Acknowledgements to

Donald Hearn & Pauline Baker: Computer Graphics with OpenGL

Version,3rd / 4th Edition, Pearson Education,2011

Edward Angel: Interactive Computer Graphics- A Top Down approach

with OpenGL, 5th edition. Pearson Education, 2008

M M Raiker, Computer Graphics using OpenGL, Filip learning/Elsevier

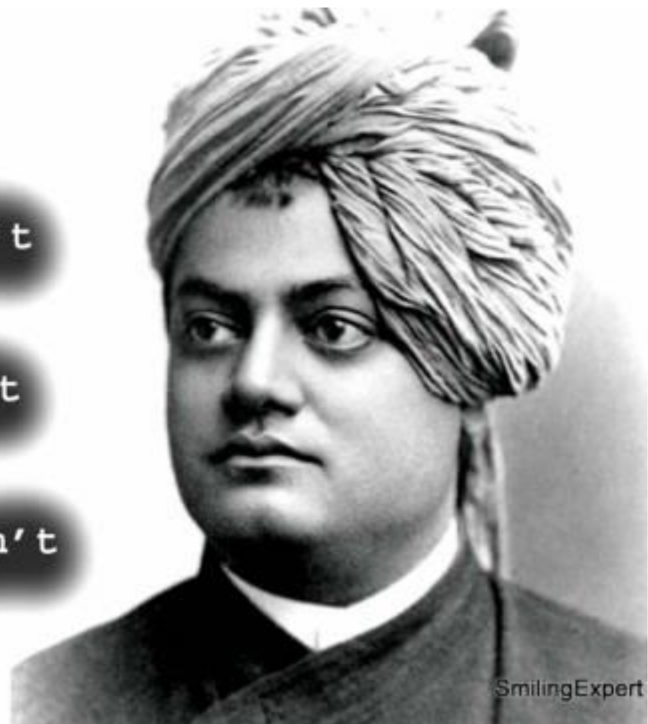
3 GOLDEN RULES

Who is Helping You, Don't Forget them.

Who is Loving you, Don't Hate them .

Who is Believing you, Don't Cheat them.

Swami Vivekananda



If you create an act, you create a habit. If you create a habit, you create a character. If you create a character, you create a destiny.

Reputation is what men and women think of us. Character is what God and the angels know of us.

1. Overview: Computer Graphics and OpenGL

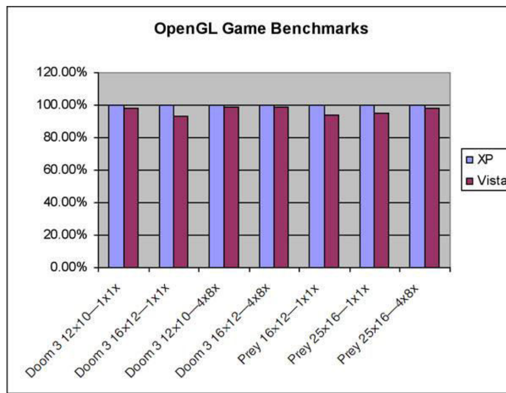
1.1 Basics of computer graphics
1.2 Application of Computer Graphics,
1.3 Video Display Devices
1.3.1 Random Scan and Raster Scan displays,
1.3.2 Color CRT monitors,
1.3.4 Flat panel displays.
1.4 Raster-scan systems:
1.4.1 Video controller,
1.4.2 Raster scan Display processor,
1.4.3 Graphics workstations and viewing systems,
1.5 Input devices,
1.6 Graphics networks,
1.7 Graphics on the internet,
1.8 Graphics software.
OpenGL:
1.9 Introduction to OpenGL ,
1.10 Coordinate reference frames,
1.11 Specifying two-dimensional world coordinate reference frames in OpenGL,
1.12 OpenGL point functions,
1.13 OpenGL line functions, point attributes,
1.14 Line attributes,
1.15 Curve attributes,
1.16 OpenGL point attribute functions,
1.17 OpenGL line attribute functions,
1.18 Line drawing algorithms(DDA, Bresenham's),
1.19 Circle generation algorithms (Bresenham's).

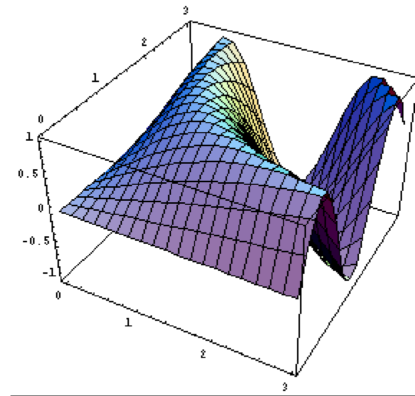
1.1 Basics of Computer Graphics

Computer graphics is an art of drawing pictures, lines, charts, etc. using computers with the help of programming. Computer graphics image is made up of number of pixels. Pixel is the smallest addressable graphical unit represented on the computer screen.

1.2 Applications of Computer Graphics

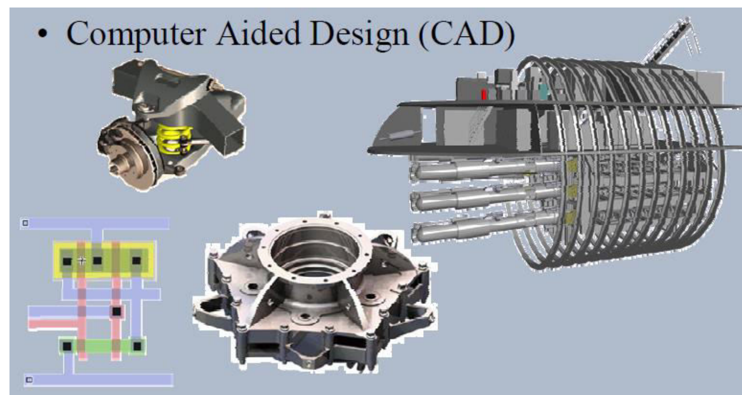
a. Graphs and Charts





- ✓ An early application for computer graphics is the display of simple data graphs usually plotted on a character printer. Data plotting is still one of the most common graphics application.
- ✓ Graphs & charts are commonly used to summarize functional, statistical, mathematical, engineering and economic data for research reports, managerial summaries and other types of publications.
- ✓ Typically examples of data plots are line graphs, bar charts, pie charts, surface graphs, contour plots and other displays showing relationships between multiple parameters in two dimensions, three dimensions, or higher-dimensional spaces

b. Computer-Aided Design



- ✓ A major use of computer graphics is in design processes-particularly for engineering and architectural systems.

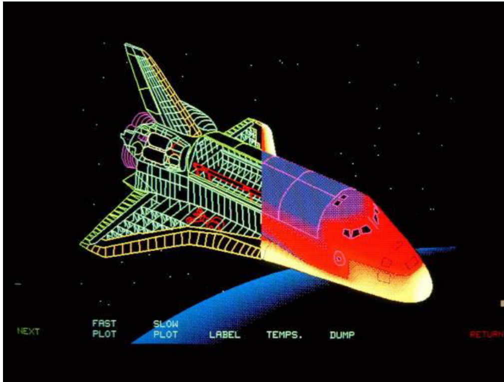
- ✓ CAD, computer-aided design or CADD, computer-aided drafting and design methods are now routinely used in the automobiles, aircraft, spacecraft, computers, home appliances.
- ✓ Circuits and networks for communications, water supply or other utilities are constructed with repeated placement of a few geographical shapes.
- ✓ Animations are often used in CAD applications. Real-time, computer animations using wire-frame shapes are useful for quickly testing the performance of a vehicle or system.

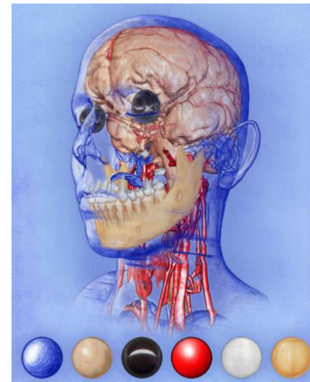
c. Virtual-Reality Environments

- ✓ Animations in virtual-reality environments are often used to train heavy-equipment operators or to analyze the effectiveness of various cabin configurations and control placements.
- ✓ With virtual-reality systems, designers and others can move about and interact with objects in various ways. Architectural designs can be examined by taking simulated "walk" through the rooms or around the outsides of buildings to better appreciate the overall effect of a particular design.
- ✓ With a special glove, we can even "grasp" objects in a scene and turn them over or move them from one place to another.

d. Data Visualizations

- ✓ Producing graphical representations for scientific, engineering and medical data sets and processes is another fairly new application of computer graphics, which is generally referred to as scientific visualization. And the term business visualization is used in connection with data sets related to commerce, industry and other nonscientific areas.



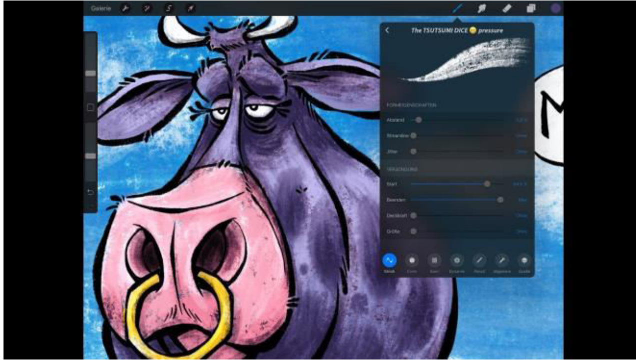


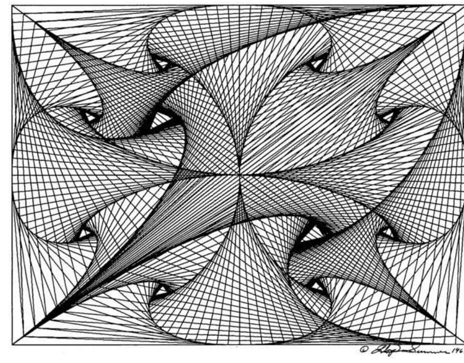
- ✓ There are many different kinds of data sets and effective visualization schemes depend on the characteristics of the data. A collection of data can contain scalar values, vectors or higher-order tensors.

e. Education and Training

- ✓ Computer generated models of physical,financial,political,social,economic & other systems are often used as educational aids.
- ✓ Models of physical processes physiological functions,equipment, such as the color coded diagram as shown in the figure, can help trainees to understand the operation of a system.
- ✓ For some training applications,special hardware systems are designed.Examples of such specialized systems are the simulators for practice sessions ,aircraft pilots,air traffic-control personnel.
- ✓ Some simulators have no video screens,for eg: flight simulator with only a control panel for instrument flying

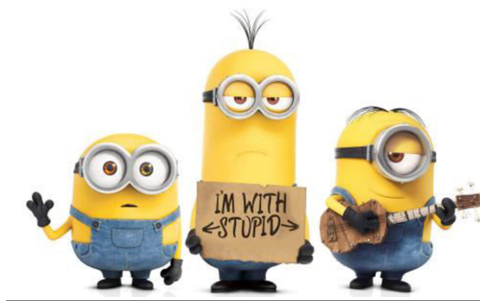
f. Computer Art





- ✓ The picture is usually painted electronically on a graphics tablet using a stylus, which can simulate different brush strokes, brush widths and colors.
- ✓ Fine artists use a variety of other computer technologies to produce images. To create pictures the artist uses a combination of 3D modeling packages, texture mapping, drawing programs and CAD software etc.
- ✓ Commercial art also uses theses "painting" techniques for generating logos & other designs, page layouts combining text & graphics, TV advertising spots & other applications.
- ✓ A common graphics method employed in many television commercials is morphing, where one object is transformed into another.

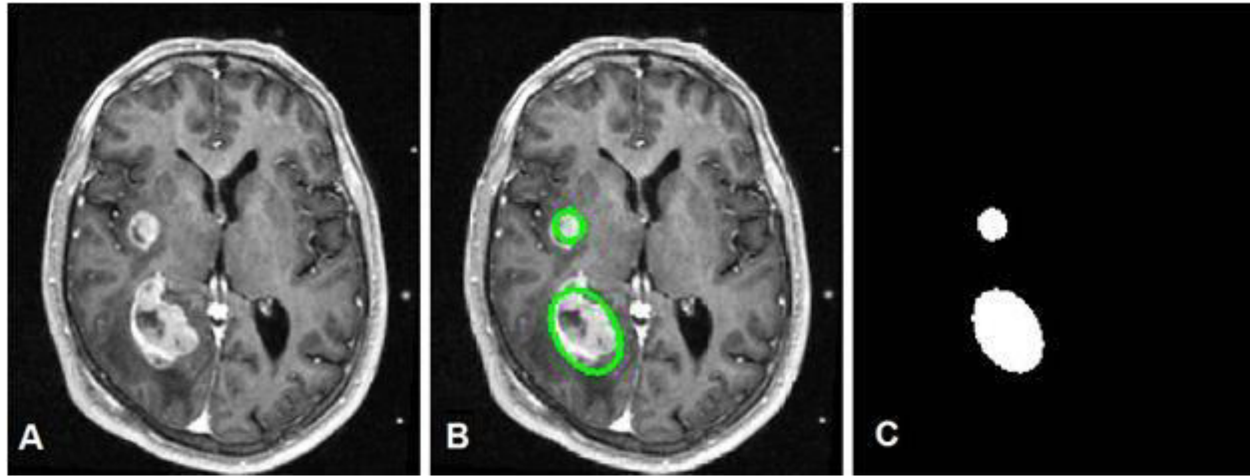
g. Entertainment



- ✓ Television production, motion pictures, and music videos routinely a computer graphics methods.
- ✓ Sometimes graphics images are combined a live actors and scenes and sometimes the films are completely generated a computer rendering and animation techniques.

- ✓ Some television programs also use animation techniques to combine computer generated figures of people, animals, or cartoon characters with the actor in a scene or to transform an actor's face into another shape.

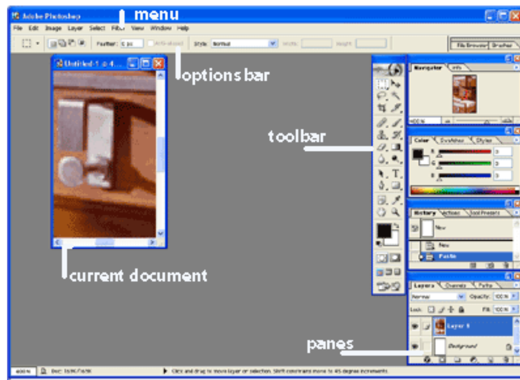
h. Image Processing



- ✓ The modification or interpretation of existing pictures, such as photographs and TV scans is called image processing.
- ✓ Methods used in computer graphics and image processing overlap, the two areas are concerned with fundamentally different operations.
- ✓ Image processing methods are used to improve picture quality, analyze images, or recognize visual patterns for robotics applications.
- ✓ Image processing methods are often used in computer graphics, and computer graphics methods are frequently applied in image processing.
- ✓ Medical applications also make extensive use of image processing techniques for picture enhancements in tomography and in simulations and surgical operations.
- ✓ It is also used in computed X-ray tomography(CT), position emission tomography(PET),and computed axial tomography(CAT).

i. Graphical User Interfaces

- ✓ It is common now for applications software to provide graphical user interface (GUI).
- ✓ A major component of graphical interface is a window manager that allows a user to display multiple, rectangular screen areas called display windows.

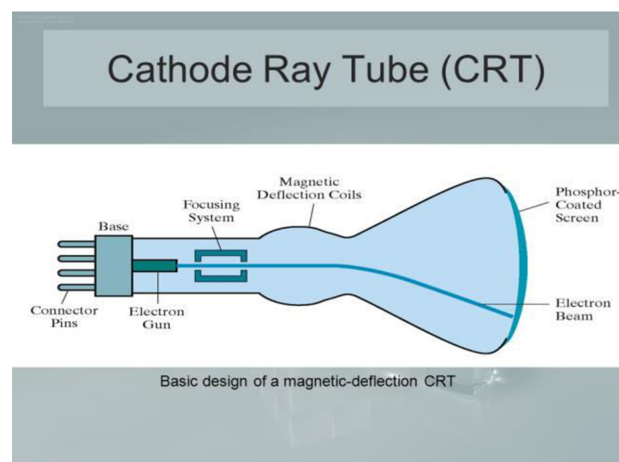


- ✓ Each screen display area can contain a different process, showing graphical or non-graphical information, and various methods can be used to activate a display window.
- ✓ Using an interactive pointing device, such as mouse, we can active a display window on some systems by positioning the screen cursor within the window display area and pressing the left mouse button.

1.3 Video Display Devices

- ✓ The primary output device in a graphics system is a video monitor.
- ✓ Historically, the operation of most video monitors was based on the standard cathoderay tube (CRT) design, but several other technologies exist.
- ✓ In recent years, flat-panel displays have become significantly more popular due to their reduced power consumption and thinner designs.

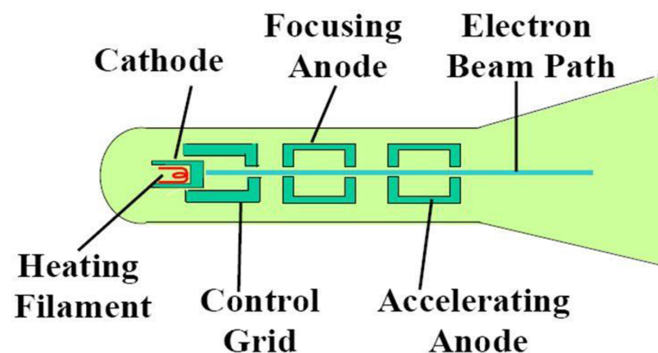
Refresh Cathode-Ray Tubes



Basic design of a magnetic-deflection CRT

- ✓ A beam of electrons, emitted by an electron gun, passes through focusing and deflection systems that direct the beam toward specified positions on the phosphor-coated screen.
- ✓ The phosphor then emits a small spot of light at each position contacted by the electron beam and the light emitted by the phosphor fades very rapidly.
- ✓ One way to maintain the screen picture is to store the picture information as a charge distribution within the CRT in order to keep the phosphors activated.
- ✓ The most common method now employed for maintaining phosphor glow is to redraw the picture repeatedly by quickly directing the electron beam back over the same screen points. This type of display is called a refresh CRT.
- ✓ The frequency at which a picture is redrawn on the screen is referred to as the refresh rate.

Operation of an electron gun with an accelarating anode



- ✓ The primary components of an electron gun in a CRT are the heated metal cathode and a control grid.
- ✓ The heat is supplied to the cathode by directing a current through a coil of wire, called the filament, inside the cylindrical cathode structure.
- ✓ This causes electrons to be “boiled off” the hot cathode surface.
- ✓ Inside the CRT envelope, the free, negatively charged electrons are then accelerated toward the phosphor coating by a high positive voltage.

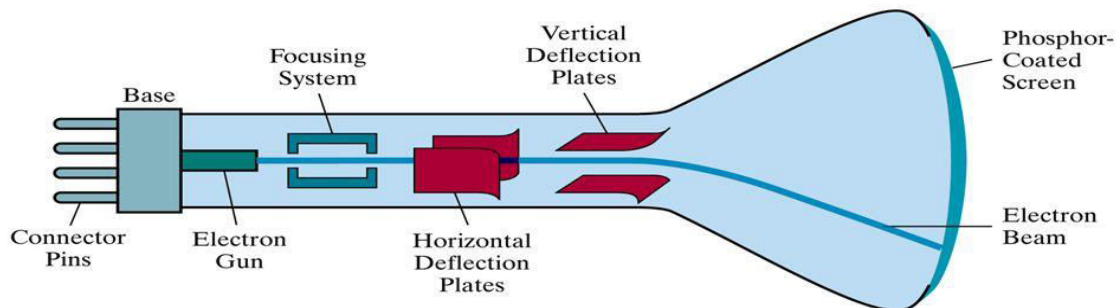
- ✓ Intensity of the electron beam is controlled by the voltage at the control grid.
- ✓ Since the amount of light emitted by the phosphor coating depends on the number of electrons striking the screen, the brightness of a display point is controlled by varying the voltage on the control grid.
- ✓ The focusing system in a CRT forces the electron beam to converge to a small cross section as it strikes the phosphor and it is accomplished with either electric or magnetic fields.
- ✓ With electrostatic focusing, the electron beam is passed through a positively charged metal cylinder so that electrons along the center line of the cylinder are in equilibrium position.
- ✓ Deflection of the electron beam can be controlled with either electric or magnetic fields.
- ✓ Cathode-ray tubes are commonly constructed with two pairs of magnetic-deflection coils
- ✓ One pair is mounted on the top and bottom of the CRT neck, and the other pair is mounted on opposite sides of the neck.
- ✓ The magnetic field produced by each pair of coils results in a traverse deflection force that is perpendicular to both the direction of the magnetic field and the direction of travel of the electron beam.
- ✓ Horizontal and vertical deflections are accomplished with these pair of coils

Electrostatic deflection of the electron beam in a CRT

- ✓ When electrostatic deflection is used, two pairs of parallel plates are mounted inside the CRT envelope where, one pair of plates is mounted horizontally to control vertical deflection, and the other pair is mounted vertically to control horizontal deflection.
- ✓ Spots of light are produced on the screen by the transfer of the CRT beam energy to the phosphor.
- ✓ When the electrons in the beam collide with the phosphor coating, they are stopped and their kinetic energy is absorbed by the phosphor.
- ✓ Part of the beam energy is converted by the friction in to the heat energy, and the remainder causes electros in the phosphor atoms to move up to higher quantum-energy levels.

- ✓ After a short time, the "excited" phosphor electrons begin dropping back to their stable ground state, giving up their extra energy as small quantum of light energy called photons.

Cathode Ray Tube (CRT)



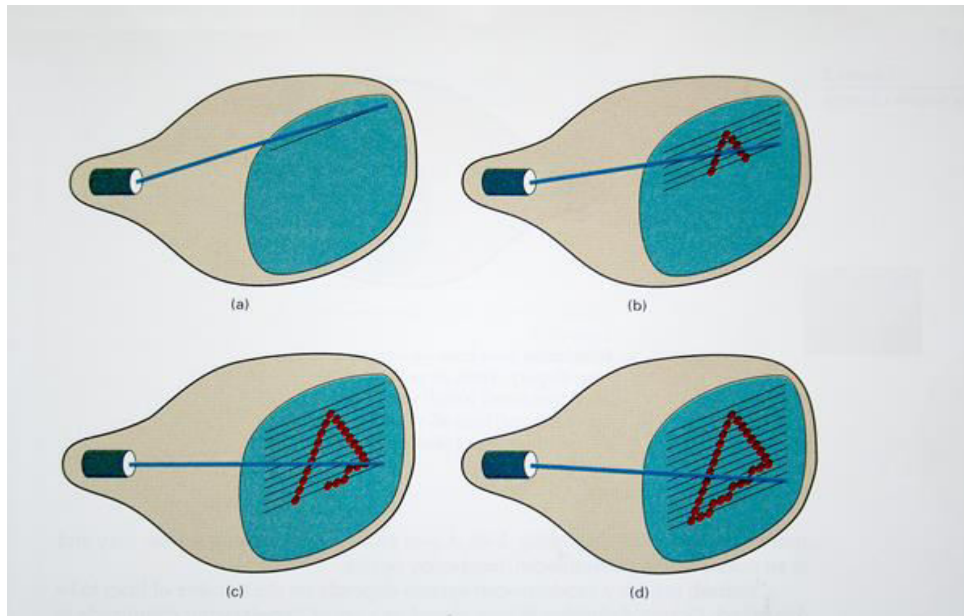
Electrostatic deflection of the electron beam in a CRT

- ✓ What we see on the screen is the combined effect of all the electrons light emissions: a glowing spot that quickly fades after all the excited phosphor electrons have returned to their ground energy level.
- ✓ The frequency of the light emitted by the phosphor is proportional to the energy difference between the excited quantum state and the ground state.
- ✓ Lower persistence phosphors required higher refresh rates to maintain a picture on the screen without flicker.
- ✓ The maximum number of points that can be displayed without overlap on a CRT is referred to as a resolution.
- ✓ Resolution of a CRT is dependent on the type of phosphor, the intensity to be displayed, and the focusing and deflection systems.
- ✓ High-resolution systems are often referred to as high-definition systems.

1.3.1 Raster-Scan Displays and Random Scan Displays

i)Raster-Scan Displays

- The electron beam is swept across the screen one row at a time from top to bottom.
- As it moves across each row, the beam intensity is turned on and off to create a pattern of illuminated spots.
- This scanning process is called refreshing. Each complete scanning of a screen is normally called a frame.
- The refreshing rate, called the frame rate, is normally 60 to 80 frames per second, or described as 60 Hz to 80 Hz.
- Picture definition is stored in a memory area called the frame buffer.
- This frame buffer stores the intensity values for all the screen points. Each screen point is called a pixel (picture element).
- Property of raster scan is Aspect ratio, which defined as number of pixel columns divided by number of scan lines that can be displayed by the system.



Case 1: In case of black and white systems

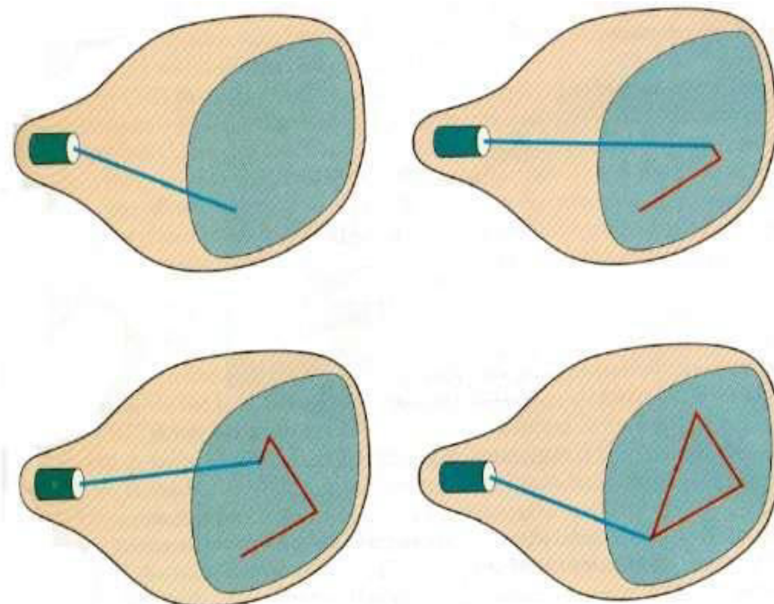
- ✓ On black and white systems, the frame buffer storing the values of the pixels is called a bitmap.
- ✓ Each entry in the bitmap is a 1-bit data which determine the on (1) and off (0) of the intensity of the pixel.

Case 2: In case of color systems

- On color systems, the frame buffer storing the values of the pixels is called a pixmap (Though now a days many graphics libraries name it as bitmap too).
- Each entry in the pixmap occupies a number of bits to represent the color of the pixel. For a true color display, the number of bits for each entry is 24 (8 bits per red/green/blue channel, each channel 28=256 levels of intensity value, ie. 256 voltage settings for each of the red/green/blue electron guns).

ii). Random-Scan Displays

- ✓ When operated as a random-scan display unit, a CRT has the electron beam directed only to those parts of the screen where a picture is to be displayed.
- ✓ Pictures are generated as line drawings, with the electron beam tracing out the component lines one after the other.
- ✓ For this reason, random-scan monitors are also referred to as vector displays (or strokewriting displays or calligraphic displays).
- ✓ The component lines of a picture can be drawn and refreshed by a random-scan system in any specified order



- ✓ A pen plotter operates in a similar way and is an example of a random-scan, hard-copy device.

- ✓ Refresh rate on a random-scan system depends on the number of lines to be displayed on that system.
- ✓ Picture definition is now stored as a set of line-drawing commands in an area of memory referred to as the display list, refresh display file, vector file, or display program
- ✓ To display a specified picture, the system cycles through the set of commands in the display file, drawing each component line in turn.
- ✓ After all line-drawing commands have been processed, the system cycles back to the first line command in the list.
- ✓ Random-scan displays are designed to draw all the component lines of a picture 30 to 60 times each second, with up to 100,000 "short" lines in the display list.
- ✓ When a small set of lines is to be displayed, each refresh cycle is delayed to avoid very high refresh rates, which could burn out the phosphor.

Difference between Raster scan system and Random scan system

Base of Difference	Raster Scan System	Random Scan System
Electron Beam	The electron beam is swept across the screen, one row at a time, from top to bottom	The electron beam is directed only to theparts of screen where a picture is to be drawn
Resolution	Its resolution is poor because raster system in contrast produces zigzag lines that are plotted as discrete point sets.	Its resolution is good because this system produces smooth lines drawings because CRT beam directly follows the line path.
Picture Definition	Picture definition is stored as a set of intensity values for all screen points,called pixels in a refresh buffer area.	Picture definition is stored as a set of line drawing instructions in a display file.
Realistic Display	The capability of this system to store intensity values for pixel makes it well suited for the realistic display of scenes	These systems are designed for line-drawing and can't display realistic shaded scenes.

	contain shadow and color pattern.	
Draw an Image	Screen points/pixels are used to draw an image	Mathematical functions are used to draw an image

1.3.2 Color CRT Monitors

- ❖ A CRT monitor displays color pictures by using a combination of phosphors that emit different-colored light.
- ❖ It produces range of colors by combining the light emitted by different phosphors.
- ❖ There are two basic techniques for color display:
    1. Beam-penetration technique
    2. Shadow-mask technique

1) Beam-penetration technique:

- ✓ This technique is used with random scan monitors.
- ✓ In this technique inside of CRT coated with two phosphor layers usually red and green.
- ✓ The outer layer of red and inner layer of green phosphor.
- ✓ The color depends on how far the electron beam penetrates into the phosphor layer.
- ✓ A beam of fast electron penetrates more and excites inner green layer while slow eletron excites outer red layer.
- ✓ At intermediate beam speed we can produce combination of red and green lights which emit additional two colors orange and yellow.
- ✓ The beam acceleration voltage controls the speed of the electrons and hence color of pixel.

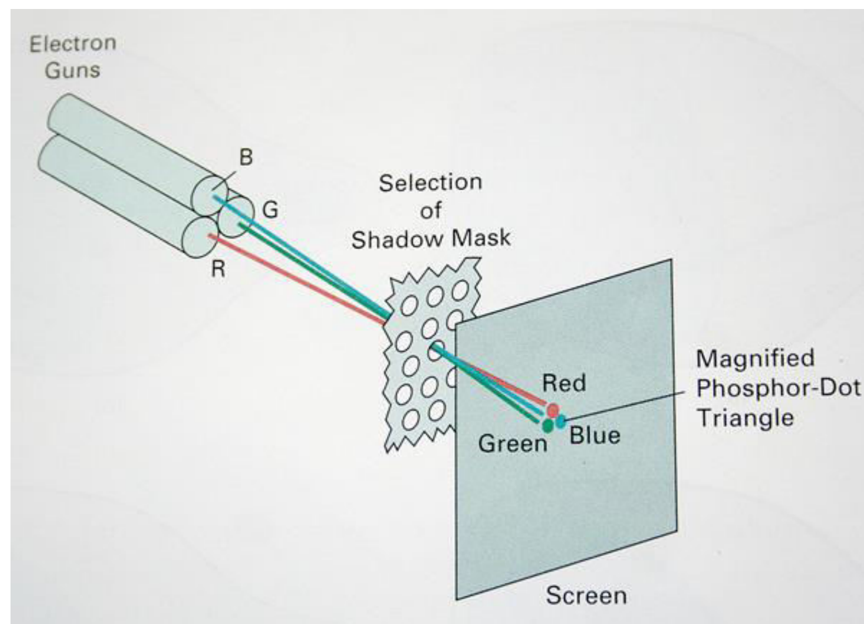
Disadvantages:

- ➢ It is a low cost technique to produce color in random scan monitors.
- ➢ It can display only four colors.
- ➢ Quality of picture is not good compared to other techniques.

2)Shadow-mask technique

- ✓ It produces wide range of colors as compared to beam-penetration technique.
- ✓ This technique is generally used in raster scan displays. Including color TV.

- ✓ In this technique CRT has three phosphor color dots at each pixel position.
- ✓ One dot for red, one for green and one for blue light. This is commonly known as Dot triangle.
- ✓ Here in CRT there are three electron guns present, one for each color dot. And a shadow mask grid just behind the phosphor coated screen.
- ✓ The shadow mask grid consists of series of holes aligned with the phosphor dot pattern.
- ✓ Three electron beams are deflected and focused as a group onto the shadow mask and when they pass through a hole they excite a dot triangle.
- ✓ In dot triangle three phosphor dots are arranged so that each electron beam can activate only its corresponding color dot when it passes through the shadow mask.
- ✓ A dot triangle when activated appears as a small dot on the screen which has color of combination of three small dots in the dot triangle.
- ✓ By changing the intensity of the three electron beams we can obtain different colors in the shadow mask CRT.



1.3.3Flat Panel Display

- ➔ The term flat panel display refers to a class of video device that have reduced volume, weight & power requirement compared to a CRT.
- ➔ As flat panel display is thinner than CRTs, we can hang them on walls or wear on our wrists.

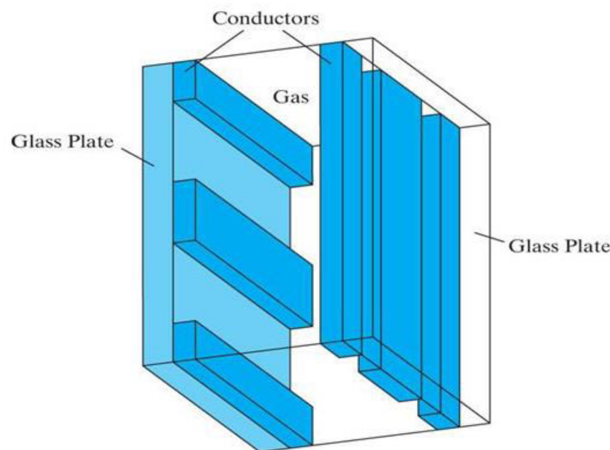
- Since we can even write on some flat panel displays they will soon be available as pocket notepads.
- We can separate flat panel display in two categories:

    1. Emissive displays: - the emissive display or emitters are devices that convert electrical energy into light. For Ex. Plasma panel, thin film electroluminescent displays and light emitting diodes.

    2. Non emissive displays: - non emissive display or non emitters use optical effects to convert sunlight or light from some other source into graphics patterns. For Ex. LCD (Liquid Crystal Display).

a) Plasma Panels displays

- This is also called gas discharge displays.
- It is constructed by filling the region between two glass plates with a mixture of gases that usually includes neon.
- A series of vertical conducting ribbons is placed on one glass panel and a set of horizontal ribbon is built into the other glass panel.

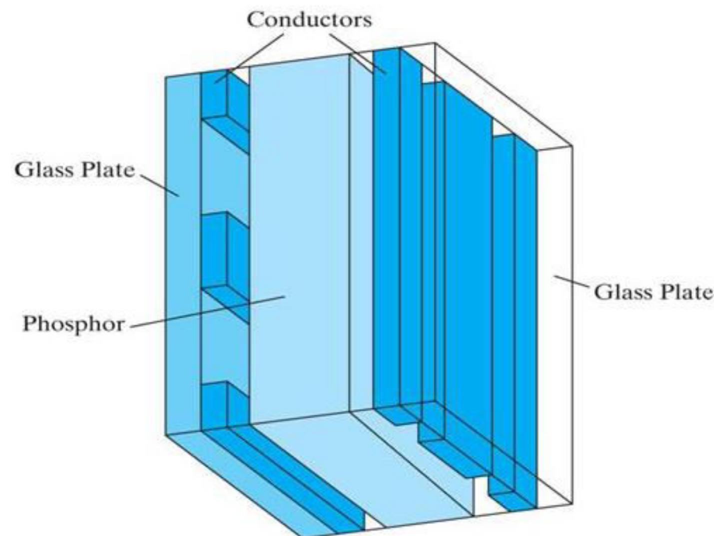


- Firing voltage is applied to a pair of horizontal and vertical conductors cause the gas at the intersection of the two conductors to break down into glowing plasma of electrons and ions.
- Picture definition is stored in a refresh buffer and the firing voltages are applied to refresh the pixel positions, 60 times per second.

- Alternating current methods are used to provide faster application of firing voltages and thus brighter displays.
- Separation between pixels is provided by the electric field of conductor.
- One disadvantage of plasma panels is they were strictly monochromatic device that means shows only one color other than black like black and white.

b.Thin Film Electroluminescent Displays

- It is similar to plasma panel display but region between the glass plates is filled with phosphors such as doped with magnesium instead of gas.
- When sufficient voltage is applied the phosphors becomes a conductor in area of intersection of the two electrodes.
- Electrical energy is then absorbed by the manganese atoms which then release the energy as a spot of light similar to the glowing plasma effect in plasma panel.
- It requires more power than plasma panel.
- In this good color and gray scale difficult to achieve.

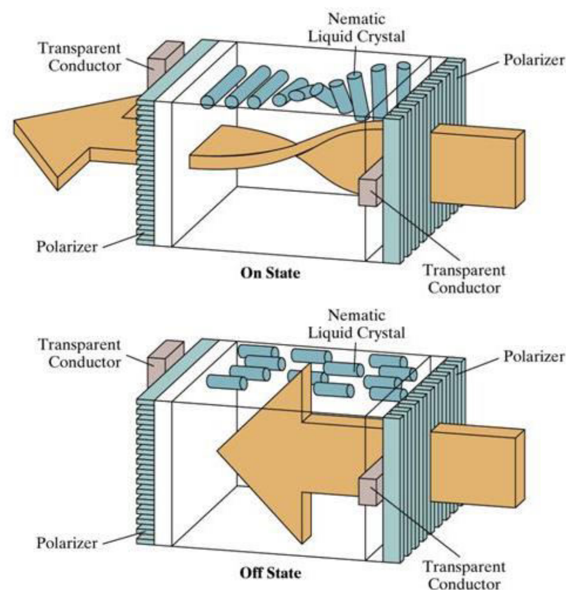


c. Light Emitting Diode (LED)

- In this display a matrix of multi-color light emitting diode is arranged to form the pixel position in the display and the picture definition is stored in refresh buffer.
- Similar to scan line refreshing of CRT information is read from the refresh buffer and converted to voltage levels that are applied to the diodes to produce the light pattern on the display.

d)Liquid Crystal Display (LCD)

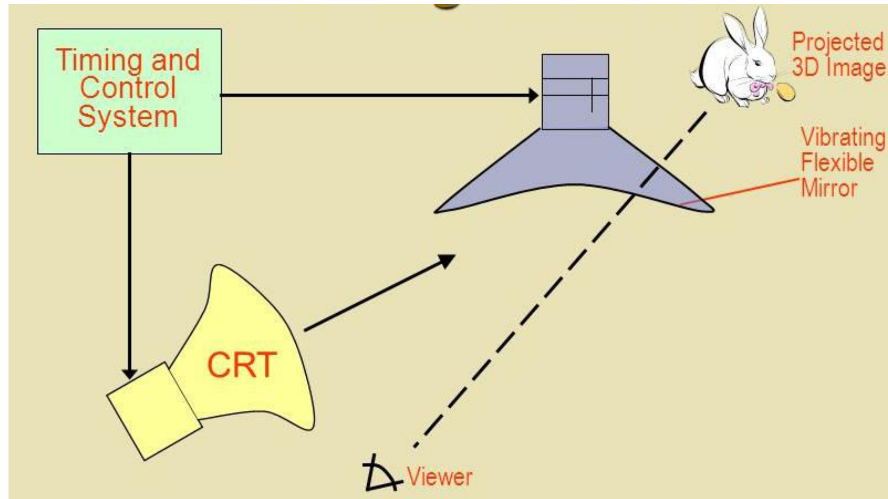
- This non emissive device produce picture by passing polarized light from the surrounding or from an internal light source through liquid crystal material that can be aligned to either block or transmit the light.
- The liquid crystal refreshes to fact that these compounds have crystalline arrangement of molecules then also flows like liquid.
- It consists of two glass plates each with light polarizer at right angles to each other sandwich the liquid crystal material between the plates.
- Rows of horizontal transparent conductors are built into one glass plate, and column of vertical conductors are put into the other plates.
- The intersection of two conductors defines a pixel position.
- In the ON state polarized light passing through material is twisted so that it will pass through the opposite polarizer.
- In the OFF state it will reflect back towards source.



Three- Dimensional Viewing Devices

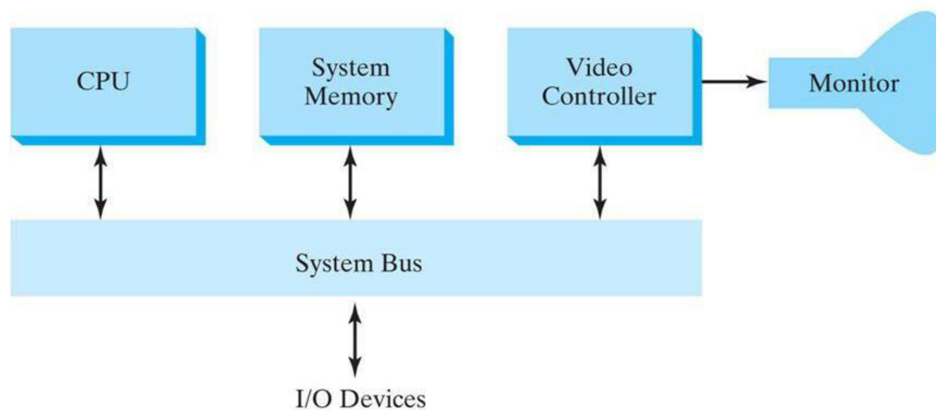
- Graphics monitors for the display of three-dimensional scenes have been devised using a technique that reflects a CRT image from a vibrating, flexible mirror As the varifocal mirror vibrates, it changes focal length.

- These vibrations are synchronized with the display of an object on a CRT so that each point on the object is reflected from the mirror into a spatial position corresponding to the distance of that point from a specified viewing location.
- This allows us to walk around an object or scene and view it from different sides.



1.4 Raster-Scan Systems

- Interactive raster-graphics systems typically employ several processing units.
- In addition to the central processing unit (CPU), a special-purpose processor, called the video controller or display controller, is used to control the operation of the display device.
- Organization of a simple raster system is shown in below Figure.

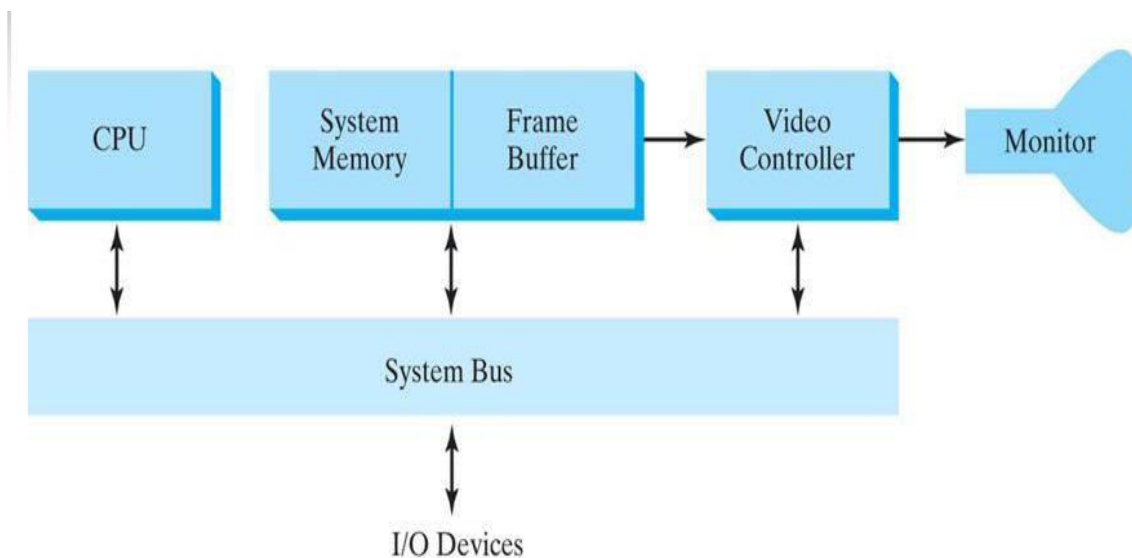


- Here, the frame buffer can be anywhere in the system memory, and the video controller accesses the frame buffer to refresh the screen.

➔ In addition to the video controller, raster systems employ other processors as coprocessors and accelerators to implement various graphics operations.

1.4.1 Video controller:

- ✓ The figure below shows a commonly used organization for raster systems.
- ✓ A fixed area of the system memory is reserved for the frame buffer, and the video controller is given direct access to the frame-buffer memory.
- ✓ Frame-buffer locations, and the corresponding screen positions, are referenced in the Cartesian coordinates.

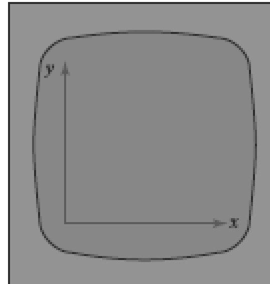


Cartesian reference frame:

- ✓ Frame-buffer locations and the corresponding screen positions, are referenced in Cartesian coordinates.
- ✓ In an application (user) program, we use the commands within a graphics software package to set coordinate positions for displayed objects relative to the origin of the
- ✓ The coordinate origin is referenced at the lower-left corner of a screen display area by the software commands, although we can typically set the origin at any convenient location for a particular application.

Working:

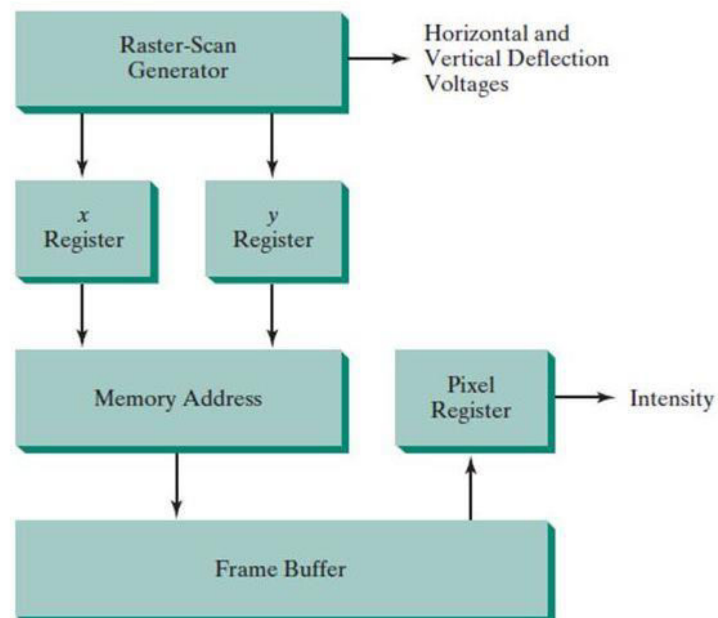
- ✓ Figure shows a two-dimensional Cartesian reference frame with the origin at the lowerleft screen corner.



- ✓ The screen surface is then represented as the first quadrant of a two-dimensional system with positive x and y values increasing from left to right and bottom of the screen to the top respectively.
- ✓ Pixel positions are then assigned integer x values that range from 0 to xmax across the screen, left to right, and integer y values that vary from 0 to ymax, bottom to top.

Basic Video Controller Refresh Operations

- ✓ The basic refresh operations of the video controller are diagrammed



- ✓ Two registers are used to store the coordinate values for the screen pixels.

- ✓ Initially, the x register is set to 0 and the y register is set to the value for the top scan line.
- ✓ The contents of the frame buffer at this pixel position are then retrieved and used to set the intensity of the CRT beam.
- ✓ Then the x register is incremented by 1, and the process is repeated for the next pixel on the top scan line.
- ✓ This procedure continues for each pixel along the top scan line.
- ✓ After the last pixel on the top scan line has been processed, the x register is reset to 0 and the y register is set to the value for the next scan line down from the top of the screen.
- ✓ The procedure is repeated for each successive scan line.
- ✓ After cycling through all pixels along the bottom scan line, the video controller resets the registers to the first pixel position on the top scan line and the refresh process starts over

a.Speed up pixel position processing of video controller:

- ✓ Since the screen must be refreshed at a rate of at least 60 frames per second,the simple procedure illustrated in above figure may not be accommodated by RAM chips if the cycle time is too slow.
- ✓ To speed up pixel processing, video controllers can retrieve multiple pixel values from the refresh buffer on each pass.
- ✓ When group of pixels has been processed, the next block of pixel values is retrieved from the frame buffer.

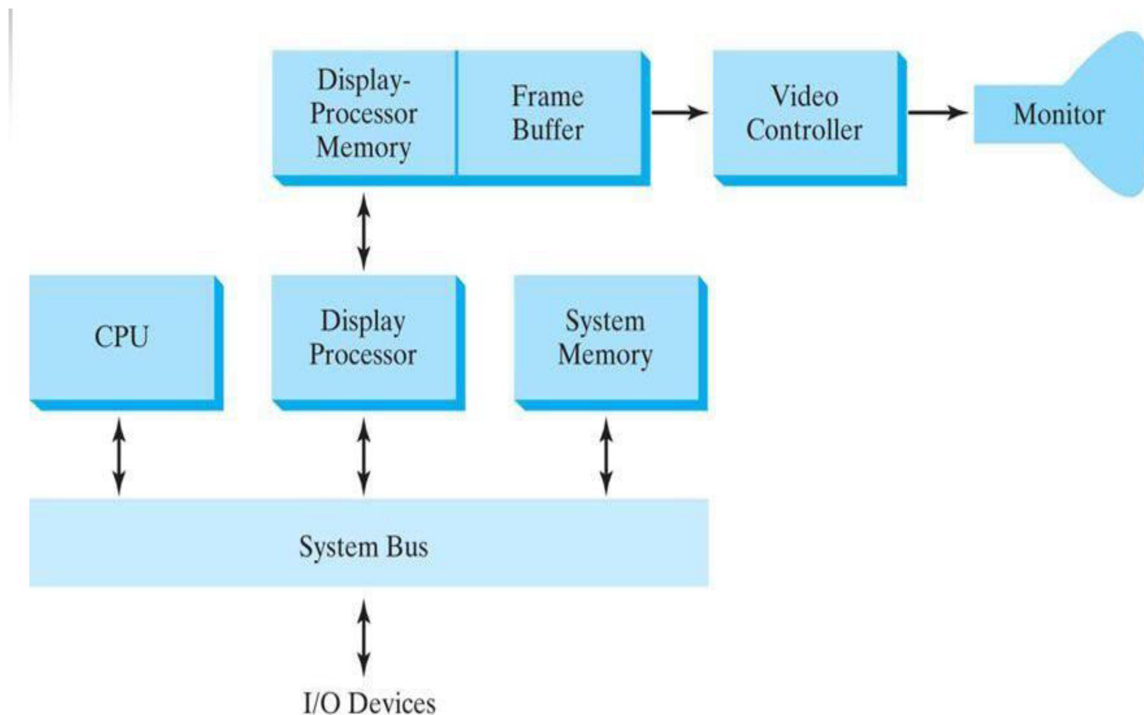
Advantages of video controller:

- ✓ A video controller can be designed to perform a number of other operations.
- ✓ For various applications, the video controller can retrieve pixel values from different memory areas on different refresh cycles.
- ✓ This provides a fast mechanism for generating real-time animations.
- ✓ Another video-controller task is the transformation of blocks of pixels, so that screen areas can be enlarged, reduced, or moved from one location to another during the refresh cycles.
- ✓ In addition, the video controller often contains a lookup table, so that pixel values in the frame buffer are used to access the lookup table. This provides a fast method for changing screen intensity values.

✓ Finally, some systems are designed to allow the video controller to mix the framebuffer image with an input image from a television camera or other input device

b) Raster-Scan Display Processor

✓ Figure shows one way to organize the components of a raster system that contains a separate display processor, sometimes referred to as a graphics controller or a display coprocessor.



✓ The purpose of the display processor is to free the CPU from the graphics chores.

✓ In addition to the system memory, a separate display-processor memory area can be provided.

Scan conversion:

✓ A major task of the display processor is digitizing a picture definition given in an application program into a set of pixel values for storage in the frame buffer.

✓ This digitization process is called scan conversion.

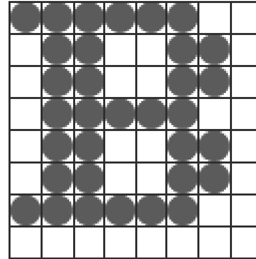
Example 1: displaying a line

➔ Graphics commands specifying straight lines and other geometric objects are scan converted into a set of discrete points, corresponding to screen pixel positions.

➔ Scan converting a straight-line segment.

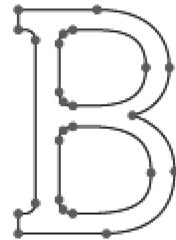
Example 2: displaying a character

- Characters can be defined with rectangular pixel grids
- The array size for character grids can vary from about 5 by 7 to 9 by 12 or more for higher-quality displays.
- A character grid is displayed by superimposing the rectangular grid pattern into the frame buffer at a specified coordinate position.



<u>Using outline:</u>

- For characters that are defined as outlines, the shapes are scan-converted into the frame buffer by locating the pixel positions closest to the outline.



<u>Additional operations of Display processors:</u>

- Display processors are also designed to perform a number of additional operations.
- These functions include generating various line styles (dashed, dotted, or solid), displaying color areas, and applying transformations to the objects in a scene.
- Display processors are typically designed to interface with interactive input devices, such as a mouse.

<u>Methods to reduce memory requirements in display processor:</u>

- In an effort to reduce memory requirements in raster systems, methods have been devised for organizing the frame buffer as a linked list and encoding the color information.
- One organization scheme is to store each scan line as a set of number pairs.

- ➔ Encoding methods can be useful in the digital storage and transmission of picture information

i)Run-length encoding:

- ❋ The first number in each pair can be a reference to a color value, and the second number can specify the number of adjacent pixels on the scan line that are to be displayed in that color.
- ❋ This technique, called run-length encoding, can result in a considerable saving in storage space if a picture is to be constructed mostly with long runs of a single color each.
- ❋ A similar approach can be taken when pixel colors change linearly.

ii)Cell encoding:

- ❋ Another approach is to encode the raster as a set of rectangular areas (cell encoding).

Disadvantages of encoding:

- ❖ The disadvantages of encoding runs are that color changes are difficult to record and storage requirements increase as the lengths of the runs decrease.
- ❖ In addition, it is difficult for the display controller to process the raster when many short runs are involved.
- ❖ Moreover, the size of the frame buffer is no longer a major concern, because of sharp declines in memory costs

1.4.3 Graphics workstations and viewing systems

- ✓ Most graphics monitors today operate as raster-scan displays, and both CRT and flat panel systems are in common use.
- ✓ Graphics workstation range from small general-purpose computer systems to multi monitor facilities, often with ultra –large viewing screens.
- ✓ High-definition graphics systems, with resolutions up to 2560 by 2048, are commonly used in medical imaging, air-traffic control, simulation, and CAD.
- ✓ Many high-end graphics workstations also include large viewing screens, often with specialized features.

- ✓ Multi-panel display screens are used in a variety of applications that require "wall-sized" viewing areas. These systems are designed for presenting graphics displays at meetings, conferences, conventions, trade shows, retail stores etc.
- ✓ A multi-panel display can be used to show a large view of a single scene or several individual images. Each panel in the system displays one section of the overall picture
- ✓ A large, curved-screen system can be useful for viewing by a group of people studying a particular graphics application.
- ✓ A 360 degree paneled viewing system in the NASA control-tower simulator, which is used for training and for testing ways to solve air-traffic and runway problems at airports.

1.5 Input Devices

- ➢ Graphics workstations make use of various devices for data input.Most systems have keyboards and mouses,while some other systems have trackball,spaceball,joystick,button boxes,touch panels,image scanners and voice systems.

Keyboard:

- ➢ Keyboard on graphics system is used for entering text strings,issuing certain commands and selecting menu options.
- ➢ Keyboards can also be provided with features for entry of screen coordinates,menu selections or graphics functions.
- ➢ General purpose keyboard uses function keys and cursor-control keys.
- ➢ Function keys allow user to select frequently accessed operations with a single keystroke.Cursor-control keys are used for selecting a displayed object or a location by positioning the screen cursor.

Button Boxes and Dials:

- ➢ Buttons are often used to input predefined functions .Dials are common devices for entering scalar values.
- ➢ Numerical values within some defined range are selected for input with dial rotations.

Mouse Devices:

- Mouse is a hand-held device,usually moved around on a flat surface to position the screen cursor.wheeler or roolers on the bottom of the mouse used to record the amount and direction of movement.
- Some of the mouses uses optical sensors,which detects movement across the horizontal and vertical grid lines.
- Since a mouse can be picked up and put down,it is used for making relative changes in the position of the screen.
- Most general purpose graphics systems now include a mouse and a keyboard as the primary input devices.

Trackballs and Spaceballs:

- A trackball is a ball device that can be rotated with the fingers or palm of the hand to produce screen cursor movement.
- Laptop keyboards are equipped with a trackball to eliminate the extra space required by a mouse.
- Spaceball is an extension of two-dimensional trackball concept.
- Spaceballs are used for three-dimensional positioning and selection operations in virtual-reality systems,modeling,animation,CAD and other applications.

Joysticks:

- Joystick is used as a positioning device,which uses a small vertical lever(stick) mounded on a base.It is used to steer the screen cursor around and select screen position with the stick movement.
- A push or pull on the stick is measured with strain gauges and converted to movement of the screen cursor in the direction of the applied pressure.

Data Gloves:

- Data glove can be used to grasp a virtual object.The glove is constructed with a series of sensors that detect hand and finger motions.
- Input from the glove is used to position or manipulate objects in a virtual scene.

Digitizers:

- Digitizer is a common device for drawing,painting or selecting positions.
- Graphics tablet is one type of digitizer,which is used to input 2-dimensional coordinates by activating a hand cursor or stylus at selected positions on a flat surface.
- A hand cursor contains cross hairs for sighting positions and stylus is a pencil-shaped device that is pointed at positions on the tablet.

Image Scanners:

- Drawings,graphs,photographs or text can be stored for computer processing with an image scanner by passing an optical scanning mechanism over the information to be stored.
- Once we have the representation of the picture, then we can apply various image-processing method to modify the representation of the picture and various editing operations can be performed on the stored documents.

Touch Panels:

- Touch panels allow displayed objects or screen positions to be selected with the touch of a finger.
- Touch panel is used for the selection of processing options that are represented as a menu of graphical icons.
- Optical touch panel-uses LEDs along one vertical and horizontal edge of the frame.
- Acoustical touch panels generates high-frequency sound waves in horizontal and vertical directions across a glass plate.

Light Pens:

- Light pens are pencil-shaped devices used to select positions by detecting the light coming from points on the CRT screen.
- To select positions in any screen area with a light pen,we must have some nonzero light intensity emitted from each pixel within that area.
- Light pens sometimes give false readings due to background lighting in a room.

Voice Systems:

- Speech recognizers are used with some graphics workstations as input devices for voice commands.The voice system input can be used to initiate operations or to enter data.
- A dictionary is set up by speaking command words several times,then the system analyses each word and matches with the voice command to match the pattern

1.6 Graphics Networks

- So far, we have mainly considered graphics applications on an isolated system with a single user.
- Multiuser environments & computer networks are now common elements in many graphics applications.
- Various resources, such as processors, printers, plotters and data files can be distributed on a network & shared by multiple users.
- A graphics monitor on a network is generally referred to as a graphics server.
- The computer on a network that is executing a graphics application is called the client.
- A workstation that includes processors, as well as a monitor and input devices can function as both a server and a client.

1.7 Graphics on Internet

- ✓ A great deal of graphics development is now done on the Internet.
- ✓ Computers on the Internet communicate using TCP/IP.
- ✓ Resources such as graphics files are identified by URL (Uniform resource locator).
- ✓ The World Wide Web provides a hypertext system that allows users to loacate and view documents, audio and graphics.
- ✓ Each URL sometimes also called as universal resource locator.
- ✓ The URL contains two parts Protocol- for transferring the document, and Server- contains the document.

1.8 Graphics Software

- ✓ There are two broad classifications for computer-graphics software

1. Special-purpose packages: Special-purpose packages are designed for nonprogrammers

   Example: generate pictures, graphs, charts, painting programs or CAD systems in some application area without worrying about the graphics procedure
2. General programming packages: general programming package provides a library of graphics functions that can be used in a programming language such as C, C++, Java, or FORTRAN.

   Example: GL (Graphics Library), OpenGL, VRML (Virtual-Reality Modeling Language), Java 2D And Java 3D

NOTE: A set of graphics functions is often called a computer-graphics application programming interface (CG API)

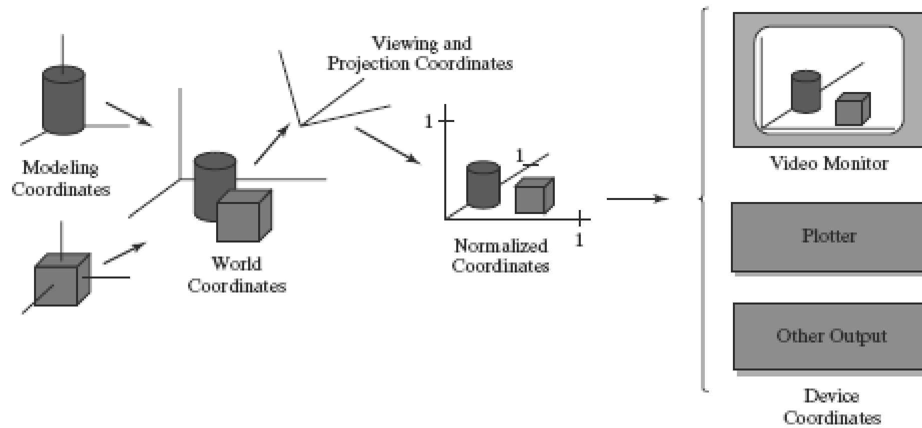
1.10 Coordinate Representations

- ✓ To generate a picture using a programming package we first need to give the geometric descriptions of the objects that are to be displayed known as coordinates.
- ✓ If coordinate values for a picture are given in some other reference frame (spherical, hyperbolic, etc.), they must be converted to Cartesian coordinates.
- ✓ Several different Cartesian reference frames are used in the process of constructing and displaying
- ✓ First we define the shapes of individual objects, such as trees or furniture, These reference frames are called modeling coordinates or local coordinates
- ✓ Then we place the objects into appropriate locations within a scene reference frame called world coordinates.
- ✓ After all parts of a scene have been specified, it is processed through various output-device reference frames for display. This process is called the viewing pipeline.
- ✓ The scene is then stored in normalized coordinates. Which range from −1 to 1 or from 0 to 1 Normalized coordinates are also referred to as normalized device coordinates.
- ✓ The coordinate systems for display devices are generally called device coordinates, or screen coordinates.

NOTE: Geometric descriptions in modeling coordinates and world coordinates can be given in

floating-point or integer values.

- ✓ Example: Figure briefly illustrates the sequence of coordinate transformations from modeling coordinates to device coordinates for a display



$$(x_{mc}, y_{mc}, z_{mc}) \rightarrow (x_{wc}, y_{wc}, z_{wc}) \rightarrow (x_{vc}, y_{vc}, z_{vc}) \rightarrow (x_{pc}, y_{pc}, z_{pc})$$
$$\rightarrow (x_{nc}, y_{nc}, z_{nc}) \rightarrow (x_{dc}, y_{dc})$$

1.11 Graphics Functions

- ➔ It provides users with a variety of functions for creating and manipulating pictures
- ➔ The basic building blocks for pictures are referred to as graphics output primitives
- ➔ Attributes are properties of the output primitives
- ➔ We can change the size, position, or orientation of an object using geometric transformations
- ➔ Modeling transformations, which are used to construct a scene.
- ➔ Viewing transformations are used to select a view of the scene, the type of projection to be used and the location where the view is to be displayed.
- ➔ Input functions are used to control and process the data flow from these interactive devices(mouse, tablet and joystick)
- ➔ Graphics package contains a number of tasks .We can lump the functions for carrying out many tasks by under the heading control operations.

<u>Software Standards</u>

- ✓ The primary goal of standardized graphics software is portability.

- ✓ In 1984, Graphical Kernel System (GKS) was adopted as the first graphics software standard by the International Standards Organization (ISO)
- ✓ The second software standard to be developed and approved by the standards organizations was Programmer's Hierarchical Interactive Graphics System (PHIGS).
- ✓ Extension of PHIGS, called PHIGS+, was developed to provide 3-D surface rendering capabilities not available in PHIGS.
- ✓ The graphics workstations from Silicon Graphics, Inc. (SGI), came with a set of routines called GL (Graphics Library)

<u>Other Graphics Packages</u>

- ✓ Many other computer-graphics programming libraries have been developed for

1. general graphics routines
2. Some are aimed at specific applications (animation, virtual reality, etc.)
   Example: Open Inventor Virtual-Reality Modeling Language (VRML).

We can create 2-D scenes with in Java applets (java2D, Java 3D)

1.12 Introduction To OpenGL

- ✓ OpenGL basic(core) library :-A basic library of functions is provided in OpenGL for specifying graphics primitives, attributes, geometric transformations, viewing transformations, and many other operations.

<u>Basic OpenGL Syntax</u>

- ➔ Function names in the OpenGL basic library (also called the OpenGL core library) are prefixed with gl. The component word first letter is capitalized.
- ➔ For eg:- glBegin, glClear, glCopyPixels, glPolygonMode
- ➔ Symbolic constants that are used with certain functions as parameters are all in capital letters, preceded by "GL", and component are separated by underscore.
- ➔ For eg:- GL_2D, GL_RGB, GL_CCW, GL_POLYGON, GL_AMBIENT_AND_DIFFUSE.

- ➔ The OpenGL functions also expect specific data types. For example, an OpenGL function parameter might expect a value that is specified as a 32-bit integer. But the size of an integer specification can be different on different machines.
- ➔ To indicate a specific data type, OpenGL uses special built-in, data-type names, such as GLbyte, GLshort, GLint, GLfloat, GLdouble, Glboolean

Related Libraries

- ➔ In addition to OpenGL basic(core) library(prefixed with gl), there are a number of associated libraries for handling special operations:-

  1) OpenGL Utility(GLU):- Prefixed with "glu". It provides routines for setting up viewing and projection matrices, describing complex objects with line and polygon approximations, displaying quadrics and B-splines using linear approximations, processing the surface-rendering operations, and other complex tasks.

  -Every OpenGL implementation includes the GLU library

  2) Open Inventor:- provides routines and predefined object shapes for interactive three-dimensional applications which are written in C++.

