ELEMENTS OF CIVIL ENGINEERING AND MECHANICS [As per Choice Based Credit System (CBCS) scheme] (Effective from the academic year 2015 -2016) SEMESTER - I/II			
Subject Code	15CIV13/23	IA Marks	20
Number of Lecture Hours/Week	04	Exam Marks	80
Total Number of Lecture Hours	50	Exam Hours	03
CREDITS - 04			

COURSE OBJECTIVES:

The objectives of this course is to make students to learn basics of Civil Engineering concepts and infrastructure development, solve problems involving Forces, loads and Moments and know their applications in allied subjects. It is a pre-requisite for several courses involving Forces, Moments, Centroids, Moment of inertia and Kinematics.

Particulars	Hours
Module 1: Introduction to Civil Engineering &Engineering Mechanics	**10**
Introduction to Civil Engineering Scope of different fields of Civil Engineering - Surveying, Building Materials, Construction Technology, Geotechnical Engineering, Structural Engineering, Hydraulics, WaterResources and Irrigation Engineering, Transportation Engineering, Environmental Engineering.	01
Infrastructure: Types of infrastructure, Role of Civil Engineer in theInfrastructural Development, Effect of the infrastructural facilities onsocio-economic development of a country.	01
Roads: Classification of Roads and their functions, Comparison of Flexible and Rigid Pavements (Advantages and Limitations)	01

Bridges: Types of Bridges and Culverts, RCC, Steel and Composite Bridges	01
Dams: Different types of Dams based on Material, Structural behavior and functionality with simple sketches.	01
Introduction to Engineering Mechanics: Basic idealizations - Particle, Continuum and Rigid body; Newton's laws□Force and its characteristics, types of forces-Gravity, Lateral and its distribution on surfaces, Classification of force systems, Principle of physical independence, superposition, transmissibility of forces, , Introduction to SI units. Couple, Moment of a couple, Characteristics of couple, Moment of a force, Equivalent force - Couple system; Numerical problems on moment of forces and couples, on equivalent force - couple system.	02 03
Module 2: Analysis of Concurrent Force Systems	**10**
Concepts: Resultants and Equilibrium Composition of forces - Definition of Resultant; Composition of coplanar -concurrent force system, Parallelogram Law of forces, Principle of resolved parts;	03
Numerical problems on composition of coplanar concurrent force systems. Equilibrium of forces - Definition of Equilibrant; Conditions of static equilibrium for different force systems, Lami's theorem; Numerical problems on equilibrium of coplanar – concurrent and non-concurrent force systems.	03
Application- Static Friction in rigid bodies in contact Types of friction, Laws of static friction, Limiting friction, Angle of friction, angle of repose; Impending motion on horizontal and inclined planes; Numerical Problems on single and two blocks on inclined planes	02 02

1. Know basics of Civil Engineering, its scope of study, knowledge about Roads, Bridges and Dams;
2. Comprehend the action of Forces, Moments and other loads on systems of rigid bodies;
3. Compute the reactive forces and the effects that develop as a result of the external loads;
4. Locate the Centroid and compute the Moment of Inertia of regular cross-sections.
5. Express the relationship between the motion of bodies and
6. Equipped to pursue studies in allied courses in Mechanics.

Question Paper Pattern:

- 10 Questions are to be set such that 2 questions are selected from each module.
- 2 Questions are to be set under respective modules.
- Intra module questions are to be set such that the questions should cover the entire module and further, should be answerable for the set marks.
- Each question should be set for 16 marks (Preferably 8 marks each)
- Not more than 3 sub questions are to be set under any main question
- Students should answer 5 full questions selecting at least 1 from each module.

TEXT BOOKS

1. Elements of Civil Engineering and Engineering Mechanics by M.N. Shesha Prakash and Ganesh. B. Mogaveer, PHI Learning, 3rd Revised edition (2014)
2. Engineering Mechanics-Statics and Dynamics by A Nelson, Tata McGraw Hill Education Private Ltd, New Delhi, 2009.
3. Elements of Civil Engineering (IV Edition) by S.S. Bhavikatti, New Age International Publisher, New Delhi, 3rd edition 2009.

REFERENCES

1. Engineering Mechanics by S.Timoshenko,D.H.Young, and J.V.Rao, TATA McGraw-Hill Book Company, New Delhi
2. Beer FP and Johnson ER, **"Mechanics for Engineers- Dynamics and Statics"**- 3rd SI Metric edition, Tata McGraw Hill. - 2008
3. Shames IH, **"Engineering Mechanics – Statics & Dynamics"**- PHI – 2009

Date 10/2/17

MODULE 1

Introduction to Civil Engineering and Engineering Mechanics

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- CIVIL ENGINEERING : It is the oldest branch of proffessional engineering course where civil engineers are concerned with public projects.

Scope of different fields of Civil Engineering :-

→ Surveying
→ Building Materials
→ Construction Technology
→ Surveying Geotechnical Engineering
→ Structural Engineering
→ Hydraulics
→ Water Resources and Irrigation Engineering
→ Transportation Engineering
→ Environmental Engineering

- DAMS :- A dam is a barrier that impounds water or underground streams.

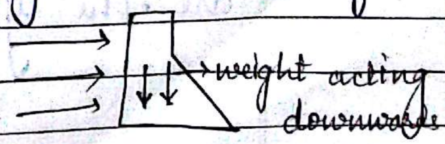
They suppress floods, provide water for irrigation, human consumption, industrial use, aquaculture, Hydropower generation, etc.

Classification of dams based on :-

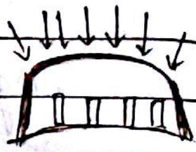
(1) Material of Construction - (i) Rigid Dam (constructed with stone, masonry, concrete, steel or timber.
(ii) Non-Rigid Dams (Embankment Dams) (constructed with earth, tailings, rockfill, etc).

(2) Structural Behaviour (i) Gravity Dam (resists the force acting on it by its own weight, constructed by concrete or masonry).

→ Bhakra Dam,
[found/constructed where rocks are competent and stable]

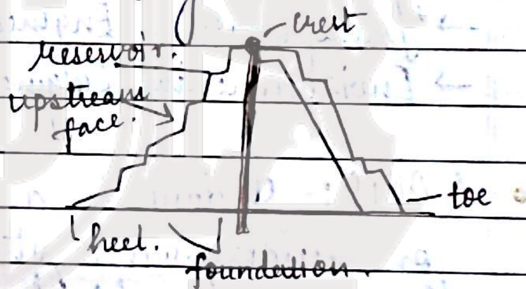


(ii) Arch Dam (curved masonry which resists the force acting on it by arch action.
Eg:- 1. Idukki Dam (convex upstream).



(iii) Buttress Dam (held up by a series of support) (flat or curved face, it is a gravity dam reinforced by structural supports).

(iv) Embankment Dam (resists the forces acting on it by its shear strength and to some extent also by its own weight.



(3) Functionality :- (i) Storage Dams
(ii) Diversion Dams
(iii) Detention Dams
(iv) Debris Dams.
(v) Coffer Dam

• BRIDGES AND CULVERTS :- Culvert is a tunnel structure constructed under roadways or railways to provide cross drainage or to take electrical or other cables.
Eg: Arch, beam, cable-stayed, suspension and truss are the types of bridges.

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Imp

• Component of Roads :-

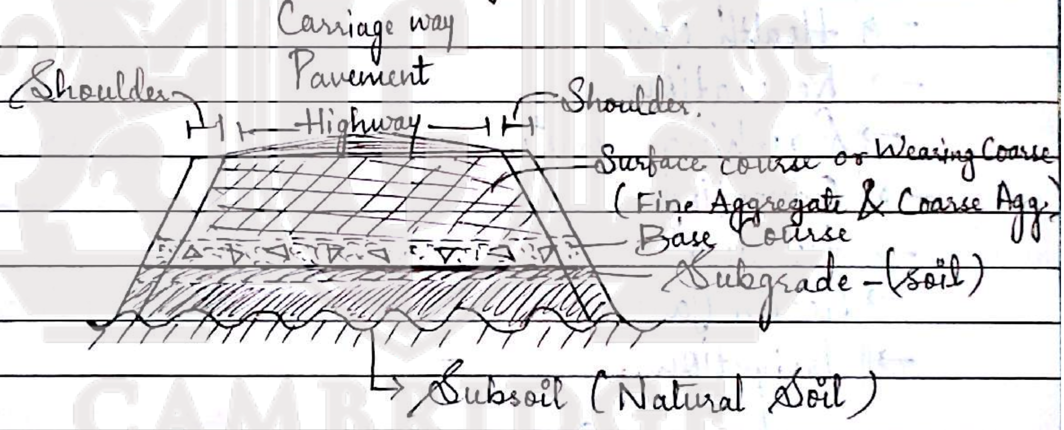
Width of the road (Single Lane) – 3.8 mtrs
(Double Lane) – 3.5 + 3.5 = 7 mtrs

Width of the vehicles (4 wheelers) – 2.44 mtrs

Width of the drainage (Shoulders) – (2 – 5) mtrs

Width of Footpath – 2 mtrs (min.)

Verticle distance for dividing footpath and road – (10-15) cm
(Kerb)



✓ Subsoil – Natural Soil / Mud
✓ Subgrade – Soil
✓ Subway / Base Coarse – Small Stones and Big Stones ie Fine Aggregate and Coarse Aggregate
✓ Surface Course or Wearing Course – Tar (Bitumen) – Asphalt or concrete road.
✓ Camber – Instrument to check whether water will flow to the drainage or not.

Traffic separators, Footpath, Kerb, median. Curves, Cycle tracks, Carriage way, Parking lane

Guard Stones and Guard Rails, Fencing.

→ Shoulder is useful for drainage and parking of vehicles.

* Recreation facilities – Amusement Parks
Swimming Pools.

* Roles of Civil Engineers :-
→ Providing shelter
→ laying ordinary village.
→ construction of bridges, irrigation tanks.

• Socio-economic development of a contry :-
→ Transportation
→ Health care
→ Recreation
→ Drinking water
→ Education
→ Housing
→ Health Care.
→ Irrigation.

Subsoil – Weak, Moisture Sensitive
Subgrade – Moderate Strength, Free Draining

→ Flexible Pavement	Rigid Pavement
* Bitumen roads.	* Concrete roads.
* Life span is more	* Life span is less.
* We can use within 24 hrs.	* We can use after 14-15 days of construction

- Explain the Classification of Roads :-

According to Season :-
(1) All ~~wth~~ weather road - National Highways

(2) Fair weather road - Earthen Roads.

According to Surface Characteristics :-
(1) Bituminous

(2) Concrete road.

(3) Earthen Road.

(4) ~~B~~ Water ~~Board~~ Bound Mechaderm. - stone roads.

According to Functional Variations (Nagpur Road Plan)
(1) National Highways
(2) State highways
(3) Major District roads
(4) Other District roads
(5) Village roads

According to Transport Planning :-
(1) Primary - National Highways and State Highways
(2) Secondary - Major District Roads.
(3) Tertiary - Other district roads and Village roads.

According to Urban Roads :-

(1) Artirials – Majestic to Banshankri.

(2) Sub artirials

(3) Collector street roads. – roads in front of houses.

(4) Express ways. – Zero traffic roads.

NH_3 - Bombay - Agra.

NH_1 → Delhi - Ambala - Amritsar.

NH_9 → Pune - Hyderabad.

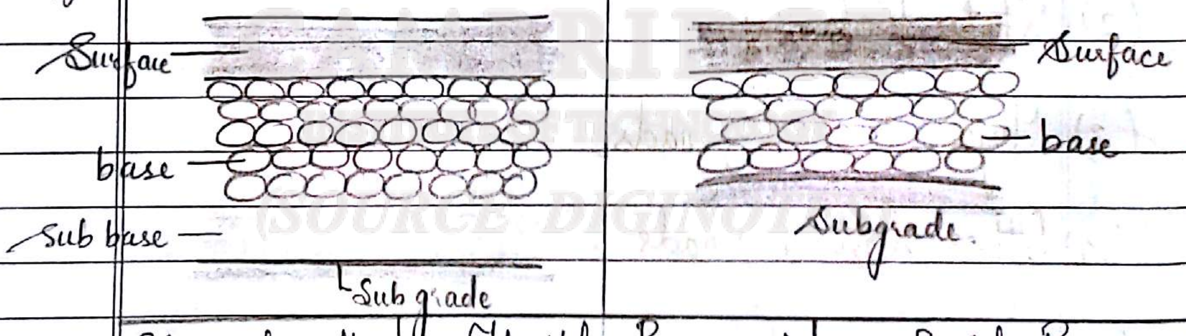
NH_{48} → Bangalore - Mangalore

NH_7 - Bangalore - Vijayawada - Hyderabad.

- Highway running over an embankment :

Imp Q - Differentiate b/w flexible pavement and rigid pavement.

Flexible Pavement (Bitumen)	Rigid Pavement (Concrete)



	Characteristics	Flexible Pavement	Rigid Pavement
1	Load Transfer	grain to grain interaction	slab action
2	Life span	15-20 years	> 40 years.
3	Initial cost	Less	more
4	Camber (Slope)	Steep (⌒)	Mild (‾)

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5	Stage construction	Possible	Impossible
6	Surface characteristics	Layers are pervious (absorption capacity is less)	Layers are impervious
7	Glare and night visibility	Night vision poor due to black colour.	Night vision bright due to white colour.
8	Traffic division	Open traffic from next day construction	Takes longer time after construction of use
9	Strength	less	more strong

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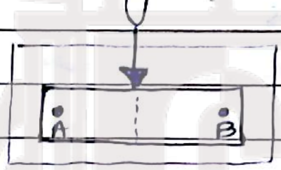
- ENGINEERING MECHANICS :- The branch of science which mainly deals with study of body when the body ~~Rigid Body : The body~~ is at rest or in motion when an external force is acting on a body.

→ Rigid Body : The body doesn't alter its shape and size.

OR

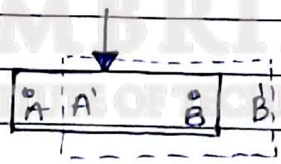
The distance b/w the two points of a body will not change on the application of force.



→ Deformal Body :- The body that alters its shape or size.

OR.

The distance b/w the 2 points is changed on the application of load.



→ Particle :- Defined as which is having infinely small volume and entire mass of the body is assumed to be concentrated at a point.

→ Continumn :- The bodies are assumed to have continuous distribution of matter even though they are made up of atoms, molecules etc. Then the effect on a body is treated as continumn.

→ Idealization or Assumptions in engineering mechanics

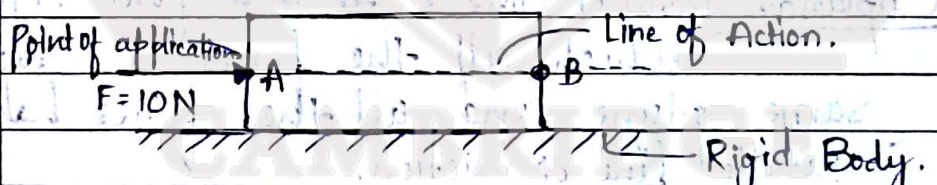
(1) A body consists of continuous distribution of matter
(2) A body is considered as perfectly rigid.
(3) Particle has mass but not size
(4) The force act through a very small point.

