine work, } W_T &= 1 (h_2 - h_{3s}) + (1 - m) (h_3 - h_{4s}) \\ &= 1 (2802.3 - 2452.65) + (1 - 0.177) (2452.68 - 1862.1) \\ &= 835.67 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Pump work, } W_P &= (1 - m) (h_{6s} - h_5) + 1 (h_{1s} - h_7) \\ &= (1 - 0.177) (121.65 - 121.4) + 1 (538.34 - 535.4) \\ &= 3.146 \text{ kJ/kg} \end{aligned}$$

$$\therefore W_{net} = W_T - W_P = 832.52 \text{ kJ/kg}$$

$$\begin{aligned} \text{Heat supplied, } Q_H &= 1 (h_2 - h_{1s}) \\ &= 1 (2802.3 - 538.34) \\ &= 2263.96 \text{ kJ/kg} \end{aligned}$$

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{832.52}{2263.96} = 0.368 \text{ or } \mathbf{36.8\%}$$

c) **SSC:**

$$SSC = \frac{3600}{W_{net}} = 4.324 \text{ kg / kWh}$$

2. In problem (3), also calculate the increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration)

$$\text{Solution: } T_{m1} \text{ (with regeneration)} = \frac{h_2 - h_1}{S_2 - S_1} = \frac{2263.96}{(6.1838 - 1.6072)} = 494.68 \text{ k}$$

$$T_{m1} \text{ (without regeneration)} = \frac{h_2 - h_6}{S_2 - S_6} = \frac{2802.3 - 121.65}{(6.1838 - 0.4225)} = 465.29 \text{ k}$$

$$\therefore \text{Increase in } T_{m1} \text{ due to regeneration} = 494.68 - 465.29 = \mathbf{29.39^0K}$$

$$W_T \text{ (without regeneration)} = h_2 - h_4 = 2802.3 - 1862.1 = 940.2 \text{ kJ/kg}$$

$$\begin{aligned} W_P \text{ (without regeneration)} &= (h_1 - h_5) = v_5 (30 - 0.04) 10^2 \\ &= 0.001004 (29.96) 10^2 = 3.01 \text{ kJ/kg} \end{aligned}$$

$$\therefore h_1 = 3.01 + 121.4 = 124.41 \text{ kJ/kg}$$

$$\therefore \eta_{th} \text{ (without regeneration)} = \frac{W_{net}}{Q_H} = \frac{(940.2 - 3.01)}{2802.3 - 124.41} = 0.349$$

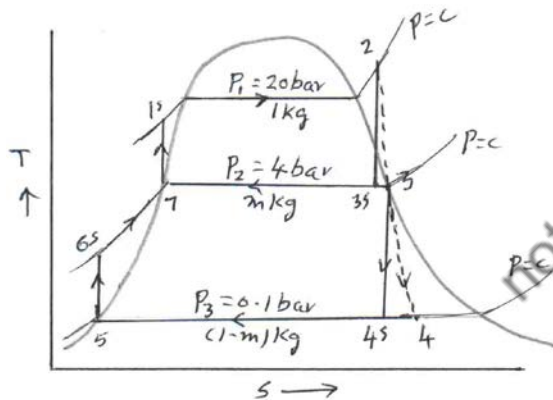
$$\therefore \text{Increase in } \eta_{th} \text{ due to regeneration} = 0.368 - 0.349 = 0.018 \quad \text{i.e., 1.8\%}$$

$$\text{Steam rate (without regeneration)} = 3.84 \text{ kg/kWh}$$

$$\therefore \text{Increase in steam rate due to regeneration} = 4.324 - 3.84 = 0.484 \text{ kg/kWh}$$

3. Steam at 20 bar and 300°C is supplied to a turbine in a cycle and is bled at 4 bar. The bled-steam just comes out saturated. This steam heats water in an open heater to its saturation state. The rest of the steam in the turbine expands to a condenser pressure of 0.1 bar. Assuming the turbine efficiency to be the same before and after bleeding, find: a) the turbine η and the steam quality at the exit of the last stage; b) the mass flow rate of bled steam 1kg of steam flow at the turbine inlet; c) power output / (kg/s) of steam flow; and d) overall cycle η .

Solution:



$$P_1 = 20 \text{ bar} \quad t_1 = 300^\circ\text{C} \quad P_2 = 4 \text{ bar} \quad P_3 = 0.1 \text{ bar}$$

From steam tables,

$$\text{For } P_1 = 20 \text{ bar and } t_1 = 300^\circ\text{C}$$

$$v_2 = 0.12550 \quad h_2 = 3025.0 \quad S_2 = 6.7696$$

$$\text{For } P_2 = 4 \text{ bar, } h_3 = 2737.6, t_s = 143.63$$

$$h_f = 604.7, h_{fg} = 2132.9, S_f = 1.7764, S_{fg} = 5.1179, S_g = 6.8943$$

$$\text{For } P_3 = 0.1 \text{ bar, } 45.83, 191.8, 2392.9, 2584.8, 0.6493, 7.5018, 8.1511$$

$$\text{We have, } S_2 = S_{3s} \text{ i.e., } 6.7696 = 1.7764 + x_3 (5.1179)$$

$$\therefore x_3 = 0.976$$

$$\therefore h_{3s} = 604.7 + 0.976 (2132.9) = 2685.63 \text{ kJ/kg}$$

$$\therefore \eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{3025 - 2737.6}{3025 - 2685.63} = 0.847$$

$$S_3 = S_{4s} \quad \text{i.e., } 6.8943 = 0.6493 + x_4 (7.5018)$$

$$\therefore x_{4s} = 0.832$$

$$\therefore h_{4s} = 191.8 + 0.832 (2392.9) = 2183.81 \text{ kJ/kg}$$

But η_t is same before and after bleeding i.e., $\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}}$

$$\text{i.e., } 0.847 = \frac{2737.6 - h_4}{2737.6 - 2183.81}$$

$$\therefore h_4 = 2268.54 \text{ kJ/kg}$$

$$\therefore h_4 = h_{f4} + x_4 h_{fg4} \quad \therefore x_4 = 0.868$$

b) Applying energy balance to open heater, $mh_3 + (1 - m) h_{6s} = 1 (h_7)$

$$\therefore m = \frac{h_7 - h_6}{h_3 - h_6}$$

Condensate pump work, $(h_{6s} - h_5) = v_5 (P_3 - P_2)$
 $= 0.0010102 (3.9) 10^2 = 0.394 \text{ kJ/kg}$

$$\therefore h_{6s} = 191.8 + 0.394 = 192.19 \text{ kJ/kg}$$

Similarly, $h_{1s} = h_7 + v_7 (P_1 - P_2)$
 $= 604.7 + -.0010839 (16) 10^2 = 606.43 \text{ kJ/kg}$

$$\therefore m = \frac{604.7 - 192.19}{2737.6 - 192.19} = 0.162$$

c) Power output or $W_T = (h_2 - h_3) + (1 - m) (h_3 - h_4)$
 $= (3025 - 2737.6) + (1 - 0.162) (2737.6 - 2268.54)$
 $= 680.44 \text{ kJ/kg}$

For 1kg/s of steam, $W_T = 680.44 \text{ kW}$

d) Overall thermal efficiency, $\eta_0 = \frac{W_{net}}{Q_H}$

$$W_P = (1 - m) (h_{6s} - h_5) + 1 (h_{1s} - h_7)$$

$$= (1 - 0.162) (192.19 - 191.8) + 1 (606.43 - 604.7)$$

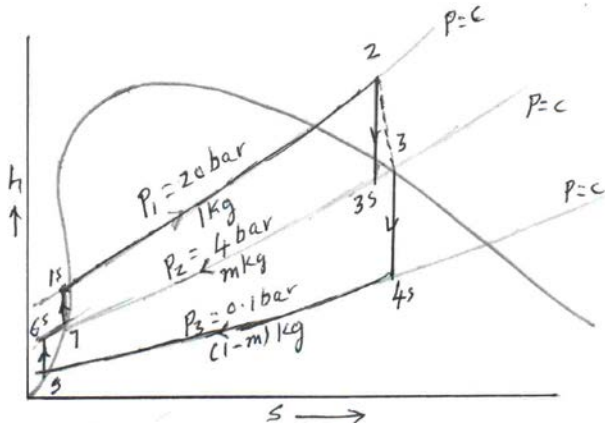
$$= 2.057 \text{ kJ/kg}$$

$$W_{net} = 680.44 - 2.057 = 678.38 \text{ kJ/kg}$$

$$Q_H = 1 (h_2 - h_{1s}) = (3025 - 606.43) = 2418.57 \text{ kJ/kg}$$

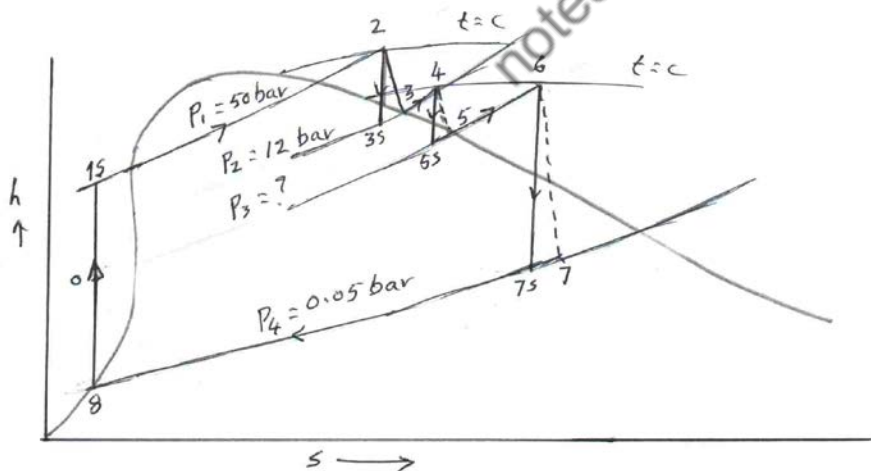
$$\therefore \eta_0 = \frac{678.38}{2418.57} = 0.2805$$

Using Moiller Diagram



4. Steam at 50 bar, 350°C expands to 12 bar in a HP stage, and is dry saturated at the stage exit. This is now reheated to 280°C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280°C . Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.

Solution:



$$P_1 = 50 \text{ bar} \quad t_2 = 350^{\circ}\text{C} \quad P_2 = 12 \text{ bar} \quad t_4 = 280^{\circ}\text{C}, \quad t_6 = 280^{\circ}\text{C}$$

$$P_3 = ? \quad P_4 = 0.05 \text{ bar}$$

From Mollier diagram

$$h_2 = 3070 \text{ kJ/kg} \quad h_{3s} = 2755 \text{ kJ/kg} \quad h_3 = 2780 \text{ kJ/kg} \quad h_4 = 3008 \text{ kJ/kg}$$

$$(a) \eta_t \text{ for HP stage} = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{3070 - 2780}{3070 - 2755}$$

$$= 0.921$$

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$$\begin{aligned} h_2 - h_3 &= h_4 - h_5 \\ \text{i.e., } 3070 - 2780 &= 3008 - h_5 \\ \therefore h_5 &= 2718 \text{ kJ/kg} \end{aligned}$$

Since state 5 is on the saturation line, we find from Mollier chart, $P_3 = 2.6$ bar, Also from Mollier chart, $h_{5s} = 2708$ kJ/kg, $h_6 = 3038$ kJ/kg, $h_{7s} = 2368$ kJ/kg

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93 \text{ kJ/kg}$$

$$\begin{aligned} \therefore \text{At a pressure } 0.05 \text{ bar, } h_7 &= h_{f7} + x_7 h_{fg7} \\ 2420.93 &= 137.8 + x_7 (2423.8) \\ \therefore x_7 &= 0.941 \end{aligned}$$

$$\begin{aligned} \text{Total power output} &= (h_2 - h_3) + (h_4 - h_5) + (h_6 - h_7) \\ &= (3070 - 2780) + (3008 - 2718) + (3038 - 2420.93) \\ &= 1197.07 \text{ kJ/kg} \end{aligned}$$

\therefore Total power output /kg of steam = 1197.07 kW

For $P_4 = 0.05$ bar from steam tables, $h_8 = 137.8$ kJ/kg;

$$\begin{aligned} W_P &= 0.0010052 (50 - 0.05) 10^2 = 5.021 \text{ kJ/kg} \\ &= h_8 - h_{1s} \\ \therefore h_{1s} &= 142.82 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Heat supplied, } Q_H &= (h_2 - h_{1s}) + (h_4 - h_3) + (h_6 - h_5) \\ &= (3070 - 142.82) + (3008 - 2780) + (3038 - 2718) \\ &= 3475.18 \text{ kJ/kg} \end{aligned}$$

$$W_{\text{net}} = W_T - W_P = 1197.07 - 5.021 = 1192.05 \text{ kJ/kg}$$

$$\therefore \eta_{th} = \frac{W_{\text{net}}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m}$$

$$\therefore T_m = 465.6 \text{ K}$$

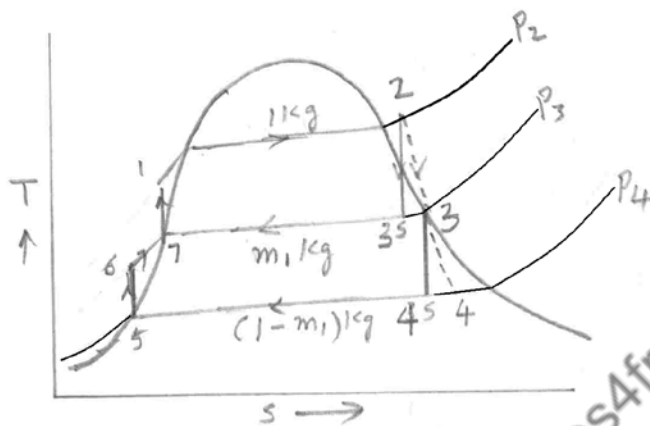
Or

$$T_m = \frac{h_2 - h_{1s}}{S_2 - S_{1s}} = \frac{3070 - 142.82}{6.425 - 0.4763} = 492 \text{ K}$$

$$SSC = \frac{3600}{1192.05} = 3.02 \text{ kg / kWh}$$

5. Steam at 30 bar and 350°C is supplied to a steam turbine in a practical regenerative cycle and the steam is bled at 4 bar. The bled steam comes out as dry saturated steam and heats the feed water in an open feed water heater to its saturated liquid state. The rest of the steam in the turbine expands to condenser pressure of 0.1 bar. Assuming the turbine η to be same before and after bleeding determine (i) the turbine η , (ii) steam quality at inlet to condenser, (iii) mass flow rate of bled steam per unit mass rate at turbine inlet and (iv) the cycle η .

Solution:



$$P_2 = 30 \text{ bar} \quad t_2 = 350^\circ\text{C} \quad P_3 = 4 \text{ bar} \quad P_4 = 0.1 \text{ bar}$$

$$h_3 = h_g \text{ at } P_3 = 4 \text{ bar}, = 2737.6 \text{ kJ/kg}$$

From superheated steam tables,

$$h_2 = h_3 = h_g \Big|_{P_3=4\text{bar}} = 2737.6 \text{ kJ/kg}$$

$$h_2 = h \Big|_{P_2=30\text{bar} \ \& \ t_2=350^\circ\text{C}} = 3117.5 \text{ kJ/kg} \quad \text{and} \quad S_2 = 6.7471 \text{ kJ/kg-K}$$

$$h_5 = h_f \Big|_{P_5=0.1\text{bar}} = 191.8 \text{ kJ/kg}$$

$$h_7 = h_f \Big|_{P_7=4\text{bar}} = 604.7 \text{ kJ/kg}$$

Process 2-3s is isentropic, i.e., $S_2 = S_{3s}$

$$6.7471 = 1.7764 + x_{3s} (5.1179)$$

$$\therefore x_{3S} = 0.971$$

$$\begin{aligned}\therefore h_{3S} &= h_{f3} + x_{3S} h_{fg3} \\ &= 604.7 + 0.971 (2132.9) \\ &= 2676.25 \text{ kJ/kg}\end{aligned}$$

Process 3-4s is isentropic i.e., $S_3 = S_{4S}$

$$\text{i.e., } 6.8943 = 0.6493 + x_{4S} (7.5018)$$

$$\therefore x_{4S} = 0.832$$

$$\therefore h_{4S} = 191.8 + 0.832 (2392.9) = 2183.8 \text{ kJ/kg}$$

Given, η_t (before bleeding) = η_t (after bleeding)

$$\text{We have, } \eta_t \text{ (before bleeding)} = \frac{h_2 - h_3}{h_2 - h_{3S}} = \frac{3117.5 - 2737.6}{3117.5 - 2676.25} = 0.86$$

$$\therefore 0.86 = \frac{h_3 - h_4}{h_3 - h_{4S}} = \frac{2737.6 - h_4}{2737.6 - 2183.8} \quad \therefore h_4 = 2261.33 \text{ kJ/kg}$$

But $h_4 = h_{f4} + x_4 h_{fg4}$

$$2261.33 = 191.8 + x_4 (2392.9)$$

$$\therefore x_4 = 0.865$$

i.e., Dryness fraction at entry to condenser = $x_4 = \mathbf{0.865}$

iii) Let m kg of steam is bled. Applying energy balance to FWH,

$$mh_3 + (1 - m) h_6 = h_7$$

$$\begin{aligned}\text{We have } W_{P1} &= (h_6 - h_5) = \int v dp \\ &= 0.0010102 (4 - 0.1) 10^5 / 10^3 \\ &= 0.394 \text{ kJ/kg}\end{aligned}$$

$$\therefore h_6 = 0.394 + 191.8 = 192.19 \text{ kJ/kg}$$

Substituting,

$$m(2737.6) + (1 - m) 192.19 = 604.7$$

$$\therefore m = 0.162 \text{ kg}$$

$$\text{Also, } W_{P2} = (h_1 - h_7) = \int v dp$$

$$= 0.0010839 \times (30 - 4) 10^2$$

$$= 2.82 \text{ kJ/kg}$$

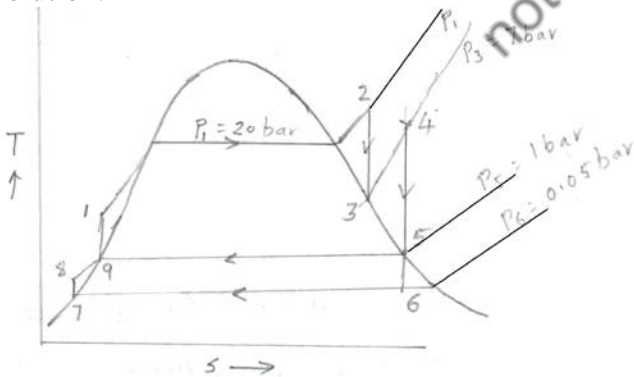
$$\therefore h_1 = 2.82 + 604.7 = 607.52 \text{ kJ/kg}$$

$$\therefore \eta_{\text{cycle}} = \frac{W_T = W_P}{Q_H} = \frac{[(h_2 - h_3) + (1 - m)(h_3 - h_4)] - [(1 - m)(h_6 - h_5) + (h_1 - h_2)]}{(h_2 - h_1)}$$

$$\eta_{\text{cycle}} = 0.31$$

6. In an ideal reheat regenerative cycle, the high pressure turbine receives steam at 20 bar, 300°C. After expansion to 7 bar, the steam is reheated to 300°C and expands in an intermediate pressure turbine to 1 bar. A fraction of steam is now extracted for feed water heating in an open type FWH. The remaining steam expands in a low pressure turbine to a final pressure of 0.05 bar. Determine (i) cycle thermal η , (ii) specific steam consumption, (iii) quality of steam entering condenser.