  3) Window-system libraries:- To create graphics we need display window. We cannot create the display window directly with the basic OpenGL functions since it contains only device-independent graphics functions, and window-management operations are device-dependent. However, there are several window-system libraries that supports OpenGL functions for a variety of machines.

  Eg:- Apple GL(AGL), Windows-to-OpenGL(WGL), Presentation Manager to OpenGL(PGL), GLX.

  4) OpenGL Utility Toolkit(GLUT):- provides a library of functions which acts as interface for interacting with any device specific screen-windowing system, thus making our program device-independent. The GLUT library functions are prefixed with "glut".

Header Files

- ✓ In all graphics programs, we will need to include the header file for the OpenGL core library.

- In windows to include OpenGL core libraries and GLU we can use the following header files:-
  #include <windows.h> //precedes other header files for including Microsoft windows ver of OpenGL libraries
  #include<GL/gl.h>
  #include <GL/glu.h>
- The above lines can be replaced by using GLUT header file which ensures gl.h and glu.h are included correctly,
- #include <GL/glut.h> //GL in windows
- In Apple OS X systems, the header file inclusion statement will be,
- #include <GLUT/glut.h>

Display-Window Management Using GLUT

- We can consider a simplified example, minimal number of operations for displaying a picture.

Step 1: initialization of GLUT

- We are using the OpenGL Utility Toolkit, our first step is to initialize GLUT.
- This initialization function could also process any command line arguments, but we will not need to use these parameters for our first example programs.
- We perform the GLUT initialization with the statement
  glutInit (&argc, argv);

Step 2: title

- We can state that a display window is to be created on the screen with a given caption for the title bar. This is accomplished with the function
  glutCreateWindow ("An Example OpenGL Program");
- where the single argument for this function can be any character string that we want to use for the display-window title.

Step 3: Specification of the display window

- Then we need to specify what the display window is to contain.
- For this, we create a picture using OpenGL functions and pass the picture definition to the GLUT routine glutDisplayFunc, which assigns our picture to the display window.

- Example: suppose we have the OpenGL code for describing a line segment in a procedure called lineSegment.
- Then the following function call passes the line-segment description to the display window:

  glutDisplayFunc (lineSegment);

Step 4: one more GLUT function

- But the display window is not yet on the screen.
- We need one more GLUT function to complete the window-processing operations.
- After execution of the following statement, all display windows that we have created, including their graphic content, are now activated:

  glutMainLoop ( );

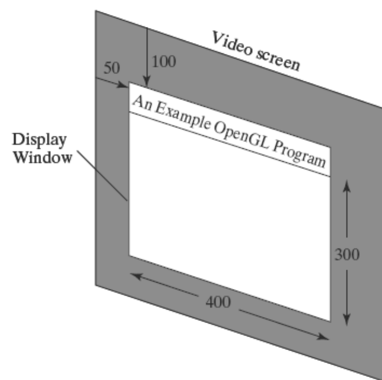
- This function must be the last one in our program. It displays the initial graphics and puts the program into an infinite loop that checks for input from devices such as a mouse or keyboard.

Step 5: these parameters using additional GLUT functions

- Although the display window that we created will be in some default location and size, we can set these parameters using additional GLUT functions.

GLUT Function 1:

- ➔ We use the glutInitWindowPosition function to give an initial location for the upper left corner of the display window.
- ➔ This position is specified in integer screen coordinates, whose origin is at the upper-left corner of the screen.



GLUT Function 2:

After the display window is on the screen, we can reposition and resize it.

GLUT Function 3:

- ➔ We can also set a number of other options for the display window, such as buffering and a choice of color modes, with the glutInitDisplayMode function.
- ➔ Arguments for this routine are assigned symbolic GLUT constants.
- ➔ Example: the following command specifies that a single refresh buffer is to be used for the display window and that we want to use the color mode which uses red, green, and blue (RGB) components to select color values:

  glutInitDisplayMode (GLUT_SINGLE | GLUT_RGB);
- ➔ The values of the constants passed to this function are combined using a logical or operation.
- ➔ Actually, single buffering and RGB color mode are the default options.
- ➔ But we will use the function now as a reminder that these are the options that are set for our display.
- ➔ Later, we discuss color modes in more detail, as well as other display options, such as double buffering for animation applications and selecting parameters for viewing threedimensional scenes.

A Complete OpenGL Program

- ➔ There are still a few more tasks to perform before we have all the parts that we need for a complete program.

Step 1: to set background color

- ➔ For the display window, we can choose a background color.
- ➔ Using RGB color values, we set the background color for the display window to be white, with the OpenGL function:

  glClearColor (1.0, 1.0, 1.0, 0.0);
- ➔ The first three arguments in this function set the red, green, and blue component colors to the value 1.0, giving us a white background color for the display window.
- ➔ If, instead of 1.0, we set each of the component colors to 0.0, we would get a black background.

- ➔ The fourth parameter in the glClearColor function is called the alpha value for the specified color. One use for the alpha value is as a “blending” parameter
- ➔ When we activate the OpenGL blending operations, alpha values can be used to determine the resulting color for two overlapping objects.
- ➔ An alpha value of 0.0 indicates a totally transparent object, and an alpha value of 1.0 indicates an opaque object.
- ➔ For now, we will simply set alpha to 0.0.
- ➔ Although the glClearColor command assigns a color to the display window, it does not put the display window on the screen.

Step 2: to set window color

- ➔ To get the assigned window color displayed, we need to invoke the following OpenGL function:

  glClear (GL_COLOR_BUFFER_BIT);
- ➔ The argument GL COLOR BUFFER BIT is an OpenGL symbolic constant specifying that it is the bit values in the color buffer (refresh buffer) that are to be set to the values indicated in the glClearColor function. (OpenGL has several different kinds of buffers that can be manipulated.

Step 3: to set color to object

- ➔ In addition to setting the background color for the display window, we can choose a variety of color schemes for the objects we want to display in a scene.
- ➔ For our initial programming example, we will simply set the object color to be a dark green

  glColor3f (0.0, 0.4, 0.2);
- ➔ The suffix 3f on the glColor function indicates that we are specifying the three RGB color components using floating-point (f) values.
- ➔ This function requires that the values be in the range from 0.0 to 1.0, and we have set red = 0.0, green = 0.4, and blue = 0.2.

Example program

- For our first program, we simply display a two-dimensional line segment.
- To do this, we need to tell OpenGL how we want to "project" our picture onto the display window because generating a two-dimensional picture is treated by OpenGL as a special case of three-dimensional viewing.
- So, although we only want to produce a very simple two-dimensional line, OpenGL processes our picture through the full three-dimensional viewing operations.
- We can set the projection type (mode) and other viewing parameters that we need with the following two functions:

```
glMatrixMode (GL_PROJECTION);
gluOrtho2D (0.0, 200.0, 0.0, 150.0);
```

- This specifies that an orthogonal projection is to be used to map the contents of a twodimensional rectangular area of world coordinates to the screen, and that the x-coordinate values within this rectangle range from 0.0 to 200.0 with y-coordinate values ranging from 0.0 to 150.0.
- Whatever objects we define within this world-coordinate rectangle will be shown within the display window.
- Anything outside this coordinate range will not be displayed.
- Therefore, the GLU function gluOrtho2D defines the coordinate reference frame within the display window to be (0.0, 0.0) at the lower-left corner of the display window and (200.0, 150.0) at the upper-right window corner.
- For now, we will use a world-coordinate rectangle with the same aspect ratio as the display window, so that there is no distortion of our picture.
- Finally, we need to call the appropriate OpenGL routines to create our line segment.
- The following code defines a two-dimensional, straight-line segment with integer,
- Cartesian endpoint coordinates (180, 15) and (10, 145).

```
glBegin (GL_LINES);
glVertex2i (180, 15);
glVertex2i (10, 145);
glEnd ( );
```

- Now we are ready to put all the pieces together:

The following OpenGL program is organized into three functions.

- ➔ init: We place all initializations and related one-time parameter settings in function init.
- ➔ lineSegment: Our geometric description of the "picture" that we want to display is in function lineSegment, which is the function that will be referenced by the GLUT function glutDisplayFunc.
- ➔ main function main function contains the GLUT functions for setting up the display window and getting our line segment onto the screen.
- ➔ glFlush: This is simply a routine to force execution of our OpenGL functions, which are stored by computer systems in buffers in different locations,depending on how OpenGL is implemented.
- ➔ The procedure lineSegment that we set up to describe our picture is referred to as a display callback function.
- ➔ And this procedure is described as being "registered" by glutDisplayFunc as the routine to invoke whenever the display window might need to be redisplayed.

Example: if the display window is moved.

Following program to display window and line segment generated by this program:

```
#include <GL/glut.h> // (or others, depending on the system in use)
void init (void)
{
        glClearColor (1.0, 1.0, 1.0, 0.0);          // Set display-window color to white.
        glMatrixMode (GL_PROJECTION);    // Set projection parameters.
        gluOrtho2D (0.0, 200.0, 0.0, 150.0);
}
void lineSegment (void)
{
        glClear (GL_COLOR_BUFFER_BIT);      // Clear display window.
        glColor3f (0.0, 0.4, 0.2);                      // Set line segment color to green.
        glBegin (GL_LINES);
        glVertex2i (180, 15);                         // Specify line-segment geometry.
        glVertex2i (10, 145);
        glEnd ( );
```

```
    glFlush ( ); // Process all OpenGL routines as quickly as possible.
}
void main (int argc, char** argv)
{
    glutInit (&argc, argv);                         // Initialize GLUT.
    glutInitDisplayMode (GLUT_SINGLE | GLUT_RGB);   // Set display mode.
    glutInitWindowPosition (50, 100);     // Set top-left display-window position.
    glutInitWindowSize (400, 300);       // Set display-window width and height.
    glutCreateWindow ("An Example OpenGL Program"); // Create display window.
    init ( );                            // Execute initialization procedure.
    glutDisplayFunc (lineSegment);       // Send graphics to display window.
    glutMainLoop ( );                    // Display everything and wait.
}
```

1.13 Coordinate Reference Frames

To describe a picture, we first decide upon

- A convenient Cartesian coordinate system, called the world-coordinate reference frame, which could be either 2D or 3D.
- We then describe the objects in our picture by giving their geometric specifications in terms of positions in world coordinates.
- Example: We define a straight-line segment with two endpoint positions, and a polygon is specified with a set of positions for its vertices.
- These coordinate positions are stored in the scene description along with other info about the objects, such as their color and their coordinate extents
- Co-ordinate extents :Co-ordinate extents are the minimum and maximum x, y, and z values for each object.
- A set of coordinate extents is also described as a bounding box for an object.
- Ex:For a 2D figure, the coordinate extents are sometimes called its bounding rectangle.
- Objects are then displayed by passing the scene description to the viewing routines which identify visible surfaces and map the objects to the frame buffer positions and then on the video monitor.

- The scan-conversion algorithm stores info about the scene, such as color values, at the appropriate locations in the frame buffer, and then the scene is displayed on the output device.

Screen co-ordinates:

- ✓ Locations on a video monitor are referenced in integer screen coordinates, which correspond to the integer pixel positions in the frame buffer.
- ✓ Scan-line algorithms for the graphics primitives use the coordinate descriptions to determine the locations of pixels
- ✓ Example: given the endpoint coordinates for a line segment, a display algorithm must calculate the positions for those pixels that lie along the line path between the endpoints.
- ✓ Since a pixel position occupies a finite area of the screen, the finite size of a pixel must be taken into account by the implementation algorithms.
- ✓ For the present, we assume that each integer screen position references the centre of a pixel area.
- ✓ Once pixel positions have been identified the color values must be stored in the frame buffer

Assume we have available a low-level procedure of the form

i)setPixel (x, y);

- stores the current color setting into the frame buffer at integer position(x, y), relative to the position of the screen-coordinate origin

ii)getPixel (x, y, color);

- Retrieves the current frame-buffer setting for a pixel location;
- Parameter color receives an integer value corresponding to the combined RGB bit codes stored for the specified pixel at position (x,y).
- Additional screen-coordinate information is needed for 3D scenes.
- For a two-dimensional scene, all depth values are 0.

Absolute and Relative Coordinate Specifications

Absolute coordinate:

- So far, the coordinate references that we have discussed are stated as absolute coordinate values.
- This means that the values specified are the actual positions within the coordinate system in use.

Relative coordinates:

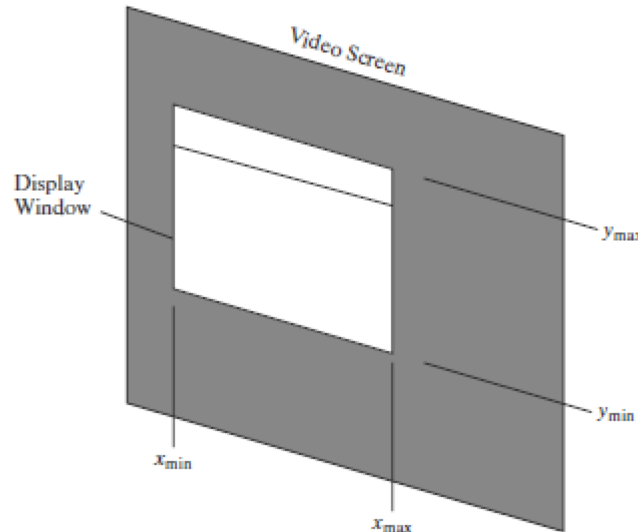
- However, some graphics packages also allow positions to be specified using relative coordinates.
- This method is useful for various graphics applications, such as producing drawings with pen plotters, artist's drawing and painting systems, and graphics packages for publishing and printing applications.
- Taking this approach, we can specify a coordinate position as an offset from the last position that was referenced (called the current position).

Specifying a Two-Dimensional World-Coordinate Reference Frame in OpenGL

- The gluOrtho2D command is a function we can use to set up any 2D Cartesian reference frames.
- The arguments for this function are the four values defining the x and y coordinate limits for the picture we want to display.
- Since the gluOrtho2D function specifies an orthogonal projection, we need also to be sure that the coordinate values are placed in the OpenGL projection matrix.
- In addition, we could assign the identity matrix as the projection matrix before defining the world-coordinate range.
- This would ensure that the coordinate values were not accumulated with any values we may have previously set for the projection matrix.
- Thus, for our initial two-dimensional examples, we can define the coordinate frame for the screen display window with the following statements

```
glMatrixMode (GL_PROJECTION);
glLoadIdentity ( );
gluOrtho2D (xmin, xmax, ymin, ymax);
```

- The display window will then be referenced by coordinates (xmin, ymin) at the lower-left corner and by coordinates (xmax, ymax) at the upper-right corner, as shown in Figure below



- We can then designate one or more graphics primitives for display using the coordinate reference specified in the gluOrtho2D statement.
- If the coordinate extents of a primitive are within the coordinate range of the display window, all of the primitive will be displayed.
- Otherwise, only those parts of the primitive within the display-window coordinate limits will be shown.
- Also, when we set up the geometry describing a picture, all positions for the OpenGL primitives must be given in absolute coordinates, with respect to the reference frame defined in the gluOrtho2D function.

1.14 OpenGL Functions

Geometric Primitives:

- It includes points, line segments, polygon etc.
- These primitives pass through geometric pipeline which decides whether the primitive is visible or not and also how the primitive should be visible on the screen etc.
- The geometric transformations such rotation, scaling etc can be applied on the primitives which are displayed on the screen.The programmer can create geometric primitives as shown below:

```
glBegin(type);

        glVertex*();
        glVertex*();
            •
            •
            •
        glVertex*();

glEnd():
```

where:

glBegin indicates the beginning of the object that has to be displayed

glEnd indicates the end of primitive

1.15 OpenGL Point Functions

- The type within glBegin() specifies the type of the object and its value can be as follows:

  GL_POINTS

- Each vertex is displayed as a point.
- The size of the point would be of at least one pixel.
- Then this coordinate position, along with other geometric descriptions we may have in our scene, is passed to the viewing routines.
- Unless we specify other attribute values, OpenGL primitives are displayed with a default size and color.
- The default color for primitives is white, and the default point size is equal to the size of a single screen pixel

Syntax:

Case 1:

glBegin (GL_POINTS);

glVertex2i (50, 100);

glVertex2i (75, 150);

glVertex2i (100, 200);

```
glEnd ( );
```

Case 2:

- we could specify the coordinate values for the preceding points in arrays such as

```
int point1 [ ] = {50, 100};
int point2 [ ] = {75, 150};
int point3 [ ] = {100, 200};
```

and call the OpenGL functions for plotting the three points as

```
glBegin (GL_POINTS);
glVertex2iv (point1);
glVertex2iv (point2);
glVertex2iv (point3);
glEnd ( );
```

Case 3:

- specifying two point positions in a three dimensional world reference frame. In this case, we give the coordinates as explicit floating-point values:

```
glBegin (GL_POINTS);
glVertex3f (-78.05, 909.72, 14.60);
glVertex3f (261.91, -5200.67, 188.33);
glEnd ( );
```

1.16 OpenGL LINE FUNCTIONS

- Primitive type is GL_LINES
- Successive pairs of vertices are considered as endpoints and they are connected to form an individual line segments.
- Note that successive segments usually are disconnected because the vertices are processed on a pair-wise basis.
- we obtain one line segment between the first and second coordinate positions and another line segment between the third and fourth positions.
- if the number of specified endpoints is odd, so the last coordinate position is ignored.

Case 1: Lines

glBegin (GL_LINES);

glVertex2iv (p1);

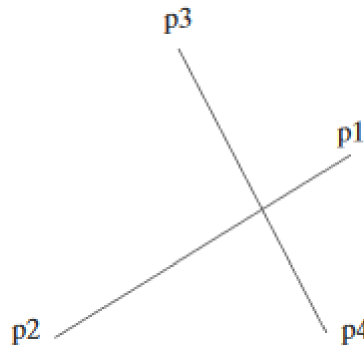
glVertex2iv (p2);

glVertex2iv (p3);

glVertex2iv (p4);

glVertex2iv (p5);

glEnd ( );



Case 2: GL_LINE_STRIP:

Successive vertices are connected using line segments. However, the final vertex is not connected to the initial vertex.

glBegin (GL_LINES_STRIP);

glVertex2iv (p1);

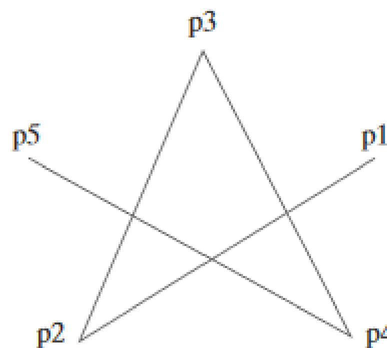
glVertex2iv (p2);

glVertex2iv (p3);

glVertex2iv (p4);

glVertex2iv (p5);

glEnd ( );



Case 3: GL_LINE_LOOP:

Successive vertices are connected using line segments to form a closed path or loop i.e., final vertex is connected to the initial vertex.

glBegin (GL_LINES_LOOP);

glVertex2iv (p1);

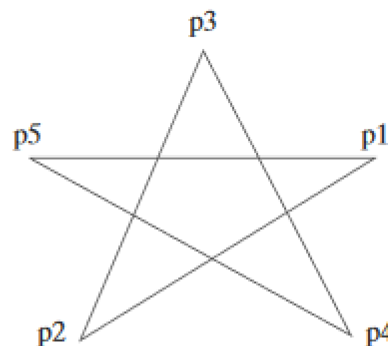
glVertex2iv (p2);

glVertex2iv (p3);

glVertex2iv (p4);

glVertex2iv (p5);

glEnd ( );



1.16 Point Attributes

- ➔ Basically, we can set two attributes for points: color and size.
- ➔ In a state system: The displayed color and size of a point is determined by the current values stored in the attribute list.
- ➔ Color components are set with RGB values or an index into a color table.
- ➔ For a raster system: Point size is an integer multiple of the pixel size, so that a large point is displayed as a square block of pixels

<u>Opengl Point-Attribute Functions</u>

<u>Color:</u>

- ➔ The displayed color of a designated point position is controlled by the current color values in the state list.
- ➔ Also, a color is specified with either the glColor function or the glIndex function.

<u>Size:</u>

- ➔ We set the size for an OpenGL point with

  glPointSize (size);

  and the point is then displayed as a square block of pixels.
- ➔ Parameter size is assigned a positive floating-point value, which is rounded to an integer (unless the point is to be antialiased).
- ➔ The number of horizontal and vertical pixels in the display of the point is determined by parameter size.
- ➔ Thus, a point size of 1.0 displays a single pixel, and a point size of 2.0 displays a 2×2 pixel array.
- ➔ If we activate the antialiasing features of OpenGL, the size of a displayed block of pixels will be modified to smooth the edges.
- ➔ The default value for point size is 1.0.

<u>Example program:</u>

- ➔ Attribute functions may be listed inside or outside of a glBegin/glEnd pair.
- ➔ Example: the following code segment plots three points in varying colors and sizes.

➔ The first is a standard-size red point, the second is a double-size green point, and the third is a triple-size blue point:

Ex:

```
glColor3f (1.0, 0.0, 0.0);
        glBegin (GL_POINTS);
        glVertex2i (50, 100);
        glPointSize (2.0);
        glColor3f (0.0, 1.0, 0.0);
        glVertex2i (75, 150);
        glPointSize (3.0);
        glColor3f (0.0, 0.0, 1.0);
        glVertex2i (100, 200);
glEnd ( );
```

1.17 Line-Attribute Functions OpenGL

➔ In OpenGL straight-line segment with three attribute settings: line color, line-width, and line style.

➔ OpenGL provides a function for setting the width of a line and another function for specifying a line style, such as a dashed or dotted line.

<u>OpenGL Line-Width Function</u>

➔ Line width is set in OpenGL with the function

Syntax: glLineWidth (width);

➔ We assign a floating-point value to parameter width, and this value is rounded to the nearest nonnegative integer.

➔ If the input value rounds to 0.0, the line is displayed with a standard width of 1.0, which is the default width.

➔ Some implementations of the line-width function might support only a limited number of widths, and some might not support widths other than 1.0.

- ➔ That is, the magnitude of the horizontal and vertical separations of the line endpoints, deltax and deltay, are compared to determine whether to generate a thick line using vertical pixel spans or horizontal pixel spans.

OpenGL Line-Style Function

- ➔ By default, a straight-line segment is displayed as a solid line.
- ➔ But we can also display dashed lines, dotted lines, or a line with a combination of dashes and dots.
- ➔ We can vary the length of the dashes and the spacing between dashes or dots.
- ➔ We set a current display style for lines with the OpenGL function:

Syntax: glLineStipple (repeatFactor, pattern);

Pattern:

- ➔ Parameter pattern is used to reference a 16-bit integer that describes how the line should be displayed.
- ➔ 1 bit in the pattern denotes an "on" pixel position, and a 0 bit indicates an "off" pixel position.
- ➔ The pattern is applied to the pixels along the line path starting with the low-order bits in the pattern.
- ➔ The default pattern is 0xFFFF (each bit position has a value of 1),which produces a solid line.

repeatFactor

- ➔ Integer parameter repeatFactor specifies how many times each bit in the pattern is to be repeated before the next bit in the pattern is applied.
- ➔ The default repeat value is 1.

Polyline:

- ➔ With a polyline, a specified line-style pattern is not restarted at the beginning of each segment.

➔ It is applied continuously across all the segments, starting at the first endpoint of the polyline and ending at the final endpoint for the last segment in the series.

Example:

➔ For line style, suppose parameter pattern is assigned the hexadecimal representation 0x00FF and the repeat factor is 1.

➔ This would display a dashed line with eight pixels in each dash and eight pixel positions that are “off” (an eight-pixel space) between two dashes.

➔ Also, since low order bits are applied first, a line begins with an eight-pixel dash starting at the first endpoint.

➔ This dash is followed by an eight-pixel space, then another eight-pixel dash, and so forth, until the second endpoint position is reached.

Activating line style:

- Before a line can be displayed in the current line-style pattern, we must activate the line-style feature of OpenGL.

  glEnable (GL_LINE_STIPPLE);

- If we forget to include this enable function, solid lines are displayed; that is, the default pattern 0xFFFF is used to display line segments.
- At any time, we can turn off the line-pattern feature with

  glDisable (GL_LINE_STIPPLE);

- This replaces the current line-style pattern with the default pattern (solid lines).

Example Code:

```
typedef struct { float x, y; } wcPt2D;
wcPt2D dataPts [5];
void linePlot (wcPt2D dataPts [5])
{
        int k;
        glBegin (GL_LINE_STRIP);
        for (k = 0; k < 5; k++)
                glVertex2f (dataPts [k].x, dataPts [k].y);
```

```
        glFlush ( );
        glEnd ( );
}
/* Invoke a procedure here to draw coordinate axes. */
glEnable (GL_LINE_STIPPLE); /* Input first set of (x, y) data values. */
glLineStipple (1, 0x1C47); // Plot a dash-dot, standard-width polyline.
linePlot (dataPts);
/* Input second set of (x, y) data values. */
glLineStipple (1, 0x00FF); / / Plot a dashed, double-width polyline.
glLineWidth (2.0);
linePlot (dataPts);
/* Input third set of (x, y) data values. */
glLineStipple (1, 0x0101); // Plot a dotted, triple-width polyline.
glLineWidth (3.0);
linePlot (dataPts);
glDisable (GL_LINE_STIPPLE);
```

1.18 Curve Attributes

- ➔ Parameters for curve attributes are the same as those for straight-line segments.
- ➔ We can display curves with varying colors, widths, dot-dash patterns, and available pen or brush options.
- ➔ Methods for adapting curve-drawing algorithms to accommodate attribute selections are similar to those for line drawing.
- ➔ Raster curves of various widths can be displayed using the method of horizontal or vertical pixel spans.

  Case 1: Where the magnitude of the curve slope $|m| <= 1.0$, we plot vertical spans;

  Case 2: when the slope magnitude $|m| > 1.0$, we plot horizontal spans.

<u>Different methods to draw a curve:</u>

<u>Method 1:</u> Using circle symmetry property, we generate the circle path with vertical spans in the octant from x = 0 to x = y, and then reflect pixel positions about the line y = x to y=0

Method 2: Another method for displaying thick curves is to fill in the area between two Parallel curve paths, whose separation distance is equal to the desired width. We could do this using the specified curve path as one boundary and setting up the second boundary either inside or outside the original curve path. This approach, however, shifts the original curve path either inward or outward, depending on which direction we choose for the second boundary.

Method 3: The pixel masks discussed for implementing line-style options could also be used in raster curve algorithms to generate dashed or dotted patterns

Method 4: Pen (or brush) displays of curves are generated using the same techniques discussed for straight-line segments.

Method 5: Painting and drawing programs allow pictures to be constructed interactively by using a pointing device, such as a stylus and a graphics tablet, to sketch various curve shapes.

1.19 Line Drawing Algorithm

- ✓ A straight-line segment in a scene is defined by coordinate positions for the endpoints of the segment.
- ✓ To display the line on a raster monitor, the graphics system must first project the endpoints to integer screen coordinates and determine the nearest pixel positions along the line path between the two endpoints then the line color is loaded into the frame buffer at the corresponding pixel coordinates
- ✓ The Cartesian slope-intercept equation for a straight line is

  y=m * x +b------------>(1)

  with m as the slope of the line and b as the y intercept.
- ✓ Given that the two endpoints of a line segment are specified at positions (x0,y0) and (xend, yend) ,as shown in fig.

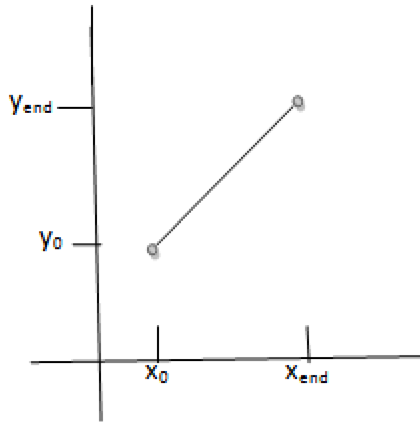


fig. Line path between endpoint positions (x_0,y_0) and (x_{end}, y_{end}).

- ✓ We determine values for the slope m and y intercept b with the following equations:

  m=(yend - y0)/(xend - x0)----------------->(2)

  b=y0 - m.x0-------------->(3)

- ✓ Algorithms for displaying straight line are based on the line equation (1) and calculations given in eq(2) and (3).
- ✓ For given x interval δx along a line, we can compute the corresponding y interval δy from eq.(2) as

  δy=m. δx----------------->(4)

- ✓ Similarly, we can obtain the x interval δx corresponding to a specified δy as

  δx=δy/m------------------>(5)

- ✓ These equations form the basis for determining deflection voltages in analog displays, such as vector-scan system, where arbitrarily small changes in deflection voltage are possible.
- ✓ For lines with slope magnitudes
  - ➔ |m|<1, δx can be set proportional to a small horizontal deflection voltage with the corresponding vertical deflection voltage set proportional to δy from eq.(4)
  - ➔ |m|>1, δy can be set proportional to a small vertical deflection voltage with the corresponding horizontal deflection voltage set proportional to δx from eq.(5)
  - ➔ |m|=1, δx=δy and the horizontal and vertical deflections voltages are equal

DDA Algorithm (DIGITAL DIFFERENTIAL ANALYZER)

- ➔ The DDA is a scan-conversion line algorithm based on calculating either δy or δx.

- ➔ A line is sampled at unit intervals in one coordinate and the corresponding integer values nearest the line path are determined for the other coordinate
- ➔ DDA Algorithm has three cases so from equation i.e.., $m=(y_{k+1} - y_k)/(x_{k+1} - x_k)$

Case1:

if $m<1$, x increment in unit intervals

i.e..,$x_{k+1}=x_k+1$

then, $m=(y_{k+1} - y_k)/(x_{k+1} - x_k)$

$\mathbf{m= y_{k+1} - y_k}$

$\mathbf{y_{k+1} = y_k + m}$------------>(1)

- ➔ where k takes integer values starting from 0,for the first point and increases by 1 until final endpoint is reached. Since m can be any real number between 0.0 and 1.0,

Case2:

if $m>1$, y increment in unit intervals

i.e.., $y_{k+1} = y_k + 1$

then, $m= (y_k + 1- y_k)/(x_{k+1} - x_k)$

$\mathbf{m(x_{k+1} - x_k)=1}$

$\mathbf{x_{k+1} =(1/m)+ x_k}$-----------------(2)

Case3:

if $m=1$,both x and y increment in unit intervals

i.e..,$x_{k+1}=x_k + 1$ and $y_{k+1} = y_k + 1$

Equations (1) and (2) are based on the assumption that lines are to be processed from the left endpoint to the right endpoint. If this processing is reversed, so that the starting endpoint is at the right, then either we have $\delta x=-1$ and

$y_{k+1} = y_k - m$-----------------(3)

or(when the slope is greater than 1)we have $\delta y=-1$ with

$x_{k+1} = x_k - (1/m)$----------------(4)

- ➔ Similar calculations are carried out using equations (1) through (4) to determine the pixel positions along a line with negative slope. thus, if the absolute value of the slope is less than 1 and the starting endpoint is at left ,we set δx==1 and calculate y values with eq(1).
- ➔ when starting endpoint is at the right(for the same slope),we set δx=-1 and obtain y positions using eq(3).
- ➔ This algorithm is summarized in the following procedure, which accepts as input two integer screen positions for the endpoints of a line segment.
- ➔ if m<1,where x is incrementing by 1

  $y_{k+1} = y_k + m$
- ➔ So initially x=0,Assuming (x0,y0)as initial point assigning x= x0,y=y0 which is the starting point .
  - o Illuminate pixel(x, round(y))
  - o x1= x+ 1 , y1=y + 1
  - o Illuminate pixel(x1,round(y1))
  - o x2= x1+ 1 , y2=y1 + 1
  - o Illuminate pixel(x2,round(y2))
  - o Till it reaches final point.
- ➔ if m>1,where y is incrementing by 1

  $x_{k+1} =(1/m)+ x_k$
- ➔ So initially y=0,Assuming (x0,y0)as initial point assigning x= x0,y=y0 which is the starting point .
  - o Illuminate pixel(round(x),y)
  - o x1= x+(1/m) ,y1=y
  - o Illuminate pixel(round(x1),y1)
  - o x2= x1+ (1/m) , y2=y1
  - o Illuminate pixel(round(x2),y2)
  - o Till it reaches final point.

- ➔ The DDA algorithm is faster method for calculating pixel position than one that directly implements .

- ➔ It eliminates the multiplication by making use of raster characteristics, so that appropriate increments are applied in the x or y directions to step from one pixel position to another along the line path.
- ➔ The accumulation of round off error in successive additions of the floating point increment, however can cause the calculated pixel positions to drift away from the true line path for long line segments. Furthermore ,the rounding operations and floating point arithmetic in this procedure are still time consuming.
- ➔ we improve the performance of DDA algorithm by separating the increments m and 1/m into integer and fractional parts so that all calculations are reduced to integer operations.

```
#include <stdlib.h>
#include <math.h>
inline int round (const float a)
{
        return int (a + 0.5);
}
void lineDDA (int x0, int y0, int xEnd, int yEnd)
{
        int dx = xEnd - x0, dy = yEnd - y0, steps, k;
        float xIncrement, yIncrement, x = x0, y = y0;
        if (fabs (dx) > fabs (dy))
                steps = fabs (dx);
        else
                steps = fabs (dy);
        xIncrement = float (dx) / float (steps);
        yIncrement = float (dy) / float (steps);
        setPixel (round (x), round (y));
        for (k = 0; k < steps; k++) {
                x += xIncrement;
                y += yIncrement;
                setPixel (round (x), round (y));
        }
```

}

Bresenham's Algorithm:

- ➔ It is an efficient raster scan generating algorithm that uses incremental integral calculations
- ➔ To illustrate Bresenham's approach, we first consider the scan-conversion process for lines with positive slope less than 1.0.
- ➔ Pixel positions along a line path are then determined by sampling at unit x intervals. Starting from the left endpoint (x0, y0) of a given line, we step to each successive column (x position) and plot the pixel whose scan-line y value is closest to the line path.

- ➔ Consider the equation of a straight line y=mx+c where m=dy/dx

Bresenham's Line-Drawing Algorithm for |m| < 1.0

1. Input the two line endpoints and store the left endpoint in (x0, y0).
2. Set the color for frame-buffer position (x0, y0); i.e., plot the first point.
3. Calculate the constants Δx, Δy, 2Δy, and 2Δy − 2Δx, and obtain the starting value for the decision parameter as

$$p_0 = 2\Delta y - \Delta x$$

4. At each xk along the line, starting at k = 0, perform the following test:

If pk < 0, the next point to plot is (xk + 1, yk) and

$$p_{k+1} = p_k + 2\Delta y$$

Otherwise, the next point to plot is (xk + 1, yk + 1) and

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

5. Repeat step 4 Δx − 1 more times.

Note:

If |m|>1.0

Then

$$p_0 = 2\Delta x - \Delta y$$

and

If pk < 0, the next point to plot is (xk , yk +1) and

$$p_{k+1} = p_k + 2\Delta x$$

Otherwise, the next point to plot is (xk + 1, yk + 1) and

$$p_{k+1} = p_k + 2\Delta x - 2\Delta y$$

Code:

```
#include <stdlib.h>
#include <math.h>
/* Bresenham line-drawing procedure for |m| < 1.0. */
void lineBres (int x0, int y0, int xEnd, int yEnd)
{
        int dx = fabs (xEnd - x0), dy = fabs(yEnd - y0);
        int p = 2 * dy - dx;
        int twoDy = 2 * dy, twoDyMinusDx = 2 * (dy - dx);
        int x, y;
        /* Determine which endpoint to use as start position. */
        if (x0 > xEnd) {
                x = xEnd;
                y = yEnd;
                xEnd = x0;
        }
        else {
                x = x0;
                y = y0;
        }
        setPixel (x, y);
        while (x < xEnd) {
                x++;
                if (p < 0)
                p += twoDy;
```

```
                else {
                        y++;
                        p += twoDyMinusDx;
                }
                setPixel (x, y);
        }
}
```

Properties of Circles

- A circle is defined as the set of points that are all at a given distance r from a center position (x_c , y_c).
- For any circle point (x, y), this distance relationship is expressed by the Pythagorean theorem in Cartesian coordinates as

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

- We could use this equation to calculate the position of points on a circle circumference by stepping along the x axis in unit steps from $x_c - r$ to $x_c + r$ and calculating the corresponding y values at each position as

$$y = y_c \pm \sqrt{r^2 - (x_c - x)^2}$$

- One problem with this approach is that it involves considerable computation at each step. Moreover, the spacing between plotted pixel positions is not uniform.
- We could adjust the spacing by interchanging x and y (stepping through y values and calculating x values) whenever the absolute value of the slope of the circle is greater than 1; but this simply increases the computation and processing required by the algorithm.
- Another way to eliminate the unequal spacing is to calculate points along the circular boundary using polar coordinates r and θ
- Expressing the circle equation in parametric polar form yields the pair of equations

$$x = x_c + r\cos\theta$$
$$y = y_c + r\sin\theta$$

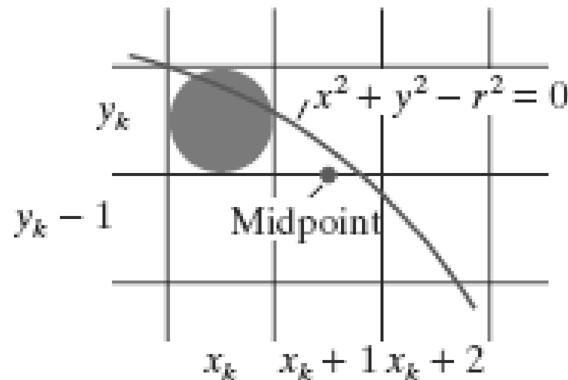
Midpoint Circle Algorithm

- ➔ Midpoint circle algorithm generates all points on a circle centered at the origin by incrementing all the way around circle.
- ➔ The strategy is to select which of 2 pixels is closer to the circle by evaluating a function at the midpoint between the 2 pixels
- ➔ To apply the midpoint method, we define a circle function as

$$f_{circ}(x, y) = x^2 + y^2 - r^2$$

- ➔ To summarize, the relative position of any point (x, y) can be determined by checking the sign of the circle function as follows:

$$f_{circ}(x, y) \begin{cases} < 0, & \text{if } (x, y) \text{ is inside the circle boundary} \\ = 0, & \text{if } (x, y) \text{ is on the circle boundary} \\ > 0, & \text{if } (x, y) \text{ is outside the circle boundary} \end{cases}$$



Eight way symmetry

- ➔ The shape of the circle is similar in each quadrant.
- ➔ Therefore ,if we determine the curve positions in the first quadrant ,we can generate the circle positions in the second quadrant of xy plane.
- ➔ The circle sections in the third and fourth quadrant can be obtained from sections in the first and second quadrant by considering the symmetry along X axis

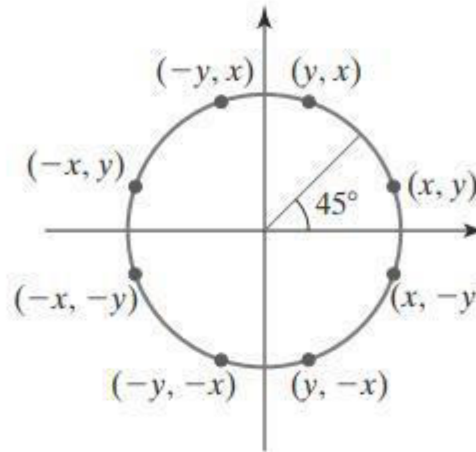


FIGURE 13
Symmetry of a circle. Calculation of a circle point (x, y) in one octant yields the circle points shown for the other seven octants.

- Conside the circle centered at the origin,if the point (x, y) is on the circle,then we can compute 7 other points on the circle as shown in the above figure.
- Our decision parameter is the circle function evaluated at the midpoint between these two pixels:

$$p_k = f_{\text{circ}}\left(x_k + 1, y_k - \frac{1}{2}\right)$$
$$= (x_k + 1)^2 + \left(y_k - \frac{1}{2}\right)^2 - r^2$$

- Successive decision parameters are obtained using incremental calculations.
- We obtain a recursive expression for the next decision parameter by evaluating the circle function at sampling position $x_{k+1} + 1 = x_k + 2$:

$$p_{k+1} = f_{\text{circ}}\left(x_{k+1} + 1, y_{k+1} - \frac{1}{2}\right)$$
$$= [(x_k + 1) + 1]^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - r^2$$

or

$$p_{k+1} = p_k + 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

where y_{k+1} is either y_k or $y_k - 1$, depending on the sign of p_k.

➔ The initial decision parameter is obtained by evaluating the circle function at the start position $(x_0, y_0) = (0, r)$:

$$p_0 = f_{circ}\left(1, r - \frac{1}{2}\right)$$
$$= 1 + \left(r - \frac{1}{2}\right)^2 - r^2$$
$$p_0 = \frac{5}{4} - r$$

➔ If the radius r is specified as an integer, we can simply round p0 to

p0 = 1 − r (for r an integer)

because all increments are integers.

Midpoint Circle Algorithm

1. Input radius r and circle center (x_c, y_c), then set the coordinates for the first point on the circumference of a circle centered on the origin as

$$(x_0, y_0) = (0, r)$$

2. Calculate the initial value of the decision parameter as

$$p0 = 1 - r$$

3. At each x_k position, starting at $k = 0$, perform the following test:

If $p_k < 0$, the next point along the circle centered on (0, 0) is (x_k+1, y_k) and

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

Otherwise, the next point along the circle is $(x_k + 1, y_k - 1)$ and

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$

where $2x_{k+1} = 2x_k + 2$ and $2y_{k+1} = 2y_k - 2$.

4. Determine symmetry points in the other seven octants.

5. Move each calculated pixel position (x, y) onto the circular path centered at (x_c, y_c) and plot the coordinate values as follows:

$$x = x + x_c,\ y = y + y_c$$

6. Repeat steps 3 through 5 until $x \geq y$.

Code:

```
void draw_pixel(GLint cx, GLint cy)
{
        glColor3f(0.5,0.5,0.0);
        glBegin(GL_POINTS);
                glVertex2i(cx, cy);
        glEnd();
}

void plotpixels(GLint h, GLint k, GLint x, GLint y)
{
        draw_pixel(x+h, y+k);
        draw_pixel(-x+h, y+k);
        draw_pixel(x+h, -y+k);
        draw_pixel(-x+h, -y+k);
        draw_pixel(y+h, x+k);
        draw_pixel(-y+h, x+k);
        draw_pixel(y+h, -x+k);
        draw_pixel(-y+h, -x+k);
}

void circle_draw(GLint xc, GLint yc, GLint r)
{
        GLint d=1-r, x=0,y=r;
        while(y>x)
        {
                plotpixels(xc, yc, x, y);
                if(d<0) d+=2*x+3;
                else
                {
```

```
                d+=2*(x-y)+5;
                --y;
            }
            ++x;
        }
        plotpixels(xc, yc, x, y);
}
```

Acknowledgements to

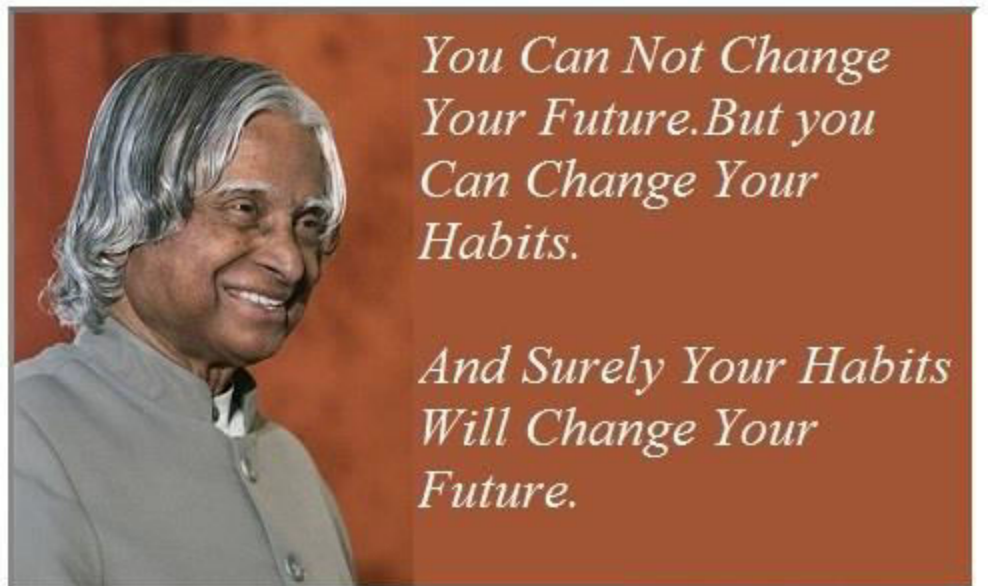
Donald Hearn & Pauline Baker: Computer Graphics with OpenGL

Version,3rd / 4th Edition, Pearson Education,2011

Edward Angel: Interactive Computer Graphics- A Top Down approach

with OpenGL, 5th edition. Pearson Education, 2008

M M Raiker, Computer Graphics using OpenGL, Filip learning/Elsevier



2.1 Fill area Primitives:

2.1.1 Introduction

2.1.2 Polygon fill-areas,

2.1.3 OpenGL polygon Fill Area Functions,

2.1.4 Fill area attributes,

2.1.5 General scan line polygon fill algorithm,

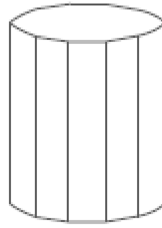
2.1.6 OpenGL fill-area Attribute functions.

2.1.1 Introduction

- An useful construct for describing components of a picture is an area that is filled with some solid color or pattern.
- A picture component of this type is typically referred to as a **fill area or a filled area**.
- Any fill-area shape is possible, graphics libraries generally do not support specifications for arbitrary fill shapes
- Figure below illustrates a few possible fill-area shapes.

- Graphics routines can more efficiently process polygons than other kinds of fill shapes because polygon boundaries are described with linear equations.
- When lighting effects and surface-shading procedures are applied, an approximated curved surface can be displayed quite realistically.
- Approximating a curved surface with polygon facets is sometimes referred to as ***surface tessellation,*** **or fitting the surface with a *polygon mesh.***

- Below figure shows the side and top surfaces of a metal cylinder approximated in an outline form as a polygon mesh.



- Displays of such figures can be generated quickly as *wire-frame* views, showing only the polygon edges to give a general indication of the surface structure
- Objects described with a set of polygon surface patches are usually referred to as standard graphics objects, or just graphics objects.

2.1.2 Polygon Fill Areas

- ✓ A polygon is a plane figure specified by a set of three or more coordinate positions, called *vertices,* that are connected in sequence by straight-line segments, called the *edges* or *sides* of the polygon.
- ✓ It is required that the polygon edges have no common point other than their endpoints.
- ✓ Thus, by definition, a polygon must have all its vertices within a single plane and there can be no edge crossings
- ✓ Examples of polygons include triangles, rectangles, octagons, and decagons
- ✓ Any plane figure with a closed-polyline boundary is alluded to as a polygon, and one with no crossing edges is referred to as a *standard polygon* or a *simple polygon*

Problem:

- ➢ For a computer-graphics application, it is possible that a designated set of polygon vertices do not all lie exactly in one plane
- ➢ This is due to roundoff error in the calculation of numerical values, to errors in selecting coordinate positions for the vertices, or, more typically, to approximating a curved surface with a set of polygonal patches

Solution:

- ➢ To divide the specified surface mesh into triangles

Polygon Classifications

- ✓ Polygons are classified into two types
  1. Convex Polygon and
  2. Concave Polygon

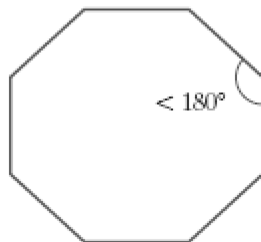
Convex Polygon:

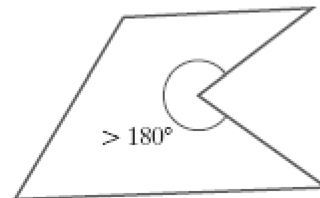
- ✓ The polygon is convex if all interior angles of a polygon are less than or equal to 180◦, where an interior angle of a polygon is an angle inside the polygon boundary that is formed by two adjacent edges
- ✓ An equivalent definition of a convex polygon is that its interior lies completely on one side of the infinite extension line of any one of its edges.
- ✓ Also, if we select any two points in the interior of a convex polygon, the line segment joining the two points is also in the interior.

Concave Polygon:

- ✓ A polygon that is not convex is called a concave polygon.

Te below figure shows convex and concave polygon





- ✓ The term degenerate polygon is often used to describe a set of vertices that are collinear or that have repeated coordinate positions.

Problems in concave polygon:

- ➔ Implementations of fill algorithms and other graphics routines are more complicated

Solution:

- ➔ It is generally more efficient to split a concave polygon into a set of convex polygons before processing

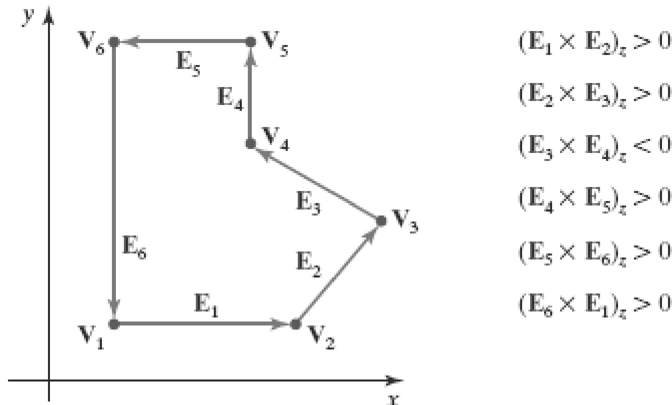
Identifying Concave Polygons

Characteristics:

- A concave polygon has at least one interior angle greater than 180◦.
- The extension of some edges of a concave polygon will intersect other edges, and
- Some pair of interior points will produce a line segment that intersects the polygon boundary

Identification algorithm 1

- Identifying a concave polygon by calculating cross-products of successive pairs of edge vectors.
- If we set up a vector for each polygon edge, then we can use the cross-product of adjacent edges to test for concavity. All such vector products will be of the same sign (positive or negative) for a convex polygon.
- Therefore, if some cross-products yield a positive value and some a negative value, we have a concave polygon



$(E_1 \times E_2)_z > 0$

$(E_2 \times E_3)_z > 0$

$(E_3 \times E_4)_z < 0$

$(E_4 \times E_5)_z > 0$

$(E_5 \times E_6)_z > 0$

$(E_6 \times E_1)_z > 0$

Identification algorithm 2

- Look at the polygon vertex positions relative to the extension line of any edge.
- If some vertices are on one side of the extension line and some vertices are on the other side, the polygon is concave.

Splitting Concave Polygons

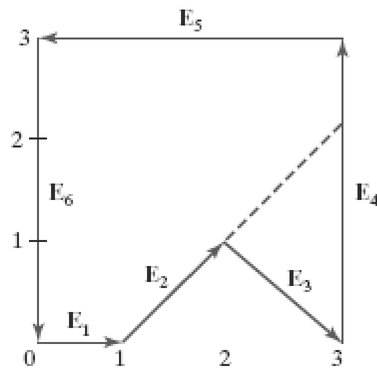
✓ Split concave polygon it into a set of convex polygons using edge vectors and edge cross-products; or, we can use vertex positions relative to an edge extension line to determine which vertices are on one side of this line and which are on the other.

Vector method

- ➔ First need to form the edge vectors.
- ➔ Given two consecutive vertex positions, V_k and V_{k+1}, we define the edge vector between them as

$$E_k = V_{k+1} - V_k$$

- ➔ Calculate the cross-products of successive edge vectors in order around the polygon perimeter.
- ➔ If the z component of some cross-products is positive while other cross-products have a negative z component, the polygon is concave.
- ➔ We can apply the vector method by processing edge vectors in counterclockwise order If any cross-product has a negative z component (as in below figure), the polygon is concave and we can split it along the line of the first edge vector in the cross-product pair



$E1 = (1, 0, 0)$ $E2 = (1, 1, 0)$

$E3 = (1, -1, 0)$ $E4 = (0, 2, 0)$

$E5 = (-3, 0, 0)$ $E6 = (0, -2, 0)$

- ➔ Where the z component is 0, since all edges are in the xy plane.
- ➔ The crossproduct $E_j \times E_k$ for two successive edge vectors is a vector perpendicular to the xy plane with z component equal to $E_{jx}E_{ky} - E_{kx}E_{jy}$:
- ➔ The values for the above figure is as follows

$E1 \times E2 = (0, 0, 1)$ $E2 \times E3 = (0, 0, -2)$

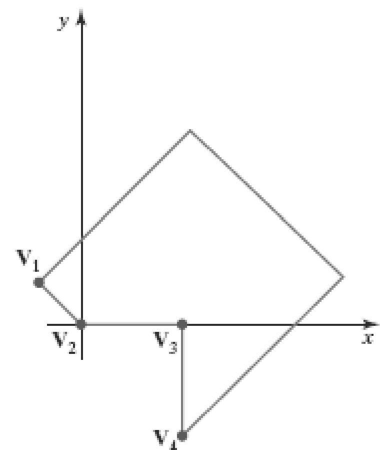
$E3 \times E4 = (0, 0, 2)$ $E4 \times E5 = (0, 0, 6)$

$E5 \times E6 = (0, 0, 6)$ $E6 \times E1 = (0, 0, 2)$

- Since the cross-product $E2 \times E3$ has a negative z component, we split the polygon along the line of vector E2.
- The line equation for this edge has a slope of 1 and a y intercept of -1 . No other edge cross-products are negative, so the two new polygons are both convex.

Rotational method

- Proceeding counterclockwise around the polygon edges, we shift the position of the polygon so that each vertex V_k in turn is at the coordinate origin.
- We rotate the polygon about the origin in a clockwise direction so that the next vertex V_{k+1} is on the x axis.
- If the following vertex, V_{k+2}, is below the x axis, the polygon is concave.
- We then split the polygon along the x axis to form two new polygons, and we repeat the concave test for each of the two new polygons



Splitting a Convex Polygon into a Set of Triangles

- Once we have a vertex list for a convex polygon, we could transform it into a set of triangles.
- First define any sequence of three consecutive vertices to be a new polygon (a triangle).
- The middle triangle vertex is then deleted from the original vertex list .
- The same procedure is applied to this modified vertex list to strip off another triangle.
- We continue forming triangles in this manner until the original polygon is reduced to just three vertices, which define the last triangle in the set.
- Concave polygon can also be divided into a set of triangles using this approach, although care must be taken that the new diagonal edge formed by joining the first and third selected vertices does not cross the concave portion of the polygon, and that the three selected vertices at each step form an interior angle that is less than 180◦

Identifying interior and exterior region of polygon

- We may want to specify a complex fill region with intersecting edges.
- For such shapes, it is not always clear which regions of the *xy* plane we should call "interior" and which regions.
- We should designate as "exterior" to the object boundaries.
- Two commonly used algorithms
    1. Odd-Even rule and
    2. The nonzero winding-number rule.

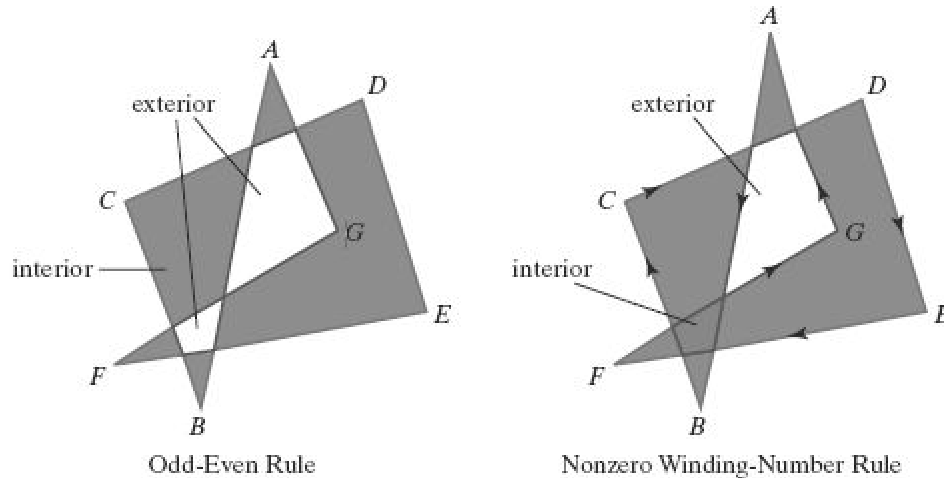
Inside-Outside Tests

- ✓ Also called the *odd-parity rule* or the *even-odd rule.*
- ✓ Draw a line from any position P to a distant point outside the coordinate extents of the closed polyline.
- ✓ Then we count the number of line-segment crossings along this line.
- ✓ If the number of segments crossed by this line is odd, then P is considered to be an *interior* point Otherwise, P is an *exterior* point
- ✓ We can use this procedure, for example,to fill the interior region between two concentric circles or two concentric polygons with a specified color.

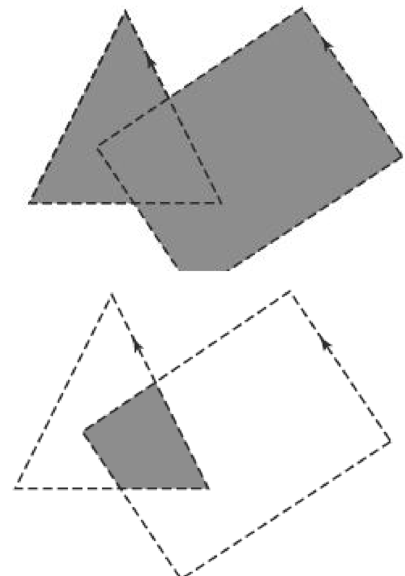
Nonzero Winding-Number rule

- ✓ This counts the number of times that the boundary of an object "winds" around a particular point in the counterclockwise direction termed as winding number,
- ✓ Initialize the winding number to 0 and again imagining a line drawn from any position P to a distant point beyond the coordinate extents of the object.
- ✓ The line we choose must not pass through any endpoint coordinates.
- ✓ As we move along the line from position P to the distant point, we count the number of object line segments that cross the reference line in each direction
- ✓ We add 1 to the winding number every time we intersect a segment that crosses the line in the direction from right to left, and we subtract 1 very time we intersect a segment that crosses from left to right

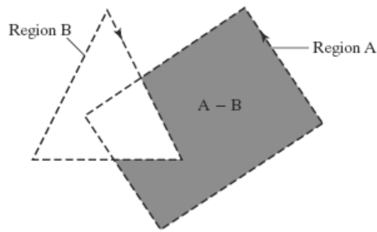
- ✓ If the winding number is nonzero, P is considered to be an interior point. Otherwise, P is taken to be an exterior point



- ✓ The nonzero winding-number rule tends to classify as interior some areas that the odd-even rule deems to be exterior.
- ✓ Variations of the nonzero winding-number rule can be used to define interior regions in other ways define a point to be interior if its winding number is positive or if it is negative; or we could use any other rule to generate a variety of fill shapes
- ✓ Boolean operations are used to specify a fill area as a combination of two regions
- ✓ One way to implement Boolean operations is by using a variation of the basic winding-number rule consider the direction for each boundary to be counterclockwise, the union of two regions would consist of those points whose winding number is positive



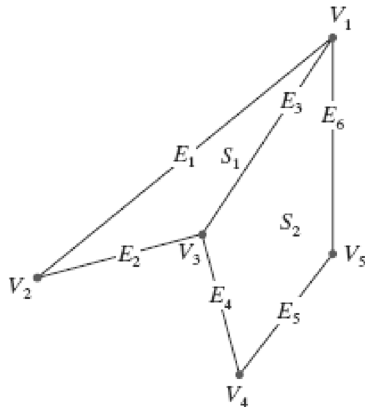
- ✓ The intersection of two regions with counterclockwise boundaries would contain those points whose winding number is greater than 1,



- To set up a fill area that is the difference of two regions (say, A − B), we can enclose region A with a counterclockwise border and B with a clockwise border

Polygon Tables

- ✓ The objects in a scene are described as sets of polygon surface facets
- ✓ The description for each object includes coordinate information specifying the geometry for the polygon facets and other surface parameters such as color, transparency, and light-reflection properties.
- ✓ The data of the polygons are placed into tables that are to be used in the subsequent processing, display, and manipulation of the objects in the scene
- ✓ These polygon data tables can be organized into two groups:
  1. Geometric tables and
  2. Attribute tables
- ✓ Geometric data tables contain vertex coordinates and parameters to identify the spatial orientation of the polygon surfaces.
- ✓ Attribute information for an object includes parameters specifying the degree of transparency of the object and its surface reflectivity and texture characteristics
- ✓ Geometric data for the objects in a scene are arranged conveniently in three lists: a vertex table, an edge table, and a surface-facet table.
- ✓ Coordinate values for each vertex in the object are stored in the vertex table.
- ✓ The edge table contains pointers back into the vertex table to identify the vertices for each polygon edge.
- ✓ And the surface-facet table contains pointers back into the edge table to identify the edges for each polygon



VERTEX TABLE	
V_1:	x_1, y_1, z_1
V_2:	x_2, y_2, z_2
V_3:	x_3, y_3, z_3
V_4:	x_4, y_4, z_4
V_5:	x_5, y_5, z_5

EDGE TABLE	
E_1:	V_1, V_2
E_2:	V_2, V_3
E_3:	V_3, V_1
E_4:	V_3, V_4
E_5:	V_4, V_5
E_6:	V_5, V_1

SURFACE-FACET TABLE	
S_1:	E_1, E_2, E_3
S_2:	E_3, E_4, E_5, E_6

- ✓ The object can be displayed efficiently by using data from the edge table to identify polygon boundaries.
- ✓ An alternative arrangement is to use just two tables: a vertex table and a surface-facet table this scheme is less convenient, and some edges could get drawn twice in a wire-frame display.
- ✓ Another possibility is to use only a surface-facet table, but this duplicates coordinate information, since explicit coordinate values are listed for each vertex in each polygon facet. Also the relationship between edges and facets would have to be reconstructed from the vertex listings in the surface-facet table.
- ✓ We could expand the edge table to include forward pointers into the surface-facet table so that a common edge between polygons could be identifiedmore rapidly the vertex table could be expanded to reference corresponding edges, for faster information retrieval

E_1:	V_1, V_2, S_1
E_2:	V_2, V_3, S_1
E_3:	V_3, V_1, S_1, S_2
E_4:	V_3, V_4, S_2
E_5:	V_4, V_5, S_2
E_6:	V_5, V_1, S_2

- ✓ Because the geometric data tables may contain extensive listings of vertices and edges for complex objects and scenes, it is important that the data be checked for consistency and completeness.
- ✓ Some of the tests that could be performed by a graphics package are

  (1) that every vertex is listed as an endpoint for at least two edges,

(2) that every edge is part of at least one polygon,

(3) that every polygon is closed,

(4) that each polygon has at least one shared edge, and

(5) that if the edge table contains pointers to polygons, every edge referenced by a polygon pointer has a reciprocal pointer back to the polygon.

Plane Equations

- Each polygon in a scene is contained within a plane of infinite extent.
- The general equation of a plane is

$$Ax + By + Cz + D = 0$$

Where,

- ➔ (x, y, z) is any point on the plane, and
- ➔ The coefficients A, B, C, and D (called *plane parameters*) are constants describing the spatial properties of the plane.

- We can obtain the values of A, B, C, and D by solving a set of three plane equations using the coordinate values for three noncollinear points in the plane for the three successive convex-polygon vertices, $(x1, y1, z1)$, $(x2, y2, z2)$, and $(x3, y3, z3)$, in a counterclockwise order and solve the following set of simultaneous linear plane equations for the ratios A/D, B/D, and C/D:

$$(A/D)x_k + (B/D)y_k + (C/D)z_k = -1, \quad k = 1, 2, 3$$

- The solution to this set of equations can be obtained in determinant form, using Cramer's rule, as

$$A = \begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix} \qquad B = \begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{vmatrix}$$

$$C = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad D = -\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

- Expanding the determinants, we can write the calculations for the plane coefficients in the form

$$A = y_1(z_2 - z_3) + y_2(z_3 - z_1) + y_3(z_1 - z_2)$$
$$B = z_1(x_2 - x_3) + z_2(x_3 - x_1) + z_3(x_1 - x_2)$$
$$C = x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$$
$$D = -x_1(y_2z_3 - y_3z_2) - x_2(y_3z_1 - y_1z_3) - x_3(y_1z_2 - y_2z_1)$$

- It is possible that the coordinates defining a polygon facet may not be contained within a single plane.
- We can solve this problem by dividing the facet into a set of triangles; or we could find an approximating plane for the vertex list.
- One method for obtaining an approximating plane is to divide the vertex list into subsets, where each subset contains three vertices, and calculate plane parameters *A*, *B*, *C*, *D*for each subset.