Imp • Define Force, Explain the characteristics of force with a neat figure.

Ans Force :- It is defined as acc. to Newton's second law, the action or agent which changes or tends to change the state of rest of a uniform motion of body in a straight line

OR

also called push or pull effect on a body and it is a vector quatity.



The 4 characteristics are:

(1) Magnitude - From fig 10 N is magnitude
(2) Direction - Arrow indicates direction
(3) Point of Application - From fig. point A is point of application.
(4) Line of Action - From fig. AB is line of action.

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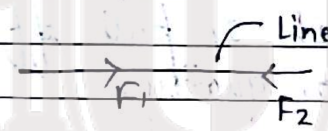
Imp • With a neat figure explain the classification of force system.

Ans- The diff classification force system are :-

(1) Collinear
(2) Coplanar Parallel Force
(3) Coplanar concurrent force.
(4) Coplanar non-concurrent force.
(5) Non coplanar parallel Force
(6) Non-coplanar concurrent Force.
(7) Non-coplanar non-concurrent Force.

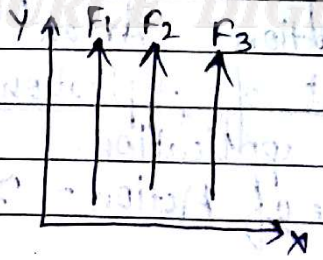
Collinear Force :- It is a force system in which all the forces are acting in same line of action.



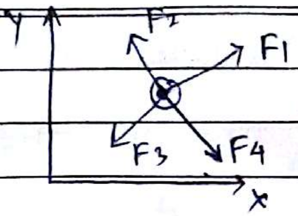
Eg: Tug of War.

Coplanar Parallel Force :- It is a force system in which all the forces are lying on same plane and all the forces have parallel line of action.

Eg :- Forces or load and the support reaction of the wheel.

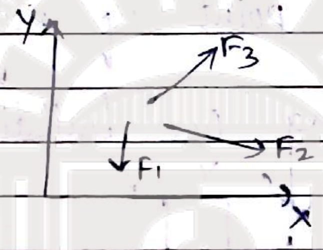


Coplanar Concurrent Force :- It is a force system in which all forces are acting in same plane and line of action meet at the same point. (point of origin is same.



Eg: Forces in the rope.

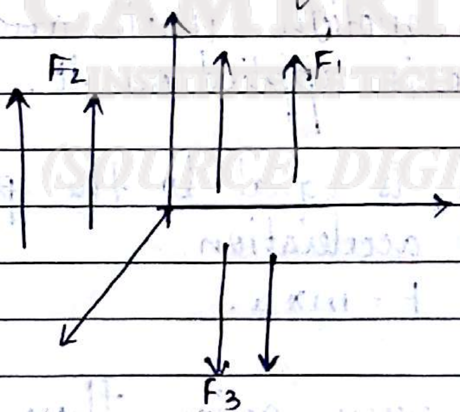
Coplanar Non-concurrent forces :- It is a forces system are acting in same plane and the line of action meets at diff. points.



Eg: Forces acting on building frame.

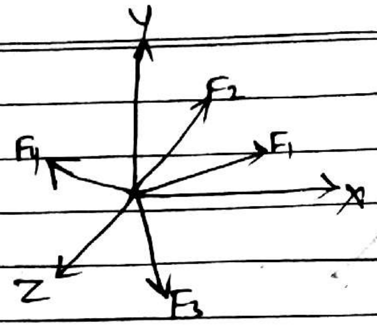
Non-Coplanar Parallel forces :- It is a force system in which all forces are acting in diff plane and also all forces have parallel line of action.

Eg:- Forces acting at a point of contact of bench with floor in a classroom.



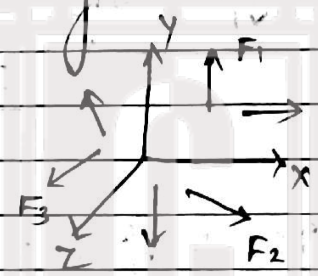
Non-Coplanar Concurrent forces :- It is a force system in which all forces are acting in different planes and also all forces meet at same point.

Eg: Forces acting on a tripod stand which a camera is mounted on it.



Non-coplanare nonconcurrent forces :- It is a force system in which all forces acting on different planes and all the forces will meet at different point of origin.

Eg: Forces acting on building frame.



- NEWTONS LAWS:-

1st LAW :- The body will be in the state of rest or motion until and unless any external force is applied on it.

2nd LAW :- Force is equal to the product of mass and acceleration.

$$F = m \times a.$$

3rd LAW :- For every action there is an equal and opposite reaction.

- INTRODUCTION TO SI UNITS :-

→ The international system of units formly recognised by General conference of weight in 1960.

→ India also adoptted SI units.

→ SI unit consists of 7 base units ~~to~~, 2 supplementary units and a no. of derived units.

→ The base units are length, mass, time, etc.

Force – Newton
Mass – kg.
Velocity – ms^{-1}
Length – m.
Energy – Joule.
Acceleration – ms^{-2}.
Power – Watt.

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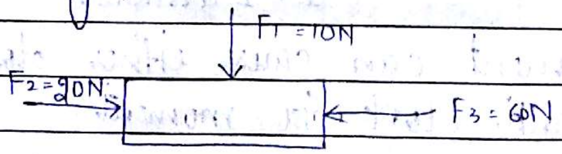
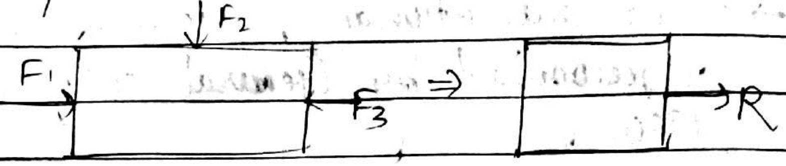
- PRINCIPLES OF ENGINEERING MECHANICS :-

(1) Principle of physical independence :- If two or more forces act on body or an object, every force produce its own effect on object independent of the remaining forces.



(2) Principle of Super position :- If two or more forces act on a body, then the combined effect due to all the forces is the vector addition of individual forces.

The net affect can be represented by a single force ie resultant (R)



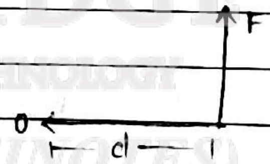
$$\boxed{\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3}$$

(3) <u>Principle of Transmissibility</u> :- The state of rest or uniform motion of a rigid body is unaltered if point of application is transmitted to any other point along the same line of action.

A shifted to D.

Imp • <u>MOMENT OF FORCE</u> : Defined as the rotational effect on a ~~free~~ body. Mathematically it is defined as product of magnitude of Force and the ⊥ distance ~~from~~ / ~~r/block~~ of the force from the point.

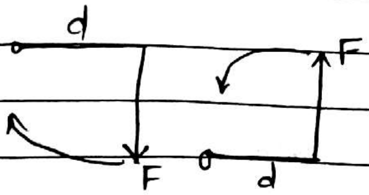
$$\boxed{M_0 = F \times d}$$



F = Magnitude of Force

d = ⊥ distance.

The moment can cause either close wise moment or anti-clock wise moment.

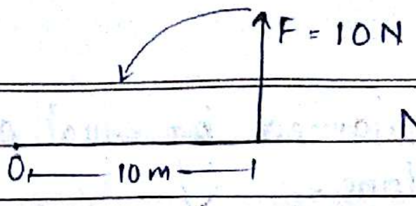


$$\boxed{M_0 = -F \times d}$$

$$\boxed{M_0 = +F \times d}$$

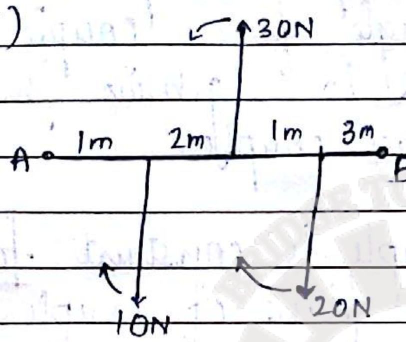
The unit of moment is N-metre.

Eg: (i)



$M_0 = 10 \times 10$
$= -100$ N-m {Anticlockwise}

(ii)

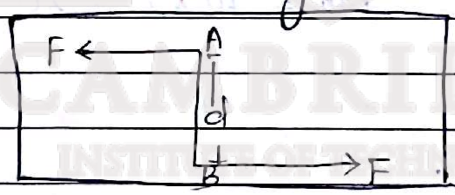


$M_A = +20 \times 4m = 80$ N-m.
$= -30 \times 3m = -90$ N-m
$= 10 \times 1m = 10$ N-m //

Imp • DEFINE COUPLE. EXPLAIN THE CHARACTERISTICS OF COUPLE:

Couple: When 2 equal and opposite parallel forces acting on a body and some distance apart then these 2 forces constitute a couple.

Couple has tendency to rotate a body.

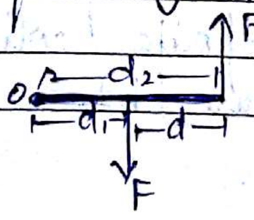


Characteristics :-

- Algebraic sum of forces constituting the couple is 0.

$-F + F = 0$

- The algebraic sum of the moment of forces constituting a couple about any point is equal to moment of couple itself.



$M_0 = +F \times d_1 - F \times d_2$
$M_0 = F \times (d_1 - d_2) = Fd$ //

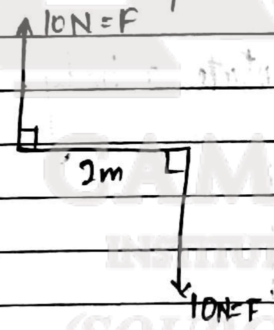
- Couple can be balanced by equal and opposite forces in same plane.

- Any no. of coplanar force couple can be reduced to a single force couple whose magnitude is equal to algebraic sum of moment of all force couple.

- The moment of couple is constant for any point choosen in plane of couple ~~is constant~~

- Any two couples whose moment are equal and with same sign are equivalent.

- MOMENT OF COUPLE: It is defined as product of either one of the force and the ⊥ force b/w them. From fig



$M_o = 10 \times 2 = 20$ Nm

INTO

- RESOLUTION OF FORCE INTO FORCE & COUPLE:

STEPS:

→ Figure shows the body subjected to force F at A now it is reqd. to shift this force at B.

(1) Keeping the force F at A, superimpose an another 2 equal and opposite collinear forces at B as shown in figure (b).

(2) Now the force at A and downward force at B will constitute a couple.

(3) From the figure C, the force F is shifted at a point B along with a couple ie $M_B = F \times d$.

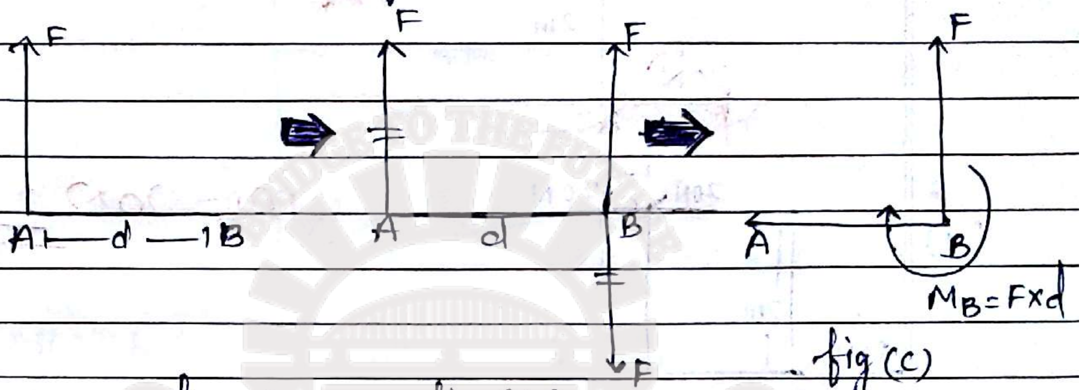
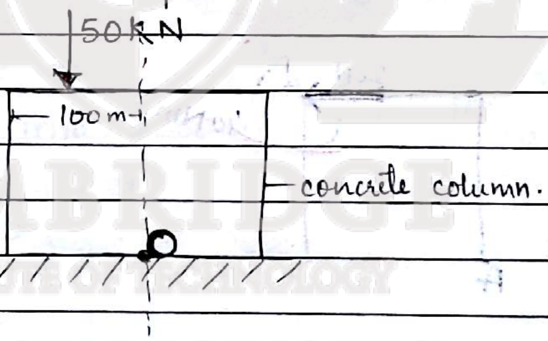


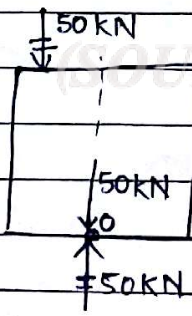
fig (a) fig (b) fig (c)

Q.1- A concrete column is carrying a force of 50kN as shown in figure. Replace the system of force couple system at a point O.

Ans-

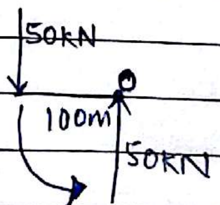


(1)



Superimposing 50 kN at O.

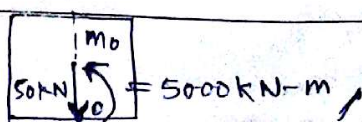
(2)



$M_o = 50 \times 100$

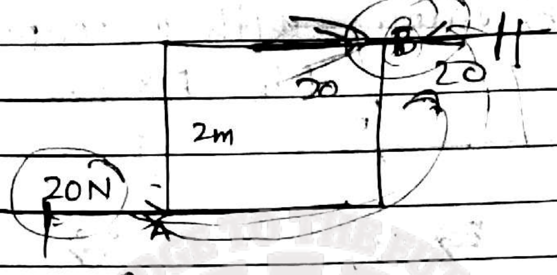
$M_o = 5000 \text{ kN-m}$

(3)

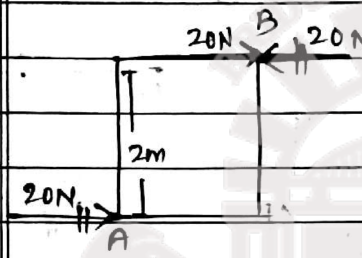


Q-2 Replace the force couple system @ B as shown in figure :-

Ans-

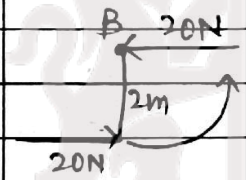


(1)



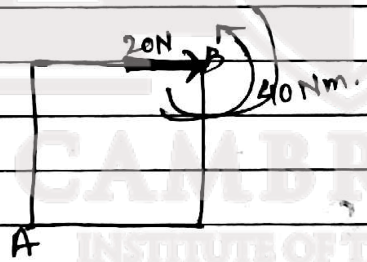
$M_B = 20 \times 2 = 40$ N-m

(2)



$M_B = \circlearrowleft 20 \times 2 = 40$ Nm

(3)



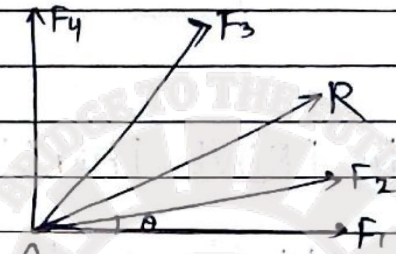
4/3/17

MODULE 2

Analysis of Concurrent Force Systems

Date: / /

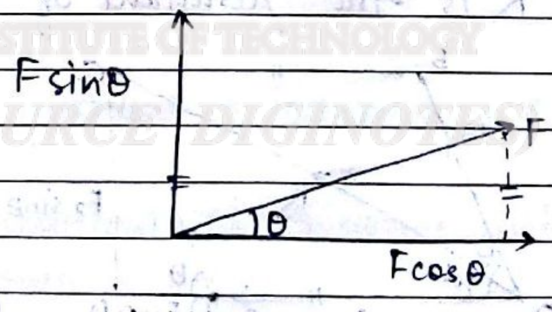
- RESULTANT: Resultant forces, when a no. of concurrent forces acting on a body, it is possible to find a single force which can produce a same effect that can be produced by other forces. Such a force is called as resultant forces.