Solution:



$$h_2 = h|_{20 \text{ bar}, 300^\circ \text{C}} = 3025 \text{ kJ/kg} \text{ and } s_2 = 6.7696 \text{ kJ/kg-K}$$

Process 2-3 is isentropic

$$\text{i.e., } S_2 = S_3$$

$$6.7696 = 1.9918 + x_3(4.7134)$$

$$\therefore x_3 = 1.014$$

i.e., state 3 can be approximated as dry saturated.

$$\therefore h_3 = h|_{7\text{bar, dry sat.}} = 2762\text{kJ/kg}$$

$$\therefore h_4 = h|_{7\text{bar, }300^\circ\text{C}} = 3059.8\text{kJ/kg} \text{ and } s_4 = 7.2997\text{ kJ/kg-K}$$

Process 4-5 is isentropic i.e., $S_4 = S_5$

$$7.2997 = 1.3027 + x_5 (6.0571)$$

$$\therefore x_5 = 0.99$$

$$\therefore h_5 = h_{f5} + x_5 h_{fg5} = 417.5 + 0.99 (2257.9) = 2652.9\text{ kJ/kg}$$

Process 5-6 is isentropic i.e., $S_5 = S_6$

$$7.2997 = 0.4763 + x_6 (7.9197)$$

$$\therefore x_6 = 0.862$$

$$\therefore h_6 = 137.8 + 0.862 (2423.8) = 2226.1\text{ kJ/kg}$$

$$h_7 = h_f|_{0.05\text{bar}} = 137.8\text{ kJ/kg}$$

Neglecting W_{P1} , $h_8 = h_7$, Also neglecting W_{P2} , $h_9 = h_1$

$$\therefore h_9 = h_f|_{1\text{bar}} = 417.5\text{ kJ/kg}$$

Applying energy balance to FWH

$$mh_5 + (1 - m) h_8 = h_9$$

$$\text{i.e., } m (2652.9) + (1 - m) 137.8 = 417.5 \quad \therefore m = 0.111\text{ kg/kg of steam}$$

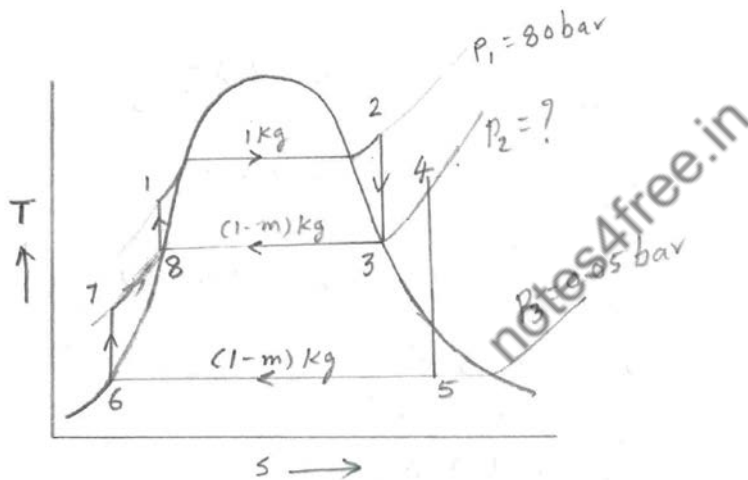
$$(i) \quad \eta_c = \frac{(h_2 - h_3) + (h_4 - h_5) + (1 - m)(h_5 - h_6)}{(h_2 - h_1) + (h_4 - h_3)} = 0.35$$

$$(ii) \quad SSC = \frac{3600}{W_{net}} = 3.57\text{kg/kWh}$$

(iii) Quality of steam entering condenser, $x_6 = 0.862$

7. The net power output of a regenerative – reheat cycle power plant is 80mW. Steam enters the high pressure turbine at 80 bar, 500°C and expands to a pressure P_2 and emerges as dry vapour. Some of the steam goes to an open feed water heater and the balance is reheated at 400°C at constant pressure P_2 and then expanded in the low pressure turbine to 0.05 bar. Determine (i) the reheat pressure P_2 , (ii) the mass of bled steam per kg boiler steam, (iii) the steam flow rate in HP turbine, (iv) cycle η . Neglect pump work. Sketch the relevant lines on h-s diagram. Assume expansion in the turbines as isentropic.

Solution:



$$P = 80000 \text{ kW} \quad P_1 = 80 \text{ bar} \quad t_2 = 500^\circ\text{C} \quad P_2 = ? \quad t_3 = 400^\circ\text{C}$$

$$P_3 = 0.05 \text{ bar} \quad m = ? \quad \dot{m}_s = ? \quad \eta_{\text{cycle}} = ?$$

$$h_2 = h \Big|_{80 \text{ bar}, 500^\circ\text{C}} = 3398.8 \text{ kJ/kg} \text{ and } s_2 = 6.7262$$

Process 2-3 is isentropic i.e., $S_2 = S_3 = 6.7262 \text{ kJ/kg-K}$

Given state 3 is dry saturated i.e., $S_3 = 6.7262 = S_g \Big|_{P_2}$

From table A – 1, for dry saturated steam, at $P = 6.0 \text{ bar}$, $S_g = 6.7575$

and at $P = 7.0$ bar, $S_g = 6.7052$

Using linear interpolation,

$$\Delta P = \frac{6.0 - 7.0}{(6.7575 - 6.7052)} x (6.7262 - 6.7052) = 0.402 \text{ bar}$$

$$\therefore (i) P_2 = 6 + 0.402 = 6.402 \text{ bar}$$

$$\therefore h_3 = h \Big|_{P_2=6.4\text{bar}}$$

From table A - 1, For $P = 6$ bar $h_g = 2755.5$ $S_g = 6.7575$

For $P = 7$ bar, $h_g = 2762.0$ $S_g = 6.7052$

$$\therefore \text{For } P = 6.4 \text{ bar} \Rightarrow \frac{2762 - 2755.5}{1} x (0.4) + 2755.5 = 2758.1 \text{ kJ / kg}$$

$$\therefore h_3 = 2758.1 \text{ kJ/kg}$$

$$h_4 = h \Big|_{6.4\text{bar}, 400^\circ\text{C}}$$

From superheated steam tables, For $P = 6.0$ bar, $h = 3270.6$ $s = 7.709$

$P = 7.0$ bar, $h = 3269.0$ $s = 7.6362$

$$\therefore \text{For } 6.4 \text{ bar, } h_4 \Rightarrow 3269.96 \text{ kJ/kg}$$

$$S_4 \Rightarrow 7.6798 \text{ kJ/kg-K}$$

Process 4-5 is isentropic, $S_4 = S_5$

$$\text{i.e., } 7.6798 = 0.4763 + x_5 (7.9197)$$

$$\therefore x_5 = 0.909$$

$$\therefore h_5 = 137.8 + 0.909 (2423.8) = 2342.41 \text{ kJ/kg}$$

$$h_6 = h_f \Big|_{0.05\text{bar}} = 137.8 \text{ kJ / kg}$$

$h_7 = h_6$ (since W_{P1} is neglected)

$$h_8 = h_f \Big|_{6.4\text{bar}} = 681.1 \text{ kJ / kg}$$

$h_1 = h_8$ (since W_{P2} is neglected)

(ii) Applying energy balance to FWH,

$$mh_3 + (1 - m) h_7 = h_8$$

$$m (2758.1) + (1 - m) 137.8 = 681.1$$

$$\therefore m = 0.313 \text{ kg/kg of steam}$$

$$\begin{aligned} \text{(iii) } W_1 = W_{HP} &= (h_2 - h_3) = (3398.8 - 2758.1) \\ &= 640.7 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} W_2 = W_{LP} &= (1 - m) (h_4 - h_5) \\ &= (1 - 0.313) (3269.96 - 2342.41) \\ &= 637.2 \text{ kJ/kg} \end{aligned}$$

$$\therefore W_{net} = W_1 + W_2 = 1277.9 \text{ kJ/kg}$$

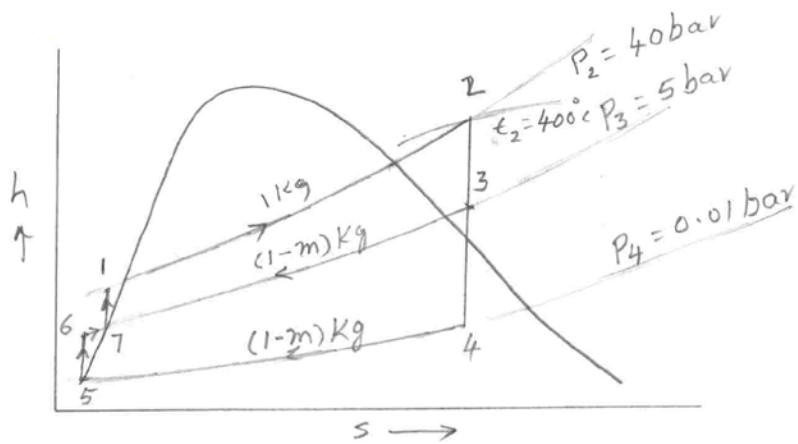
$$\therefore \text{Steam flow rate through HP turbine} = \frac{\text{Power}}{W_{net}} = \frac{80000}{1277.9} = 62.6 \text{ kg/s}$$

$$\text{(iv) } \eta_{\text{cycle}} = ? \quad Q_H = (h_2 - h_1) + (1 - m) (h_4 - h_3) = 3069.35 \text{ kJ/kg}$$

$$\therefore \eta_{\text{cycle}} = \frac{W_{net}}{Q_H} = \frac{1277.9}{3069.35} = 0.42$$

8. In a single heater regenerative cycle, the steam enters the turbine at 30 bar, 400°C and the exhaust pressure is 0.01 bar. The feed water heater is a direct contact type which operates at 5 bar. Find (i) thermal η and the steam rate of the cycle, (ii) the increase in mean temperature of heat addition, η and steam rate as compared to the Rankine cycle without regeneration. Pump work may neglected.

Solution:



$$P_2 = 40 \text{ bar} \quad t_2 = 400^\circ\text{C} \quad P_4 = 0.01 \text{ bar} \quad P_3 = 5 \text{ bar}$$

From h-s diagram,

$$h_2 = h|_{30 \text{ bar}, 400^\circ\text{C}} = 3230 \text{ kJ/kg}$$

$$h_3 = 2790 \text{ kJ/kg}$$

$$h_4 = 1930 \text{ kJ/kg}$$

$$h_5 = 29.3 \text{ kJ/kg}$$

$$h_7 = 640.1 \text{ kJ/kg}$$

Since pump work may neglect, $h_6 = h_5$ & $h_1 = h_7$

(i) $\eta_{\text{cycle}} = ?$

Let m = mass of steam bled per kg boiler steam

Applying SFEE to FWH,

$$mh_3 + (1 - m) h_6 = h_7$$

$$m (2790) + (1 - m) 29.3 = 640.1$$

$\therefore m = 0.221 \text{ kg/kg}$ of boiler steam

$$\begin{aligned} W_T &= (h_2 - h_3) + (1 - m) (h_3 - h_4) \\ &= (3230 - 2790) + (1 - 0.221) (2790 - 1930) \\ &= 1109.73 \text{ kJ/kg} \end{aligned}$$

$$Q_H = (h_2 - h_1) = (3230 - 640.1)$$

$$= 2589.9 \text{ kJ/kg}$$

$$\therefore \eta_{\text{cycle}} = \frac{W_T}{Q_H} = 0.428 \quad \text{Since } W_P \text{ is neglected}$$

$$(ii) \text{ steam rate} = \frac{3600}{W_T} = 3.24 \text{ kg / kWh}$$

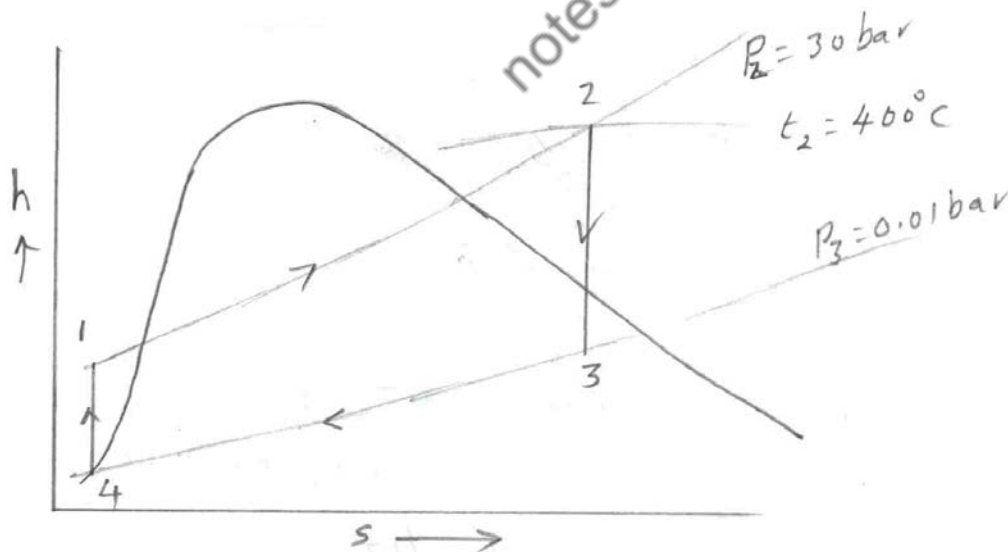
$$(iii) \text{ Mean temperature of heat addition, } \Delta T_m = \frac{Q_H}{s_2 - s_5}$$

From h-s diagram, $s_2 = 6.83 \text{ kJ/kg-K}$

From steam tables, $s_5 = 0.1060 \text{ kJ/kg-K}$

$$\therefore \Delta T_m = \frac{2589.9}{(6.83 - 0.106)} = 385.2^\circ \text{K}$$

Case (ii) Rankine cycle without Regeneration:



From h-s diagram,

$$h_2 = 3230 \text{ kJ/kg}$$

$$h_3 = 1930 \text{ kJ/kg}$$

$$h_4 = 29.3 \text{ kJ/kg}$$

$$h_1 = h_4$$

$$S_2 = 6.83 \text{ kJ/kg-K}$$

$$S_4 = 0.1060 \text{ kJ/kg-K}$$

$$(i) \quad \eta_{cycle} = \frac{W_T}{Q_H} = \frac{(h_2 - h_3)}{(h_2 - h_1)}$$

$$= \frac{1300}{3200.7} = 0.41$$

$$(ii) \quad \text{Steam rate} = \frac{3600}{W_T} = 2.76 \text{ kg / kWh}$$

$$(iii) \quad \text{Mean temperature of heat addition, } \Delta T_m = \frac{3200.7}{(6.83 - 0.106)} = 476^0 \text{ K}$$

Comparison	ΔT_m	η_{cycle}	Steam rate
Rankine cycle with regeneration	385.2 K	0.428	3.24 kg/kWh
Rankine cycle without regeneration	476 ⁰ K	0.41	2.76kg/kWh
\therefore Increase w.r.t Rankine cycle	- 0.19 i.e., (-19%)	0.044 (4.4%)	0.174 (17.4%)

Refrigeration

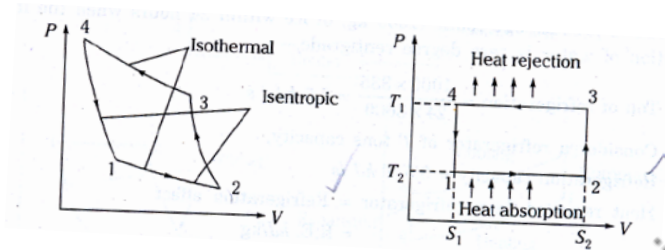
Definition

Refrigeration is the process of providing and maintaining temperature of the system below that of the surrounding atmosphere.

Carnot Cycle

The reversed carnot cycle can be considered in refrigeration system.

$$C.O.P = \frac{T_2}{T_2 - T_1} \text{ where } T_2 < T_1$$



Unit of Refrigeration

The common unit used in the field of refrigeration is known as Ton of refrigeration.

A ton of refrigeration is defined as the quantity of heat required to be removed to produce one ton (1000kg) of ice within 24 hours when the initial condition of water is 0°C

$$\text{Ton of refrigeration} = \frac{1000 \times 335}{24 \times 3600} = 3.5 \text{ kJ/s}$$

Consider a refrigerator of T tons capacity,

Refrigeration capacity = 3.5 kJ/s

Heat removed from

refrigerator = Refrigeration effect = R.E. kJ/s

Power of the compressor = work/kg of refrigerant x

mass flow rate

Air Refrigeration system working on Bell-coleman cycle

In air refrigeration system, air is used as the refrigerant which always remains in the gaseous phase. The heat removed consists only of sensible heat and as a result, the coefficient of performance (C.O.P) is low.

The various processes are:

Process 1-2:

The air leaving the evaporator enters a compressor. Where it is compressed isentropically to higher pressure and temperature.

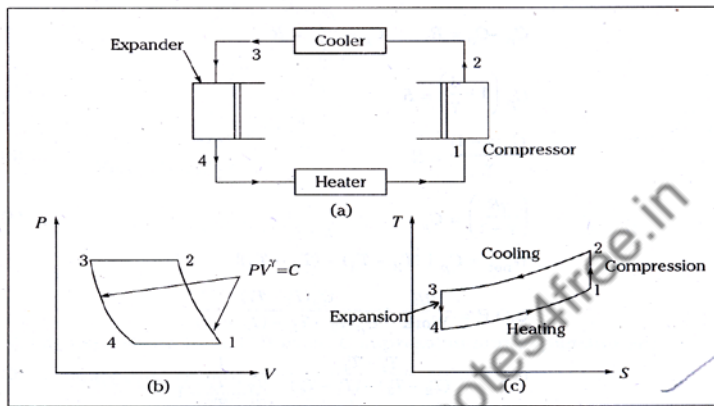
Process 2-3:

This high pressure, high temperature air, then enters a cooler where it is cooled at constant pressure to a low temperature.

Process 3-4: This high pressure, low temperature air is then expanded in an expander to lower pressure and temperature in a isentropic manner. At point 4, the temperature of the air will be lowest.

Process 4-1: This low temperature air is then passed through the heater coils where it absorbs heat from the space to be cooled namely the refrigerator and the air gets heated back to the initial temperature, but in the process, it cools the refrigerator. And the cycle repeats.

Air refrigeration system



Expression C.O.P when compression and expansion are Isentropic

Refrigeration Effect = Heat removed from the refrigerator

$$= C_p (T_1 - T_4) \text{ kJ / kg}$$

$$\text{Work input} = W_C - W_E = \gamma \left(\frac{P_2 V_2 - P_1 V_1}{\gamma - 1} \right) - \gamma \left(\frac{P_3 V_3 - P_4 V_4}{\gamma - 1} \right)$$

$$\text{Work input} = W_C - W_E = \left(\frac{\gamma}{\gamma - 1} \right) [R(T_2 - T_1) - R(T_3 - T_4)]$$

$$W_{net} = \left(\frac{\gamma R}{\gamma - 1} \right) [(T_2 - T_1) - (T_3 - T_4)]$$

But $C_p = \frac{\gamma R}{\gamma - 1}$

$$W_{net} = C_p [(T_2 - T_1) - (T_3 - T_4)]$$

Process 1-2 is isentropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \quad \text{-----(2)}$$

Process 3-4 is isentropic

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \quad \text{-----(3)}$$

From (2) and (3)

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$C.O.P = \frac{RE}{Work} = \frac{C_p(T_1 - T_4)}{C_p[(T_2 - T_1) - (T_3 - T_4)]}$$

$$C.O.P = \frac{(T_1 - T_4)}{[(T_2 - T_1) - (T_3 - T_4)]} = \frac{1}{\frac{T_2 - T_3}{T_1 - T_4} - 1} \quad \text{(1)}$$

$$\frac{T_2}{T_3} = \frac{T_1}{T_4}$$

$$\frac{T_2}{T_3} - 1 = \frac{T_1}{T_4} - 1$$

$$\frac{T_2 - T_3}{T_3} = \frac{T_1 - T_4}{T_4}$$

$$\frac{T_2 - T_3}{T_1 - T_4} = \frac{T_3}{T_4} \quad \text{-----(4)}$$

From (1) and (4)

$$C.O.P = \frac{1}{\frac{T_3}{T_4} - 1}$$

$$C.O.P = \frac{T_4}{T_3 - T_4}$$

For Polytropic process

Net work

$$W_{\text{net}} = \left(\frac{n}{n-1} \right) \left(\frac{\gamma-1}{\gamma} \right) C_p [(T_2 - T_1) - (T_3 - T_4)]$$

$$COP = \frac{T_4}{(T_3 - T_4) \left(\frac{n}{n-1} \right) \left(\frac{\gamma-1}{\gamma} \right)}$$

Advantages of air refrigeration system

1. Air is cheap, easily available.
2. It is not flammable.
3. For a given capacity, weight of air refrigeration system is less compared to other system and hence it is widely used for aircraft cooling.

Disadvantages

1. Since heat removed by air consists only of sensible heat, weight of air required is high.
2. C.O.P of the system is low compared to other systems.

Problem 1

A cold storage is to be maintained at -5°C (268k) while the surroundings are at 35°C . The heat leakage from the surroundings into the cold storage is estimated to be 29kW. The actual C.O.P of the refrigeration plant is one third of an ideal plant working between the same temperatures. Find the power required to drive the plant. (VTU Jan 2007)

Solution :-

$$T_1 = 35^\circ\text{C} = 308\text{k} \quad T_2 = 5^\circ\text{C} = 268\text{k}$$

C.O.P of the ideal plant is nothing but

C.O.P based on carnot cycle.

$$\begin{aligned} \therefore \text{C.O.P ideal} &= \frac{T_2}{T_1 - T_2} \\ &= \frac{268}{308 - 268} = 6.7 \end{aligned}$$

$$\begin{aligned} \text{Actual C.O.P} &= \frac{1}{3} \text{ideal C.O.P} \\ &= \frac{1}{3} \times 6.7 = 2.233 \end{aligned}$$

Q2 = The heat removed from low temperature reservoir (cold storage) must be equal to heat leakage from surroundings to the cold storage (which is 29kW)

$$Q_2 = 29 \text{ kW}$$

$$\text{Actual C.O.P} = \frac{Q_2}{W}$$

$$W = \frac{Q_2}{\text{Actual C.O.P}} = \frac{29}{2.233}$$

$$\text{Power required} = 12.98 \text{ kW}$$

Problem 2

A refrigeration machine of 6 tones capacity working on Bell coleman cycle has an upper limit pressure of 5.2 bar. The pressure and temperature at the start of the compression are 1 bar and 18°C respectively. The cooled compressed air enters the expander at 41°C. assuming both expansion and compression to be adiabatic with an index of 1.4.