Front and Back Polygon Faces

- The side of a polygon that faces into the object interior is called the back face, and the visible, or outward, side is the front face .
- Every polygon is contained within an infinite plane that partitions space into two regions.
- Any point that is not on the plane and that is visible to the front face of a polygon surface section is said to be *in front of* (or *outside*) the plane, and, thus, outside the object.
- And any point that is visible to the back face of the polygon is *behind* (or *inside*) the plane.
- Plane equations can be used to identify the position of spatial points relative to the polygon facets of an object.
- For any point (x, y, z) not on a plane with parameters *A*, *B*, *C*, *D*, we have

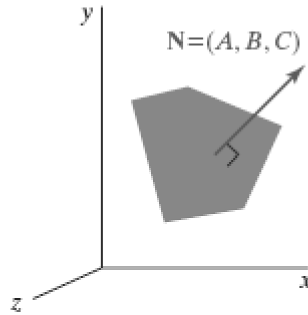
  $$Ax + B\,y + C\,z + D \mathrel{!}= 0$$

- Thus, we can identify the point as either behind or in front of a polygon surface contained within that plane according to the sign (negative or positive) of

  $Ax + By + Cz + D$:

  if $Ax + B\,y + C\,z + D < 0$, the point (x, y, z) is behind the plane

  if $Ax + B\,y + C\,z + D > 0$, the point (x, y, z) is in front of the plane
- Orientation of a polygon surface in space can be described with the normal vector for the plane containing that polygon



- The normal vector points in a direction from inside the plane to the outside; that is, from the back face of the polygon to the front face.
- Thus, the normal vector for this plane is N = (1, 0, 0), which is in the direction of the positive x axis.
- That is, the normal vector is pointing from inside the cube to the outside and is perpendicular to the plane $x = 1$.
- The elements of a normal vector can also be obtained using a vector crossproduct Calculation.
- We have a convex-polygon surface facet and a right-handed Cartesian system, we again select any three vertex positions,V1,V2, and V3, taken in counterclockwise order when viewing from outside the object toward the inside.
- Forming two vectors, one from V1 to V2 and the second from V1 to V3, we calculate N as the vector cross-product:

$$N = (V2 - V1) \times (V3 - V1)$$

- This generates values for the plane parameters A, B, and C.We can then obtain the value for parameter D by substituting these values and the coordinates in

$Ax + By + Cz + D = 0$

- The plane equation can be expressed in vector form using the normal N and the position P of any point in the plane as

$$N \cdot P = -D$$

2.1.3 OpenGL Polygon Fill-Area Functions

- ✓ A glVertex function is used to input the coordinates for a single polygon vertex, and a complete polygon is described with a list of vertices placed between a glBegin/glEnd pair.
- ✓ By default, a polygon interior is displayed in a solid color, determined by the current color settings we can fill a polygon with a pattern and we can display polygon edges as line borders around the interior fill.
- ✓ There are six different symbolic constants that we can use as the argument in the glBegin function to describe polygon fill areas
- ✓ In some implementations of OpenGL, the following routine can be more efficient than generating a fill rectangle using glVertex specifications:

```
glRect* (x1, y1, x2, y2);
```

- ✓ One corner of this rectangle is at coordinate position (*x*1, *y*1), and the opposite corner of the rectangle is at position (*x*2, *y*2).
- ✓ Suffix codes for glRect specify the coordinate data type and whether coordinates are to be expressed as array elements.
- ✓ These codes are i (for integer), s (for short), f (for float), d (for double), and v (for vector).
- ✓ Example

```
glRecti (200, 100, 50, 250);
```

If we put the coordinate values for this rectangle into arrays, we can generate the same square with the following code:

```
int vertex1 [ ] = {200, 100};
int vertex2 [ ] = {50, 250};
glRectiv (vertex1, vertex2);
```

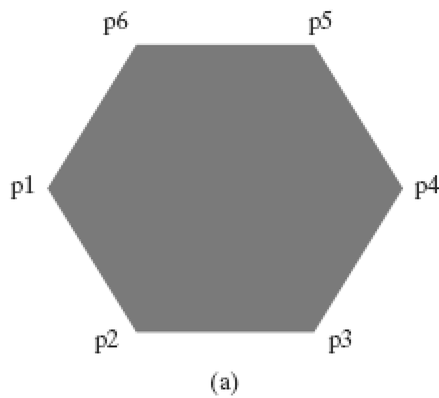
Polygon

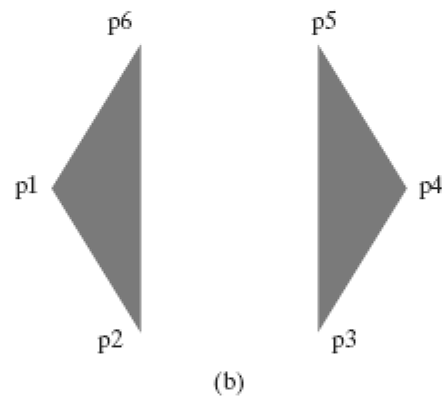
- ❖ With the OpenGL primitive constant GL POLYGON, we can display a single polygon fill area.
- ❖ Each of the points is represented as an array of (*x*, *y*) coordinate values:

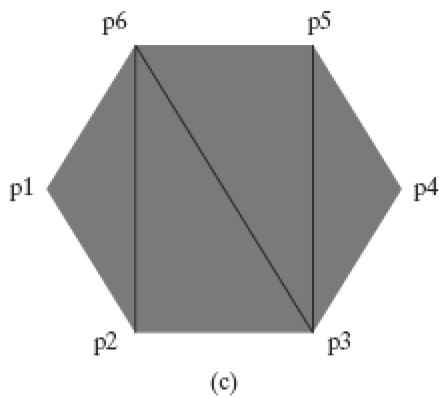
```
glBegin (GL_POLYGON);
    glVertex2iv (p1);
```

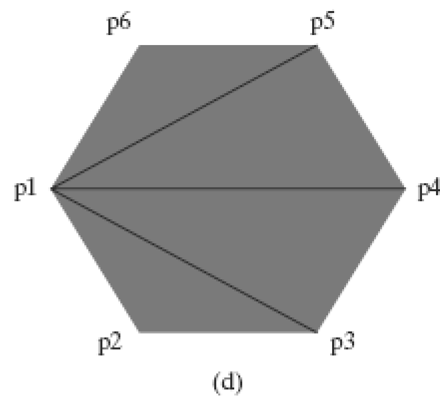
```
            glVertex2iv (p2);
            glVertex2iv (p3);
            glVertex2iv (p4);
            glVertex2iv (p5);
            glVertex2iv (p6);
        glEnd ( );
```

- A polygon vertex list must contain at least three vertices. Otherwise, nothing is displayed.









(a) A single convex polygon fill area generated with the primitive constant GL POLYGON. (b) Two unconnected triangles generated with GL TRIANGLES.

(c) Four connected triangles generated with GL TRIANGLE STRIP.

(d) Four connected triangles generated with GL TRIANGLE FAN.

Triangles

- Displays the trianlges.

- Three primitives in triangles, GL_TRIANGLES, GL_TRIANGLE_FAN, GL_TRIANGLE_STRIP

```
glBegin (GL_TRIANGLES);
    glVertex2iv (p1);
    glVertex2iv (p2);
    glVertex2iv (p6);
    glVertex2iv (p3);
    glVertex2iv (p4);
    glVertex2iv (p5);
glEnd ( );
```

- In this case, the first three coordinate points define the vertices for one triangle, the next three points define the next triangle, and so forth.
- For each triangle fill area, we specify the vertex positions in a counterclockwise order triangle strip

```
glBegin (GL_TRIANGLE_STRIP);
    glVertex2iv (p1);
    glVertex2iv (p2);
    glVertex2iv (p6);
    glVertex2iv (p3);
    glVertex2iv (p5);
    glVertex2iv (p4);
glEnd ( );
```

- Assuming that no coordinate positions are repeated in a list of N vertices, we obtain $N - 2$ triangles in the strip. Clearly, we must have $N \geq 3$ or nothing is displayed.
- Each successive triangle shares an edge with the previously defined triangle, so the ordering of the vertex list must be set up to ensure a consistent display.
- Example, our first triangle (n = 1) would be listed as having vertices (p1, p2, p6). The second triangle (n = 2) would have the vertex ordering (p6, p2, p3). Vertex ordering for the third triangle (n = 3) would be (p6, p3, p5). And the fourth triangle (n = 4) would be listed in the polygon tables with vertex ordering (p5, p3, p4).

Triangle Fan

- Another way to generate a set of connected triangles is to use the “fan” Approach

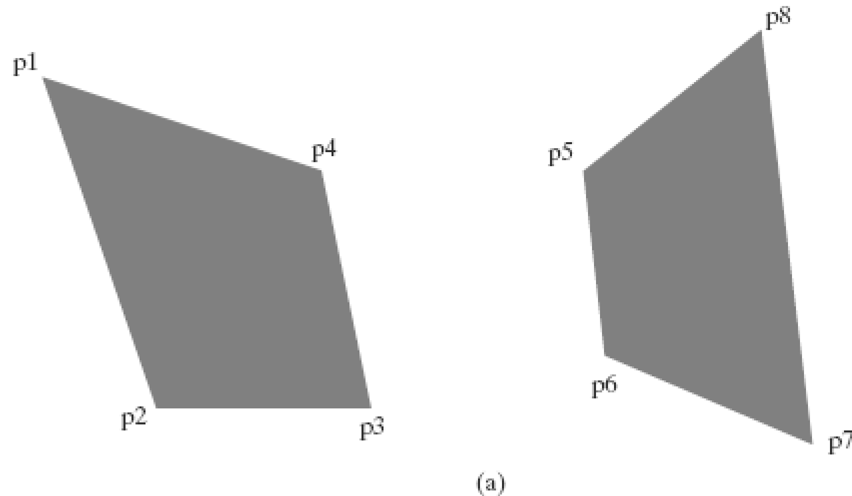
```
glBegin (GL_TRIANGLE_FAN);
    glVertex2iv (p1);
    glVertex2iv (p2);
    glVertex2iv (p3);
    glVertex2iv (p4);
    glVertex2iv (p5);
    glVertex2iv (p6);
glEnd ( );
```

- For N vertices, we again obtain $N-2$ triangles, providing no vertex positions are repeated, and we must list at least three vertices be specified in the proper order to define front and back faces for each triangle correctly.
- Therefore, triangle 1 is defined with the vertex list (p1, p2, p3); triangle 2 has the vertex ordering (p1, p3, p4); triangle 3 has its vertices specified in the order (p1, p4, p5); and triangle 4 is listed with vertices (p1, p5, p6).

Quadrilaterals

- ✓ OpenGL provides for the specifications of two types of quadrilaterals.
- ✓ With the GL QUADS primitive constant and the following list of eight vertices, specified as two-dimensional coordinate arrays, we can generate the display shown in Figure (a):

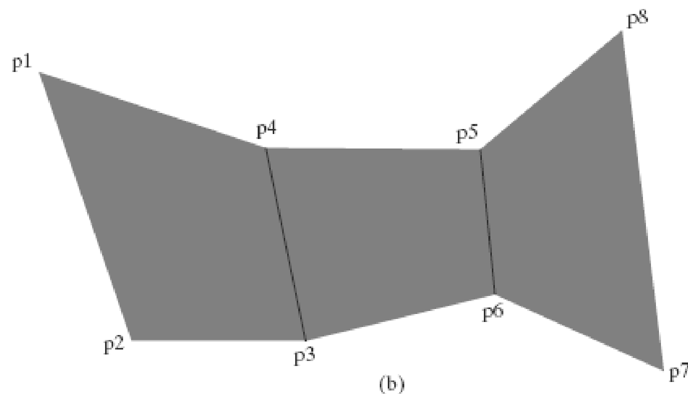
```
glBegin (GL_QUADS);
    glVertex2iv (p1);
    glVertex2iv (p2);
    glVertex2iv (p3);
    glVertex2iv (p4);
    glVertex2iv (p5);
    glVertex2iv (p6);
    glVertex2iv (p7);
    glVertex2iv (p8);
glEnd ( );
```



(a)

- ✓ Rearranging the vertex list in the previous quadrilateral code example and changing the primitive constant to GL QUAD STRIP, we can obtain the set of connected quadrilaterals shown in Figure (b):

```
glBegin (GL_QUAD_STRIP);
    glVertex2iv (p1);
    glVertex2iv (p2);
    glVertex2iv (p4);
    glVertex2iv (p3);
    glVertex2iv (p5);
    glVertex2iv (p6);
    glVertex2iv (p8);
    glVertex2iv (p7);
glEnd ( );
```



(b)

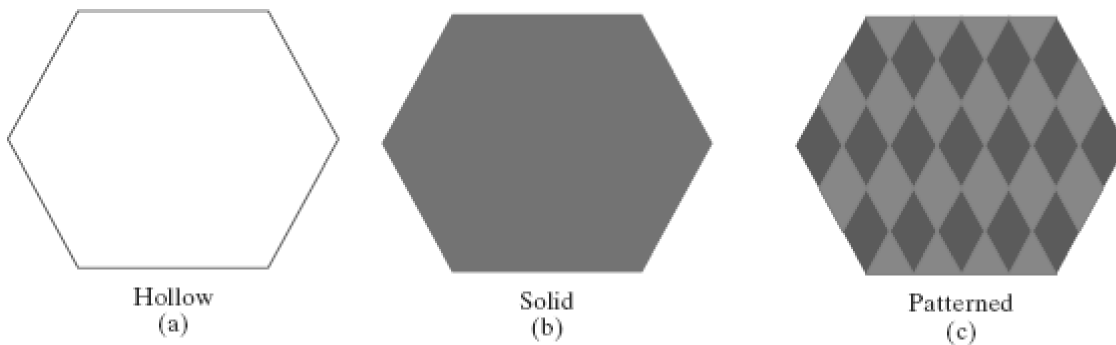
- ✓ For a list of *N* vertices, we obtain $N/2-1$ quadrilaterals, providing that $N \geq 4$. Thus, our first quadrilateral ($n = 1$) is listed as having a vertex ordering of (p1, p2, p3, p4). The second quadrilateral (n=2) has the vertex ordering (p4, p3, p6, p5), and the vertex ordering for the third quadrilateral (n=3) is (p5, p6, p7, p8).

2.1.4 Fill-Area Attributes

- ➔ We can fill any specified regions, including circles, ellipses, and other objects with curved boundaries

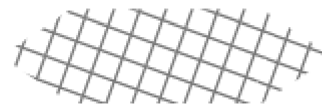
Fill Styles

- ➔ A basic fill-area attribute provided by a general graphics library is the display style of the interior.
- ➔ We can display a region with a single color, a specified fill pattern, or in a "hollow" style by showing only the boundary of the region



- ➔ We can also fill selected regions of a scene using various brush styles, color-blending combinations, or textures.
- ➔ For polygons, we could show the edges in different colors, widths, and styles; and we can select different display attributes for the front and back faces of a region.
- ➔ Fill patterns can be defined in rectangular color arrays that list different colors for different positions in the array.
- ➔ An array specifying a fill pattern is a *mask* that is to be applied to the display area.
- ➔ The mask is replicated in the horizontal and vertical directions until the display area is filled with nonoverlapping copies of the pattern.
- ➔ This process of filling an area with a rectangular pattern is called tiling, and a rectangular fill pattern is sometimes referred to as a tiling pattern predefined fill patterns are available in a system, such as the *hatch* fill patterns

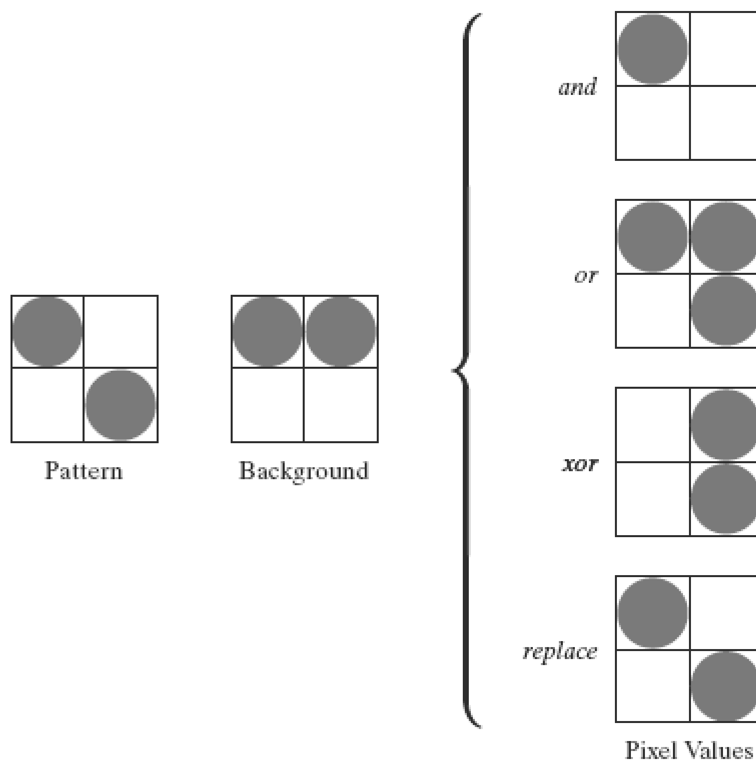
Diagonal
Hatch Fill


Diagonal
Crosshatch Fill

➔ Hatch fill could be applied to regions by drawing sets of line segments to display either single hatching or crosshtching

Color-Blended Fill Regions

- Color-blended regions can be implemented using either transparency factors to control the blending of background and object colors, or using simple logical or replace operations as shown in figure



- The *linear soft-fill algorithm* repaints an area that was originally painted by merging a foreground color F with a single background color B, where F != B.
- The current color P of each pixel within the area to be refilled is some linear combination of F and B:

$$P = tF + (1 - t)B$$

- Where the transparency factor t has a value between 0 and 1 for each pixel.
- For values of t less than 0.5, the background color contributes more to the interior color of the region than does the fill color.
- If our color values are represented using separate red, green, and blue (RGB) components, each component of the colors, with

$$P = (P_R, P_G, P_B),\ F = (F_R, F_G, F_B),\ B = (B_R, B_G, B_B) \text{ is used}$$

- We can thus calculate the value of parameter t using one of the RGB color components as follows:

$$t = \frac{P_k - B_k}{F_k - B_k}$$

Where $k = R, G$, or B; and $Fk != Bk$.

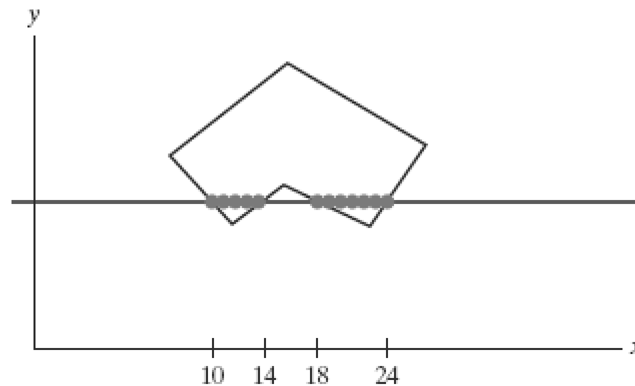
- When two background colors $B1$ and $B2$ are mixed with foreground color F, the resulting pixel color P is

$$P = t0F + t1B1 + (1 - t0 - t1)B2$$

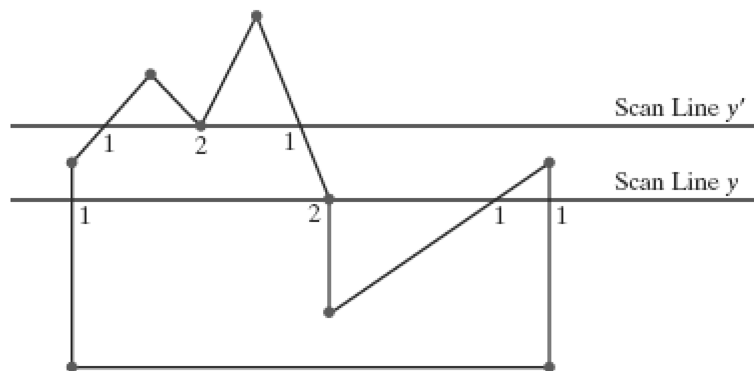
- Where the sum of the color-term coefficients $t0$, $t1$, and $(1 - t0 - t1)$ must equal 1.
- With three background colors and one foreground color, or with two background and two foreground colors, we need all three RGB equations to obtain the relative amounts of the four colors.

2.1.5 General Scan-Line Polygon-Fill Algorithm

- ➔ A scan-line fill of a region is performed by first determining the intersection positions of the boundaries of the fill region with the screen scan lines.
- ➔ Then the fill colors are applied to each section of a scan line that lies within the interior of the fill region.
- ➔ The simplest area to fill is a polygon because each scanline intersection point with a polygon boundary is obtained by solving a pair of simultaneous linear equations, where the equation for the scan line is simply y = constant.

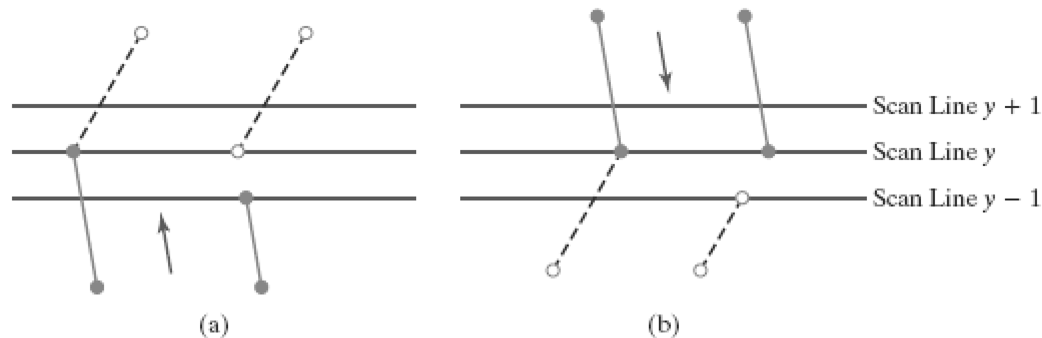


- ➔ Figure above illustrates the basic scan-line procedure for a solid-color fill of a polygon.
- ➔ For each scan line that crosses the polygon, the edge intersections are sorted from left to right, and then the pixel positions between, and including, each intersection pair are set to the specified fill color the fill color is applied to the five pixels from $x = 10$ to $x = 14$ and to the seven pixels from $x = 18$ to $x = 24$.
- ➔ Whenever a scan line passes through a vertex, it intersects two polygon edges at that point.
- ➔ In some cases, this can result in an odd number of boundary intersections for a scan line.



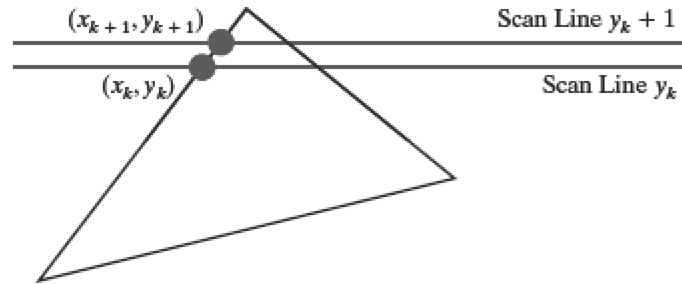
- ➔ Scan line *y*’ intersects an even number of edges, and the two pairs of intersection points along this scan line correctly identify the interior pixel spans.
- ➔ But scan line *y* intersects five polygon edges.
- ➔ Thus, as we process scan lines, we need to distinguish between these cases.
- ➔ For scan line *y*, the two edges sharing an intersection vertex are on opposite sides of the scan line.
- ➔ But for scan line *y*’, the two intersecting edges are both above the scan line.

- ➔ Thus, a vertex that has adjoining edges on opposite sides of an intersecting scan line should be counted as just one boundary intersection point.
- ➔ If the three endpoint *y* values of two consecutive edges monotonically increase or decrease, we need to count the shared (middle) vertex as a single intersection point for the scan line passing through that vertex.
- ➔ Otherwise, the shared vertex represents a local extremum (minimum or maximum) on the polygon boundary, and the two edge intersections with the scan line passing through that vertex can be added to the intersection list.
- ➔ One method for implementing the adjustment to the vertex-intersection count is to shorten some polygon edges to split those vertices that should be counted as one intersection.
- ➔ We can process nonhorizontal edges around the polygon boundary in the order specified, either clockwise or counterclockwise.
- ➔ Adjusting endpoint y values for a polygon, as we process edges in order around the polygon perimeter. The edge currently being processed is indicated as a solid line



In (a), the y coordinate of the upper endpoint of the current edge is decreased by 1. In (b), the y coordinate of the upper endpoint of the next edge is decreased by 1.

- ➔ Coherence properties can be used in computer-graphics algorithms to reduce processing.
- ➔ Coherence methods often involve incremental calculations applied along a single scan line or between successive scan lines



- The slope of this edge can be expressed in terms of the scan-line intersection coordinates:

$$m = \frac{y_{k+1} - y_k}{x_{k+1} - x_k}$$

- Because the change in y coordinates between the two scan lines is simply

$$y_{k+1} - y_k = 1$$

- The x-intersection value x_{k+1} on the upper scan line can be determined from the x-intersection value x_k on the preceding scan line as

$$x_{k+1} = x_k + \frac{1}{m}$$

- Each successive x intercept can thus be calculated by adding the inverse of the slope and rounding to the nearest integer.
- Along an edge with slope m, the intersection x_k value for scan line k above the initial scan line can be calculated as

$$x_k = x_0 + k/m$$

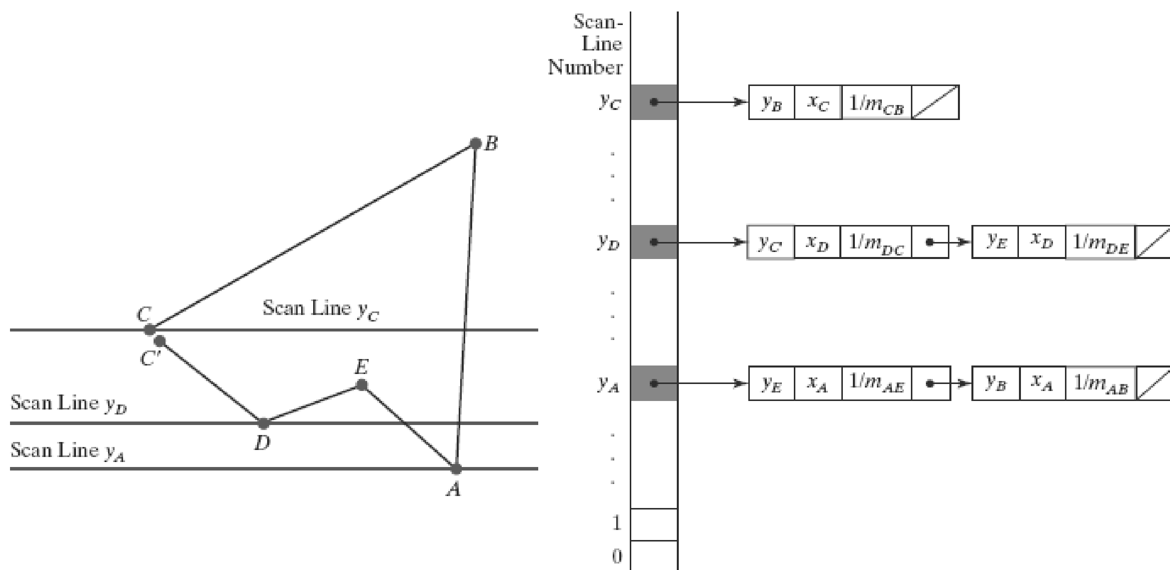
Where, m is the ratio of two integers

$$m = \frac{\Delta y}{\Delta x}$$

- Where Δx and Δy are the differences between the edge endpoint x and y coordinate values.
- Thus, incremental calculations of x intercepts along an edge for successive scan lines can be expressed as

$$x_{k+1} = x_k + \frac{\Delta x}{\Delta y}$$

- ➔ To perform a polygon fill efficiently, we can first store the polygon boundary in a *sorted edge table* that contains all the information necessary to process the scan lines efficiently.
- ➔ Proceeding around the edges in either a clockwise or a counterclockwise order, we can use a bucket sort to store the edges, sorted on the smallest *y* value of each edge, in the correct scan-line positions.
- ➔ Only nonhorizontal edges are entered into the sorted edge table.
- ➔ Each entry in the table for a particular scan line contains the maximum *y* value for that edge, the *x*-intercept value (at the lower vertex) for the edge, and the inverse slope of the edge. For each scan line, the edges are in sorted order fromleft to right



- ➔ We process the scan lines from the bottom of the polygon to its top, producing an *active edge list* for each scan line crossing the polygon boundaries.
- ➔ The active edge list for a scan line contains all edges crossed by that scan line, with iterative coherence calculations used to obtain the edge intersections
- ➔ Implementation of edge-intersection calculations can be facilitated by storing *Δx* and Δ*y* values in the sorted edge list

2.1.6 OpenGL Fill-Area Attribute Functions

- ➔ We generate displays of filled convex polygons in four steps:
  1. Define a fill pattern.
  2. Invoke the polygon-fill routine.

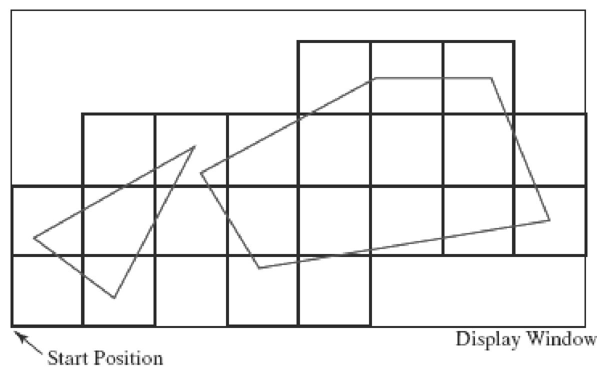
3. Activate the polygon-fill feature of OpenGL.
4. Describe the polygons to be filled.

➔ A polygon fill pattern is displayed up to and including the polygon edges. Thus, there are no boundary lines around the fill region unless we specifically add them to the display

OpenGL Fill-Pattern Function

- To fill the polygon with a pattern in OpenGL, we use a 32 × 32 bit mask.
- A value of 1 in the mask indicates that the corresponding pixel is to be set to the current color, and a 0 leaves the value of that frame-buffer position unchanged.
- The fill pattern is specified in unsigned bytes using the OpenGL data type Glubyte

  GLubyte fillPattern [ ] = { 0xff, 0x00, 0xff, 0x00, ... };
- The bits must be specified starting with the bottom row of the pattern, and continuing up to the topmost row (32) of the pattern.
- This pattern is replicated across the entire area of the display window, starting at the lower-left window corner, and specified polygons are filled where the pattern overlaps those polygons



- Once we have set a mask, we can establish it as the current fill pattern with the function

  glPolygonStipple (fillPattern);
- We need to enable the fill routines before we specify the vertices for the polygons that are to be filled with the current pattern

  glEnable (GL_POLYGON_STIPPLE);
- Similarly, we turn off pattern filling with

  glDisable (GL_POLYGON_STIPPLE);

OpenGL Texture and Interpolation Patterns

- Another method for filling polygons is to use texture patterns.
- This can produce fill patterns that simulate the surface appearance of wood, brick, brushed steel, or some other material.
- We assign different colors to polygon vertices.
- Interpolation fill of a polygon interior is used to produce realistic displays of shaded surfaces under various lighting conditions.
- The polygon fill is then a linear interpolation of the colors at the vertices:

```
glShadeModel (GL_SMOOTH);
glBegin (GL_TRIANGLES);
        glColor3f (0.0, 0.0, 1.0);
        glVertex2i (50, 50);
        glColor3f (1.0, 0.0, 0.0);
        glVertex2i (150, 50);
        glColor3f (0.0, 1.0, 0.0);
        glVertex2i (75, 150);
glEnd ( );
```

OpenGL Wire-Frame Methods

- We can also choose to show only polygon edges. This produces a wire-frame or hollow display of the polygon; or we could display a polygon by plotting a set of points only at the vertex positions.
- These options are selected with the function

  glPolygonMode (face, displayMode);
- We use parameter face to designate which face of the polygon that we want to show as edges only or vertices only.
- This parameter is then assigned either

  GL_FRONT, GL_BACK, or GL_FRONT_AND_BACK.
- If we want only the polygon edges displayed for our selection, we assign the constant GL_LINE to parameter displayMode.

- To plot only the polygon vertex points, we assign the constant GL_POINT to parameter displayMode.
- Another option is to display a polygon with both an interior fill and a different color or pattern for its edges.
- The following code section fills a polygon interior with a green color, and then the edges are assigned a red color:

```
glColor3f (0.0, 1.0, 0.0);
/* Invoke polygon-generating routine. */
glColor3f (1.0, 0.0, 0.0);
glPolygonMode (GL_FRONT, GL_LINE);
/* Invoke polygon-generating routine again. */
```

- For a three-dimensional polygon (one that does not have all vertices in the *xy* plane), this method for displaying the edges of a filled polygon may produce gaps along the edges.
- This effect, sometimes referred to as **stitching.**
- One way to eliminate the gaps along displayed edges of a three-dimensional polygon is to shift the depth values calculated by the fill routine so that they do not overlap with the edge depth values for that polygon.
- We do this with the following two OpenGL functions:

```
glEnable (GL_POLYGON_OFFSET_FILL);
glPolygonOffset (factor1, factor2);
```

- The first function activates the offset routine for scan-line filling, and the second function is used to set a couple of floating-point values factor1 and factor2 that are used to calculate the amount of depth offset.
- The calculation for this depth offset is

depthOffset = factor1 · maxSlope + factor2 · const

Where,

maxSlope is the maximum slope of the polygon and

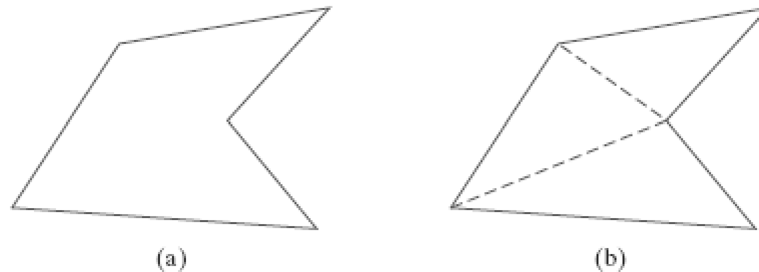
const is an implementation constant

- As an example of assigning values to offset factors, we can modify the previous code segment as follows:

```
glColor3f (0.0, 1.0, 0.0);
```

```
glEnable (GL_POLYGON_OFFSET_FILL);
glPolygonOffset (1.0, 1.0);
/* Invoke polygon-generating routine. */
glDisable (GL_POLYGON_OFFSET_FILL);
glColor3f (1.0, 0.0, 0.0);
glPolygonMode (GL_FRONT, GL_LINE);
/* Invoke polygon-generating routine again. */
```

- ➔ Another method for eliminating the stitching effect along polygon edges is to use the OpenGL stencil buffer to limit the polygon interior filling so that it does not overlap the edges.
- ➔ To display a concave polygon using OpenGL routines, we must first split it into a set of convex polygons.
- ➔ We typically divide a concave polygon into a set of triangles. Then we could display the triangles.



Dividing a concave polygon (a) into a set of triangles (b) produces triangle edges (dashed) that are interior to the original polygon.

- ➔ Fortunately, OpenGL provides a mechanism that allows us to eliminate selected edges from a wire-frame display.
- ➔ So all we need do is set that bit flag to "off" and the edge following that vertex will not be displayed.
- ➔ We set this flag for an edge with the following function:

glEdgeFlag (flag)

- ➔ To indicate that a vertex does not precede a boundary edge, we assign the OpenGL constant GL_FALSE to parameter flag.

- ➔ This applies to all subsequently specified vertices until the next call to glEdgeFlag is made.
- ➔ The OpenGL constant GL_TRUE turns the edge flag on again, which is the default.
- ➔ As an illustration of the use of an edge flag, the following code displays only two edges of the defined triangle

```
glPolygonMode (GL_FRONT_AND_BACK, GL_LINE);
glBegin (GL_POLYGON);
        glVertex3fv (v1);
        glEdgeFlag (GL_FALSE);
        glVertex3fv (v2);
        glEdgeFlag (GL_TRUE);
        glVertex3fv (v3);
glEnd ( );
```

- ➔ Polygon edge flags can also be specified in an array that could be combined or associated with a vertex array.
- ➔ The statements for creating an array of edge flags are

```
glEnableClientState (GL_EDGE_FLAG_ARRAY);
glEdgeFlagPointer (offset, edgeFlagArray);
```

OpenGL Front-Face Function

- ➢ We can label selected surfaces in a scene independently as front or back with the function

```
glFrontFace (vertexOrder);
```

- ➢ If we set parameter vertexOrder to the OpenGL constant GL_CW, then a subsequently defined polygon with a clockwise ordering.
- ➢ The constant GL_CCW labels a counterclockwise ordering of polygon vertices as front-facing, which is the default ordering.r its vertices is considered to be front-facing

2.2 2DGeometric Transformations:

2.2.1 Basic 2D Geometric Transformations,

2.2.2 Matrix representations and homogeneous coordinates.

2.2.3 Inverse transformations,

2.2.4 2DComposite transformations,

2.2.5 Other 2D transformations,

2.2.6 Raster methods for geometric transformations,

2.2.7 OpenGL raster transformations

2.2.8 OpenGL geometric transformations function,

Two-Dimensional Geometric Transformations

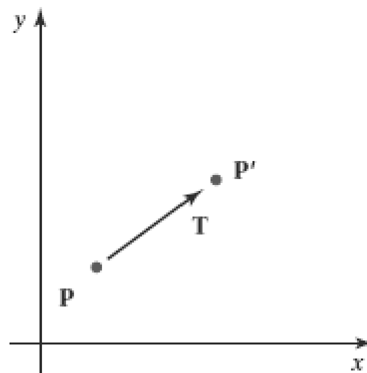
Operations that are applied to the geometric description of an object to change its position, orientation, or size are called **geometric transformations.**

2.2.1 Basic Two-Dimensional Geometric Transformations

The geometric-transformation functions that are available in all graphics packages are those for translation, rotation, and scaling.

Two-Dimensional Translation

- We perform a **translation** on a single coordinate point by adding offsets to its coordinates so as to generate a new coordinate position.
- We are moving the original point position along a straight-line path to its new location.
- To translate a two-dimensional position, we add **translation distances** tx and ty to the original coordinates (x, y) to obtain the new coordinate position (x', y') as shown in Figure



- The translation values of x' and y' is calculated as

$$x' = x + t_x, \qquad y' = y + t_y$$

- The translation distance pair (t_x, t_y) is called a **translation vector** or **shift vector Column vector representation is given as**

$$\mathbf{P} = \begin{bmatrix} x \\ y \end{bmatrix}, \qquad \mathbf{P}' = \begin{bmatrix} x' \\ y' \end{bmatrix}, \qquad \mathbf{T} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

- This allows us to write the two-dimensional translation equations in the matrix Form

$$\mathbf{P}' = \mathbf{P} + \mathbf{T}$$

- Translation is a *rigid-body transformation* that moves objects without deformation.

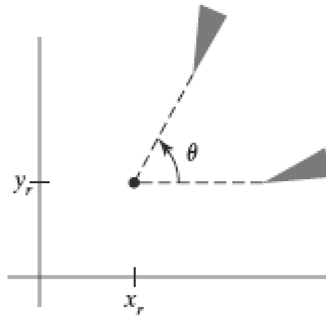
Code:

```
class wcPt2D {
        public:
        GLfloat x, y;
};
void translatePolygon (wcPt2D * verts, GLint nVerts, GLfloat tx, GLfloat ty)
{
    GLint k;
    for (k = 0; k < nVerts; k++) {
        verts [k].x = verts [k].x + tx;
        verts [k].y = verts [k].y + ty;
    }
    glBegin (GL_POLYGON);
    for (k = 0; k < nVerts; k++)
        glVertex2f (verts [k].x, verts [k].y);
    glEnd ( );
}
```

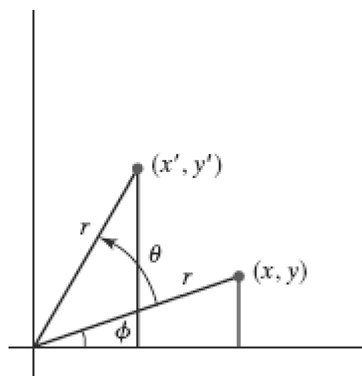
Two-Dimensional Rotation

- ✓ We generate a **rotation** transformation of an object by specifying a **rotation axis** and a **rotation angle.**

- ✓ A two-dimensional rotation of an object is obtained by repositioning the object along a circular path in the *xy* plane.
- ✓ In this case, we are rotating the object about a rotation axis that is perpendicular to the *xy* plane (parallel to the coordinate *z* axis).
- ✓ Parameters for the two-dimensional rotation are the rotation angle θ and a position (xr, yr), called the **rotation point** (or **pivot point**), about which the object is to be rotated



- ✓ A positive value for the angle θ defines a counterclockwise rotation about the pivot point, as in above Figure , and a negative value rotates objects in the clockwise direction.
- ✓ The angular and coordinate relationships of the original and transformed point positions are shown in Figure



- ✓ In this figure, r is the constant distance of the point from the origin, angle φ is the original angular position of the point from the horizontal, and θ is the rotation angle.
- ✓ we can express the transformed coordinates in terms of angles θ and φ as

$$x' = r\cos(\phi + \theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta$$
$$y' = r\sin(\phi + \theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta$$

- ✓ The original coordinates of the point in polar coordinates are

$$x = r\cos\phi, \qquad y = r\sin\phi$$

- ✓ Substituting expressions of x and y in the eaquations of x' and y' we get

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

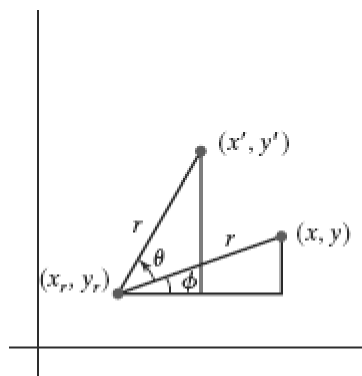
- ✓ We can write the rotation equations in the matrix form

P' = R· P

Where the rotation matrix is,

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

- ✓ Rotation of a point about an arbitrary pivot position is illustrated in Figure



- ✓ The transformation equations for rotation of a point about any specified rotation position (*xr* , *yr*):

$$x' = x_r + (x - x_r)\cos\theta - (y - y_r)\sin\theta$$
$$y' = y_r + (x - x_r)\sin\theta + (y - y_r)\cos\theta$$

Code:

```
class wcPt2D {
        public:
        GLfloat x, y;
};
void rotatePolygon (wcPt2D * verts, GLint nVerts, wcPt2D pivPt, GLdouble theta)
{
    wcPt2D * vertsRot;
    GLint k;
    for (k = 0; k < nVerts; k++) {
```

```
        vertsRot [k].x = pivPt.x + (verts [k].x - pivPt.x) * cos (theta) - (verts [k].y -
        pivPt.y) * sin (theta);
        vertsRot [k].y = pivPt.y + (verts [k].x - pivPt.x) * sin (theta) + (verts [k].y -
        pivPt.y) * cos (theta);
    }
    glBegin (GL_POLYGON);
    for (k = 0; k < nVerts; k++)
        glVertex2f (vertsRot [k].x, vertsRot [k].y);
    glEnd ( );
}
```

Two-Dimensional Scaling

- ✓ To alter the size of an object, we apply a **scaling** transformation.
- ✓ A simple twodimensional scaling operation is performed by multiplying object positions (x, y) by **scaling factors** *sx* and *sy* to produce the transformed coordinates (x', y'):

$$x' = x \cdot s_x, \qquad y' = y \cdot s_y$$

- ✓ The basic two-dimensional scaling equations can also be written in the following matrix form

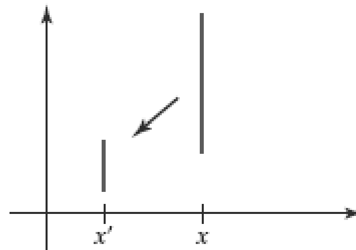
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{S} \cdot \mathbf{P}$$

Where **S** is the 2 × 2 scaling matrix

- ✓ Any positive values can be assigned to the scaling factors *sx* and *sy*.
- ✓ Values less than 1 reduce the size of objects
- ✓ Values greater than 1 produce enlargements.
- ✓ Specifying a value of 1 for both *sx* and *sy* leaves the size of objects unchanged.
- ✓ When *sx* and *sy* are assigned the same value, a **uniform scaling** is produced, which maintains relative object proportions.

- ✓ Unequal values for *sx* and *sy* result in a **differential scaling** that is often used in design applications.
- ✓ In some systems, negative values can also be specified for the scaling parameters. This not only resizes an object, it reflects it about one or more of the coordinate axes.
- ✓ Figure below illustrates scaling of a line by assigning the value 0.5 to both *sx* and *sy*



- ✓ We can control the location of a scaled object by choosing a position, called the **fixed point,** that is to remain unchanged after the scaling transformation.
- ✓ Coordinates for the fixed point, (x_f , y_f), are often chosen at some object position, such as its centroid but any other spatial position can be selected.
- ✓ For a coordinate position (*x*, *y*), the scaled coordinates (*x'*, *y'*) are then calculated from the following relationships:

$$x' - x_f = (x - x_f)s_x, \qquad y' - y_f = (y - y_f)s_y$$

- ✓ We can rewrite Equations to separate the multiplicative and additive terms as

$$x' = x \cdot s_x + x_f(1 - s_x)$$
$$y' = y \cdot s_y + y_f(1 - s_y)$$

- ✓ Where the additive terms $x_f(1 - s_x)$ and $y_f(1 - s_y)$ are constants for all points in the object.

Code:

```
class wcPt2D {
        public:
        GLfloat x, y;
};
void scalePolygon (wcPt2D * verts, GLint nVerts, wcPt2D fixedPt, GLfloat sx, GLfloat sy)
{
    wcPt2D vertsNew;
```

```
        GLint k;
        for (k = 0; k < nVerts; k++) {
                vertsNew [k].x = verts [k].x * sx + fixedPt.x * (1 - sx);
                vertsNew [k].y = verts [k].y * sy + fixedPt.y * (1 - sy);
        }
        glBegin (GL_POLYGON);
        for (k = 0; k < nVerts; k++)
                glVertex2f (vertsNew [k].x, vertsNew [k].y);
        glEnd ( );
}
```

2.2.2 Matrix Representations and Homogeneous Coordinates

- ✓ Each of the three basic two-dimensional transformations (translation, rotation, and scaling) can be expressed in the general matrix form

$$P' = M_1 \cdot P + M_2$$

- ✓ With coordinate positions **P** and **P'** represented as column vectors.
- ✓ Matrix **M**1 is a 2 × 2 array containing multiplicative factors, and **M**2 is a two-element column matrix containing translational terms.
- ✓ For translation, **M**1 is the identity matrix.
- ✓ For rotation or scaling, **M**2 contains the translational terms associated with the pivot point or scaling fixed point.

Homogeneous Coordinates

- ➢ Multiplicative and translational terms for a two-dimensional geometric transformation can be combined into a single matrix if we expand the representations to 3 × 3 matrices
- ➢ We can use the third column of a transformation matrix for the translation terms, and all transformation equations can be expressed as matrix multiplications.
- ➢ We also need to expand the matrix representation for a two-dimensional coordinate position to a three-element column matrix

- A standard technique for accomplishing this is to expand each twodimensional coordinate-position representation (x, y) to a three-element representation (xh, yh, h), called **homogeneous coordinates,** where the **homogeneous parameter** h is a nonzero value such that

$$x = \frac{x_h}{h}, \qquad y = \frac{y_h}{h}$$

- A general two-dimensional homogeneous coordinate representation could also be written as ($h \cdot x$, $h \cdot y$, h).
- A convenient choice is simply to set $h = 1$. Each two-dimensional position is then represented with homogeneous coordinates (x, y, 1).
- The term *homogeneous coordinates* is used in mathematics to refer to the effect of this representation on Cartesian equations.

Two-Dimensional Translation Matrix

- ✓ The homogeneous-coordinate for translation is given by

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- ✓ This translation operation can be written in the abbreviated form

$$\mathbf{P}' = \mathbf{T}(t_x, t_y) \cdot \mathbf{P}$$

with $\mathbf{T}(tx, ty)$ as the 3 × 3 translation matrix

Two-Dimensional Rotation Matrix

- ✓ Two-dimensional rotation transformation equations about the coordinate origin can be expressed in the matrix form

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{R}(\theta) \cdot \mathbf{P}$$

- ✓ The rotation transformation operator $\mathbf{R}(\theta)$ is the 3 × 3 matrix with rotation parameter θ.

Two-Dimensional Scaling Matrix

- ✓ A scaling transformation relative to the coordinate origin can now be expressed as the matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{S}(s_x, s_y) \cdot \mathbf{P}$$

- ✓ The scaling operator S(sx, sy) is the 3 × 3 matrix with parameters sx and sy

2.2.3 Inverse Transformations

- ❖ For translation,we obtain the inverse matrix by negating the translation distances. Thus, if we have two-dimensional translation distances *tx* and *ty*, the inverse translation matrix is

$$\mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

- ❖ An inverse rotation is accomplished by replacing the rotation angle by its negative.
- ❖ A two-dimensional rotation through an angle θ about the coordinate origin has the inverse transformation matrix

$$\mathbf{R}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- ❖ We form the inverse matrix for any scaling transformation by replacing the scaling parameters with their reciprocals. the inverse transformation matrix is

$$\mathbf{S}^{-1} = \begin{bmatrix} \frac{1}{s_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.2.4 Two-Dimensional Composite Transformations

- ✓ Forming products of transformation matrices is often referred to as a **concatenation,** or **composition,** of matrices if we want to apply two transformations to point position **P**, the transformed location would be calculated as

$$\begin{aligned} \mathbf{P}' &= \mathbf{M}_2 \cdot \mathbf{M}_1 \cdot \mathbf{P} \\ &= \mathbf{M} \cdot \mathbf{P} \end{aligned}$$

- ✓ The coordinate position is transformed using the composite matrix**M**, rather than applying the individual transformations **M**1 and then**M**2.

<u>Composite Two-Dimensional Translations</u>

- ✓ If two successive translation vectors ($t1x$, $t1y$) and ($t2x$, $t2y$) are applied to a twodimensional coordinate position **P**, the final transformed location **P'** is calculated as

$$\begin{aligned} \mathbf{P}' &= \mathbf{T}(t_{2x}, t_{2y}) \cdot \{\mathbf{T}(t_{1x}, t_{1y}) \cdot \mathbf{P}\} \\ &= \{\mathbf{T}(t_{2x}, t_{2y}) \cdot \mathbf{T}(t_{1x}, t_{1y})\} \cdot \mathbf{P} \end{aligned}$$

where **P** and **P'** are represented as three-element, homogeneous-coordinate column vectors

- ✓ Also, the composite transformation matrix for this sequence of translations is

$$\begin{bmatrix} 1 & 0 & t_{2x} \\ 0 & 1 & t_{2y} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_{1x} \\ 0 & 1 & t_{1y} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{1x} + t_{2x} \\ 0 & 1 & t_{1y} + t_{2y} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{T}(t_{2x}, t_{2y}) \cdot \mathrm{T}(t_{1x}, t_{1y}) = \mathrm{T}(t_{1x} + t_{2x}, t_{1y} + t_{2y})$$

<u>Composite Two-Dimensional Rotations</u>

- ✓ Two successive rotations applied to a point **P** produce the transformed position

$$\begin{aligned} \mathrm{P}' &= \mathrm{R}(\theta_2) \cdot \{\mathrm{R}(\theta_1) \cdot \mathrm{P}\} \\ &= \{\mathrm{R}(\theta_2) \cdot \mathrm{R}(\theta_1)\} \cdot \mathrm{P} \end{aligned}$$

- ✓ By multiplying the two rotation matrices, we can verify that two successive rotations are additive:

$$\mathbf{R}(\theta 2) \cdot \mathbf{R}(\theta 1) = \mathbf{R}(\theta 1 + \theta 2)$$

- So that the final rotated coordinates of a point can be calculated with the composite rotation matrix as

$$\mathbf{P'} = \mathbf{R}(\theta 1 + \theta 2) \cdot \mathbf{P}$$

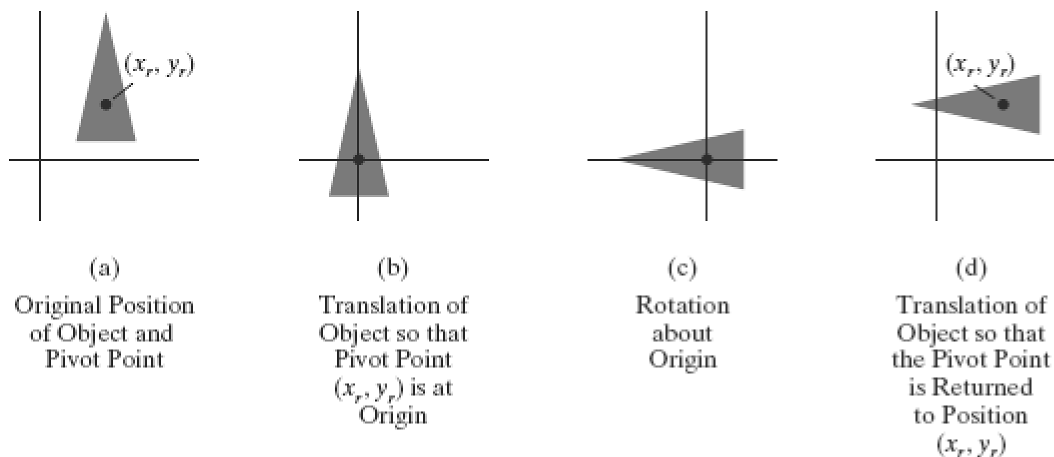
Composite Two-Dimensional Scalings

- Concatenating transformation matrices for two successive scaling operations in two dimensions produces the following composite scaling matrix

$$\begin{bmatrix} s_{2x} & 0 & 0 \\ 0 & s_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_{1x} & 0 & 0 \\ 0 & s_{1y} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{1x} \cdot s_{2x} & 0 & 0 \\ 0 & s_{1y} \cdot s_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{S}(s_{2x}, s_{2y}) \cdot \mathbf{S}(s_{1x}, s_{1y}) = \mathbf{S}(s_{1x} \cdot s_{2x}, \; s_{1y} \cdot s_{2y})$$

General Two-Dimensional Pivot-Point Rotation



(a) Original Position of Object and Pivot Point

(b) Translation of Object so that Pivot Point (x_r, y_r) is at Origin

(c) Rotation about Origin

(d) Translation of Object so that the Pivot Point is Returned to Position (x_r, y_r)

- We can generate a two-dimensional rotation about any other pivot point (*xr* , *yr*) by performing the following sequence of translate-rotate-translate operations:

  1. Translate the object so that the pivot-point position is moved to the coordinate origin.

  2. Rotate the object about the coordinate origin.

  3. Translate the object so that the pivot point is returned to its original position.

- The composite transformation matrix for this sequence is obtained with the concatenation

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

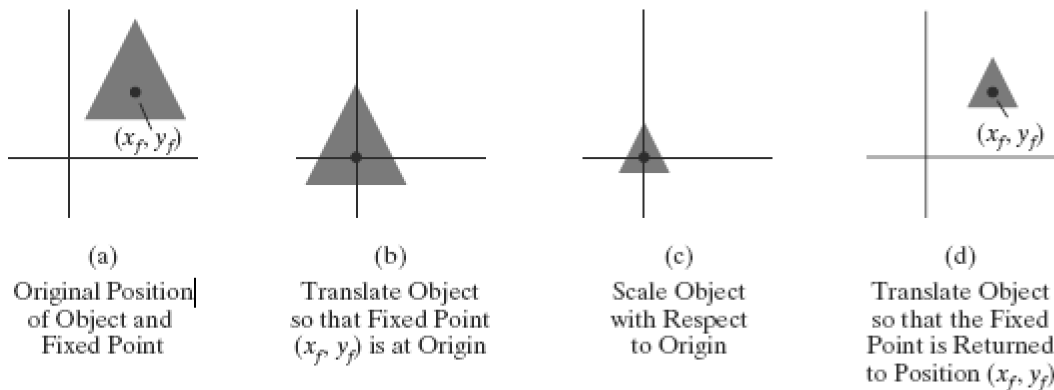
$$= \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta) + y_r \sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta) - x_r \sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

which can be expressed in the form

$$\mathbf{T}(x_r, y_r) \cdot \mathbf{R}(\theta) \cdot \mathbf{T}(-x_r, -y_r) = \mathbf{R}(x_r, y_r, \theta)$$

where $\mathbf{T}(-xr, -yr) = \mathbf{T}^{-1}(xr, yr)$.

General Two-Dimensional Fixed-Point Scaling



(a) Original Position of Object and Fixed Point (b) Translate Object so that Fixed Point (x_f, y_f) is at Origin (c) Scale Object with Respect to Origin (d) Translate Object so that the Fixed Point is Returned to Position (x_f, y_f)

- ✓ To produce a two-dimensional scaling with respect to a selected fixed position $(x f, yf)$, when we have a function that can scale relative to the coordinate origin only. This sequence is

  1. Translate the object so that the fixed point coincides with the coordinate origin.

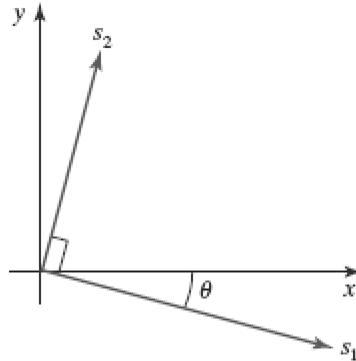
  2. Scale the object with respect to the coordinate origin.

  3. Use the inverse of the translation in step (1) to return the object to its original position.
- ✓ Concatenating the matrices for these three operations produces the required scaling matrix: $\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & x_f(1-s_x) \\ 0 & s_y & y_f(1-s_y) \\ 0 & 0 & 1 \end{bmatrix}$

$$\mathbf{T}(x_f, y_f) \cdot \mathbf{S}(s_x, s_y) \cdot \mathbf{T}(-x_f, -y_f) = \mathbf{S}(x_f, y_f, s_x, s_y)$$

General Two-Dimensional Scaling Directions

- ✓ Parameters sx and sy scale objects along the x and y directions.
- ✓ We can scale an object in other directions by rotating the object to align the desired scaling directions with the coordinate axes before applying the scaling transformation.
- ✓ Suppose we want to apply scaling factors with values specified by parameters $s1$ and $s2$ in the directions shown in Figure



- ✓ The composite matrix resulting from the product of these three transformations is

$$\mathbf{R}^{-1}(\theta) \cdot \mathbf{S}(s_1, s_2) \cdot \mathbf{R}(\theta) = \begin{bmatrix} s_1 \cos^2\theta + s_2 \sin^2\theta & (s_2 - s_1)\cos\theta\sin\theta & 0 \\ (s_2 - s_1)\cos\theta\sin\theta & s_1 \sin^2\theta + s_2\cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix Concatenation Properties

Property 1:

- ✓ Multiplication of matrices is associative.
- ✓ For any three matrices,**M**1,**M**2, and**M**3, the matrix product **M**3 · **M**2 · **M**1 can be performed by first multiplying **M**3 and **M**2 or by first multiplying**M**2 and **M**1:

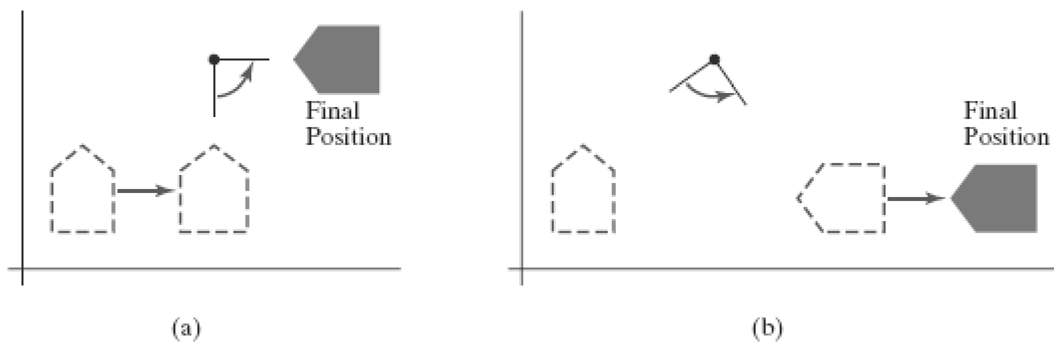
$$\mathbf{M}3 \cdot \mathbf{M}2 \cdot \mathbf{M}1 = (\mathbf{M}3 \cdot \mathbf{M}2) \cdot \mathbf{M}1 = \mathbf{M}3 \cdot (\mathbf{M}2 \cdot \mathbf{M}1)$$

- ✓ We can construct a composite matrix either by multiplying from left to right (premultiplying) or by multiplying from right to left (postmultiplying)

Property 2:

- ✓ Transformation products, on the other hand, may not be commutative. The matrix product**M**2 ·**M**1 is not equal to**M**1 ·**M**2, in general.

- ✓ This means that if we want to translate and rotate an object, we must be careful about the order in which the composite matrix is evaluated



- ✓ Reversing the order in which a sequence of transformations is performed may affect the transformed position of an object. In (a), an object is first translated in the x direction, then rotated counterclockwise through an angle of 45◦. In (b), the object is first rotated 45◦ counterclockwise, then translated in the x direction.

<u>General Two-Dimensional Composite Transformations and Computational Efficiency</u>

- ✓ A two-dimensional transformation, representing any combination of translations, rotations, and scalings, can be expressed as

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} rs_{xx} & rs_{xy} & trs_x \\ rs_{yx} & rs_{yy} & trs_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- ✓ The four elements *rsjk* are the multiplicative rotation-scaling terms in the transformation, which involve only rotation angles and scaling factors if an object is to be scaled and rotated about its centroid coordinates (*xc* , *yc*) and then translated, the values for the elements of the composite transformation matrix are

$$\mathbf{T}(t_x, t_y) \cdot \mathbf{R}(x_c, y_c, \theta) \cdot \mathbf{S}(x_c, y_c, s_x, s_y)$$
$$= \begin{bmatrix} s_x \cos\theta & -s_y \sin\theta & x_c(1 - s_x \cos\theta) + y_c s_y \sin\theta + t_x \\ s_x \sin\theta & s_y \cos\theta & y_c(1 - s_y \cos\theta) - x_c s_x \sin\theta + t_y \\ 0 & 0 & 1 \end{bmatrix}$$

- ✓ Although the above matrix requires nine multiplications and six additions, the explicit calculations for the transformed coordinates are

$$x' = x \cdot rs_{xx} + y \cdot rs_{xy} + trs_x, \qquad y' = x \cdot rs_{yx} + y \cdot rs_{yy} + trs_y$$

- ✓ We need actually perform only four multiplications and four additions to transform coordinate positions.
- ✓ Because rotation calculations require trigonometric evaluations and several multiplications for each transformed point, computational efficiency can become an important consideration in rotation transformations
- ✓ If we are rotating in small angular steps about the origin, for instance, we can set cos θ to 1.0 and reduce transformation calculations at each step to two multiplications and two additions for each set of coordinates to be rotated.
- ✓ These rotation calculations are

$$x' = x - y \sin\theta, \quad y' = x \sin\theta + y$$

Two-Dimensional Rigid-Body Transformation

- ➔ If a transformation matrix includes only translation and rotation parameters, it is a **rigid-body transformation matrix.**
- ➔ The general form for a two-dimensional rigid-body transformation matrix is

$$\begin{bmatrix} r_{xx} & r_{xy} & tr_x \\ r_{yx} & r_{yy} & tr_y \\ 0 & 0 & 1 \end{bmatrix}$$

where the four elements *r jk* are the multiplicative rotation terms, and the elements *trx* and *try* are the translational terms

- ➔ A rigid-body change in coordinate position is also sometimes referred to as a **rigid-motion** transformation.
- ➔ In addition, the above matrix has the property that its upper-left 2 × 2 submatrix is an *orthogonal matrix*.
- ➔ If we consider each row (or each column) of the submatrix as a vector, then the two row vectors (*rxx*, *rxy*) and (*ryx*, *ryy*) (or the two column vectors) form an orthogonal set of unit vectors.
- ➔ Such a set of vectors is also referred to as an *orthonormal* vector set. Each vector has unit length as follows

$$r_{xx}^2 + r_{xy}^2 = r_{yx}^2 + r_{yy}^2 = 1$$

and the vectors are perpendicular (their dot product is 0):

$$r_{xx}r_{yx} + r_{xy}r_{yy} = 0$$

➔ Therefore, if these unit vectors are transformed by the rotation submatrix, then the vector (*rxx*, *rxy*) is converted to a unit vector along the *x* axis and the vector (*ryx*, *ryy*) is transformed into a unit vector along the *y* axis of the coordinate system

$$\begin{bmatrix} r_{xx} & r_{xy} & 0 \\ r_{yx} & r_{yy} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{xx} \\ r_{xy} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} r_{xx} & r_{xy} & 0 \\ r_{yx} & r_{yy} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{yx} \\ r_{yy} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

➔ For example, the following rigid-body transformation first rotates an object through an angle θ about a pivot point (*xr* , *yr*) and then translates the object

$$\mathbf{T}(t_x, t_y) \cdot \mathbf{R}(x_r, y_r, \theta) = \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta) + y_r \sin\theta + t_x \\ \sin\theta & \cos\theta & y_r(1-\cos\theta) - x_r \sin\theta + t_y \\ 0 & 0 & 1 \end{bmatrix}$$

➔ Here, orthogonal unit vectors in the upper-left 2×2 submatrix are (cos θ, −sin θ) and (sin θ, cos θ).

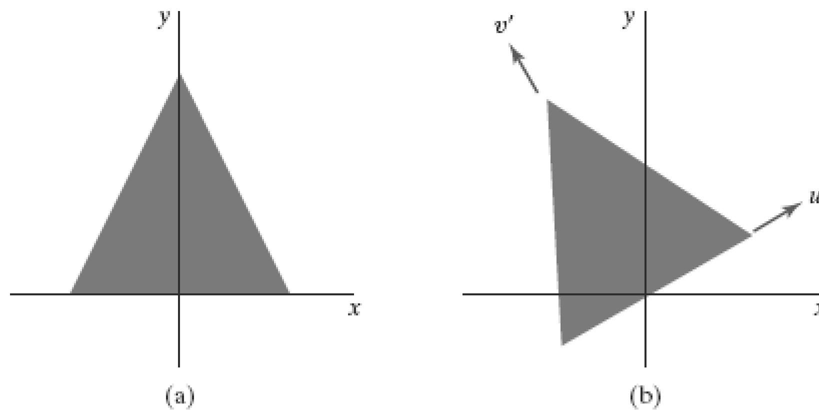
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta \\ -\sin\theta \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Constructing Two-Dimensional Rotation Matrices

✓ The orthogonal property of rotation matrices is useful for constructing the matrix when we know the final orientation of an object, rather than the amount of angular rotation necessary to put the object into that position.

✓ We might want to rotate an object to align its axis of symmetry with the viewing (camera) direction, or we might want to rotate one object so that it is above another object.

✓ Figure shows an object that is to be aligned with the unit direction vectors **u_** and **v**



The rotation matrix for revolving an object from position (a) to position (b) can be constructed with the values of the unit orientation vectors u' and v' relative to the original orientation.

2.2.5 Other Two-Dimensional Transformations

Two such transformations

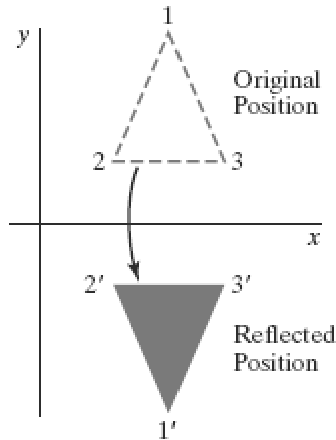
1. Reflection and
2. Shear.