The process of determining the resultant force of a given system of forces is known as composition of forces.

→ Resultant can be determined by graphical and analytical method.

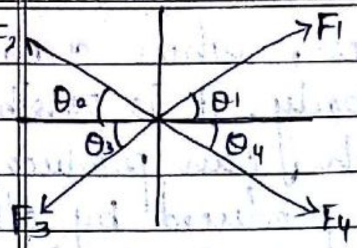
- RESOLUTION OF FORCES: Method of resolving a single force into 2 component is known as resolution of forces



horizontal component = $F\cos\theta$.
vertical component = $F\sin\theta$.

Imp
- METHOD OF RESOLVING NUMBER OF CO-PLANAR CONCURRENT FORCES:

$$\sqrt{\Sigma(HC)^2 + \Sigma(VC)^2}$$



$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

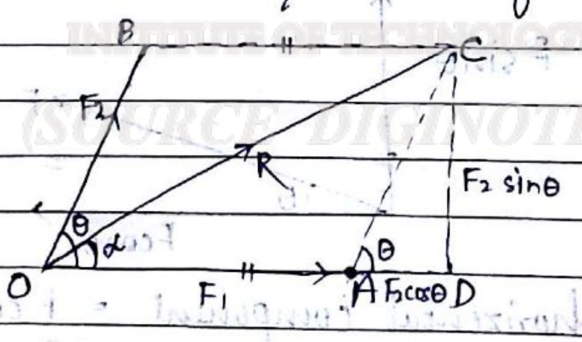
$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

Imp • STATE AND PROVE PARALLELOGRAM LAW OF FORCES:

This law is applicable to ditermine the

Defn: It states that if 2 forces acting at a point having both magnitude and direction representing two sides of a parallelogram then the resultant of this 2 forces having both magnitude and direction representing diagonal of ||gm. passing through same point.

Proof:- Let F_1 and F_2 are 2 forces acting on line OB & OA and R is the resultant of these 2 forces.



In ΔOCD

$$OC^2 = OD^2 + CD^2$$

$$R^2 = (OA + AD)^2 + F_2^2 \sin^2\theta.$$

$$R^2 = (F_1 + F_2\cos\theta)^2 + F_2^2 \sin^2\theta$$

$$R^2 = F_1^2 + F_2^2\cos^2\theta + 2F_1F_2\cos\theta + F_2^2\sin^2\theta$$

$$R^2 = F_1^2 + F_2^2(\cos^2\theta + \sin^2\theta) + 2F_1F_2\cos\theta.$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

In ΔOCD :- $\tan\alpha = \dfrac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$

$$\alpha = \tan^{-1}\left(\frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}\right)$$

Case I :- If $\theta = 90°$.

$\alpha = \tan^{-1}\left(\dfrac{F_2}{F_1}\right)$, $R = \sqrt{F_1^2 + F_2^2}$

Case II $\theta = 0$.

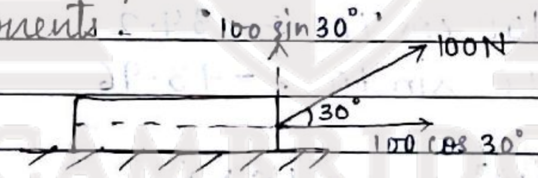
$\alpha = \tan^{-1}(0) = 0$, $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2}$

Case III : $\theta = 180°$

$R = F_2 - F_1$, $\alpha = 0°$.

Q.1- A force 100 N is acting on the body as shown in fig.
Resolve the force into vertical and horizontal components :-

Ans-

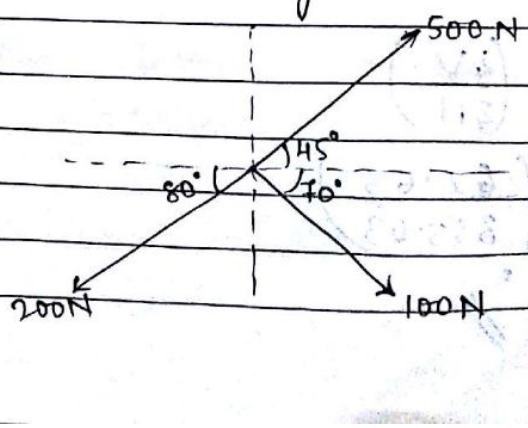


Vertical Component = $100 \sin 30° = 50$ N

Horizontal Component = $100 \cos 30 = 86.60$ N

Q.2 Find the resultant of 3 forces acting on a point O' as shown in fig :-

Ans-



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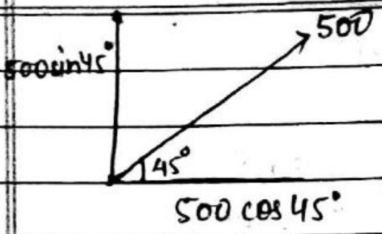
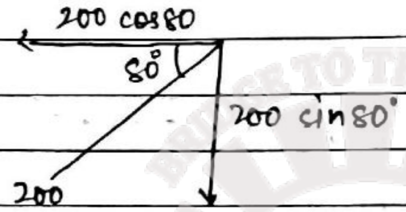


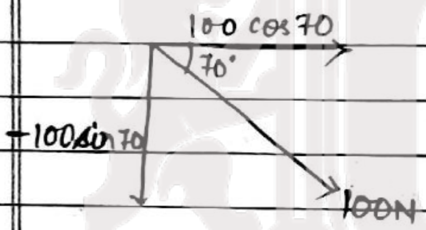
$H = 500 \times \cos 45 = 353.55$

$V = 500 \sin 45 = 353.55.$



$H = -200 \cos 80 = -34.72.$

$V = -200 \sin 80 = -196.96.$



$H = 100 \cos 70 = 34.2$

$V = -100 \sin 70 = -93.96.$

$\therefore R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$\Sigma H = 353.03.$

$\Sigma V = 62.63.$

$R = 35850 \text{ N}.$

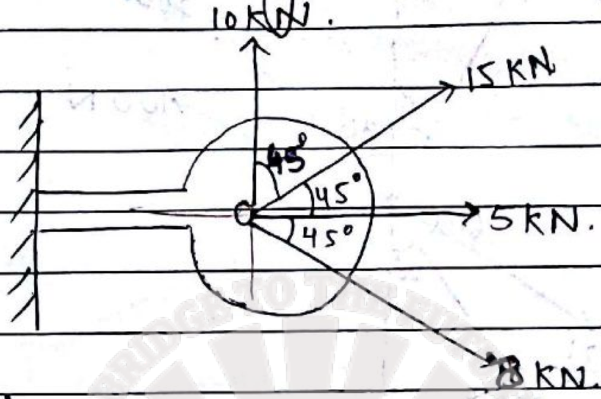
$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

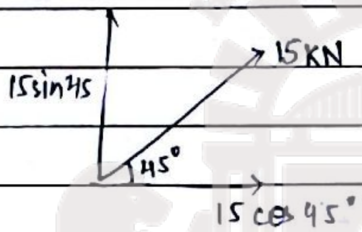
$\alpha = \tan^{-1}\left(\frac{62.63}{353.03}\right)$

$\alpha = 10.06°$ //

Q3- Find the value of resultant for the system of forces shown in fig :-

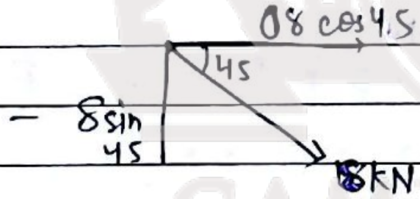
Ans-





$H = 15 \cos 45° = 10.60$ KN.

$V = 15 \sin 45 = 10.60$ KN



$H = 8 \cos 45 = 5.65$ KN

$V = -8 \sin 45 = -5.65$. KN

$\therefore \Sigma H = 5 + 10.60 + 5.65 = 21.25$ KN.

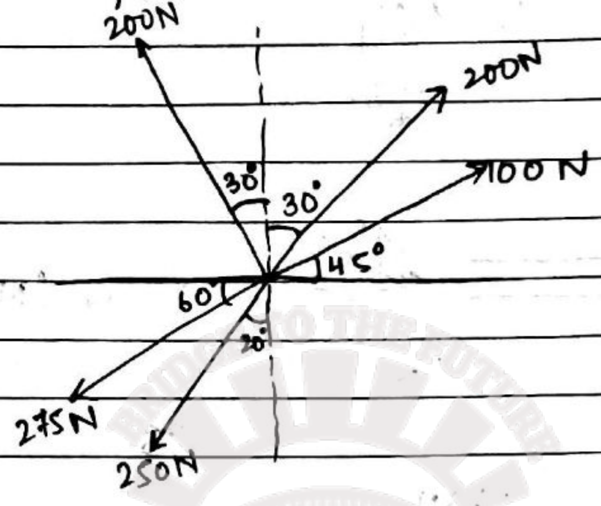
$\Sigma V = 10 + 10.60 - 5.65. = 14.95$ KN.

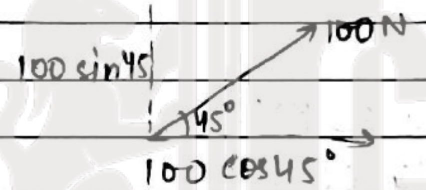
$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 25.98$ KN.

$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 35.12°$ //

Q. 4- Find the resultant

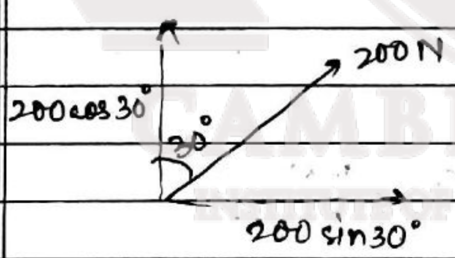
Ans-





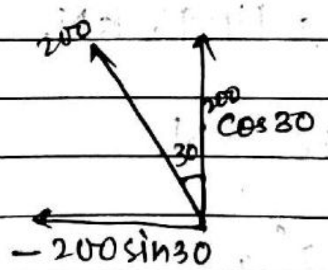
$H = 100 \cos 45 = 70.71$ N

$V = 100 \sin 45 = 70.71$ N



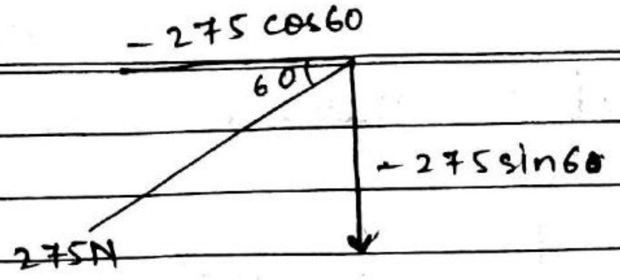
$H = 200 \sin 30^\circ = 100$

$V = 200 \cos 30 = 173.21$



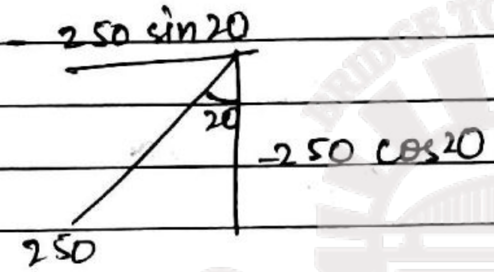
$H = -200 \sin 30 = -100$

$V = 200 \cos 30 = 173.21$



$H = -275\cos 60 = -137.50$

$V = -275\sin 60 = -238.16$



$H = -250\sin 20 = -85.51$

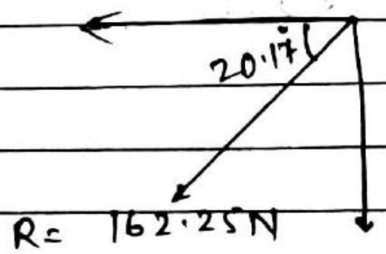
$V = -250\cos 20 = -234.92$

$\therefore \Sigma H = -157.30\ N$

$\Sigma V = -55.95\ N$

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 162.25N$

$\tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 20.17^\circ$

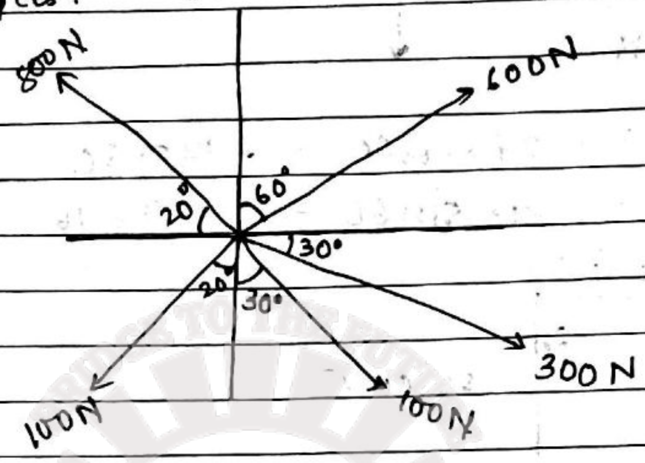


III rd Quad.

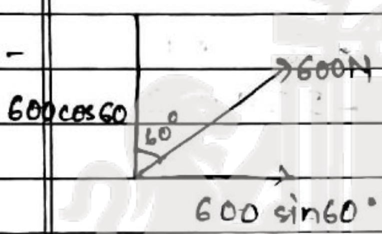
papergrid

Date: / /

Q.5- Find magnitude of the resultant for the given system of forces.