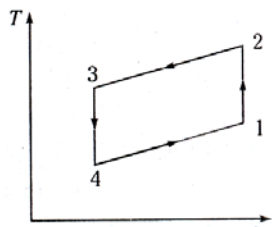
Calculate:-

- (i) Co-efficient of performance.
- (ii) Quantity of air circulated per minute.
- (iii) Piston displacement of compressor and expander
- (iv) Bore of compression and expansion cylinder when the unit runs at 240 rpm and is double acting with stroke length = 200 mm
- (v) Power required to drive the unit

Solution :-

$$T_1 = 18^\circ\text{C} \quad P_1 = 1 \text{ bar}$$

$$T_3 = 41^\circ\text{C} \quad P_2 = 5.2 \text{ bar}$$



$$\begin{aligned} \text{Work input} &= C_p [(T_2 - T_1) - (T_3 - T_4)] \\ &= 1.005 [(466 - 291) - (314 - 196)] = 57 \text{ kJ / kg} \end{aligned}$$

$$\begin{aligned} \text{C.O.P} &= \frac{\text{Re griferation effect}}{\text{Work input}} \\ &= \frac{95.42}{57} = 1.67 \end{aligned}$$

$$\text{Re frigeration capacity} = 6 \text{ tons} = 6 \times 3.5 = 21 \text{ kJ/s}$$

$$\begin{aligned} \text{Mass of air/sec} &= \frac{\text{Re griferation capacity}}{\text{R.E}} \\ &= \frac{21}{95.42} = 0.22 \text{ kg / s} \end{aligned}$$

$$\begin{aligned} \text{Power required} &= \text{workdone/kg of air} \times \text{Mass of air/sec} \\ &= 57 \times 0.22 = 12.54 \text{ kW} \end{aligned}$$

$$\text{Mass of air/min} = 0.22 \times 60 = 13.2 \text{ kg/min}$$

$$V_1 = \frac{mRT_1}{P_1} = \frac{13.2 \times 0.287 \times 291}{1 \times 10^2} = 11 \text{ m}^3 / \text{min}$$

$$\text{Piston displacement of compressor } V_1 = 11 \text{ m}^3 / \text{min}$$

$$V_4 = \frac{mRT_4}{P_4} = \frac{13.2 \times 0.287 \times 196}{1 \times 10^2} = 7.42 \text{ m}^3 / \text{min}$$

$$\text{Piston displacement of expander } V_4 = 7.42 \text{ m}^3 / \text{min}$$

$$\text{But } V_1 = 2 \frac{\pi}{4} d_1^2 LN$$

$$11 = 2 \frac{\pi}{4} d_1^2 \times 0.2 \times 240$$

$$d_1 = \text{diameter of compressor cylinder} = 0.38 \text{ m} = 38 \text{ cm}$$

$$V_4 = 2 \frac{\pi}{4} d_2^2 LN$$

$$7.42 = 2 \frac{\pi}{4} d_1^2 \times 0.2 \times 240$$

$$d_1 = \text{diameter of expander cylinder} = 0.313 \text{ m} = 31.3 \text{ cm}$$

Problem3 An air refrigerator system operating on Bell Coleman cycle, takes in air from cold room at 268 K and compresses it from 1 bar to 5.5 bar the index of compression being 1.25. the compressed air is cooled to 300 K. the ambient temperature is 20°C. Air expands in expander where the index of expansion is 1.35.

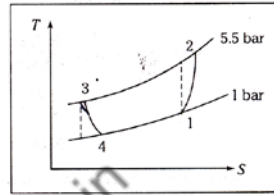
Calculate:

- i) **C.O.P of the system**
- ii) **Quantity of air circulated per minute for production of 1500 kg of ice per day at 0°C from water at 20°C.**
- iii) **Capacity of the plant.**

Solution

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = 268(5.5)^{\frac{1.25-1}{1.25}}$$

$$= 376.8\text{K}$$



$$T_4 = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{\gamma-1}{\gamma}} = 300 \left(\frac{1}{5.5} \right)^{\frac{1.35-1}{1.35}} = 192.83\text{K}$$

$$W_C = \left(\frac{n}{n-1} \right) \left(\frac{\gamma-1}{\gamma} \right) C_p (T_2 - T_1)$$

$$= \left(\frac{1.25}{1.25-1} \right) \left(\frac{1.4-1}{1.4} \right) 1.005(376.8 - 268) = 156.2\text{kJ / kg}$$

$$W_E = \left(\frac{n}{n-1} \right) \left(\frac{\gamma-1}{\gamma} \right) C_p (T_3 - T_4)$$

$$= \left(\frac{1.35}{1.35-1} \right) \left(\frac{1.4-1}{1.4} \right) 1.005(300 - 192.83) = 118.69\text{kJ / kg}$$

$$\text{Network} = W_C - W_E = 156.2 - 118.69 = 37.5\text{kJ / kg}$$

$$R.E = C_p (T_1 - T_4) = 1.005(268 - 192.83) = 75.54\text{kJ / s}$$

$$C.O.P = \frac{R.E}{\text{work}} = \frac{75.54}{37.5} = 2$$

$$\begin{aligned} \text{Heat extracted/kg of ice} &= C_{pw}(20-0) + L \\ &= 4.187(20) + 335 = 418.74 \text{kJ/kg} \end{aligned}$$

$$\text{Mass of ice produced/sec} = \frac{1500}{24 \times 3600} = 0.0173 \text{kg/s}$$

$$\text{Actual heat extracted/sec} = 418.74 \times 0.0173$$

$$\text{or Refrigeration capacity} = 7.26 \text{kJ/s} = \frac{7.26}{3.5} = 2.02 \text{tons}$$

$$\begin{aligned} \text{Mass flow rate} &= \frac{\text{Refrigeration Capacity}}{\text{Refrigeration effect}} = \frac{7.26}{75.54} \\ &= 0.096 \text{kg/s} \end{aligned}$$

Problem 4

An air refrigeration system is to be designed according to the following specifications

Pressure of air at compressor inlet = 101 kPa

Pressure of work at compressor outlet = 404 kPa

Pressure loss in the inter cooler = 12 kPa

Pressure loss in the cold chamber = 3 kPa

Temperature of air at compressor inlet = -7°

Temperature of air at turbine inlet = 27°

Isentropic efficiency of compressor = 85%

Isentropic efficiency of turbine = 85%

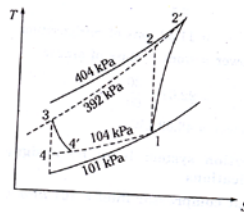
Determine

- C.O.P of cycle
- Power required to produce 1 ton of refrigeration
- Mass flow rate of air required for 1 ton of refrigeration

Solution :-

$$T_1 = -7^\circ\text{C} \quad P_1 = 101 \text{kPa}$$

$$T_3 = 27^\circ\text{C} \quad \eta_T = 0.85; \eta_C = 0.85$$



$$\text{Process 1-2 is isentropic, Hence } T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$= 266 \left(\frac{404}{101} \right)^{\frac{1.4-1}{1.4}} = 395.4 \text{K}$$

$$\eta_C = \frac{T_2 - T_1}{T'_2 - T_1} \text{ or } T'_2 - T_1 = \frac{395.4 - 266}{0.88}$$

$$T'_2 = 418.2k$$

$$P_4 - P_1 = 0.03P_1 \quad \therefore P_4 = 1.03P_1 = 1.03 \times 101 = 104kPa$$

$$P_2 - P_3 = 0.03P_2 \quad \therefore P_3 = 0.97P_2 = 0.97 \times 404 = 392kPa$$

$$\text{Process 3-4 is isentropic, } \therefore T_4 = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{\gamma-1}{\gamma}}$$

$$= 300 \left(\frac{104}{392} \right)^{\frac{1.4-1}{1.4}} = 202.3K$$

$$\eta_E = \frac{T_3 - T'_4}{T_3 - T_4} \quad \therefore T'_4 = T_3 - \eta_T (T_3 - T_4)$$

$$T'_4 = 300 - 0.85 \times [300 - 205.3] = 216.53k$$

$$\text{Refrigeration effect/kg of air} = C_p (T_1 - T'_4)$$

$$= 1.005 \times [266 - 216.53] = 50.47kJ/kg$$

$$\text{Compressor work/kg of air} = C_p (T'_2 - T_1)$$

$$= 1.005 \times [418.2 - 266] = 152.96kJ/kg$$

$$\text{Turbine work/kg of air } W_T = C_p (T_3 - T_4)$$

$$= 1.005 \times [300 - 216.53] = 84.9kJ/kg$$

$$\text{Net work Input/kg of air } W_{net} = W_C - W_T$$

$$= 152.96 - 80.9 = 72.06kJ/kg$$

$$C.O.P = \frac{RE}{Work} = \frac{46.73}{72.06} = 0.73$$

Power required per tons of refrigeration

$$= \frac{\text{Refrigeration capacity}}{C.O.P}$$

Refrigeration capacity = 1 ton = 3.5kJ/s

$$\text{Mass of air} = \frac{\text{Refrigeration capacity}}{RE}$$

$$= \frac{3.5}{50.47} = 0.075kg/s$$

$$\text{Power} = W_{net} \times \text{mass of air} / \text{sec} = 72.06 \times 0.075 = 5.42kW$$

Psychrometrics and Air-conditioning Systems

Problem 1

Moist air at 30°C, 1.01325 bar has a relative humidity of 80%. Determine without using the psychrometry chart

- 1) Partial pressures of water vapour and air
- 2) Specific humidity
- 3) Specific Volume and
- 4) Dew point temperature (V.T.U. July 2004)

Solution: At 30°C from table $p_{vs} = 4.2461 \text{ kPa}$

$$\phi = \frac{p_v}{p_{vs}}$$

$$p_v = 0.8 \times 4.2461 = 3.397 \text{ kPa}$$

$$\omega = \frac{0.622 p_v}{p - p_v} = 0.622 \times \frac{3.397}{101.325 - 3.397}$$
$$= 0.213 \text{ kg/kg of dry air.}$$

Corresponding to $p_v = 3.397 \text{ kPa}$ from tables, we get dew point temperature = 28.9°C

Problem 2:

Atmospheric air at 101.325 kPa has 30°C DBT and 15°C DPT. Without using the psychrometric chart, using the property values from the table, Calculate

1. Partial pressure of air and water vapour
2. Specific humidity
3. Relative humidity
4. Vapour density and
5. Enthalpy of moist air

Solution:

$$p = 101.325 \text{ kPa} = 1.01325 \text{ bar}$$

$$\text{DBT} = 30^\circ\text{C},$$

$$\text{DPT} = 15^\circ\text{C}$$

From table

$$\text{Corresponding to DBT} = 30^\circ\text{C}, \text{ we have } p_{vs} = 0.042461 \text{ bar}$$

$$\text{Corresponding to DPT} = 15^\circ\text{C}, \text{ we have } p_v = 0.017051 \text{ bar}$$

$$\text{Partial pressure of air} = p - p_v = 1.01325 - 0.017051$$
$$= 0.984274 \text{ bar}$$

$$\begin{aligned} \text{Specific humidity} &= 0.622 \frac{p_v}{p_a} = \frac{0.622 \times 0.017051}{0.984274} \\ &= 0.01077 \text{ kg/kg of dry air} \end{aligned}$$

$$\begin{aligned} \text{Relative humidity} &= \frac{p_v}{p_{vs}} = \frac{0.017051}{0.042461} = 0.4015 \\ &= 40.15\% \end{aligned}$$

$$\begin{aligned} \text{Enthalpy} &= 1.005t_{db} + \omega(2500 + 1088t_{db}) \\ &= 1.005 \times 30 + 0.010775(2500 + 1.88 \times 30) \\ &= 57.69 \text{ kJ/kg of dry air} \end{aligned}$$

$$\begin{aligned} \text{Specific volume of dry air, } v_a &= \frac{RT}{P} \\ &= \frac{0.2872 \times 303}{0.98425 \times 100} = 0.874 \text{ m}^3 / \text{kg} \end{aligned}$$

$$\text{Vapour density } \rho_w = \frac{\omega}{v_a} = \frac{0.010775}{0.847} = 0.12 \text{ kg/m}^3$$

Problem 3:

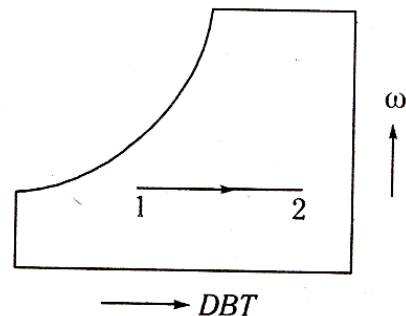
Air at 30°C DBT and 25°C WBT is heated to 40°C. if the air is 300 m³/min, find the amount of heat added/min and RH and WBT of air. Take air pressure to be 1 bar

Solution:

At 25°C WBT from tables $p_{wswb} = 0.03166$ bar

$$\begin{aligned} \therefore p_v &= (P_{VS})_{wbt} - \frac{(p - p_{vswb})(t_{db} - t_{wb})}{1547 - 1.44t_{wb}} \\ &= 0.03166 - \frac{(1 - 0.03166)(30 - 25)}{1547 - 1.44 \times 25} \\ &= 0.0284 \text{ bar} \end{aligned}$$

$$\begin{aligned} \omega_1 &= 0.622 \frac{p_v}{p - p_v} \\ &= 0.622 \left(\frac{0.0284}{1 - 0.0284} \right) \\ &= 0.0179 \text{ kJ/kg of dry air} \end{aligned}$$



At 40°C DBT

$$P_{vs} = 0.07375 \text{ bar}$$

During sensible heating ω and p_v remain constant

$$p_v = 0.0284 \text{ bar}$$

$$\begin{aligned} RH = \phi &= \frac{p_v}{p_{vs}} = \frac{0.0284}{0.07375} \\ &= 0.385 = 38.5\% \end{aligned}$$

$$\begin{aligned} H_2 &= 1.005 \times 40 + 0.0179(2500 + 1.88 \times 40) \\ &= 86.29 \text{ kJ/kg of dry air} \end{aligned}$$

$$\begin{aligned} \text{Weight of } 300 \text{ m}^3 / \text{min of air} &= \frac{(p - p_v)V}{RT} \\ &= \frac{(1 - 0.0284) \times 300 \times 10^2}{0.287 \times 303} = 335.18 \text{ kg/min} \end{aligned}$$

$$\therefore \text{Heat added/min} = 335.18(86.29 - 76) = 3449 \text{ kJ/min}$$

From chart WBT = 27.2°C

Problem 4:

One stream of air at 5.5 m³/min at 15°C and 60% RH flows into another stream of air at 35 m³/min at 25°C and 70% RH, calculate for the mixture

1) Dry bulb temperature, 2) Wet bulb temperature 3) Specific Humidity and 4) Enthalpy

Solution:

For air at 15°C and 60%RH, $V = 5.5 \text{ m}^3/\text{min}$

$$\therefore p_{vs} = 0.017051 \text{ bar}$$

$$RH = \phi = \frac{p_v}{p_{vs}}$$

$$\therefore p_v = 0.6 \times 0.017051 = 0.01023 \text{ bar}$$

$$\text{Mass of air} = \frac{(p - p_v)V}{RT} = \frac{(1.01325 - 0.01023) \times 10^2 \times 5.5}{0.287 \times 288}$$

$$m_1 = 6.672 \text{ kg/min}$$

$$\begin{aligned} \omega_1 &= \frac{0.622 p_v}{(p - p_v)} = \frac{0.622 \times 0.01023}{(1.01325 - 0.01023)} \\ &= 0.006343 \text{ kg/kg of dry air} \end{aligned}$$

$$\begin{aligned}
 H_1 &= 1.005t_{db} + \omega_1(2500 + 1.88t_{db}) \\
 &= 1.008 \times 18 + 0.006343(2500 + 1.88 \times 15) \\
 &= 34.12 \text{ J/kg of dry air}
 \end{aligned}$$

For air at 25°C and 70% RH, $V = 35 \text{ m}^3 / \text{min}$

$$P_{vs} = 0.03169 \text{ bar}$$

$$\phi = RH = \frac{p_v}{P_{vs}}$$

$$p_v = 0.03169 \times 0.7 = 0.02218 \text{ bar}$$

$$\begin{aligned}
 H_1 &= 1.005t_{db} + \omega_1(2500 + 1.88t_{db}) \\
 &= 1.008 \times 18 + 0.006343(2500 + 1.88 \times 15) \\
 &= 34.12 \text{ J/kg of dry air}
 \end{aligned}$$

For air at 25°C and 70% RH, $V = 35 \text{ m}^3 / \text{min}$

$$P_{vs} = 0.03169 \text{ bar}$$

$$\phi = RH = \frac{p_v}{P_{vs}}$$

$$p_v = 0.03169 \times 0.7 = 0.02218 \text{ bar}$$

$$\text{Mass of air} = \frac{(1.01325 - 0.02218 \times 10^2) \times 35}{0.287 \times 298}$$

$$m_2 = 40.55 \text{ kg} \cdot \text{min}$$

$$\omega_2 = \frac{0.622 \times 0.02218}{(1.01325 - 0.02218)} = 0.01392 \text{ kg / kg of dry air}$$

$$H_2 = (1.005 \times 25) + 0.01392(2500 + 1.88 \times 25)$$

$$H_2 = 60.59 \text{ kJ / kg of dry air}$$

Mass of dry air / Unit mass of moist air

$$m_{a1} = \frac{m_1}{1 + \omega_1} = \frac{6.672}{1 + 0.006343} = 6.6299$$

$$\text{Since } m_{a2} = \frac{m_2}{1 + \omega_2} = \frac{40.55}{1 + 0.01392} = 39.993$$

Then enthalpy of the mixed air,

$$\begin{aligned}
 H_{\text{mix}} &= \frac{m_{a1}(H_1) + m_{a2}(H_2)}{m_1 + m_2} \\
 &= \frac{6.6299(34.12) + 39.993(60.56)}{6.672 + 40.55} \\
 &= 55.96 \text{ kJ/kg of dry air}
 \end{aligned}$$

Specific Humidity of the mixed air,

$$\begin{aligned}\omega_{\text{mix}} &= \frac{m_{a1}(\omega_1) + m_{a2}(\omega_2)}{m_1 + m_2} \\ &= \frac{(6.6299 \times 0.006343) + (39.993 \times 0.01932)}{6.672 + 40.55} \\ &= 0.01268 \text{ kg/kg of dry air}\end{aligned}$$

$$\begin{aligned}\text{But } H_{\text{mix}} &= 1.005t_{db} + \omega_{\text{mix}}(2500 + 1.88t_{db}) \\ 55.96 &= 1.005xt_{db} + 0.01234(2500 + 1.88t_{db})\end{aligned}$$

$$t_{db} = 24.42^\circ\text{C}$$

DBT of the mixture = 24.42°C

From chart WBT = 19°C

RH = 67%

Problem 5:

An air conditioning system is designed under the following conditions

Outdoor conditions: 30°C DBT , 75% RH

Required indoor conditions: 22°C DBT , 70% RH

Amount of Free air circulated $3.33 \text{ m}^3/\text{s}$

Coil dew point temperature $\text{DPT} = 14^\circ$

The required condition is achieved first by cooling and dehumidification and then by heating. Estimate

- 1) The capacity of the cooling coil in tons of refrigeration
- 2) Capacity of the heating coil in kW
- 3) The amount of water vapour removed in kg/hr

Solution:

Locate point 'a' 30°C DBT , 75% RH out door condition

Locate point 'd' 22°C DBT , 70% RH required condition

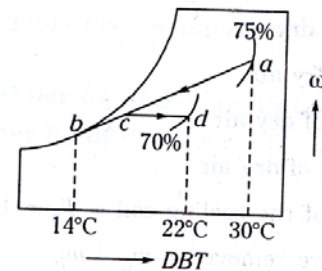
Locate point 'b' 14°C DPT , coil surface temperature

Join ab

at d, draw a horizontal line to cut the line ab at point c.

ac \rightarrow cooling and dehumidification

cd \rightarrow heating



From chart

$$H_a = 83 \text{ kJ / kg of air}$$

$$H_b = 40 \text{ kJ / kg of air}$$

$$H_d = 53 \text{ kJ / kg of air}$$

$$H_c = 48 \text{ kJ / kg of air}$$

$$W_a = 0.0202 \text{ kg / kg of dry air}$$

$$W_c = W_d = 0.0118 \text{ kg / kg of dry air}$$

$$V_{sa} = 0.88 \text{ m}^3 / \text{kg}$$

$$\text{Mass of air} = \frac{V}{V_a} = \frac{3.33}{0.88} = 3.78 \text{ kg / s}$$

$$\begin{aligned} \text{Capacity of cooling coil} &= \frac{m_a (H_a - H_c)}{3.5} \\ &= \frac{3.78(83 - 48)}{3.5} = 37.84 \text{ tons of refrigeration} \end{aligned}$$

$$\begin{aligned} \text{Capacity of heating coil} &= m_a (H_d - H_c) \\ &= 3.78(53 - 48) = 18.92 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Amount of water vapour removed} &= m_a (\omega_a - \omega_d) 3600 \\ &= 3.78(0.0202 - 0.0118) 3600 \\ &= 114.3 \text{ kg/hr} \end{aligned}$$

Problem 6:

A summer air conditioning system for hot and humid weather (DBT=32°C and 70% RH)

Consists in passing the atmosphere air over a cooling coil where the air is cooled and dehumidified. The air leaving the cooling coil is saturated at the coil temperature. It is then sensibly heated to the required comfort condition of 24°C and 50% RH by passing it over an electric heater then delivered to the room.