Reflection

- ✓ A transformation that produces a mirror image of an object is called a **reflection.**
- ✓ For a two-dimensional reflection, this image is generated relative to an **axis of reflection** by rotating the object 180◦ about the reflection axis**.**
- ✓ Reflection about the line $y = 0$ (the x axis) is accomplished with the transformation Matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

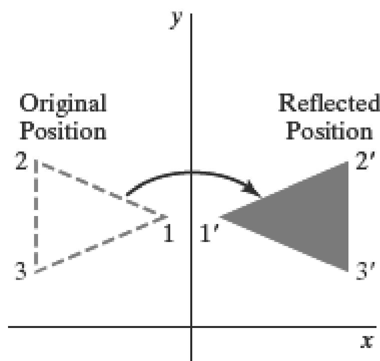
- ✓ This transformation retains x values, but "flips" the y values of coordinate positions.
- ✓ The resulting orientation of an object after it has been reflected about the x axis is shown in Figure



- ✓ A reflection about the line $x = 0$ (the y axis) flips x coordinates while keeping y coordinates the same. The matrix for this transformation is

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

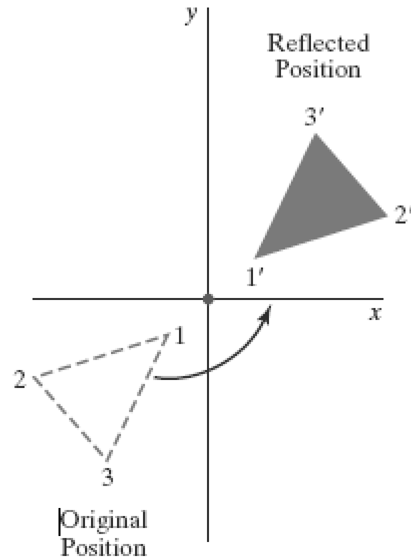
- ✓ Figure below illustrates the change in position of an object that has been reflected about the line $x = 0$.



- ✓ We flip both the x and y coordinates of a point by reflecting relative to an axis that is perpendicular to the xy plane and that passes through the coordinate origin the matrix representation for this reflection is

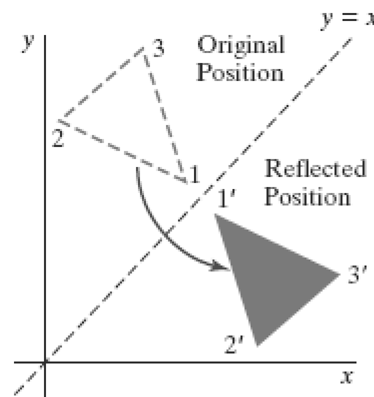
$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- ✓ An example of reflection about the origin is shown in Figure



- If we choose the reflection axis as the diagonal line $y = x$ (Figure below), the reflection matrix is

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- To obtain a transformation matrix for reflection about the diagonal $y = -x$, we could concatenate matrices for the transformation sequence:

  (1) clockwise rotation by 45◦,

  (2) reflection about the y axis, and

  (3) counterclockwise rotation by 45◦.

  The resulting transformation matrix is

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Shear

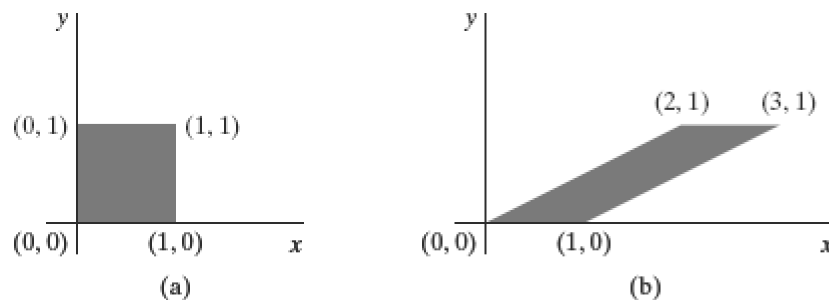
- ✓ A transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other is called a **shear.**
- ✓ Two common shearing transformations are those that shift coordinate *x* values and those that shift *y* values. An *x*-direction shear relative to the *x* axis is produced with the transformation Matrix

$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which transforms coordinate positions as

$$x' = x + sh_x \cdot y, \qquad y' = y$$

- ✓ Any real number can be assigned to the shear parameter *shx* Setting parameter *shx* to the value 2, for example, changes the square into a parallelogram is shown below. Negative values for *shx* shift coordinate positions to the left.



A unit square (a) is converted to a parallelogram (b) using the x -direction shear with shx = 2.

- ✓ We can generate *x*-direction shears relative to other reference lines with

$$\begin{bmatrix} 1 & sh_x & -sh_x \cdot y_{\text{ref}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, coordinate positions are transformed as

$$x' = x + sh_x(y - y_{\text{ref}}), \qquad y' = y$$

- A y-direction shear relative to the line $x = x$ref is generated with the transformation Matrix

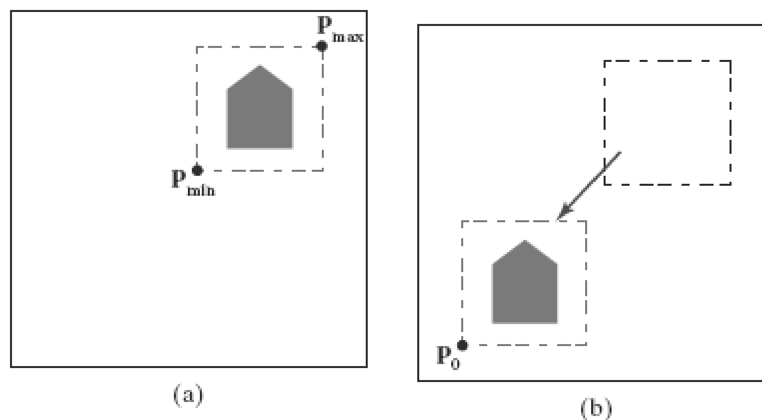
$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & -sh_y \cdot x_{\text{ref}} \\ 0 & 0 & 1 \end{bmatrix}$$

which generates the transformed coordinate values

$$x' = x, \qquad y' = y + sh_y(x - x_{\text{ref}})$$

2.2.6 Raster Methods for Geometric Transformations

- Raster systems store picture information as color patterns in the frame buffer.
- Therefore, some simple object transformations can be carried out rapidly by manipulating an array of pixel values
- Few arithmetic operations are needed, so the pixel transformations are particularly efficient.
- Functions that manipulate rectangular pixel arrays are called *raster operations* and moving a block of pixel values from one position to another is termed a *block transfer,* a *bitblt,* or a *pixblt*.
- Figure below illustrates a two-dimensional translation implemented as a block transfer of a refresh-buffer area



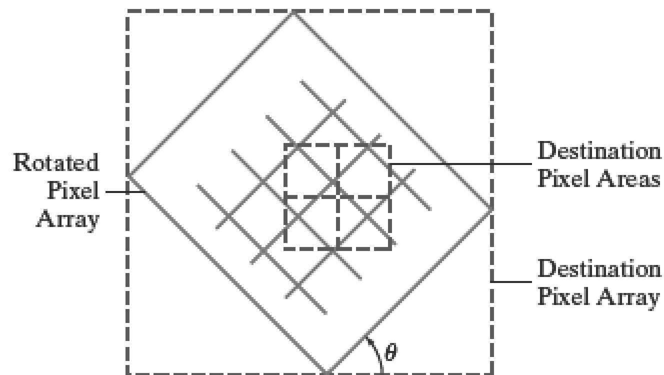
Translating an object from screen position (a) to the destination position shown in (b) by moving a rectangular block of pixel values. Coordinate positions Pmin and Pmax specify the limits of the rectangular block to be moved, and P0 is the destination reference position.

- Rotations in 90-degree increments are accomplished easily by rearranging the elements of a pixel array.
- We can rotate a two-dimensional object or pattern 90◦ counterclockwise by reversing the pixel values in each row of the array, then interchanging rows and columns.
- A 180◦ rotation is obtained by reversing the order of the elements in each row of the array, then reversing the order of the rows.
- Figure below demonstrates the array manipulations that can be used to rotate a pixel block by 90◦ and by 180◦.

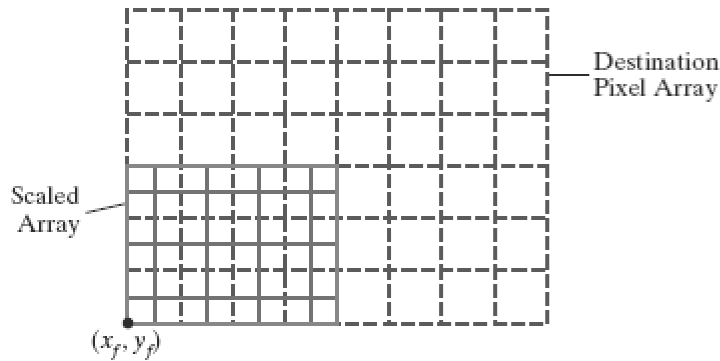
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix} \qquad \begin{bmatrix} 3 & 6 & 9 & 12 \\ 2 & 5 & 8 & 11 \\ 1 & 4 & 7 & 10 \end{bmatrix} \qquad \begin{bmatrix} 12 & 11 & 10 \\ 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

(a) (b) (c)

- For array rotations that are not multiples of 90◦, we need to do some extra processing.
- The general procedure is illustrated in Figure below.



- Each destination pixel area is mapped onto the rotated array and the amount of overlap with the rotated pixel areas is calculated.
- A color for a destination pixel can then be computed by averaging the colors of the overlapped source pixels, weighted by their percentage of area overlap.
- Pixel areas in the original block are scaled, using specified values for *sx* and *sy*, and then mapped onto a set of destination pixels.
- The color of each destination pixel is then assigned according to its area of overlap with the scaled pixel areas



2.2.7 OpenGL Raster Transformations

- A translation of a rectangular array of pixel-color values from one buffer area to another can be accomplished in OpenGL as the following copy operation:

  glCopyPixels (xmin, ymin, width, height, GL_COLOR);

- The first four parameters in this function give the location and dimensions of the pixel block; and the OpenGL symbolic constant **GL_COLOR** specifies that it is color values are to be copied.

- A block of RGB color values in a buffer can be saved in an array with the function

glReadPixels (xmin, ymin, width, height, GL_RGB, GL_UNSIGNED_BYTE, colorArray);

- If color-table indices are stored at the pixel positions, we replace the constant GL RGB with GL_COLOR_INDEX.

- To rotate the color values, we rearrange the rows and columns of the color array, as described in the previous section. Then we put the rotated array back in the buffer with **glDrawPixels (width, height, GL_RGB, GL_UNSIGNED_BYTE, colorArray);**

- A two-dimensional scaling transformation can be performed as a raster operation in OpenGL by specifying scaling factors and then invoking either **glCopyPixels** or **glDrawPixels**.
- For the raster operations, we set the scaling factors with **glPixelZoom (sx, sy);**

- We can also combine raster transformations with logical operations to produce various effects with the *exclusive or* operator

2.2.8 OpenGL Functions for Two-Dimensional Geometric Transformations

- ✓ To perform a translation, we invoke the translation routine and set the components for the three-dimensional translation vector.
- ✓ In the rotation function, we specify the angle and the orientation for a rotation axis that intersects the coordinate origin.
- ✓ In addition, a scaling function is used to set the three coordinate scaling factors relative to the coordinate origin. In each case, the transformation routine sets up a 4 × 4 matrix that is applied to the coordinates of objects that are referenced after the transformation call

Basic OpenGL Geometric Transformations

➔ A 4× 4 translation matrix is constructed with the following routine:

glTranslate* (tx, ty, tz);

- ✓ Translation parameters **tx**, **ty**, and **tz** can be assigned any real-number values, and the single suffix code to be affixed to this function is either **f** (float) or **d** (double).
- ✓ For two-dimensional applications, we set **tz** = 0.0; and a two-dimensional position is represented as a four-element column matrix with the *z* component equal to 0.0.
- ✓ example: **glTranslatef (25.0, -10.0, 0.0);**

➔ Similarly, a 4 × 4 rotation matrix is generated with

glRotate* (theta, vx, vy, vz);

- ✓ where the vector **v** = (**vx**, **vy**, **vz**) can have any floating-point values for its components.
- ✓ This vector defines the orientation for a rotation axis that passes through the coordinate origin.
- ✓ If **v** is not specified as a unit vector, then it is normalized automatically before the elements of the rotation matrix are computed.

- ✓ The suffix code can be either **f** or **d**, and parameter **theta** is to be assigned a rotation angle in degree.
- ✓ For example, the statement: **glRotatef (90.0, 0.0, 0.0, 1.0);**

➔ We obtain a 4 × 4 scaling matrix with respect to the coordinate origin with the following routine:

glScale* (sx, sy, sz);

- ✓ The suffix code is again either **f** or **d**, and the scaling parameters can be assigned any real-number values.
- ✓ Scaling in a two-dimensional system involves changes in the *x* and *y* dimensions, so a typical two-dimensional scaling operation has a *z* scaling factor of 1.0
- ✓ **Example: glScalef (2.0, -3.0, 1.0);**

<u>OpenGL Matrix Operations</u>

- ✓ The glMatrixMode routine is used to set the *projection mode which designates the matrix that is to be used for the projection transformation.*
- ✓ We specify the *modelview mode* with the statement

  glMatrixMode (GL_MODELVIEW);

  - which designates the 4×4 modelview matrix as the **current matrix**
  - Two other modes that we can set with the **glMatrixMode** function are the *texture mode* and the *color mode*.
  - The texture matrix is used for mapping texture patterns to surfaces, and the color matrix is used to convert from one color model to another.
  - The default argument for the **glMatrixMode** function is **GL_MODELVIEW**.
- ✓ With the following function, we assign the identity matrix to the current matrix:

  glLoadIdentity ( );
- ✓ Alternatively, we can assign other values to the elements of the current matrix using

  glLoadMatrix* (elements16);
- ✓ A single-subscripted, 16-element array of floating-point values is specified with parameter **elements16**, and a suffix code of either **f** or **d** is used to designate the data type
- ✓ The elements in this array must be specified in *column-major* order
- ✓ To illustrate this ordering, we initialize the modelview matrix with the following code:

```
glMatrixMode (GL_MODELVIEW);
GLfloat elems [16];
GLint k;
for (k = 0; k < 16; k++)
        elems [k] = float (k);
glLoadMatrixf (elems);
```

Which produces the matrix

$$\mathbf{M} = \begin{bmatrix} 0.0 & 4.0 & 8.0 & 12.0 \\ 1.0 & 5.0 & 9.0 & 13.0 \\ 2.0 & 6.0 & 10.0 & 14.0 \\ 3.0 & 7.0 & 11.0 & 15.0 \end{bmatrix}$$

- ✓ We can also concatenate a specified matrix with the current matrix as follows:

  glMultMatrix* (otherElements16);

- ✓ Again, the suffix code is either **f** or **d**, and parameter **otherElements16** is a 16-element, single-subscripted array that lists the elements of some other matrix in column-major order.
- ✓ Thus, assuming that the current matrix is the modelview matrix, which we designate as **M**, then the updated modelview matrix is computed as

  $\mathbf{M} = \mathbf{M} \cdot \mathbf{M}'$

- ✓ The **glMultMatrix** function can also be used to set up any transformation sequence with individually defined matrices.
- ✓ For example,

```
glMatrixMode (GL_MODELVIEW);
glLoadIdentity ( ); // Set current matrix to the identity.
glMultMatrixf (elemsM2); // Postmultiply identity with matrix M2.
glMultMatrixf (elemsM1); // Postmultiply M2 with matrix M1.
```

produces the following current modelview matrix:

$\mathbf{M} = \mathbf{M}2 \cdot \mathbf{M}1$

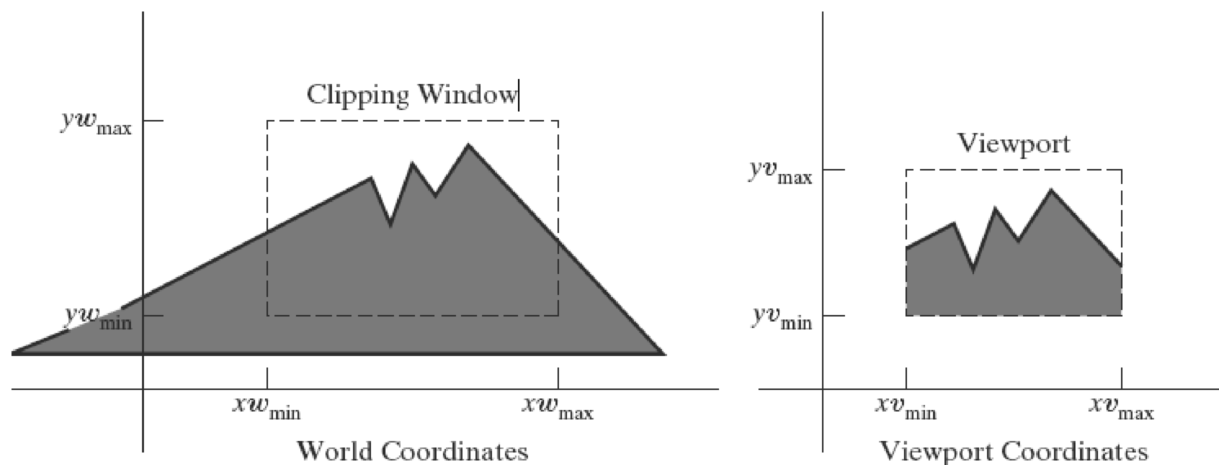
2.3 Two Dimensional Viewing

2.3.1 2D viewing pipeline

2.3.1 OpenGL 2D viewing functions.

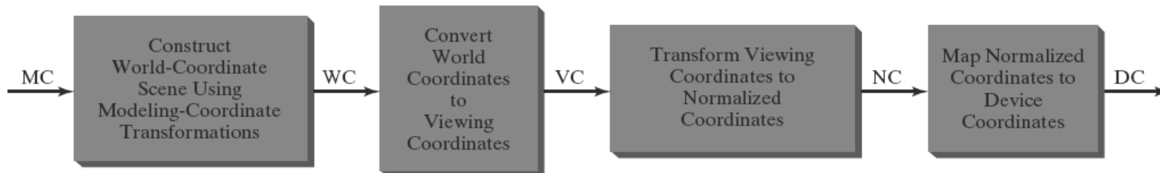
2.3.1 The Two-Dimensional Viewing Pipeline

- A section of a two-dimensional scene that is selected for display is called a clipping Window.
- Sometimes the clipping window is alluded to as the *world window* or the *viewing window*
- Graphics packages allow us also to control the placement within the display window using another "window" called the **viewport**.
- The clipping window selects *what* we want to see; the viewport indicates *where* it is to be viewed on the output device.
- By changing the position of a viewport, we can view objects at different positions on the display area of an output device
- Usually, clipping windows and viewports are rectangles in standard position, with the rectangle edges parallel to the coordinate axes.
- We first consider only rectangular viewports and clipping windows, as illustrated in Figure



Viewing Pipeline

- The mapping of a two-dimensional, world-coordinate scene description to device coordinates is called a **two-dimensional viewing transformation**.
- This transformation is simply referred to as the *window-to-viewport transformation* or the *windowing transformation*
- We can describe the steps for two-dimensional viewing as indicated in Figure



- Once aworld-coordinate scene has been constructed,wecould set up a separate two-dimensional, **viewingcoordinate reference frame** for specifying the clipping window.
- To make the viewing process independent of the requirements of any output device, graphics systems convert object descriptions to normalized coordinates and apply the clipping routines.
- Systems use normalized coordinates in the range from 0 to 1, and others use a normalized range from −1 to 1.
- At the final step of the viewing transformation, the contents of the viewport are transferred to positions within the display window.
- Clipping is usually performed in normalized coordinates.
- This allows us to reduce computations by first concatenating the various transformation matrices

2.3.2 OpenGL Two-Dimensional Viewing Functions

- The GLU library provides a function for specifying a two-dimensional clipping window, and we have GLUT library functions for handling display windows.

OpenGL Projection Mode

- ✓ Before we select a clipping window and a viewport in OpenGL, we need to establish the appropriate mode for constructing the matrix to transform from world coordinates to screen coordinates.

- ✓ We must set the parameters for the clipping window as part of the projection transformation.
- ✓ Function:

  glMatrixMode (GL_PROJECTION);
- ✓ We can also set the initialization as

  glLoadIdentity ( );

  This ensures that each time we enter the projection mode, the matrix will be reset to the identity matrix so that the new viewing parameters are not combined with the previous ones

GLU Clipping-Window Function

- ✓ To define a two-dimensional clipping window, we can use the GLU function:

  gluOrtho2D (xwmin, xwmax, ywmin, ywmax);
- ✓ This function specifies an orthogonal projection for mapping the scene to the screen the orthogonal projection has no effect on our two-dimensional scene other than to convert object positions to normalized coordinates.
- ✓ Normalized coordinates in the range from −1 to 1 are used in the OpenGL clipping routines.
- ✓ Objects outside the normalized square (and outside the clipping window) are eliminated from the scene to be displayed.
- ✓ If we do not specify a clipping window in an application program, the default coordinates are $(xw\text{min}, yw\text{min}) = (-1.0, -1.0)$ and $(xw\text{max}, yw\text{max}) = (1.0, 1.0)$.
- ✓ Thus the default clipping window is the normalized square centered on the coordinate origin with a side length of 2.0.

OpenGL Viewport Function

- ✓ We specify the viewport parameters with the OpenGL function

  glViewport (xvmin, yvmin, vpWidth, vpHeight);

  Where,

  ➔ **xvmin** and **yvmin** specify the position of the lowerleft corner of the viewport relative to the lower-left corner of the display window,

➔ **vpWidth** and **vpHeight** are pixel width and height of the viewport

- ✓ Coordinates for the upper-right corner of the viewport are calculated for this transformation matrix in terms of the viewport width and height:

$$xv_{max} = xv_{min} + vpWidth, \qquad yv_{max} = yv_{min} + vpHeight$$

- ✓ Multiple viewports can be created in OpenGL for a variety of applications.
- ✓ We can obtain the parameters for the currently active viewport using the query function

glGetIntegerv (GL_VIEWPORT, vpArray);

where,

➔ **vpArray** is a single-subscript, four-element array.

Creating a GLUT Display Window

- ✓ The GLUT library interfaces with any window-management system, we use the GLUT routines for creating and manipulating display windows so that our example programs will be independent of any specific machine.
- ✓ We first need to initialize GLUT with the following function:

glutInit (&argc, argv);

- ✓ We have three functions inGLUTfor defining a display window and choosing its dimensions and position:

1. **glutInitWindowPosition (xTopLeft, yTopLeft);**

➔ gives the integer, screen-coordinate position for the top-left corner of the display window, relative to the top-left corner of the screen

2. **glutInitWindowSize (dwWidth, dwHeight);**

➔ we choose a width and height for the display window in positive integer pixel dimensions.

➔ If we do not use these two functions to specify a size and position, the default size is 300 by 300 and the default position is (−1, −1), which leaves the positioning of the display window to the window-management system

3. **glutCreateWindow ("Title of Display Window");**
   ➔ creates the display window, with the specified size and position, and assigns a title, although the use of the title also depends on the windowing system

Setting the GLUT Display-Window Mode and Color

- ✓ Various display-window parameters are selected with the GLUT function

1. **glutInitDisplayMode (mode);**
   ➔ We use this function to choose a color mode (RGB or index) and different buffer combinations, and the selected parameters are combined with the logical **or** operation.

2. **glutInitDisplayMode (GLUT_SINGLE | GLUT_RGB);**
   ➔ The color mode specification **GLUT_RGB** is equivalent to **GLUT_RGBA.**

3. **glClearColor (red, green, blue, alpha);**
   ➔ A background color for the display window is chosen in RGB mode with the OpenGL routine

4. **glClearIndex (index);**
   ➔ This function sets the display window color using color-index mode,
   ➔ Where parameter **index** is assigned an integer value corresponding to a position within the color table.

GLUT Display-Window Identifier

- ✓ Multiple display windows can be created for an application, and each is assigned a positive-integer **display-window identifier,** starting with the value 1 for the first window that is created.
- ✓ Function:

windowID = glutCreateWindow ("A Display Window");

Deleting a GLUT Display Window

- ✓ If we know the display window's identifier, we can eliminate it with the statement

  glutDestroyWindow (windowID);

Current GLUT Display Window

- ✓ When we specify any display-window operation, it is applied to the **current display window,** which is either the last display window that we created or the one**.**
- ✓ we select with the following command

  glutSetWindow (windowID);

- ✓ We can query the system to determine which window is the current display window:

  currentWindowID = glutGetWindow ( );

  ➔ A value of 0 is returned by this function if there are no display windows or if the current display window was destroyed

Relocating and Resizing a GLUT Display Window

- ✓ We can reset the screen location for the current display window with the function

  glutPositionWindow (xNewTopLeft, yNewTopLeft);

- ✓ Similarly, the following function resets the size of the current display window:

  glutReshapeWindow (dwNewWidth, dwNewHeight);

- ✓ With the following command, we can expand the current display window to fill the screen:

  glutFullScreen ( );

- ✓ Whenever the size of a display window is changed, its aspect ratio may change and objects may be distorted from their original shapes. We can adjust for a change in display-window dimensions using the statement

  glutReshapeFunc (winReshapeFcn);

Managing Multiple GLUT Display Windows

- ✓ The GLUT library also has a number of routines for manipulating a display window in various ways.

- ✓ We use the following routine to convert the current display window to an icon in the form of a small picture or symbol representing the window:

  glutIconifyWindow ( );

- ✓ The label on this icon will be the same name that we assigned to the window, but we can change this with the following command:

  glutSetIconTitle ("Icon Name");

- ✓ We also can change the name of the display window with a similar command:

  glutSetWindowTitle ("New Window Name");

- ✓ We can choose any display window to be in front of all other windows by first designating it as the current window, and then issuing the "pop-window" command:

  glutSetWindow (windowID);

  glutPopWindow ( );

- ✓ In a similar way, we can "push" the current display window to the back so that it is behind all other display windows. This sequence of operations is

  glutSetWindow (windowID);

  glutPushWindow ( );

- ✓ We can also take the current window off the screen with

  glutHideWindow ( );

- ✓ In addition, we can return a "hidden" display window, or one that has been converted to an icon, by designating it as the current display window and then invoking the function

  glutShowWindow ( );

GLUT Subwindows

- ✓ Within a selected display window, we can set up any number of second-level display windows, which are called *subwindows.*
- ✓ We create a subwindow with the following function:

  glutCreateSubWindow (windowID, xBottomLeft, yBottomLeft, width, height);

- ✓ Parameter **windowID** identifies the display window in which we want to set up the subwindow.

- ✓ Subwindows are assigned a positive integer identifier in the same way that first-level display windows are numbered, and we can place a subwindow inside another subwindow.
- ✓ Each subwindow can be assigned an individual display mode and other parameters. We can even reshape, reposition, push, pop, hide, and show subwindows

Selecting a Display-Window Screen-Cursor Shape

- ✓ We can use the following GLUT routine to request a shape for the screen cursor that is to be used with the current window:

  glutSetCursor (shape);

  where, shape can be

  ➔ **GLUT_CURSOR_UP_DOWN :** an up-down arrow.
  ➔ **GLUT_CURSOR_CYCLE:** A rotating arrow is chosen
  ➔ **GLUT_CURSOR_WAIT:** a wristwatch shape.
  ➔ **GLUT_CURSOR_DESTROY:** a skull and crossbones

Viewing Graphics Objects in a GLUT Display Window

- ✓ After we have created a display window and selected its position, size, color, and other characteristics, we indicate what is to be shown in that window
- ✓ Then we invoke the following function to assign something to that window:

  glutDisplayFunc (pictureDescrip);
- ✓ This routine, called **pictureDescrip** for this example, is referred to as a *callback function* because it is the routine that is to be executed whenever GLUT determines that the display-window contents should be renewed.
- ✓ We may need to call **glutDisplayFunc** after the **glutPopWindow** command if the display window has been damaged during the process of redisplaying the windows.
- ✓ In this case, the following function is used to indicate that the contents of the current display window should be renewed:

  glutPostRedisplay ( );

<u>Executing the Application Program</u>

- ✓ When the program setup is complete and the display windows have been created and initialized, we need to issue the final GLUT command that signals execution of the program:

  glutMainLoop ( );

<u>Other GLUT Functions</u>

- ✓ Sometimes it is convenient to designate a function that is to be executed when there are no other events for the system to process. We can do that with

  glutIdleFunc (function);

- ✓ Finally, we can use the following function to query the system about some of the current state parameters:

  glutGet (stateParam);

- ✓ This function returns an integer value corresponding to the symbolic constant we select for its argument.
- ✓ For example, for the stateParam we can have the values
  - ➔ **GLUT_WINDOW_X**: obtains the *x*-coordinate position for the top-left corner of the current display window
  - ➔ **GLUT_WINDOW_WIDTH** or **GLUT_SCREEN_WIDTH** : retrieve the current display-window width or the screen width with**.**

Acknowledgements to

Donald Hearn & Pauline Baker: Computer Graphics with OpenGL

Version,3rd / 4th Edition, Pearson Education,2011

Edward Angel: Interactive Computer Graphics- A Top Down approach

with OpenGL, 5th edition. Pearson Education, 2008

M M Raiker, Computer Graphics using OpenGL, Filip learning/Elsevier

3.1 Clipping:

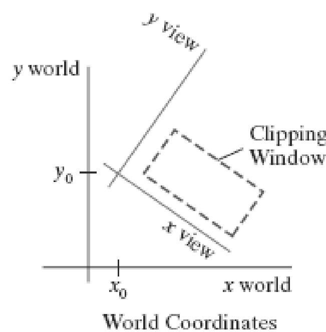
3.1.1Clipping window,
3.1.2 Normalization and Viewport transformations,
3.1.3 Clipping algorithms:
- 2D point clipping,
- 2D line clipping algorithms: cohen-sutherland line clipping.
- Polygon fill area clipping: Sutherland Hodgeman polygon clipping algorithm.

3.1.1 The Clipping Window

- We can design our own clipping window with any shape, size, and orientation we choose.
- But clipping a scene using a concave polygon or a clipping window with nonlinear boundaries requires more processing than clipping against a rectangle.
- Rectangular clipping windows in standard position are easily defined by giving the coordinates of two opposite corners of each rectangle

Viewing-Coordinate Clipping Window

✓ A general approach to the two-dimensional viewing transformation is to set up a *viewing-coordinate system* within the world-coordinate frame



✓ We choose an origin for a two-dimensional viewing-coordinate frame at some world position $\mathbf{P}_0 = (x_0, y_0)$, and we can establish the orientation using a world vector **V** that defines the *y*view direction.

✓ Vector **V** is called the two-dimensional **view up vector.**

- ✓ An alternative method for specifying the orientation of the viewing frame is to give a rotation angle relative to either the *x* or *y* axis in the world frame.
- ✓ The first step in the transformation sequence is to translate the viewing origin to the world origin.
- ✓ Next, we rotate the viewing system to align it with the world frame.
- ✓ Given the orientation vector **V**, we can calculate the components of unit vectors $\mathbf{v} = (v_x, v_y)$ and $\mathbf{u} = (u_x, u_y)$ for the *y*view and *x*view axes, respectively.

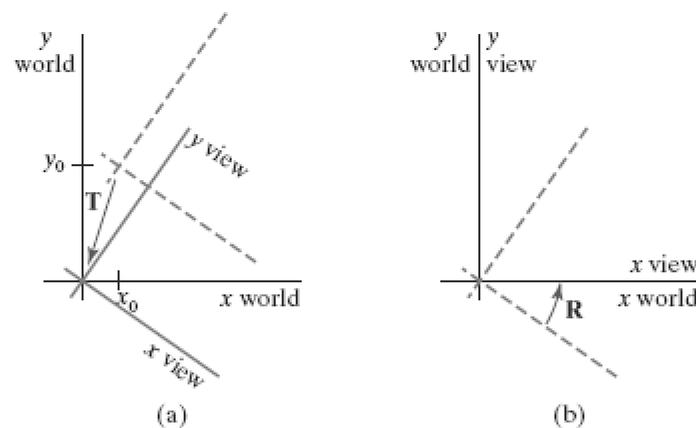
$$\mathbf{M}_{WC,VC} = \mathbf{R} \cdot \mathbf{T}$$

Where,

T is the translation matrix,

R is the rotation matrix

- ✓ A viewing-coordinate frame is moved into coincidence with the world frame is shown in below figure

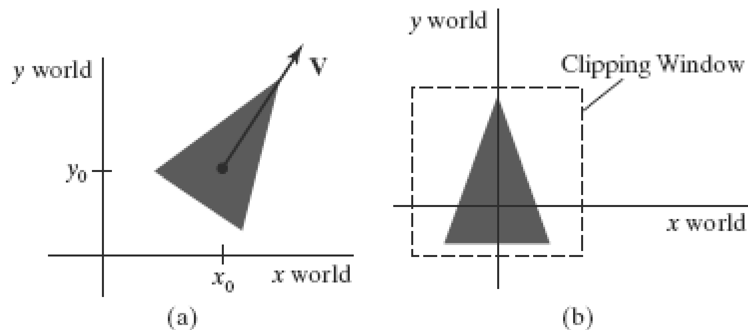


(a) applying a translation matrix **T** to move the viewing origin to the world origin, then (b) applying a rotation matrix **R** to align the axes of the two systems.

World-Coordinate Clipping Window

- ➢ A routine for defining a standard, rectangular clipping window in world coordinates is typically provided in a graphics-programming library.
- ➢ We simply specify two world-coordinate positions, which are then assigned to the two opposite corners of a standard rectangle.

- Once the clipping window has been established, the scene description is processed through the viewing routines to the output device.
- Thus, we simply rotate (and possibly translate) objects to a desired position and set up the clipping window all in world coordinates.



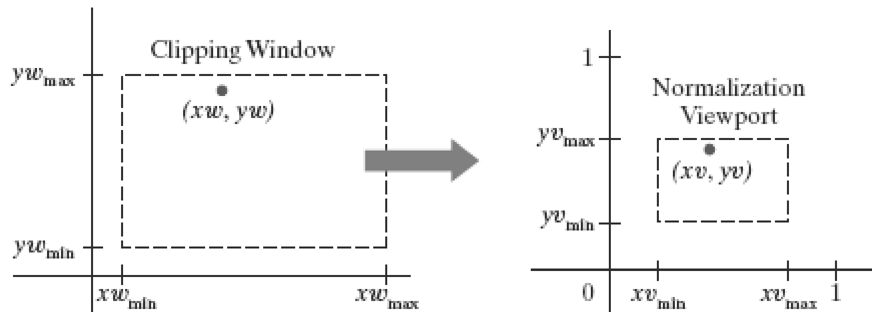
A triangle
(a), with a selected reference point and orientation vector, is translated and rotated to position
(b) within a clipping window.

3.1.2 Normalization and Viewport Transformations

- The viewport coordinates are often given in the range from 0 to 1 so that the viewport is positioned within a unit square.
- After clipping, the unit square containing the viewport is mapped to the output display device

Mapping the Clipping Window into a Normalized Viewport

- ✓ We first consider a viewport defined with normalized coordinate values between 0 and 1.
- ✓ Object descriptions are transferred to this normalized space using a transformation that maintains the same relative placement of a point in the viewport as it had in the clipping window Position (x_w, y_w) in the clipping window is mapped to position (x_v, y_v) in the associated viewport.



- ✓ To transform the world-coordinate point into the same relative position within the viewport, we require that

$$\frac{xv - xv_{min}}{xv_{max} - xv_{min}} = \frac{xw - xw_{min}}{xw_{max} - xw_{min}}$$
$$\frac{yv - yv_{min}}{yv_{max} - yv_{min}} = \frac{yw - yw_{min}}{yw_{max} - yw_{min}}$$

- ✓ Solving these expressions for the viewport position (x_v, y_v), we have

$x_v = s_x x_w + t_x$

$y_v = s_y y_w + t_y$

Where the scaling factors are

$$s_x = \frac{xv_{max} - xv_{min}}{xw_{max} - xw_{min}}$$
$$s_y = \frac{yv_{max} - yv_{min}}{yw_{max} - yw_{min}}$$

and the translation factors are

$$t_x = \frac{xw_{max}xv_{min} - xw_{min}xv_{max}}{xw_{max} - xw_{min}}$$
$$t_y = \frac{yw_{max}yv_{min} - yw_{min}yv_{max}}{yw_{max} - yw_{min}}$$

- ✓ We could obtain the transformation from world coordinates to viewport coordinates with the following sequence:

1. Scale the clipping window to the size of the viewport using a fixed-point position of (xw_{min}, yw_{min}).

2. Translate (xw_{min}, yw_{min}) to (xv_{min}, yv_{min}).

- ✓ The scaling transformation in step (1) can be represented with the two dimensional Matrix

$$\mathbf{S} = \begin{bmatrix} s_x & 0 & xw_{\min}(1 - s_x) \\ 0 & s_y & yw_{\min}(1 - s_y) \\ 0 & 0 & 1 \end{bmatrix}$$

- ✓ The two-dimensional matrix representation for the translation of the lower-left corner of the clipping window to the lower-left viewport corner is

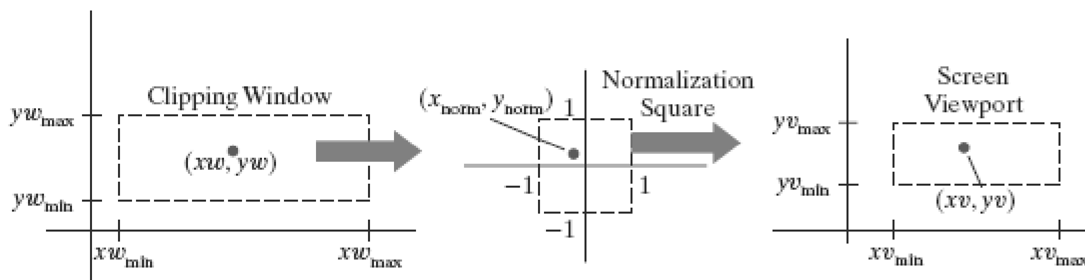
$$\mathbf{T} = \begin{bmatrix} 1 & 0 & xv_{\min} - xw_{\min} \\ 0 & 1 & yv_{\min} - yw_{\min} \\ 0 & 0 & 1 \end{bmatrix}$$

- ✓ And the composite matrix representation for the transformation to the normalized viewport is

$$\mathbf{M}_{\text{window, normviewp}} = \mathbf{T} \cdot \mathbf{S} = \begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Mapping the Clipping Window into a Normalized Square

- ✓ Another approach to two-dimensional viewing is to transform the clipping window into a normalized square, clip in normalized coordinates, and then transfer the scene description to a viewport specified in screen coordinates.
- ✓ This transformation is illustrated in Figure below with normalized coordinates in the range from −1 to 1



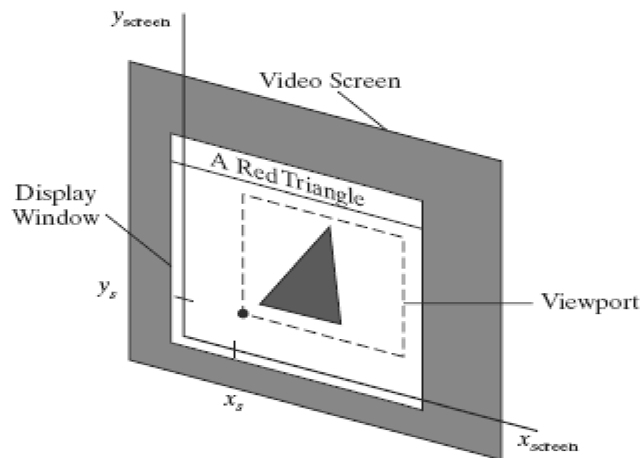
- ✓ The matrix for the normalization transformation is obtained by substituting −1 for $xv_{\min}$ and $yv_{\min}$ and substituting +1 for $xv_{\max}$ and $yv_{\max}$.

$$\mathbf{M}_{\text{window, normsquare}} = \begin{bmatrix} \frac{2}{xw_{\max} - xw_{\min}} & 0 & -\frac{xw_{\max} + xw_{\min}}{xw_{\max} - xw_{\min}} \\ 0 & \frac{2}{yw_{\max} - yw_{\min}} & -\frac{yw_{\max} + yw_{\min}}{yw_{\max} - yw_{\min}} \\ 0 & 0 & 1 \end{bmatrix}$$

- ✓ Similarly, after the clipping algorithms have been applied, the normalized square with edge length equal to 2 is transformed into a specified viewport.
- ✓ This time, we get the transformation matrix by substituting −1 for $xw_{\min}$ and $yw_{\min}$ and substituting +1 for $xw_{\max}$ and $yw_{\max}$

$$\mathbf{M}_{\text{normsquare, viewport}} = \begin{bmatrix} \frac{xv_{\max} - xv_{\min}}{2} & 0 & \frac{xv_{\max} + xv_{\min}}{2} \\ 0 & \frac{yv_{\max} - yv_{\min}}{2} & \frac{yv_{\max} + yv_{\min}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

- ✓ Typically, the lower-left corner of the viewport is placed at a coordinate position specified relative to the lower-left corner of the display window. Figure below demonstrates the positioning of a viewport within a display window.



Display of Character Strings

- ✓ Character strings can be handled in one of two ways when they are mapped through the viewing pipeline to a viewport.
- ✓ The simplest mapping maintains a constant character size.

- ✓ This method could be employed with bitmap character patterns.
- ✓ But outline fonts could be transformed the same as other primitives; we just need to transform the defining positions for the line segments in the outline character shape

Split-Screen Effects and Multiple Output Devices

- ✓ By selecting different clipping windows and associated viewports for a scene, we can provide simultaneous display of two or more objects, multiple picture parts, or different views of a single scene.
- ✓ It is also possible that two or more output devices could be operating concurrently on a particular system, and we can set up a clipping-window/viewport pair for each output device.
- ✓ A mapping to a selected output device is sometimes referred to as a **workstation transformation**

3.1.3 Clipping Algorithms

- ➔ Any procedure that eliminates those portions of a picture that are either inside or outside a specified region of space is referred to as a **clipping algorithm** or simply **clipping.**
- ➔ The most common application of clipping is in the viewing pipeline, where clipping is applied to extract a designated portion of a scene (either two-dimensional or three-dimensional) for display on an output device.
- ➔ Different objects clipping are

1. Point clipping
2. Line clipping (straight-line segments)
3. Fill-area clipping (polygons)
4. Curve clipping
5. Text clipping

Two-Dimensional Point Clipping

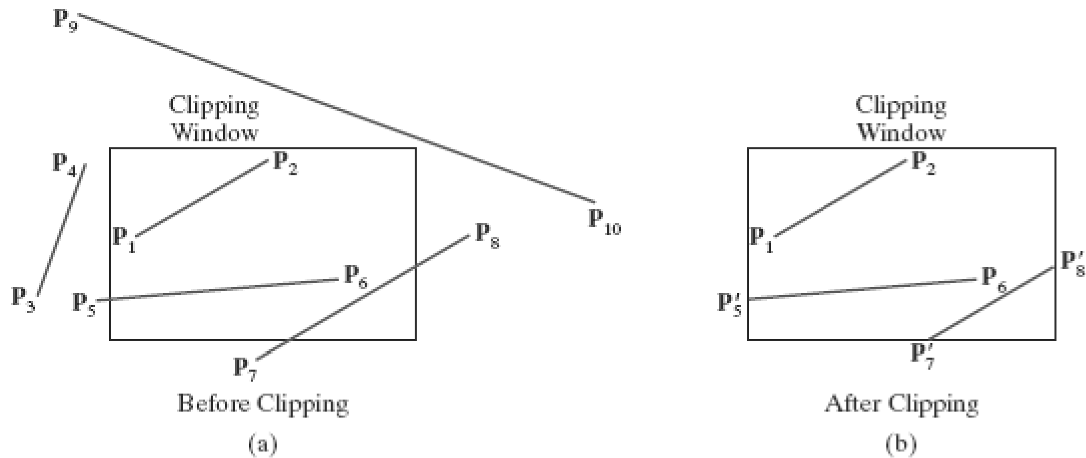
- ❖ For a clipping rectangle in standard position, we save a two-dimensional point **P** = *(x, y)* for display if the following inequalities are satisfied:

$$xw_{\min} \leq x \leq xw_{\max} \quad \text{and} \quad yw_{\min} \leq y \leq yw_{\max}$$

❖ If any of these four inequalities is not satisfied, the point is clipped

Two-Dimensional Line Clipping

✓ Clipping straight-line segments using a standard rectangular clipping window.



✓ A line-clipping algorithm processes each line in a scene through a series of tests and intersection calculations to determine whether the entire line or any part of it is to be saved.

✓ The expensive part of a line-clipping procedure is in calculating the intersection positions of a line with the window edges.

✓ Therefore, a major goal for any line-clipping algorithm is to minimize the intersection calculations.

✓ To do this, we can first perform tests to determine whether a line segment is completely inside the clipping window or completely outside.

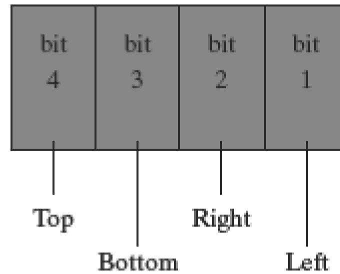
✓ It is easy to determine whether a line is completely inside a clipping window, but it is more difficult to identify all lines that are entirely outside the window.

✓ One way to formulate the equation for a straight-line segment is to use the following parametric representation, where the coordinate positions (x_0, y_0) and (x_{end}, y_{end}) designate the two line endpoints:

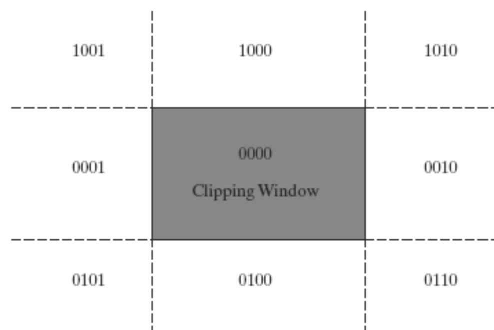
$$x = x_0 + u(x_{end} - x_0)$$
$$y = y_0 + u(y_{end} - y_0) \qquad 0 \le u \le 1$$

Cohen-Sutherland Line Clipping

- ✓ Processing time is reduced in the Cohen-Sutherland method by performing more tests before proceeding to the intersection calculations.
- ✓ Initially, every line endpoint in a picture is assigned a four-digit binary value, called a **region code,** and each bit position is used to indicate whether the point is inside or outside one of the clipping-window boundaries.



- ✓ A possible ordering for the clipping window boundaries corresponding to the bit positions in the Cohen-Sutherland endpoint region code.
- ✓ Thus, for this ordering, the rightmost position (bit 1) references the left clipping-window boundary, and the leftmost position (bit 4) references the top window boundary.
- ✓ A value of 1 (or *true*) in any bit position indicates that the endpoint is outside that window border. Similarly, a value of 0 (or *false*) in any bit position indicates that the endpoint is not outside (it is inside or on) the corresponding window edge.
- ✓ Sometimes, a region code is referred to as an "**out**" **code** because a value of 1 in any bit position indicates that the spatial point is outside the corresponding clipping boundary.
- ✓ The nine binary region codes for identifying the position of a line endpoint, relative to the clipping-window boundaries.



- ✓ Bit values in a region code are determined by comparing the coordinate values (x, y) of an endpoint to the clipping boundaries.

- ✓ Bit 1 is set to 1 if $x < xw_{min}$, and the other three bit values are determined similarly.
- ✓ To determine a boundary intersection for a line segment, we can use the slopeintercept form of the line equation.
- ✓ For a line with endpoint coordinates (x_0, y_0) and (x_{end}, y_{end}), the y coordinate of the intersection point with a vertical clipping border line can be obtained with the calculation

$$y = y_0 + m(x - x_0)$$

Where the x value is set to either xw_{min} or xw_{max}, and the slope of the line is calculated as

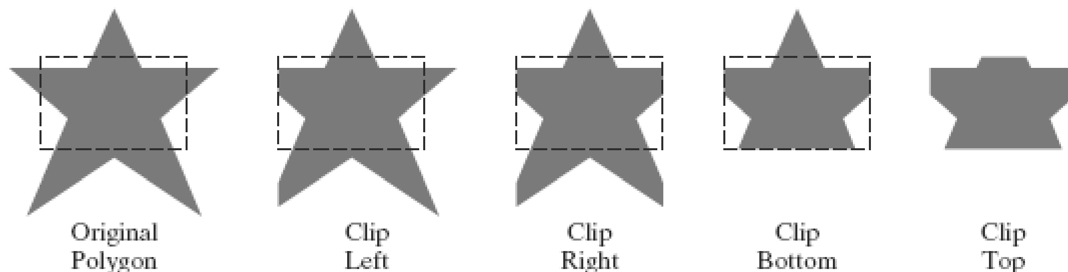
$$m = (y_{end} - y_0)/(x_{end} - x_0).$$

- ✓ Similarly, if we are looking for the intersection with a horizontal border, the x coordinate can be calculated as

$$x = x_0 + y - y_0/m ,$$ with y set either to yw_{min} or to yw_{max}.

Polygon Fill-Area Clipping

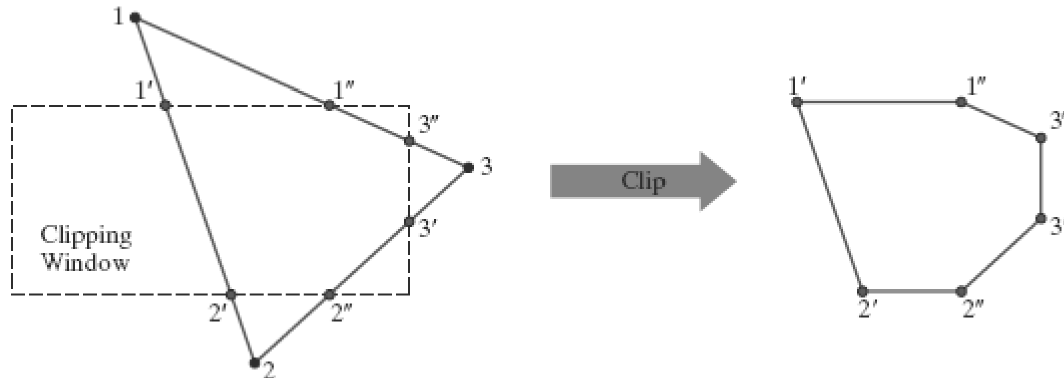
- ➔ To clip a polygon fill area, we cannot apply a line-clipping method to the individual polygon edges directly because this approach would not, in general, produce a closed polyline.
- ➔ We can process a polygon fill area against the borders of a clipping window using the same general approach as in line clipping.
- ➔ We need to maintain a fill area as an entity as it is processed through the clipping stages.
- ➔ Thus, we can clip a polygon fill area by determining the new shape for the polygon as each clipping-window edge is processed, as demonstrated



- ➔ When we cannot identify a fill area as being completely inside or completely outside the clipping window, we then need to locate the polygon intersection positions with the clipping boundaries.

➔ One way to implement convex-polygon clipping is to create a new vertex list at each clipping boundary, and then pass this new vertex list to the next boundary clipper.

➔ The output of the final clipping stage is the vertex list for the clipped polygon



Sutherland--Hodgman Polygon Clipping

❖ An efficient method for clipping a convex-polygon fill area, developed by Sutherland and Hodgman, is to send the polygon vertices through each clipping stage so that a single clipped vertex can be immediately passed to the next stage.

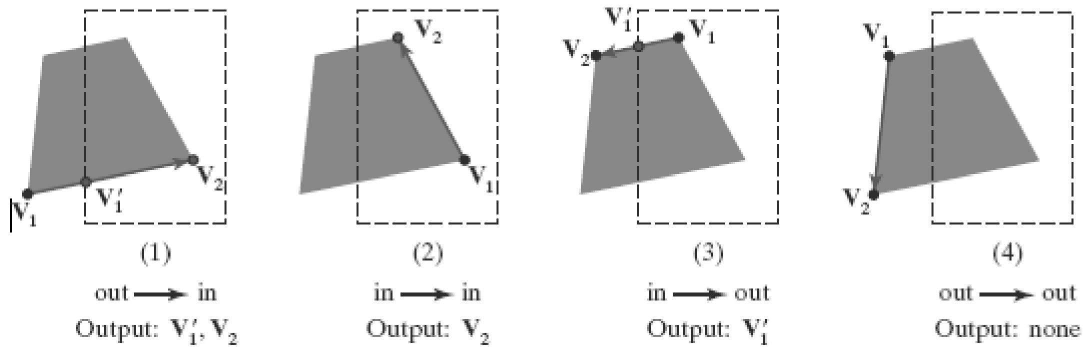
❖ The final output is a list of vertices that describe the edges of the clipped polygon fill area the basic Sutherland-Hodgman algorithm is able to process concave polygons when the clipped fill area can be described with a single vertex list.

❖ The general strategy in this algorithm is to send the pair of endpoints for each successive polygon line segment through the series of clippers (left, right, bottom, and top)

❖ There are four possible cases that need to be considered when processing a polygon edge against one of the clipping boundaries.

1. One possibility is that the first edge endpoint is outside the clipping boundary and the second endpoint is inside.
2. Or, both endpoints could be inside this clipping boundary.
3. Another possibility is that the first endpoint is inside the clipping boundary and the second endpoint is outside.
4. And, finally, both endpoints could be outside the clipping boundary

❖ To facilitate the passing of vertices from one clipping stage to the next, the output from each clipper can be formulated as shown in Figure below



The selection of vertex edge of intersection for each clipper is given as follows

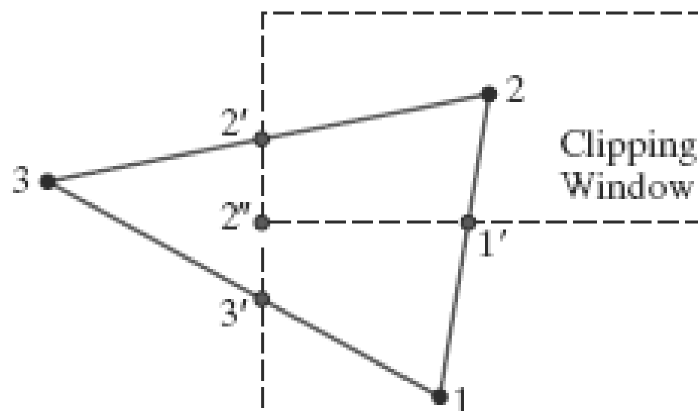
1. If the first input vertex is outside this clipping-window border and the second vertex is inside, both the intersection point of the polygon edge with the window border and the second vertex are sent to the next clipper.

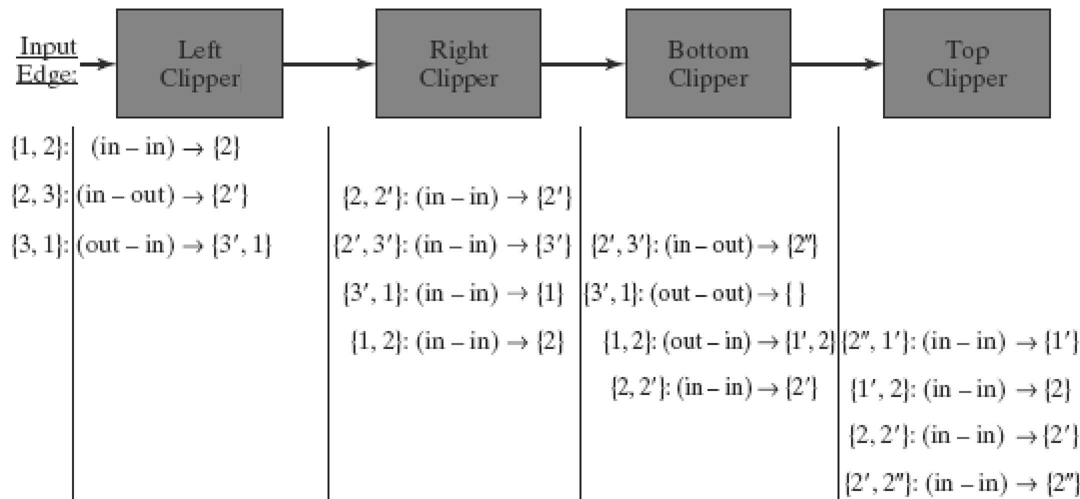
2. If both input vertices are inside this clipping-window border, only the second vertex is sent to the next clipper.

3. If the first vertex is inside this clipping-window border and the second vertex is outside, only the polygon edge-intersection position with the clipping-window border is sent to the next clipper.

4. If both input vertices are outside this clipping-window border, no vertices are sent to the next clipper.

Example





- When a concave polygon is clipped with the Sutherland-Hodgman algorithm, extraneous lines may be displayed.
- This occurs when the clipped polygon should have two or more separate sections. But since there is only one output vertex list, the last vertex in the list is always joined to the first vertex.
- There are several things we can do to display clipped concave polygons correctly.
- For one, we could split a concave polygon into two or more convexpolygons and process each convex polygon separately using the Sutherland- Hodgman algorithm
- Another possibility is to modify the Sutherland- Hodgman method so that the final vertex list is checked for multiple intersection points along any clipping-window boundary.
- If we find more than two vertex positions along any clipping boundary, we can separate the list of vertices into two or more lists that correctly identify the separate sections of the clipped fill area.
- A third possibility is to use a more general polygon clipper that has been designed to process concave polygons correctly

3.2 3DGeometric Transformations:

3.2.1 3D Geometric Transformations
3.2.2 3D Translation,
3.2.3 Rotation,
3.2.4 Scaling,
3.2.5 Composite 3D Transformations,
3.2.6 Other 3D Transformations,
3.2.7 Affine Transformations,
3.2.8 Opengl Geometric Transformations

3.2.1 Three-Dimensional Geometric Transformations

- Methods for geometric transformations in three dimensions are extended from two dimensional methods by including considerations for the *z* coordinate.
- A three-dimensional position, expressed in homogeneous coordinates, is represented as a four-element column vector

3.2.2 Three-Dimensional Translation

- A position **P** = *(x, y, z)* in three-dimensional space is translated to a location **P'**= *(x', y', z')* by adding translation distances *tx*, *ty*, and *tz* to the Cartesian coordinates of **P**:

$$x' = x + t_x, \qquad y' = y + t_y, \qquad z' = z + t_z$$

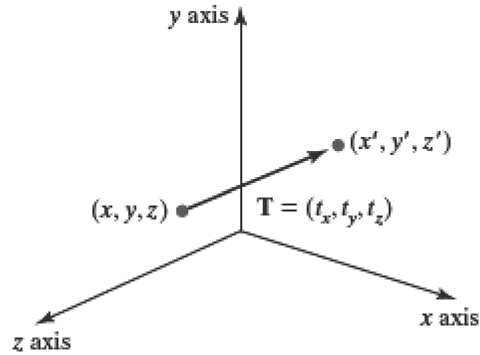
- We can express these three-dimensional translation operations in matrix form

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

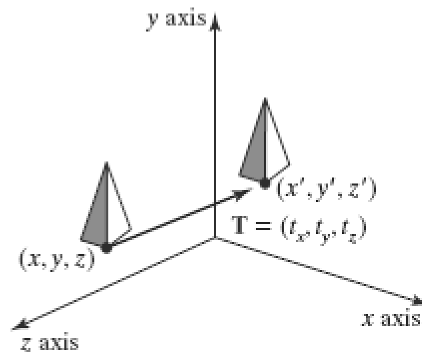
or

$$\mathbf{P}' = \mathbf{T} \cdot \mathbf{P}$$

- Moving a coordinate position with translation vector T = (tx , ty , tz) .



- Shifting the position of a three-dimensional object using translation vector **T**.



CODE:

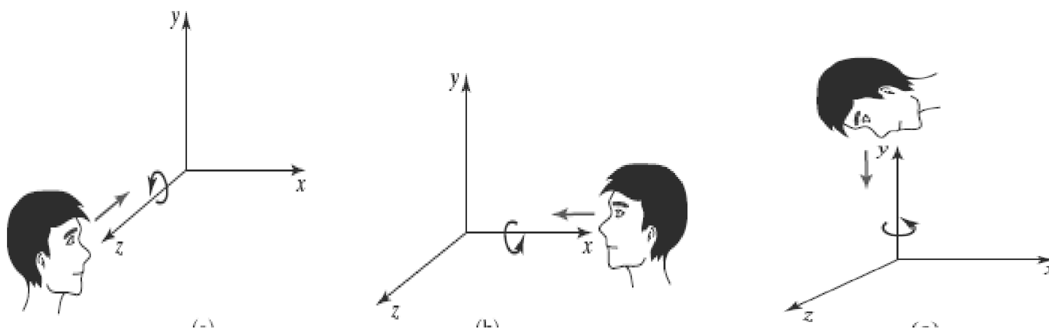
```
typedef GLfloat Matrix4x4 [4][4];
/* Construct the 4 x 4 identity matrix. */
void matrix4x4SetIdentity (Matrix4x4 matIdent4x4)
{
        GLint row, col;
        for (row = 0; row < 4; row++)
        for (col = 0; col < 4 ; col++)
        matIdent4x4 [row][col] = (row == col);
}
void translate3D (GLfloat tx, GLfloat ty, GLfloat tz)
{
        Matrix4x4 matTransl3D;
        /* Initialize translation matrix to identity. */
        matrix4x4SetIdentity (matTransl3D);
```

```
        matTransl3D [0][3] = tx;
        matTransl3D [1][3] = ty;
        matTransl3D [2][3] = tz;
}
```

- An inverse of a three-dimensional translation matrix is obtained by negating the translation distances *tx*, *ty*, and *tz*

3.2.3 Three-Dimensional Rotation

- ✓ By convention, positive rotation angles produce counterclockwise rotations about a coordinate axis.
- ✓ Positive rotations about a coordinate axis are counterclockwise, when looking along the positive half of the axis toward the origin.



<u>Three-Dimensional Coordinate-Axis Rotations</u>

Along z axis:

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

- ✓ In homogeneous-coordinate form, the three-dimensional *z*-axis rotation equations are

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

✓ Transformation equations for rotations about the other two coordinate axes can be obtained with a cyclic permutation of the coordinate parameters *x*, *y*, and *z*

$$x \rightarrow y \rightarrow z \rightarrow x$$

Along x axis

$$y' = y\cos\theta - z\sin\theta$$
$$z' = y\sin\theta + z\cos\theta$$
$$x' = x$$

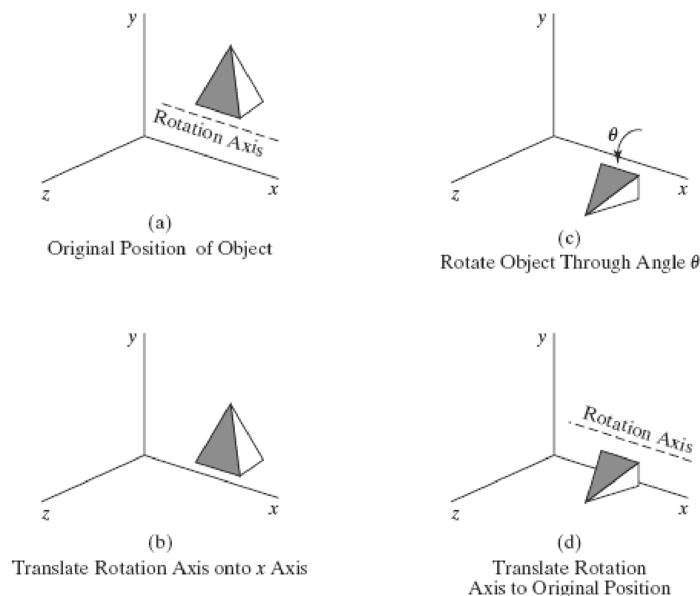
Along y axis

$$z' = z\cos\theta - x\sin\theta$$
$$x' = z\sin\theta + x\cos\theta$$
$$y' = y$$

✓ An inverse three-dimensional rotation matrix is obtained in the same by replacing θ with $-\theta$.

General Three-Dimensional Rotations

✓ A rotation matrix for any axis that does not coincide with a coordinate axis can be set up as a composite transformation involving combinations of translations and the coordinate-axis rotations the following transformation sequence is used:



1. Translate the object so that the rotation axis coincides with the parallel coordinate axis.

2. Perform the specified rotation about that axis.

3. Translate the object so that the rotation axis is moved back to its original position.

- ✓ A coordinate position **P** is transformed with the sequence shown in this figure as

$$\mathbf{P}' = \mathbf{T}^{-1} \cdot \mathbf{R}_x(\theta) \cdot \mathbf{T} \cdot \mathbf{P}$$

Where the composite rotation matrix for the transformation is

$$\mathbf{R}(\theta) = \mathbf{T}^{-1} \cdot \mathbf{R}_x(\theta) \cdot \mathbf{T}$$

- ✓ When an object is to be rotated about an axis that is not parallel to one of the coordinate axes, we must perform some additional transformations we can accomplish the required rotation in five steps:

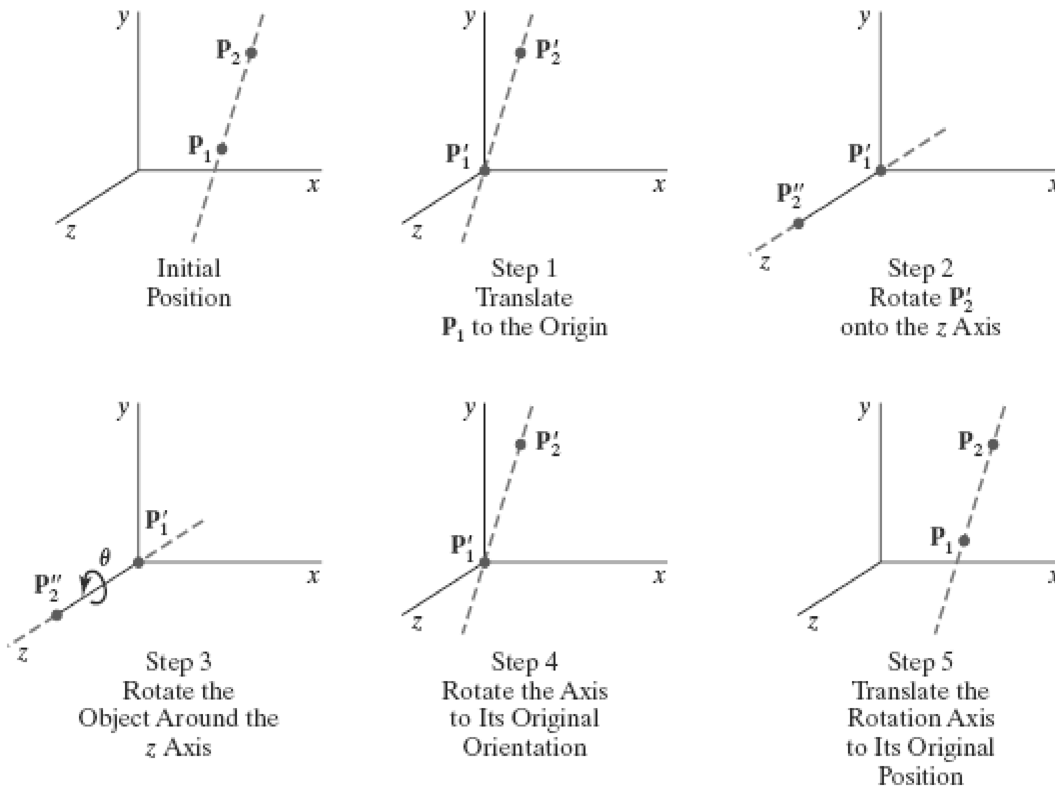
1. Translate the object so that the rotation axis passes through the coordinate origin.