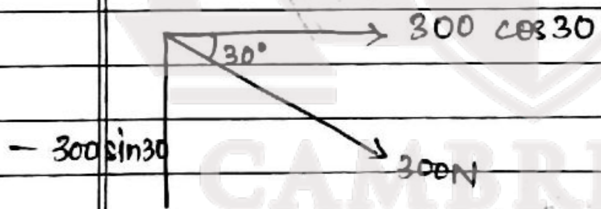


Ans -



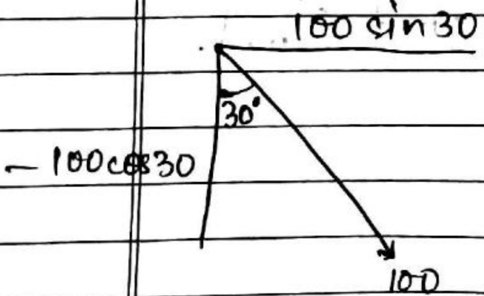
$H = 600 \sin 60 = 519.62$ N

$V = 600 \cos 60 = 300$ N



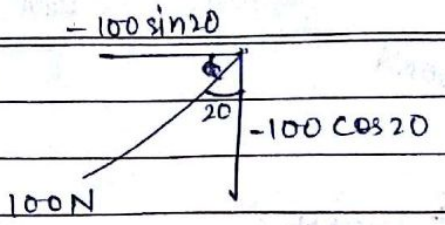
$H = 300 \cos 30 = 259.81$ N

$V = -300 \sin 30 = -150$ N



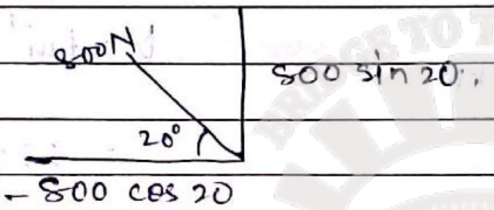
$H = 100 \sin 30 = 50$

$V = -100 \cos 30 = -86.60$.



$H = -100 \sin 20 = -34.20\ N$

$V = -100 \cos 20 = -93.97\ N.$



$H = -800 \cos 20 = -751.75 N$

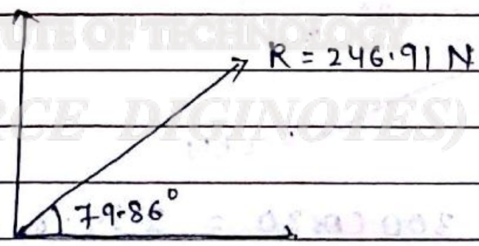
$V = 800 \sin 20 = 273.62\ N.$

$\therefore \Sigma H = 43.48\ N.$

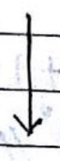
$\Sigma V = 243.05\ N.$

$\therefore R = \sqrt{\Sigma H^2 + \Sigma V^2} = 246.91\ N$ //

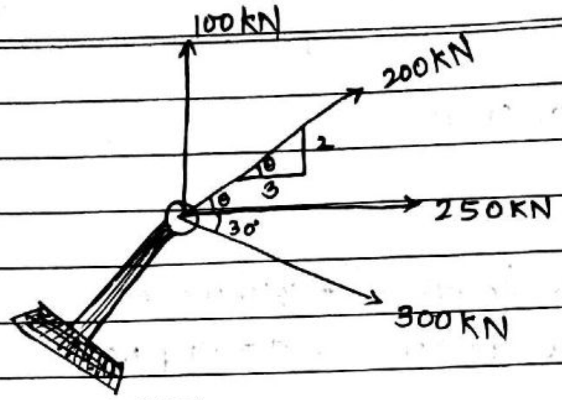
$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 79.86°$ //



Q.6- Force acting on a board ~~acting on~~ as shown in figure. Determine the magnitude and direction of the resultant figure.



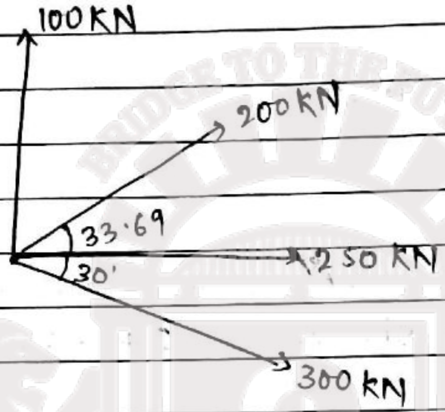
Ans -

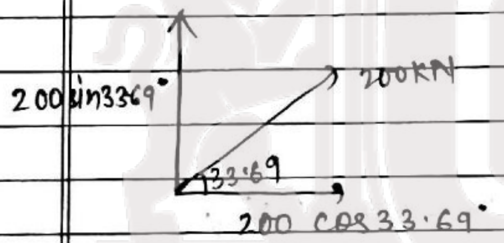


$\tan\theta = \frac{2}{3}$

$\theta = \tan^{-1}\frac{2}{3}$

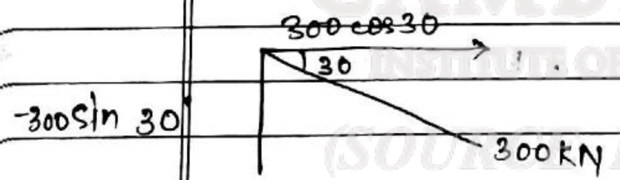
$= 33.69^\circ$.





$H = 200 \cos 33.69 = 166.41$ KN.

$V = 200 \sin 33.69 = 110.93$ KN.



$H = 300 \cos 30 = 259.8$ KN

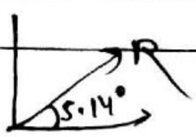
$V = -300 \sin 30 = -150$ KN.

$\Sigma H = 250 + 259.8 + 166.41 = 676.21$ KN.

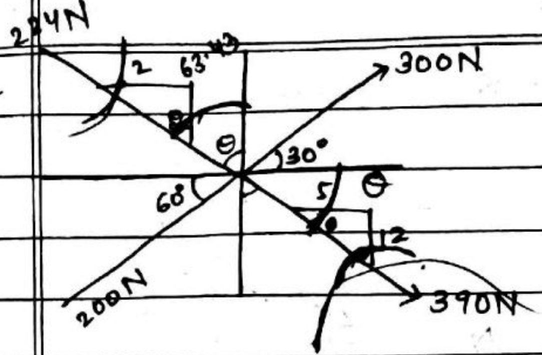
$\Sigma V = 100 + 110.93 - 150 = 60.93$ KN.

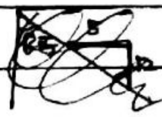
$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 679.09$ KN.

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 5.14^\circ$ //



Q-





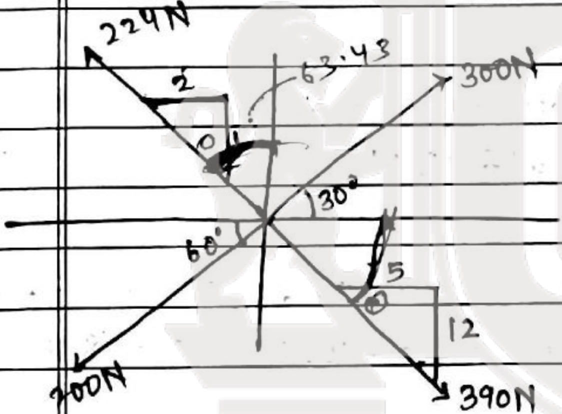
$\tan\theta = \frac{5}{12}$

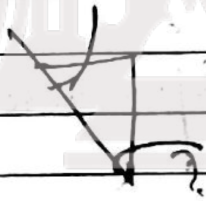
$\theta = 6.5$

$\tan\theta = \frac{2}{1}$

$\theta = \tan^{-1}(2) = 63.43.$

$\tan\theta = \frac{12}{5} = 67.38.$

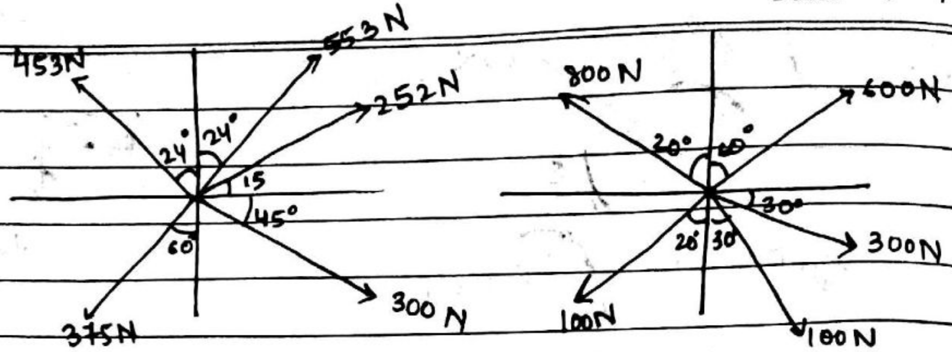




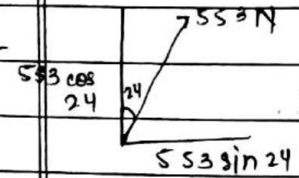
$\tan\theta = \frac{2}{1}$

$\theta = 63.43.$

Q-

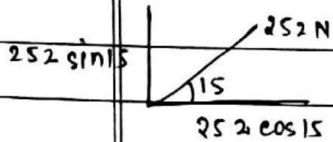


Ans-



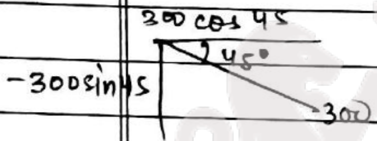
$H = 553 \sin 24 = 224.92\ N$

$V = 553 \cos 24 = 505.19\ N$



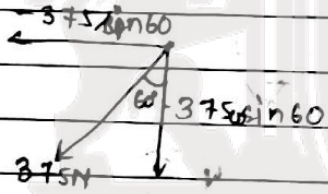
$H = 252 \cos 15 = 243.41\ N$

$V = 252 \sin 15 = 65.22\ N.$



$H = 300 \cos 45 = 212.13\ N$

$V = -212.13\ N.$



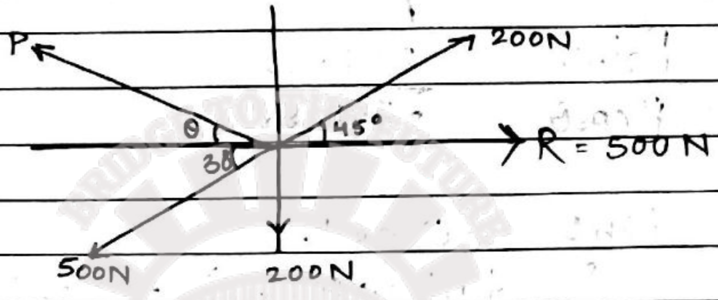
$H = -375 \sin 60 = -324.7\ N.$

$V = -375 \cos 60 = -187.5\ N$

0/3/17

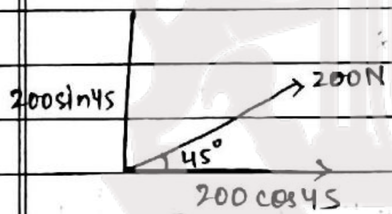
Q.1- **Imp** 4-coplanar forces acting @ a point as shown in figure. One of the forces is unknown whose magnitude is P. The resultant has a magnitude of 500 N and is acting along x-axis. Determine the unknown force P.

Ans-



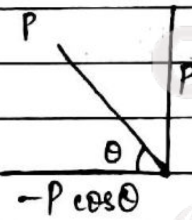
"$\boxed{\Sigma H = R = 500 N}$" —— ①

"$\boxed{\Sigma V = 0 N}$" —— ②



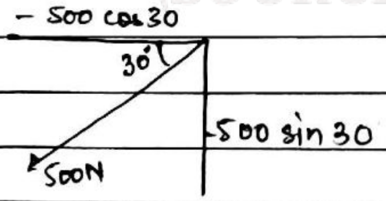
$H = 200 \cos 45 = 141.42$ N

$V = 200 \sin 45 = 141.42$ N.



$H = -P\cos\theta$.

$V = P\sin\theta$.



$H = -500 \cos 30 = -433.0$

$V = -500 \sin 30 = -250.$

$\Sigma H = 141.42 - P\cos\theta - 433.0.$

$500 = -P\cos\theta - 291.58.$ [From eq ①].

⇒ $+P\cos\theta = -291.58 - 500 = -791.58$

⇒ $P\cos\theta = -791.58$ —— ③

$\Sigma V = 141.42 + P\sin\theta - 250 + 200$

$0 = -308.58 + P\sin\theta$

$\Rightarrow P\sin\theta = 308.58$ — (4)

Eq (4) ÷ Eq (3)

$$\frac{P\sin\theta}{P\cos\theta} = \frac{308.58}{-791.58}$$

$\tan\theta = -0.389$

$\theta = \tan^{-1}(+0.389) = 21.25^\circ$

Squaring and adding eq (3) and (4) :-

$P^2\sin^2\theta + P^2\cos^2\theta = 721820.5128$

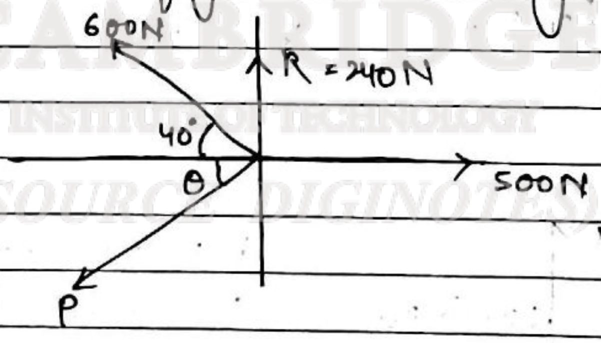
$P^2(1) = 721820.5128$

$P = 849.60\ N$

Imp

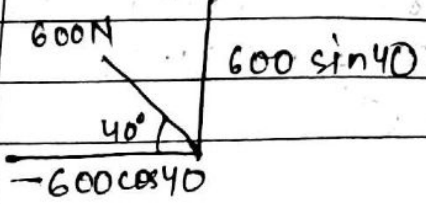
Q.2 A force system as shown in fig. has resultant of 240 N acting upwards along y-axis. Find the value of P and its inclination.

Ans-



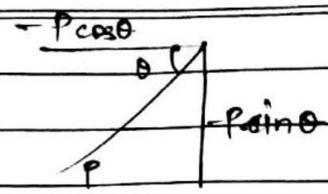
"$\Sigma V = 240\ N = R$" — (1)

$\Sigma H = 0$ — (2)



$H = -600\cos 40 = -459.6\ N$

$V = 600\sin 40 = 385.6\ N$



$H = -P\cos\theta$.
$V = -P\sin\theta$.

$\therefore \Sigma H = 500 - 459.6 - P\cos\theta$.
$0 = 40.4 - P\cos\theta$.
$\Rightarrow P\cos\theta = 40.4$ —— (3).

$\Sigma V = 385.6 - P\sin\theta$
$240 = 385.6 - P\sin\theta$.
$P\sin\theta = 385.6 - 240 = 145.6$. —— (4).

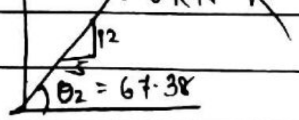
$\frac{P\sin\theta}{P\cos\theta} = \frac{145.6}{40.4}$.
$\tan\theta = 3.603$
$\theta = \tan^{-1}(3.603) = 74.48°$ //

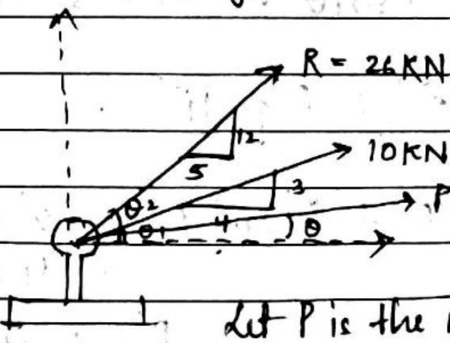
Squaring and adding eq (3) and (4) :-

$P^2\cos^2\theta + P^2\sin^2\theta = 22831.52$.
$P^2 = 22831.52$.
$P = 151.10\ N$ //

Imp Q.3 26 KN forces is a resultant 2 forces, one of which is shown in figure. Determine the other force.