Sketch the flow diagram of the arrangement and represent the process undergone by the air on a skeleton psychrometric chart and determine

- 1) The temperature of the cooling coil
- 2) The amount of moisture removed per kg of dry air in the cooling coil.
- 3) The heat removed per kg of dry air in the cooling coil and
- 4) The heat added per kg of dry air in the heating coil

From chart

$$H_a = 86 \text{ kJ / kg of air}$$

$$H_b = 38 \text{ kJ / kg of air}$$

$$H_c = 48.5 \text{ kJ / kg of air}$$

$$\omega_a = 0.021 \text{ kg / kg of dry air}$$

$$\omega_b = 0.0092 \text{ kg / kg of dry air}$$

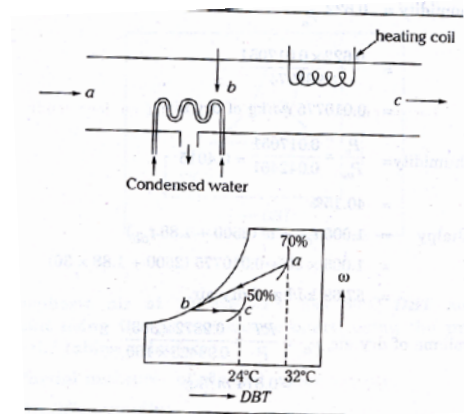
The temperature of the cooling

$$\text{coil} = T_b = 13^\circ\text{C}$$

$$\begin{aligned} \text{Amount of moisture removed} &= \omega_a - \omega_b \\ &= 0.021 - 0.0092 = 0.0108 \text{ kg/kg of dry air} \end{aligned}$$

$$\begin{aligned} \text{Heat removed} &= H_a - H_b = 86 - 38 \\ &= 48 \text{ kJ/kg of dry air} \end{aligned}$$

$$\begin{aligned} \text{Heat added} &= H_c - H_b = 48.5 - 38 \\ &= 10.5 \text{ kJ/kg of dry air} \end{aligned}$$



Locate point 'a' 32°C, 70%RH out door condition

Locate point 'c' 24°C DBT, 50%RH required condition

At c draw a horizontal line to cut the saturation

line at point 'b'

Join ab

ab → cooling and dehumidification

bc → heating

Problem 7

It is required to design an air conditioning plant for an office room with the following conditions.

Outdoor conditions: 14°CDBT, 10°CWBWBT

Required conditions: 20°CDBT, 60% RH

Amount of air circulated 0.3m³/min/person

Starting capacity of the office= 60

The required condition is achieved first by heating and then by adiabatic humidifying. Determine the following.

Heating capacity of the coil in kW and the surface temperature required, if the by pass factor of the coil is 0.4

Capacity of the humidifier.

Locate point 'a' 14°C, and 10°CWBWBT (out door condition)

Locate point 'c' 20°C DBT, 60%RH required condition

At a draw a horizontal line

At 'c' draw a constant enthalpy line to cut the
horizontal line at point 'b'

Join ab

ab → heating

bc → *adiabatic* humidification

From chart

$$H_a = 30 \text{ kJ/kg of air}$$

$$H_b = H_c = 43 \text{ kJ/kg of air}$$

$$\omega_a = \omega_b = 0.006 \text{ kg/kg of dry air}$$

$$\omega_c = 0.00875 \text{ kg/kg of dry air}$$

$$\text{Specific volume } V_{sa} = 0.8175 \text{ m}^3/\text{kg}$$

$$\text{Volume of air supplied} = V = \frac{0.3 \times 60}{60} = 0.3 \text{ m}^3/\text{sec}$$

$$\begin{aligned} \text{Weight of air supplied } m_a &= \frac{V}{V_a} = \frac{0.3}{0.8175} \\ &= 0.3669 \text{ kg/sec} \end{aligned}$$

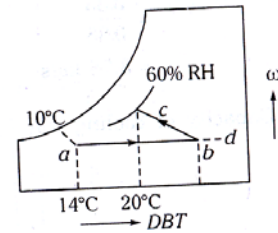
$$\begin{aligned} \text{Capacity of the heating coil} &= m_a (H_b - H_a) \\ &= 0.3669(43 - 30) = 4.77 \text{ kW} \end{aligned}$$

From chart $T_b = 26.5^\circ\text{C}$

Let coil surface temperature be T_d

$$\begin{aligned} \text{By passing factor} &= \frac{T_d - T_b}{T_d - T_a} = 0.4 T_d - 5.6 = T_d - 26.5 \\ 0.4 &= \frac{T_d - 26.5}{T_d - 1.4} \\ T_d &= 34.8^\circ\text{C} \end{aligned}$$

$$\begin{aligned} \text{Capacity of the humidifier} &= m_a (\omega_c - \omega_b) \times 3600 \\ &= 0.3669(0.00875 - 0.006) \times 3600 \\ &= 3.63 \text{ kg/hour} \end{aligned}$$



Problem 8

An air conditioned system is to be designed for a hall of 200 seating capacity when the following conditions are given:

Atmospheric condition = 30°C DBT and 50% RH

Indoor condition = 22°C DBT and 60% RH

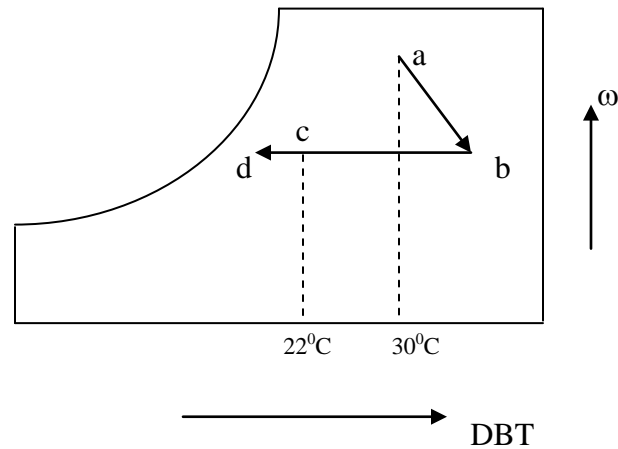
Volume of air required = $0.4 \text{ m}^3/\text{min}/\text{person}$

The required condition is achieved first by chemical dehumidification and after that by sensible cooling.

Find the following .

- DBT of the air leaving the dehumidifier.
- The quantity of water vapour removed in the dehumidifier per hour.
- The capacity of cooling coil in tons of refrigeration.
- Surface temperature of the coil if the by pass factor of the coil is 0.25.

Solution:



Locate point 'a', 30⁰C DBT, 50% RH, the atmospheric condition.
 Locate point 'c', 22⁰C DBT, 60% RH, the required indoor condition.
 "Since chemical dehumidification process follows constant enthalpy line"
 at a draw a line parallel to constant enthalpy line.

At 'c' draw a constant ω line to cut the previous line at point b.

a) DBT of air leaving the dehumidifier $T_b = 40.5^0\text{C}$

From chart

$$H_b = H_a = 65\text{kJ/kg}, \quad \omega_a = 0.013 \text{ kg/kg of dry air}$$

$$H_c = 45 \text{ kJ/kg}, \quad \omega_b = 0.009 \text{ kg/kg of dry air}$$

$$V_{sa} = 0.875 \text{ m}^3/\text{min}$$

$$\text{Volume of air} = 200 \times 0.4 = 80 \text{ m}^3/\text{min}$$

$$W_a = \text{Weight of air} = V/V_{sa} = 80/0.875 = 91.42 \text{ kg/min}$$

b) Quantity of water vapour removed/hour = $W_a(\omega_a - \omega_b)60$
 $= 91.42(0.013 - 0.009)60 = 21.94 \text{ kg/hr}$

c) Capacity of cooling coil = $W_a(H_a - H_b) / (60 \times 3.5) = 91.42(65 - 45) / (60 \times 3.5)$
 $= 8.7 \text{ tons}$

d) By pass factor = $(T_c - T_d) / (T_b - T_d) = 0.25$

$T_d = \text{Temperature of cooling coil} = 15.83^0\text{C}$

Problem 9

An air conditioned system is to be designed for a cinema hall of 1000 seating capacity when the following conditions are given:

Outdoor condition = 11⁰C DBT and 70% RH

Required indoor condition = 20⁰C DBT and 60% RH

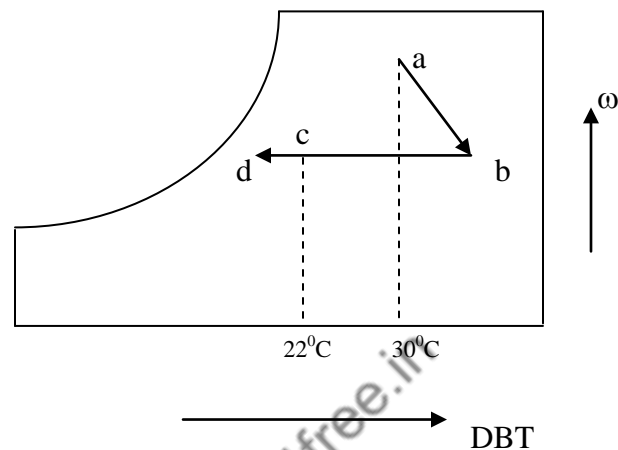
Amount of air required = $0.3\text{m}^3/\text{min}/\text{person}$

The required condition is achieved first by heating, then by humidifying and finally by heating. The condition of air coming out of the humidifier is 75% RH.

Find the following .

- a) Heating capacity of the first heater in kW and condition of the air coming out of the first heater in kW and condition of the air

Solution:



Locate point 'a', 30°C DBT, 50% RH, the atmospheric condition.

Locate point 'c', 22°C DBT, 60% RH, the required indoor condition.

“Since chemical dehumidification process follows constant enthalpy line”
at a draw a line parallel to constant enthalpy line.

At 'c' draw a constant ω line to cut the previous line at point b.

- e) DBT of air leaving the dehumidifier $T_b = 40.5^{\circ}\text{C}$

From chart

$$H_b = H_a = 65\text{kJ/kg}, \quad \omega_a = 0.013 \text{ kg/kg of dry air}$$

$$H_c = 45 \text{ kJ/kg}, \quad \omega_b = 0.009 \text{ kg/kg of dry air}$$

$$V_{sa} = 0.875 \text{ m}^3/\text{min}$$

$$\text{Volume of air} = 200 \times 0.4 = 80 \text{ m}^3/\text{min}$$

$$W_a = \text{Weight of air} = V/V_{sa} = 80/0.875 = 91.42 \text{ kg/min}$$

- f) Quantity of water vapour removed/hour = $W_a(\omega_a - \omega_b)60$
 $= 91.42(0.013 - 0.009)60 = 21.94 \text{ kg/hr}$

- g) Capacity of cooling coil = $W_a(H_a - H_b)/(60 \times 3.5) = 91.42(65 - 45)/(60 \times 3.5)$
 $= 8.7 \text{ tons}$

- h) By pass factor = $(T_c - T_d)/(T_b - T_d) = 0.25$

$$T_d = \text{Temperature of cooling coil} = 15.83^{\circ}\text{C}$$

Reciprocating Compressor

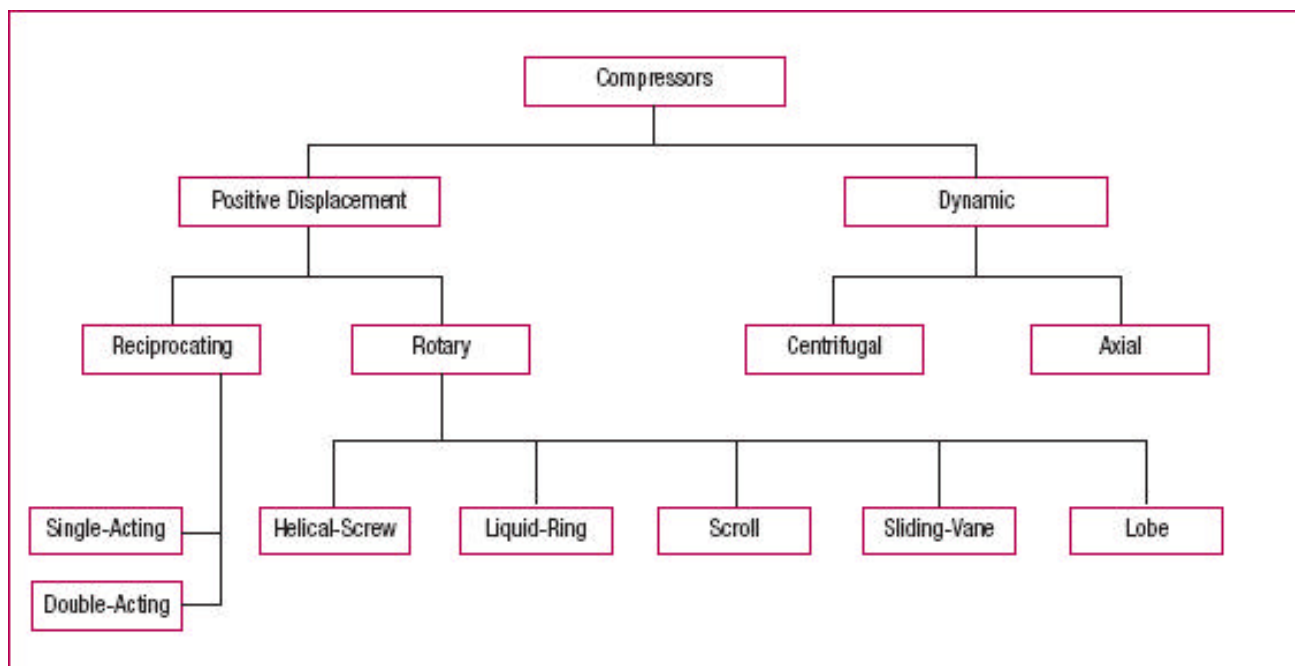
5.1 INTRODUCTION

Compressors are work absorbing devices which are used for increasing pressure of fluid at the expense of work done on fluid.

The compressors used for compressing air are called air compressors. Compressors are invariably used for all applications requiring high pressure air. Some of popular applications of compressor are, for driving pneumatic tools and air operated equipments, spray painting, compressed air engine, supercharging surface cleaning, refrigeration and air conditioning, chemical industry etc. compressors are supplied with low pressure air (or any fluid) at inlet which comes out as high pressure air (or any fluid) at outlet. Work required for increasing pressure of air is available from the prime mover driving the compressor. Generally, electric motor, internal combustion engine or steam engine, turbine etc. are used as prime movers. Compressors are similar to fans and blowers but differ in terms of pressure ratios. Fan is said to have pressure ratio up to 1.1 and blowers have pressure ratio between 1.1 to 4 while compressors have pressure ratios more than 4.

5.2 CLASSIFICATION OF COMPRESSORS

Table-5.1 Types of Compressors



Compressors can be classified in the following different ways.

- (a) **Based on principle of operation:** Based on the principle of operation compressors can be classified as.
- (i) Positive displacement compressor.
 - (ii) Non-positive displacement compressors.

In positive displacement compressors the compression is realized by displacement of solid boundary and preventing fluid by solid boundary from flowing back in the direction of pressure gradient. Due to solid wall displacement these are capable of providing quite large pressure ratios. Positive displacement compressors can be further classified based on the type of mechanism used for compression. These can be

- (i) Reciprocating type positive displacement compressors
- (ii) Rotary type positive displacement compressors.

Reciprocating compressors generally, employ piston-cylinder arrangement where displacement of piston in cylinder causes rise in pressure. Reciprocating compressors are capable of giving large pressure ratios but the mass handling capacity is limited or small. Reciprocating compressors may also be single acting compressor or double acting compressor. Single acting compressor has one delivery stroke per revolution while in double acting there are two delivery strokes per revolution of crank shaft. Rotary compressors employing positive displacement have a rotary part whose boundary causes positive displacement of fluid and thereby compression. Rotary compressors of this type are available in the names as given below;

- (i) Roots blower
- (ii) Vane type compressors

Rotary compressors of above type are capable of running at higher speed and can handle large mass flow rate than reciprocating compressors of positive displacement type.

Non-positive displacement compressors, also called as steady flow compressors use dynamic action of solid boundary for realizing pressure rise. Here fluid is not contained in definite volume and subsequent volume reduction does not occur as in case of positive displacement compressors. Non-positive displacement compressor may be of 'axial flow type' or 'centrifugal type' depending upon type of flow in compressor.

(b) **Based on number of stages:** Compressors may also be classified on the basis of number of stages. Generally, the number of stages depend upon the maximum delivery pressure. Compressors can be single stage or multistage. Normally maximum compression ratio of 5 is realized in single stage compressors. For compression ratio more than 5 the multistage compressors are used.

Type values of maximum delivery pressures generally available from different type of compressor are,

- (i) Single stage Compressor, for delivery pressure upto 5 bar.
- (ii) Two stage Compressor, for delivery pressure between 5 to 35 bar
- (iii) Three stage Compressor, for delivery pressure between 35 to 85 bar.
- (iv) Four stage compressor, for delivery pressure more than 85 bar

(c) **Based on Capacity of compressors :** Compressors can also be classified depending upon the capacity of Compressor or air delivered per unit time. Typical values of capacity for different compressors are given as;

- (i) Low capacity compressors, having air delivery capacity of $0.15 \text{ m}^3/\text{s}$ or less
- (ii) Medium capacity compressors, having air delivery capacity between 0.15 to $5 \text{ m}^3/\text{s}$.
- (iii) High capacity compressors, having air delivery capacity more than $5 \text{ m}^3/\text{s}$

(d) Based on highest pressure developed: Depending upon the maximum pressure available from compressor they can be classified as low pressure, medium pressure, high pressure and super high pressure compressors. Typical values of maximum pressure developed for different compressors are as under:

- (i) Low pressure compressor, having maximum pressure upto 1 bar
- (ii) Medium pressure compressor, having maximum pressure from 1 bar to 8 bar
- (iii) High pressure compressor, having maximum pressure from 8 to 10 bar
- (iv) Super high pressure compressor, having maximum pressure more than 10 bar.

5.3 Reciprocating Compressors

Reciprocating Compressor has piston cylinder arrangement as shown Fig.5.1

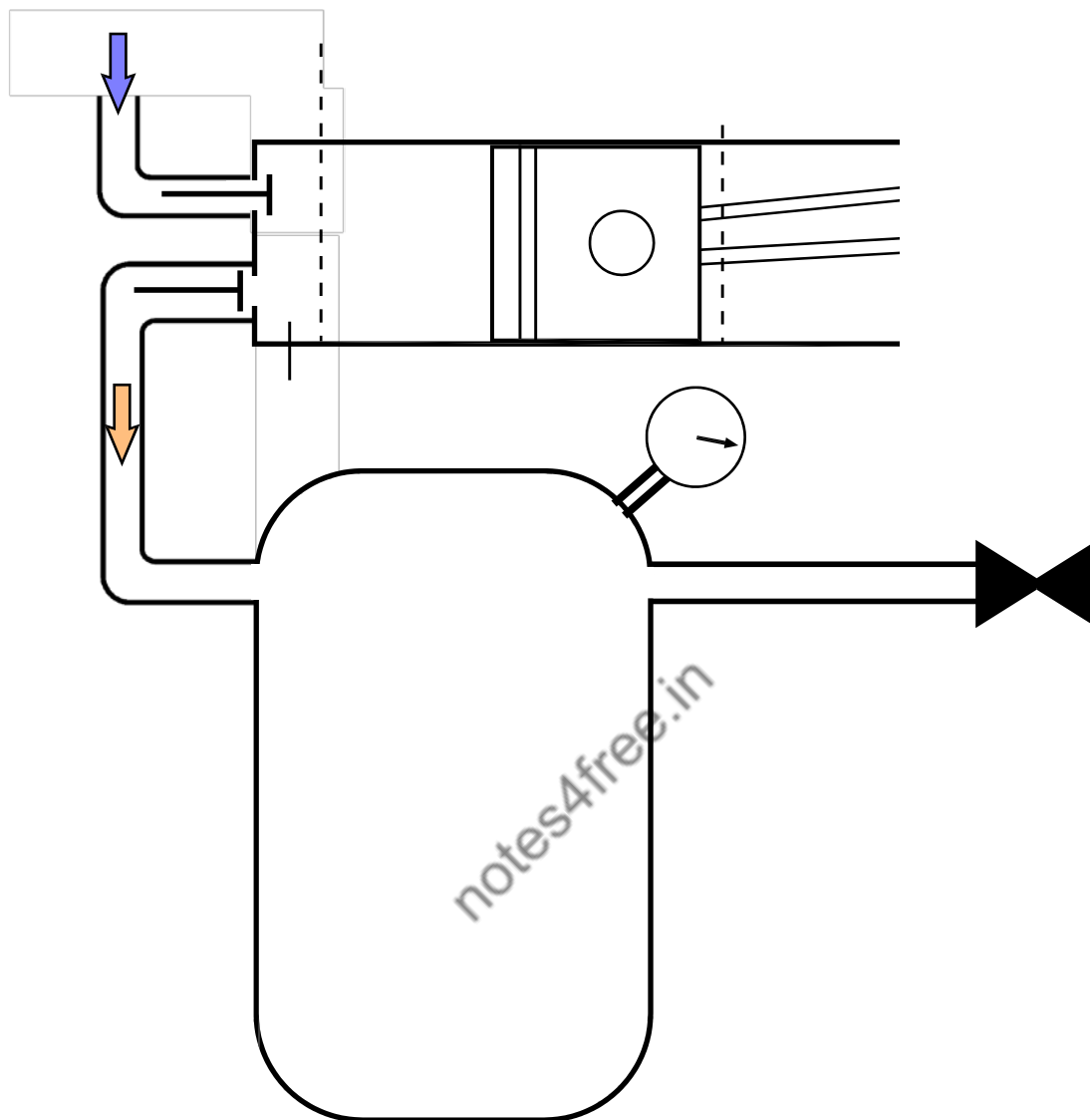


Fig.5.1 Reciprocating Compressor

Reciprocating Compressor has piston, cylinder, inlet valve, exit valve, connecting rod, crank, piston pin, crank pin and crank shaft. Inlet valve and exit valves may be of spring loaded type which get opened and closed due to pressure differential across them. Let us consider piston to be at top dead centre (TDC) and move towards bottom dead centre (BDC). Due to this piston movement from TDC to BDC suction pressure is created causing opening of inlet valve. With this opening of inlet valve and suction pressure the atmospheric air enters the cylinder.