2. Rotate the object so that the axis of rotation coincides with one of the coordinate axes.

3. Perform the specified rotation about the selected coordinate axis.

4. Apply inverse rotations to bring the rotation axis back to its original orientation.

5. Apply the inverse translation to bring the rotation axis back to its original spatial position.



- Components of the rotation-axis vector are then computed as

$$\mathbf{V} = \mathbf{P}2 - \mathbf{P}1$$
$$= (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

- The unit rotation-axis vector **u** is

$$\mathbf{u} = \frac{\mathbf{V}}{|\mathbf{V}|} = (a, b, c)$$

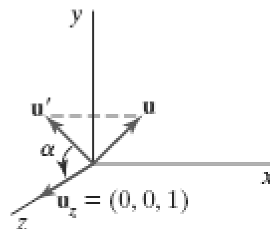
Where the components *a*, *b*, and *c* are the direction cosines for the rotation axis

$$a = \frac{x_2 - x_1}{|\mathbf{V}|}, \qquad b = \frac{y_2 - y_1}{|\mathbf{V}|}, \qquad c = \frac{z_2 - z_1}{|\mathbf{V}|}$$

- The first step in the rotation sequence is to set up the translation matrix that repositions the rotation axis so that it passes through the coordinate origin.
- Translation matrix is given by

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Because rotation calculations involve sine and cosine functions, we can use standard vector operations to obtain elements of the two rotation matrices.
- A vector dot product can be used to determine the cosine term, and a vector cross product can be used to calculate the sine term.
- Rotation of u around the x axis into the x z plane is accomplished by rotating u' (the projection of u in the y z plane) through angle α onto the z axis.



- If we represent the projection of **u** in the *yz* plane as the vector **u**'= (0, *b*, *c*), then the cosine of the rotation angle α can be determined from the dot product of **u**' and the unit vector $\mathbf{u}_z$ along the *z* axis:

$$\cos\alpha = \frac{\mathbf{u}' \cdot \mathbf{u}_z}{|\mathbf{u}'|\,|\mathbf{u}_z|} = \frac{c}{d}$$

where d is the magnitude of **u**’

$$d = \sqrt{b^2 + c^2}$$

- The coordinate-independent form of this cross-product is

$$\mathbf{u}' \times \mathbf{u}_z = \mathbf{u}_x\, |\mathbf{u}'|\, |\mathbf{u}_z| \sin\alpha$$

- and the Cartesian form for the cross-product gives us

$$\mathbf{u}' \times \mathbf{u}_z = \mathbf{u}_x \cdot b$$

- Equating the above two equations

$$d \sin\alpha = b$$

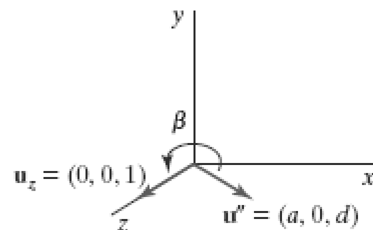
or

$$\sin\alpha = \frac{b}{d}$$

- We have determined the values for cos α and sin α in terms of the components of vector **u**, the matrix elements for rotation of this vector about the x axis and into the xz plane

$$\mathbf{R}_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{c}{d} & -\frac{b}{d} & 0 \\ 0 & \frac{b}{d} & \frac{c}{d} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Rotation of unit vector u” (vector u after rotation into the x z plane) about the y axis. Positive rotation angle β aligns u” with vector uz .



- We can determine the cosine of rotation angle β from the dot product of unit vectors **u**’’ and **u***z*. Thus,

$$\cos\beta = \frac{\mathbf{u}'' \cdot \mathbf{u}_z}{|\mathbf{u}''|\,|\mathbf{u}_z|} = d$$

- Comparing the coordinate-independent form of the cross-product

$$\mathbf{u}'' \times \mathbf{u}_z = \mathbf{u}_y\,|\mathbf{u}''|\,|\mathbf{u}_z|\sin\beta$$

with the Cartesian form

$$\mathbf{u}'' \times \mathbf{u}_z = \mathbf{u}_y \cdot (-a)$$

- we find that

$$\sin\beta = -a$$

- The transformation matrix for rotation of **u**" about the *y* axis is

$$\mathbf{R}_y(\beta) = \begin{bmatrix} d & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The specified rotation angle θ can now be applied as a rotation about the z axis as follows:

$$\mathbf{R}_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The transformation matrix for rotation about an arbitrary axis can then be expressed as the composition of these seven individual transformations:

$$\mathbf{R}(\theta) = \mathbf{T}^{-1} \cdot \mathbf{R}_x^{-1}(\alpha) \cdot \mathbf{R}_y^{-1}(\beta) \cdot \mathbf{R}_z(\theta) \cdot \mathbf{R}_y(\beta) \cdot \mathbf{R}_x(\alpha) \cdot \mathbf{T}$$

- The composite matrix for any sequence of three-dimensional rotations is of the form

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The upper-left 3 × 3 submatrix of this matrix is orthogonal

$$\mathbf{R} \cdot \begin{bmatrix} r_{11} \\ r_{12} \\ r_{13} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{R} \cdot \begin{bmatrix} r_{21} \\ r_{22} \\ r_{23} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{R} \cdot \begin{bmatrix} r_{31} \\ r_{32} \\ r_{33} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

- Assuming that the rotation axis is not parallel to any coordinate axis, we could form the following set of local unit vectors

$$\mathbf{u}'_z = \mathbf{u}$$
$$\mathbf{u}'_y = \frac{\mathbf{u} \times \mathbf{u}_x}{|\mathbf{u} \times \mathbf{u}_x|}$$
$$\mathbf{u}'_x = \mathbf{u}'_y \times \mathbf{u}'_z$$

- If we express the elements of the unit local vectors for the rotation axis as

$$\mathbf{u}'_x = (u'_{x1}, u'_{x2}, u'_{x3})$$
$$\mathbf{u}'_y = (u'_{y1}, u'_{y2}, u'_{y3})$$
$$\mathbf{u}'_z = (u'_{z1}, u'_{z2}, u'_{z3})$$

- Then the required composite matrix, which is equal to the product $\mathbf{R}_y(\beta) \cdot \mathbf{R}_x(\alpha)$, is

$$\mathbf{R} = \begin{bmatrix} u'_{x1} & u'_{x2} & u'_{x3} & 0 \\ u'_{y1} & u'_{y2} & u'_{y3} & 0 \\ u'_{z1} & u'_{z2} & u'_{z3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>Quaternion Methods for Three-Dimensional Rotations</u>

- ✓ A more efficient method for generating a rotation about an arbitrarily selected axis is to use a quaternion representation for the rotation transformation.
- ✓ Quaternions, which are extensions of two-dimensional complex numbers, are useful in a number of computer-graphics procedures, including the generation of fractal objects.
- ✓ One way to characterize a quaternion is as an ordered pair, consisting of a *scalar part* and a *vector part:*

$$q = (s, \mathbf{v})$$

- ✓ *A rotation about any axis passing through the coordinate origin is accomplished by first setting up a unit quaternion with the scalar and vector parts as follows:*

$$s = \cos\frac{\theta}{2}, \qquad \mathbf{v} = \mathbf{u}\sin\frac{\theta}{2}$$

✓ Any point position **P** that is to be rotated by this quaternion can be represented in quaternion notation as

$$\mathbf{P} = (0, \mathbf{p})$$

✓ Rotation of the point is then carried out with the quaternion operation

$$\mathbf{P}' = q\mathbf{P}q^{-1}$$

where $q^{-1} = (s, -\mathbf{v})$ is the inverse of the unit quaternion q

✓ This transformation produces the following new quaternion:

$$\mathbf{P}' = (0, \mathbf{p}')$$

✓ The second term in this ordered pair is the rotated point position **p'**, which is evaluated with vector dot and cross-products as

$$\mathbf{p}' = s^2\mathbf{p} + \mathbf{v}(\mathbf{p} \cdot \mathbf{v}) + 2s(\mathbf{v} \times \mathbf{p}) + \mathbf{v} \times (\mathbf{v} \times \mathbf{p})$$

✓ Designating the components of the vector part of q as $\mathbf{v} = (a, b, c)$, we obtain the elements for the composite rotation matrix

$$\mathbf{M}_R(\theta) = \begin{bmatrix} 1 - 2b^2 - 2c^2 & 2ab - 2sc & 2ac + 2sb \\ 2ab + 2sc & 1 - 2a^2 - 2c^2 & 2bc - 2sa \\ 2ac - 2sb & 2bc + 2sa & 1 - 2a^2 - 2b^2 \end{bmatrix}$$

✓ Using the following trigonometric identities to simplify the terms

$$\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 1 - 2\sin^2\frac{\theta}{2} = \cos\theta, \qquad 2\cos\frac{\theta}{2}\sin\frac{\theta}{2} = \sin\theta$$

we can rewrite Matrix as

$$\mathbf{M}_R(\theta) = \begin{bmatrix} u_x^2(1-\cos\theta) + \cos\theta & u_x u_y(1-\cos\theta) - u_z\sin\theta & u_x u_z(1-\cos\theta) + u_y\sin\theta \\ u_y u_x(1-\cos\theta) + u_z\sin\theta & u_y^2(1-\cos\theta) + \cos\theta & u_y u_z(1-\cos\theta) - u_x\sin\theta \\ u_z u_x(1-\cos\theta) - u_y\sin\theta & u_z u_y(1-\cos\theta) + u_x\sin\theta & u_z^2(1-\cos\theta) + \cos\theta \end{bmatrix}$$

3.2.4 Three-Dimensional Scaling

✓ The matrix expression for the three-dimensional scaling transformation of a position **P** = *(x, y, z) is given by*

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- ✓ The three-dimensional scaling transformation for a point position can be represented as

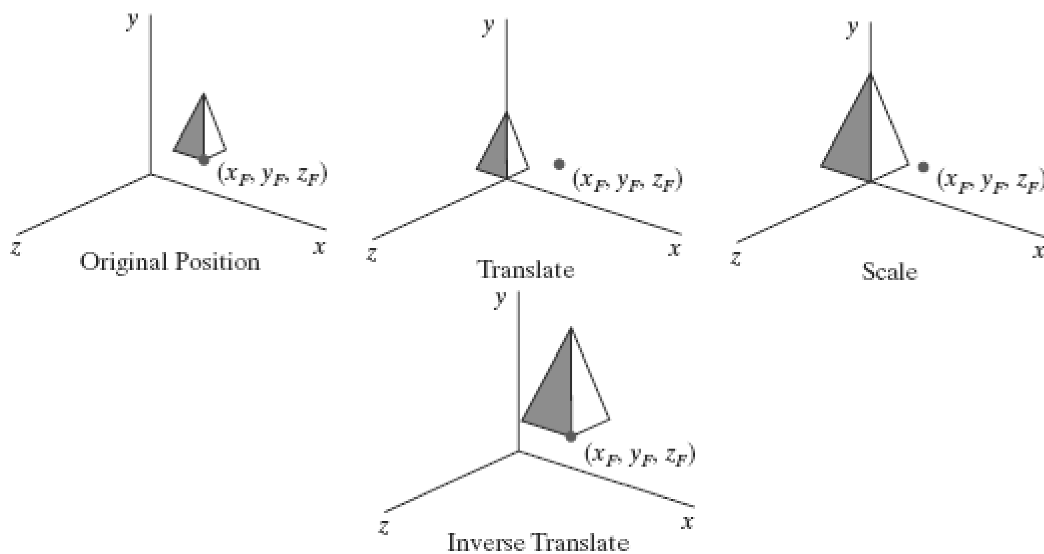
$$\mathbf{P}' = \mathbf{S} \cdot \mathbf{P}$$

where scaling parameters s_x, s_y, and s_z are assigned any positive values.

- ✓ Explicit expressions for the scaling transformation relative to the origin are

$$x' = x \cdot s_x, \qquad y' = y \cdot s_y, \qquad z' = z \cdot s_z$$

- ✓ Because some graphics packages provide only a routine that scales relative to the coordinate origin, we can always construct a scaling transformation with respect to any selected *fixed position* (x_f, y_f, z_f) using the following transformation sequence:
  1. Translate the fixed point to the origin.
  2. Apply the scaling transformation relative to the coordinate origin
  3. Translate the fixed point back to its original position.
- ✓ This sequence of transformations is demonstrated



$$\mathbf{T}(x_f, y_f, z_f) \cdot \mathbf{S}(s_x, s_y, s_z) \cdot \mathbf{T}(-x_f, -y_f, -z_f) = \begin{bmatrix} s_x & 0 & 0 & (1 - s_x)x_f \\ 0 & s_y & 0 & (1 - s_y)y_f \\ 0 & 0 & s_z & (1 - s_z)z_f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

CODE:

```
class wcPt3D
{
	private:
	GLfloat x, y, z;
	public:
	/* Default Constructor:
	* Initialize position as (0.0, 0.0, 0.0).
	*/
	wcPt3D ( ) {
		x = y = z = 0.0;
	}
	setCoords (GLfloat xCoord, GLfloat yCoord, GLfloat zCoord) {
		x = xCoord;
		y = yCoord;
		z = zCoord;
	}
	GLfloat getx ( ) const {
		return x;
	}
	GLfloat gety ( ) const {
		return y;
	}
	GLfloat getz ( ) const {
		return z;
	}
};
typedef float Matrix4x4 [4][4];
void scale3D (GLfloat sx, GLfloat sy, GLfloat sz, wcPt3D fixedPt)
{
	Matrix4x4 matScale3D;
```

```
    /* Initialize scaling matrix to identity. */
    matrix4x4SetIdentity (matScale3D);
    matScale3D [0][0] = sx;
    matScale3D [0][3] = (1 - sx) * fixedPt.getx ( );
    matScale3D [1][1] = sy;
    matScale3D [1][3] = (1 - sy) * fixedPt.gety ( );
    matScale3D [2][2] = sz;
    matScale3D [2][3] = (1 - sz) * fixedPt.getz ( );
}
```

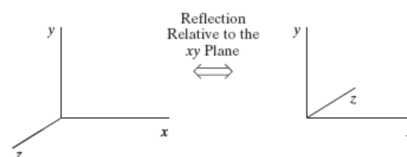
3.2.5 Composite Three-Dimensional Transformations

- We form a composite threedimensional transformation by multiplying the matrix representations for the individual operations in the transformation sequence.
- We can implement a transformation sequence by concatenating the individual matrices from right to left or from left to right, depending on the order in which the matrix representations are specified

3.2.6 Other Three-Dimensional Transformations

Three-Dimensional Reflections

- A reflection in a three-dimensional space can be performed relative to a selected *reflection axis* or with respect to a *reflection plane*.
- Reflections with respect to a plane are similar; when the reflection plane is a coordinate plane (x_y, x_z, or y_z), we can think of the transformation as a 180◦ rotation in four-dimensional space with a conversion between a left-handed frame and a right-handed frame
- An example of a reflection that converts coordinate specifications froma right handed system to a left-handed system is shown below



➔ The matrix representation for this reflection relative to the *xy* plane is

$$M_{\text{zreflect}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Three-Dimensional Shears

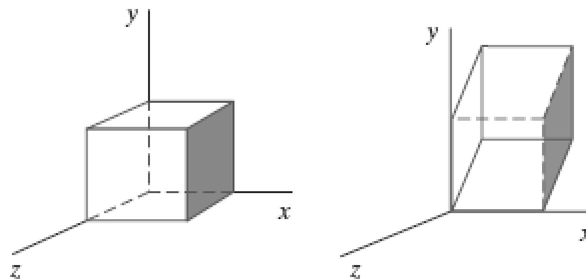
➔ These transformations can be used to modify object shapes.

➔ For three-dimensional we can also generate shears relative to the *z* axis.

➔ A general *z*-axis shearing transformation relative to a selected reference position is produced with the following matrix:

$$M_{\text{zshear}} = \begin{bmatrix} 1 & 0 & \text{sh}_{zx} & -\text{sh}_{zx} \cdot z_{\text{ref}} \\ 0 & 1 & \text{sh}_{zy} & -\text{sh}_{zy} \cdot z_{\text{ref}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

➔ The Below figure shows the shear transformation of a cube



A unit cube (a) is sheared relative to the origin (b) by Matrix 46, with $\text{sh}_{zx} = \text{sh}_{zy} = 1$.

3.2.7 Affine Transformations

❖ A coordinate transformation of the form

$$x' = a_{xx}x + a_{xy}y + a_{xz}z + b_x$$
$$y' = a_{yx}x + a_{yy}y + a_{yz}z + b_y$$
$$z' = a_{zx}x + a_{zy}y + a_{zz}z + b_z$$

is called an **affine transformation**

- Affine transformations (in two dimensions, three dimensions, or higher dimensions) have the general properties that parallel lines are transformed into parallel lines, and finite points map to finite points.
- Translation, rotation, scaling, reflection,andshear are examples of affine transformations.
- Another example of an affine transformation is the conversion of coordinate descriptions for a scene from one reference system to another because this transformation can be described as a combination of translation and rotation

3.2.8 OpenGL Geometric-Transformation Functions

OpenGL Matrix Stacks

glMatrixMode:

- used to select the modelview composite transformation matrix as the target of subsequent OpenGL transformation calls
- four modes: modelview, projection, texture, and color
- the top matrix on each stack is called the “current matrix”**.**
- for that mode. the **modelview matrix stack** is the 4 × 4 composite matrix that combines the viewing transformations and the various geometric transformations that we want to apply to a scene.
- OpenGL supports a modelview stack depth of at least 32,

glGetIntegerv (GL_MAX_MODELVIEW_STACK_DEPTH, stackSize);

- determine the number of positions available in the modelview stack for a particular implementation of OpenGL.
- It returns a single integer value to array **stackSize**
- **other OpenGL symbolic constants:** GL_MAX_PROJECTION_STACK_DEPTH, GL_MAX_TEXTURE_STACK_DEPTH, or GL_MAX_COLOR_STACK_DEPTH**.**
- We can also find out how many matrices are currently in the stack with

glGetIntegerv (GL_MODELVIEW_STACK_DEPTH, numMats);

We have two functions available in OpenGL for processing the matrices in a stack

glPushMatrix ( );

Copy the current matrix at the top of the active stack and store that copy in the second stack position

glPopMatrix ( );

which destroys the matrix at the top of the stack, and the second matrix in the stack becomes the current matrix

3.3 Illumination and Color

3.3.1 Illumination models
3.3.2 Light sources,
3.3.3 Basic illumination models-Ambient light, diffuse reflection, specular and phong model,
3.3.4 Corresponding openGL functions.
3.3.5 Properties of light,
3.3.6 Color models, RGB and CMY color models.
.

3.3.1 Illumination Models

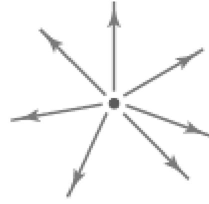
- ✓ An **illumination model,** also called a **lighting model** (and sometimes referred to as a *shading model*), is used to calculate the color of an illuminated position on the surface of an object

3.3.2 Light Sources

- ✓ Any object that is emitting radiant energy is a **light source** that contributes to the lighting effects for other objects in a scene.
- ✓ We can model light sources with a variety of shapes and characteristics, and most emitters serve only as a source of illumination for a scene.
- ✓ A light source can be defined with a number of properties. We can specify its position, the color of the emitted light, the emission direction, and its shape.
- ✓ We could set up a light source that emits different colors in different directions.
- ✓ We assign light emitting properties using a single value for each of the red, green, and blue (RGB) color components, which we can describe as the amount, or the "intensity," of that color component.

<u>Point Light Sources</u>

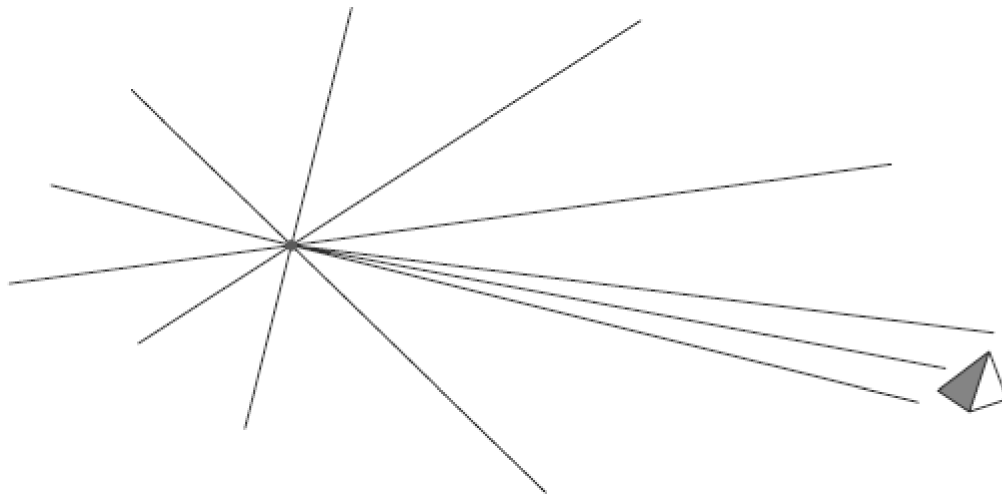
- ❖ The simplest model for an object that is emitting radiant energy is a **point light source** with a single color, specified with three RGB components



- ❖ A point source for a scene by giving its position and the color of the emitted light. light rays are generated along radially diverging paths from the single-color source position.
- ❖ This light-source model is a reasonable approximation for sources whose dimensions are small compared to the size of objects in the scene

Infinitely Distant Light Sources

- ❖ A large light source, such as the sun, that is very far from a scene can also be approximated as a point emitter, but there is little variation in its directional effects.
- ❖ The light path from a distant light source to any position in the scene is nearly constant



- ❖ We can simulate an infinitely distant light source by assigning it a color value and a fixed direction for the light rays emanating from the source.
- ❖ The vector for the emission direction and the light-source color are needed in the illumination calculations, but not the position of the source.

Radial Intensity Attenuation

- ➢ As radiant energy from a light source travels outwards through space, its amplitude at any distance d_l from the source is attenuated by the factor $1/d^2$ a surface close to the light

source receives a higher incident light intensity from that source than a more distant surface.

- However, using an attenuation factor of $1/d_l^2$ with a point source does not always produce realistic pictures.
- The factor $1/d_l^2$ tends to produce too much intensity variation for objects that are close to the light source, and very little variation when d_l is large
- We can attenuate light intensities with an inverse quadratic function of *dl* that includes a linear term:

$$f_{\text{radatten}}(d_l) = \frac{1}{a_0 + a_1 d_l + a_2 d_l^2}$$

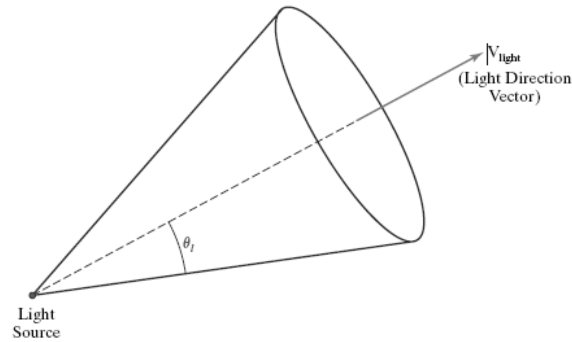
- The numerical values for the coefficients, a_0, a_1, and a_2, can then be adjusted to produce optimal attenuation effects.
- We cannot apply intensity-attenuation calculation 1 to a point source at “infinity,” because the distance to the light source is indeterminate.
- We can express the intensity-attenuation function as

$$f_{l,\text{radatten}} = \begin{cases} 1.0, & \text{if source is at infinity} \\ \dfrac{1}{a_0 + a_1 d_l + a_2 d_l^2}, & \text{if source is local} \end{cases}$$

<u>Directional Light Sources and Spotlight Effects</u>

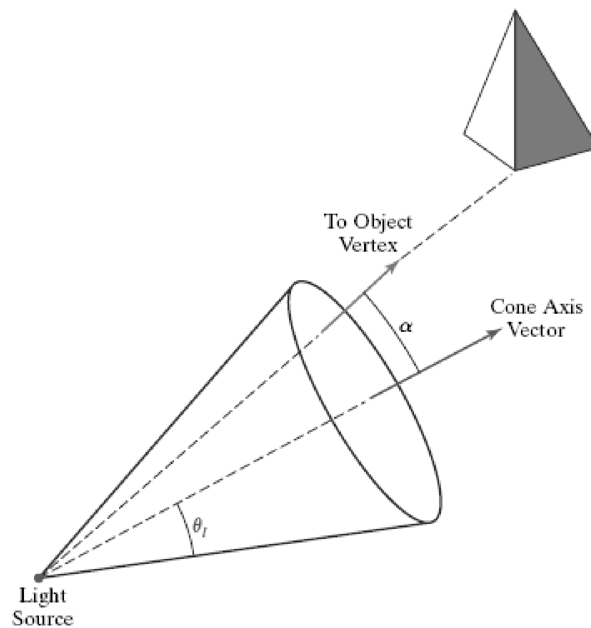
- A local light source can be modified easily to produce a directional, or spotlight, beam of light.
- If an object is outside the directional limits of the light source, we exclude it from illumination by that source
- One way to set up a directional light source is to assign it a vector direction and an angular limit θ_l measured from that vector direction, in addition to its position and color
- We can denote $\mathbf{V}_{\text{light}}$ as the unit vector in the light-source direction and $\mathbf{V}_{\text{obj}}$ as the unit vector in the direction from the light position to an object position.

Then $\mathbf{V}_{\text{obj}} \cdot \mathbf{V}_{\text{light}} = \cos\alpha$



where angle α is the angular distance of the object from the light direction vector.

➔ If we restrict the angular extent of any light cone so that $0^\circ < \theta_l \leq 90^\circ$, then the object is within the spotlight if $\cos \alpha \geq \cos \theta_l$, as shown



➔ . If $\mathbf{V}_{obj} \cdot \mathbf{V}_{light} < \cos \theta_l$, however, the object is outside the light cone.

Angular Intensity Attenuation

- For a directional light source, we can attenuate the light intensity angularly about the source as well as radially out from the point-source position
- This allows intensity decreasing as we move farther from the cone axis.
- A commonly used angular intensity-attenuation function for a directional light source is

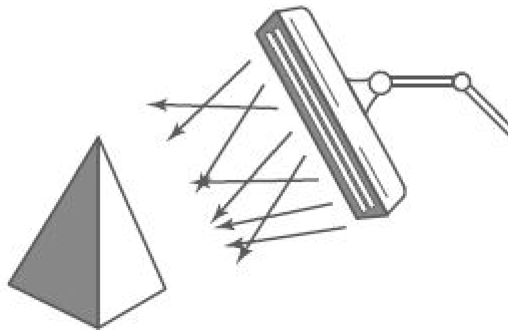
$$f_{\text{angatten}}(\phi) = \cos^{a_l}\phi, \qquad 0^\circ \le \phi \le \theta$$

- Where the attenuation exponent *al* is assigned some positive value and angle φ is measured from the cone axis.
- The greater the value for the attenuation exponent *al* , the smaller the value of the angular intensity-attenuation function for a given value of angle$\varphi > 0^\circ$.
- There are several special cases to consider in the implementation of the angular-attenuation function.
- There is no angular attenuation if the light source is not directional (not a spotlight).
- We can express the general equation for angular attenuation as

$$f_{l,\text{angatten}} = \begin{cases} 1.0, & \text{if source is not a spotlight} \\ 0.0, & \text{if } \mathbf{V}_{\text{obj}} \cdot \mathbf{V}_{\text{light}} = \cos\alpha < \cos\theta_l \\ & \quad \text{(object is outside the spotlight cone)} \\ (\mathbf{V}_{\text{obj}} \cdot \mathbf{V}_{\text{light}})^{a_l}, & \text{otherwise} \end{cases}$$

Extended Light Sources and the Warn Model

- ✓ When we want to include a large light source at a position close to the objects in a scene, such as the long neon lamp, we can approximate it as a lightemitting surface



- ✓ One way to do this is to model the light surface as a grid of directional point emitters.
- ✓ We can set the direction for the point sources so that objects behind the light-emitting surface are not illuminated.
- ✓ We could also include other controls to restrict the direction of the emitted light near the edges of the source

- ✓ The **Warn model** provides a method for producing studio lighting effects using sets of point emitters with various parameters to simulate the barn doors, flaps, and spotlighting controls employed by photographers.
- ✓ Spotlighting is achieved with the cone of light discussed earlier, and the flaps and barn doors provide additional directional control

3.3.3 Basic Illumination Models

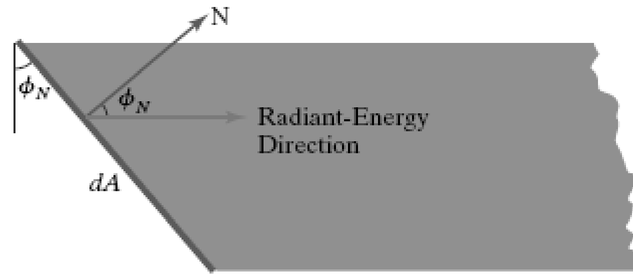
- ➢ Light-emitting objects in a basic illumination model are generally limited to point sources many graphics packages provide additional functions for dealing with directional lighting (spotlights) and extended light sources.

Ambient Light

- ➢ This produces a uniform ambient lighting that is the same for all objects, and it approximates the global diffuse reflections from the various illuminated surfaces.
- ➢ Reflections produced by ambient-light illumination are simply a form of diffuse reflection, and they are independent of the viewing direction and the spatial orientation of a surface.
- ➢ However, the amount of the incident ambient light that is reflected depends on surface optical properties, which determine how much of the incident energy is reflected and how much is absorbed

Diffuse Reflection

- ➢ The incident light on the surface is scattered with equal intensity in all directions, independent of the viewing position.
- ➢ Such surfaces are called **ideal diffuse reflectors** They are also referred to as **Lambertian reflectors,** because the reflected radiant light energy fromany point on the surface is calculated with **Lambert's cosine law.**
- ➢ This law states that the amount of radiant energy coming from any small surface area dAin a direction φN relative to the surface normal is proportional to cos φN



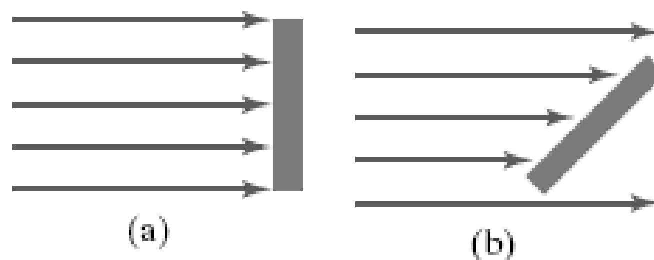
- The intensity of light in this direction can be computed as the ratio of the magnitude of the radiant energy per unit time divided by the projection of the surface area in the radiation direction:

$$\begin{aligned}\text{Intensity} &= \frac{\text{radiant energy per unit time}}{\text{projected area}} \\ &\propto \frac{\cos\phi_N}{dA\cos\phi_N} \\ &= \text{constant}\end{aligned}$$

- Assuming that every surface is to be treated as an ideal diffuse reflector (Lambertian), we can set a parameter *kd* for each surface that determines the fraction of the incident light that is to be scattered as diffuse reflections.
- This parameter is called the **diffuse-reflection coefficient** or the **diffuse reflectivity. T**he ambient contribution to the diffuse reflection at any point on a surface is simply

$$I_{\text{ambdiff}} = k_d I_a$$

- The below figure illustrates this effect, showing a beam of light rays incident on two equal-area plane surface elements with different spatial orientations relative to the illumination direction from a distant source



A surface that is perpendicular to the direction of the incident light (a) is more illuminated than an equal-sized surface at an oblique angle (b) to the incoming light direction.

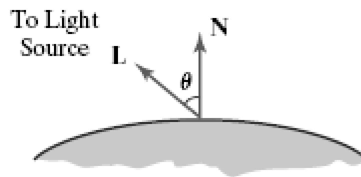
- We can model the amount of incident light on a surface from a source with intensity I_l as

$$I_{l,\text{incident}} = I_l \cos\theta$$

- We can model the diffuse reflections from a light source with intensity I_l using the calculation

$$\begin{aligned} I_{l,\text{diff}} &= k_d I_{l,\text{incident}} \\ &= k_d I_l \cos\theta \end{aligned}$$

- At any surface position, we can denote the unit normal vector as **N** and the unit direction vector to a point source as **L**,



- The diffuse reflection equation for single point-source illumination at a surface position can be expressed in the form

$$I_{l,\text{diff}} = \begin{cases} k_d I_l (\mathbf{N} \cdot \mathbf{L}), & \text{if } \mathbf{N} \cdot \mathbf{L} > 0 \\ 0.0, & \text{if } \mathbf{N} \cdot \mathbf{L} \leq 0 \end{cases}$$

- The unit direction vector **L** to a nearby point light source is calculated using the surface position and the light-source position:

$$\mathbf{L} = \frac{\mathbf{P}_{\text{source}} - \mathbf{P}_{\text{surf}}}{|\mathbf{P}_{\text{source}} - \mathbf{P}_{\text{surf}}|}$$

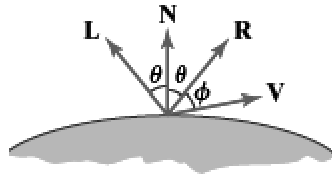
- We can combine the ambient and point-source intensity calculations to obtain an expression for the total diffuse reflection at a surface position
- Using parameter k_a , we can write the total diffuse-reflection equation for a single point source as

$$I_{\text{diff}} = \begin{cases} k_a I_a + k_d I_l (\mathbf{N} \cdot \mathbf{L}), & \text{if } \mathbf{N} \cdot \mathbf{L} > 0 \\ k_a I_a, & \text{if } \mathbf{N} \cdot \mathbf{L} \leq 0 \end{cases}$$

- Where both k_a and k_d depend on surface material properties and are assigned values in the range from 0 to 1.0 for monochromatic lighting effects

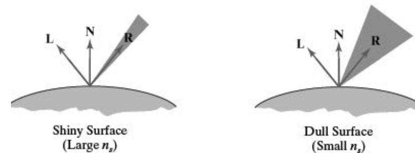
Specular Reflection and the Phong Model

- ✓ The bright spot, or specular reflection, that we can see on a shiny surface is the result of total, or near total, reflection of the incident light in a concentrated region around the **specular-reflection angle**.
- ✓ The below figure shows the specular reflection direction for a position on an illuminated surface

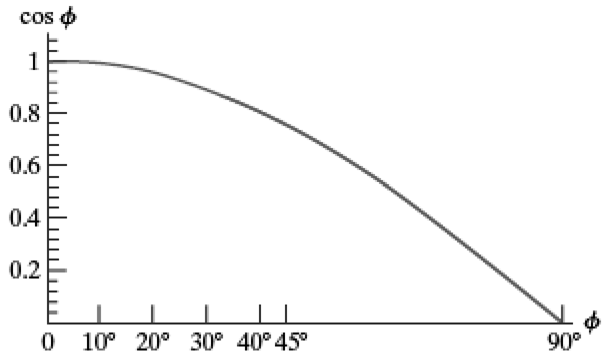


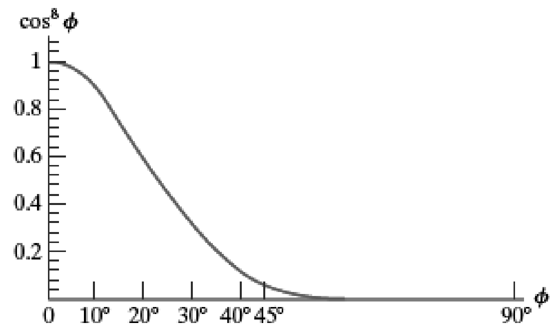
1. **N** represents: unit normal surface vector The specular reflection angle equals the angle of the incident light, with the two angles measured on opposite sides of the unit normal surface vector **N**
2. **R**represents the unit vector in the direction of ideal specular reflection,
3. **L** is the unit vector directed toward the point light source, and
4. **V**is the unit vector pointing to the viewer fromthe selected surface position.

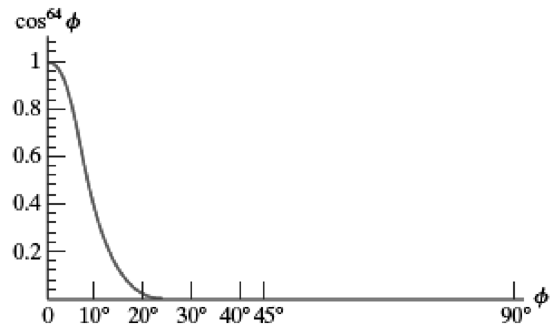
- ✓ Angle φ is the viewing angle relative to the specular-reflection direction **R**
- ✓ An empirical model for calculating the specular reflection range, developed by Phong Bui Tuong and called the **Phong specular-reflection model** or simply the **Phon G model,** sets the intensity of specular reflection proportional to $\cos^{ns} \varphi$
- ✓ **Angle φ** can be assigned values in the range 0◦ to 90◦, so that cos φ varies from 0 to 1.0.
- ✓ The value assigned to the specular-reflection exponent *ns* is determined by the type of surface that we want to display.
- ✓ A very shiny surface is modeled with a large value for *ns* (say, 100 or more), and smaller values (down to 1) are used for duller surfaces.
- ✓ For a perfect reflector, *ns* is infinite. For a rough surface, such as chalk or cinderblock, *ns* is assigned a value near 1.

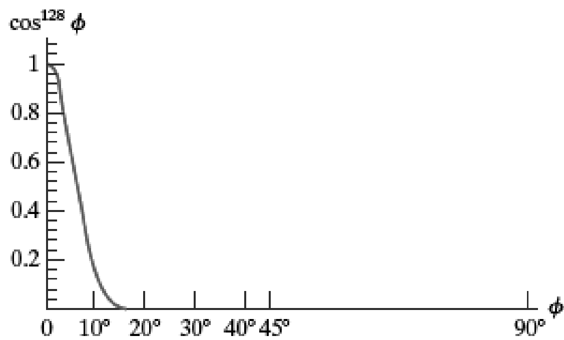


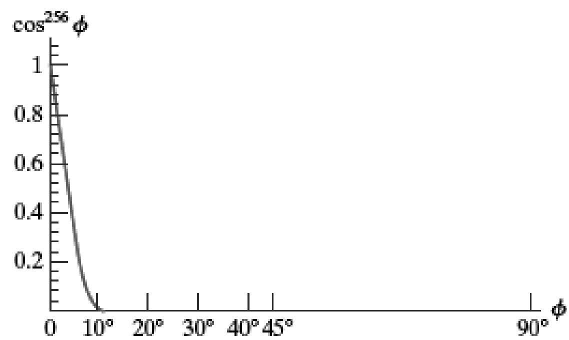
- ✓ Plots of cosns φ using five different values for the specular exponent n_s .



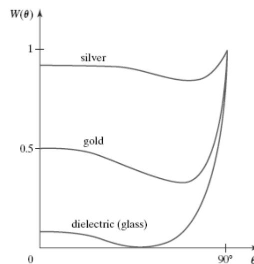








- ✓ We can approximately model monochromatic specular intensity variations using a **specular-reflection coefficient,** *W(θ)*, for each surface.
- ✓ In general, *W(θ)* tends to increase as the angle of incidence increases. At θ = 90◦, all the incident light is reflected (*W(θ)* = 1).



- Using the spectral-reflection function $W(\theta)$, we can write the Phong specular-reflection model as

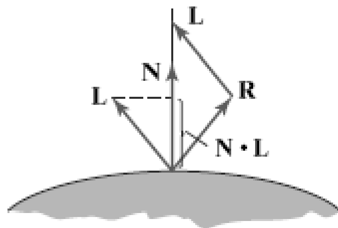
$$I_{l,\text{spec}} = W(\theta) I_l \cos^{n_s} \phi$$

where I_l is the intensity of the light source, and φ is the viewing angle relative to the specular-reflection direction **R**.

- Because**V**and **R** are unit vectors in the viewing and specular-reflection directions, we can calculate the value of cos φ with the dot product **V·R**.
- In addition, no specular effects are generated for the display of a surface if **V** and **L** are on the same side of the normal vector **N** or if the light source is behind the surface
- We can determine the intensity of the specular reflection due to a point light source at a surface position with the calculation

$$I_{l,\text{spec}} = \begin{cases} k_s I_l (\mathbf{V} \cdot \mathbf{R})^{n_s}, & \text{if } \mathbf{V} \cdot \mathbf{R} > 0 \quad \text{and} \quad \mathbf{N} \cdot \mathbf{L} > 0 \\ 0.0, & \text{if } \mathbf{V} \cdot \mathbf{R} \le 0 \quad \text{or} \quad \mathbf{N} \cdot \mathbf{L} \le 0 \end{cases}$$

- The direction for **R**, the reflection vector, can be computed from the directions for vectors **L** and **N**.



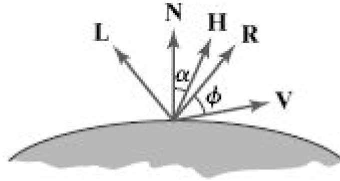
- The projection of **L** onto the direction of the normal vector has a magnitude equal to the dot product**N·L**, which is also equal to the magnitude of the projection of unit vector **R** onto the direction of **N**.
- Therefore, from this diagram, we see that

  R + **L** = *(*2**N·L***)***N**

  and the specular-reflection vector is obtained as

  R = *(*2**N·L***)***N** – **L**
- A somewhat simplified Phong model is obtained using the **halfway vector H** between **L** and **V** to calculate the range of specular reflections.

- ✓ If we replace **V·R** in the Phong model with the dot product**N·H**, this simply replaces the empirical cos φ calculation with the empirical cos α calculation



- ✓ The halfway vector is obtained as

$$\mathbf{H} = \frac{\mathbf{L} + \mathbf{V}}{|\mathbf{L} + \mathbf{V}|}$$

- ✓ For nonplanar surfaces, **N·H** requires less computation than **V·R** because the calculation of **R** at each surface point involves the variable vector **N**.

3.3.4 OpenGL Illumination Functions

OpenGL Point Light-Source Function

glLight* (lightName, lightProperty, propertyValue);

- ➔ A suffix code of **i** or **f** is appended to the function name, depending on the data type of the property value
- ➔ **lightName:** GL_LIGHT0, GL_LIGHT1, GL_LIGHT2, . . . , GL_LIGHT7
- ➔ **lightProperty:** must be assigned one of the OpenGL symbolic property constants

glEnable (lightName);→ turn on that light with the command

glEnable (GL_LIGHTING);→ activate the OpenGL lighting routines

Specifying an OpenGL Light-Source Position and Type

GL_POSITION:

- ➔ specifies light-source position
- ➔ this symbolic constant is used to set two light-source properties at the same time: the light-source position and the *light-source type*

Example:

GLfloat light1PosType [ ] = {2.0, 0.0, 3.0, 1.0};

```
GLfloat light2PosType [ ] = {0.0, 1.0, 0.0, 0.0};
glLightfv (GL_LIGHT1, GL_POSITION, light1PosType);
glEnable (GL_LIGHT1);
glLightfv (GL_LIGHT2, GL_POSITION, light2PosType);
glEnable (GL_LIGHT2);
```

Specifying OpenGL Light-Source Colors

➔ Unlike an actual light source, an OpenGL light has three different color properties the symbolic color-property constants **GL_AMBIENT, GL_DIFFUSE,** and **GL_SPECULAR**

Example:

```
GLfloat blackColor [ ] = {0.0, 0.0, 0.0, 1.0};
GLfloat whiteColor [ ] = {1.0, 1.0, 1.0, 1.0};
glLightfv (GL_LIGHT3, GL_AMBIENT, blackColor);
glLightfv (GL_LIGHT3, GL_DIFFUSE, whiteColor);
glLightfv (GL_LIGHT3, GL_SPECULAR, whiteColor);
```

Specifying Radial-Intensity Attenuation Coefficients

➔ For an OpenGL Light Source we could assign the radial-attenuation coefficient values as

```
glLightf (GL_LIGHT6, GL_CONSTANT_ATTENUATION, 1.5);
glLightf (GL_LIGHT6, GL_LINEAR_ATTENUATION, 0.75);
glLightf (GL_LIGHT6, GL_QUADRATIC_ATTENUATION, 0.4);
```

OpenGL Directional Light Sources (Spotlights)

➔ There are three OpenGL property constants for directional effects: **GL_SPOT_DIRECTION, GL_SPOT_CUTOFF**, and **GL_SPOT_EXPONENT**

```
GLfloat dirVector [ ] = {1.0, 0.0, 0.0};
glLightfv (GL_LIGHT3, GL_SPOT_DIRECTION, dirVector);
glLightf (GL_LIGHT3, GL_SPOT_CUTOFF, 30.0);
glLightf (GL_LIGHT3, GL_SPOT_EXPONENT, 2.5);
```

OpenGL Global Lighting Parameters

glLightModel* (paramName, paramValue);

- ➔ We append a suffix code of i or f, depending on the data type of the parameter value.
- ➔ In addition, for vector data, we append the suffix code v.
- ➔ Parameter paramName is assigned an OpenGL symbolic constant that identifies the global property to be set, and parameter paramValue is assigned a single value or set of values.

```
globalAmbient [ ] = {0.0, 0.0, 0.3, 1.0);
glLightModelfv (GL_LIGHT_MODEL_AMBIENT, globalAmbient);
```

glLightModeli (GL_LIGHT_MODEL_LOCAL_VIEWER, GL_TRUE);

- ➔ turn off this default and use the actual viewing position (which is the viewing-coordinate origin) to calculate V

Texture

- ➔ patterns are combined only with the nonspecular color, and then the two colors are combined.
- ➔ We select this two-color option with

glLightModeli (GL_LIGHT_MODEL_COLOR_CONTROL,
GL_SEPARATE_SPECULAR_COLOR);

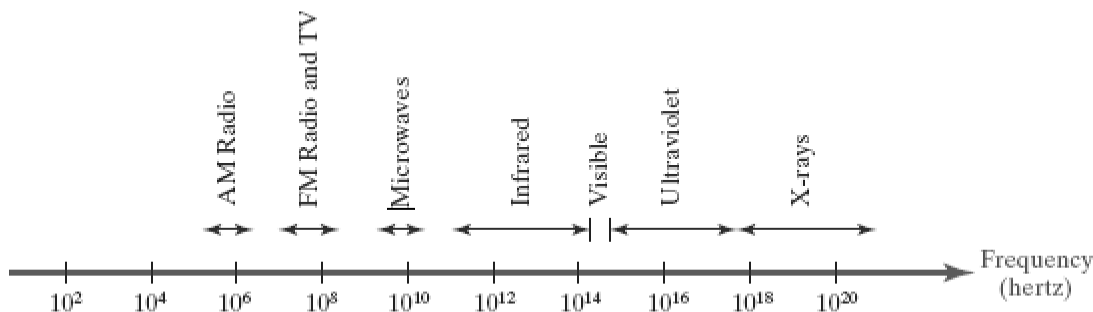
Color Models

3.3.5 Properties of Light

- ✓ We can characterize light as radiant energy, but we also need other concepts to describe our perception of light.

The Electromagnetic Spectrum

- ✓ Color is electromagnetic radiation within a narrow frequency band.
- ✓ Some of the other frequency groups in the electromagnetic spectrum are referred to as radio waves, microwaves, infrared waves, and X-rays. The frequency is shown below



- ✓ Each frequency value within the visible region of the electromagnetic spectrum corresponds to a distinct **spectral color.**
- ✓ At the low-frequency end (approximately 3.8×1014 hertz) are the red colors, and at the high-frequency end (approximately 7.9 × 1014 hertz) are the violet colors.
- ✓ In the wave model of electromagnetic radiation, light can be described as oscillating transverse electric and magnetic fields propagating through space.
- ✓ The electric and magnetic fields are oscillating in directions that are perpendicular to each other and to the direction of propagation.
- ✓ For one spectral color (a monochromatic wave), the wavelength and frequency are inversely proportional to each other, with the proportionality constant as the speed of light (*c*):

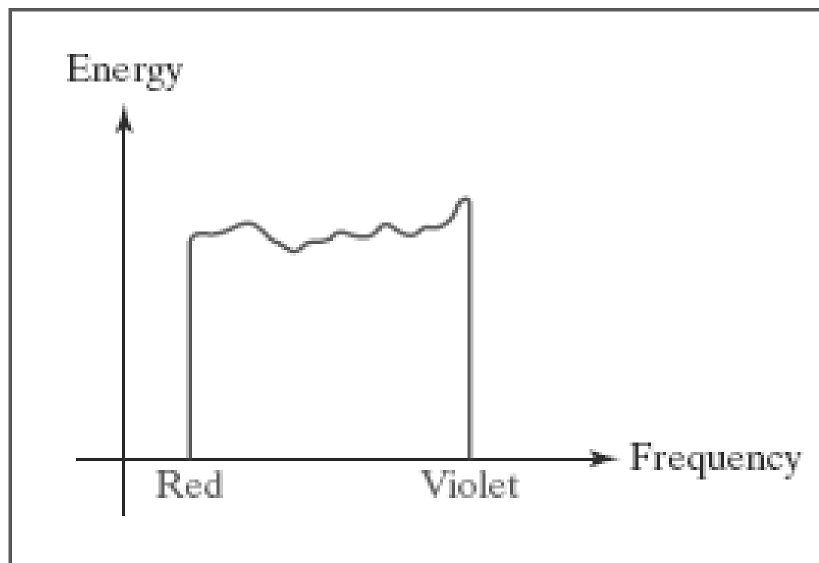
$$c = \lambda f$$

- ✓ A light source such as the sun or a standard household light bulb emits all frequencies within the visible range to produce white light.

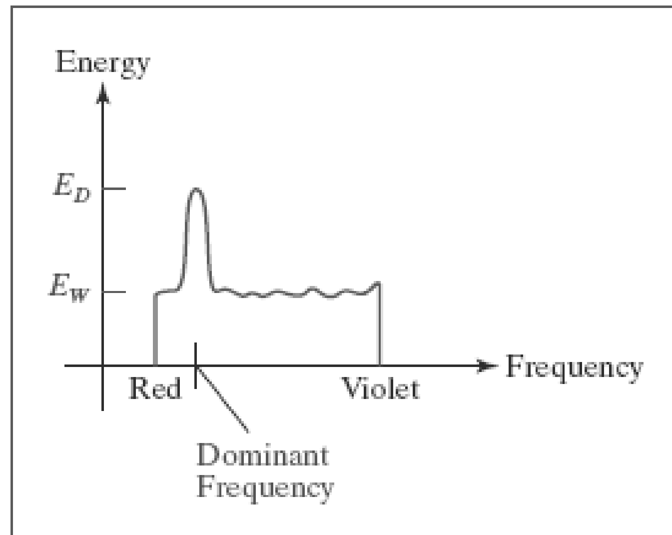
- ✓ When white light is incident upon an opaque object, some frequencies are reflected and some are absorbed.
- ✓ If low frequencies are predominant in the reflected light, the object is described as red. In this case, we say that the perceived light has a **dominant frequency** (or **dominant wavelength**) at the red end of the spectrum.
- ✓ The dominant frequency is also called the **hue,** or simply the **color,** of the light.

Psychological Characteristics of Color

- Other properties besides frequency are needed to characterize our perception of Light
- **Brightness:** which corresponds to the total light energy and can be quantified as the luminance of the light.
- **Purity**, or the **saturation** of the light: Purity describes how close a light appears to be to a pure spectral color, such as red.
- **chromaticity**, is used to refer collectively to the two properties describing color characteristics: purity and dominant frequency (hue).
- We can calculate the brightness of the source as the area under the curve, which gives the total energy density emitted.
- Purity (saturation) depends on the difference between *ED* and *EW*
- Below figure shows *Energy distribution for a white light source*



- Below figure shows, Energy distribution for a light source with a dominant frequency near the red end of the frequency range.



3.3.7 Color Models

- Any method for explaining the properties or behavior of color within some particular context is called a **color model.**

Primary Colors

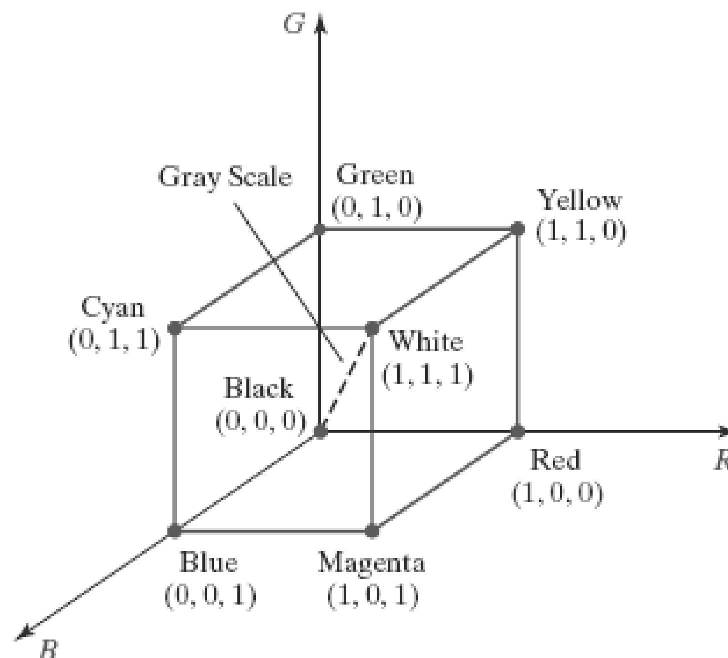
- The hues that we choose for the sources are called the **primary colors,** and the **color gamut** for the model is the set of all colors that we can produce from the primary colors.
- Two primaries that produce white are referred to as **complementary colors.**
- Examples of complementary color pairs are red and cyan, green and magenta, and blue and yellow

Intuitive Color Concepts

- An artist creates a color painting by mixing color pigments with white and black pigments to form the various shades, tints, and tones in the scene.
- Starting with the pigment for a "pure color" ("pure hue"), the artist adds a black pigment to produce different **shades** of that color.
- **Tones** of the color are produced by adding both black and white pigments.

The RGB Color Model

- According to the *tristimulus theory* of vision, our eyes perceive color through the stimulation of three visual pigments in the cones of the retina.
- One of the pigments is most sensitive to light with a wavelength of about 630 nm (red), another has its peak sensitivity at about 530 nm (green), and the third pigment is most receptive to light with a wavelength of about 450 nm (blue).
- The three primaries red, green, and blue, which is referred to as the *RGB color model*.
- We can represent this model using the unit cube defined on *R*, *G*, and *B* axes, as shown in Figure



- The origin represents black and the diagonally opposite vertex, with coordinates (1, 1, 1), is white the RGB color scheme is an additive model.
- Each color point within the unit cube can be represented as a weighted vector sum of the primary colors, using unit vectors **R**, **G**, and **B**:

$$C(\lambda) = (R, G, B) = R\mathbf{R} + G\,\mathbf{G} + B\,\mathbf{B}$$

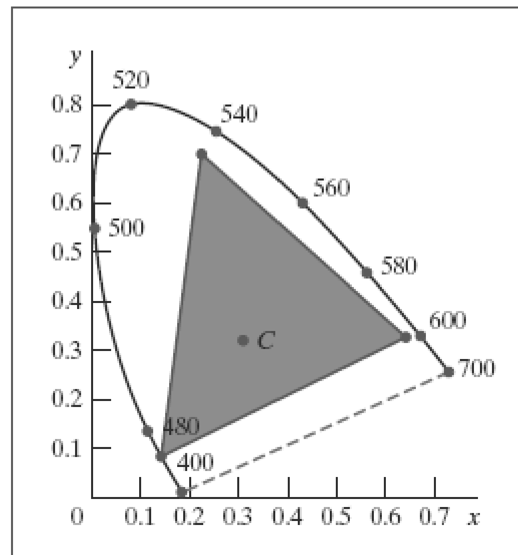
where parameters *R*, *G*, and *B* are assigned values in the range from 0 to 1.0

- Chromaticity coordinates for the National Television System Committee (NTSC) standard RGB phosphors are listed in Table

RGB (x, y) Chromaticity Coordinates

	NTSC Standard	CIE Model	Approx. Color Monitor Values
R	(0.670, 0.330)	(0.735, 0.265)	(0.628, 0.346)
G	(0.210, 0.710)	(0.274, 0.717)	(0.268, 0.588)
B	(0.140, 0.080)	(0.167, 0.009)	(0.150, 0.070)

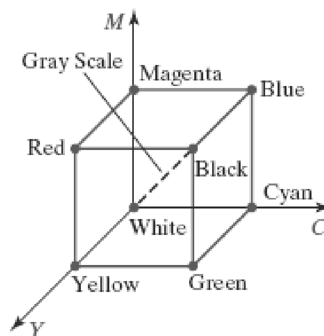
- Below figure shows the approximate color gamut for the NTSC standard RGB primaries



The CMY and CMYK Color Models

The CMY Parameters

- A subtractive color model can be formed with the three primary colors cyan, magenta, and yellow
- A unit cube representation for the CMY model is illustrated in Figure



- In the CMY model, the spatial position (1, 1, 1) represents black, because all components of the incident light are subtracted.
- The origin represents white light.
- Equal amounts of each of the primary colors produce shades of gray along the main diagonal of the cube.
- A combination of cyan and magenta ink produces blue light, because the red and green components of the incident light are absorbed.
- Similarly, a combination of cyan and yellow ink produces green light, and a combination of magenta and yellow ink yields red light.
- The CMY printing process often uses a collection of four ink dots, which are arranged in a close pattern somewhat as an RGB monitor uses three phosphor dots.
- Thus, in practice, the CMY color model is referred to as the CMYK model, where *K* is the black color parameter.
- One ink dot is used for each of the primary colors (cyan, magenta, and yellow), and one ink dot is black

<u>Transformations Between CMY and RGB Color Spaces</u>

- We can express the conversion from an RGB representation to a CMY representation using the following matrix transformation:

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- Where the white point in RGB space is represented as the unit column vector.
- And we convert from a CMY color representation to an RGB representation using the matrix transformation

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} C \\ M \\ Y \end{bmatrix}$$

Acknowledgements to

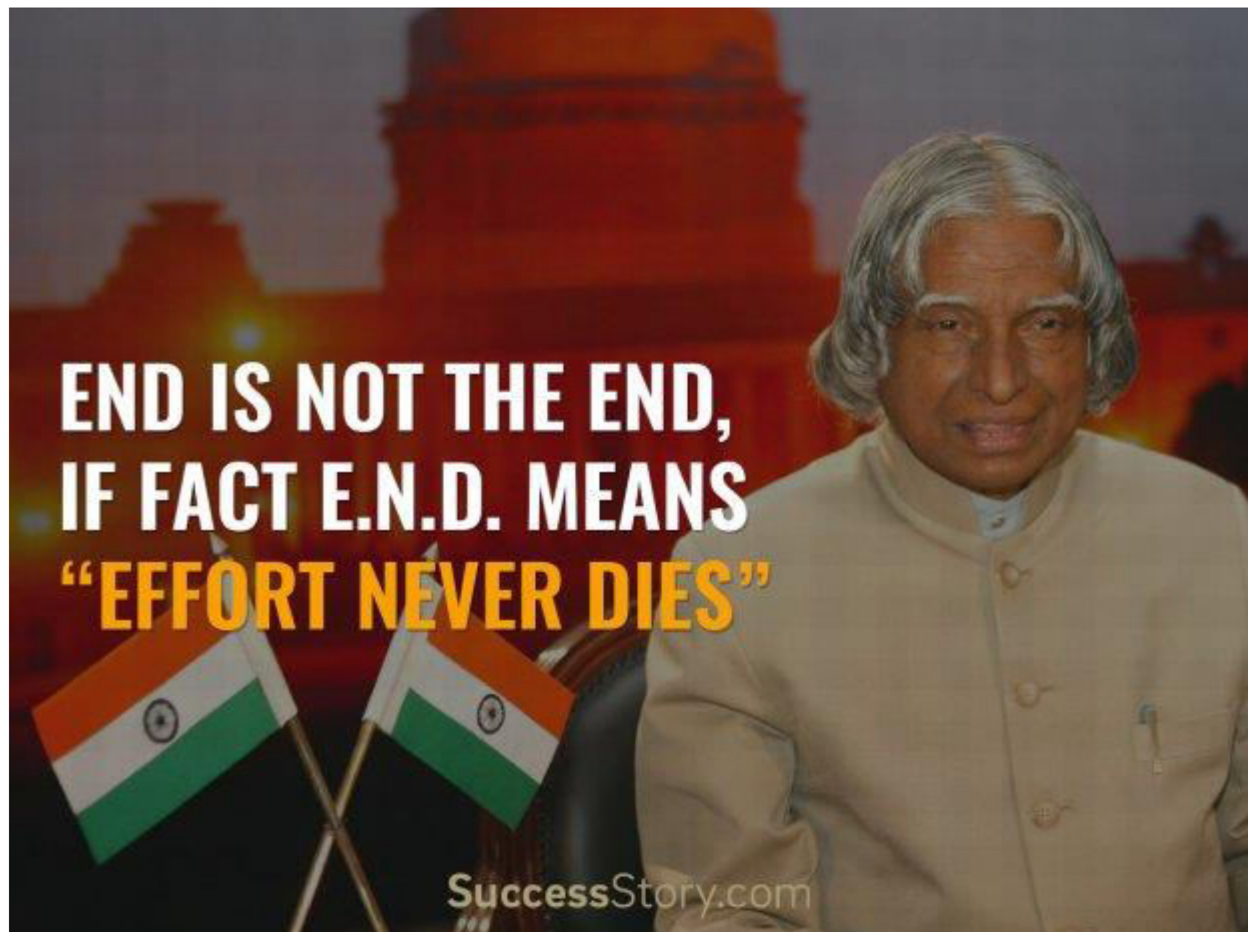
Donald Hearn & Pauline Baker: Computer Graphics with OpenGL

Version,3rd / 4th Edition, Pearson Education,2011

Edward Angel: Interactive Computer Graphics- A Top Down approach

with OpenGL, 5th edition. Pearson Education, 2008

M M Raiker, Computer Graphics using OpenGL, Filip learning/Elsevier



4 3D Viewing and Visible Surface Detection

4.1 3D viewing concepts,
4.2 3D viewing pipeline,
4.3 3D viewing coordinate parameters ,
4.4 Transformation from world to viewing coordinates,
4.5 Projection transformation,
4.6 Orthogonal projections,
4.7 Perspective projections,
4.8 The viewport transformation and 3D screen coordinates.
4.9 OpenGL 3D viewing functions.
Visible Surface Detection Methods:
4.10 Classification of visible surface Detection algorithms,
4.11Back face detection,
4.12 Depth buffer method and
4.13 OpenGL visibility detection functions.

Three-Dimensional Viewing

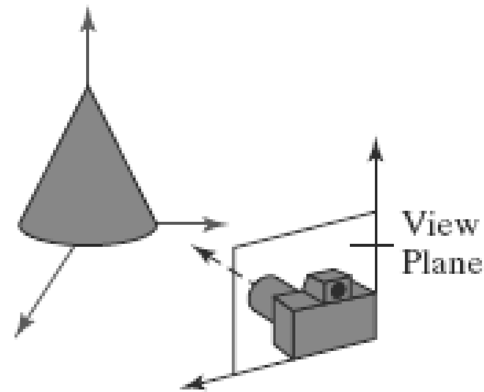
4.1 Overview of Three-Dimensional Viewing Concepts

- When we model a three-dimensional scene, each object in the scene is typically defined with a set of surfaces that form a closed boundary around the object interior.
- In addition to procedures that generate views of the surface features of an object, graphics packages sometimes provide routines for displaying internal components or cross-sectional views of a solid object.
- Many processes in three-dimensional viewing, such as the clipping routines, are similar to those in the two-dimensional viewing pipeline.
- But three-dimensional viewing involves some tasks that are not present in twodimensional Viewing

Viewing a Three-Dimensional Scene

- To obtain a display of a three-dimensional world-coordinate scene, we first set up a coordinate reference for the viewing, or "camera," parameters.

- This coordinate reference defines the position and orientation for a *view plane* (or *projection plane*) that corresponds to a camera film plane as shown in below figure.

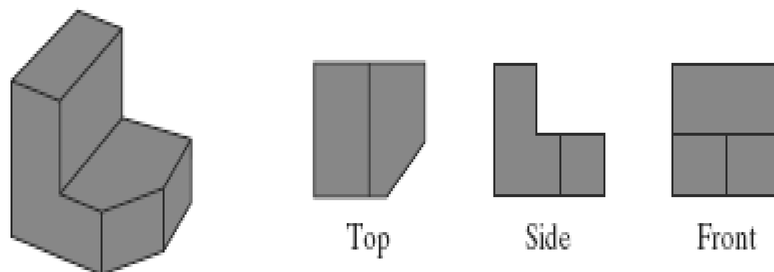


Coordinate reference for obtaining a selected view of a three-dimensional scene.

- We can generate a view of an object on the output device in wireframe (outline) form, or we can apply lighting and surface-rendering techniques to obtain a realistic shading of the visible surfaces Projections

Two methods:

1. One method for getting the description of a solid object onto a view plane is to project points on the object surface along parallel lines. This technique, called *parallel projection*



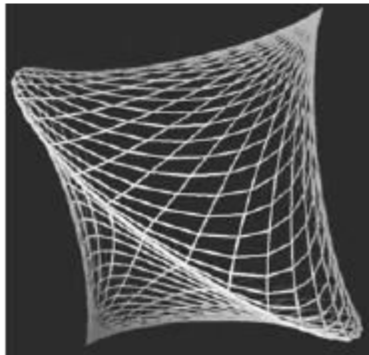
Three parallel-projection views of an object, showing relative proportions from different viewing positions

2. Another method for generating a view of a three-dimensional scene is to project points to the view plane along converging paths. This process, called a *perspective projection,*

causes objects farther from the viewing position to be displayed smaller than objects of the same size that are nearer to the viewing position

Depth Cueing

- ✓ Depth information is important in a three-dimensional scene so that we can easily identify, for a particular viewing direction, which is the front and which is the back of each displayed object.
- ✓ There are several ways in which we can include depth information in the two-dimensional representation of solid objects.
- ✓ A simple method for indicating depth with wire-frame displays is to vary the brightness of line segments according to their distances from the viewing position which is termed as depth cueing.