Ans.



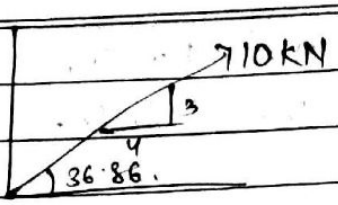


Let P is the unknown forces

$\theta_2 = \tan^{-1}\left(\frac{12}{5}\right)$

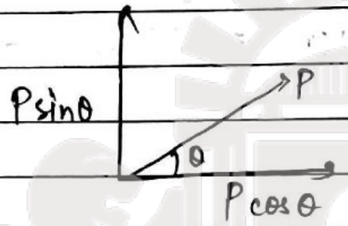
$\theta_2 = 67.38$

$H = 26\cos 67.38 = 10\ kN. = R_x$
$V = 26\sin 67.38 = 23.9\ KN = R_y$



$$\theta_1 = \tan^{-1}\left(\frac{3}{4}\right) = 36.86^\circ$$

$$H = 10\cos 36.86 = 8.00\ kN$$
$$V = 10\sin 36.86 = 5.99\ KN.$$



$$H = P\cos\theta$$
$$V = P\sin\theta$$

$$\Sigma H = R_x$$

$$\therefore P\cos\theta + 8 = 10$$
$$P\cos\theta = 2\ kN \quad \text{——} \ (3)$$

$$\Sigma V = R_y$$
$$\therefore P\sin\theta + 5.99 = 23.9$$
$$P\sin\theta = 23.9 - 5.99 = 17.91\ kN \quad \text{——} \ (4)$$

Eq $(3)^2$ + Eq $(4)^2$.

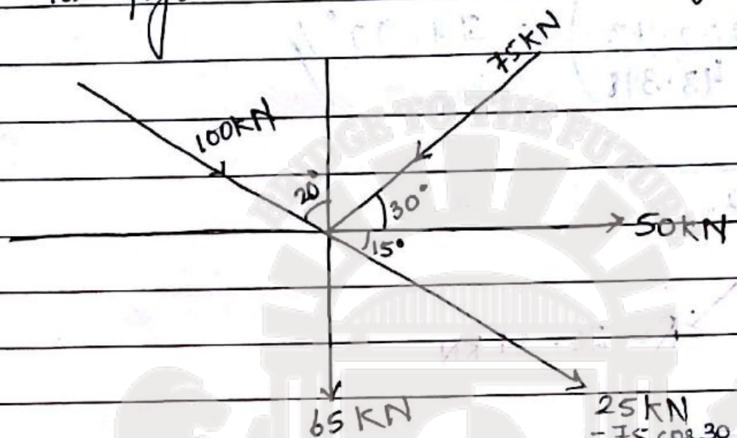
$$\therefore P^2\sin^2\theta + P^2\cos^2\theta = 2^2 + 17.91^2$$
$$= 4 + 320.7681$$
$$P^2 = 324.7681$$
$$P = \sqrt{324.7681} = 18.02\ kN.$$

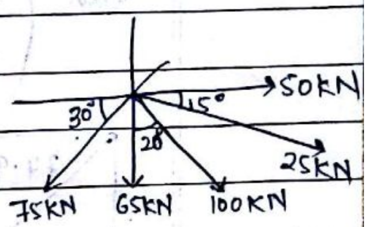
$\tan\theta = \frac{17.91}{2} = 8.955$

$\theta = \tan^{-1}(8.955) = 83.62^\circ$ //

Q.4 Find the resultant of the given system of forces shown in fig:

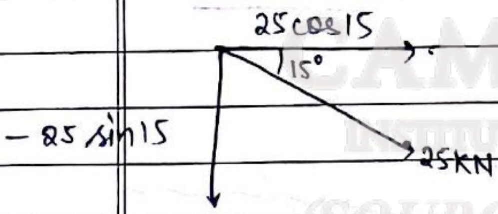
Ans-





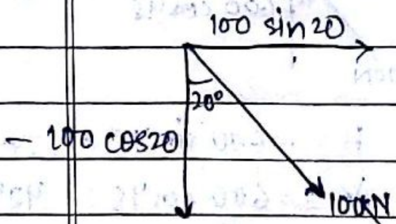
$H = -75\cos 30 = -64.95$ KN

$V = -75\sin 30 = -37.5$ KN.



$H = 25\cos 15 = 24.148$ KN

$V = -25\sin 15 = -6.47$ KN



$H = 100\sin 20 = 34.20$

$V = -100\cos 20 = -93.96.$

$\therefore \Sigma H = 50 + 34.20 + 24.148 - 64.95 = 43.398.$

$\Sigma V = -65 - 37.5 - 6.47 - 93.96 = -202.93$

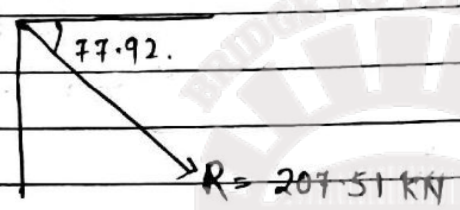
Date: / /

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$= \sqrt{(43.398)^2 + (-202.93)^2} = 207.51 \text{ KN.}$$

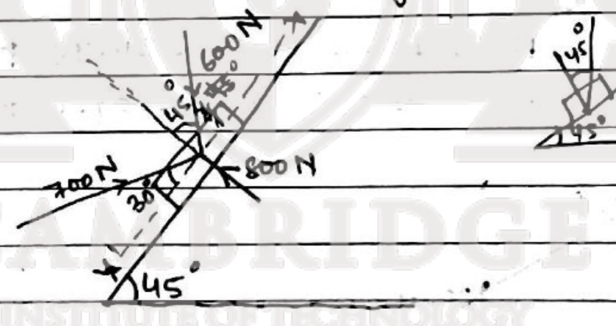
$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

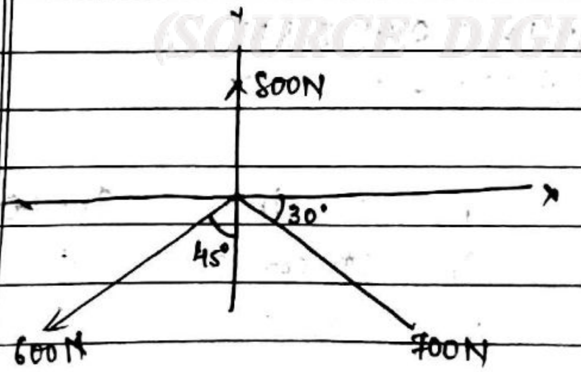
$$= \tan^{-1}\left(\frac{202.93}{43.398}\right) = 77.92^\circ$$

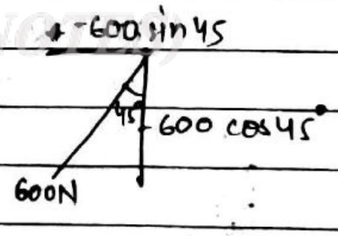


Q.5- Determine the resultant of system of forces acting on body as shown in figure :- (take the coordinate direction as shown in figure :-

Ans-

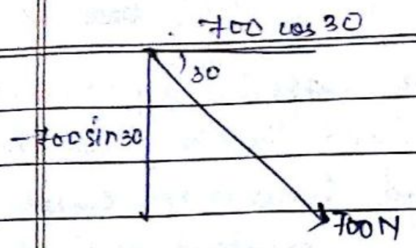






$H = -600 \sin 45 = -424.26$

$V = -600 \cos 45 = -424.26.$



$H = 700 \cos 30 = 606.21$

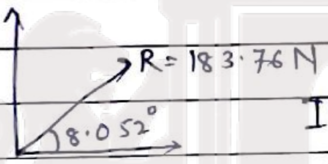
$V = -700 \sin 30 = -350$

$\therefore \ \Sigma H = 606.21 - 424.26 = 181.95$

$\Sigma V = 800 - 350 - 424.26 = 25.74$.

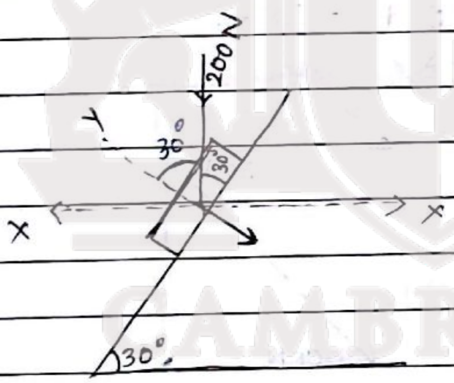
$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 183.76 \text{ N}$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 8.052^\circ$



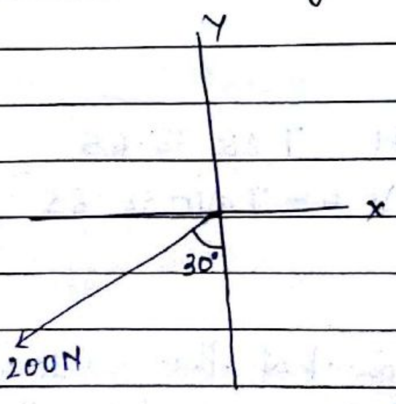
Ist Quad.

Q.6 (16/3)



The force is acting on block. Resolve the force into horizontal and vertical component.

Ans-

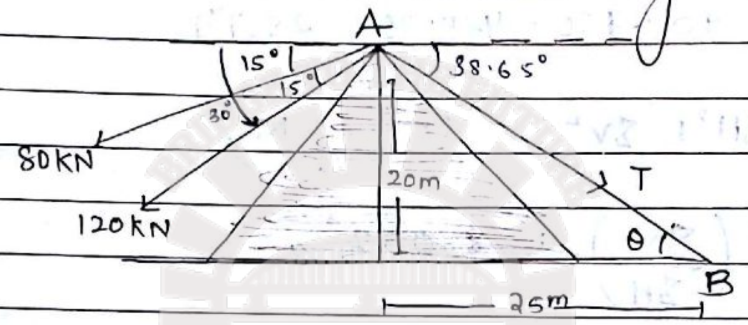


$H = -200 \sin 30^\circ = -100 \text{ N}$

$V = -200 \cos 30^\circ = -173.20 \text{N}$

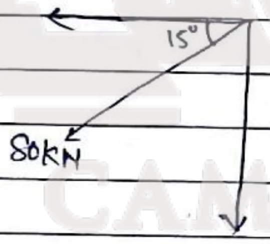
Q.7 (Imp) An electrical transmission tower supported by two cables carrying a load of 80 kN and 120 kN as shown in fig. Determine the reqd. tension in cable AB so that all the three cables are extended vertically downwards. Also find the resultant of 3 cables when it (resultant) is acting downwards.

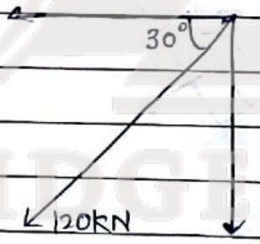
Ans-



$\tan\theta = \frac{20}{25}$

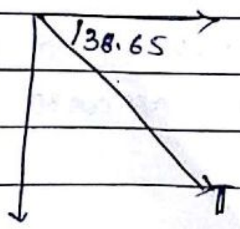
$\theta = \tan^{-1}\left(\frac{20}{25}\right) = 38.65$





$H = -80\cos 15$
$= -77.27$
$V = -80\sin 15$
$= -20.70$

$H = -120\cos 30 = -103.92$
$V = -120\sin 30 = -60$



$H = T\cos 38.65$
$V = -T\sin 38.65$

From the ques, we know that the resultant of all forces acting vertically downward so that

$\Sigma H = 0$
$\Sigma V = R$

$\Rightarrow \Sigma H = 0$

$\Rightarrow -77.27 - 103.92 + T\cos 38.65 = 0$

$\Rightarrow -181.19 + T(0.78) = 0$

$\Rightarrow T = \frac{181.19}{0.78} = 232.29\ kN$ //

$\Sigma V = R.$

$\Rightarrow -20.70 - 60 - T(0.624) = R$

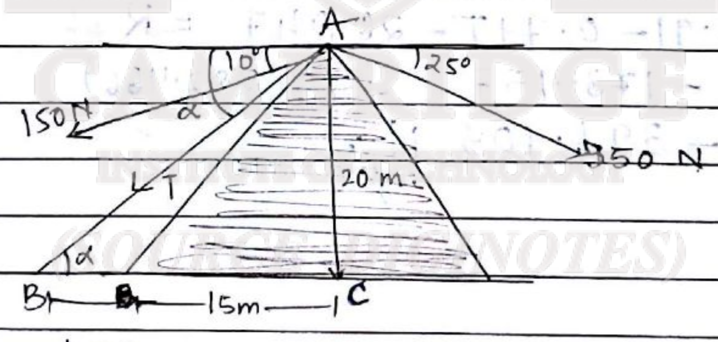
$\Rightarrow -80.7 - 144.94 = R$

$\therefore R = -225.648\ kN$ //

(−) indicates the resultant is acting vertically downwards.

Q.8 Two cables attached at the top of the tower carrying an another cable AB. Determine the tension in cable AB such that the resultant of all the forces in 3 cable acts vertically downwards and also determine the resultant of forces.

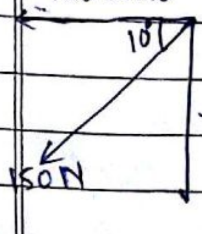
Ans-



In ΔABC :-

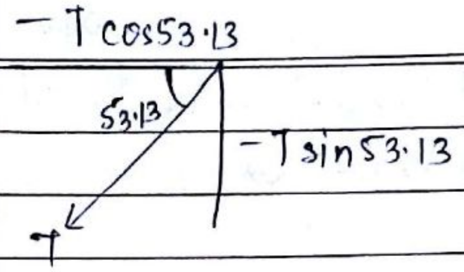
$$\tan\alpha = \frac{20}{15} = \frac{4}{3}$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$ //



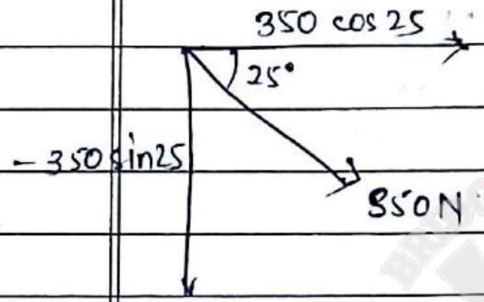
$H = -150\cos 10 = -147.72$

$V = -150\sin 10 = -26.047$



$H = -T\cos 53.13 = -0.600\,T$

$V = -T\sin 53.13 = -0.799\,T$



$H = 350\cos 25 = 317.20$

$V = -350\sin 25 = -147.91$

$\Sigma V = R$

$\Sigma H = 0$

$\therefore \Sigma H = 0$

$\Rightarrow -0.6T + 317.20 - 147.72 = 0$

$\Rightarrow +0.6T = +169.48$

$\Rightarrow T = \frac{169.48}{0.6} = 282.466\ N$ //

$\Sigma V = R$

$\Rightarrow -147.91 - 0.79T - 26.047 = R$

$\therefore R = -173.957 - 0.79(282.466)$

$\Rightarrow R = -397.105\ N$ //

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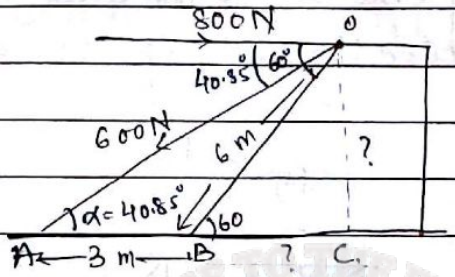
Q.9. Determine the resultant of forces acting at a structure at a point O.