Air gets into cylinder during this stroke and is subsequently compressed in next stroke with both inlet valve and exit valve closed. Both inlet valve and exit valves are of plate type and spring loaded so as to operate automatically as and when sufficient pressure difference is available to cause deflection in spring of valve plates to open them. After piston reaching BDC it reverses its motion and compresses the air inducted in previous stroke. Compression is continued till the pressure of air inside becomes sufficient to cause deflection in exit valve. At the moment when exit valve plate gets lifted the exhaust of compressed air takes place. This piston again reaches TDC from where downward piston movement is again accompanied by suction. This is how reciprocating compressor. Keeps on working as flow device. In order to counter for the heating of piston-cylinder arrangement during compression the provision of cooling the cylinder is there in the form of cooling jackets in the body. Reciprocating compressor described above has suction, compression and discharge as three prominent processes getting completed in two strokes of piston or one revolution of crank shaft.

5.4 Thermodynamic Analysis

Compression of air in compressor may be carried out following number of thermodynamic processes such as isothermal compression, polytropic compressor or adiabatic compressor. Fig.16.3 shows the thermodynamic cycle involved in compressor. Theoretical cycle is shown neglecting clearance volume but in actual cycle clearance volume can not be negligible. Clearance volume is necessary in order to prevent collision of piston with cylinder head, accommodating valve mechanism etc., Compression process is shown by process 1-2, $1-2^1$, $1-2^{11}$ and $1-2^{111}$ following isothermal, polytropic and adiabatic processes.

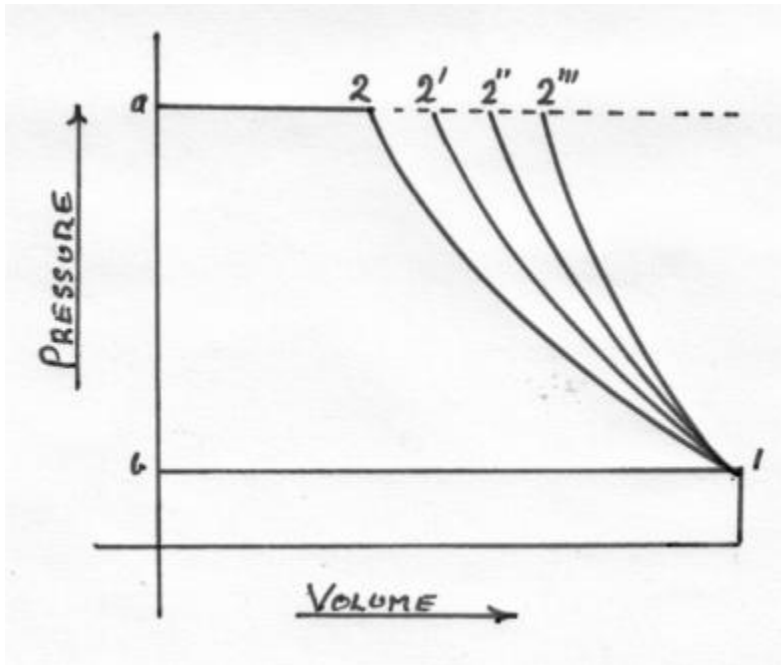


Fig.5.2 P-V diagram for Reciprocating Compressor without Clearance

On P-V diagram process 4-1 shows the suction process followed by compression during 1-2 and discharge through compressor is shown by process 2-3.

Air enters compressor at pressure p_1 and is compressed upto p_2 . Compression work requirement can be estimated from the area below the each compression process. Area on p-V diagram shows that work requirement shall be minimum with isothermal process 1-2". Work requirement is maximum with process 1-2 ie., adiabatic process. As a designer one shall be interested in a compressor having minimum compression work requirement. Therefore, ideally compression should occur isothermally for minimum work input. In practice it is not possible to have isothermal compression because constancy of temperature during compression can not be realized. Generally, compressors run at substantially high speed while isothermal compression requires compressor to run at very slow speed so that heat evolved during compression is dissipated out and temperature remains constant. Actually due to high speed running of compressor the compression process may be assumed to be near adiabatic or polytropic process following law of compression as $pV^n=C$ with of 'n' varying between 1.25 to 1.35 for air. Compression process following three processes is also shown on T-s diagram in Fig.16.4. it is thus obvious that actual compression process should be compared with isothermal compression process. A mathematical parameter called isothermal efficiency is defined for quantifying the degree of deviation of actual compression process from ideal compression process. Isothermal efficiency is defined by the ratio is isothermal work and actual indicated work in reciprocating compressor.

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Actual indicated Work}}$$

Practically, compression process is attempted to be closed to isothermal process by air/water cooling, spraying cold water during compression process. In case of multistage compression process the compression in different stages is accompanied by intercooling in between the stages. $P_2 V_2$

Mathematically, for the compression work following polytropic process, $PV^n=C$. Assuming negligible clearance volume the cycle work done.

$W_c = \text{Area on p-V diagram}$

$$\begin{aligned} W_c &= \left[p_2 V_2 + \left(\frac{p_2 V_2 - p_1 V_1}{n-1} \right) \right] - p_1 V_1 & 1 \\ &= \left[\left(\frac{n}{n-1} \right) [p_2 V_2 - p_1 V_1] \right] \\ &= \left(\frac{n}{n-1} \right) (p_1 V_1) \left[\frac{p_2 V_2}{p_1 V_1} - 1 \right] \\ &= \left(\frac{n}{n-1} \right) (p_1 V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \\ &= \left(\frac{n}{n-1} \right) (mRT_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \\ &= \left(\frac{n}{n-1} \right) (mR)(T_2 - T_1) \end{aligned}$$

In case of compressor having isothermal compression process, $n = 1$, ie., $p_1 V_1 = p_2 V_2$

$$W_{iso} = p_2 V_2 + p_1 V_1 \ln r - p_1 V_1$$

$$W_{iso} = p_1 V_1 \ln r, \quad \text{where, } r = \frac{V_1}{V_2}$$

In case of compressor having adiabatic compression process,

$$W_{adiabatic} = \left(\frac{\gamma}{\gamma-1} \right) (mR)(T_2 - T_1)$$

Or

$$W_{adiabatic} = (mC_p)(T_2 - T_1)$$

$$W_{adiabatic} = (m)(h_2 - h_1)$$

$$\eta_{iso} = \frac{p_1 V_1 \ln r}{\left(\frac{n}{n-1}\right)(p_1 V_1) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]}$$

The isothermal efficiency of a compressor should be close to 100% which means that actual compression should occur following a process close to isothermal process. For this the mechanism be derived to maintain constant temperature during compression process. Different arrangements which can be used are:

- (i) Faster heat dissipation from inside of compressor to outside by use of fins over cylinder. Fins facilitate quick heat transfer from air being compressed to atmosphere so that temperature rise during compression can be minimized.
- (ii) Water jacket may be provided around compressor cylinder so that heat can be picked by cooling water circulating through water jacket. Cooling water circulation around compressor regulates rise in temperature to great extent.
- (iii) The water may also be injected at the end of compression process in order to cool the air being compressed. This water injection near the end of compression process requires special arrangement in compressor and also the air gets mixed with water and needs to be separated out before being used. Water injection also contaminates the lubricant film inner surface of cylinder and may initiate corrosion etc, The water injection is not popularly used.
- (iv) In case of multistage compression in different compressors operating serially, the air leaving one compressor may be cooled upto ambient state or somewhat high temperature before being injected into subsequent compressor. This cooling of fluid being compressed between two consecutive compressors is called intercooling and is frequently used in case of multistage compressors.

Considering clearance volume: With clearance volume the cycle is represented on Fig.5.3 The work done for compression of air polytropically can be given by the area enclosed in cycle 1-2-3-4. Clearance volume in compressors varies from 1.5% to 35% depending upon type of compressor.

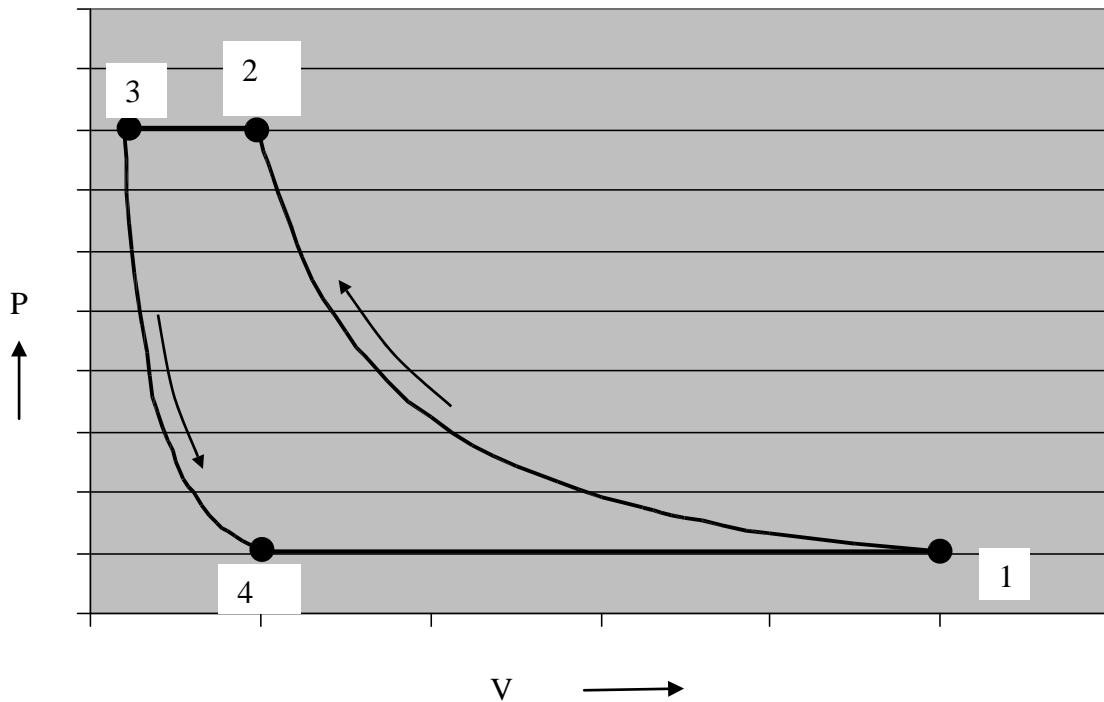


Fig.5.3 P-V diagram for Reciprocating Compressor with Clearance

$W_{c, \text{with CV}} = \text{Area } 1234$

$$= \left(\frac{n}{n-1} \right) (p_1 V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] - \left(\frac{n}{n-1} \right) (p_4 V_4) \left[\left(\frac{p_3}{p_4} \right)^{\frac{n-1}{n}} - 1 \right]$$

Here $P_1 = P_4$, $P_2 = P_3$

$$= \left(\frac{n}{n-1} \right) (p_1 V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] - \left(\frac{n}{n-1} \right) (p_1 V_4) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \left(\frac{n}{n-1} \right) (p_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] (V_1 - V_4)$$

In the cylinder of reciprocating compressor $(V_1 - V_4)$ shall be the actual volume of air delivered per cycle. $V_d = V_1 - V_4$. This $(V_1 - V_4)$ is actually the volume of air inhaled in the cycle and delivered subsequently.

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(p_1 V_d) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

If air is considered to behave as perfect gas then pressure, temperature, volume and mass can be inter related using perfect gas equation. The mass at state 1 may be given as m_1 mass at state 2 shall be m_1 , but at state 3 after delivery mass reduces to m_2 and at state 4 it shall be m_2 .

$$\text{So, at state 1, } p_1 V_1 = m_1 R T_1$$

$$\text{at state 2, } p_2 V_2 = m_1 R T_2$$

$$\text{at state 3, } p_3 V_3 = m_2 R T_3 \text{ or } p_2 V_3 = m_2 R T_3$$

$$\text{at state 4, } p_4 V_4 = m_2 R T_4 \text{ or } p_1 V_4 = m_2 R T_4$$

Ideally there shall be no change in temperature during suction and delivery i.e., $T_4 = T_1$ and $T_2 = T_3$ from earlier equation

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(p_1) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right] (V_1 - V_4)$$

Temperature and pressure can be related as,

$$\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \frac{T_2}{T_1} \quad \text{and} \quad \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} = \frac{T_4}{T_3} \quad \Longrightarrow \quad \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \frac{T_4}{T_3}$$

Substituting

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1 R T_1 - m_2 R T_4) \left[\frac{T_2}{T_1} - 1 \right]$$

Substituting for constancy of temperature during suction and delivery.

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1 R T_1 - m_2 R T_1) \left[\frac{T_2 - T_1}{T_1} \right]$$

Or

$$W_{c,withCV} = \left(\frac{n}{n-1}\right)(m_1 - m_2) R (T_2 - T_1)$$

Thus $(m_1 - m_2)$ denotes the mass of air sucked or delivered. For unit mass of air delivered the work done per kg of air can be given as,

$$W_{e,withCV} = \left(\frac{n}{n-1} \right) R(T_2 - T_1) \quad \text{per kg of air}$$

Thus from above expressions it is obvious that the clearance volume reduces the effective swept volume i.e., the mass of air handled but the work done per kg of air delivered remains unaffected.

From the cycle work estimated as above the theoretical power required for running compressor shall be,

For single acting compressor running with N rpm, power input required, assuming clearance volume.

$$Power_{required} = \left[\left(\frac{n}{n-1} \right) \left[\left(\frac{p_2}{p_1} \right)^{\left(\frac{n-1}{n} \right)} - 1 \right] p_1 (V_1 - V_4) \right] (N)$$

For double acting compressor, Power

$$Power_{required} = \left[\left(\frac{n}{n-1} \right) \left[\left(\frac{p_2}{p_1} \right)^{\left(\frac{n-1}{n} \right)} - 1 \right] p_1 (V_1 - V_4) \right] (2N)$$

Volumetric efficiency: Volumetric efficiency of compressor is the measure of the deviation from volume handling capacity of compressor. Mathematically, the volumetric efficiency is given by the ratio of actual volume of air sucked and swept volume of cylinder. Ideally the volume of air sucked should be equal to the swept volume of cylinder, but it is not so in actual case. Practically the volumetric efficiency lies between 60 to 90%.

Volumetric efficiency can be overall volumetric efficiency and absolute volumetric efficiency as given below.

$$\text{Overall volumetric efficiency} = \frac{\text{Volume of free air sucked in cylinder}}{\text{Swept volume of LP cylinder}}$$

$$(\text{Volumetric efficiency})_{\text{freeaircondition}} = \frac{\text{Volume of free air sucked in cylinder}}{(\text{Swept volume of LP cylinder})_{\text{freeaircondition}}}$$

Here free air condition refers to the standard conditions. Free air condition may be taken as 1 atm or 1.01325 bar and 15°C or 288K. consideration for free air is necessary as otherwise the different compressors can not be compared using volumetric efficiency because specific volume or density of air varies with altitude. It may be seen that a compressor at datum level (sea level) shall deliver large mass than the same compressor at high altitude.

This concept is used for giving the capacity of compressor in terms of ‘free air delivery’ (FAD). “Free air delivery is the volume of air delivered being reduced to free air conditions”.

In case of air the free air delivery can be obtained using perfect gas equation as,

$$\frac{p_a V_a}{T_a} = \frac{p_1 (V_1 - V_4)}{T_1} = \frac{p_2 (V_2 - V_3)}{T_2}$$

Where subscript a or p_a , V_a , T_a denote properties at free air conditions

$$V_a = \frac{p_1 T_a}{p_a} \frac{p_1 (V_1 - V_4)}{T_1} = \text{FAD per cycle}$$

This volume V_a gives ‘free air delivered’ per cycle by the compressor.

Absolute volumetric efficiency can be defined, using NTP conditions in place of free air conditions.

$$\eta_{vol} = \frac{\text{FAD}}{\text{Swept volume}} = \frac{V_a}{(V_1 - V_2)} = \frac{p_1 T_a (V_1 - V_4)}{p_a T_1 (V_1 - V_3)}$$

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ \frac{(V_s + V_c) - V_4}{V_s} \right\}$$

Here V_s is the swept volume = $V_1 - V_3$

V_c is the clearance volume = V_3

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ 1 + \left(\frac{V_c}{V_s} \right) - \left(\frac{V_4}{V_s} \right) \right\}$$

$$\text{Here } \frac{V_4}{V_s} = \frac{V_4}{V_c} \cdot \frac{V_c}{V_s} = \left(\frac{V_4}{V_3} \cdot \frac{V_c}{V_s} \right)$$

Let the ratio of clearance volume to swept volume be given by $C = \frac{V_c}{V_s}$

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ 1 + C - C \left(\frac{V_4}{V_3} \right) \right\}$$

$$\eta_{vol} = \left(\frac{p_1 T_a}{p_a T_1} \right) \left\{ 1 + C - C \left(\frac{p_2}{p_1} \right)^{1/n} \right\}$$

Volumetric efficiency depends on ambient pressure and temperature, suction pressure and temperature, ratio of clearance to swept volume, and pressure limits. Volumetric efficiency increases with decrease in pressure ratio in compressor.

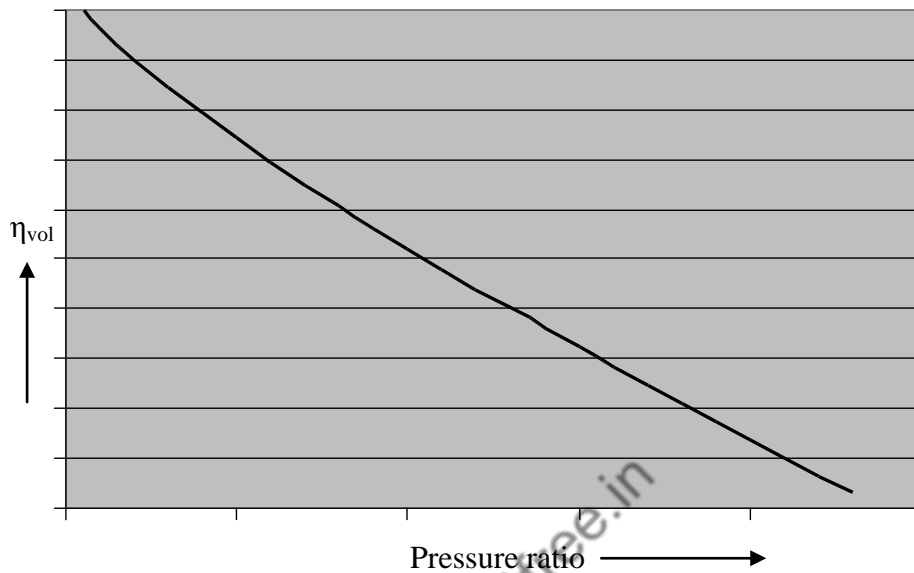


Fig.5.4 Volumetric efficiency v/s Pressure ratio

Multistage Compression

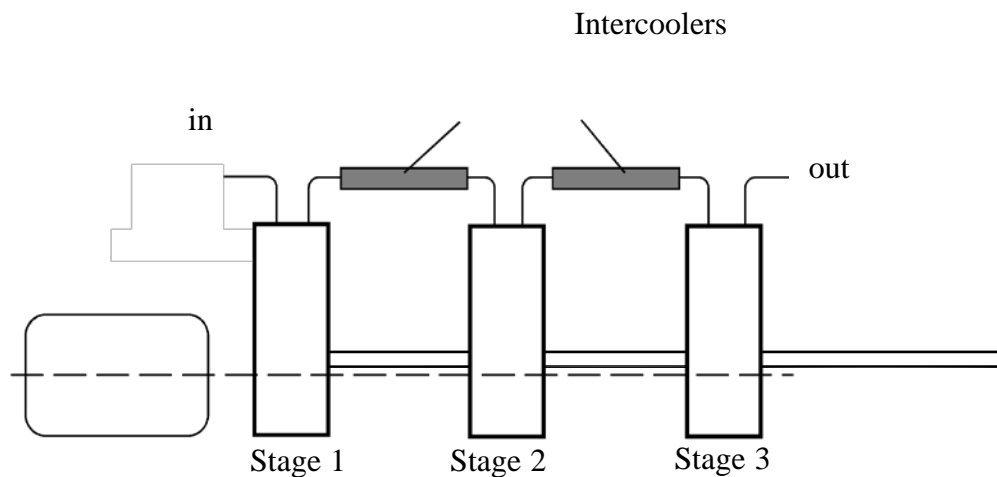


Fig.5.5 Multistage Compressor with inter coolers

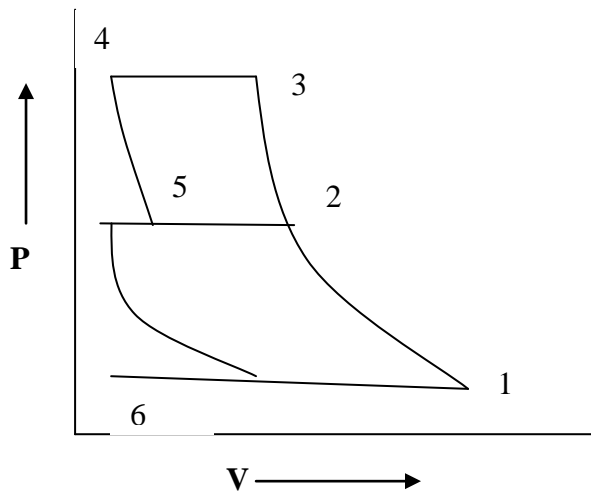


Fig.5.6 P-V diagram for Multistage Compressor

Multistage compression refers to the compression process completed in more than one stage i.e., a part of compression occurs in one cylinder and subsequently compressed air is sent to subsequent cylinders for further compression. In case it is desired to increase the compression ratio of compressor then multi-stage compression becomes inevitable. If we look at the expression for volumetric efficiency then it shows that the volumetric efficiency decreases with increase in pressure ratio. This aspect can also be explained using p-V representation shown in Fig.5.6.