A wire-frame object displayed with depth cueing, so that the brightness of lines decreases from the front of the object to the back

- ✓ The lines closest to the viewing position are displayed with the highest intensity, and lines farther away are displayed with decreasing intensities.
- ✓ Depth cueing is applied by choosing a maximum and a minimum intensity value and a range of distances over which the intensity is to vary.
- ✓ Another application of depth cuing is modeling the effect of the atmosphere on the perceived intensity of objects

Identifying Visible Lines and Surfaces

- ➢ One approach is simply to highlight the visible lines or to display them in a different color. Another technique, commonly used for engineering drawings, is to display the

nonvisible lines as dashed lines. Or we could remove the nonvisible lines from the display

Surface Rendering

- We set the lighting conditions by specifying the color and location of the light sources, and we can also set background illumination effects.
- Surface properties of objects include whether a surface is transparent or opaque and whether the surface is smooth or rough.
- We set values for parameters to model surfaces such as glass, plastic, wood-grain patterns, and the bumpy appearance of an orange.

Exploded and Cutaway Views

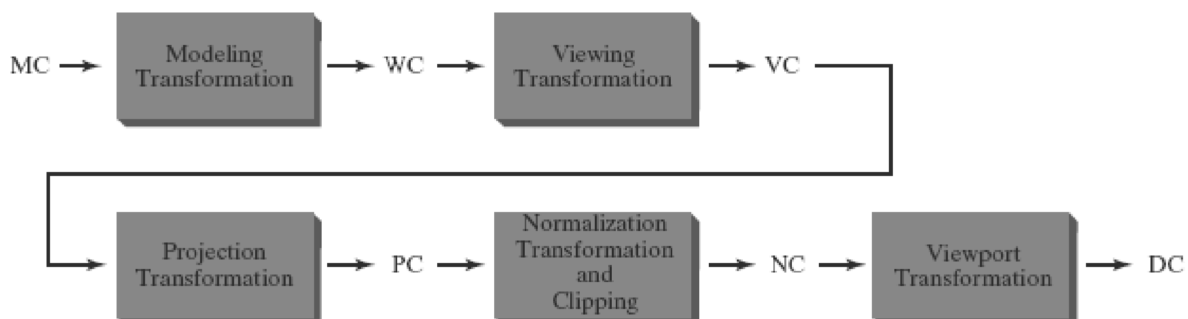
- Exploded and cutaway views of such objects can then be used to show the internal structure and relationship of the object parts.
- An alternative to exploding an object into its component parts is a cutaway view, which removes part of the visible surfaces to show internal structure

Three-Dimensional and Stereoscopic Viewing

- Three-dimensional views can be obtained by reflecting a raster image from a vibrating, flexible mirror.
- The vibrations of the mirror are synchronized with the display of the scene on the cathode ray tube (CRT).
- As the mirror vibrates, the focal length varies so that each point in the scene is reflected to a spatial position corresponding to its depth.
- Stereoscopic devices present two views of a scene: one for the left eye and the other for the right eye.
- The viewing positions correspond to the eye positions of the viewer. These two views are typically displayed on alternate refresh cycles of a raster monitor

4.2 The Three-Dimensional Viewing Pipeline

- ✓ First of all, we need to choose a viewing position corresponding to where we would place a camera.
- ✓ We choose the viewing position according to whether we want to display a front, back, side, top, or bottom view of the scene.
- ✓ We could also pick a position in the middle of a group of objects or even inside a single object, such as a building or a molecule.
- ✓ Then we must decide on the camera orientation.
- ✓ Finally, when we snap the shutter, the scene is cropped to the size of a selected clipping window, which corresponds to the aperture or lens type of a camera, and light from the visible surfaces is projected onto the camera film.
- ✓ Some of the viewing operations for a three-dimensional scene are the same as, or similar to, those used in the two-dimensional viewing pipeline.
- ✓ A two-dimensional viewport is used to position a projected view of the three dimensional scene on the output device, and a two-dimensional clipping window is used to select a view that is to be mapped to the viewport.
- ✓ Clipping windows, viewports, and display windows are usually specified as rectangles with their edges parallel to the coordinate axes.
- ✓ The viewing position, view plane, clipping window, and clipping planes are all specified within the viewing-coordinate reference frame.

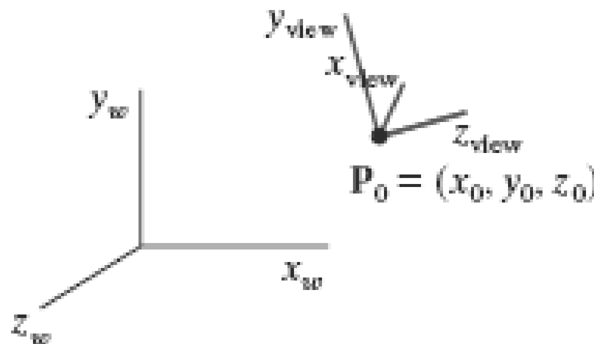


- ✓ Figure above shows the general processing steps for creating and transforming a three-dimensional scene to device coordinates.
- ✓ Once the scene has been modeled in world coordinates, a viewing-coordinate system is selected and the description of the scene is converted to viewing coordinates

- ✓ A two-dimensional clipping window, corresponding to a selected camera lens, is defined on the projection plane, and a three-dimensional clipping region is established.
- ✓ This clipping region is called the view volume.
- ✓ Projection operations are performed to convert the viewing-coordinate description of the scene to coordinate positions on the projection plane.
- ✓ Objects are mapped to normalized coordinates, and all parts of the scene outside the view volume are clipped off.
- ✓ The clipping operations can be applied after all device-independent coordinate transformations.
- ✓ We will assume that the viewport is to be specified in device coordinates and that normalized coordinates are transferred to viewport coordinates, following the clipping operations.
- ✓ The final step is to map viewport coordinates to device coordinates within a selected display window

4.3 Three-Dimensional Viewing-Coordinate Parameters

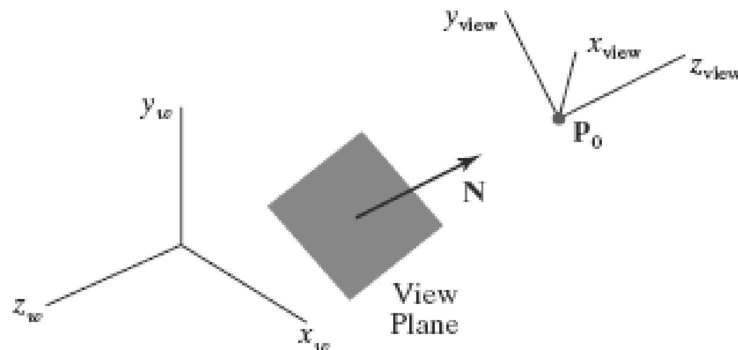
- ➢ Select a world-coordinate position $P_0 = (x_0, y_0, z_0)$ for the viewing origin, which is called the view point or viewing position and we specify a view-up vector V, which defines the y_{view} direction.
- ➢ Figure below illustrates the positioning of a three-dimensional viewing-coordinate frame within a world system.



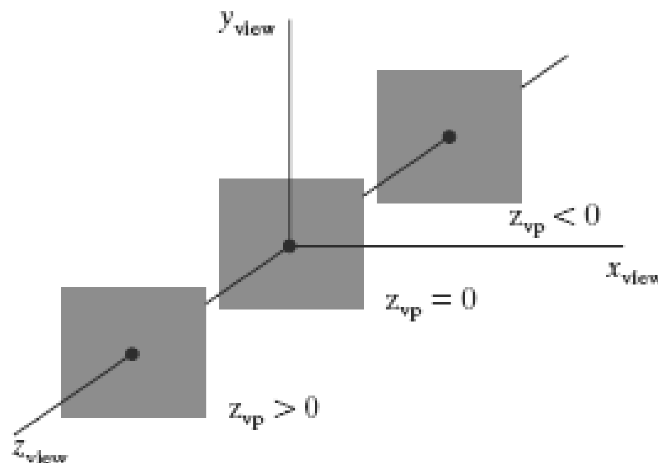
A right-handed viewing-coordinate system, with axes x_{view}, y_{view}, and z_{view}, relative to a right-handed world-coordinate frame.

The View-Plane Normal Vector

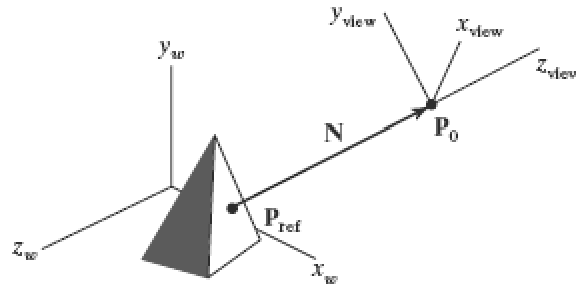
- ✓ Because the viewing direction is usually along the *z*view axis, the view plane, also called the projection plane, is normally assumed to be perpendicular to this axis.
- ✓ Thus, the orientation of the view plane, as well as the direction for the positive *z*view axis, can be definedwith a view-plane normal vector N,



- ✓ An additional scalar parameter is used to set the position of the view plane at some coordinate value z_{vp} along the z_{view} axis,



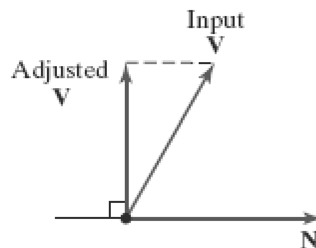
- ✓ This parameter value is usually specified as a distance from the viewing origin along the direction of viewing, which is often taken to be in the negative z_{view} direction.
- ✓ Vector N can be specified in various ways. In some graphics systems, the direction for N is defined to be along the line from the world-coordinate origin to a selected point position.
- ✓ Other systems take N to be in the direction from a reference point P_{ref} to the viewing origin P_0,



Specifying the view-plane normal vector N as the direction from a selected reference point P_{ref} to the viewing-coordinate origin P_0.

The View-Up Vector

- Once we have chosen a view-plane normal vector N, we can set the direction for the view-up vector V.
- This vector is used to establish the positive direction for the y_{view} axis.
- Usually, V is defined by selecting a position relative to the world-coordinate origin, so that the direction for the view-up vector is from the world origin to this selected position



- Because the view-plane normal vector N defines the direction for the z_{view} axis, vector V should be perpendicular to N.
- But, in general, it can be difficult to determine a direction for V that is precisely perpendicular to N.
- Therefore, viewing routines typically adjust the user-defined orientation of vector V,

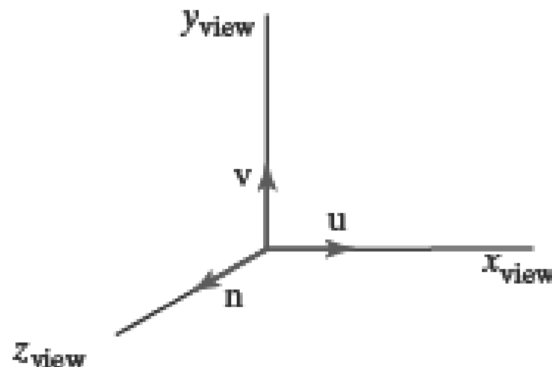
The uvn Viewing-Coordinate Reference Frame

- ✓ Left-handed viewing coordinates are sometimes used in graphics packages, with the viewing direction in the positive z_{view} direction.

- ✓ With a left-handed system, increasing z_{view} values are interpreted as being farther from the viewing position along the line of sight.
- ✓ But right-handed viewing systems are more common, because they have the same orientation as the world-reference frame.
- ✓ Because the view-plane normal N defines the direction for the z_{view} axis and the view-up vector V is used to obtain the direction for the y_{view} axis, we need only determine the direction for the x_{view} axis.
- ✓ Using the input values for N and V,we can compute a third vector, U, that is perpendicular to both N and V.
- ✓ Vector U then defines the direction for the positive x_{view} axis.
- ✓ We determine the correct direction for U by taking the vector cross product of V and N so as to form a right-handed viewing frame.
- ✓ The vector cross product of N and U also produces the adjusted value for V, perpendicular to both N and U, along the positive y_{view} axis.
- ✓ Following these procedures, we obtain the following set of unit axis vectors for a right-handed viewing coordinate system.

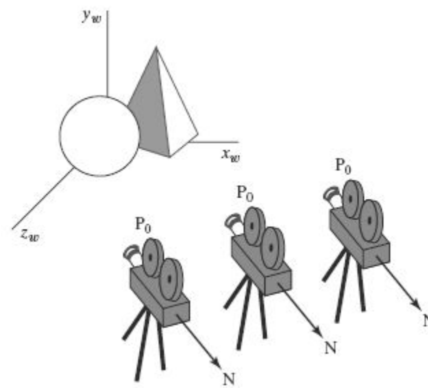
$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = (n_x, n_y, n_z)$$
$$\mathbf{u} = \frac{\mathbf{V} \times \mathbf{n}}{|\mathbf{V} \times \mathbf{n}|} = (u_x, u_y, u_z)$$
$$\mathbf{v} = \mathbf{n} \times \mathbf{u} = (v_x, v_y, v_z)$$

- ✓ The coordinate system formed with these unit vectors is often described as a uvn viewing-coordinate reference frame

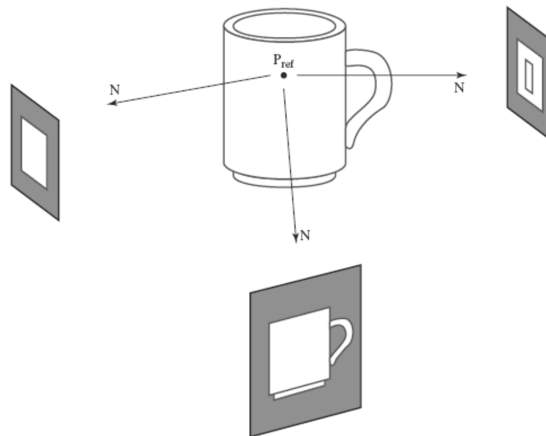


Generating Three-Dimensional Viewing Effects

- ✓ By varying the viewing parameters, we can obtain different views of objects in a scene.
- ✓ we could change the direction of N to display objects at positions around the viewing-coordinate origin.
- ✓ We could also vary N to create a composite display consisting of multiple views from a fixed camera position.
- ✓ In interactive applications, the normal vectorNis the viewing parameter that is most often changed. Of course,when we change the direction for N,we also have to change the other axis vectors to maintain a right-handed viewing-coordinate system.
- ✓ If we want to simulate an animation panning effect, as when a camera moves through a scene or follows an object that is moving through a scene, we can keep the direction for N fixed as we move the view point,



- ✓ Alternatively, different views of an object or group of objects can be generated using geometric transformations without changing the viewing parameters



4.4 Transformation from World to Viewing Coordinates

- ✓ In the three-dimensional viewing pipeline, the first step after a scene has been constructed is to transfer object descriptions to the viewing-coordinate reference frame.
- ✓ This conversion of object descriptions is equivalent to a sequence of transformations that superimposes the viewing reference frame onto the world frame

  1. Translate the viewing-coordinate origin to the origin of the worldcoordinate system.
  2. Apply rotations to align the $x$view, $y$view, and $z$view axes with the world x_w, y_w, and z_w axes, respectively.

- ✓ The viewing-coordinate origin is at world position $P_0 = (x_0, y_0, z_0)$. Therefore, the matrix for translating the viewing origin to the world origin is

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- ✓ For the rotation transformation, we can use the unit vectors u, v, and n to form the composite rotation matrix that superimposes the viewing axes onto the world frame. This transformation matrix is

$$\mathbf{R} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  where the elements of matrix R are the components of the uvn axis vectors.

- ✓ The coordinate transformation matrix is then obtained as the product of the preceding translation and rotation matrices:

$$\mathbf{M}_{WC,\,VC} = \mathbf{R} \cdot \mathbf{T} = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{u} \cdot \mathbf{P}_0 \\ v_x & v_y & v_z & -\mathbf{v} \cdot \mathbf{P}_0 \\ n_x & n_y & n_z & -\mathbf{n} \cdot \mathbf{P}_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

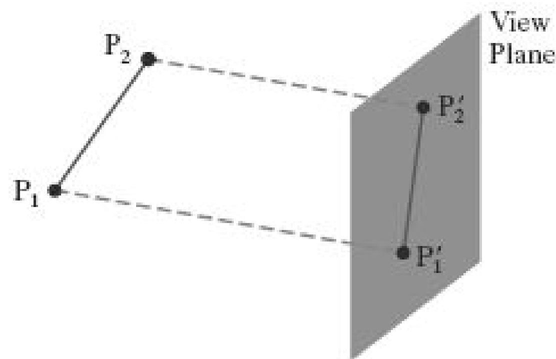
- ✓ Translation factors in this matrix are calculated as the vector dot product of each of the u, v, and n unit vectors with P_0, which represents a vector from the world origin to the viewing origin.

- ✓ These matrix elements are evaluated as

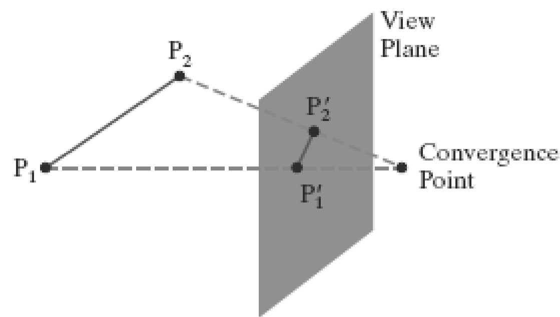
$$-\mathbf{u} \cdot \mathbf{P}_0 = -x_0 u_x - y_0 u_y - z_0 u_z$$
$$-\mathbf{v} \cdot \mathbf{P}_0 = -x_0 v_x - y_0 v_y - z_0 v_z$$
$$-\mathbf{n} \cdot \mathbf{P}_0 = -x_0 n_x - y_0 n_y - z_0 n_z$$

4.5 Projection Transformations

- ➔ Graphics packages generally support both parallel and perspective projections.
- ➔ In a parallel projection, coordinate positions are transferred to the view plane along parallel lines.
- ➔ A parallel projection preserves relative proportions of objects, and this is the method used in computeraided drafting and design to produce scale drawings of three-dimensional objects.
- ➔ All parallel lines in a scene are displayed as parallel when viewed with a parallel projection.
- ➔ There are two general methods for obtaining a parallel-projection view of an object: We can project along lines that are perpendicular to the view plane, or we can project at an oblique angle to the view plane

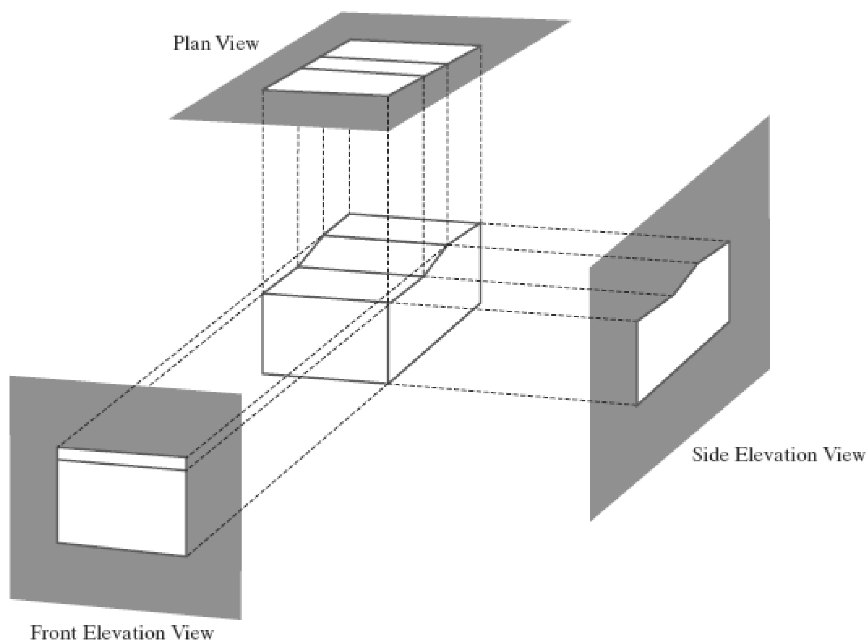


- ➔ For a perspective projection, object positions are transformed to projection coordinates along lines that converge to a point behind the view plane.
- ➔ Unlike a parallel projection, a perspective projection does not preserve relative proportions of objects.
- ➔ But perspective views of a scene are more realistic because distant objects in the projected display are reduced in size.



4.6 Orthogonal Projections

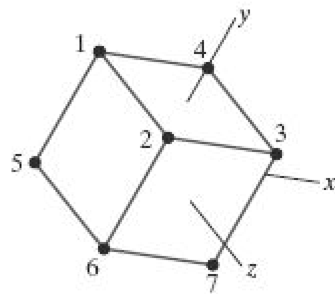
- ➔ A transformation of object descriptions to a view plane along lines that are all parallel to the view-plane normal vector N is called an orthogonal projection also termed as orthographic projection.
- ➔ This produces a parallel-projection transformation in which the projection lines are perpendicular to the view plane.
- ➔ Orthogonal projections are most often used to produce the front, side, and top views of an object

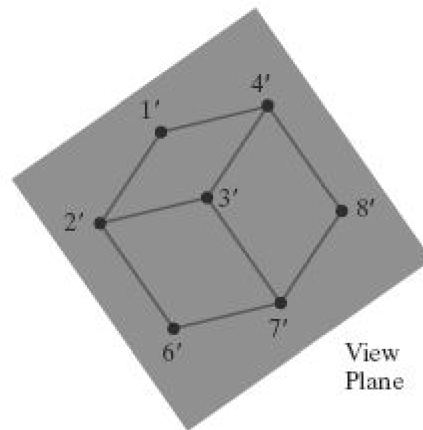


- ➔ Front, side, and rear orthogonal projections of an object are called *elevations;* and a top orthogonal projection is called a *plan view*

Axonometric and Isometric Orthogonal Projections

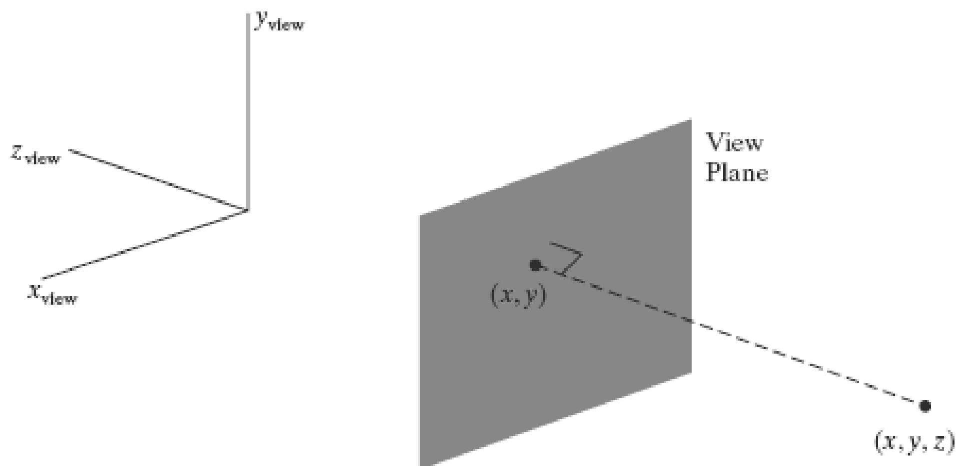
- We can also form orthogonal projections that display more than one face of an object. Such views are called axonometric orthogonal projections.
- The most commonly used axonometric projection is the isometric projection, which is generated by aligning the projection plane (or the object) so that the plane intersects each coordinate axis in which the object is defined, called the *principal axes,* at the same distance from the origin





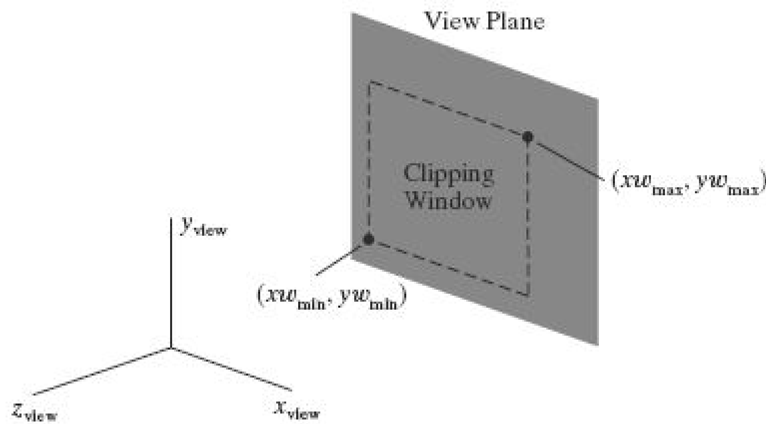
Orthogonal Projection Coordinates

- With the projection direction parallel to the z_{view} axis, the transformation equations for an orthogonal projection are trivial. For any position (x, y, z) in viewing coordinates, as in Figure below, the projection coordinates are $x_p = x$, $y_p = y$

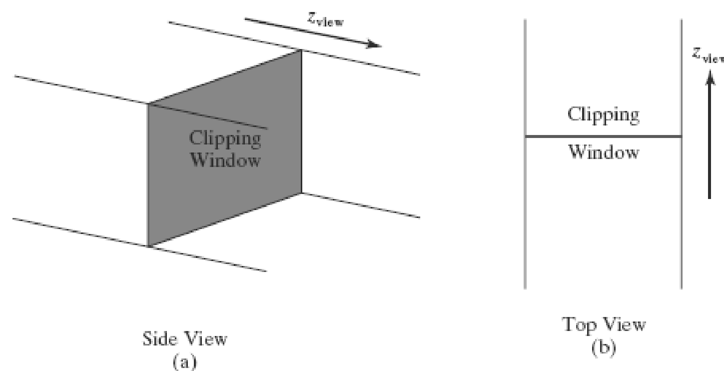


Clipping Window and Orthogonal-Projection View Volume

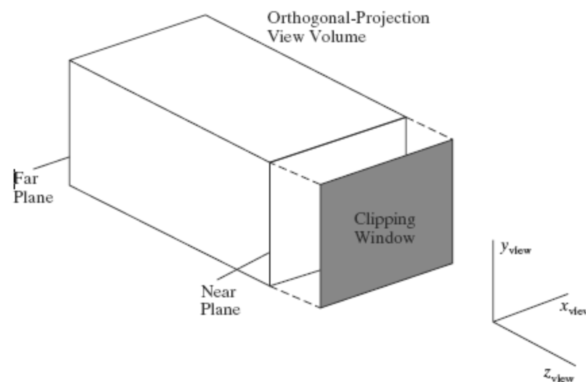
- In OpenGL, we set up a clipping window for three-dimensional viewing just as we did for two-dimensional viewing, by choosing two-dimensional coordinate positions for its lower-left and upper-right corners.
- For three-dimensional viewing, the clipping window is positioned on the view plane with its edges parallel to the x_{view} and y_{view} axes, as shown in Figure below . If we want to use some other shape or orientation for the clipping window, we must develop our own viewing procedures



- The edges of the clipping window specify the *x* and *y* limits for the part of the scene that we want to display.
- These limits are used to form the top, bottom, and two sides of a clipping region called the orthogonal-projection view volume.
- Because projection lines are perpendicular to the view plane, these four boundaries are planes that are also perpendicular to the view plane and that pass through the edges of the clipping window to form an infinite clipping region, as in Figure below.

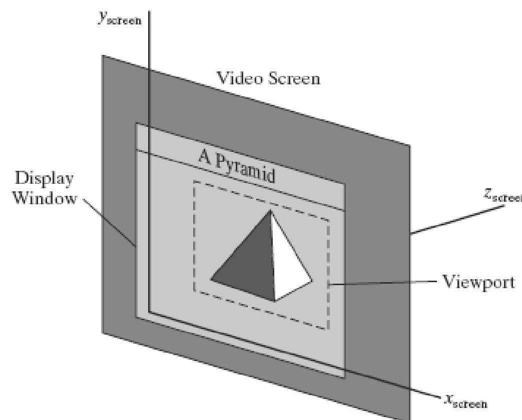


- These two planes are called the near-far clipping planes, or the front-back clipping planes.
- The near and far planes allow us to exclude objects that are in front of or behind the part of the scene that we want to display.
- When the near and far planes are specified, we obtain a finite orthogonal view volume that is a *rectangular parallelepiped*, as shown in Figure below along with one possible placement for the view plane

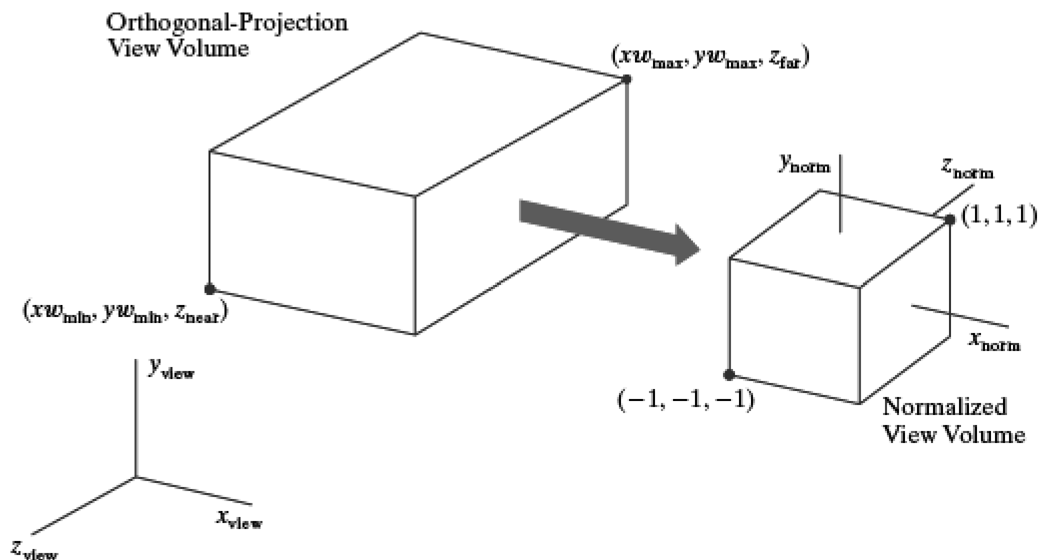


Normalization Transformation for an Orthogonal Projection

- Once we have established the limits for the view volume, coordinate descriptions inside this rectangular parallelepiped are the projection coordinates, and they can be mapped into a normalized view volume without any further projection processing.
- Some graphics packages use a unit cube for this normalized view volume, with each of the *x*, *y*, and *z* coordinates normalized in the range from 0 to 1.
- Another normalization-transformation approach is to use a symmetric cube, with coordinates in the range from −1 to 1



- We can convert projection coordinates into positions within a left-handed normalized-coordinate reference frame, and these coordinate positions will then be transferred to lefthanded screen coordinates by the viewport transformation.
- To illustrate the normalization transformation, we assume that the orthogonal-projection view volume is to be mapped into the symmetric normalization cube within a left-handed reference frame.
- Also, *z*-coordinate positions for the near and far planes are denoted as *z*near and *z*far, respectively. Figure below illustrates this normalization transformation



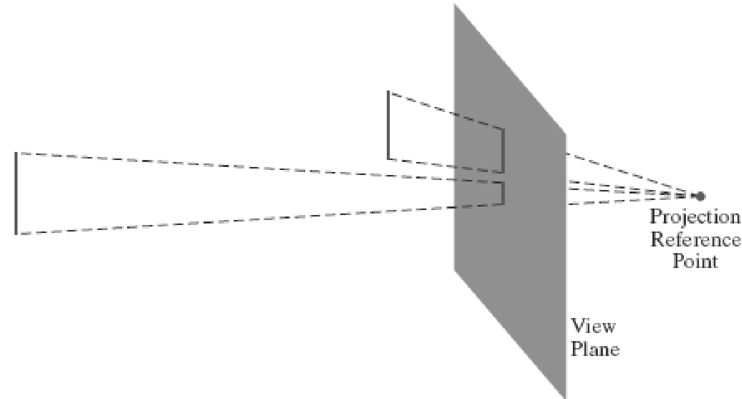
- The normalization transformation for the orthogonal view volume is

$$\mathbf{M}_{ortho,norm} = \begin{bmatrix} \frac{2}{xw_{max} - xw_{min}} & 0 & 0 & -\frac{xw_{max} + xw_{min}}{xw_{max} - xw_{min}} \\ 0 & \frac{2}{yw_{max} - yw_{min}} & 0 & -\frac{yw_{max} + yw_{min}}{yw_{max} - yw_{min}} \\ 0 & 0 & \frac{-2}{z_{near} - z_{far}} & \frac{z_{near} + z_{far}}{z_{near} - z_{far}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.7 Perspective Projections

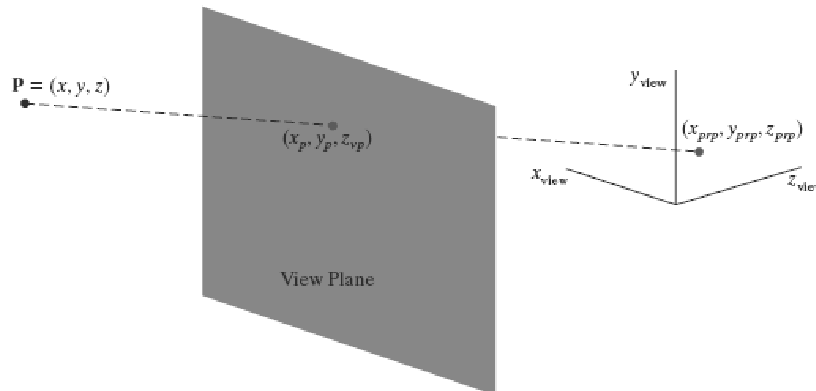
- ✓ We can approximate this geometric-optics effect by projecting objects to the view plane along converging paths to a position called the projection reference point (or center of projection).

- ✓ Objects are then displayed with foreshortening effects, and projections of distant objects are smaller than the projections of objects of the same size that are closer to the view plane



Perspective-Projection Transformation Coordinates

- ✓ Figure below shows the projection path of a spatial position (x, y, z) to a general projection reference point at $(x_{prp}, y_{prp}, z_{prp})$.



- ✓ The projection line intersects the view plane at the coordinate position (x_p, y_p, z_{vp}), where z_{vp} is some selected position for the view plane on the z_{view} axis.
- ✓ We can write equations describing coordinate positions along this perspective-projection line in parametric form as

$$x' = x - (x - x_{prp})u$$
$$y' = y - (y - y_{prp})u \qquad 0 \le u \le 1$$
$$z' = z - (z - z_{prp})u$$

- ✓ On the view plane, $z' = z_{vp}$ and we can solve the z' equation for parameter u at this position along the projection line:

$$u = \frac{z_{vp} - z}{z_{prp} - z}$$

- ✓ Substituting this value of u into the equations for x' and y', we obtain the general perspective-transformation equations

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + x_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) + y_{prp}\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$

<u>Perspective-Projection Equations: Special Cases</u>

<u>Case 1:</u>

➔ To simplify the perspective calculations, the projection reference point could be limited to positions along the *z*view axis, then

$x_{prp} = y_{prp} = 0$:

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right), \qquad y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right)$$

<u>Case 2:</u>

➔ Sometimes the projection reference point is fixed at the coordinate origin, and

$(x_{prp}, y_{prp}, z_{prp}) = (0, 0, 0)$:

$$x_p = x\left(\frac{z_{vp}}{z}\right), \qquad y_p = y\left(\frac{z_{vp}}{z}\right)$$

<u>Case 3:</u>

➔ If the view plane is the *uv* plane and there are no restrictions on the placement of the projection reference point, then we have

$z_{vp} = 0$:

$$x_p = x\left(\frac{z_{prp}}{z_{prp} - z}\right) - x_{prp}\left(\frac{z}{z_{prp} - z}\right)$$

$$y_p = y\left(\frac{z_{prp}}{z_{prp} - z}\right) - y_{prp}\left(\frac{z}{z_{prp} - z}\right)$$

Case 4:

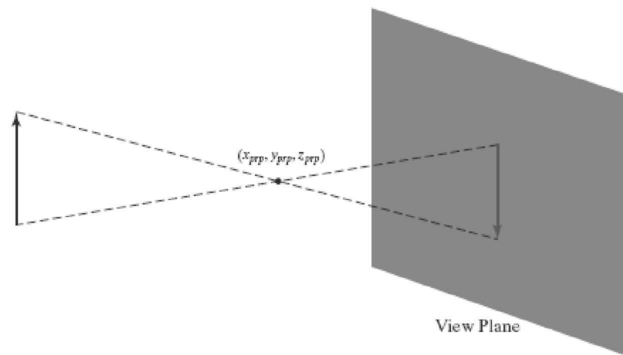
➔ With the *uv* plane as the view plane and the projection reference point on the z_{view} axis, the perspective equations are

$x_{prp} = y_{prp} = z_{vp} = 0$:

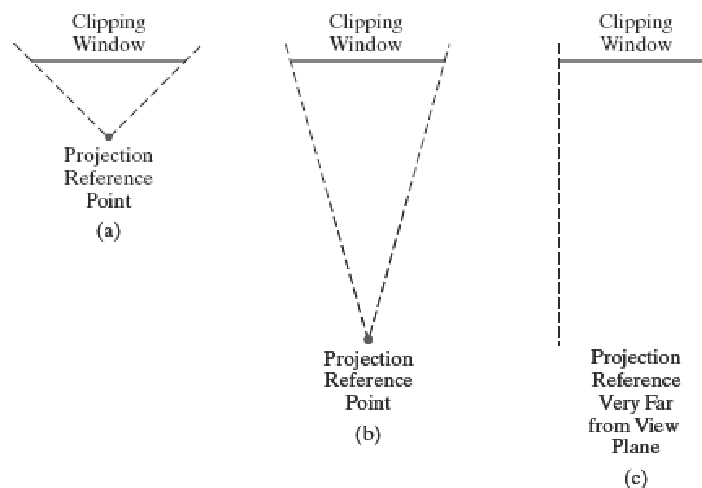
$$x_p = x\left(\frac{z_{prp}}{z_{prp} - z}\right), \qquad y_p = y\left(\frac{z_{prp}}{z_{prp} - z}\right)$$

✓ The view plane is usually placed between the projection reference point and the scene, but, in general, the view plane could be placed anywhere except at the projection point.

✓ If the projection reference point is between the view plane and the scene, objects are inverted on the view plane (refer below figure)



✓ Perspective effects also depend on the distance between the projection reference point and the view plane, as illustrated in Figure below.

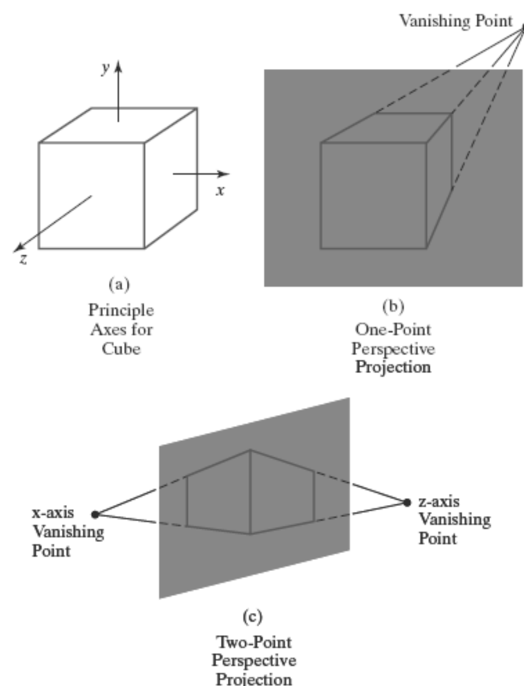


- ✓ If the projection reference point is close to the view plane, perspective effects are emphasized; that is, closer objects will appearmuchlarger thanmore distant objects of the same size.
- ✓ Similarly, as the projection reference point moves farther from the view plane, the difference in the size of near and far objects decreases

Vanishing Points for Perspective Projections

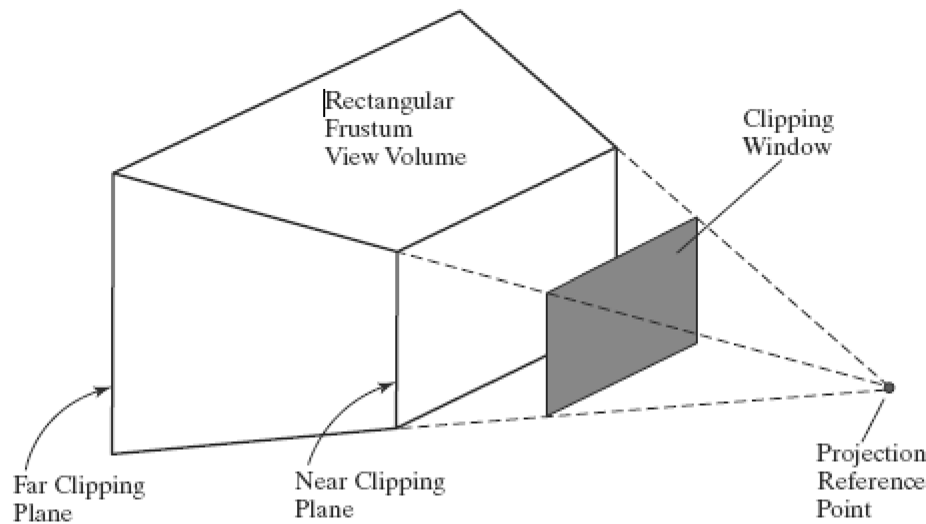
- ➢ The point at which a set of projected parallel lines appears to converge is called a vanishing point.
- ➢ Each set of projected parallel lines has a separate vanishing point.
- ➢ For a set of lines that are parallel to one of the principal axes of an object, the vanishing point is referred to as a principal vanishing point.
- ➢ We control the number of principal vanishing points (one, two, or three) with the orientation of the projection plane, and perspective projections are accordingly classified as one-point, two-point, or three-point projections

Principal vanishing points for perspective-projection views of a cube. When the cube in (a) is projected to a view plane that intersects only the z axis, a single vanishing point in the z direction (b) is generated. When the cube is projected to a view plane that intersects both the z and x axes, two vanishing points (c) are produced.
Perspective-Projection View Volume



- ➢ A perspective-projection view volume is often referred to as a pyramid of vision because it approximates the *cone of vision* of our eyes or a camera.

- The displayed view of a scene includes only those objects within the pyramid, just as we cannot see objects beyond our peripheral vision, which are outside the cone of vision.
- By adding near and far clipping planes that are perpendicular to the $z$view axis (and parallel to the view plane), we chop off parts of the infinite, perspectiveprojection view volume to form a truncated pyramid, or frustum, view volume



- But with a perspective projection, we could also use the near clipping plane to take out large objects close to the view plane that could project into unrecognizable shapes within the clipping window.
- Similarly, the far clipping plane could be used to cut out objects far from the projection reference point that might project to small blots on the view plane.

<u>Perspective-Projection Transformation Matrix</u>

- ✓ We can use a three-dimensional, homogeneous-coordinate representation to express the perspective-projection equations in the form

$$x_p = \frac{x_h}{h}, \qquad y_p = \frac{y_h}{h}$$

where the homogeneous parameter has the value

$$h = z_{prp} - z$$

$$x_h = x(z_{prp} - z_{vp}) + x_{prp}(z_{vp} - z)$$

$$y_h = y(z_{prp} - z_{vp}) + y_{prp}(z_{vp} - z)$$

- ✓ The perspective-projection transformation of a viewing-coordinate position is then accomplished in two steps.
- ✓ First, we calculate the homogeneous coordinates using the perspective-transformation matrix:

$$\mathbf{P}_h = \mathbf{M}_{\text{pers}} \cdot \mathbf{P}$$

Where,

P_h is the column-matrix representation of the homogeneous point (x_h, y_h, z_h, h) and

P is the column-matrix representation of the coordinate position $(x, y, z, 1)$.

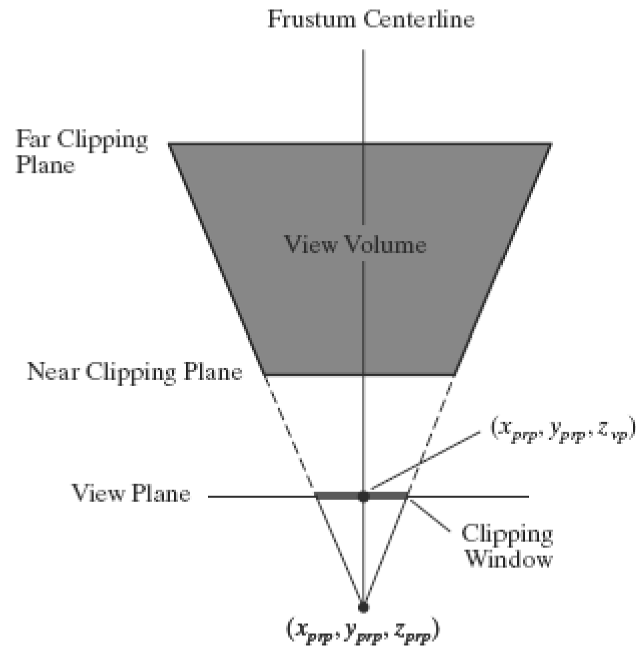
- ✓ Second, after other processes have been applied, such as the normalization transformation and clipping routines, homogeneous coordinates are divided by parameter h to obtain the true transformation-coordinate positions.
- ✓ The following matrix gives one possible way to formulate a perspective-projection matrix.

$$\mathbf{M}_{\text{pers}} = \begin{bmatrix} z_{prp} - z_{vp} & 0 & -x_{prp} & x_{prp}z_{prp} \\ 0 & z_{prp} - z_{vp} & -y_{prp} & y_{prp}z_{prp} \\ 0 & 0 & s_z & t_z \\ 0 & 0 & -1 & z_{prp} \end{bmatrix}$$

- ✓ Parameters s_z and t_z are the scaling and translation factors for normalizing the projected values of z-coordinates.
- ✓ Specific values for s_z and t_z depend on the normalization range we select.

Symmetric Perspective-Projection Frustum

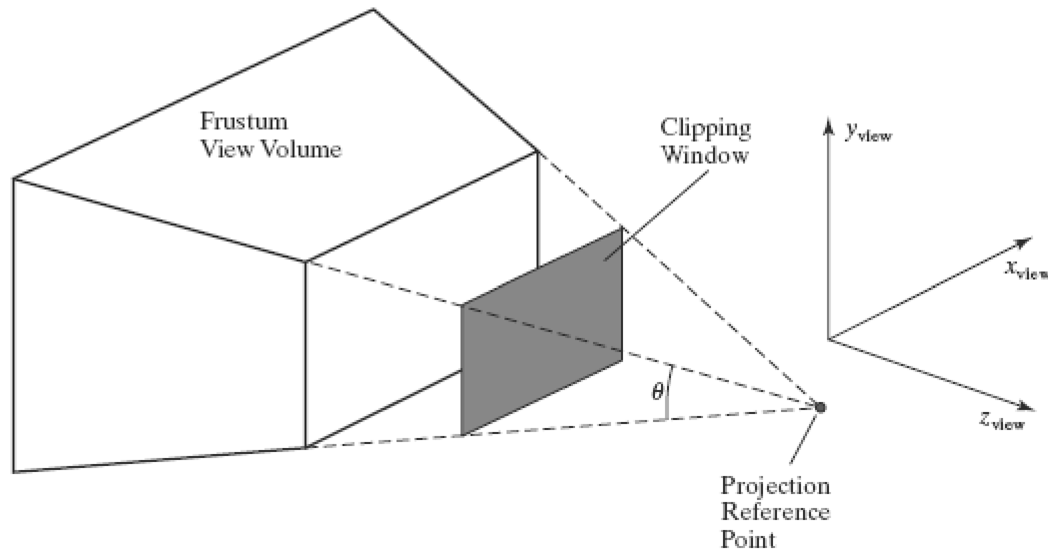
- ✓ The line from the projection reference point through the center of the clipping window and on through the view volume is the centerline for a perspectiveprojection frustum.
- ✓ If this centerline is perpendicular to the view plane, we have a symmetric frustum (with respect to its centerline)



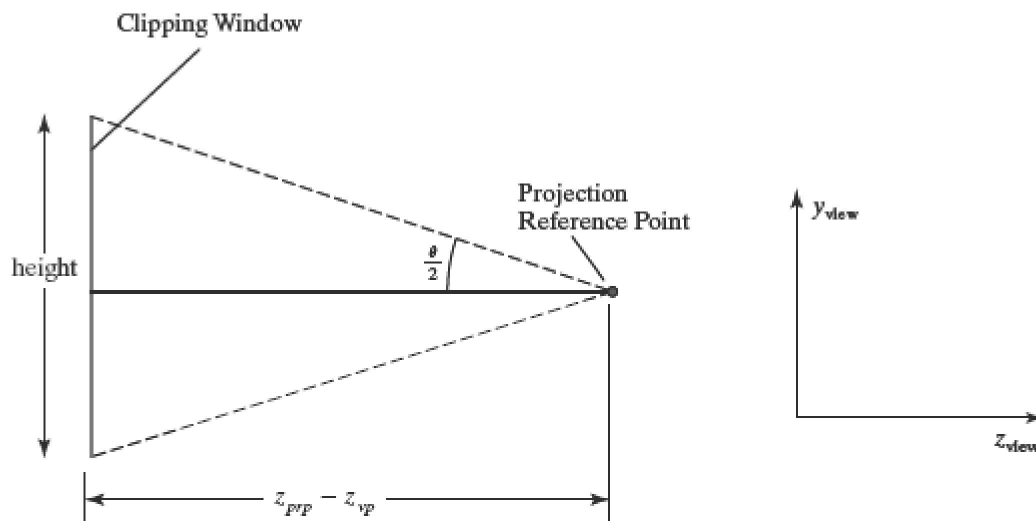
- Because the frustum centerline intersects the view plane at the coordinate location $(x_{prp}, y_{prp}, z_{vp})$, we can express the corner positions for the clipping window in terms of the window dimensions:

$$xw_{\min} = x_{prp} - \frac{\text{width}}{2}, \qquad xw_{\max} = x_{prp} + \frac{\text{width}}{2}$$
$$yw_{\min} = y_{prp} - \frac{\text{height}}{2}, \qquad yw_{\max} = y_{prp} + \frac{\text{height}}{2}$$

- Another way to specify a symmetric perspective projection is to use parameters that approximate the properties of a camera lens.
- A photograph is produced with a symmetric perspective projection of a scene onto the film plane.
- Reflected light rays from the objects in a scene are collected on the film plane from within the “cone of vision” of the camera.
- This cone of vision can be referenced with a field-of-view angle, which is a measure of the size of the camera lens.
- A large field-of-view angle, for example, corresponds to a wide-angle lens.
- In computer graphics, the cone of vision is approximated with a symmetric frustum, and we can use a field-of-view angle to specify an angular size for the frustum.



- ✓ For a given projection reference point and view-plane position, the field-of view angle determines the height of the clipping window from the right triangles in the diagram of Figure below, we see that



$$\tan\left(\frac{\theta}{2}\right) = \frac{\text{height}/2}{z_{prp} - z_{vp}}$$

- ✓ so that the clipping-window height can be calculated as

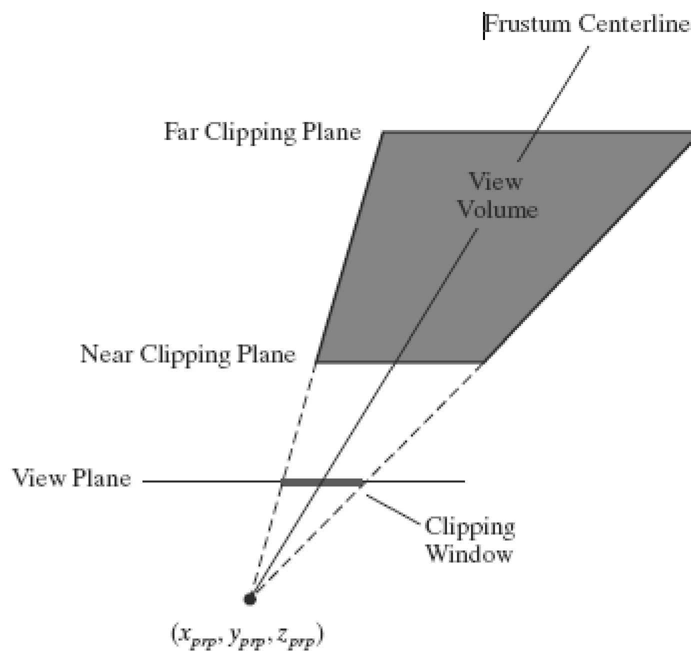
$$\text{height} = 2(z_{prp} - z_{vp})\tan\left(\frac{\theta}{2}\right)$$

- ✓ Therefore, the diagonal elements with the value $z_{prp} - z_{vp}$ could be replaced by either of the following two expressions

$$z_{prp} - z_{vp} = \frac{\text{height}}{2}\cot\left(\frac{\theta}{2}\right)$$
$$= \frac{\text{width}\cdot\cot(\theta/2)}{2\cdot\text{aspect}}$$

Oblique Perspective-Projection Frustum

- ✓ If the centerline of a perspective-projection view volume is not perpendicular to the view plane, we have an oblique frustum



- ✓ In this case, we can first transform the view volume to a symmetric frustum and then to a normalized view volume.
- ✓ An oblique perspective-projection view volume can be converted to a sym metric frustum by applying a *z*-axis shearing-transformation matrix.
- ✓ This transformation shifts all positions on any plane that is perpendicular to the *z* axis by an amount that is proportional to the distance of the plane from a specified *z*- axis reference position.
- ✓ The computations for the shearing transformation, as well as for the perspective and normalization transformations, are greatly reduced if we take the projection reference point to be the viewing-coordinate origin.

- Taking the projection reference point as $(x_{prp}, y_{prp}, z_{prp}) = (0, 0, 0)$, we obtain the elements of the required shearing matrix as

$$\mathbf{M}_{z\,\text{shear}} = \begin{bmatrix} 1 & 0 & \text{sh}_{zx} & 0 \\ 0 & 1 & \text{sh}_{zy} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- We need to choose values for the shearing parameters such that

$$\begin{bmatrix} 0 \\ 0 \\ z_{\text{near}} \\ 1 \end{bmatrix} = \mathbf{M}_{z\,\text{shear}} \cdot \begin{bmatrix} \frac{xw_{\min} + xw_{\max}}{2} \\ \frac{yw_{\min} + yw_{\max}}{2} \\ z_{\text{near}} \\ 1 \end{bmatrix}$$

- Therefore, the parameters for this shearing transformation are

$$\text{sh}_{zx} = -\frac{xw_{\min} + xw_{\max}}{2\, z_{\text{near}}}$$
$$\text{sh}_{zy} = -\frac{yw_{\min} + yw_{\max}}{2\, z_{\text{near}}}$$

- Similarly, with the projection reference point at the viewing-coordinate origin and with the near clipping plane as the view plane, the perspective-projection matrix is simplified to

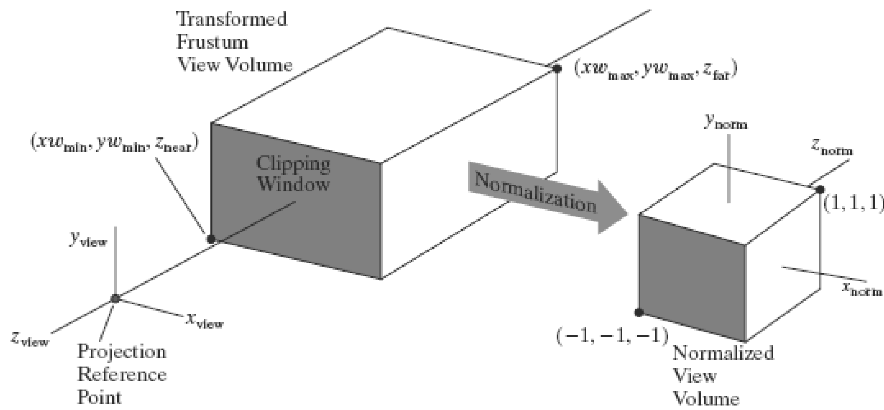
$$\mathbf{M}_{\text{pers}} = \begin{bmatrix} -z_{\text{near}} & 0 & 0 & 0 \\ 0 & -z_{\text{near}} & 0 & 0 \\ 0 & 0 & s_z & t_z \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

- Concatenating the simplified perspective-projection matrix with the shear matrix we have

$$\mathbf{M}_{\text{obliquepers}} = \mathbf{M}_{\text{pers}} \cdot \mathbf{M}_{z\,\text{shear}}$$
$$= \begin{bmatrix} -z_{\text{near}} & 0 & \frac{xw_{\min} + xw_{\max}}{2} & 0 \\ 0 & -z_{\text{near}} & \frac{yw_{\min} + yw_{\max}}{2} & 0 \\ 0 & 0 & s_z & t_z \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Normalized Perspective-Projection Transformation Coordinates

- When we divide the homogeneous coordinates by the homogeneous parameter h, we obtain the actual projection coordinates, which are orthogonal-projection coordinates
- The final step in the perspective transformation process is to map this parallelepiped to a *normalized view volume*.
- The transformed frustum view volume, which is a rectangular parallelepiped, is mapped to a symmetric normalized cube within a left-handed reference frame



- Because the centerline of the rectangular parallelepiped view volume is now the z_{view} axis, no translation is needed in the x and y normalization transformations: We require only the x and y scaling parameters relative to the coordinate origin.
- The scaling matrix for accomplishing the xy normalization is

$$\mathbf{M}_{xy\,\text{scale}} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Concatenating the xy-scalingmatrix produces the following normalization matrix for a perspective-projection transformation.

$$\mathbf{M}_{\text{normpers}} = \mathbf{M}_{xy\,\text{scale}} \cdot \mathbf{M}_{\text{obliquepers}}$$

$$= \begin{bmatrix} -z_{\text{near}} s_x & 0 & s_x \dfrac{xw_{\min} + xw_{\max}}{2} & 0 \\ 0 & -z_{\text{near}} s_y & s_y \dfrac{yw_{\min} + yw_{\max}}{2} & 0 \\ 0 & 0 & s_z & t_z \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

➔ From this transformation, we obtain the homogeneous coordinates:

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \mathbf{M}_{\text{normpers}} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

And the projection coordinates are

$$x_p = \frac{x_h}{h} = \frac{-z_{\text{near}} s_x x + s_x (xw_{\text{min}} + xw_{\text{max}})/2}{-z}$$
$$y_p = \frac{y_h}{h} = \frac{-z_{\text{near}} s_y y + s_y (yw_{\text{min}} + yw_{\text{max}})/2}{-z}$$
$$z_p = \frac{z_h}{h} = \frac{s_z z + t_z}{-z}$$

➔ To normalize this perspective transformation, we want the projection coordinates to be $(x_p, y_p, z_p) = (-1, -1, -1)$ when the input coordinates are $(x, y, z) = (x_{w\text{min}}, y_{w\text{min}}, z_{\text{near}})$, and we want the projection coordinates to be $(x_p, y_p, z_p) = (1, 1, 1)$ when the input coordinates are $(x, y, z) = (x_{w\text{max}}, y_{w\text{max}}, z_{\text{far}})$.

$$s_x = \frac{2}{xw_{\text{max}} - xw_{\text{min}}}, \qquad s_y = \frac{2}{yw_{\text{max}} - yw_{\text{min}}}$$
$$s_z = \frac{z_{\text{near}} + z_{\text{far}}}{z_{\text{near}} - z_{\text{far}}}, \qquad t_z = \frac{2\, z_{\text{near}}\, z_{\text{far}}}{z_{\text{near}} - z_{\text{far}}}$$

➔ And the elements of the normalized transformation matrix for a general perspective-projection are

$$\mathbf{M}_{\text{normpers}} = \begin{bmatrix} \frac{-2z_{\text{near}}}{xw_{\text{max}} - xw_{\text{min}}} & 0 & \frac{xw_{\text{max}} + xw_{\text{min}}}{xw_{\text{max}} - xw_{\text{min}}} & 0 \\ 0 & \frac{-2z_{\text{near}}}{yw_{\text{max}} - yw_{\text{min}}} & \frac{yw_{\text{max}} + yw_{\text{min}}}{yw_{\text{max}} - yw_{\text{min}}} & 0 \\ 0 & 0 & \frac{z_{\text{near}} + z_{\text{far}}}{z_{\text{near}} - z_{\text{far}}} & -\frac{2z_{\text{near}} z_{\text{far}}}{z_{\text{near}} - z_{\text{far}}} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

4.8 The Viewport Transformation and Three-Dimensional Screen Coordinates

- ✓ Once we have completed the transformation to normalized projection coordinates, clipping can be applied efficiently to the symmetric cube then the contents of the normalized view volume can be transferred to screen coordinates.
- ✓ Positions throughout the three-dimensional view volume also have a depth (z coordinate), and we need to retain this depth information for the visibility testing and surface-rendering algorithms
- ✓ If we include this z renormalization, the transformation from the normalized view volume to three dimensional screen coordinates is

$$\mathbf{M}_{\text{normviewvol,3D screen}} = \begin{bmatrix} \frac{xv_{\max} - xv_{\min}}{2} & 0 & 0 & \frac{xv_{\max} + xv_{\min}}{2} \\ 0 & \frac{yv_{\max} - yv_{\min}}{2} & 0 & \frac{yv_{\max} + yv_{\min}}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- ✓ In normalized coordinates, the $z_{\text{norm}} = -1$ face of the symmetric cube corresponds to the clipping-window area. And this face of the normalized cube is mapped to the rectangular viewport, which is now referenced at $z$screen = 0.
- ✓ Thus, the lower-left corner of the viewport screen area is at position $(xv_{\min}, yv_{\min}, 0)$ and the upper-right corner is at position $(xv_{\max}, yv_{\max}, 0)$.

4.9 OpenGL Three-Dimensional Viewing Functions

OpenGL Viewing-Transformation Function

glMatrixMode (GL_MODELVIEW);

- ➔ a matrix is formed and concatenated with the current modelview matrix, We set the modelview mode with the statement above

gluLookAt (x0, y0, z0, xref, yref, zref, Vx, Vy, Vz);

- ➔ Viewing parameters are specified with the above GLU function.

- This function designates the origin of the viewing reference frame as the world-coordinate position $P_0 = (x_0, y_0, z_0)$, the reference position as $P_{ref} = (x_{ref}, y_{ref}, z_{ref})$, and the view-up vector as $V = (V_x, V_y, V_z)$.
- If we do not invoke the gluLookAt function, the default OpenGL viewing parameters are

$$P_0 = (0, 0, 0)$$
$$P_{ref} = (0, 0, -1)$$
$$V = (0, 1, 0)$$

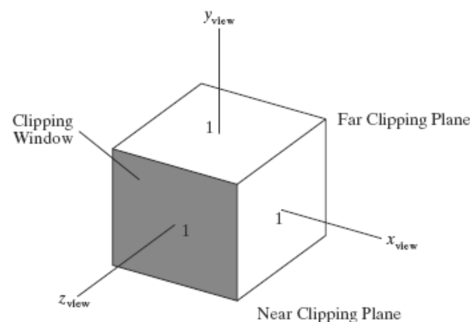
OpenGL Orthogonal-Projection Function

glMatrixMode (GL_PROJECTION);

- set up a projection-transformation matrix.
- Then, when we issue any transformation command, the resulting matrix will be concatenated with the current projection matrix.

glOrtho (xwmin, xwmax, ywmin, ywmax, dnear, dfar);

- Orthogonal-projection parameters are chosen with the function
- All parameter values in this function are to be assigned double-precision, floating point Numbers
- Function glOrtho generates a parallel projection that is perpendicular to the view plane
- Parameters d_{near} and d_{far} denote distances in the negative z_{view} direction from the viewing-coordinate origin
- We can assign any values (positive, negative, or zero) to these parameters, so long as $d_{near} < d_{far}$.
- Exa: glOrtho (-1.0, 1.0, -1.0, 1.0, -1.0, 1.0);



OpenGL General Perspective-Projection Function

glFrustum (xwmin, xwmax, ywmin, ywmax, dnear, dfar);

- ➔ specify a perspective projection that has either a symmetric frustum view volume or an oblique frustum view volume
- ➔ All parameters in this function are assigned double-precision, floating-point numbers.
- ➔ The first four parameters set the coordinates for the clipping window on the near plane, and the last two parameters specify the distances from the coordinate origin to the near and far clipping planes along the negative z_{view} axis.

OpenGL Viewports and Display Windows

glViewport (xvmin, yvmin, vpWidth, vpHeight);

- ➔ A rectangular viewport is defined.
- ➔ The first two parameters in this function specify the integer screen position of the lower-left corner of the viewport relative to the lower-left corner of the display window.
- ➔ And the last two parameters give the integer width and height of the viewport.
- ➔ To maintain the proportions of objects in a scene, we set the aspect ratio of the viewport equal to the aspect ratio of the clipping window.
- ➔ Display windows are created and managed with GLUT routines. The default viewport in OpenGL is the size and position of the current display window

OpenGL Three-Dimensional Viewing Program Example

```
#include <GL/glut.h>
GLint winWidth = 600, winHeight = 600; // Initial display-window size.
GLfloat x0 = 100.0, y0 = 50.0, z0 = 50.0; // Viewing-coordinate origin.
GLfloat xref = 50.0, yref = 50.0, zref = 0.0; // Look-at point.
GLfloat Vx = 0.0, Vy = 1.0, Vz = 0.0; // View-up vector.
/* Set coordinate limits for the clipping window: */
GLfloat xwMin = -40.0, ywMin = -60.0, xwMax = 40.0, ywMax = 60.0;
/* Set positions for near and far clipping planes: */
GLfloat dnear = 25.0, dfar = 125.0;
```

```
void init (void)
{
        glClearColor (1.0, 1.0, 1.0, 0.0);
        glMatrixMode (GL_MODELVIEW);
        gluLookAt (x0, y0, z0, xref, yref, zref, Vx, Vy, Vz);
        glMatrixMode (GL_PROJECTION);
        glFrustum (xwMin, xwMax, ywMin, ywMax, dnear, dfar);
}
void displayFcn (void)
{
        glClear (GL_COLOR_BUFFER_BIT);
        glColor3f (0.0, 1.0, 0.0); // Set fill color to green.
        glPolygonMode (GL_FRONT, GL_FILL);
        glPolygonMode (GL_BACK, GL_LINE); // Wire-frame back face.
        glBegin (GL_QUADS);
                glVertex3f (0.0, 0.0, 0.0);
                glVertex3f (100.0, 0.0, 0.0);
                glVertex3f (100.0, 100.0, 0.0);
                glVertex3f (0.0, 100.0, 0.0);
        glEnd ( );
        glFlush ( );
}
void reshapeFcn (GLint newWidth, GLint newHeight)
{
        glViewport (0, 0, newWidth, newHeight);
        winWidth = newWidth;
        winHeight = newHeight;
}
void main (int argc, char** argv)
{
        glutInit (&argc, argv);
```

```
    glutInitDisplayMode (GLUT_SINGLE | GLUT_RGB);
    glutInitWindowPosition (50, 50);
    glutInitWindowSize (winWidth, winHeight);
    glutCreateWindow ("Perspective View of A Square");
    init ( );
    glutDisplayFunc (displayFcn);
    glutReshapeFunc (reshapeFcn);
    glutMainLoop ( );
}
```

Visible-Surface Detection Methods

4.10 Classification of Visible-Surface Detection Algorithms

- We can broadly classify visible-surface detection algorithms according to whether they deal with the object definitions or with their projected images.
- **Object-space methods:** compares objects and parts of objects to each other within the scene definition to determine which surfaces, as a whole, we should label as visible**.**
- **Image-space methods:** visibility is decided point by point at each pixel position on the projection plane**.**
- Although there are major differences in the basic approaches taken by the various visible-surface detection algorithms, most use sorting and coherence methods to improve performance.
- Sorting is used to facilitate depth comparisons by ordering the individual surfaces in a scene according to their distance from the view plane.
- Coherence methods are used to take advantage of regularities in a scene.