Ans-



$$\cos 60 = \frac{BC}{OB}$$

$$0.5 = \frac{BC}{6}$$

$$BC = 6 \times 0.5 = 3 \text{ m}$$

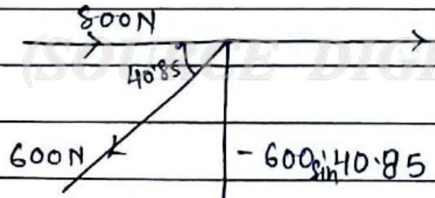
$$\sin 60 = \frac{OC}{OB}$$

$$0.86 \times 6 = OC$$

$$OC = 5.19 \text{ m}$$

$$\tan \alpha = \frac{OC}{AC} = \frac{5.19}{6}$$

$$\alpha = \tan^{-1} = 40.85°$$



$$H = -600 \cos 40.85 = -453.85.$$

$$V = -600 \sin 40.85 = -392.44.$$

$$R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{(346.15)^2 + (-392.44)^2}$$

$$= 523.28 \text{ N}$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H} \Rightarrow \theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 48.58°$$

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Calcy :- $a_1 = \cos 30$ $a_2 = \sin 30$

$b_1 = \cos 45$ $b_2 = -\sin 45$

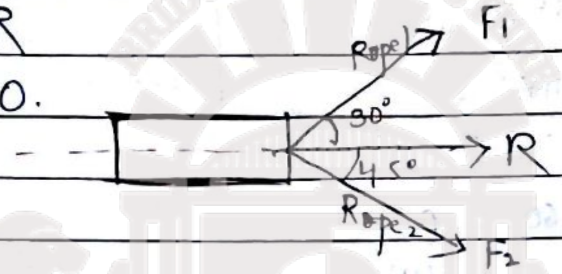
$c_1 = 5$ $c_2 = 0$

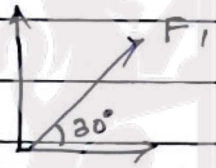
Q.10 A block is pulled by 2 ropes as shown in fig. If resultant of the 2 forces is 5 kN and directed along the axis of the block. Determine the tension in each of the rope.

Ans.

$\Sigma H = R$

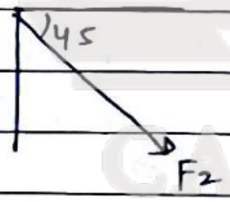
$\Sigma V = 0.$





$H = F_1 \cos 30° = 0.86 F_1$

$V = F_1 \sin 30 = 0.5 F_1.$



$H = F_2 \cos 45 = 0.70 F_2.$

$V = -F_2 \sin 45 = -0.70 F_2.$

$\Sigma H = R$

~~$F_1 \cos 30 + 0$~~

$0.86 F_1 + 0.70 F_2 = 5$

$0.5 F_1 - 0.70 F_2 = 0.$

$F_1 = 3.67 \text{ kN}$

$F_2 = 2.62 \text{ kN}$ //

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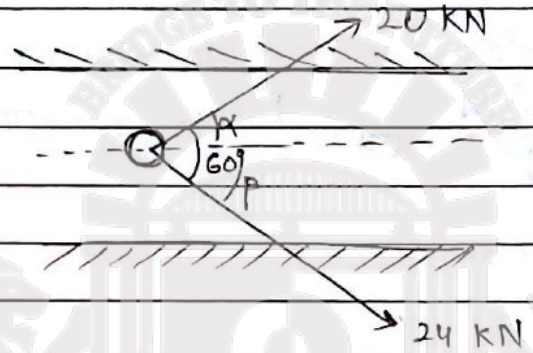
Q.11- Two locomotives moving on opposite bank of a canal and carried by two rope 11 to the banks to pull. a vessel as shown in figure. The force in the ropes are 20 kN and 24 kN by the total angle b/w them is 60°. Find the resultant pull on the vessel and the angles α and β.

Ans-



$F_1 = 20$ kN

$F_2 = 24$ kN

$\theta = 60°$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{20^2 + 24^2 + 2 \cdot 20 \cdot 24\cos 60}$

$R = 38.15$ kN

$\alpha = \tan^{-1}\left(\frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}\right)$

$= \tan^{-1}\left(\frac{24 \sin 60}{20 + 24\cos 60}\right)$

$= \tan^{-1}\left(\frac{20.78}{32}\right)$

$\alpha = \tan^{-1}\left(\frac{20.78}{32}\right) = 32.99°$

$\alpha + \beta = 60°$

$\Rightarrow \beta = 60° - \alpha$

$= 27.01°$

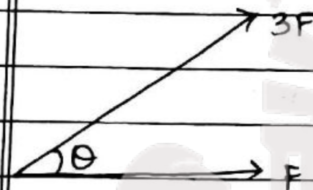
Q.12 – The resultant of 2 forces, one of which is 3 times the other force is 300 N. When the direction of small force is reversed, the resultant is 200 N. Determine the 2 forces and the angle b/w them.

Ans.

~~$F_1 = 3F_2$.~~
~~$F_1 = 300\,N$~~
~~$\therefore 3F_2 = 300$~~
~~$\Rightarrow F_2 = 100\,N$.~~
~~$R = 200\,N$~~

$\left.\begin{matrix} F \\ 3F \end{matrix}\right\} R = 300\,N$

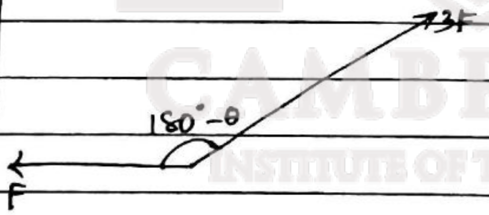
$(-)\left.\begin{matrix} F \\ 3F \end{matrix}\right\} R = 200\,N$



$R = \sqrt{F^2 + (3F)^2 + 2(F)(3F)\cos\theta}$.

$(300)^2 = F^2 + 9F^2 + 6F^2\cos\theta$.

$\Rightarrow 90000 = 10F^2 + 6F^2\cos\theta$ —— ①



$200^2 = F^2 + 9F^2 + 6F^2\cos(180-\theta)$

$\Rightarrow 40000 = 10F^2 - 6F^2\cos\theta$ —— ②

$\therefore$ Adding eq ① and ② :-

$10F^2 + 6F^2\cos\theta = 90000$
$10F^2 - 6F^2\cos\theta = 40000$

$20F^2 = 130000$

$F^2 = \frac{130000}{20} = 6500$

$F = 80.6\,N$

$\therefore 3F = 3 \times 80.6$

$= 241.86\ N$

(1) $\Rightarrow 90000 = 65000 + 39000 \cos\theta$.

$\Rightarrow 39000 \cos\theta = 25000$

$\Rightarrow \theta = \cos^{-1}\left(\frac{25000}{39000}\right) = 50.13^\circ$

EQUILIBRIUM OF FORCES.

18/3/17

- **Define Equilibrium. What are the conditions for equilibrium**

→ Any system of forces acting on a body is said to be in equilibrium when the resultant of the force is 0 and the algebraic sum of the forces should be equal to zero.

→ A body is said to be in state of equilibrium, the body should be in rest under action of forces.

Conditions for equilibrium :-

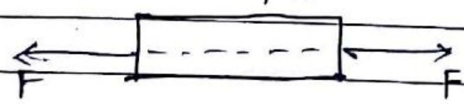
→ If the body should be in equilibrium, it should satisfy the following conditions :-

$$\Sigma H = 0$$
$$\Sigma V = 0$$
$$R = 0$$
$$\Sigma M = 0$$

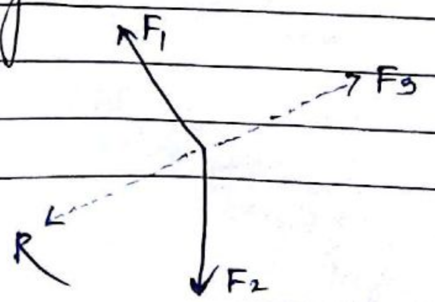
- **PRINCIPLE OF EQUILIBRIUM FOR DIFF. FORCE SYSTEM :-**

→ 2 Force System

If the body is acted upon 2 forces, then for equilibrium, the forces should be in equal in magnitude and opposite in direction.



→ 3 Force System

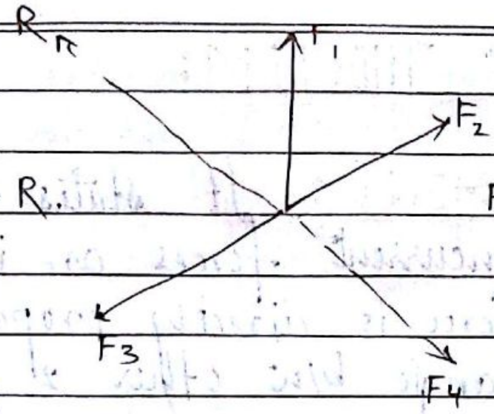


F_1 & $F_2 = R$

$R = F_3$

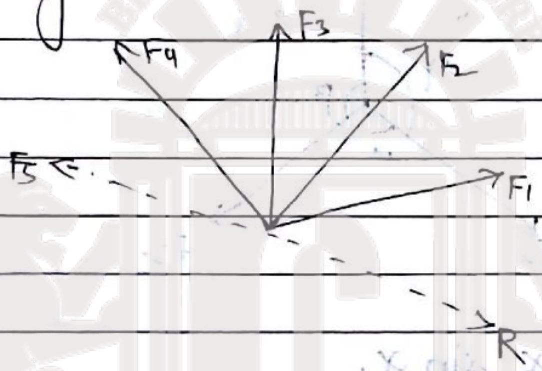
→ 4 force system

F_1 & F_2 & F_3 = R.
R = F_4. //



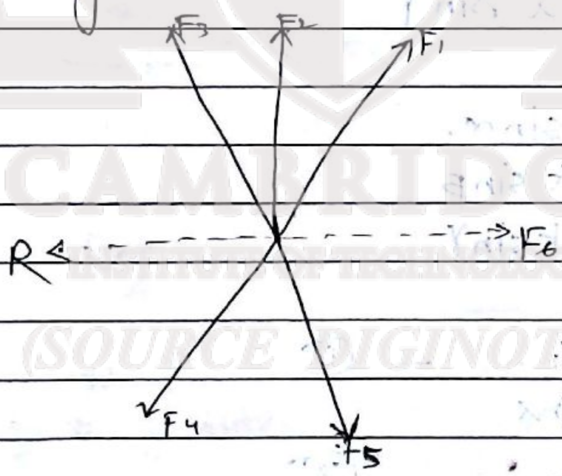
F_1 & F_2 = R_1
F_3 & F_4 = R_2
R_1 = R_2.

→ 5 force system



F_1 & F_2 & F_3 & F_4 = R.
F_5 = R //

• 6 force system:-



F_1 & F_2 & F_3, = R_1
F_4 & F_5 & F_6 = R_2
R_1 = R_2.

F_1 & F_2 & F_3 & F_4 & F_5 = R
R = F_6 //

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Imp

LAMI'S THEOREM:

STATEMENT :- It states that if 3 coplanar concurrent forces are in equilibrium then each force is directly proportional to sine of the angle b/w other 2 forces.

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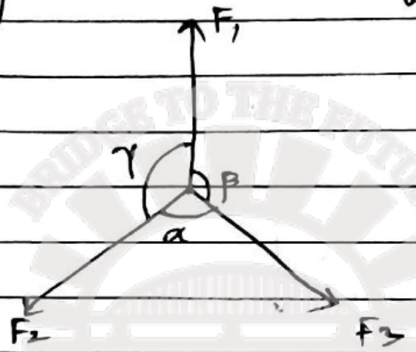

$F_1 \propto \sin\alpha$.

$F_2 \propto \sin\beta$

$F_3 \propto \sin\gamma$

$\Rightarrow F_1 = k\sin\alpha$.

$\Rightarrow F_2 = k\sin\beta$

$\Rightarrow F_3 = k\sin\gamma$

$\Rightarrow k = \frac{F_1}{\sin\alpha}$ — (1)

$\Rightarrow k = \frac{F_2}{\sin\beta}$ — (2)

$\Rightarrow k = \frac{F_3}{\sin\gamma}$ — (3)

From (1), (2) and (3)

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma} = k}$$

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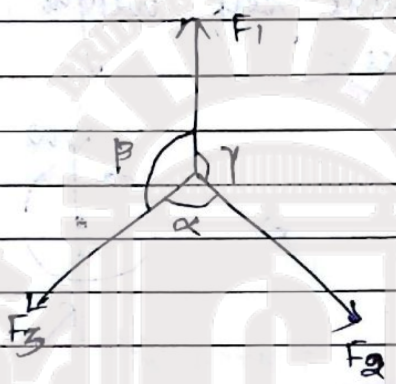
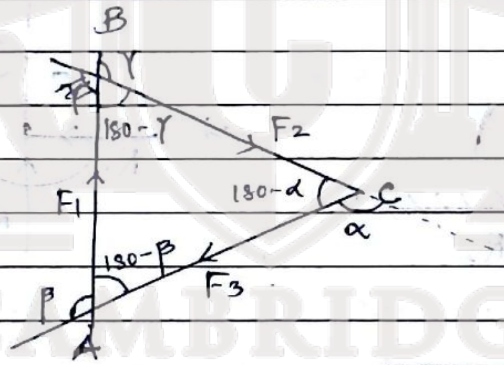
Consider the 3 forces are in equilibrium. Then by converse law of Δ, the forces can be represented by sines of the Δ by both magnitude and direction.