Therefore, the volumetric efficiency reduces with increasing pressure ratio in compressor with single stage compression. Also for getting the same amount of free air delivery the size of cylinder is to be increased with increasing pressure ratio. The increase in pressure ratio also requires sturdy structure from mechanical strength point of view for withstanding large pressure difference.

The solution to number of difficulties discussed above lies in using the multistage compression where compression occurs in parts in different cylinders one after the other. Fig.16.6b, shows the multistage compression occurring in two stages. Here first stage of compression occurs in cycle 12671 and after first stage compression partly compressed enters second stage of compression and occurs in cycle 2345. In case of multistage compression the compression in first stage occurs at low temperature and subsequent compression in following stages occurs at higher temperature. The compression work requirement depends largely upon the average temperature during compression. Higher average temperature

during compression has larger work requirement compared to low temperature so it is always desired to keep the low average temperature during compression.

Apart from the cooling during compression the temperature of air at inlet to compressor can be reduced so as to reduce compression work. In multistage compression the partly compressed air leaving first stage is cooled upto ambient air temperature in intercooler and then sent to subsequent cylinder (stage) for compression. Thus, intercoolers when put between the stages reduce the compression work and compression is called intercooled compression. Intercooling is called perfect when temperature at inlet to subsequent stages of compression is reduced to ambient temperature. Fig.16.6c, shows multi-stage (two stage) intercooled compression. Intercooling between two stages causes temperature drop from 2 to 2' i.e discharge from first stage (at 2) is cooled upto the ambient temperature stage (at 2') which lies on isothermal compression process 1-2'-3''. In the absence of intercooling the discharge from first stage shall enter at 2. Final discharge from second stage occurs at 3' in case of intercooled compression compared to discharge at 3 in case of non-intercooled compression. Thus, intercooling offers reduced work requirement by the amount shown by area 22'3'3 on p-V diagram. If the intercooling is not perfect then the inlet state to second/subsequent stage shall not lie on the isothermal compression process line and this stage shall lie between actual discharge state from first stage and isothermal compression line.

Fig.16.7 shows the schematic of multi stage compressor (double stage) with inter cooler between stage T-s representation is shown in Fig.16.8. The total work requirement for running this shall be algebraic summation of work required for low pressure (LP) and high pressure (HP) stages. The size of HP cylinder is smaller than LP cylinder as HP cylinder handles high pressure air having smaller specific volume.

Mathematical analysis of multistage compressor is done with following assumptions:

- (i) Compression in all the stages is done following same index of compression and there is no pressure drop in suction and delivery pressures in each stage. Suction and delivery pressure remains constant in the stages.
- (ii) There is perfect intercooling between compression stages.

- (iii) Mass handled in different stages is same i.e, mass of air in LP and HP stages are same.
- (iv) Air behaves as perfect gas during compression.

From combined p-V diagram the compressor work requirement can be given as,

$$\text{Work requirement in LP cylinder, } W_{LP} = \left(\frac{n}{n-1}\right) P_1 V_1 \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\}$$

$$\text{Work requirement in HP cylinder, } W_{HP} = \left(\frac{n}{n-1}\right) P_2 V_2 \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\}$$

For perfect intercooling, $p_1 V_1 = p_2 V_2'$ and

$$W_{HP} = \left(\frac{n}{n-1}\right) P_2 V_2' \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\}$$

Therefore, total work requirement, $W_c = W_{LP} + W_{HP}$, for perfect inter cooling

$$W_c = \left(\frac{n}{n-1}\right) \left[P_1 V_1 \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\} + P_2 V_2' \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\} \right]$$

$$= \left(\frac{n}{n-1}\right) \left[P_1 V_1 \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\} + P_1 V_1 \left\{ \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 1 \right\} \right]$$

$$W_c = \left(\frac{n}{n-1}\right) P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} + \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} - 2 \right]$$

Power = $W_c \times N$ - Watts

If we look at compressor work then it shows that with the initial and final pressures p_1 and P_2' remaining same the intermediate pressure p_2 may have value floating between p_1 and P_2' and change the work requirement W_c . Thus, the compressor work can be optimized with respect to intermediate pressure P

2. Mathematically, it can be differentiated with respect to P_2 .

$$\frac{dW_c}{dP_2} = \frac{d}{dP_2} \left[\left(\frac{n}{n-1} \right) P_1 V_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left(\frac{P_2'}{P_2} \right)^{\frac{n-1}{n}} - 2 \right\} \right]$$

$$\frac{dW_c}{dP_2} = \left[\left(\frac{n}{n-1} \right) P_1 V_1 \frac{d}{dP_2} \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} + \left(\frac{P_2'}{P_2} \right)^{\frac{n-1}{n}} - 2 \right\} \right]$$

$$\frac{dW_c}{dP_2} = \left(\frac{n}{n-1} \right) P_1 V_1 \left\{ \left(\frac{n-1}{n} \right) P_1^{\frac{1-n}{n}} \cdot P_2^{\frac{-1}{n}} - \left(\frac{n-1}{n} \right) \cdot P_2^{\frac{1-n}{n}} \cdot P_2^{\frac{1-2n}{n}} \right\}$$

Equating to zero yields

$$P_1^{\frac{1-n}{n}} \cdot P_2^{\frac{-1}{n}} = P_2^{\frac{1-n}{n}} \cdot P_2^{\frac{1-2n}{n}}$$

$$P_2^{\frac{-2+2n}{n}} = P_2^{\frac{1-n}{n}} \cdot P_1^{\frac{n-1}{n}}$$

$$P_2^{2\left(\frac{n-1}{n}\right)} = (P_1 \cdot P_2)^{\left(\frac{n-1}{n}\right)}$$

$$P_2^2 = (P_1 \cdot P_2), P_2 = \sqrt{P_1 \cdot P_2}$$

Pressure ratio in Ist stage = Pressure ratio in IInd stage

Thus, it is established that the compressor work requirement shall be minimum when the pressure ratio in each stage is equal.

In case of multiple stages, say i number of stages, for the delivery and suction pressures of P_{i+1} and P_1 the optimum stage pressure ratio shall be,

$$\text{Optimum stage pressure ratio} = \left(\frac{P_{i+1}}{P_1} \right)^{\frac{1}{i}} \text{ for pressure at stages being } P_1, P_2, P_3, \dots, P_{i-1}, P_i,$$

P_{i+1}

Minimum work required in two stage compressor can be given by

$$W_{C,\min} = \left(\frac{n}{n-1} \right) P_1 V_1 \cdot 2 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{(n-1)}{n}} - 1 \right\}$$

For i number of stages, minimum work,

$$W_{C,\min} = i \cdot \left(\frac{n}{n-1} \right) P_1 V_1 \left\{ \left(\frac{P_{i+1}}{P_1} \right)^{\frac{(n-1)}{n \cdot i}} - 1 \right\}$$

It also shows that for optimum pressure ratio the work required in different stages remains same for the assumptions made for present analysis. Due to pressure ratio being equal in all stages the temperature ratios and maximum temperature in each stage remains same for perfect intercooling.

Cylinder dimensions: In case of multistage compressor the dimension of cylinders can be estimated basing upon the fact that the mass flow rate of air across the stages remains same. For perfect intercooling the temperature of air at suction of each stage shall be same.

If the actual volume sucked during suction stroke is V_1, V_2, V_3, \dots for different stages they by perfect gas law, $P_1 V_1 = RT_1, P_2 V_2 = RT_2, P_3 V_3 = RT_3$

For perfect intercooling

$$P_1 V_1 = RT_1, P_2 V_2 = RT_1, P_3 V_3 = RT_1$$

$$P_1 V_1 = P_2 V_2 = RT_2, P_3 V_3 = \dots$$

If the volumetric efficiency of respective stages in $\eta_{V_1}, \eta_{V_2}, \eta_{V_3}, \dots$

Then theoretical volume of cylinder1, $V_{1,th} = \frac{V_1}{\eta_{V_1}}; V_1 = \eta_{V_1} \cdot V_{1,th}$

Cylinder 2, $V_{2,th} = \frac{V_2}{\eta_{V_2}}; V_2 = \eta_{V_2} \cdot V_{2,th}$

Cylinder 3, $V_{3,th} = \frac{V_3}{\eta_{V_3}}; V_3 = \eta_{V_3} \cdot V_{3,th}$

Substituting,

$$P_1 \cdot \eta_{V_1} \cdot V_{1,th} = P_2 \cdot \eta_{V_2} \cdot V_{2,th} = P_3 \cdot \eta_{V_3} \cdot V_{3,th} = \dots$$

Theoretical volumes of cylinder can be given using geometrical dimensions of cylinder as diameters $D_1, D_2, D_3 \dots$ and stroke lengths $L_1, L_2, L_3 \dots$

Or
$$V_{1,th} = \frac{\pi}{4} \cdot D_1^2 \cdot L_1$$

$$V_{2,th} = \frac{\pi}{4} \cdot D_2^2 \cdot L_2$$

$$V_{3,th} = \frac{\pi}{4} \cdot D_3^2 \cdot L_3$$

Or
$$P_1 \cdot \eta_{V_1} \cdot \frac{\pi}{4} \cdot D_1^2 \cdot L_1 = P_2 \cdot \eta_{V_2} \cdot \frac{\pi}{4} \cdot D_2^2 \cdot L_2$$

$$= P_3 \cdot \eta_{V3} \cdot \frac{\pi}{4} \cdot D_3^2 \cdot L_3 = \dots$$

$$P_1 \cdot \eta_{V1} \cdot \frac{\pi}{4} \cdot D_1^2 \cdot L_1 = P_2 \cdot \eta_{V2} \cdot \frac{\pi}{4} \cdot D_2^2 \cdot L_2$$

$$= P_3 \cdot \eta_{V3} \cdot D_3^2 \cdot L_3 = \dots$$

If the volumetric efficiency is same for all cylinders, i.e. $\eta_{V1} = \eta_{V2} = \eta_{V3} = \dots$ and stroke for all cylinder is same i.e. $L_1 = L_2 = L_3 = \dots$

Then,
$$D_1^2 P_1 = D_2^2 P_2 = D_3^2 P_3 = \dots$$

These generic relations may be used for getting the ratio of diameters of cylinders of multistage compression.

Energy balance: Energy balance may be applied on the different components constituting multistage compression.

For LP stage the steady flow energy equation can be written as below:

$$m \cdot h_1 + W_{LP} = m \cdot h_2 + Q_{LP}$$

$$Q_{LP} = W_{LP} - m(h_2 - h_1)$$

$$Q_{LP} = W_{LP} - m \cdot C_p (T_2 - T_1)$$

For intercooling (Fig. 5.5) between LP and HP stage steady flow energy equation shall be;

$$m \cdot h_2 = m \cdot h_{2'} + Q_{Int}$$

$$Q_{Int} = m(h_2 - h_{2'})$$

$$Q_{Int} = m \cdot C_p (T_2 - T_{2'})$$

For HP stage (Fig.5.5) the steady flow energy equation yields.

$$m \cdot h_{2'} + W_{HP} = m \cdot h_{3'} + Q_{HP}$$

$$Q_{HP} = W_{HP} + m(h_{2'} - h_{3'})$$

$$Q_{HP} = W_{HP} + m \cdot C_p (T_{2'} - T_{3'}) = W_{HP} - m \cdot C_p (T_{3'} - T_{2'})$$

In case of perfect intercooling and optimum pressure ratio, $T_{2'} = T_1$ and $T_2 = T_{3'}$.

Hence for these conditions,

$$Q_{LP} = W_{LP} - m \cdot C_p (T_2 - T_1)$$

$$Q_{Int} = m \cdot C_p (T_2 - T_1)$$

$$Q_{HP} = W_{HP} - m \cdot C_p (T_2 - T_1)$$

Total heat rejected during compression shall be the sum of heat rejected during compression and heat extracted in intercooler for perfect intercooling.

Heat rejected during compression for polytropic process $= \left(\frac{\gamma - n}{\gamma - 1} \right) \times Work$

UNIT-5 Air Compressors

Review of equations

Work done in a single stage compressor

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] = \frac{n}{n-1} mR(T_2 - T_1)$$

Work done in a two stage compressor for perfect inter cooling

$$= \frac{2n}{n-1} P_1 V_1 \left[\left(\frac{P_3}{P_1} \right)^{\frac{n-1}{2n}} - 1 \right]$$

Work done in a two stage compressor

$$\eta_v = \frac{P_1 T_a}{P_a T_1} \left[1 + c - c \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}} \right]$$

Volumetric Efficiency

$$= \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_2 V_2 \left[\left(\frac{P_2}{P_2} \right)^{\frac{n-1}{n}} - 1 \right]$$

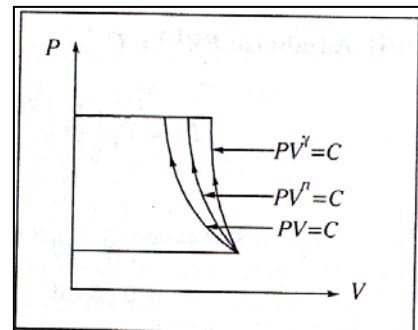
Problem 1

A single stage acting air compressor 30 cm bore and 40 cm stroke is running at a speed of 100 RPM. It takes in air at 1 bar and 20°C and drive it when the compresses it to a pressure of 5 bar. Find the power required to drive it when compression is (i) isothermal (ii) $PV^{1.2} = C$ and (iii) adiabatic. Also find the isothermal efficiencies for the cases (ii) and (iii) Neglect clearance.

$$\begin{aligned} N_1 &= 100 \text{ RPM}, d = 30 \text{ cm} \\ L &= 40 \text{ cm} \quad P_1 = 1 \text{ bar} \\ T_1 &= 20^\circ \text{C} \quad P_2 = 5 \text{ bar} \\ V_s &= \frac{\Pi}{4} d^2 L = \frac{\Pi}{4} 0.3^2 \times 0.4 \\ &= 0.028 \text{ m}^3 / \text{cycle} \end{aligned}$$

$$V_1 = V_s = 0.028 \times \frac{100}{60} = 0.047 \text{ m}^3 / \text{s}$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{1 \times 10^2 \times 0.047}{0.287 \times 293} = 0.055 \text{ kg/s}$$



(i) Isothermal $PV = C$

$$\begin{aligned}\text{Power} &= P_1 V_1 \ln \frac{P_2}{P_1} = mRT_1 \ln \frac{P_2}{P_1} \\ &= 0.055 \times 0.287 \times 293 \ln \frac{5}{1} = 7.56 \text{ kW}\end{aligned}$$

(i) $PV^{1.2} = C$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 293 \left(\frac{5}{1} \right)^{\frac{1.2-1}{1.2}} = 383.14 \text{ K}$$

$$\begin{aligned}\text{Power} &= \frac{n}{n-1} (P_2 V_2 - P_1 V_1) = \frac{n}{n-1} \times mR(T_2 - T_1) \\ &= \frac{1.2}{1.2-1} \times 0.055 \times 0.287 (383.14 - 293) = 8.53 \text{ kW}\end{aligned}$$

$$\begin{aligned}\text{Isothermal efficiency} &= \frac{\text{Isothermal Power}}{\text{Actual power}} \\ &= \frac{7.56}{8.53} = 0.8854\end{aligned}$$

$$\eta_{\text{Isothermal}} = 88.54 \%$$

(iii) Adiabatic $PV^{1.4} = C$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = 293 \left(\frac{5}{1} \right)^{\frac{1.4-1}{1.4}} = 464 \text{ K}$$

$$\text{Isothermal efficiency} = \frac{\text{Isothermal Power}}{\text{Actual power}}$$

$$= \frac{7.56}{8.53} = 0.8854$$

$$\eta_{\text{Isothermal}} = 88.54 \%$$

(iii) Adiabatic $PV^{1.4} = C$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = 293 \left(\frac{5}{1} \right)^{\frac{1.4-1}{1.4}} = 464 \text{ K}$$

$$\text{Power} = \frac{\gamma}{\gamma-1} \times mR(T_2 - T_1)$$

$$= \frac{1.4}{1.4-1} \times 0.055 \times 0.287(464 - 293) = 9.45 \text{ kW}$$

$$\eta_{\text{Isothermal}} = \frac{7.56}{9.45} = 0.80$$

$$= 80 \%$$

Problem 2

A single acting stage acting air compressor with clearance running at 360 rpm has a bore of 10 cm. The compression and expansion are polytropic with $n = 1.25$ for each. The clearance volume is 80 cm^3 . If the suction and delivery pressures are 98.1 kPa and 706.32 kPa absolute, find the free air at 101 kPa and 15°C delivered per minute. What is the work done per cycle? The temperature at the beginning of compression may be taken as 30°C . Find also the power required to drive the compressor.

$$T_a = 15^\circ\text{C}, \quad d = 10 \text{ cm}$$

$$L = 8.5 \text{ cm} \quad P_1 = 98.1 \text{ kPa}$$

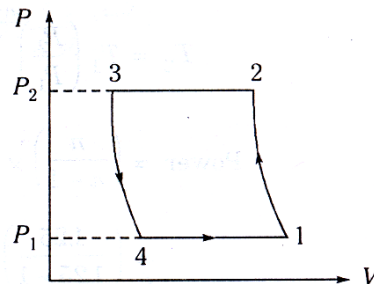
$$T_1 = 30^\circ\text{C} \quad P_2 = 706.32 \text{ kPa}$$

$$P_a = 101 \text{ kPa}$$

$$V_s = \frac{\Pi}{4} d^2 L = \frac{\Pi}{4} 0.1^2 \times 0.085$$

$$= 6.675 \times 10^{-4} \text{ m}^3$$

$$V_c = 80 \text{ cm}^3 = 0.8 \times 10^{-4} \text{ m}^3$$



$$\text{Clearance ratio } C = \frac{V_c}{V_s} = \frac{0.8 \times 10^{-4}}{6.675 \times 10^{-4}} = 0.12$$

Volumetric efficiency

$$\eta_v = \frac{P_1 T_a}{P_a T_1} \left[1 + c - c \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}} \right]$$

$$= \frac{98.1 \times 288}{101 \times 303} \left[1 + 0.12 - 0.12 \left(\frac{706.32}{98.1} \right)^{\frac{1}{1.25}} \right] = 0.496$$

$$m = \frac{P_a T_a}{R T_a} = \frac{101 \times 0.1193}{0.287 \times 288}$$

$$= 0.1457 \text{ kg/min}$$

$$\eta_v = \frac{V_a}{V_s}$$

$$\therefore \text{Volume of free air } V_a = 0.496 \times 6.675 \times 10^{-4}$$

$$= 0.0003314 \text{ m}^3 / \text{cycle}$$

$$= 0.0003314 \times 360 = 0.1193 \text{ m}^3 / \text{min}$$

$$m = \frac{0.1457}{60} = 0.00242 \text{ kg/sec}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 303 \left(\frac{706.32}{98.1} \right)^{\frac{1.25-1}{1.25}} = 449.67 \text{ K}$$

$$\text{Power} = \left(\frac{n}{n-1} \right) \times mR(T_2 - T_1)$$

$$= \left(\frac{1.25}{1.25-1} \right) \times 0.00242 \times 0.287(449.67 - 303)$$

$$= 0.5 \text{ kJ/s or kW}$$

$$\text{Workdone/cycle} = \frac{0.5}{360/60} = 0.0084 \text{ kJ/cycle}$$

Problem 3

A single stage double acting air compressor is required to deal with 17 m³/min of air measured at 1 bar and 15°C. the pressure and temperature at the end of suction is 0.98 bar and 30°C. The delivery pressure is 6.3bar. The rpm of the compressor is 500.assuming a clearance volume of 5% of the stroke volume, laws of the compression and expansion as PV^{1.32}=C, calculate the necessary stroke of volume, temperature of the air delivered and power of the compressor.