4.11 Back-Face Detection

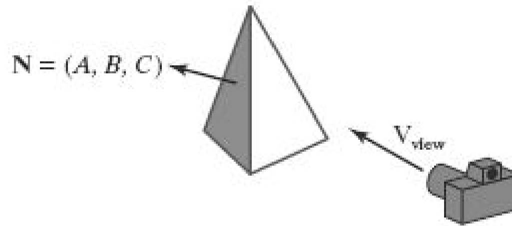
- ✓ A fast and simple object-space method for locating the back faces of a polyhedron is based on front-back tests. A point (x, y, z) is behind a polygon surface if

$$A_x + B_y + C_z + D < 0$$

where A, B,C, and $D$are the plane parameters for the polygon

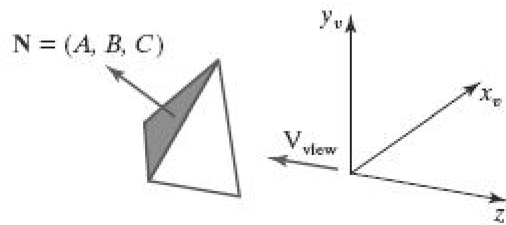
- ✓ We can simplify the back-face test by considering the direction of the normal vector N for a polygon surface. If V_{view} is a vector in the viewing direction from our camera position, as shown in Figure below, then a polygon is a back face if

$$\mathbf{V_{view} . N > 0}$$

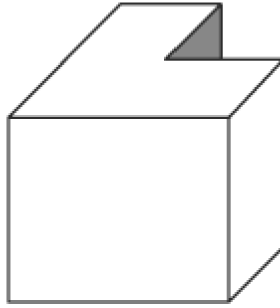


- ✓ In a right-handed viewing system with the viewing direction along the negative *zv* axis (Figure below), a polygon is a back face if the *z* component, *C*, of its normal vector N satisfies $C < 0$.
- ✓ Also, we cannot see any face whose normal has *z* component $C = 0$, because our viewing direction is grazing that polygon. Thus, in general, we can label any polygon as a back face if its normal vector has a *z* component value that satisfies the inequality

$$C <= 0$$



- ✓ Similar methods can be used in packages that employ a left-handed viewing system. In these packages, plane parameters *A*, *B*, *C*, and *D* can be calculated from polygon vertex coordinates specified in a clockwise direction.
- ✓ Inequality 1 then remains a valid test for points behind the polygon.
- ✓ By examining parameter *C* for the different plane surfaces describing an object, we can immediately identify all the back faces.
- ✓ For other objects, such as the concave polyhedron in Figure below, more tests must be carried out to determine whether there are additional faces that are totally or partially obscured by other faces



- ✓ In general, back-face removal can be expected to eliminate about half of the polygon surfaces in a scene from further visibility tests.

4.12 Depth-Buffer Method

- ❖ A commonly used image-space approach for detecting visible surfaces is the depth-buffer method, which compares surface depth values throughout a scene for each pixel position on the projection plane.
- ❖ The algorithm is usually applied to scenes containing only polygon surfaces, because depth values can be computed very quickly and the method is easy to implement.
- ❖ This visibility-detectionapproach is also frequently alluded to as the *z-buffer method,* because object depth is usually measured along the *z* axis of a viewing system

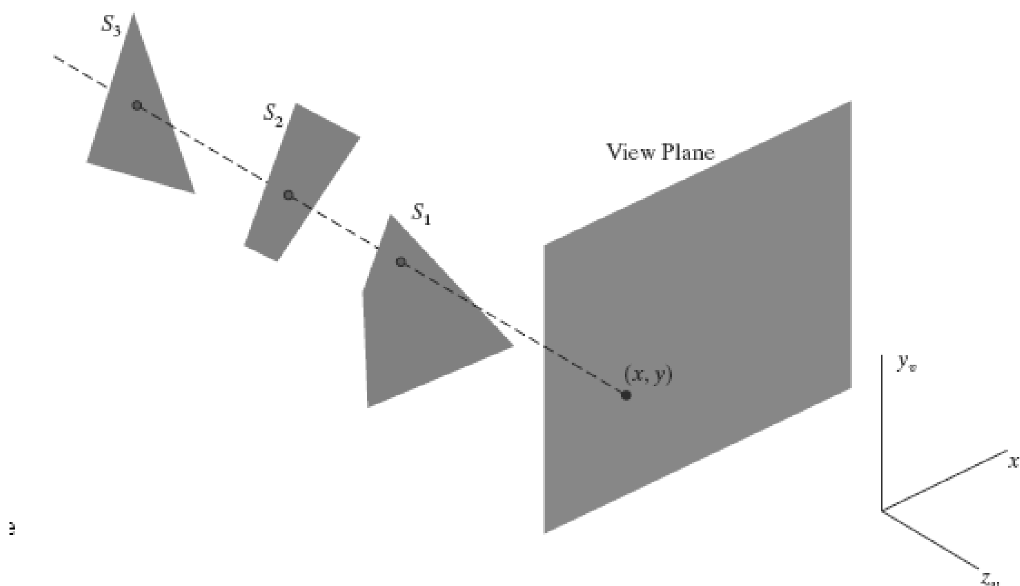


Figure above shows three surfaces at varying distances along the orthographic projection line from position *(x, y)* on a view plane.

- These surfaces can be processed in any order.
- If a surface is closer than any previously processed surfaces, its surface color is calculated and saved, along with its depth.
- The visible surfaces in a scene are represented by the set of surface colors that have been saved after all surface processing is completed
- As implied by the name of this method, two buffer areas are required. A depth buffer is used to store depth values for each *(x, y)* position as surfaces are processed, and the frame buffer stores the surface-color values for each pixel position.

Depth-Buffer Algorithm

1. Initialize the depth buffer and frame buffer so that for all buffer positions *(x, y)*,

depthBuff (x, y) = 1.0, frameBuff (x, y) = backgndColor

2. Process each polygon in a scene, one at a time, as follows:
   - For each projected *(x, y)* pixel position of a polygon, calculate the depth *z* (if not already known).
   - If $z <$ depthBuff (x, y), compute the surface color at that position and set

depthBuff (x, y) = z, frameBuff (x, y) = surfColor (x, y)

After all surfaces have been processed, the depth buffer contains depth values for the visible surfaces and the frame buffer contains the corresponding color values for those surfaces.

- Given the depth values for the vertex positions of any polygon in a scene, we can calculate the depth at any other point on the plane containing the polygon.
- At surface position *(x, y)*, the depth is calculated from the plane equation as

$$z = \frac{-Ax - By - D}{C}$$

- If the depth of position *(x, y)* has been determined to be *z*, then the depth *z'* of the next position *(x + 1, y)* along the scan line is obtained as

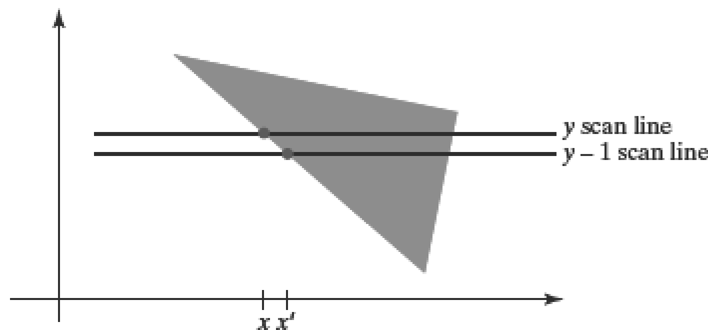
$$z' = \frac{-A(x+1) - By - D}{C}$$

$$z' = z - \frac{A}{C}$$

- The ratio $-A/C$ is constant for each surface, so succeeding depth values across a scan line are obtained from preceding values with a single addition.
- We can implement the depth-buffer algorithm by starting at a top vertex of the polygon.
- Then, we could recursively calculate the *x*-coordinate values down a left edge of the polygon.
- The *x* value for the beginning position on each scan line can be calculated from the beginning (edge) *x* value of the previous scan line as

$$x' = x - \frac{1}{m}$$

where *m* is the slope of the edge (Figure below).



- Depth values down this edge are obtained recursively as

$$z' = z + \frac{A/m + B}{C}$$

- If we are processing down a vertical edge, the slope is infinite and the recursive calculations reduce to

$$z' = z + \frac{B}{C}$$

- One slight complication with this approach is that while pixel positions are at integer *(x, y)* coordinates, the actual point of intersection of a scan line with the edge of a polygon may not be.
- As a result, it may be necessary to adjust the intersection point by rounding its fractional part up or down, as is done in scan-line polygon fill algorithms.
- An alternative approach is to use a midpoint method or Bresenham-type algorithm for determining the starting *x* values along edges for each scan line.

- The method can be applied to curved surfaces by determining depth and color values at each surface projection point.
- In addition, the basic depth-buffer algorithm often performs needless calculations.
- Objects are processed in an arbitrary order, so that a color can be computed for a surface point that is later replaced by a closer surface.

4.13 OpenGL Visibility-Detection Functions

OpenGL Polygon-Culling Functions

- Back-face removal is accomplished with the functions

  glEnable (GL_CULL_FACE);

  glCullFace (mode);

  - ➔ where parameter mode is assigned the value GL_BACK, GL_FRONT, GL_FRONT_AND_BACK
  - ➔ By default, parameter mode in the glCullFace function has the value GL_BACK
  - ➔ The culling routine is turned off with

    glDisable (GL_CULL_FACE);

OpenGL Depth-Buffer Functions

- To use the OpenGL depth-buffer visibility-detection routines, we first need to modify the GL Utility Toolkit (GLUT) initialization function for the display mode to include a request for the depth buffer, as well as for the refresh buffer

  glutInitDisplayMode (GLUT_SINGLE | GLUT_RGB | GLUT_DEPTH);
- Depth buffer values can then be initialized with

  glClear (GL_DEPTH_BUFFER_BIT);

  - the preceding initialization sets all depth-buffer values to the maximum value 1.0 by default
- The OpenGL depth-buffer visibility-detection routines are activated with the following function:

  glEnable (GL_DEPTH_TEST);

  And we deactivate the depth-buffer routines with

  glDisable (GL_DEPTH_TEST);

- We can also apply depth-buffer visibility testing using some other initial value for themaximumdepth, and this initial value is chosen with theOpenGLfunction:

  glClearDepth (maxDepth);

  - Parameter maxDepth can be set to any value between 0.0 and 1.0.
  - Projection coordinates in OpenGL are normalized to the range from −1.0 to 1.0, and the depth values between the near and far clipping planes are further normalized to the range from 0.0 to 1.0.
- As an option, we can adjust these normalization values with

  glDepthRange (nearNormDepth, farNormDepth);

  - By default, nearNormDepth = 0.0 and farNormDepth = 1.0.
  - But with the glDepthRange function, we can set these two parameters to any values within the range from 0.0 to 1.0, including nearNormDepth > farNormDepth
- Another option available in OpenGL is the test condition that is to be used for the depth-buffer routines.We specify a test condition with the following function:

  glDepthFunc (testCondition);

  - Parameter testCondition can be assigned any one of the following eight symbolic constants: GL_LESS, GL_GREATER, GL_EQUAL, GL_NOTEQUAL, GL_LEQUAL, GL_GEQUAL, GL_NEVER (no points are processed), and GL_ALWAYS.
  - The default value for parameter testCondition is GL_LESS.
- We can also set the status of the depth buffer so that it is in a read-only state or in a read-write state. This is accomplished with

  glDepthMask (writeStatus);

  - When writeStatus = GL_TRUE (the default value), we can both read from and write to the depth buffer.
  - With writeStatus = GL_FALSE, the write mode for the depth buffer is disabled and we can retrieve values only for comparison in depth testing.

OpenGL Wire-Frame Surface-Visibility Methods

- ✓ A wire-frame display of a standard graphics object can be obtained in OpenGL by requesting that only its edges are to be generated.
- ✓ We do this by setting the polygon-mode function as, for example:

  glPolygonMode (GL_FRONT_AND_BACK, GL_LINE);

  But this displays both visible and hidden edges

```
glEnable (GL_DEPTH_TEST);
glPolygonMode (GL_FRONT_AND_BACK, GL_LINE);
glColor3f (1.0, 1.0, 1.0);
/* Invoke the object-description routine. */
glPolygonMode (GL_FRONT_AND_BACK, GL_FILL);
glEnable (GL_POLYGON_OFFSET_FILL);
glPolygonOffset (1.0, 1.0);
glColor3f (0.0, 0.0, 0.0);
/* Invoke the object-description routine again. */
glDisable (GL_POLYGON_OFFSET_FILL);
```

OpenGL Depth-Cueing Function

- ✓ We can vary the brightness of an object as a function of its distance from the viewing position with

  glEnable (GL_FOG);

  glFogi (GL_FOG_MODE, GL_ LINEAR);

This applies the linear depth function to object colors using d_{min} = 0.0 and d_{max} = 1.0. But we can set different values for *d*min and *d*max with the following function calls:

glFogf (GL_FOG_START, minDepth);

glFogf (GL_FOG_END, maxDepth);

- ➔ In these two functions, parameters minDepth and maxDepth are assigned floating-point values, although integer values can be used if we change the function suffix to i.
- ➔ We can use the glFog function to set an atmosphere color that is to be combined with the color of an object after applying the linear depthcueing function

Difference Between perspective projection and parallel projection

Perspective projection	Parallel projection
The center of projection is at a finite distance from the viewing plane	Center of projection at infinity results with a parallel projection
Explicitly specify: center of projection	Direction of projection is specified
Size of the object is inversely proportional to the distance of the object from the center of projection	No change in the size of object
Produces realistic views but does not preserve relative proportion of objects	A parallel projection oreserves relative proportion of objects, but does not give us a reaistic representation of the appearance of object.
Not useful for recording exact shape and measurements of the object	Used for exact measurement
Parallel lines do not in general project as parallel	Parallel lines do remain parallel

Acknowledgements to

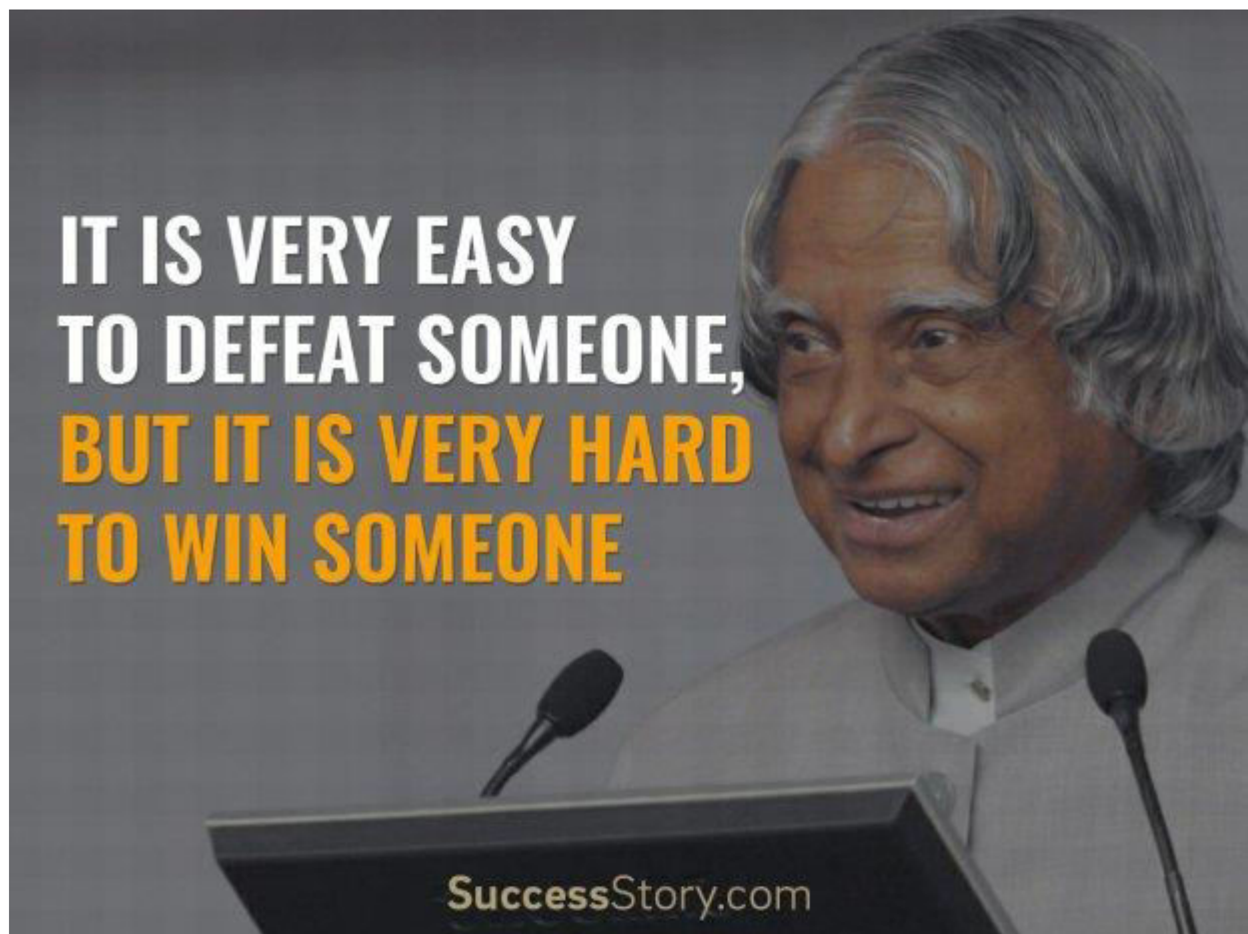
Donald Hearn & Pauline Baker: Computer Graphics with OpenGL

Version,3rd / 4th Edition, Pearson Education,2011

Edward Angel: Interactive Computer Graphics- A Top Down approach

with OpenGL, 5th edition. Pearson Education, 2008

M M Raiker, Computer Graphics using OpenGL, Filip learning/Elsevier



5.1 INPUT AND INTERACTION

Interaction; Input devices;
Clients and servers; Display lists;
Display lists and modeling; Programming
event-driven input; Menus; Picking;
A simple CAD program;
Building interactive models;
Animating interactive programs;
Design of interactive programs;
Logic operations.

5.1.1INTERACTION

- In the field of computer graphics, interaction refers to the manner in which the application program communicates with input and output devices of the system.
- For e.g. Image varying in response to the input from the user.
- **OpenGL doesn't directl**y support interaction in order to maintain portability. However, OpenGL provides the GLUT library. This library supports interaction with the keyboard, mouse etc and hence enables interaction. The GLUT library is compatible with many operating systems such as X windows, Current Windows, Mac OS etc and hence indirectly ensures the portability of OpenGL.

5.1.2 INPUT DEVICES

✓ Input devices are the devices which provide input to the computer graphics application program. Input devices can be categorized in two ways:

1. Physical input devices
2. Logical input devices

PHYSICAL INPUT DEVICES

- ✓ Physical input devices are the input devices which has the particular hardware architecture.
- ✓ The two major categories in physical input devices are:
- ***Key board devices*** like standard keyboard, flexible keyboard, handheld keyboard etc. These are used to provide character input like letters, numbers, symbols etc.
- ***Pointing devices*** like mouse, track ball, light pen etc. These are used to specify the position on the computer screen.

1. KEYBOARD: It is a general keyboard which has set of characters. We make use of ASCII value to represent the character i.e. it interacts with the programmer by passing the ASCII value of key pressed by programmer. Input can be given either single character of array of characters to the program.

Figure: Computer Keyboard

2. MOUSE AND TRACKBALL: These are pointing devices used to specify the position. Mouse and trackball interacts with the application program by passing the position of the clicked button. Both these devices are similar in use and construction. In these devices, the motion of the ball is converted to signal sent back to the computer by pair of encoders inside the device. These encoders measure motion in 2-orthogonal directions.

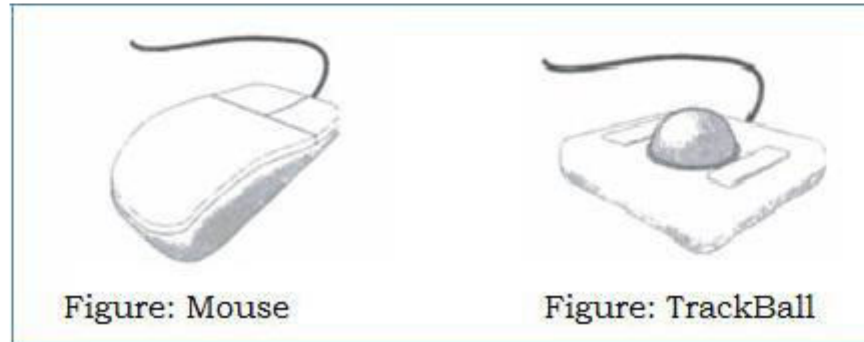


Figure: Mouse Figure: TrackBall

The values passed by the pointing devices can be considered as positions and converted to a 2-D location in either screen or world co-ordinates. Thus, as a mouse moves across a surface, the integrals of the velocities yield x,y values that can be converted to indicate the position for a cursor on the screen as shown below:

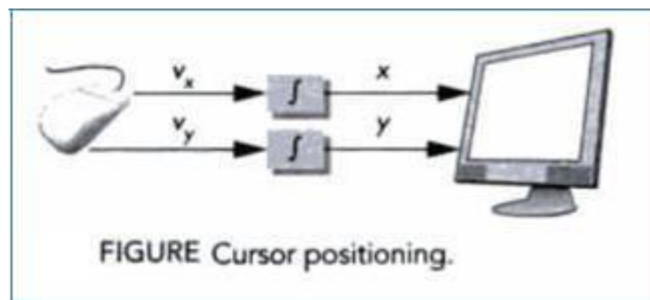


FIGURE Cursor positioning.

These devices are ***relative positioning devices*** because changes in the position of the ball yield a position in the user program.

3. <u>DATA TABLETS:</u> It provides absolute positioning. It has rows and columns of wires embedded under its surface. The position of the stylus is determined through electromagnetic interactions between signals travelling through the wires and sensors in the stylus.

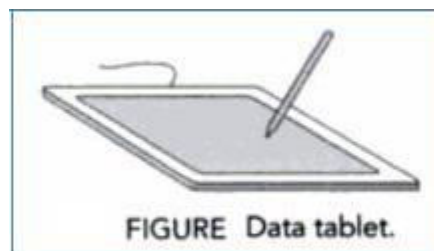


FIGURE Data tablet.

4. **LIGHT PEN:** It consists of light-sensing device such as **"**photocell**"**. The light pen is held at the front of the CRT. When the electron beam strikes the phosphor, the light is emitted from the CRT. If it exceeds the threshold then light sensing device of the light pen sends a signal to the computer specifying the position.

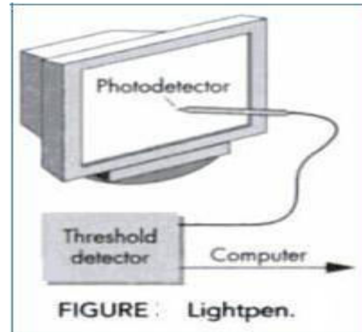


FIGURE : Lightpen.

The major disadvantage is that it has the difficulty in obtaining a position that corresponds to a dark area of the screen

5. **JOYSTICK:** The motion of the stick in two orthogonal directions is encoded, interpreted as two velocities and integrated to indentify a screen location. The integration implies that if the stick is left in its resting position, there is no change in cursor position. The faster the stick is moved from the resting position; the faster the screen location changes. Thus, joystick is *variable sensitivity device*.

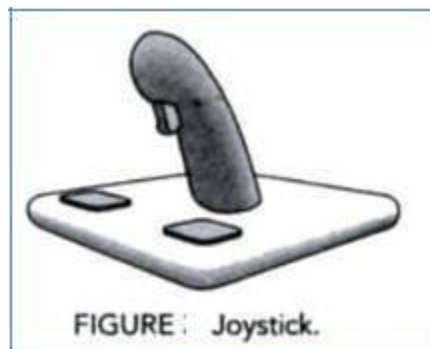


FIGURE : Joystick.

The advantage is that it is designed using mechanical elements such as springs and dampers which offer resistance to the user while pushing it. Such mechanical feel is suitable for application such as the flight simulators, game controllers etc.

6. **<u>SPACE BALL:</u>** It is a 3-Dimensional input device which looks like a joystick with a ball on the end of the stick.

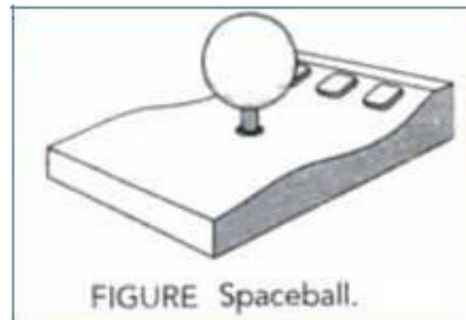


FIGURE Spaceball.

Stick doesn't move rather pressure sensors in the ball measure the forces applied by the user. The space ball can measure not only three direct forces (up-down, front-back, left-right) but also three independent twists. So totally device measures six independent values and thus has six degree of freedom.

Other 3-Dimensional devices such as laser scanners, measure 3-D positions directly. Numerous tracking systems used in virtual reality applications sense the position of the user and so on.

<u>LOGICAL INPUT DEVICES</u>

- ➔ These are characterized by its high-level interface with the application program rather than by its physical characteristics.
- ➔ Consider the following fragment of C code:

```
int x;
scanf("%d",&x);
printf("%d",x);
```

- ➔ The above code reads and then writes an integer. Although we run this program on workstation providing input from keyboard and seeing output on the display screen, the use of scanf() and printf() requires no knowledge of the properties of physical devices such as keyboard codes or resolution of the display.

➔ *These are logical functions that are defined by how they handle input or output character strings from the perspective of C program.*

➔ From logical devices perspective inputs are from inside the application program. *The two major characteristics describe the logical behavior of input devices are as follows:*

- ***The measurements that the device returns to the user program***
- ***The time when the device returns those measurements***

API defines six classes of logical input devices which are given below:

1. **STRING:** A string device is a logical device that provides the ASCII values of input characters to the user program. This logical device is usually implemented by means of physical keyboard.

2. **LOCATOR:** A locator device provides a position in world coordinates to the user program. It is usually implemented by means of pointing devices such as mouse or track ball.

3. **PICK:** A pick device returns the identifier of an object on the display to the user program. It is usually implemented with the same physical device as the locator but has a separate software interface to the user program. In OpenGL, we can use a process of selection to accomplish picking.

4. **CHOICE:** A choice device allows the user to select one of a discrete number of options. In OpenGL, we can use various widgets provided by the window system. A widget is a graphical interactive component provided by the window system or a toolkit. The Widgets include menus, scrollbars and graphical buttons. For example, a menu with n selections acts as a choice **device, allowing user to select one of 'n' alternatives.**

5. **VALUATORS:** They provide analog input to the user program on some graphical systems; there are boxes or dials to provide value.

6. **STROKE:** A stroke device returns array of locations. Example, pushing down a mouse button starts the transfer of data into specified array and releasing of button ends this transfer.

INPUT MODES

Input devices can provide input to an application program in terms of two entities:

1. ***Measure*** of a device is what the device returns to the application program.
2. ***Trigger*** of a device is a physical input on the device with which the user can send signal to the computer

Example 1: The measure of a keyboard is a single character or array of characters where as the trigger is the enter key.

Example 2: The measure of a mouse is the position of the cursor whereas the trigger is when the mouse button is pressed.

The application program can obtain the measure and trigger in ***three distinct modes***:

1. REQUEST MODE: In this mode, measure of the device is not returned to the program until the device is triggered.

- For example, consider a typical C program which reads a character input using scanf(). When the program needs the input, it halts when it encounters the scanf() statement and waits while user type characters at the terminal. The data is placed in a keyboard buffer (measure) whose contents are returned to the program only after enter key (trigger) is pressed.
- Another example, consider a logical device such as locator, we can move out pointing device to the desired location and then trigger the device with its button, the trigger will cause the location to be returned to the application program.

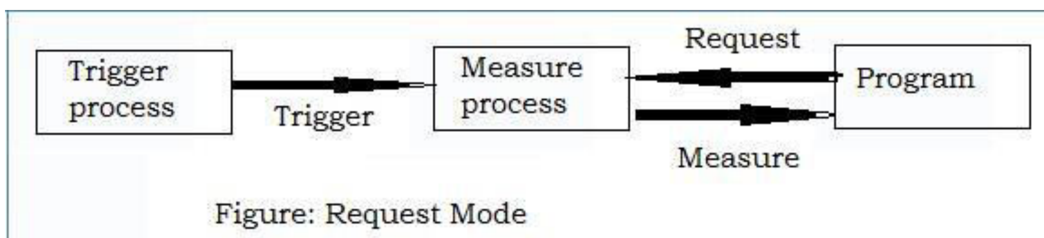


Figure: Request Mode

2. **SAMPLE MODE:** In this mode, input is immediate. As soon as the function call in the user program is executed, the measure is returned. Hence no trigger is needed.

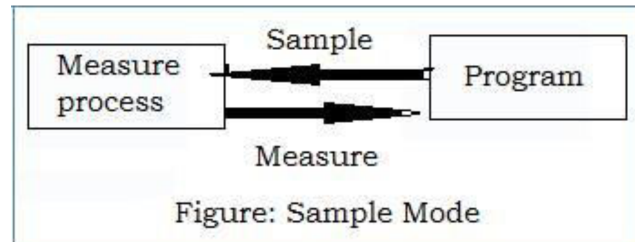


Figure: Sample Mode

Both request and sample modes are useful for the situation if and only if there is a single input device from which the input is to be taken. However, in case of flight simulators or computer games variety of input devices are used and these mode cannot be used. Thus, event mode is used.

3. **EVENT MODE:** This mode can handle the multiple interactions.

- Suppose that we are in an environment with multiple input devices, each with its own trigger and each running a measure process.
- Whenever a device is triggered, an event is generated.The device measure including the identifier for the device is placed in an event queue.
- If the queue is empty, then the application program will wait until an event occurs. If there is an event in a queue, the program can look at the first event type and then decide what to do.

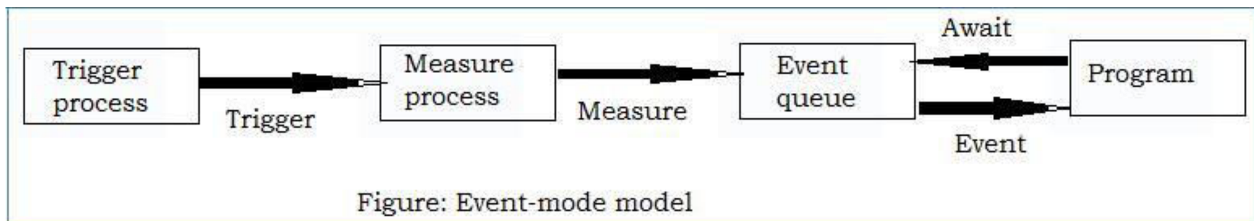


Figure: Event-mode model

Another approach is to associate a function when an event occurs, which is called as "call back."

5.1.3 CLIENT AND SERVER

- The computer graphics architecture is based on the client-server model. I.e., if computer graphics is to be useful for variety of real applications, it must function well in a world of distributed computing and network.
  - In this architecture the building blocks are entities called as **"*servers*"** perform the tasks requested by the **"*client*"**
  - Servers and clients can be distributed over a network or can be present within a single system. Today most of the computing is done in the form of distributed based and network based as shown below:

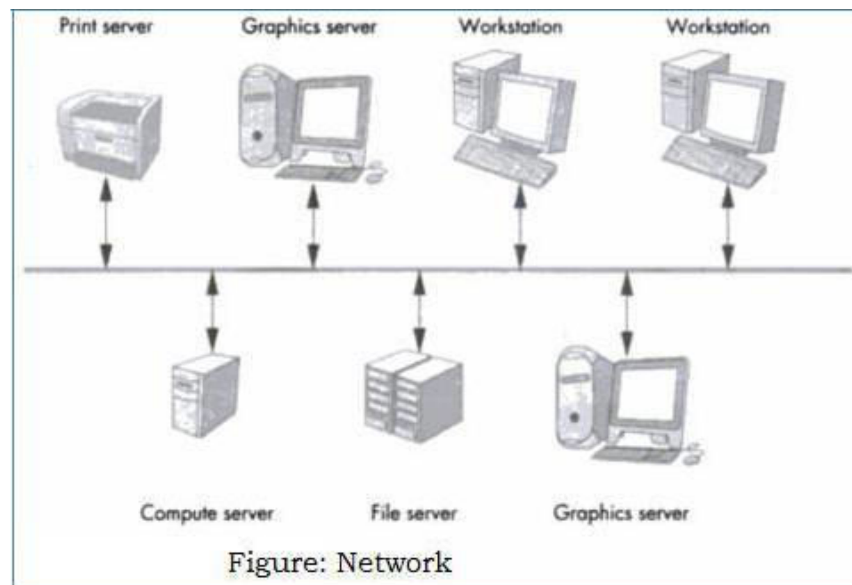


Figure: Network

  - Most popular examples of servers are print servers **–** which allow using high speed printer devices among multiple users. File servers **–** allow users to share files and programs.
- Users or clients will make use of these services with the help of user programs or client programs. The OpenGL application programs are the client programs that use the services provided by the graphics server.
  - Even if we have single user isolated system, the interaction would be configured as client-server model.

5.1.4 DISPLAY LISTS

The original architecture of a graphical system was based on a general-purpose computer connected to a display. The architecture is shown in the next page.

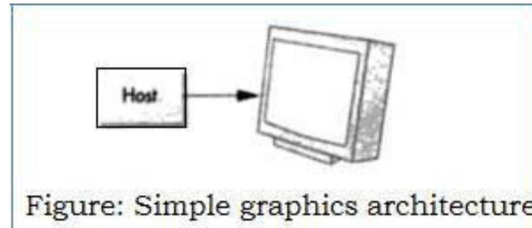


Figure: Simple graphics architecture

At that time, the disadvantage is that system was slow and expensive. Therefore, a special purpose computer is build which is known as ***"display processor"***.

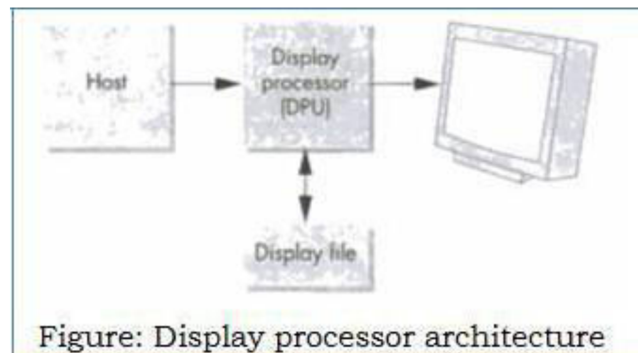


Figure: Display processor architecture

The user program is processed by the host computer which results a compiled list of instruction that was then sent to the display processor, where the instruction are stored in a display memory called as ***"display file"*** or ***"display list"***. Display processor executes its display list contents repeatedly at a sufficient high rate to produce flicker-free image.

There are two modes in which objects can be drawn on the screen:

1. **<u>IMMEDIATE MODE:</u>** This mode sends the complete description of the object which needs to be drawn to the graphics server and no data can be retained. i.e., to redisplay the same object, the program must re-send the information. The information includes vertices, attributes, primitive types, viewing details.

2. <u>RETAINED MODE:</u> This mode is offered by the display lists. The object is defined once and its description is stored in a display list which is at the server side and redisplay of the object can be done by a simple function call issued by the client to the server.

NOTE: The main disadvantage of using display list is it requires memory at the server architecture and server efficiency decreases if the data is changing regularly.

<u>DEFINITION AND EXECUTION OF DISPLAY LISTS</u>

- ➔ Display lists are defined similarly to the geometric primitives. i.e., glNewList() at the beginning and glEndList() at the end is used to define a display list.
- ➔ Each display list must have a unique identifier – an integer that is usually a macro defined in the C program by means of #define directive to an appropriate name for the object in the list. *For example, the following code defines red box:*

```
#define BOX 1 /* or some other unused integer */

glNewList(BOX, GL_COMPILE);
    glBegin(GL_POLYGON);
        glColor3f(1.0, 0.0, 0.0);
        glVertex2f(-1.0, -1.0);
        glVertex2f( 1.0, -1.0);
        glVertex2f( 1.0,  1.0);
        glVertex2f(-1.0,  1.0);
    glEnd();
glEndList();
```

- ➔ The flag GL_COMPILE indicates the system to send the list to the server but not to display its contents. If we want an immediate display of the contents while the list is being constructed then GL_COMPILE_AND_EXECUTE flag is set.

➔ Each time if the client wishes to redraw the box on the display, it need not resend the entire description. Rather, it can call the following function:

glCallList(Box)

➔ The Box can be made to appear at different places on the monitor by changing the projection matrix as shown below:

```
glMatrixMode(GL_PROJECTION);
for(i= 1 ; i<5; i++)
{
    glLoadIdentity();
    gluOrtho2D(-2.0*i  , 2.0*i , -2.0*i , 2.0*i );
    glCallList(BOX);
}
```

➔ OpenGL provides an API to retain the information by using ***stack*** **–** It is a data structure in which the item placed most recently is removed first [LIFO].

➔ *We can save the present values of the attributes and the matrices by pushing them into the stack, usually the below function calls are placed at the beginning of the display list,*

glPushAttrib(GL_ALL_ATTRIB_BITS);
glPushMatrix();

➢ *We can retrieve these values by popping them from the stack, usually the below function calls are placed at the end of the display list,*

glPopAttrib();
glPopMatrix();

➢ We can create multiple lists with consecutive identifiers more easily using:

glGenLists (number)

➢ We can display multiple display lists by using single funciton call:

glCallLists()

TEXT AND DISPLAY LISTS

- *There are two types of text* i.e., ***raster text and stroke text*** which can be generated.
- For example, let us consider a raster text character is to be drawn of size 8x13 pattern of bits. It takes 13 bytes to store each character.
- If we define a stroke font using only line segments, each character requires a different number of lines.

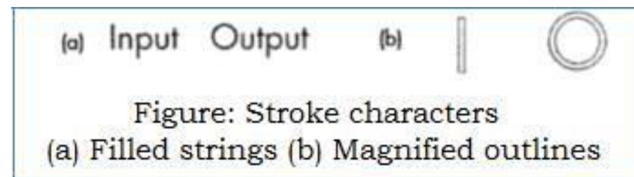


Figure: Stroke characters
(a) Filled strings (b) Magnified outlines

- From the above figure we can observe to draw letter 'I' is fairly simple, whereas drawing 'O' requires many line segments to get sufficiently smooth.
- So, on the average we need more than 13 bytes per character to represent stroke font. The performance of the graphics system will be degraded for the applications that require large quantity of text.
- A more efficient strategy is to define the font once, using a display list for each char and then store in the server. We define a function OurFont() which will draw any ASCII character stored in variable 'c'.
- The function may have the form

```
void OurFont(char c)
    {
            switch(c)
            {
                case 'a':
                    ...
                break;
                case 'A':
                    ...
                break;
                    ...
            }
    }
```

- For the character 'O' the code sequence is of the form as shown below:

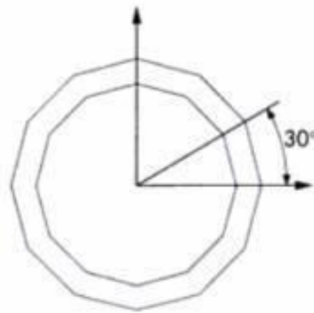


FIGURE Drawing of the letter "O."

```
case 'O':
    glTranslatef(0.5, 0.5, 0.0); /* move to center */
    glBegin(GL_QUAD_STRIP);
    for (i=0; i<=12; i++)  /* 12 vertices */
    {
          angle = 3.14159 /6.0 * i; /* 30 degrees in radians */
          glVertex2f(0.4*cos(angle)+0.5; 0.4*sin(angle)+0.5);
          glVertex2f(0.5*cos(angle)+0.5, 0.5*sin(angle)+0.5);
    }
    glEnd();
    break;
```

- The above code approximates the circle with 12 quadrilaterals.
- When we want to generate a 256-character set, the required code using OurFont() is as follows

```
base = glGenLists(256);
for(i=0;i<256;i++) {
        glNewList(base+i, GL_COMPILE);
                OurFont(i);
        glEndList();
}
```

- To display char from the list, offset is set by using glListBase(base) function. The drawing of a string is accomplished in the server by the following function, char *text_string;

```
glCallLists((GLint) strlen (text_string), GL_BYTE, text_string);
```

- The glCallLists has three arguments: (1) indicates number of lists to be executed (2) indicates the type (3) is a pointer to an array of a type given by second argument.

FONTS IN GLUT

- GLUT provides raster and stroke fonts; they do not make use of display lists.

- ***glutStrokeCharacter(GLUT_STROKE_MONO_ROMAN, int character);*** provides proportionally space characters. Position of a character is done by using a translation before the character function is called.
- ***glutBitmapCharacter(GLUT_BITMAP_8_BY_13, int character);*** produces the bitmap characters of size 8x13.

5.1.5 DISPLAY LIST AND MODELING

- Display list can call other display list. Therefore, they are powerful tools for building hierarchical models that can incorporate relationships among parts of a model.
- Consider a simple face modeling system that can produce images as follows:

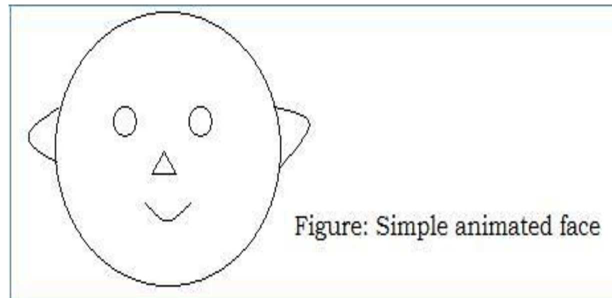


Figure: Simple animated face

- Each face has two identical eyes, two identical ears, one nose, one mouth & an outline. We can specify these parts through display lists which is given below:

```
#define EYE 1
        glNewList(EYE);
        /*eye code*/
        glEndList();

//Similarly code for ears, nose, mouth, outline

#define FACE 2
        glNewList(FACE);
//code for outline
                glTranslatef(...);
                glCallList(EYE); //left-eye
                glTranslatef(...);
                glCallList(EYE); //right-eye
                glTranslatef(...);
                glCallList(NOSE);
//similarly code for ears and mouth
        glEndList();
```

5.1.6 PROGRAMMING EVENT DRIVEN INPUT

- The various events can be recognized by the window system and call back function can be called for each of these events.

USING POINTING DEVICES

➔ Pointing devices like mouse, trackball, data tablet allow programmer to indicate a position on the display.

➔ There are two types of event associated with pointing device, which is conventionally assumed to be mouse but could be trackball or data tablet also.

1. **MOVE EVENT –** is generated when the mouse is move with one of the button being pressed. If the mouse is moved without a button being pressed, this event is called as **"*passive move event*"**.
2. **MOUSE EVENT –** is generated when one of the mouse buttons is either pressed or released.

➔ The information returned to the application program includes button that generated the event, state of the button after event (up or down), position (x,y) of the cursor. Programming a mouse event involves two steps:

1. The mouse callback function must be defined in the form: ***void myMouse(int button, int state, int x, int y)*** is written by the programmer.

For example,

```
void myMouse(int button, int state, int x, int y)
{
        if(button==GLUT_LEFT_BUTTON && state == GLUT_DOWN)
                        exit(0);
}
```

The above code ensures whenever the left mouse button is pressed down, execution of the program gets terminated.

2. Register the defined mouse callback function in the main function, by means of GLUT function:
   glutMouseFunc(myMouse);

Write an OpenGL program to display square when a left button is pressed and to exit the program if right button is pressed.

```
#include<stdio.h>
#include<stdlib.h>
#include<GL/glut.h>
int wh=500, ww=500;
float siz=3;
void myinit()
{
	glClearColor(1.0,1.0,1.0,1.0);
	glViewPort(0,0,w,h)
	glMatrixMode(GL_PROJECTION);
	glLoadIdentity();
	gluOrtho2D(0,(GLdouble) ww, 0, (GLdouble) wh);
	glMatrixMode(GL_MODELVIEW);
	glColor3f(1,0,0);
}

void drawsq (int x, int y)
{
	y=wh-y;
	glBegin(GL_POLYGON);
		glVertex2f(x+siz, y+siz);
```

```
            glVertex2f(x-siz, y+siz);
            glVertex2f(x-siz, y-siz);
            glVertex2f(x+siz, y-siz);
        glEnd();
        glFlush();
}
void display()
 {
        glClear(GL_COLOR_BUFFER_BIT);
}

void myMouse(int button, int state, int x, int y)
{
        if(button==GLUT_LEFT_BUTTON && state == GLUT_DOWN)
                        drawsq(x,y);
        if(button==GLUT_RIGHT_BUTTON && state == GLUT_DOWN)
                        exit(0);
}
void main(int argc, char **argv)
 {
        glutInit(&argc, argv);
        glutInitDisplayMode(GLUT_RGB|GLUT_SINGLE);
        glutInitWindowSize(wh,ww);
        glutCreateWindow("square");
        glutDisplayFunc(display);
        glutMouseFunc(myMouse);
        myinit();
        glutMainLoop();
}
```

KEYBOARD EVENTS

- ➔ Keyboard devices are input devices which return the ASCII value to the user program. *Keyboard events are generated when the mouse is in the window and one of the keys is pressed or released.*
- ➔ GLUT supports following two functions:
- glutKeyboardFunc() is the callback for events generated by pressing a key
- glutKeyboardUpFunc() is the callback for events generated by releasing a key.
- ➔ The information returned to the program includes ASCII value of the key pressed and the position (x,y) of the cursor when the key was pressed. Programming keyboard event involves two steps:

1. The keyboard callback function must be defined in the form:
   void mykey (unsigned char key, int x, int y)
   is written by the application programmer.

For example,

```
void mykey(unsigned char key, int x, int y)
{
                    if(key== 'q' || key== 'Q')
                          exit(0);
}
```

The above code ensures when 'Q' or 'q' key is pressed, the execution of the program gets terminated.

2. The keyboard callback function must be registered in the main functionby means of GLUT function:
   glutKeyboardFunc(mykey);

WINDOW EVENTS

- *A window event is occurred when the corner of the window is dragged to new position or size of window is minimized or maximized by using mouse.*
- The information returned to the program includes the height and width of newly resized window. Programming the window event involves two steps:

1. Window call back function must be defined in the form:

- ***void myReshape(GLsizei w, GLsizei h)*** is written by the application programmer.
- Let us consider drawing square as an example, the square of same size must be drawn regardless of window size.

```
void myReshape(GLsizei w, GLsizei h)
{
        glMatrixMode(GL_PROJECTION);
        glLoadIdentity();
        gluOrtho2D(0,(GLdouble) w, 0, (GLdouble) h);
        glMatrixMode(GL_MODELVIEW);
        glLoadIdentity();
        glViewPort(0,0,w,h)
        /*save new window size in global  variables*/
         ww=w;
        wh=h;
}
```

2. The window callback function must be registered in the main function,

glutReshapeFunc(myReshape);

THE DISPLAY AND IDLE CALLBACKS

- Display callback is specified by GLUT using ***glutDisplayFunc(myDisplay)***. It is invoked when GLUT determines that window should be redisplayed. Re-execution of the display function can be achieved by using ***glutPostRedisplay()***.
- The idle callback is invoked when there are no other events. It is specified by GLUT using ***glutIdleFunc(myIdle)***.

WINDOW MANAGEMENT

- GLUT also supports multiple windows of a given window. We can create a second top-level window as follows:

  id = glutCreateWindow("second window");
- The returned integer value allows us to select this window as the current window.

  i.e., glutSetWindow(id);

NOTE: The second window can have different properties from other window by invoking the glutInitDisplayMode before glutCreateWindow.

5.1.7 MENUS

- Menus are an important feature of any application program. OpenGL provides a feature called ***"Pop-up-menus"*** using which sophisticated interactive applications can be created.
- Menu creation involves the following steps:

1. *Define the actions corresponding to each entry in the menu.*
2. *Link the menu to a corresponding mouse button.*
3. *Register a callback function for each entry in the menu.*

```
glutCreateMenu(demo_menu);
glutAddMenuEntry("quit",1);
glutAddMenuEntry("increase square size", 2);
glutAddMenuEntry("decrease square size", 3);
glutAttachMenu(GLUT_RIGHT_BUTTON);
```

- The glutCreateMenu() registers the callback function demo_menu. The function glutAddMenuEntry() adds the entry in the menu whose name is pased in first argument and the second argument is the identifier passed to the callback when the entry is selected.

```
void demo_menu(int id)
{
      switch(id)
      {
            case 1: exit(0);
            break;
            case 2: size = 2 * size;
            break;
            case 3: if(size > 1) size = size/2;
            break;
      }
      glutPostRedisplay( );
}
```

- GLUT also supports the creation of hierarchical menus which is given below:

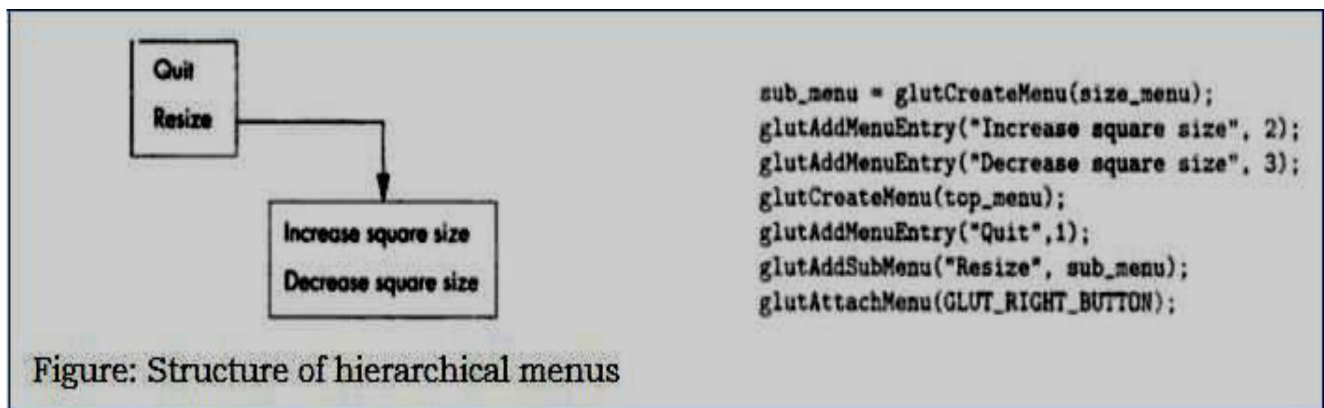


Figure: Structure of hierarchical menus

5.1.8 PICKING

- *Picking is the logical input operation that allows the user to identify an object on the display.*
- The action of picking uses pointing device but the information returned to the application program is the identifier of an object not a position.

- It is difficult to implement picking in modern system because of graphics pipeline architecture. Therefore, converting from location on the display to the corresponding primitive is not direct calculation.
- There are at least three ways to deal with this difficulty:
    - ***Selection:***
        - It involves adjusting the clipping region and viewport such that we can keep track of which primitives lies in a small clipping region and are rendered into region near the cursor.
        - These primitives are sent into a ***hit list*** that can be examined later by the user program.
    - ***Bounding boxes or extents:***
        - In this approach, the extent of an object is the smallest rectangle aligned with the coordinate axis that contain object.
        - For 2D, it is relatively easy to determine the rectangle in screen coordinates that corresponds to rectangle point in object coordinates.
        - For 3D, bounding box is right parallelepiped. If application program maintains simple data structure to relate objects and bounding boxes, approximate picking can be done within application program.
    - ***Usage of back buffer and extra rendering:***
        - When we use double buffering it has two color buffers: front and back buffers. The contents present in the front buffer is displayed, whereas contents in back buffer is not displayed so we can use back buffer for other than rendering the scene
- Picking can be performed in four steps that are initiated by user defined pick function in the application:

  - We draw the objects into back buffer with the pick colors.
  - We get the position of the mouse using the mouse callback.
  - Use glReadPixels() to find the color at the position in the frame buffer corresponding to the mouse position.
  - We search table of colors to find the object corresponds to the color read.

PICKING AND SELECTION MODE

- The difficulty in implementing the picking is we cannot go backward directly from the position of the mouse to the primitives.
- OpenGL provides **"**selection mode**"** to do picking. The glRenderMode() is used to choose select mode by passing GL_SELECT value.
- When we enter selection mode and render a scene, each primitive within the clipping volume generates a message called **"*hit*"** that is stored in a buffer called **"*name stack*"**.
- The following functions are used in selection mode:
  - ***void glSelectBuffer(GLsizei n, GLuint *buff) :*** specifies array **buffer of size 'n' in which to place selection data.**
  - ***void glInitNames() :*** initializes the name stack.
  - ***void glPushName(GLuint name) :*** pushes name on the name stack.
  - ***void glPopName() :*** pops the top name from the name stack.
  - ***void glLoadName(GLuint name) :*** replaces the top of the name stack with name.

- OpenGL allow us to set clipping volume for picking using gluPickMatrix() which is applied before gluOrtho2D.
- ***gluPickMatrix(x,y,w,h,*vp) :*** creates a projection matrix for picking that restricts drawing to a w x h area and centered at (x,y) in window coordinates within the viewport vp.

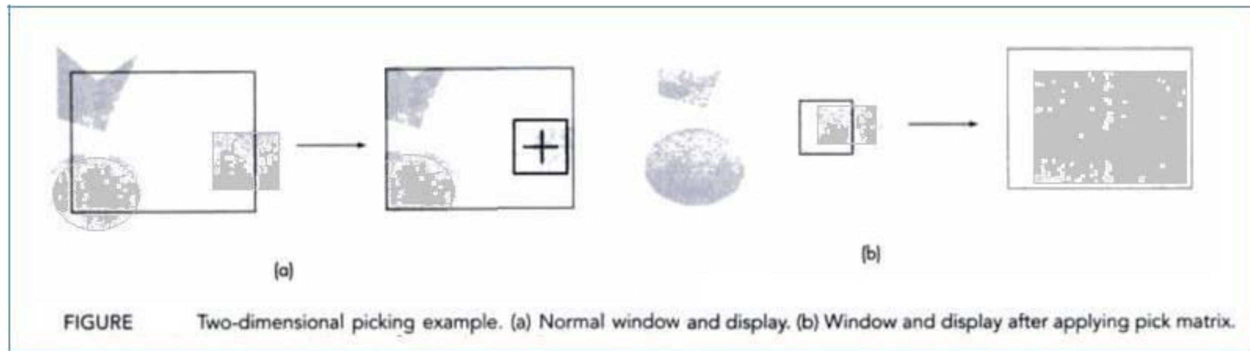


FIGURE Two-dimensional picking example. (a) Normal window and display. (b) Window and display after applying pick matrix.

(a) There is a normal window and image on the display. We also see the cursor with small box around it indicating the area in which primitive is rendered.

(b) It shows window and display after the window has been changed by gluPickMatrix.

The following code provides the implementation of picking process:

```
#include<glut.h>

void display( )
{
    glClear( GL_COLOR_BUFFER_BIT);
    draw_objects(GL_RENDER);
    glFlush();
}

void drawObjects(GLenum mode)
{
    if(mode == GL_SELECT) glLoadName(1);
    glColor3f(1.0, 0.0, 0.0);
    glRectf(-0.5, -0.5, 1.0, 1.0);

    if(mode == GL_SELECT) glLoadName(2);
    glColor3f(0.0, 0.0, 1.0);
    glRectf(-1.0, -1.0, 0.5, 0.5);
}
```

```
#define N 2 /* N x N pixels around cursor for pick area */

 void mouse(int button, int state, int x, int y )
{
    GLuint nameBuffer[SIZE]; /* define SIZE elsewhere */
    GLint hits;
    GLint viewport[4];
    If( button == GLUT_LEFT_BUTTON && state == GLUT_DOWN)
    {
        /* initialize the name stack */

        glInitNames( );
        glPushName( 0 );
        glSelectBuffer( SIZE, nameBuffer);

       /* set up viewing for selection mode */

       glGetIntegerv(GL_VIEWPORT, viewport);
       glMatrixMode(GL_PROJECTION);

       /* save original viewing matrix */

       glPushMatrix();
       glLoadIdentity( );

       /* N x N pick area around cursor */
       /* must invert mouse y to get in world coords */
       gluPickMatrix( (GLdouble) x, (GLdouble)
           ( viewport[3] - y), N, N, viewport);

      /* same clipping window as in reshape callback */

      gluOrtho2D (xmin, xmax, ymin, ymax);

      draw_objects(GL_SELECT);
      glMatrixMode(GL_PROJECTION);

      /* restore viewing matrix */

      glPopMatrix();
      glFlush();

      /* return to normal render mode */

      hits = glRenderMode(GL_RENDER);

      /* process hits from selection mode rendering */

      processHits(hits, nameBuff);

      /* normal render */

      glutPostRedisplay();
   }
}
```

```
void myReshape()
{
        glViewPort(0,0,w,h)
        glMatrixMode(GL_PROJECTION);
        glLoadIdentity();
        gluOrtho2D(0,(GLdouble) w, 0, (GLdouble) h);
        glMatrixMode(GL_MODELVIEW);
}
```

```
void processHits (GLint hits, GLuint buffer[])
{
   unsigned int i, j;
   GLuint names, *ptr;

   printf ("hits = %d\n", hits);
   ptr = (GLuint *) buffer;

    /* loop over number of hits */

   for (i = 0; i < hits; i++)
    {
        names = *ptr;

        /* skip over number of names and depths */

        ptr += 3;

        /* check each name in record */

      for (j = 0; j < names; j++)
        {
         if(*ptr==1) printf ("red rectangle\n");
         else printf ("blue rectangle\n");

        /* go to next hit record */

         ptr++;
      }
   }
}
```

```
void main(int argc, char** argv)
{
    glutInit(&argc, argv);
    glutInitDisplayMode(GLUT_SINGLE |GLUT_RGB);
```

```
    glutInitWindowSize(500,500);
    glutInitWindowPosition(100,100);
    glutCreateWindow("picking");
    glutReshapeFunc(myReshape);
    glutDisplayFunc(display);
    glutMouseFunc(Mouse);
    glClearColor(0.0,0.0,0.0,0.0);
    glutMainLoop();
}
```

5.1.9 A SIMPLE CAD PROGRAM

Applications like interactive painting, design of mechanical parts and creating characters for a game are all examples of computer-aided design (CAD). CAD programs allow –

- The use of multiple windows and viewports to display a variety of information.
- The ability to create, delete and save user-defined objects.
- Multiple modes of operations employing menus, keyboard and mouse.

For example, consider the polygon-modeling CAD program which supports following operations:

1. Creation of polygons
2. Deletion of polygons
3. Selection and movement of polygons

Refer appendix A.5 polygon modeling program for the entire code from the prescribed text (Interactive Computer Graphics by Edward Angel 5th edition)

5.1.10 BUILDING INTERACTIVE MODELS

- Using OpenGL, we can develop a program where we can do insertion, manipulation, deletion etc and we can also build a program which is quite interactive by using the concept of instancing and display lists.
  - Consider an interior design application which has items like chairs, tables and other house hold items. These items are called the basic building blocks of the application. Each occurrence of these basic items is referred to as "***instance***".
- Whenever the instances of building blocks are created by the user using the application program, the object (instance) is stored into an array called as "***instance table***". We reserve the type 0 to specify that the object no longer exists (i.e., for deletion purpose)
- *Now suppose that the user has indicated through a menu that he wishes to eliminate an object and use the mouse to locate the object.*
  - The program can now search the instance table till it finds the object as specified in the bounding box and then set its type to 0.
  - Hence, next time when the display process goes through the instance table, the object would not be displayed and thereby it appears that object has been deleted.
- Although the above strategy works fine, a better data structure to implement the instance table is using linked lists instead of arrays.

5.1.11 ANIMATING INTERACTIVE PROGRAMS

- Using OpenGl, the programmer can design interactive programs. Programs in which objects are not static rather they appear to be moving or changing is considered as "***Interactive programs***".
- Consider the following diagram:

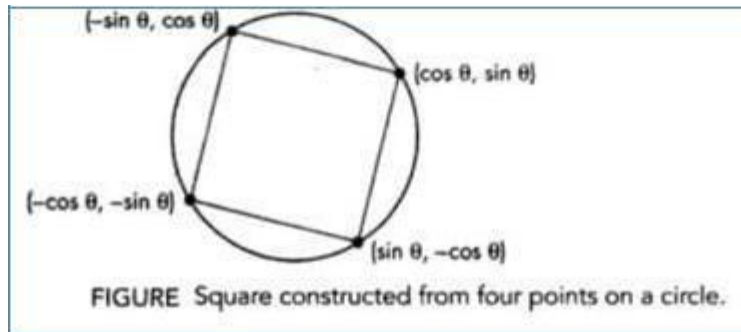


FIGURE Square constructed from four points on a circle.

- Consider a 2D point p(x,y) such that ***x = cos θ, y= sin θ***. This point would lie on a unit circle regardless of the value of θ. Thus, if we connect the above given four points we get a square which has its center as the origin. The above square can be displayed as shown below:

```
void display()
{
  glClear(GL_COLOR_BUFFER_BIT);
  glBegin(GL_POLYGON);
    thetar = theta/(3.14159/180.0); /* convert degrees to radians */
    glVertex2f(cos(thetar), sin(thetar));
    glVertex2f(-sin(thetar), cos(thetar));
    glVertex2f(-cos(thetar), -sin(thetar));
    glVertex2f(sin(thetar), -cos(thetar));
  glEnd();
}
```

- Suppose that we change the value of θ as the program is running, the square appears to rotating about its origin. If the value of θ is to be changed by a fixed amount whenever nothing else is happening then an idle callback function must be designed as shown below:

```
void idle()
{
  theta+=2; /* or some other amount */
  if(theta >= 360.0 theta-=360.0;
  glutPostRedisplay();
}
```

- The above idle callback function must be registered in the main function:

  glutIdleFunc(idle);

- Suppose that we want to turn off and turn on the rotation feature then we can write a mouse callback function as shown below:

```
void mouse(int button, int state, int x, int y)
{
  if(button==GLUT_LEFT_BUTTON&&state==GLUT_DOWN)
      glutIdleFunc(idle);
  if(button==GLUT_MIDDLE_BUTTON&&state=GLUT_DOWN)
      glutIdleFunc(NULL);
}
```

- The above mouse callback function starts the rotation of the cube when the left mouse button and when the middle button is pressed it will halt the rotation.
- The above mouse callback function must be registered in the main function as follow:

glutMouseFunc(mouse);

- However, when the above program is executed using single buffering scheme then flickering effect would be noticed on the display. This problem can be overcome using the concept of double buffering.

DOUBLE BUFFERING:

- Double buffering is a must in such animations where the primitives, attributes and viewing conditions are changing continuously.
- Double buffer consists of two buffers: front buffers and back buffers. Double buffering mode can be initialized:

glutInitDisplayMode(GLUT_RGB | GLUT_DOUBLE);

- Further in the display function, we have to include: ***glutSwapBuffers()*** to exchange the contents of front and the back buffer.
- Using this approach, the problems associated with flicker can be eliminated.

USING TIMER:

- To understand the usage of timer, consider cube rotation program and its execution is done by using fast GPU (modern GPUs can render tens of millions of primitives per second) then cube will be rendered thousands of time per second and we will see the blur on the display.
- Therefore, GLUT provides the following timer function:

glutTimerFunc(int delay, void(*timer_func (int), int value)

- Execution of this function starts timer in the event loop that delays for *delay* milliseconds. When *timer* has counted down, *timer_func* is executed the *value* parameter allow user to pass variable into the timer call back.

5.1.12 DESIGN OF INTERACTIVE PROGRAMS

The following are the features of most interactive program:

- ✓ A smooth display, showing neither flicker nor any artifacts of the refresh process.
- ✓ A variety of interactive devices on the display
- ✓ A variety of methods for entering and displaying information
- ✓ An easy to use interface that does not require substantial effort to learn
- ✓ Feedback to the user
- ✓ Tolerance for user errors
- ✓ A design that incorporates consideration of both the visual and motor properties of the human.

TOOLKITS, WIDGETS AND FRAME BUFFER:

The following two examples illustrate the limitations of geometric rendering.

1. ***Pop-up menus:*** When menu callback is invoked, the menu appears over whatever was on the display. After we make our selection, the menu disappears and screen is restored to its previous state.

2. ***Rubberbanding:*** It is a technique used to define the elastic nature of pointing device to draw primitives.

➔ Consider paint application, if we want to draw a line, we indicate only two end points of our desired line segment. i.e., after locating first point, as we move the mouse, a line segment is drawn automatically [is updated on each refresh] from first location to present position of mouse.

➔ ***Rubberbanding*** begin when mouse button is pressed and continue until button is released at that time final line segment is drawn.

➔ We cannot implement this sequence of operations using only what we have presented so for. We will explore it in next chapters.

5.1.13 LOGIC OPERATIONS

Two types of functions that define writing modes are:

1. Replacement mode 2. Exclusive OR (XOR)

- When program specifies about visible primitive then OpenGL renders it into set of color pixels and stores it in the present drawing buffer.
  - In case of default mode, consider we start with a color buffer then has been cleared to black. Later we draw a blue color rectangle of size 10 x10 pixels then 100 blue pixels are copied into the color buffer, replacing 100 black pixels. Therefore, this mode is called as **"*copy or replacement mode*"**.
- Consider the below model, where we are writing single pixel into color buffer.