By the Δ law, Δ ABC can be drawn with the sides ∥ to the forces F_1, F_2 and F_3





By applying the sine rule to the Δ ABC, we have

$F_1 \propto \sin(180 - \alpha)$

$F_2 \propto \sin(180 - \beta)$

$F_3 \propto \sin(180 - \gamma)$

$F_1 = k\sin(180 - \alpha) = k\sin\alpha$

$F_2 = k\sin(180 - \beta) = k\sin\beta$

$F_3 = k\sin(180 - \gamma) = k\sin\gamma$

} We know that $\sin(180 - \theta) = \sin\theta$

$$\therefore \frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma} = k$$

Hence Proved

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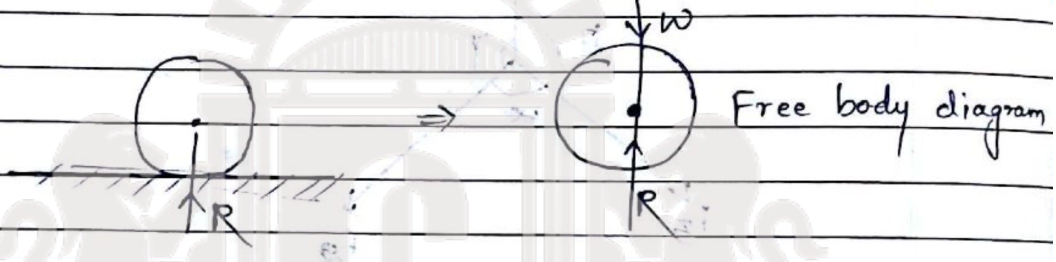
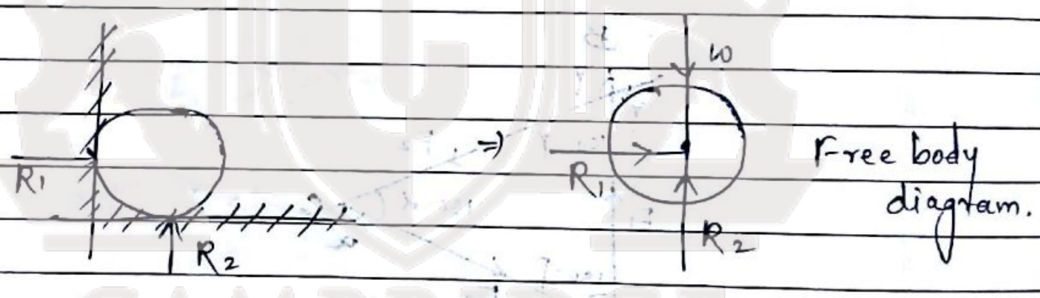
- Define free body diagram with an example :-

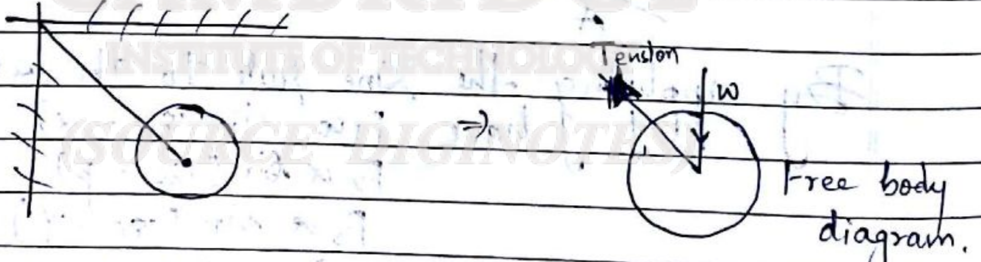
A free body diagram is nothing but a sketch shows the various forces acting on a body keeping in equilibrium by removing the contact surfaces.

The forces include self weight, reaction and other forces :-

Eg :-





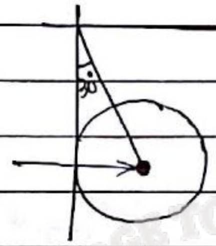


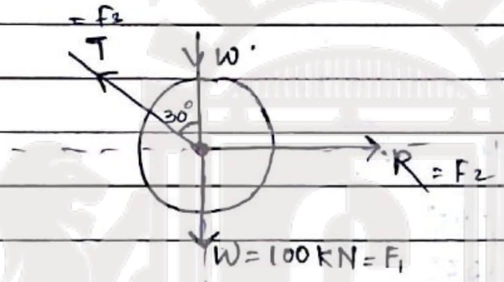
NOTE: Strings, cable, wires are always subjected to tension only, the forces act outward.

Spheres are subjected to inward reaction.

Q.1- A sphere is touching to the wall of weight 100 kN. Find the tension in the wire which is supporting the sphere.

Ans-





Using Lami's Theorem :- $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$

$\Rightarrow \frac{100}{\sin 120} = \frac{R}{\sin 150} = \frac{T}{\sin 90°}$

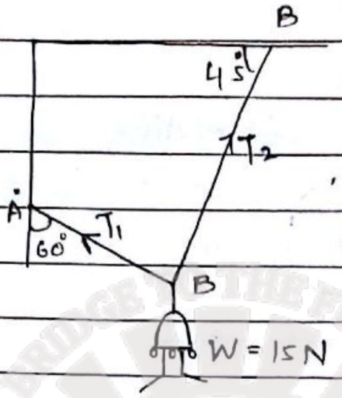
$\Rightarrow \frac{T}{1} = \frac{100}{\sin 120} = 115.47$ KN //

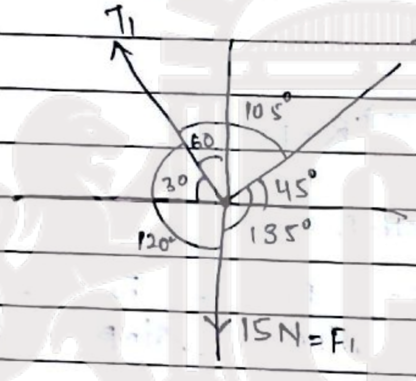
$R = \frac{100 \sin 150}{\sin 120} = \frac{\cancel{50}}{\cancel{57.73}}$ = ~~0.866 kN~~ //

= 57.73 KN //

Q.2- The figure shows, the lamp is fixed with strings. Determine the tension in strings.

Ans-





∴ Using Lami's Rule :-

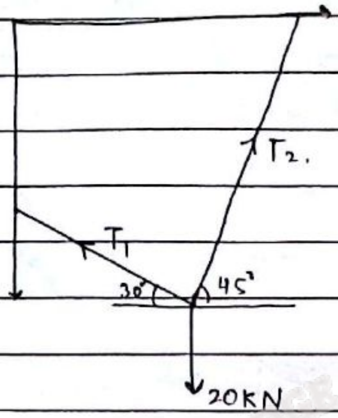
$$\frac{F_1}{\sin 105} = \frac{T_2}{\sin 120} = \frac{T_1}{\sin 135}$$

$$\Rightarrow \frac{15}{0.96} = \frac{T_1}{\sin 135}$$

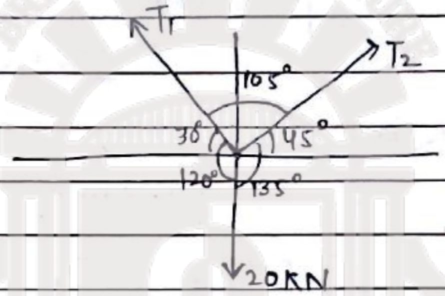
$$\Rightarrow T_1 = \frac{15 \sin 135}{\sin 105} = 10.98\text{ N}$$

$$\Rightarrow T_2 = \frac{F_1 \sin 120}{\sin 105} = \frac{15 \sin 120}{\sin 105} = 13.44\text{ N}$$

Q.3



Ans.



Using Lami's Rule :-

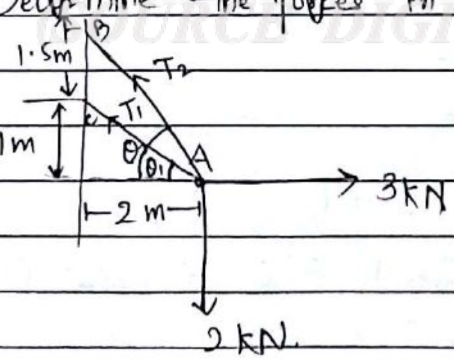
$$\Rightarrow \frac{20}{\sin 105} = \frac{T_1}{\sin 135^\circ} = \frac{T_2}{\sin 120}$$

$\Rightarrow T_1 = 14.64\ kN.$

$\Rightarrow T_2 = 17.93\ kN$ //

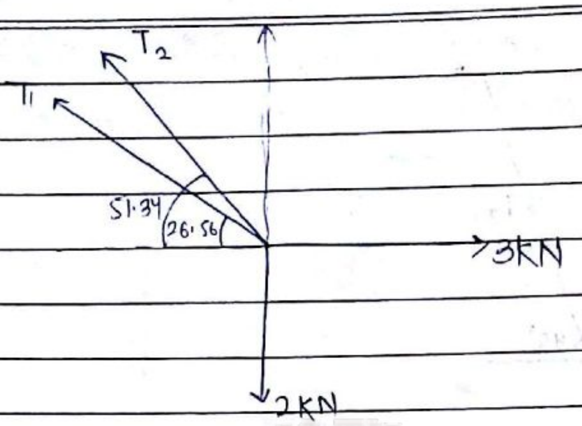
Q.4- The system of strings and forces are in equilibrium. Determine the forces in the string.

Ans-



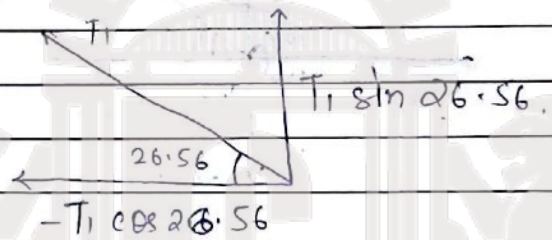
$$\theta_1 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56$$

$$\theta_2 = \tan^{-1}\left(\frac{2.5}{2}\right) = 51.34$$



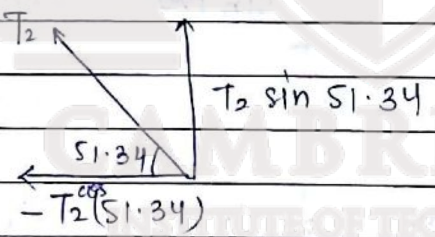
As this is in equilibrium $\Sigma H = 0$

$\Sigma V = 0$



$H = -T_1(0.86)$

$V = T_1(0.44)$



$H = -T_2(0.62)$

$V = T_2(0.78)$

$\Sigma H = -0.86T_1 - T_2(0.62) + 3$

$0.86T_1 + 0.62T_2 + 2 = 0$ — (1)

$0.44T_1 + T_2(0.78) = 0$ — (2)

$\Sigma H = 0.86T_1 + 0.62T_2 + 3.$

or $0.86T_1 + 0.62T_2 = +3$ —— (1)

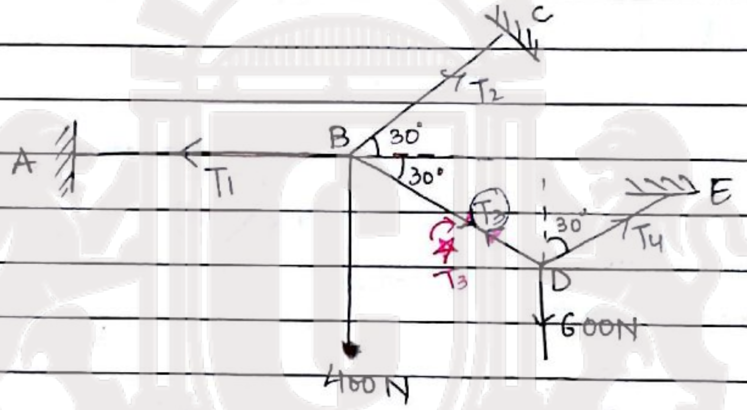
$0.44T_1 + 0.78T_2 = 2$ —— (2)

$T_1 = 2.76\,N$

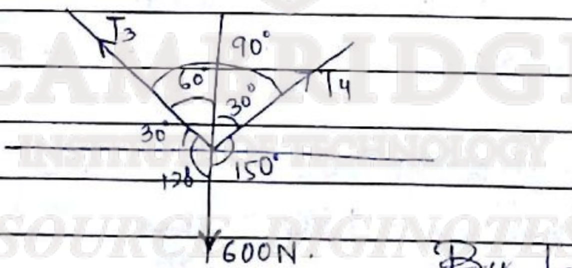
$T_2 = 1.00\,N.$

Q.5 (Imp) Determine the tension in diff parts of the strings as shown in figure:-

Ans-



FBD at D:-



By Lami's Theorem:-

$$\frac{T_3}{\sin 150^\circ} = \frac{T_4}{\sin 120^\circ} = \frac{600}{\sin 90}$$

$\Rightarrow T_3 = \frac{600 \times \sin 150}{1} = 300\,N.$

$T_4 = 600 \times \sin 120 = 519.61\,N.$

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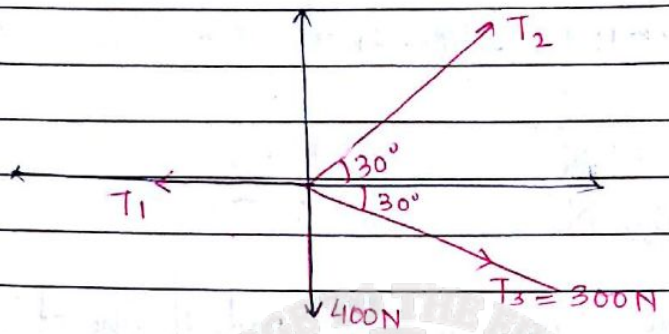
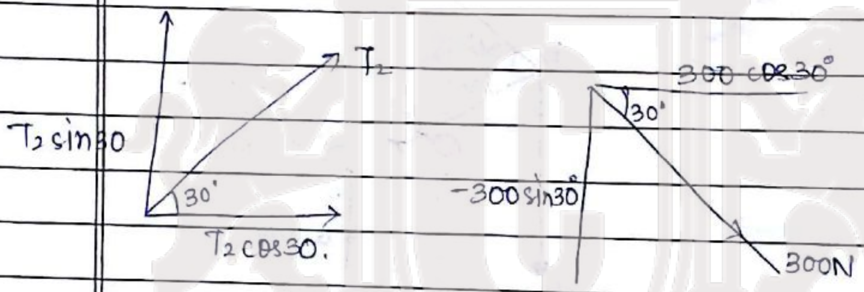
FBD at B :-



$\Sigma H = 0$

$\Sigma V = 0.$



$\Sigma H. = -T_1 + T_2 \cos 30° + 300 \cos 30°$

$\Rightarrow 0 = -T_1 + T_2 \cos 30° + 259.8$

$\Rightarrow T_1 - T_2 \cos 30 = 259.8$ ——— (1)

$\Sigma V = T_2 \sin 30 - 300 \sin 30 - 400$

$\Rightarrow 0 = T_2 \sin 30 - 150 - 400$

$\Rightarrow T_2 \sin 30 = 550$ ——— (2)

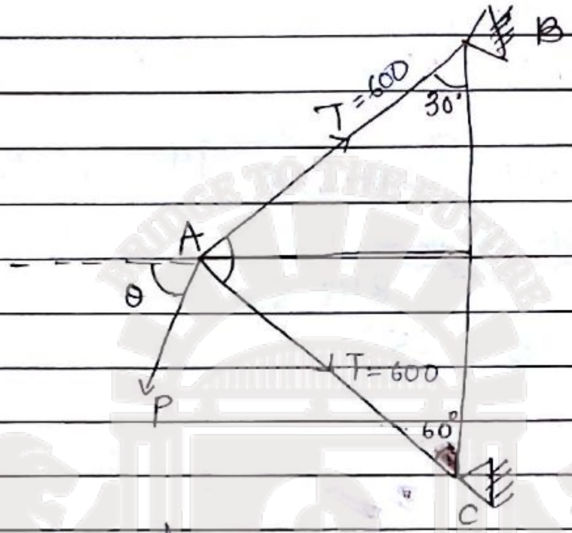
$T_2 =$ ~~519.6~~ $\frac{550}{0.5} = 1100$ //