$$T_a = 15^\circ\text{C}, \quad V_a = 17\text{m}^3/\text{min}$$

$$P_a = 1\text{bar} \quad P_1 = 0.98\text{bar}$$

$$T_1 = 30^\circ\text{C} \quad P_2 = 6.3\text{bar}$$

$$N = 500\text{rpm}$$

$$C = \frac{5}{100} = 0.05$$

$$\eta_v = \frac{P_1 T_a}{P_a T_1} \left[1 + C - C \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}} \right]$$

$$\eta_v = \frac{98.1 \times 288}{1 \times 303} \left[1 + 0.05 - 0.05 \left(\frac{6.3}{0.98} \right)^{\frac{1}{1.32}} \right] = 0.8452$$

$$\eta_v = \frac{V_a}{V_s}$$

$$V_s = \text{Stroke Volume} = \frac{17}{0.8452} = 20.11 \text{ m}^3/\text{min}$$

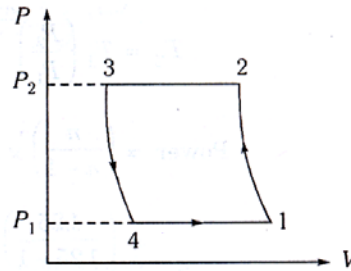
$$V_s = \frac{20.11}{500} = 0.0402 \text{ m}^3/\text{min}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 303 \left(\frac{6.3}{0.98} \right)^{\frac{1.32-1}{1.32}}$$

$$\text{Temperature of air delivered} = 475.71 \text{ K}$$

$$m = \frac{P_a T_a}{R T_a} = \frac{1 \times 100 \times 17}{0.287 \times 288} = 20.56 \text{ kg/min}$$

$$= \frac{20.56}{60} = 0.3427 \text{ kg/sec}$$

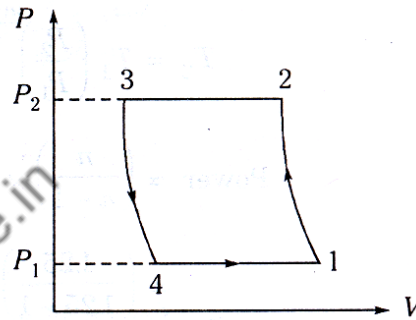


$$\begin{aligned} \text{Power} &= \left(\frac{n}{n-1} \right) \times mR(T_2 - T_1) \\ &= \left(\frac{1.32}{1.32-1} \right) \times 0.3427 \times 0.287(495.71 - 303) \\ &= 70 \text{ kJ/s or kW} \end{aligned}$$

Problem 4

A single stage double acting air compressor delivers 15 m³ of air per min of air measured at 1.013 bar and 27°C. delivers at 7bar. The condition at the end of the suction stroke are pressure 0.98 bar and temperature 4°C. The a clearance volume is 4% of the swept volume, and stroke to bore ratio is 1.3:1 and compressor runs at 300rpm. calculate the Volumetric efficiency of the compressor. Assume the index of compression and expansion to be 1.3.

$$\begin{aligned} T_a &= 27^\circ\text{C}, & V_a &= 15 \text{ m}^3 / \text{min} \\ P_a &= 1.013 \text{ bar} & P_1 &= 0.98 \text{ bar} \\ T_1 &= 40^\circ\text{C} & P_2 &= 7 \text{ bar} \\ N &= 300 \text{ rpm, Clearance ratio} & &= 0.04 \\ L/d &= 1.3 \end{aligned}$$



$$\begin{aligned} \eta_v &= \frac{P_1 T_a}{P_a T_1} \left[1 + C - C \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}} \right] \\ \eta_v &= \frac{98.1 \times 300}{1.013 \times 313} \left[1 + 0.04 - 0.04 \left(\frac{7}{0.98} \right)^{\frac{1}{1.3}} \right] = 0.796 \\ &= 79.6\% \end{aligned}$$

$$\eta_v = \frac{V_a}{V_s}$$

$$V_s = \text{Swept Volume} = \frac{15}{0.796} = 18.84 \text{ m}^3 / \text{min}$$

But for double acting compressor

$$V_s = \frac{\Pi}{4} d^2 \times L \times 2NL = 1.3d$$

$$18.84 = \frac{\Pi}{4} d^2 \times 1.3d \times 2 \times 300$$

$$d = \sqrt[3]{\frac{18.8 \times 44}{\Pi \times 1.3 \times 2 \times 300}} = 0.313 \text{ m} = 31.3 \text{ cm}$$

$$L = 1.3 \times 31.3 = 40.72 \text{ cm}$$

$$m = \frac{P_a T_a}{R T_a} = \frac{1.013 \times 100 \times 15}{0.287 \times 300} = 17.6 \text{ kg/min}$$

$$= \frac{17.6}{60} = 0.294 \text{ kg/s}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 313 \left(\frac{7}{0.98} \right)^{\frac{1.3-1}{1.3}} = 492.7 \text{ K}$$

$$\text{Power} = \left(\frac{n}{n-1} \right) \times mR(T_2 - T_1)$$

$$\text{Power} = \left(\frac{1.3}{1.3-1} \right) \times 0.294 \times 0.287(492.71 - 313)$$

$$= 65.7 \text{ kJ/s or kW}$$

$$\text{Isothermal Power} = P_1 V_1 \ln \frac{P_2}{P_1} = mRT_1 \ln \frac{P_2}{P_1}$$

$$= 0.294 \times 0.287 \times 313 \ln \frac{7}{0.98} = 51.92 \text{ kW}$$

$$\text{Isothermal efficiency} = \frac{51.92}{65.7} \times 100 = 79\%$$

Problem 5

A two-stage compressor compresses 1kg/min of air from 1bar to 42.18 bar. Initial temperature is 15°C. At the intermediate pressure the intercooling is perfect. The compression takes place according to $PV^{1.35} = C$. Neglecting the effect of clearance, determine the minimum power required to run the compressor. Also find the mass of cooling water required in the intercooler, if the temperature rise of water is limited to 5°C.

$$M = 1 \text{ kg/min} = \frac{1}{60} = 0.0166 \text{ kg/s}$$

$$P_1 = 1 \text{ bar} \quad T_1 = 15^\circ \text{C} \quad P_3 = 42.18 \text{ bar}$$

For perfect intercooling

$$P_2 = \sqrt{P_1 P_3} = \sqrt{1 \times 42.18} = 6.48 \text{ bar}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 288 \left(\frac{6.49}{1} \right)^{\frac{1.35-1}{1.35}} = 467.7 \text{ K}$$

$$\text{Maximum Power} = \left(\frac{n}{n-1} \right) \times mR(T_2 - T_1)$$

$$= \left(\frac{2 \times 1.35}{1.35 - 1} \right) \times 0.0166 \times 0.287(467.7 - 288)$$

$$= 6.6 \text{ kJ/s or kW}$$

$$\text{Heat rejected in the intercooler } Q = mC_p(T_2 - T_1)$$

$$= 0.0166 \times 1.005(467.7 - 288)$$

$$= 2.99 \text{ kJ/Sec}$$

But Q is also = $m_{\omega} \times C_{p\omega} \times$ temperature rise

$$299 = m_{\omega} \times 4.187 \times 5$$

$$m_{\omega} = 0.1432 \text{ kg/sec}$$

Problem 6

A two stage reciprocating compressor delivers 150 m³/hr of free air measured at 1.03 bar and 15°C. The final pressure 18 bar. The pressure and temperature of the air in LP cylinder before compression is 1 bar and 30°C. the diameter of the LP cylinder is twice that of HP cylinder and air enters the HP cylinder at 40°C. If compression follows the law $PV^{1.22} = C$, determine

- Intermediate pressure and power required if the intercooler is imperfect.
- Ration of cylinder diameter and minimum power required for perfect intercooling.

$$T_a = 15^\circ\text{C}, \quad V_a = 150 \text{ m}^3 / \text{min}$$

$$P_a = 1.03 \text{ bar} \quad P_1 = 1 \text{ bar}$$

$$T_1 = 30^\circ\text{C} \quad P_3 = 18 \text{ bar}$$

$$T_2 = 40^\circ\text{C}$$

Neglet the effect of clearance

(i) Imperfect intercooling

$$d_{LP} = 2d_{HP}$$

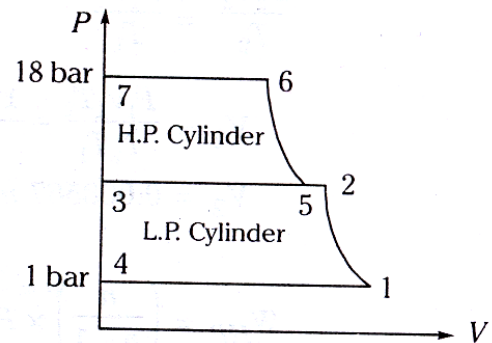
$$P_1 V_1^{1.22} = P_2 V_2^{1.22}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{1.22} = \left(\frac{\frac{\Pi}{4} d_{LP}^2 \times L}{\frac{\Pi}{4} d_{HP}^2 \times L} \right)^{1.22}$$

$$\frac{P_2}{P_1} = \left(\frac{d_{LP}}{d_{HP}} \right)^{2 \times 1.22}$$

$$P_2 = 1 \times (2)^{2.44}$$

Intercooling pressure $P_2 = 5.426 \text{ bar}$



$$\frac{P_1 V_1}{T_1} = \frac{P_a V_a}{T_a}$$

$$V_1 = \frac{P_a V_a T_1}{T_a P_1} = \frac{1.03 \times 10^2 \times 0.02916 \times 303}{1 \times 10^2 \times 288}$$

$$= 0.045 \text{ m}^3 / \text{sec}$$

$$W_{LP} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_{LP} = \frac{1.22}{1.22-1} 1 \times 10^2 \times 0.045 \left[\left(\frac{5.426}{1} \right)^{\frac{1.22-1}{1.22}} - 1 \right]$$

$$= 8.89 \text{ kW}$$

Air enters HP cylinder at $T_2 = 40^\circ\text{C}$

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$$

$$V_2 = \frac{P_1 V_1 T_2}{T_1 P_2} = \frac{1 \times 10^2 \times 0.045 \times 313}{5.426 \times 10^2 \times 303}$$

$$= 0.008567 \text{ m}^3 / \text{sec}$$

$$W_{HP} = \frac{n}{n-1} P_2 V_2 \left[\left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_{HP} = \frac{1.22}{1.22-1} 5.426 \times 10^2 \times 0.008567 \left[\left(\frac{18}{5.426} \right)^{\frac{1.22-1}{1.22}} - 1 \right]$$

$$= 6.12 \text{ kW}$$

$$\text{Total Power} = 8.89 + 6.12$$

$$= 15.01 \text{ kW}$$

Problem 7

A multi stage compressor compressing air is to be designed to elevate the pressure from 1 bar to 120 bar such that the stage pressure ratio should not exceed 4. Determine

- (i) The number of stages
- (ii) Exact stage pressure ratio
- (iii) Intermediate pressure

Solution:

$$P_1 = 1\text{bar} \quad P_{N+1} = 120\text{bar}$$

$$\text{Stage pressure ratio} = \frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = \frac{P_{N+1}}{P_N} = 4$$

Assuming the intercooling to be perfect we have

$$\frac{P_{N+1}}{P_N} = \left(\frac{P_{N+1}}{P_1} \right)^{\frac{1}{N}}$$

$$4 = \left(\frac{120}{1} \right)^{\frac{1}{N}}$$

$$N = \frac{\ln 120}{\ln 4} = 3.453$$

∴ Number of stages = 4

$$\text{Exact stage pressure ratio} = \frac{P_{N+1}}{P_N} = (120)^{\frac{1}{4}} = 3.31$$

$$\frac{P_5}{P_4} = 3.31, \quad P_4 = \frac{120}{3.31} = 36.25 \text{ bar}$$

$$\frac{P_4}{P_3} = 3.31, \quad P_3 = \frac{36.25}{3.31} = 10.95 \text{ bar}$$

$$\frac{P_3}{P_2} = 3.31, \quad P_2 = \frac{10.95}{3.31} = 3.308 \text{ bar}$$

∴ Intermediate pressures are 36.25 bar, 10.95 bar
and 3.308 bar

Problem 8

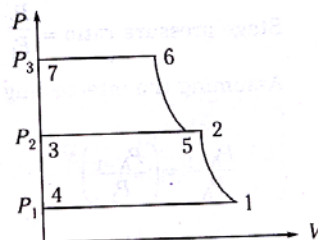
A two-stage compressor delivers air at pressure of 19 bar. The free air conditions are 1.03 bar and 25°C. The pressure of the air before compression is 0.98 bar. The intermediate pressure is 4.5 bar. The temperature of the air entering each cylinder is 35°C. The law of compression and expansion being $PV^{1.25} = C$. The clearance volume is 5% of the swept volume. Determine the volumetric efficiency and the work done per kg of air.

$$T_a = 25^\circ\text{C}, \quad P_2 = 4.5\text{bar}$$

$$P_a = 1\text{bar} \quad P_1 = 0.98\text{bar}$$

$$T_1 = 35^\circ\text{C} \quad P_3 = 19\text{bar}$$

$$\text{Clearance ratio } C = 0.05$$



$$\eta_v = \frac{P_1 T_a}{P_a T_1} \left[1 + C - C \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}} \right]$$

$$\eta_v = \frac{98.1 \times 298}{1 \times 308} \left[1 + 0.05 - 0.05 \left(\frac{4.5}{0.98} \right)^{\frac{1}{1.25}} \right] = 0.835$$

$$= 83.5\%$$

Even though the temperature of air entering each cylinder is same, work is not same in both cylinder since $P_2 \neq \sqrt{P_1 P_3}$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 308 \left(\frac{4.5}{0.98} \right)^{\frac{1.25-1}{1.25}} = 416.1 K$$

$$W_{LP} = \frac{n}{n-1} R(T_2 - T_1) = \frac{1.25}{1.25-1} \times 0.287(416.1 - 308)$$

$$= 155.12 \text{ kJ/kg}$$

Temperature of air entering HP cylinder $T_5 = 35^\circ C$

$$T_6 = T_5 \left(\frac{P_6}{P_5} \right)^{\frac{n-1}{n}} = 308 \left(\frac{19}{4.5} \right)^{\frac{1.25-1}{1.25}} = 410.82 K$$

$$W_{HP} = \frac{n}{n-1} R(T_6 - T_5) = \frac{1.25}{1.25-1} \times 0.287(410.82 - 308)$$

$$= 147.55 \text{ kJ/kg}$$

$$\text{Total work} = 155.12 + 147.55$$

$$= 302.75 \text{ kJ/kg of air}$$

Problem 9

A two-stage double acting air compressor takes in air at 1 bar and $25^\circ C$. It runs at 200 rpm. The diameter of LP cylinder is 35cm. The stroke of both LP and HP cylinders are 40cm. The clearance volume of both the cylinders is 4%. The index of compression is 1.3. The LP cylinder discharges air at a pressure of 4 bar. The air passes through the intercooler so that it enters the HP cylinder at $27^\circ C$ and 3.6 bar, finally it is discharged from the compressor at 14.4 bar.

Calculate

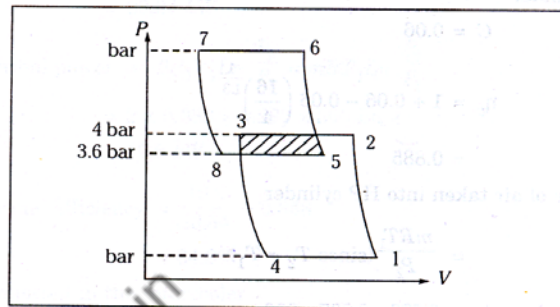
- (i) Diameter of HP cylinder
- (ii) The heat rejected in the intercooler
- (iii) The power required to drive the HP cylinder

$P_1 = 1 \text{ bar}$ $T_1 = 25^\circ\text{C}$ $N = 200$ For a double acting compressor, swept volume of LP cylinder

$$= \frac{\pi}{4} D_{LP}^2 \times L_{LP} \times \frac{2N}{60}$$

$$= \frac{\pi}{4} \times 0.35^2 \times 0.4 \times \frac{2 \times 220}{60}$$

$$V_s = 0.2822 \text{ m}^3/\text{sec}$$



Volumetric efficiency referred to the suction condition 1

$$\eta_v = 1 + C - C \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}}$$

Volume of air referred to condition 1

$$V_1 = 0.9238 \times 0.2822 = 0.26071 \text{ m}^3 / \text{S}$$

$$\text{Mass of air } m = \frac{P_1 V_1}{RT_1} = \frac{1 \times 10^5 \times 0.26071}{0.287 \times 298}$$

$$= 0.30483 \text{ kg/s}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 298 \left(\frac{4}{1} \right)^{\frac{1.3-1}{1.3}} = 410.34 \text{ K}$$

Heat rejected in the intercooler

$$= m C_p (T_2 - T_1) = 0.30483 \times 1.005 (410.34 - 298)$$

$$= 34.14 \text{ kJ/s}$$

Volume of air draw to HP cylinder

$$V_5 = \frac{mRT_5}{P_5} = \frac{0.30483 \times 0.287 \times 298}{3.6 \times 10^2} = 0.0724 \text{ m}^3 / \text{s}$$

Pressure ratio in LP cylinder = 4

Pressure ratio in HP cylinder = $14.4/3.6=4$

Since the pressure ratio and clearance percentage in both HP and LP cylinders are the same, the volumetric efficiency of the cylinders referred to the condition at the start of compression is same.

$$\begin{aligned} \therefore \text{Swept volume of HP cylinder} &= \frac{V_5}{\eta_v} = \frac{0.0724}{0.9238} \\ &= 0.07837 \text{ m}^3 / \text{s} \end{aligned}$$

$$\frac{\pi}{4} D_{HP}^2 L_{HP} \times \frac{2 \times 200}{60} = 0.7837$$

$$\begin{aligned} \frac{\pi}{4} D_{HP}^2 \times 0.4 \times \frac{2 \times 200}{60} &= 0.1934 \text{ m} \\ &= 19.34 \text{ cm} \end{aligned}$$

Since pressure ratio is same $T_6 = T_2$

Power required for HP cylinder

$$\begin{aligned} &= \left(\frac{n}{n-1} \right) \times mR(T_6 - T_5) \\ &= \left(\frac{n}{n-1} \right) \times mR(T_2 - T_1) \\ &= \left(\frac{1.3}{1.3-1} \right) \times 0.30483 \times 0.287 (410.34 - 298) \\ &= 42.58 \text{ kW} \end{aligned}$$

Problem 10

A two stage air compressor compresses air from 17°C and 1 bar to 63 bar. The air is cooled in the intercooler to 30°C and intermediate pressure is steady at 7.7 bar. The low pressure cylinder is 10 cm diameter and the stroke for both cylinders is 11.25 cm. Assuming a compression law of $PV^{1.35} = \text{constant}$, and that the volume of air at atmospheric conditions drawn in per stroke is equal to the low pressure cylinder

swept volume, find the power of the compressor while running at 250 rpm. Find also the diameter of HP cylinder.

Solution:

$$d_{LP} = 10\text{cm} \quad L = 11.25\text{cm}$$

$$P_1 = 1 \text{ bar} \quad P_2 = 7.7 \text{ bar} \quad T_1 = 17^\circ\text{C}$$

$$P_3 = 63 \text{ bar}$$

Volume of LP cylinder

$$V_1 = \pi/4 \times 0.1^2 \times 0.1125 = 0.00088 \text{ m}^3$$

$$m = P_1 V_1 / R T_1 = (1 \times 100 \times 0.00088) / (0.287 \times 290) = 0.00106 \text{ kg}$$

$$T_2 = 30^\circ\text{C}$$

Volume of air entering the HP cylinder

$$V_2 = mR T_2 / P_2 = (0.00106 \times 0.283 \times 303) / (7.7 \times 10^2) = 0.0001198 \text{ m}^3$$

$$V_2 = \pi/4 \times d_2^2 \times L$$

$$0.0001198 = \pi/4 \times d_2^2 \times 0.1125$$

$$d_2 = \text{diameter of HP cylinder}$$

$$= 0.0368 \text{ m}$$

Diameter of HP cylinder = 3.68 cm

$$\text{Work required/cycle} = (n/n-1) [P_1 V_1 \{ (P_2 / P_1)^{n-1/n} - 1 \} + P_2 V_2 (P_3 / P_2)^{n-1/n} - 1]$$

$$W = (1.35/1.35-1) [1 \times 10^2 \times 0.00088 \{ (7.7/1)^{1.35-1/1.35} - 1 \} + 7.7 \times 10^2 \times 0.0001198 \{ (63/7.7)^{1.35-1/1.35} - 1 \}]$$

$$= 0.49456 \text{ kJ/cycle}$$

$$\text{Power} = 0.49456 \times \text{Number of cycles/sec}$$

$$= 0.49456 \times 250/60$$

$$\text{Power} = 2.06 \text{ kW}$$

Problem 11

A three stage air compressor draws $8 \text{ m}^3/\text{min}$ of air at 1 bar and 18°C and delivers the same at 55 bar and 20°C . The index of compression is 1.32. The air while passing through the intercoolers and aftercoolers suffers a pressure loss of 4% and is cooled to the initial temperature. Determine the shaft power required to drive the compressor if mechanical efficiency is 85%.