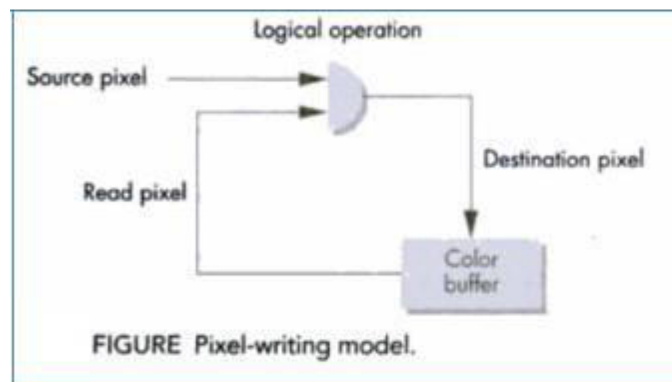


FIGURE Pixel-writing model.

- The pixel that we want to write is called as "***source pixel***".
- The pixel in the drawing buffer which gets replaced by source pixel is called as "***destination pixel***".
- In ***Exclusive-OR or (XOR) mode***, corresponding bits in each pixel are combing using XOR logical operation.
- If s and d are corresponding bits in the source and destination pixels, we **can denote the new destination bit as d'. d' = d** $\oplus$ s
- One special property of XOR operation is if we apply it twice, it returns to the original state, it returns to the original state. So, if we draw some thing in XOR mode, we can erase it by drawing it again.

$$d = (d \oplus s) \oplus s$$

- OpenGL supports all 16 logic modes, copy mode (GL_COPY) is the default. To change mode, we must enable logic operation, glEnable(GL_COLOR_LOGIC_OP) and then it can change to XOR mode glLogicOp(GL_XOR)

DRAWING ERASABLE LINES

One way to draw erasable lines is given below:

- Mouse is used to get first end point and store this in object coordinates.

```
xm = x/500.;
ym = (500-y)/500.;
```

- Again mouse is used to get second point and draw a line segment in XOR mode.

```
xmm = x/500.;
ymm = (500-y)/500.;
glLogicOp(GL_XOR);
glBegin(GL_LINES);
    glVertex2f(xm, ym);
    glVertex2f(xmm, ymm);
glLogicOp(GL_COPY);
glEnd();
glFlush();
```

- Here in the above code, copy mode is used to switch back in order to draw other objects in normal mode.
    - If we enter another point with mouse, we first draw line in XOR mode from 1st point to 2nd point and draw second line using 1st point to current point is as follows:

```
glLogicOp(GL_XOR);
glBegin(GL_LINES);
    glVertex2f(xm, ym);
    glVertex2f(xmm, ymm);
glEnd();
glFlush();
xmm = x/500.0;
ymm = (500-y)/500.0;
glBegin(GL_LINES);
    glVertex2f(xm, ym);
    glVertex2f(xmm, ymm);
glEnd();
glLogicOp(GL_COPY);
glFlush();
```

Final form of code can be written as shown below:

```
glLogicOp(GL_COPY);
glBegin(GL_LINES);
    glVertex2f(xm, ym);
    glVertex2f(xmm, ymm);
glEnd();
glFlush();
glLogicOp(GL_XOR);
```

In this example, we draw rectangle using same concept and the code for callback function are given below:

```
float xm, ym, xmm, ymm; /* the corners of the rectangle */
int first = 0; /* vertex the counter */
```

The callbacks are registered as as follows:

```
glutMouseFunc(mouse);
glutMotionFunc(move);
```

```
void mouse(int btn, int state, int x, int y)
{
    if(btn==GLUT_LEFT_BUTTON && state == GLUT_DOWN)
    {
        xm = x/500.;
        ym = (500-y)/500.;
        glColor3f(0.0, 0.0, 1.0);
        glLogicOp(GL_XOR);
        first = 0;
    }
    if(btn==GLUT_LEFT_BUTTON && state == GLUT_UP)
    {
        glRectf(xm, ym, xmm, ymm);
        glFlush();
        glColor3f(0.0, 1.0, 0.0);
        glLogicOp(GL_COPY);
        xmm = x/500.0;
        ymm = (500-y)/500.0;
        glLogicOp(GL_COPY);
        glRectf(xm, ym, xmm, ymm);
        glFlush();
    }
}
```

```
void move(int x, int y)
{
    if( first == 1)
    {
        glRectf(xm, ym, xmm, ymm);
        glFlush();
    }
    xmm = x/500.0;
    ymm = (500-y)/500.0;
    glRectf(xm, ym, xmm, ymm);
    glFlush();
    first = 1;
}
```

- For the first time, we draw a single rectangle in XOR mode.
- After that each time that we get vertex, we first erase the existing rectangle by redrawing new rectangle using new vertex.
- Finally, when mouse button is released the mouse callback is executed again which performs final erase and draw and go to replacement mode.

XOR AND COLOR

- ✓ Consider we would like to draw blue color line where 24 bit RGB values (00000000, 00000000, 11111111).
- ✓ Suppose the screen is clear to write (11111111, 11111111, 11111111) then when we draw blue line using XOR mode, then the resultant line would appear in yellow color (11111111, 11111111, 00000000) because XOR operation is applied bit-wise.
- ✓ This leads to form annoying visual effects.
- ✓ Therefore, we should use copy mode while drawing final output to get it in required color.

CURSORS AND OVERLAY PLANES

- Rubberbanding and cursors can place a significant burden on graphics system as they require the display to be updated constantly.
- Although XOR mode simplifies the process, it requires the system to read present destination pixels before computing new destination pixels.

- Alternative is to provide hardware support by providing extra bits in the color buffer by adding **“*overlay planes*”**.

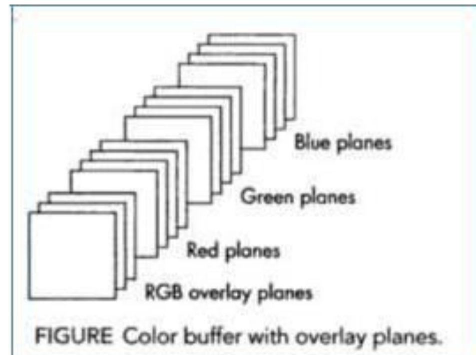


FIGURE Color buffer with overlay planes.

Therefore, typical color buffer may have 8 bits for each Red, green and blue and one red, one green and one blue overlay plane. i.e., each color will be having its own overlay plane then those values will be updated to color buffer.

FREQUENTLY ASKED QUESTIONS:

1. Explain all the available logical input devices in detail. (07M)
2. What is meant by measure and trigger of a device? Explain with the neat diagram, the various input mode models. (10M)
3. Explain the following: (i) Request Mode (ii)Sample Mode (iii)Event Mode (12M)
4. Differentiate event mode with request mode. (04M)
5. Describe logical input operation of picking in selection mode. (06M)
6. Write an OpenGL program to display square when a left button is pressed and to exit the program if right button is pressed. (10M)
7. What is display list? Give the OpenGL code segment that generates a display list defining a red triangle with vertices at (50,50) (150,50) and (100,150). (7M)
8. Using OpenGL functions, explain the structure of hierarchical menus. (06M)
9. List out the characteristics of good interactive program. Explain in detail. (08M)
10. What is double buffering? How OpenGL implements double buffering? (04M)
11. ***Write an OpenGL program on rotating or spinning a cube. (10M)***

<u>NOTE:</u> All the above questions are from previous year question papers. Do study the questions from other concepts also.

5.2 Curves:

5.2.1 Curved surfaces

5.2.2 Quadric surfaces

5.2.3 OpenGL quadric surfaces and cubic surface functions

5.2.4 Bezier spline curves

5.2.5 Bezier surfaces

5.2.6 Opengl curve functions

5.2.7 Corresponding opengl functions

5.2.1 Curved surfaces

- Sometimes it is required to generate curved objects instead of polygons, for the curved objects the equation can be expressed either in parametric form or non parametric form. Curves and surfaces can be described by parameters
- **Parametric form:**
  - ✓ When the object description is given in terms of its dimensionality parameter, the description is termed as parametric representation.
  - ✓ A curve in the plane has the form C(t) = (x(t), y(t)), and a curve in space has the form C(t) = (x(t), y(t), z(t)).
  - ✓ The functions x(t), y(t) and z(t) are called the coordinates functions.
  - ✓ The image of C(t) is called the trace of C, and C(t) is called a parametrization of C
  - ✓ A parametric curve defined by polynomial coordinate function is called a polynomial curve.
  - ✓ The degree of a polynomial curve is the highest power of the variable occurring in any coordinate function.
- **Non parametric form:**
  - ✓ When the object descriptions are directly in terms of coordinates of reference frame, then the representation is termed as non parametric.
  - ✓ Example: a surface can be described in non parametric form as:

$$f_1(x,y,z)=0 \text{ or } z=f_2(x,y)$$

- ✓ The coordinates (x, y) of points of a no parametric explicit planner curve satisfy y = f(x) or x = g(y).
- ✓ Such curve have the parametric form C(t) = (t, f(t)) or C(t) = (g(t), t).

5.2.2 Quadric surfaces

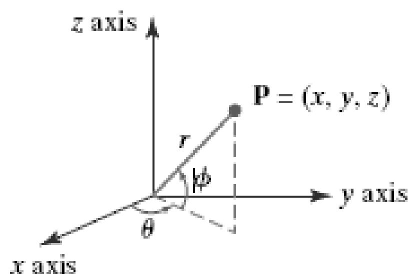
- ✓ A frequently used class of objects is the quadric surfaces, which are described with second - degree equations (quadratics).
- ✓ They include spheres, ellipsoids, tori, paraboloids, and hyperboloids.

1. <u>Sphere</u>

- ❖ A spherical surface with radius *r* centered on the coordinate origin is defined as the set of points *(x, y, z)* that satisfy the equation

$$x^2 + y^2 + z^2 = r^2$$

- ❖ We can also describe the spherical surface in parametric form, using latitude and longitude angles as shown in figure

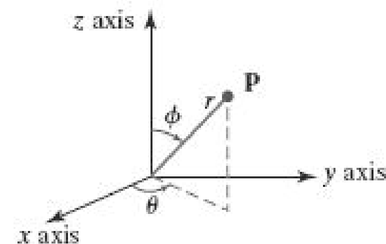


$$x = r \cos \varphi \cos \theta, -\pi/2 \le \varphi \le \pi/2$$

$$y = r \cos \varphi \sin \theta, -\pi \le \theta \le \pi$$

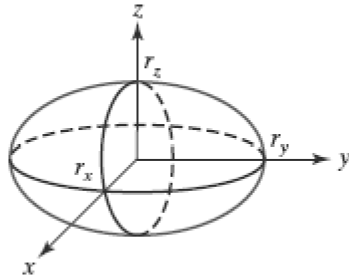
$$z = r \sin \varphi$$

- ❖ Alternatively, we could write the parametric equations using standard spherical coordinates, where angle φ is specified as the colatitudes as shown in figure



2. Ellipsoid

- ❖ An ellipsoidal surface can be described as an extension of a spherical surface where the radii in three mutually perpendicular directions can have different values



- The Cartesian representation for points over the surface of an ellipsoid centered on the origin is

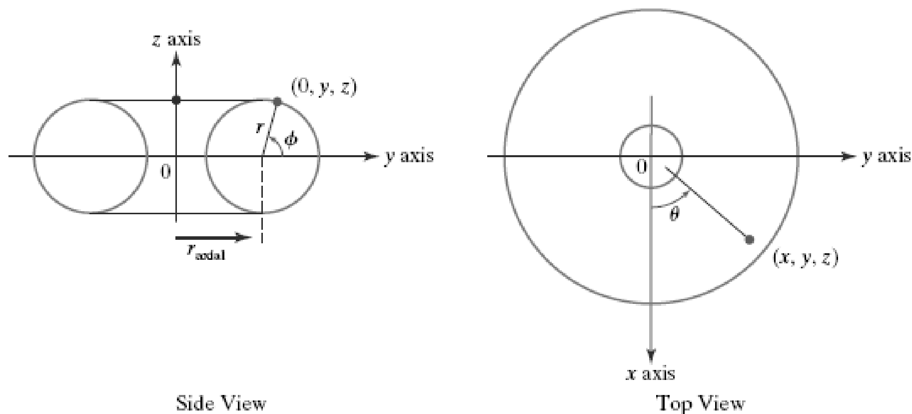
$$\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2 + \left(\frac{z}{r_z}\right)^2 = 1$$

- And a parametric representation for the ellipsoid in terms of the latitude angle φ and the longitude angle θ

$$x = rx \cos\varphi \cos\theta, -\pi/2 \le \varphi \le \pi/2$$
$$y = ry \cos\varphi \sin\theta, -\pi \le \theta \le \pi$$
$$z = rz \sin\varphi$$

3. <u>Torus</u>

- A torus is a doughnut-shaped object, as shown in fig. below.



- It can be generated by rotating a circle or other conic about a specified axis.
- The equation for the cross-sectional circle shown in the side view is given by

$$(y - raxial)^2 + z^2 = r^2$$

❖ Rotating this circle about the z axis produces the torus whose surface positions are described with the Cartesian equation

$$\left(\sqrt{x^2 + y^2} - r_{\text{axial}}\right)^2 + z^2 = r^2$$

❖ The corresponding parametric equations for the torus with a circular cross-section are

$$x = (r_{axial} + r \cos \varphi) \cos \theta, \; -\pi \le \varphi \le \pi$$

$$y = (r_{axial} + r \cos \varphi) \sin \theta, \; -\pi \le \theta \le \pi$$

$$z = r \sin \varphi$$

❖ We could also generate a torus by rotating an ellipse, instead of a circle, about the z axis. we can write the ellipse equation as

$$\left(\frac{y - r_{\text{axial}}}{r_y}\right)^2 + \left(\frac{z}{r_z}\right)^2 = 1$$

where r_{axial} is the distance along the y axis from the rotation z axis to the ellipse center. This generates a torus that can be described with the Cartesian equation

$$\left(\frac{\sqrt{x^2 + y^2} - r_{\text{axial}}}{r_y}\right) + \left(\frac{z}{r_z}\right)^2 = 1$$

❖ The corresponding parametric representation for the torus with an elliptical crosssection is

$$x = (r_{axial} + r_y \cos \varphi) \cos \theta, \; -\pi \le \varphi \le \pi$$

$$y = (r_{axial} + r_y \cos \varphi) \sin \theta, \; -\pi \le \theta \le \pi$$

$$z = r_z \sin \varphi$$

5.2.3 OpenGL Quadric-Surface and Cubic-Surface Functions

➢ A number of other three-dimensional quadric-surface objects can be displayed using functions that are included in the OpenGL Utility Toolkit (GLUT) and in the OpenGL Utility (GLU).

➢ With the GLUT functions, we can display a sphere, cone, torus, or the teapot

➢ With the GLU functions, we can display a sphere, cylinder, tapered cylinder, cone, flat circular ring (or hollow disk), and a section of a circular ring (or disk).

GLUT Quadric-Surface Functions

Sphere

Function:

glutWireSphere (r, nLongitudes, nLatitudes);

or

glutSolidSphere (r, nLongitudes, nLatitudes);

where,

- r is sphere radius which is double precision point.
- nLongitudes and nLatitudes is number of longitude and latitude lines used to approximate the sphere.

Cone

Function:

glutWireCone (rBase, height, nLongitudes, nLatitudes);

or

glutSolidCone (rBase, height, nLongitudes, nLatitudes);

where,

- rBase is the radius of cone base which is double precision point.
- height is the height of cone which is double precision point.
- nLongitudes and nLatitudes are assigned integer values that specify the number of orthogonal surface lines for the quadrilateral mesh approximation.

Torus

Function:

glutWireTorus (rCrossSection, rAxial, nConcentrics, nRadialSlices);

or

glutSolidTorus (rCrossSection, rAxial, nConcentrics, nRadialSlices);

where,

- rCrossSection radius about the coplanar z axis

- rAxial is the distance of the circle center from the *z* axis
- nConcentrics specifies the number of concentric circles (with center on the *z* axis) to be used on the torus surface,
- nRadialSlices specifies the number of radial slices through the torus surface

GLUT Cubic-Surface Teapot Function

Function:

glutWireTeapot (size);

or

glutSolidTeapot (size);

- ✓ The teapot surface is generated using OpenGL B´ezier curve functions.
- ✓ Parameter size sets the double-precision floating-point value for the maximum radius of the teapot bowl.
- ✓ The teapot is centered on the world-coordinate origin coordinate origin with its vertical axis along the *y* axis.

GLU Quadric-Surface Functions

- ❖ To generate a quadric surface using GLU functions

  1. assign a name to the quadric,

  2.activate the GLU quadric renderer, and

  3.designate values for the surface parameters
- ❖ The following statements illustrate the basic sequence of calls for displaying a wire-frame sphere centered on the world-coordinate origin:

```
GLUquadricObj *sphere1;
sphere1 = gluNewQuadric ( );
gluQuadricDrawStyle (sphere1, GLU_LINE);
gluSphere (sphere1, r, nLongitudes, nLatitudes);
```

where,

- sphere1 is the name of the object

- the quadric renderer is activated with the **gluNewQuadric** function, and then the display mode **GLU_LINE** is selected for **sphere1** with the **gluQuadricDrawStyle** command
- Parameter **r** is assigned a double-precision value for the sphere radius
- **nLongitudes** and **nLatitudes**. number of longitude lines and latitude lines

Three other display modes are available for GLU quadric surfaces

GLU_POINT: quadric surface is displayed as point plot

GLU_SILHOUETTE: quadric surface displayed will not contain shared edges between two coplanar polygon facets

GLU_FILL: quadric surface is displayed as patches of filled area.

- To produce a view of a cone, cylinder, or tapered cylinder, we replace the **gluSphere** function with

  gluCylinder (quadricName, rBase, rTop, height, nLongitudes, nLatitudes);

  - ✓ The base of this object is in the *xy* plane (*z*=0), and the axis is the *z* axis.
  - ✓ rBase is the radius at base and rTop is radius at top
  - ✓ If rTop=0.0,weget a cone; if rTop=rBase,weobtain a cylinder
  - ✓ Height is the height of the object and latitudes and longitude values will be given as nLatitude and nLongitude.

- A flat, circular ring or solid disk is displayed in the *xy* plane (*z*=0) and centered on the world-coordinate origin with

  gluDisk (ringName, rInner, rOuter, nRadii, nRings);

  - ✓ We set double-precision values for an inner radius and an outer radius with parameters **rInner** and **rOuter**. If **rInner** = 0, the disk is solid.
  - ✓ Otherwise, it is displayed with a concentric hole in the center of the disk.
  - ✓ The disk surface is divided into a set of facets with integer parameters **nRadii** and **nRings**

- We can specify a section of a circular ring with the following GLU function:

  gluPartialDisk (ringName, rInner, rOuter, nRadii, nRings, startAngle, sweepAngle);

  - ✓ **startAngle** designates an angular position in degrees in the *xy* plane measured clockwise from the positive *y* axis.
  - ✓ parameter **sweepAngle** denotes an angular distance in degrees from the **startAngle** position.
  - ✓ A section of a flat, circular disk is displayed from angular position **startAngle** to **startAngle + sweepAngle**
  - ✓ For example, if **startAngle** = 0.0 and **sweepAngle** = 90.0, then the section of the disk lying in the first quadrant of the *xy* plane is displayed.

- Allocated memory for any GLU quadric surface can be reclaimed and the surface eliminated with

  gluDeleteQuadric (quadricName);

- Also, we can define the front and back directions for any quadric surface with the following orientation function:

  gluQuadricOrientation (quadricName, normalVectorDirection);

  Where,

  Parameter **normalVectorDirection** is assigned either **GLU_OUTSIDE** or **GLU_ INSIDE**

- Another option is the generation of surface-normal vectors, as follows:

  gluQuadricNormals (quadricName, generationMode);

  Where,

  - ✓ A symbolic constant is assigned to parameter **generationMode** to indicate how surface-normal vectors should be generated. The default is **GLU_NONE.**
  - ✓ For flat surface shading (a constant color value for each surface), we use the symbolic constant **GLU_FLAT**

- ✓ When other lighting and shading conditions are to be applied, we use the constant **GLU_SMOOTH**, which generates a normal vector for each surface vertex position.

- ❖ We can designate a function that is to be invoked if an error occurs during the generation of a quadric surface:

  gluQuadricCallback (quadricName, GLU_ERROR, function);

Example Program Using GLUT and GLU Quadric-Surface Functions

```
#include <GL/glut.h>
GLsizei winWidth = 500, winHeight = 500; // Initial display-window size.
void init (void)
{
                glClearColor (1.0, 1.0, 1.0, 0.0); // Set display-window color.
}
void wireQuadSurfs (void)
{
    glClear (GL_COLOR_BUFFER_BIT); // Clear display window.
    glColor3f (0.0, 0.0, 1.0); // Set line-color to blue.
    /* Set viewing parameters with world z axis as view-up direction. */
    gluLookAt (2.0, 2.0, 2.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0);
    /* Position and display GLUT wire-frame sphere. */
    glPushMatrix ( );
    glTranslatef (1.0, 1.0, 0.0);
    glutWireSphere (0.75, 8, 6);
    glPopMatrix ( );
    /* Position and display GLUT wire-frame cone. */
    glPushMatrix ( );
    glTranslatef (1.0, -0.5, 0.5);
    glutWireCone (0.7, 2.0, 7, 6);
    glPopMatrix ( );
```

```
    /* Position and display GLU wire-frame cylinder. */
    GLUquadricObj *cylinder; // Set name for GLU quadric object.
    glPushMatrix ( );
    glTranslatef (0.0, 1.2, 0.8);
    cylinder = gluNewQuadric ( );
    gluQuadricDrawStyle (cylinder, GLU_LINE);
    gluCylinder (cylinder, 0.6, 0.6, 1.5, 6, 4);
    glPopMatrix ( );
    glFlush ( );
}
void winReshapeFcn (GLint newWidth, GLint newHeight)
{
    glViewport (0, 0, newWidth, newHeight);
    glMatrixMode (GL_PROJECTION);
    glOrtho (-2.0, 2.0, -2.0, 2.0, 0.0, 5.0);
    glMatrixMode (GL_MODELVIEW);
    glClear (GL_COLOR_BUFFER_BIT);
}
void main (int argc, char** argv)
{
    glutInit (&argc, argv);
    glutInitWindowPosition (100, 100);
    glutInitWindowSize (winWidth, winHeight);
    glutCreateWindow ("Wire-Frame Quadric Surfaces");
    init ( );
    glutDisplayFunc (wireQuadSurfs);
    glutReshapeFunc (winReshapeFcn);
    glutMainLoop ( );
}
```

5.2.4 Bézier Spline Curves

- It was developed by the French engineer Pierre Bézier for use in the design of Renault automobile bodies.
- **Bézier splines** have a number of properties that make them highly useful and convenient for curve and surface design. They are also easy to implement.
- In general, a Bézier curve section can be fitted to any number of control points, although some graphic packages limit the number of control points to four.

Bézier Curve Equations

- ✓ We first consider the general case of $n + 1$ control-point positions, denoted as $\mathbf{p}_k = (x_k, y_k, z_k)$, with k varying from 0 to n.
- ✓ These coordinate points are blended to produce the following position vector $\mathbf{P}(u)$, which describes the path of an approximating Bézier polynomial function between $\mathbf{p}_0$ and $\mathbf{p}_n$:

$$P(u) = \sum_{k=0}^{n} p_k \, BEZ_{k,n}(u), \qquad 0 \leq u \leq 1$$

- ✓ The Bézier blending functions $BEZ_{k,n}(u)$ are the *Bernstein polynomials*

$$BEZ_{k,n}(u) = C(n,k)u^k(1-u)^{n-k}$$

where parameters $C(n, k)$ are the binomial coefficients

$$C(n,k) = \frac{n!}{k!(n-k)!}$$

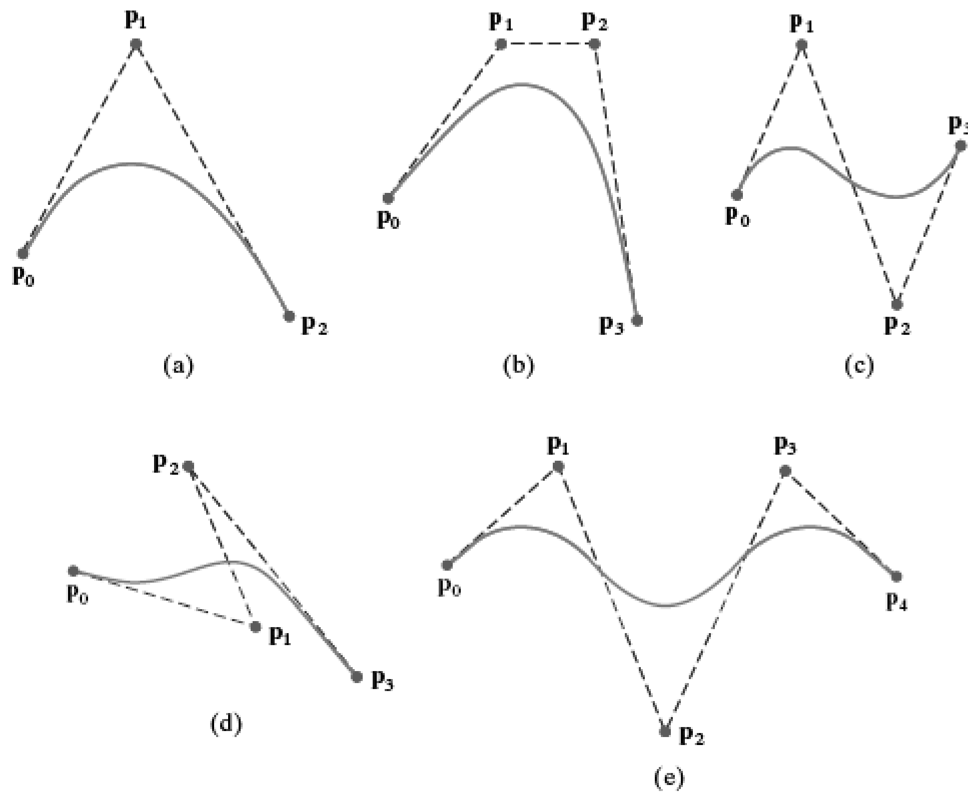
- ✓ A set of three parametric equations for the individual curve coordinates can be represented as

$$x(u) = \sum_{k=0}^{n} x_k \, BEZ_{k,n}(u)$$

$$y(u) = \sum_{k=0}^{n} y_k \, BEZ_{k,n}(u)$$

$$z(u) = \sum_{k=0}^{n} z_k \, BEZ_{k,n}(u)$$

- ✓ Below Figure demonstrates the appearance of some Bézier curves for various selections of control points in the *xy* plane ($z = 0$).



- ✓ Recursive calculations can be used to obtain successive binomial-coefficient values as

$$C(n, k) = \frac{n - k + 1}{k} C(n, k - 1)$$

for $n \geq k$. Also, the Bézier blending functions satisfy the recursive relationship

$BEZ_{k,n(u)} = (1 - u)BEZ_{k,n-1(u)} + u\, BEZ_{k-1,n-1(u)}),\ n > k \geq 1$ **(27)**

with $BEZ_{k,k} = uk$ and $BEZ_{0,k} = (1 - u)k$.

Program

```
#include <GL/glut.h>
#include <stdlib.h>
#include <math.h>
/* Set initial size of the display window. */
GLsizei winWidth = 600, winHeight = 600;
```

```
/* Set size of world-coordinate clipping window. */
GLfloat xwcMin = -50.0, xwcMax = 50.0;
GLfloat ywcMin = -50.0, ywcMax = 50.0;
class wcPt3D {
                public:
                GLfloat x, y, z;
};
void init (void)
{
        /* Set color of display window to white. */
        glClearColor (1.0, 1.0, 1.0, 0.0);
}
void plotPoint (wcPt3D bezCurvePt)
{
        glBegin (GL_POINTS);
        glVertex2f (bezCurvePt.x, bezCurvePt.y);
        glEnd ( );
}
/* Compute binomial coefficients C for given value of n. */
void binomialCoeffs (GLint n, GLint * C)
{
        GLint k, j;
        for (k = 0; k <= n; k++) {
        /* Compute n!/(k!(n - k)!). */
                C [k] = 1;
                for (j = n; j >= k + 1; j--)
                C [k] *= j;
                for (j = n - k; j >= 2; j--)
                C [k] /= j;
        }
}
```

```
void computeBezPt (GLfloat u, wcPt3D * bezPt, GLint nCtrlPts, wcPt3D * ctrlPts, GLint * C)
{
    GLint k, n = nCtrlPts - 1;
    GLfloat bezBlendFcn;
    bezPt->x = bezPt->y = bezPt->z = 0.0;
    /* Compute blending functions and blend control points. */
    for (k = 0; k < nCtrlPts; k++) {
        bezBlendFcn = C [k] * pow (u, k) * pow (1 - u, n - k);
        bezPt->x += ctrlPts [k].x * bezBlendFcn;
        bezPt->y += ctrlPts [k].y * bezBlendFcn;
        bezPt->z += ctrlPts [k].z * bezBlendFcn;
    }
}
void bezier (wcPt3D * ctrlPts, GLint nCtrlPts, GLint nBezCurvePts)
{
    wcPt3D bezCurvePt;
    GLfloat u;
    GLint *C, k;
    /* Allocate space for binomial coefficients */
    C = new GLint [nCtrlPts];
    binomialCoeffs (nCtrlPts - 1, C);
    for (k = 0; k <= nBezCurvePts; k++) {
        u = GLfloat (k) / GLfloat (nBezCurvePts);
        computeBezPt (u, &bezCurvePt, nCtrlPts, ctrlPts, C);
        plotPoint (bezCurvePt);
    }
    delete [ ] C;
}
void displayFcn (void)
{
    /* Set example number of control points and number of
```

```
    * curve positions to be plotted along the Bezier curve. */
    GLint nCtrlPts = 4, nBezCurvePts = 1000;
    wcPt3D ctrlPts [4] = { {-40.0, -40.0, 0.0}, {-10.0, 200.0, 0.0},
    {10.0, -200.0, 0.0}, {40.0, 40.0, 0.0} };
    glClear (GL_COLOR_BUFFER_BIT); // Clear display window.
    glPointSize (4);
    glColor3f (1.0, 0.0, 0.0); // Set point color to red.
    bezier (ctrlPts, nCtrlPts, nBezCurvePts);
    glFlush ( );
}
void winReshapeFcn (GLint newWidth, GLint newHeight)
{
    /* Maintain an aspect ratio of 1.0. */
    glViewport (0, 0, newHeight, newHeight);
    glMatrixMode (GL_PROJECTION);
    glLoadIdentity ( );
    gluOrtho2D (xwcMin, xwcMax, ywcMin, ywcMax);
    glClear (GL_COLOR_BUFFER_BIT);
}
void main (int argc, char** argv)
{
    glutInit (&argc, argv);
    glutInitDisplayMode (GLUT_SINGLE | GLUT_RGB);
    glutInitWindowPosition (50, 50);
    glutInitWindowSize (winWidth, winHeight);
    glutCreateWindow ("Bezier Curve");
    init ( );
    glutDisplayFunc (displayFcn);
    glutReshapeFunc (winReshapeFcn);
    glutMainLoop ( );
}
```

Properties of Bézier Curves

Property1:

- A very useful property of a Bézier curve is that the curve connects the first and last control points.
- Thus, a basic characteristic of any Bézier curve is that

  $\mathbf{P}(0) = \mathbf{p}0$

  $\mathbf{P}(1) = \mathbf{p}n$

- Values for the parametric first derivatives of a Bézier curve at the endpoints can be calculated from control-point coordinates as

$$P'(0) = -n\mathbf{p}_0 + n\mathbf{p}_1$$
$$P'(1) = -n\mathbf{p}_{n-1} + n\mathbf{p}_n$$

- The parametric second derivatives of a Bézier curve at the endpoints are calculated as

$$P''(0) = n(n-1)[(\mathbf{p}_2 - \mathbf{p}_1) - (\mathbf{p}_1 - \mathbf{p}_0)]$$
$$P''(1) = n(n-1)[(\mathbf{p}_{n-2} - \mathbf{p}_{n-1}) - (\mathbf{p}_{n-1} - \mathbf{p}_n)]$$

Property 2:

- Another important property of any Bézier curve is that it lies within the convex hull (convex polygon boundary) of the control points.
- This follows from the fact that the Bézier blending functions are all positive and their sum is always 1:

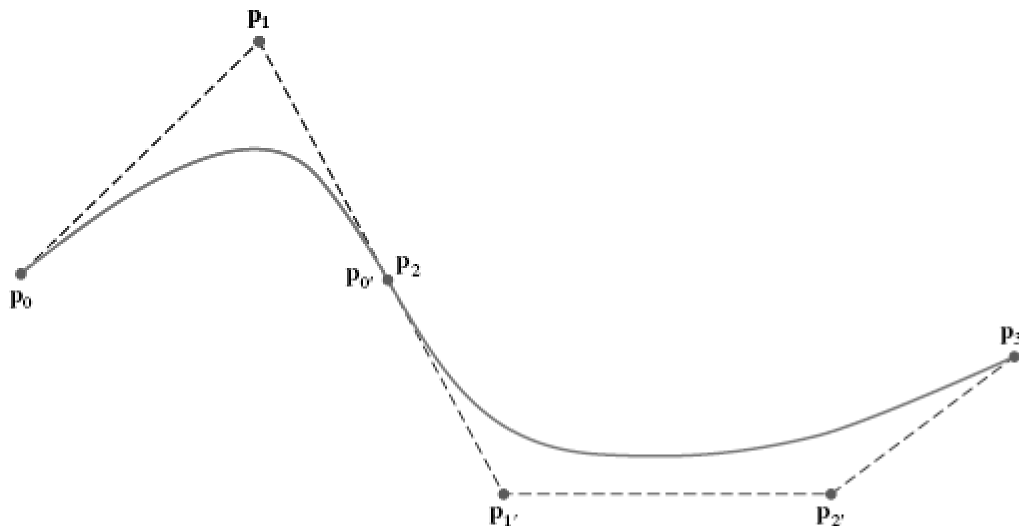
$$\sum_{k=0}^{n} \text{BEZ}_{k,n}(u) = 1$$

Other Properties:

- ❖ The basic functions are real.
- ❖ The degree of the polynomial defining the curve segment is one less than the number of defining points.
- ❖ The curve generally follows the shape of the defining polygon,
- ❖ The tangent vectors at the ends of the curve have the same direction as the first and last polygon spans respectively.

Design Techniques Using Bézier Curves

- ✓ A closed Bézier curve is generated when we set the last control-point position to the coordinate position of the first control point.
- ✓ Specifying multiple control points at a single coordinate position gives more weight to that position a single coordinate position is input as two control points, and the resulting curve is pulled nearer to this position.
- ✓ When complicated curves are to be generated, they can be formed by piecing together several Bézier sections of lower degree.
- ✓ Generating smaller Bézier-curve sections also gives us better local control over the shape of the curve.
- ✓ Because Bézier curves connect the first and last control points, it is easy to match curve sections.
- ✓ Also,Bézier curves have the important property that the tangent to the curve at an endpoint is along the line joining that endpoint to the adjacent control point to obtain first-order continuity between curve sections, we can pick control points **p**0' and **p**1'for the next curve section to be along the same straight line as control points **p**$n-1$ and **p**n of the preceding section



- ✓ If the first curve section has n control points and the next curve section has n' control points, then we match curve tangents by placing control point **p**1' at the position

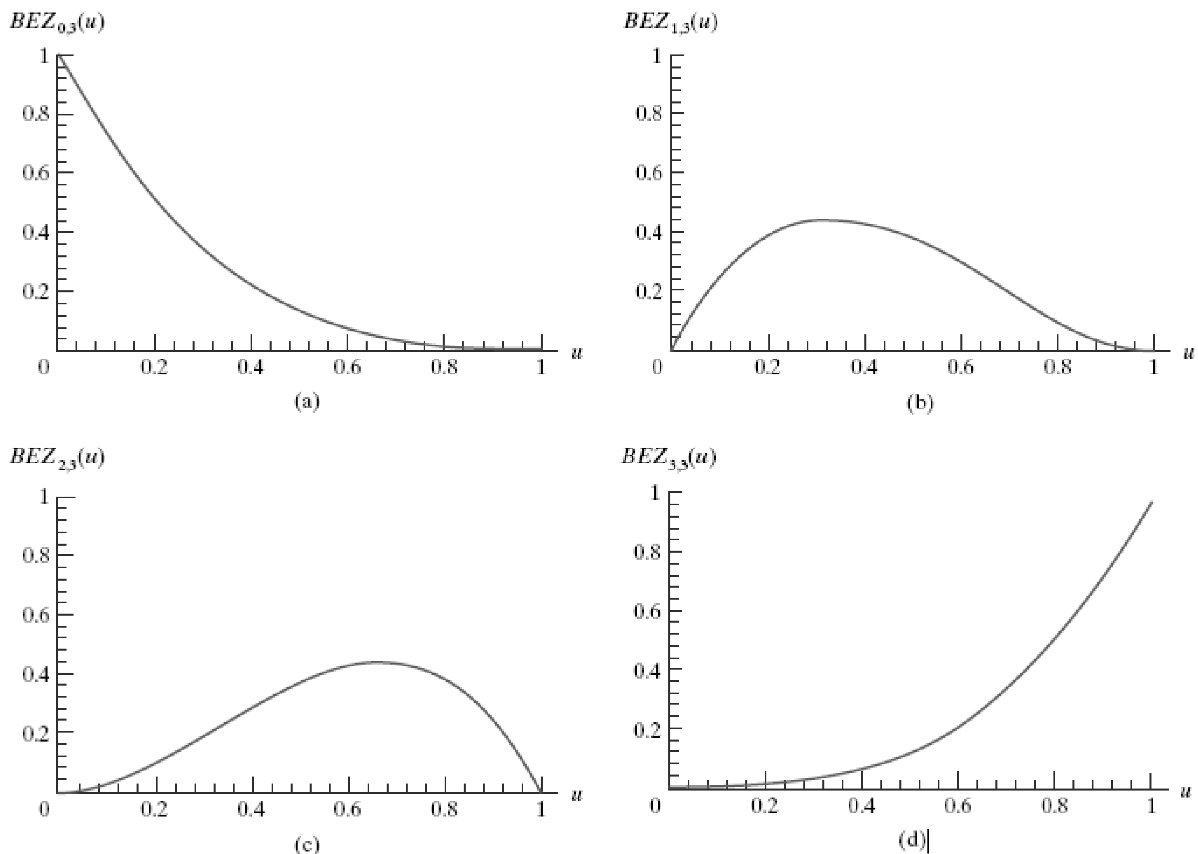
$$\mathbf{p}_{1'} = \mathbf{p}_n + \frac{n}{n'}(\mathbf{p}_n - \mathbf{p}_{n-1})$$

Cubic Bézier Curves

- ✓ Cubic Bézier curves are generated with four control points. The four blending functions for cubic Bézier curves, obtained by substituting $n = 3$ in the equations below, they are

$$\begin{aligned}
\mathrm{BEZ}_{0,3} &= (1-u)^3 \\
\mathrm{BEZ}_{1,3} &= 3u(1-u)^2 \\
\mathrm{BEZ}_{2,3} &= 3u^2(1-u) \\
\mathrm{BEZ}_{3,3} &= u^3
\end{aligned}$$

- ✓ Plots of the four cubic Bézier blending functions are given in Figure



- ✓ At the end positions of the cubic Bézier curve, the parametric first derivatives (slopes) are

$$\mathbf{P}'(0) = 3(\mathbf{p}_1 - \mathbf{p}_0), \qquad \mathbf{P}'(1) = 3(\mathbf{p}_3 - \mathbf{p}_2)$$

and the parametric second derivatives are

$$\mathbf{P}''(0) = 6(\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}_2), \qquad \mathbf{P}''(1) = 6(\mathbf{p}_1 - 2\mathbf{p}_2 + \mathbf{p}_3)$$

- ✓ A matrix formulation for the cubic-Bézier curve function is obtained by expanding the polynomial expressions for the blending functions and restructuring the equations as

$$\mathbf{P}(u) = [u^3 \quad u^2 \quad u \quad 1] \cdot \mathbf{M}_{\text{Bez}} \cdot \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix}$$

where the **Bézier matrix** is

$$\mathbf{M}_{\text{Bez}} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

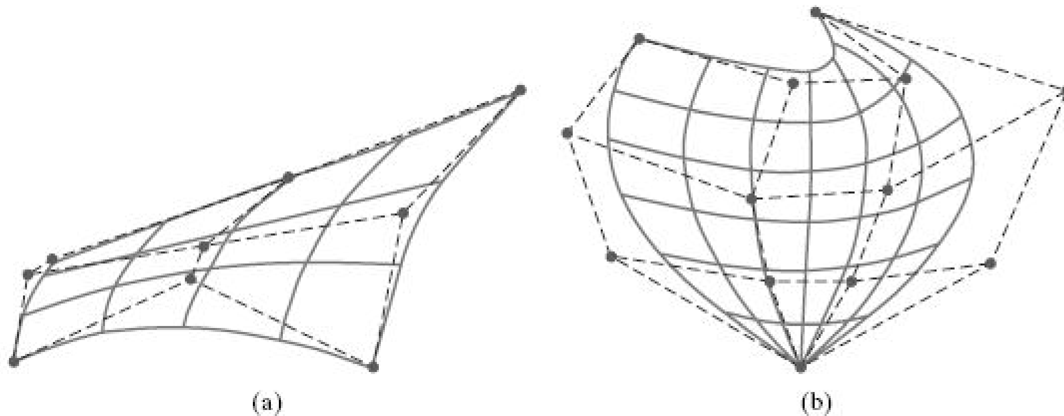
5.2.5 Bézier Surfaces

- The parametric vector function for the Bézier surface is formed as the tensor product of Bézier blending functions:

$$\mathbf{P}(u, v) = \sum_{j=0}^{m} \sum_{k=0}^{n} \mathbf{p}_{j,k} \, \text{BEZ}_{j,m}(v) \, \text{BEZ}_{k,n}(u)$$

with $p_{j,k}$ specifying the location of the $(m + 1)$ by $(n + 1)$ control points

- Figure below illustrates two Bézier surface plots. The control points are connected by dashed lines, and the solid lines show curves of constant u and constant v.
- Each curve of constant u is plotted by varying v over the interval from 0 to 1, with u fixed at one of the values in this unit interval. Curves of constant v are plotted similarly.

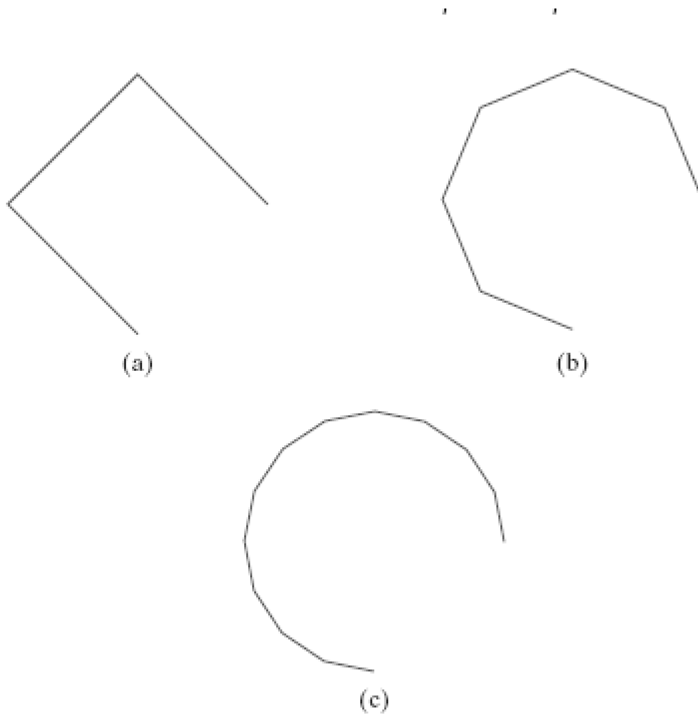


- Bézier surfaces have the same properties as Bézier curves, and they provide a convenient method for interactive design applications.
- To specify the threedimensional coordinate positions for the control points, we could first construct a rectangular grid in the *xy* "ground" plane.

- We then choose elevations above the ground plane at the grid intersections as the *z*-coordinate values for the control points.

5.2.6 OpenGL Curve Functions

- There are routines in the OpenGL Utility Toolkit (GLUT) that we can use to display some three-dimensional quadrics, such as spheres and cones, and some other shapes.
- Another method we can use to generate a display of a simple curve is to approximate it using a polyline. We just need to locate a set of points along the curve path and connect the points with straight-line segments.



- Figure above illustrates various polyline displays that could be used for a circle segment.
- A third alternative is to write our own curve-generation functions based on the algorithms with respect to line drawing and circle drawing.

5.2.7 OpenGL Approximation-Spline Functions

OpenGL Bézier-Spline Curve Functions

- We specify parameters and activate the routines for Bézier-curve display with the OpenGL functions

  glMap1* (GL_MAP1_VERTEX_3, uMin, uMax, stride, nPts, *ctrlPts);

  glEnable (GL_MAP1_VERTEX_3);

- We deactivate the routines with

  glDisable (GL_MAP1_VERTEX_3);

where,

- A suffix code of **f** or **d** is used with **glMap1** to indicate either floating-point or double precision for the data values. M
- inimum and maximum values for the curve parameter *u* are specified in **uMin** and **uMax**, although these values for a Bézier curve are typically set to 0 and 1.0, respectively.
- Bézier control points are listed in array **ctrlPts** number of elements in this array is given as a positive integer using parameter **nPts**.
- **stride** is assigned an integer offset that indicates the number of data values between the beginning of one coordinate position in array **ctrlPts** and the beginning of the next coordinate position

- A coordinate position along the curve path is calculated with

  glEvalCoord1* (uValue);

Where,

- parameter **uValue** is assigned some value in the interval from **uMin** to **uMax**.
- Function **glEvalCoord1** calculates a coordinate position using equation with the parameter value

$$u = \frac{u_{\text{value}} - u_{\text{min}}}{u_{\text{max}} - u_{\text{min}}}$$

which maps the **uValue** to the interval from 0 to 1.0.

- A spline curve is generated with evenly spaced parameter values, and OpenGL provides the following functions, which we can use to produce a set of uniformly spaced parameter values:

```
glMapGrid1* (n, u1, u2);
glEvalMesh1 (mode, n1, n2);
```

Where,

- The suffix code for **glMapGrid1** can be either **f** or **d**.
- Parameter **n** specifies the integer number of equal subdivisions over the range from **u1** to **u2**.
- Parameters **n1** and **n2** specify an integer range corresponding to **u1** and **u2**.
- Parameter **mode** is assigned either **GL POINT** or **GL LINE**, depending on whether we want to display the curve using discrete points (a dotted curve) or using straight-line segments
- In other words, with **mode = GL LINE**, the preceding OpenGL commands are equivalent to

```
glBegin (GL_LINE_STRIP);
    for (k = n1; k <= n2; k++)
        glEvalCoord1f (u1 + k * (u2 - u1) / n);
glEnd ( );
```

OpenGL Bézier-Spline Surface Functions

- Activation and parameter specification for the OpenGL Bézier-surface routines are accomplished with

```
glMap2* (GL_MAP2_VERTEX_3, uMin, uMax, uStride, nuPts, vMin,
         vMax, vStride, nvPts, *ctrlPts);
glEnable (GL_MAP2_VERTEX_3);
```

Where,

- A suffix code of **f** or **d** is used with **glMap2** to indicate either floating-point or double precision for the data values.
- For a surface, we specify minimum and maximum values for both parameter *u* and parameter *v*.

➔ If control points are to be specified using four-dimensional homogeneous coordinates, we use the symbolic constant **GL_MAP2_VERTEX_4** instead of **GL_MAP2_VERTEX_3**

➢ We deactivate the Bézier-surface routines with

glDisable {GL_MAP2_VERTEX_3}

➢ Coordinate positions on the Bézier surface can be calculated with

glEvalCoord2* (uValue, vValue);

or

glEvalCoord2*v (uvArray);

Where,

➔ Parameter **uValue** is assigned some value in the interval from **uMin** to **uMax**,

➔ Parameter **vValue** is assigned some value in the interval from **vMin** to **vMax**.

$$u = \frac{u\text{Value} - u\text{Min}}{u\text{Max} - u\text{Min}}, \qquad v = \frac{v\text{Value} - v\text{Min}}{v\text{Max} - v\text{Min}}$$

which maps each of uValue and vValue to the interval from 0 to 1.0

GLU B-Spline Curve Functions

✓ Although the GLU B-spline routines are referred to as NURBs functions, they can be used to generate B-splines that are neither nonuniform nor rational.

✓ The following statements illustrate the basic sequence of calls for displaying a B-spline curve:

```
GLUnurbsObj *curveName;
curveName = gluNewNurbsRenderer ( );
gluBeginCurve (curveName);
gluNurbsCurve (curveName, nknots, *knotVector, stride, *ctrlPts,
               degParam, GL_MAP1_VERTEX_3);
gluEndCurve (curveName);
```

✓ We eliminate a defined B-spline with

gluDeleteNurbsRenderer (curveName);

✓ A B-spline curve is divided automatically into a number of sections and displayed as a polyline by theGLUroutines.

- ✓ However, a variety of B-spline rendering options can also be selected with repeated calls to the following GLU function:

gluNurbsProperty (splineName, property, value);

<u>GLU B-Spline Surface Functions</u>

```
GLUnurbsObj *surfName
surfName = gluNewNurbsRenderer ( );
gluNurbsProperty (surfName, property1, value1);
gluNurbsProperty (surfName, property2, value2);
gluNurbsProperty (surfName, property3, value3);
...
gluBeginSurface (surfName);
gluNurbsSurface (surfName, nuKnots, uKnotVector, nvKnots, vKnotVector,
uStride, vStride, &ctrlPts [0][0][0], uDegParam, vDegParam,
GL_MAP2_VERTEX_3);
gluEndSurface (surfName);
```

- ✓ As an example of property setting, the following statements specify a wire-frame, triangularly tessellated display for a surface:

```
gluNurbsProperty (surfName, GLU_NURBS_MODE, GLU_NURBS_TESSELLATOR);
gluNurbsProperty (surfName, GLU_DISPLAY_MODE, GLU_OUTLINE_POLYGON);
```

- ✓ To determine the current value of a B-spline property, we use the following query function:

gluGetNurbsProperty (splineName, property, value);

- ✓ When the property **GLU_AUTO_LOAD_MATRIX** is set to the value **GL_FALSE**, we invoke

gluLoadSamplingMatrices (splineName, modelviewMat, projMat, viewport);

- ✓ Various events associated with spline objects are processed using

gluNurbsCallback (splineName, event, fcn);

- ✓ Data values for the **gluNurbsCallback** function are supplied by

  gluNurbsCallbackData (splineName, dataValues);

GLU Surface-Trimming Functions

- ✓ A set of one or more two-dimensional trimming curves is specified for a B-spline surface with the following statements:

```
gluBeginTrim (surfName);
gluPwlCurve (surfName, nPts, *curvePts, stride, GLU_MAP1_TRIM_2);
...
gluEndTrim (surfName);
```

Where,

- ➔ Parameter **surfName** is the name of the B-spline surface to be trimmed.
- ➔ A set of floating-point coordinates for the trimming curve is specified in array parameter **curvePts**, which contains **nPts** coordinate positions.
- ➔ An integer offset between successive coordinate positions is given in parameter **stride**

Summary of OpenGL Bezier Functions

Function	Description
`glMap1`	Specifies parameters for Bézier-curve display, color values, etc., and activate these routines using `glEnable`.
`glEvalCoord1`	Calculates a coordinate position for a Bézier curve.
`glMapGrid1`	Specifies the number of equally spaced subdivisions between two Bézier-curve parameters.
`glEvalMesh1`	Specifies the display mode and integer range for a Bézier-curve display.
`glMap2`	Specifies parameters for a Bézier-surface display, color values, etc., and activate these routines using `glEnable`.
`glEvalCoord2`	Calculates a coordinate position for a Bézier surface.
`glMapGrid2`	Specifies a two-dimensional grid of equally spaced subdivisions over a Bézier surface.
`glEvalMesh2`	Specifies the display mode and integer range for the two-dimensional Bézier-surface grid.

Summary of OpenGL B-Spline Functions

Function	Description
`gluNewNurbsRenderer`	Activates the GLU B-spline renderer for an object name that has been defined with the declaration `GLUnurbsObj *bsplineName`.
`gluBeginCurve`	Begins the assignment of parameter values for a specified B-spline curve with one or more sections.
`gluEndCurve`	Signals the end of the B-spline curve parameter specifications.
`gluNurbsCurve`	Specifies the parameter values for a named B-spline curve section.
`gluDeleteNurbsRenderer`	Eliminates a specified B-spline.
`gluNurbsProperty`	Specifies rendering options for a designated B-spline.
`gluGetNurbsProperty`	Determines the current value of a designated property for a particular B-spline.

Summary of OpenGL B-Spline Functions (*Continued*)

Function	Description
`gluBeginSurface`	Begins the assignment of parameter values for a specified B-spline surface with one or more sections.
`gluEndSurface`	Signals the end of the B-spline surface parameter specifications.
`gluNurbsSurface`	Specifies the parameter values for a named B-spline surface section.
`gluLoadSamplingMatrices`	Specifies viewing and geometric transformation matrices to be used in sampling and culling routines for a B-spline.
`gluNurbsCallback`	Specifies a callback function for a designated B-spline and associated event.
`gluNurbsCallbackData`	Specifies data values that are to be passed to the event callback function.
`gluBeginTrim`	Begins the assignment of trimming-curve parameter values for a B-spline surface.
`gluEndTrim`	Signals the end of the trimming curve parameter specifications.
`gluPwlCurve`	Specifies trimming-curve parameter values for a B-spline surface.

5.3 Animation

5.3.1 Raster methods of computer animation

5.3.2 Design of animation sequences

5.3.3 Traditional animation techniques

5.3.4 General computer animation function

5.3.5 OpenGL animation procedures

Introduction:

- To 'animate' is literally 'to give life to'.
- 'Animating' is moving something which can't move itself.
- Animation adds to graphics the dimension of time which vastly increases the amount of information which can be transmitted.
- **Computer animation** generally refers to any time sequence of visual changes in a picture.
- In addition to changing object positions using translations or rotations, a computer-generated animation could display time variations in object size, color, transparency, or surface texture.
- Two basic methods for constructing a motion sequence are
  1. **real-time animation** and
     - In a real-time computer-animation, each stage of the sequence is viewed as it is created.
     - Thus the animation must be generated at a rate that is compatible with the constraints of the refresh rate.
  2. **frame-by-frame animation**
     - For a frame-by-frame animation, each frame of the motion is separately generated and stored.
     - Later, the frames can be recorded on film, or they can be displayed consecutively on a video monitor in “real-time playback” mode.

5.3.1 Raster Methods for Computer Animation

- We can create simple animation sequences in our programs using real-time methods.
- We can produce an animation sequence on a raster-scan system one frame at a time, so that each completed frame could be saved in a file for later viewing.
- The animation can then be viewed by cycling through the completed frame sequence, or the frames could be transferred to film.
- If we want to generate an animation in real time, however, we need to produce the motion frames quickly enough so that a continuous motion sequence is displayed.
- Because the screen display is generated from successively modified pixel values in the refresh buffer, we can take advantage of some of the characteristics of the raster screen-refresh process to produce motion sequences quickly.

Double Buffering

- ✓ One method for producing a real-time animation with a raster system is to employ two refresh buffers.
- ✓ We create a frame for the animation in one of the buffers.
- ✓ Then, while the screen is being refreshed from that buffer, we construct the next frame in the other buffer.
- ✓ When that frame is complete, we switch the roles of the two buffers so that the refresh routines use the second buffer during the process of creating the next frame in the first buffer.
- ✓ When a call is made to switch two refresh buffers, the interchange could be performed at various times.
- ✓ The most straight forward implementation is to switch the two buffers at the end of the current refresh cycle, during the vertical retrace of the electron beam.
- ✓ If a program can complete the construction of a frame within the time of a refresh cycle, say 1/60 of a second, each motion sequence is displayed in synchronization with the screen refresh rate.
- ✓ If the time to construct a frame is longer than the refresh time, the current frame is displayed for two or more refresh cycles while the next animation frame is being generated.

- ✓ Similarly, if the frame construction time is 1/25 of a second, the animation frame rate is reduced to 20 frames per second because each frame is displayed three times.
- ✓ Irregular animation frame rates can occur with double buffering when the frame construction time is very nearly equal to an integer multiple of the screen refresh time the animation frame rate can change abruptly and erratically.
- ✓ One way to compensate for this effect is to add a small time delay to the program.
- ✓ Another possibility is to alter the motion or scene description to shorten the frame construction time.

<u>Generating Animations Using Raster Operations</u>

- We can also generate real-time raster animations for limited applications using block transfers of a rectangular array of pixel values.
- A simple method for translating an object from one location to another in the *xy* plane is to transfer the group of pixel values that define the shape of the object to the new location
- Sequences of raster operations can be executed to produce realtime animation for either two-dimensional or three-dimensional objects, so long as we restrict the animation to motions in the projection plane.
- Then no viewing or visible-surface algorithms need be invoked.
- We can also animate objects along two-dimensional motion paths using **color table transformations.**
- Here we predefine the object at successive positions along the motion path and set the successive blocks of pixel values to color-table entries.
- The pixels at the first position of the object are set to a foreground color, and the pixels at the other object positions are set to the background color .
- Then the animation is then accomplished by changing the color-table values so that the object color at successive positions along the animation path becomes the foreground color as the preceding position is set to the background color

5.3.2 Design of Animation Sequences

- Animation sequence in general is designed in the following steps.

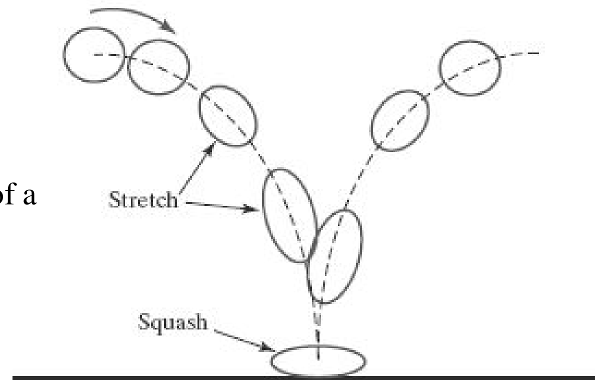
1. Storyboard layout
2. Object definitions.
3. Key-frame specifications
4. Generation of in-between frames.

- ✓ This approach of carrying out animations is applied to any other applications as well, although some applications are exceptional cases and do not follow this sequence.
- ✓ For frame-by-frame animation, every frame of the display or scene is generated separately and stored. Later, the frame recording can be done and they might be displayed consecutively in terms of movie.
- ✓ The outline of the action is storyboard. This explains the motion sequence. The storyboard consists of a set of rough structures or it could be a list of the basic ideas for the motion.
- ✓ For each participant in the action, an object definition is given. Objects are described in terms of basic shapes the examples of which are splines or polygons. The related movement associated with the objects are specified along with the shapes.
- ✓ A key frame in animation can be defined as a detailed drawing of the scene at a certain time in the animation sequence. Each object is positioned according to the time for that frame, within each key frame.
- ✓ Some key frames are selected at extreme positions and the others are placed so that the time interval between two consecutive key frames is not large. Greater number of key frames are specified for smooth motions than for slow and varying motion.
- ✓ And the intermediate frames between the key frames are In-betweens. And the Media that we use determines the number of In-betweens which are required to display the animation. A Film needs 24 frames per second, and graphics terminals are refreshed at the rate of 30 to 60 frames per second.
- ✓ Depending on the speed specified for the motion, some key frames are duplicated. For a one minutes film sequence with no duplication, we would require 288 key frames. We place the key frames a bit distant if the motion is not too complicated.
- ✓ A number of other tasks may be carried out depending upon the application requirement for example synchronization of a sound track.

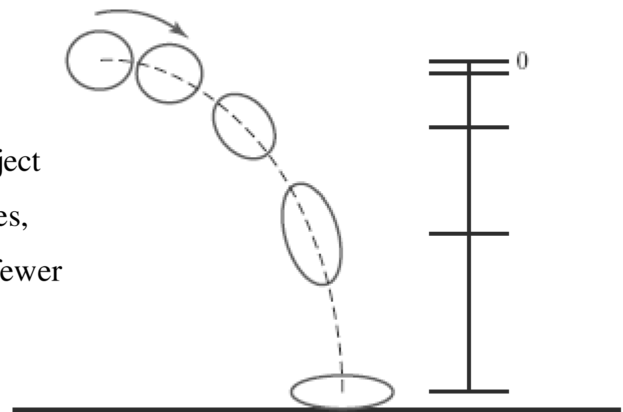
5.3.3 Traditional Animation Techniques

- ✓ Film animators use a variety of methods for depicting and emphasizing motion sequences.
- ✓ These include object deformations, spacing between animation frames, motion anticipation and follow-through, and action focusing

- ✓ One of the most important techniques for simulating acceleration effects, particularly for non rigid objects, is **squash and stretch.**
- ✓ Figure shows how this technique is used to emphasize the acceleration and deceleration of a bouncing ball. As the ball accelerates, it begins to stretch. When the ball hits the floor and stops, it is first compressed (squashed) and then stretched again as it accelerates and bounces upwards.



- ✓ Another technique used by film animators is **timing,** which refers to the spacing between motion frames. A slower moving object is represented with more closely spaced frames, and a faster moving object is displayed with fewer frames over the path of the motion.



- ✓ Object movements can also be emphasized by creating preliminary actions that indicate an **anticipation** of a coming motion

5.3.4 General Computer-Animation Functions

- ✓ Typical animation functions include managing object motions, generating views of objects, producing camera motions, and the generation of in-between frames

- ✓ Some animation packages, such as Wavefront for example, provide special functions for both the overall animation design and the processing of individual objects.
- ✓ Others are special-purpose packages for particular features of an animation, such as a system for generating in-between frames or a system for figure animation.
- ✓ A set of routines is often provided in a general animation package for storing and managing the object database. Object shapes and associated parameters are stored and updated in the database. Other object functions include those for generating the object motion and those for rendering the object surfaces
- ✓ Another typical function set simulates camera movements. Standard camera motions are zooming, panning, and tilting. Finally, given the specification for the key frames, the in-betweens can be generated automatically.

5.3.5 OpenGL Animation Procedures

- ➢ Double-buffering operations, if available, are activated using the following GLUT command:

  glutInitDisplayMode (GLUT_DOUBLE);

- ➢ This provides two buffers, called the *front buffer* and the *back buffer,* that we can use alternately to refresh the screen display
- ➢ We specify when the roles of the two buffers are to be interchanged using

  glutSwapBuffers ( );

- ➢ To determine whether double-buffer operations are available on a system, we can issue the following query:

  glGetBooleanv (GL_DOUBLEBUFFER, status);

- ➢ A value of **GL_TRUE** is returned to array parameter **status** if both front and back buffers are available on a system. Otherwise, the returned value is **GL _FALSE**.
- ➢ For a continuous animation, we can also use

  glutIdleFunc (animationFcn);

- ➢ This procedure is continuously executed whenever there are no display-window events that must be processed.
- ➢ To disable the **glutIdleFunc**, we set its argument to the value **NULL** or the value 0.

Example Program

```
#include <GL/glut.h>
#include <math.h>
#include <stdlib.h>

const double TWO_PI = 6.2831853;
GLsizei winWidth = 500, winHeight = 500; // Initial display window size.
GLuint regHex; // Define name for display list.
static GLfloat rotTheta = 0.0;

class scrPt {
	public:
		GLint x, y;
};
static void init (void)
{
	scrPt hexVertex;
	GLdouble hexTheta;
	GLint k;
	glClearColor (1.0, 1.0, 1.0, 0.0);
	/* Set up a display list for a red regular hexagon.
	* Vertices for the hexagon are six equally spaced
	* points around the circumference of a circle.
	*/
	regHex = glGenLists (1);
	glNewList (regHex, GL_COMPILE);
	glColor3f (1.0, 0.0, 0.0);
	glBegin (GL_POLYGON);
	for (k = 0; k < 6; k++) {
		hexTheta = TWO_PI * k / 6;
```

```
        hexVertex.x = 150 + 100 * cos (hexTheta);
        hexVertex.y = 150 + 100 * sin (hexTheta);
        glVertex2i (hexVertex.x, hexVertex.y);
    }
    glEnd ( );
    glEndList ( );
}
void displayHex (void)
{
    glClear (GL_COLOR_BUFFER_BIT);
    glPushMatrix ( );
    glRotatef (rotTheta, 0.0, 0.0, 1.0);
    glCallList (regHex);
    glPopMatrix ( );
    glutSwapBuffers ( );
    glFlush ( );
}
void rotateHex (void)
{
    rotTheta += 3.0;
    if (rotTheta > 360.0)
    rotTheta -= 360.0;
    glutPostRedisplay ( );
}
void winReshapeFcn (GLint newWidth, GLint newHeight)
{
    glViewport (0, 0, (GLsizei) newWidth, (GLsizei) newHeight);
    glMatrixMode (GL_PROJECTION);
    glLoadIdentity ( );
    gluOrtho2D (-320.0, 320.0, -320.0, 320.0);
    glMatrixMode (GL_MODELVIEW);
```

```
        glLoadIdentity ( );
        glClear (GL_COLOR_BUFFER_BIT);
}
void mouseFcn (GLint button, GLint action, GLint x, GLint y)
{
        switch (button) {
                case GLUT_MIDDLE_BUTTON: // Start the rotation.
                                        if (action == GLUT_DOWN)
                                        glutIdleFunc (rotateHex);
                                        break;
                case GLUT_RIGHT_BUTTON: // Stop the rotation.
                                        if (action == GLUT_DOWN)
                                        glutIdleFunc (NULL);
                                        break;
                default:
                                        break;
        }
}
void main(int argc, char ** argv)
{
        glutInit (&argc, argv);
        glutInitDisplayMode (GLUT_DOUBLE | GLUT_RGB);
        glutInitWindowPosition (150, 150);
        glutInitWindowSize (winWidth, winHeight);
        glutCreateWindow ("Animation Example");
        init ( );
        glutDisplayFunc (displayHex);
        glutReshapeFunc (winReshapeFcn);
        glutMouseFunc (mouseFcn);
        glutMainLoop ( );
}
```