Solution:

$$V_1 = 8 \text{ m}^3/\text{min} = 8/60 = 0.133 \text{ m}^3/\text{sec}$$

$$P_1 = 1 \text{ bar} \quad T_1 = 18^\circ\text{C} \quad P_4 = 55 \text{ bar}$$

The pressure drop of 4% in the intercooler is accounted for by the factor $C = 0.96$

For 3 stage compressor,

$$\begin{aligned}\text{Power} &= 3(n/n-1) \times P_1 V_1 [\{P_4/C^3 P_1\}^{n-1/3n} - 1] \\ &= 3 (1.32/1.32-1) \times 1 \times 10^2 \times 0.133 [\{55/0.96^3 \times 1\}^{1.32-1/1.32} - 1] \\ &= 283.32 \text{ kW}\end{aligned}$$

$$\text{Actual shaft power required} = \text{Power}/\eta_{\text{mech}} = 283.32/0.85 = \mathbf{333.32 \text{ kW}}$$

STEAM NOZZLES

8.1 Introduction

In the impulse steam turbine, the overall transformation of heat into mechanical work is accomplished in two distinct steps. The available energy of steam is first changed into kinetic energy, and this kinetic energy is then transformed into mechanical work. The first of these steps, viz., the transformation of available energy into kinetic energy is dealt with in this chapter.

A nozzle is a passage of varying cross-sectional area in which the potential energy of the steam is converted into kinetic energy. The increase of velocity of the steam jet at the exit of the nozzle is obtained due to decrease in enthalpy (total heat content) of the steam. The nozzle is so shaped that it will perform this conversion of energy with minimum loss.

8.2 General Forms of Nozzle Passages

A nozzle is an element whose primary function is to convert enthalpy (total heat) energy into kinetic energy. When the steam flows through a suitably shaped nozzle from zone of high pressure to one at low pressure, its velocity and specific volume both will increase.

The equation of the continuity of mass may be written thus :

...(8.1)

where m mass flow in kg/sec.,

V = velocity of steam in m/sec.,

A = area of cross-section in m^2 , and

v = specific volume of steam in m^3/kg .

In order to allow the expansion to take place properly, the area at any section of the nozzle must be such that it will accommodate the steam whatever volume and velocity may prevail at that point.

As the mass flow (m) is same at all sections of the nozzle, area of cross-section (A) varies as $\frac{1}{Vv}$. The manner in which both V and v vary depends upon the properties of the substance flowing. Hence, the contour of the passage of nozzle depends upon the nature of the substance flowing.

For example, consider a *liquid*- a substance whose specific volume v remains almost constant with change of pressure. The value of V will go on increasing with change of pressure. Thus, from eqn. (8.1), the area of cross-section should decrease with the decrease of pressure. Fig. 8-1(a) illustrates the proper contour of longitudinal section of

a nozzle suitable for liquid. This also can represent convergent nozzle for a fluid whose peculiarity is that while both velocity and specific volume increase, the rate of specific volume increase is less than that of the velocity, thus resulting in increasing value of $\frac{V}{v}$.

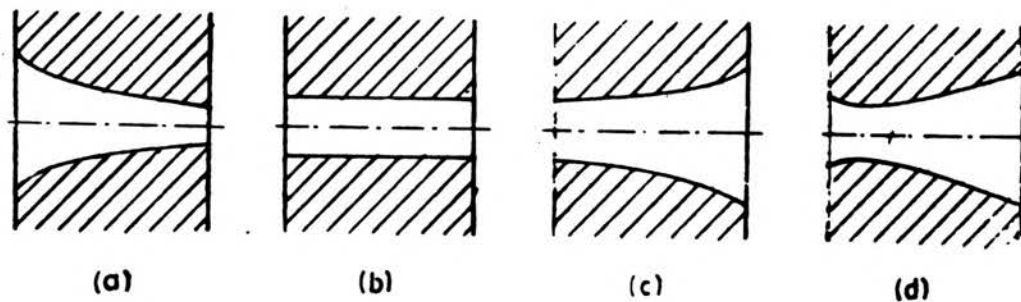


Fig. 8-1. General forms of Nozzles.

Fig. 8-1(b) represents the correct contour for some hypothetical substance for which both velocity and specific volume increase at the same rate, so that their ratio $\frac{V}{v}$ is a constant at all points. The area of cross-section should therefore, be constant at all points, and the nozzle becomes a plain tube.

Fig. 8-1(c) represents a divergent nozzle for a fluid whose peculiarity is that $\frac{V}{v}$ decreases with the drop of pressure, i.e., specific volume increases at a faster rate than velocity with the drop of pressure. The area of cross-section should increase as the pressure decreases.

Table 8-1

Properties of steam at various pressures when expanding dry saturated steam from 14 bar to 0.15 bar through a nozzle, assuming frictionless adiabatic flow.

Pressure p bar	Dryness fraction x	Enthalpy drop $H_1 - H_2$ kJ	Velocity V m/sec.	Specific Volume v_s m^3/kg	Discharge per unit area kg/m^2	Area A m^2	Diameter D metre
14	1.000	-	-	-	-	-	-
12	0.988	38.6	278	0.1633	1,723	0.00058	0.0272
10	0.974	84.1	410	0.1944	2,165	0.00046	0.0242
7	0.950	164.7	574	0.2729	*2,214	0.00045	x0.0239
3.5	0.908	309	786	0.5243	1,651	0.00061	0.0279
1.5	0.872	441.2	939	1.1593	929	0.0011	0.0374
0.70	0.840	555.6	1,054	2.365	531	0.00188	0.049
0.15	0.790	736.7	1,214	10.022	153	0.0065	0.091

* Maximum discharge per unit area

x Smallest diameter

Fig. 8-1(d) shows the general shape of convergent-divergent nozzle suitable for gases and vapours. It can be shown that in practice, while velocity and specific volume both increase from the start, velocity first increases faster than the specific volume, but after

a certain critical point, specific volume increases more rapidly than velocity. Hence the value of $\frac{V}{v}$ first increases to maximum and then decreases, necessitating a nozzle of *convergent-divergent* form. The above statement may be verified by referring to table 8-1, which shows the properties of steam at various pressures when expanding dry saturated steam from 14 bar to 0.15 bar through a nozzle, assuming frictionless adiabatic flow.

8.3 Steam Nozzles

The mass flow per second for wet steam, at a given pressure during expansion is given by

$$m = \frac{AV}{v} = \frac{AV}{xv_s} \text{ kg/sec.} \quad \dots(8.2)$$

where A = Area of cross-section in m^2 ,

V = Velocity of steam in m/sec,

v_s = Specific volume of dry saturated steam, m^3/kg ,

x = Dryness fraction of steam, and

$v = x v_s$ = Specific volume of wet steam, m^3/kg .

As the mass of steam per second (m) passing through any section of the nozzle must be constant, the area of cross-section (A) of nozzle will vary according to the variation of $\frac{V}{xv_s}$ i.e., product of A and $\frac{V}{xv_s}$ is constant. If the factor $\frac{V}{xv_s}$ increases with the drop in pressure, the cross-sectional area should decrease and hence a *convergent* shaped nozzle. The decrease of the factor $\frac{V}{xv_s}$ with pressure drop will require increasing cross-sectional area to maintain mass flow constant and hence the *divergent* shaped nozzle.

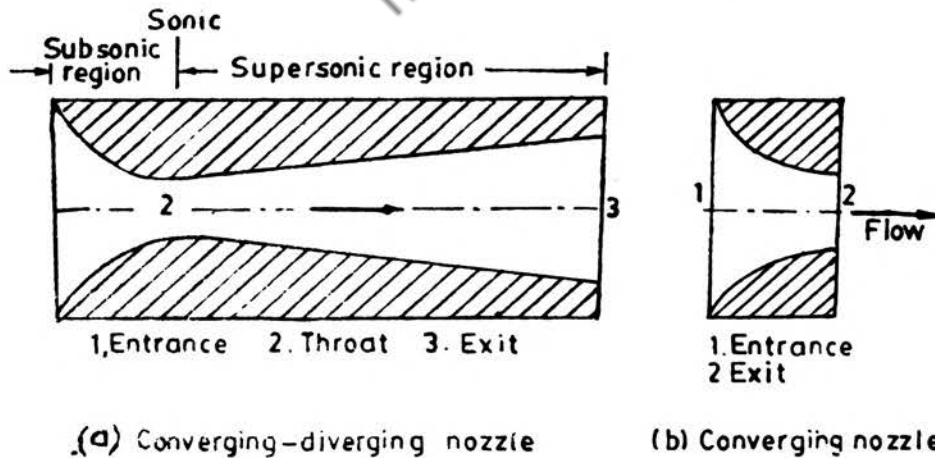


Fig. 8-2. Longitudinal sectional view of steam nozzles.

In practice at first the nozzle cross-section tapers to a smaller section in order to

allow for increasing value of $\frac{V}{xv_s}$; after this smallest diameter is reached, it will diverge to a larger cross-section. The smallest section of the nozzle is known as the *throat*.

A nozzle which first converges to throat and then diverges, as in fig. 8-2(a), is termed as *converging-diverging nozzle*. It is used for higher pressure ratio $\left(\frac{p_2}{p_1}\right)$.

Some form of nozzles finish at the throat and no diverging portion is fitted; this type shown in fig. 8-2(b), is known as *converging nozzle*. In this the greatest area is at the entrance and minimum area is at the exit which is also the throat of the nozzle. This nozzle is used when the pressure ratio, $\frac{P_2}{P_1}$ is less than 0.58 (critical).

8.4 Flow Through Steam Nozzles

From the point of view of thermodynamics, the steam flow through nozzles may be spoken as adiabatic expansion. During the flow of steam through the nozzle, heat is neither supplied nor rejected. Moreover, as the steam expands from high pressure to low pressure, the heat energy is converted into kinetic energy, i.e., work is done in expanding to increase the kinetic energy. Thus the expansion of steam through a nozzle is an adiabatic, and the flow of steam through nozzle is regarded as an adiabatic flow.

It should be noted that the expansion of steam through a nozzle is not a free expansion, and the steam is not throttled, because it has a large velocity at the end of the expansion. Work is done by the expanding steam in producing this kinetic energy.

In practice, some kinetic energy is lost in overcoming the friction between the steam and the side of the nozzle and also internal friction, which will tend to regenerate heat. The heat thus formed tends to dry the steam. About 10% to 15% of the enthalpy drop from inlet to exit is lost in friction. The effect of this friction, in resisting the flow and in drying the steam, must be taken into account in the design of steam nozzles, as it makes an appreciable difference in the results.

Another complication in the design of steam through a nozzle is due to a phenomenon known as *supersaturation*; this is due to a time lag in the condensation of the steam during the expansion. The expansion takes place very rapidly and if the steam is initially dry or superheated, it should become wet as the pressure falls, because the expansion is adiabatic. During expansion the steam does not have time to condense, but remains in an unnatural dry or superheated state, then at a certain instant, it suddenly condenses to its natural state. See illustrative problem no. 14.

Thus, the flow of steam through a nozzle may be regarded as either an ideal adiabatic (isentropic) flow, or adiabatic flow modified by friction and supersaturation.

If friction is negligible, three steps are essential in the process of expansion from pressure p_1 to p_2 :

(i) Driving of steam upto the nozzle inlet from the boiler. The 'flow-work' done on the steam is $p_1 v_1$ and results in similar volume of steam being forced through the exit to make room for fresh charge (steam).

(ii) Expansion of steam through the nozzle while pressure changes from p_1 to p_2 , the work done being $\frac{1}{n-1} (p_1 v_1 - p_2 v_2)$

where n is the index of the isentropic expansion,

v_1 = volume occupied by 1 kg of steam at entrance to nozzle, and

v_2 = volume occupied by 1 kg of steam as it leaves the nozzle.

Alternatively, this work done is equal to the change of internal energy, $\mu_1 - \mu_2$ as during isentropic expansion work is done at the cost of internal energy.

(iii) Displacement of the steam from the low pressure zone by an equal volume discharged from the nozzle. This work amounts to $p_2 v_2$ which is equal to the final flow work spent in forcing the steam out to make room for fresh charge (steam).

Thus, the new work done in increasing kinetic energy of the steam,

$$W = p_1 v_1 + \left[\frac{1}{n-1} (p_1 v_1 - p_2 v_2) \right] - p_2 v_2$$

$$W = \frac{n}{n-1} (p_1 v_1 - p_2 v_2) \quad \dots (8.3)$$

This is same as the work done during Rankine cycle.

$$\text{Alternatively, } W = p_1 v_1 + (\mu_1 - \mu_2) - p_2 v_2$$

$$= (p_1 v_1 + \mu_1) - (\mu_2 + p_2 v_2) = H_1 - H_2 \quad \dots (8.4)$$

where, H_1 and H_2 are the values of initial and final enthalpies allowing for the states of superheating or wetness as the case may be. This is exactly equivalent to the enthalpy drop equivalent to the work done during the Rankine cycle. The value of $H_1 - H_2$ may be found very rapidly from the Mollier chart ($H - \Phi$ chart) or more slowly but with greater accuracy from the steam tables.

In the design of steam nozzles the calculations to be made are :

- (i) the actual velocity attained by the steam at the exit,
- (ii) the minimum cross-sectional area (throat area) required for a given mass flow per second,
- (iii) the exit area, if the nozzle is converging-diverging, and
- (iv) the general shape of the nozzle – axial length.

8.4.1 Velocity of steam leaving nozzle : The gain of kinetic energy is equal to the enthalpy drop of the steam. The initial velocity of the steam entering the nozzle (or velocity of approach) may be neglected as being relatively very small compared with exit velocity.

For isentropic (frictionless adiabatic) flow and considering one kilogram of steam

$$\frac{V^2}{2 \times 1,000} = H_1 - H_2 = H$$

where H is enthalpy drop in kJ/kg and V = velocity of steam leaving the nozzle in m/sec.

$$\therefore V = \sqrt{2 \times 1,000H} = 44.72 \sqrt{H} \text{ m/sec.} \quad \dots (8.5)$$

Let the available enthalpy drop after deducting frictional loss be kH ,

i.e. $(1 - k)H$ is the friction loss,

$$\text{Then, } V = 44.72 \sqrt{kH} \text{ m/sec.} \quad \dots (8.6)$$

If the frictional loss in the nozzle is 15 per cent of the enthalpy drop, then $k = 0.85$.

8.4.2 Mass of steam discharged : The mass flow of steam in kg per second through a cross-sectional area A and at a pressure p_2 can be written as

$$m = \frac{AV_2}{v_2} \quad \text{where } v_2 = \text{specific volume of steam at pressure } p_2.$$

$$\text{But } v_2 = v_1 \left(\frac{p_1}{p_2} \right)^{\frac{1}{n}} = v_1 \left(\frac{p_2}{p_1} \right)^{-\frac{1}{n}} \quad \dots(8.7)$$

where, v_1 = specific volume of steam at pressure p_1 .

Using the value of velocity V from eqns. (8.3.) and (8.5),

$$m = \frac{A}{v_2} \sqrt{\left[2,000 \frac{n}{n-1} (p_1 v_1 - p_2 v_2) \right]} = \frac{A}{v_2} \sqrt{2,000 \frac{n}{n-1} p_1 v_1 \left[1 - \frac{p_2 v_2}{p_1 v_1} \right]}$$

Putting the value of v_2 from eqn. (8.7), we get,

$$m = \frac{A}{v_1 \left(\frac{p_2}{p_1} \right)^{-\frac{1}{n}}} \sqrt{2,000 \frac{n}{n-1} p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]}$$

$$m = A \sqrt{2,000 \frac{n}{n-1} \times \frac{p_1}{v_1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right]} \quad \dots (8.8)$$

8.4.3 Critical pressure ratio : Using eqn. (8.8), the rate of mass flow per unit area is given by

$$\frac{m}{A} = \sqrt{2,000 \frac{n}{n-1} \times \frac{p_1}{v_1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right]}$$

The mass flow per unit area has the maximum value at the throat which has minimum area, the value of pressure ratio $\left(\frac{p_2}{p_1} \right)$ at the throat can be evaluated from the above expression corresponding to the maximum value of $\frac{m}{A}$.

All the items of this equation are constant with the exception of the ratio $\left(\frac{p_2}{p_1} \right)$.

Hence, $\frac{m}{A}$ is maximum when $\left[\left(\frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right]$ is the maximum.

Differentiating the above expression with respect to $\left(\frac{p_2}{p_1} \right)$ and equating to zero for a maximum discharge per unit area

$$\frac{d}{d \left(\frac{p_2}{p_1} \right)} \left[\left(\frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right] = 0$$

$$\therefore \frac{2}{n} \left[\frac{p_2}{p_1} \right]^{\frac{2}{n} - 1} - \frac{n+1}{n} \left[\frac{p_2}{p_1} \right]^{\frac{n+1}{n} - 1} = 0$$

$$\text{Hence, } \left[\frac{p_2}{p_1} \right]^{\frac{2-n}{n}} = \frac{n+1}{n} \left[\frac{p_2}{p_1} \right]^{\frac{1}{n}} \text{ or } \left[\frac{p_2}{p_1} \right]^{2-n} = \left[\frac{n+1}{2} \right]^n \left(\frac{p_2}{p_1} \right)$$

$$\text{from which } \left[\frac{p_2}{p_1} \right]^{1-n} = \left[\frac{n+1}{2} \right]^n \text{ or } \frac{p_2}{p_1} = \left[\frac{2}{n+1} \right]^{\frac{n}{n-1}} \quad \dots (8.9)$$

$\frac{p_2}{p_1}$ is known as *critical pressure ratio* and depends upon the value of index n .

The following approximate values of index n and corresponding values of critical pressure ratios may be noted :

Initial condition of steam	Value of index n for isentropic expansion	Nozzle critical pressure ratio $\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$
Superheated or supersaturated	1.300	0.546
Dry saturated	1.135	0.578
Wet	1.113	0.582

Dr. Zeuner has suggested a well known equation for value of n in the adiabatic expansion of steam viz. $n = 1.035 + 0.1x_1$, where x_1 is the initial dryness fraction of steam.

The eqn. (8.9) gives the ratio between the throat pressure (p_2) and the inlet pressure (p_1) for a maximum discharge per unit area through the nozzle. The mass flow being constant for all sections of nozzle, maximum discharge per unit area occurs at the section

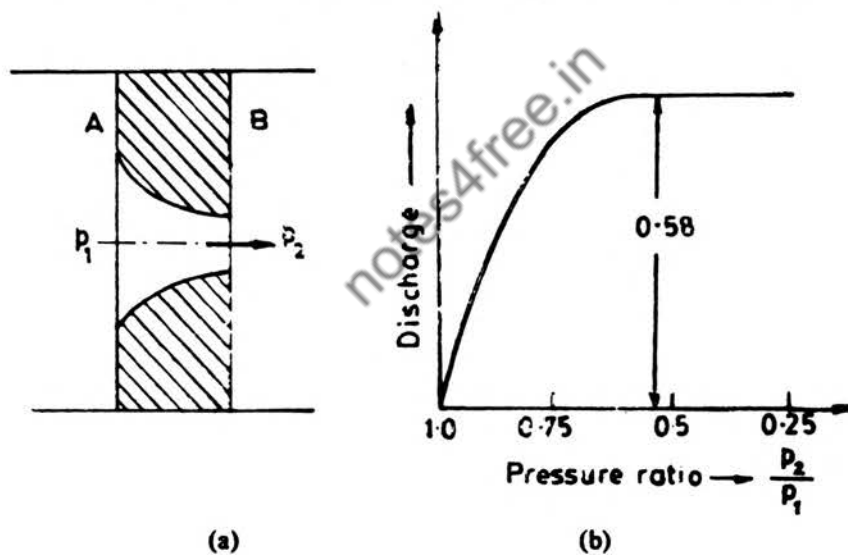


Fig. 8-3.

having minimum area, i.e., at the throat. The area of throat of all steam nozzle should be designed on this ratio. This pressure ratio at the throat is known as *critical pressure ratio*. The pressure at which the area is minimum and discharge per unit area is maximum is termed as the *critical pressure*.

The implication of the existence of a critical pressure in nozzle flow may be expressed in another way. Suppose we have two vessels A and B. A containing steam at a high and steady pressure p_1 . Suppose that the pressure in B may be varied at will. A and B are connected by a diaphragm containing a convergent nozzle, as shown in fig. 8-3(a).

Assume at first that p_2 is equal to p_1 , then there is no flow of steam through the nozzle. Now let p_2 be gradually reduced. The discharge m through the nozzle will increase as shown by the curve of fig. 8-3(b). As the pressure p_2 approaches the critical value, the discharge rate gradually approaches its maximum value, and when p_2 is reduced below the critical value, the discharge rate does not increase but remains at the same value as that at the critical pressure. The extraordinary result that p_2 can be reduced

well below the critical pressure without influencing the mass flow was first discovered by R.D. Napier.

Another explanation can be visualised as follows : the critical pressure will give velocity of steam at the throat equal to the velocity of the sound (*sonic velocity*). The flow of steam in the convergent portion of the nozzle is *sub-sonic*. Thus, to increase the velocity of steam above sonic velocity (*super sonic*) by expanding steam below critical pressure, divergent portion is necessary [fig. 8-2(a)].