$T_1 - 1100 \cos 30 = 259.8$

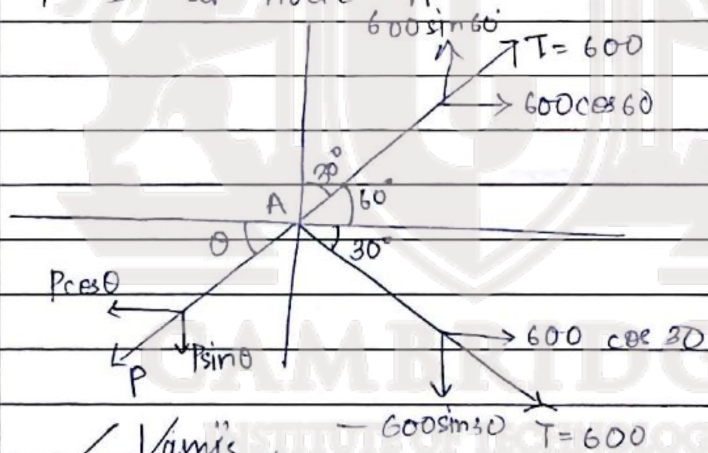
$T_1 = 259.8 + 1212.42 = 1472.2N$ //

Q.- A force P is applied on a ... The tension in strings is equal to 600 N. Determine the magnitude and direction of P

Ans -



FBD at node A



By ~~snell's~~ Lami's Law :-

$$\Sigma H = 0$$

$$\Sigma V = 0.$$

$\Rightarrow \Sigma H = 0$

$\Rightarrow 600 \cos 60 + 600 \cos 30 - P\cos\theta = 0.$

$\Rightarrow 300 + 519.61 - P\cos\theta = 0.$

$\Rightarrow 819.615 - P\cos\theta = 0$

$\Rightarrow P\cos\theta = 819.615$

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$\Sigma V = 0.$

$600 \sin 60 - 600 \sin 30 - P\sin\theta = 0$

$\Rightarrow P\sin\theta = 519.615 - 300 = 219.615.$

$\Rightarrow P^2(1) = 719999.49$

$\Rightarrow P = 848.52\ N$ //

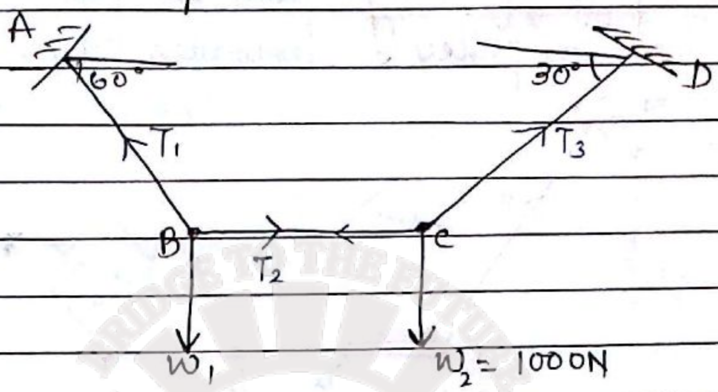
~~θ =~~ $P\sin\theta = 219.615$

$\sin\theta = \frac{219.615}{848.52} = 0.25$

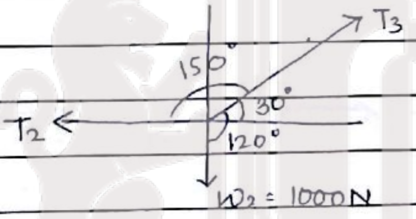
$\theta = 15^\circ$ //

Q- Find the forces in all the bias and the load w_1 to keep the system in equilibrium as shown :-

Ans.



Two nodes :-
FBD at node C :-

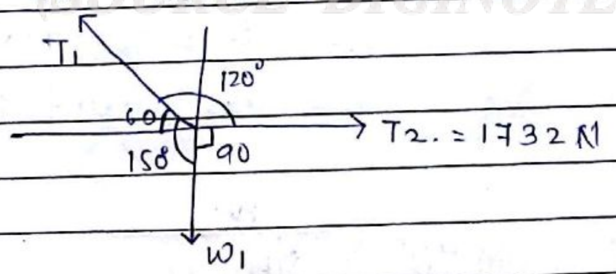


$$\frac{1000}{\sin 150} = \frac{T_2}{\sin 120} = \frac{T_3}{\sin 90^\circ}$$

$\Rightarrow$ $T_2 = \dfrac{1000 \sin 120}{\sin 150} = 1732.0 \text{ N}$

$\Rightarrow$ $T_3 = \dfrac{1000 \sin 90}{\sin 150} = 2000 \text{ N}.$

FBD at node B :-



$T_1 = 3464N$
$T_2 = 1732N$
$T_3 = 2000N$
$w_1 = 2999.91N$

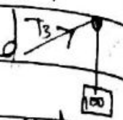
By lami's theorem

$$\frac{1732}{\sin 150^\circ} = \frac{T_1}{\sin 90} = \frac{W_1}{\sin 120^\circ}$$

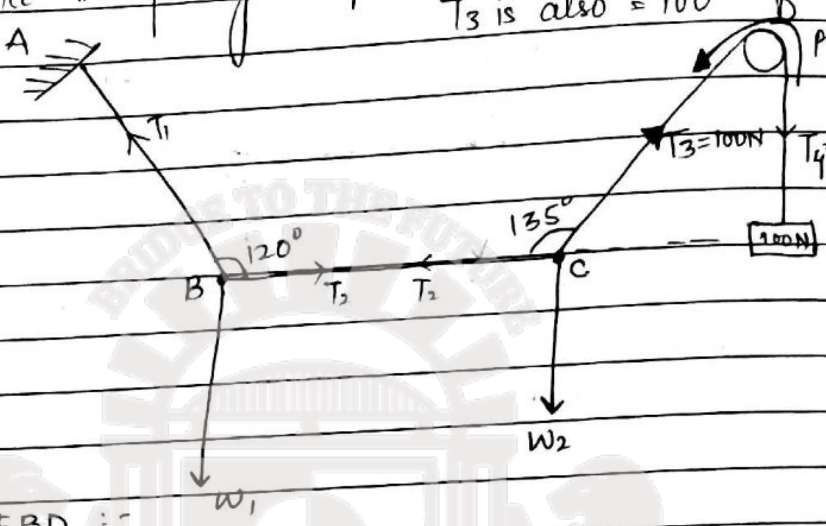
$\Rightarrow$ $w_1 = 2999.91 \text{ N}.$

$\Rightarrow$ $T_1 = 3464 \text{ N}$

Q:- (Imp) In the figure the BC position is horizontal and the pully is frictionless. Determine the tension in all parts of strings and also find w_1 and w_2.

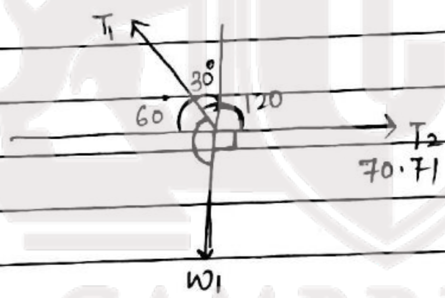


Ans - Since the pulley is frictionless $T_4 = 100$ and T_3 is also $= 100$

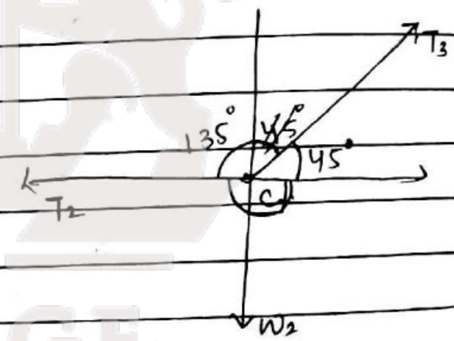


2 FBD :-

FBD at node B :-



FBD at node C :-



$$\frac{100}{\sin 90} = \frac{T_2}{\sin 135} = \frac{W_2}{\sin 135}$$

$T_2 = 70.71N$

$W_2 = 70.71N$ //

$$\frac{70.71}{\sin 150^\circ} = \frac{T_1}{\sin 90^\circ} = \frac{W_1}{\sin 120}$$

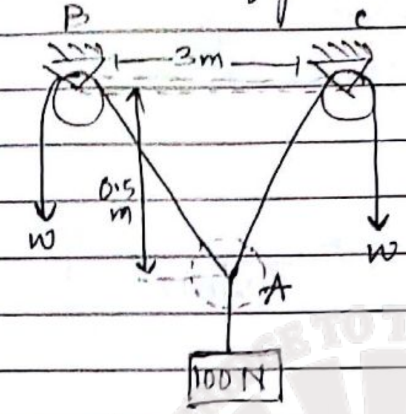
$\Rightarrow T_1 = 141.42. N$

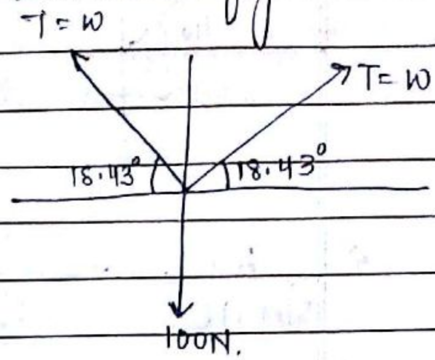
$\Rightarrow W_1 = 122.47 N.$

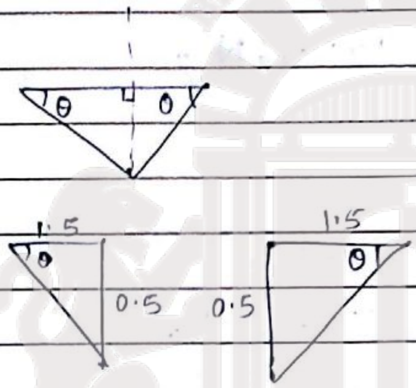
$W_2 = 70.71 N$
$W_1 = 70.71 N$
$T_1 = 141.42 N$
$W_1 = 122.47N$

Q. Find the value of W which is reqd. to maintain equilibrium configuration as shown in fig :-

Ans.





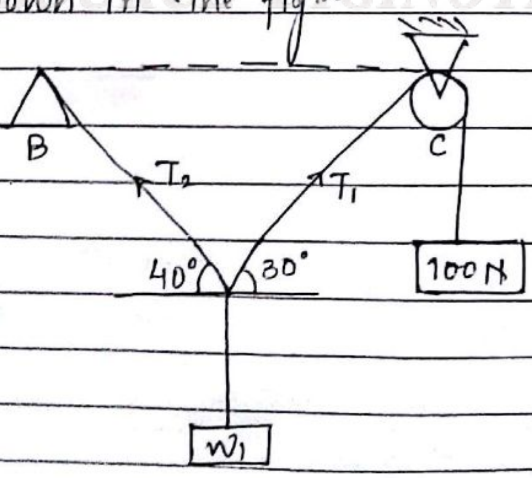


$\theta = \tan^{-1}\left(\frac{0.5}{1.5}\right) = 18.43^\circ$ //

By Lami's theorem :- $\frac{100}{\sin(143.4)} = \frac{W}{\sin(108.43)} = \frac{W}{\sin(108.43)}$

$W = 158.15\ N$ //

Q. Find the value of W_1 for the equilibrium condition as shown in the fig:-



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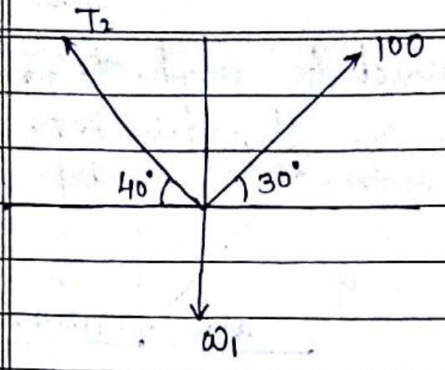
Ans-



$\Rightarrow \frac{W_1}{\sin 110} = \frac{100}{\sin 130} = \frac{T_2}{\sin 120}$

$\Rightarrow W_1 = \frac{100 \sin 110}{\sin 130}$

$\Rightarrow W_1 = \frac{93.96}{0.766} = 122.66\,N$ //

$\frac{100}{\sin 130} = \frac{T_2}{\sin 120}$

$\Rightarrow T_2 = \frac{100 \sin 120}{\sin 130} = 113.05\,N$ //

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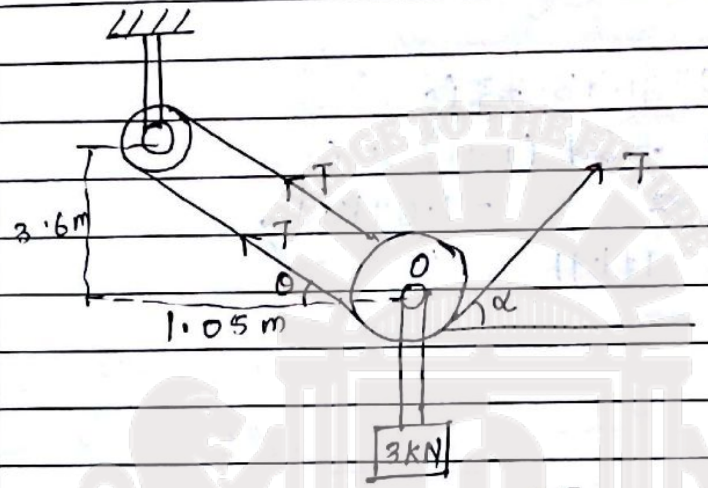
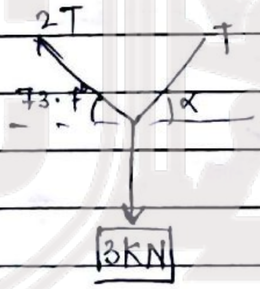
Q.9- A 3KN crate is to be supported by the rope and pulley arrangement shown in figure. Determine the magnitude and direction of the force T, which should be exerted at the free end of the rope.



FBD at node O :-

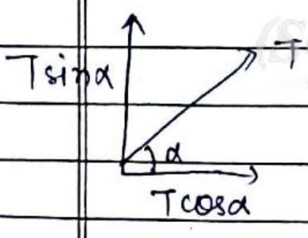


$\tan\theta = \frac{3.6}{1.05}$

$\theta = 73.7°$

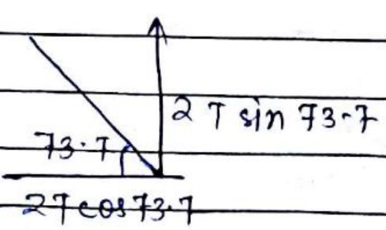
$\Sigma H = 0$

$\Sigma V = 0$



$H = T\cos\alpha$

$V = T\sin\alpha.$



$H = -2T\cos 73.7$

$V = 2T\sin 73.7$

$\Sigma H = 0.$

$\Rightarrow T\cos\alpha - 2T\cos 73.7 = 0$

$\Rightarrow \cancel{T}\cos\alpha = 2\cancel{T}\cos 73.7$

$\Rightarrow \alpha = \cos^{-1}(2\cos 73.7)$

$\Rightarrow \alpha = \cos^{-1}(0.56) = 55.9°$ //

$\Rightarrow \Sigma V = 0$

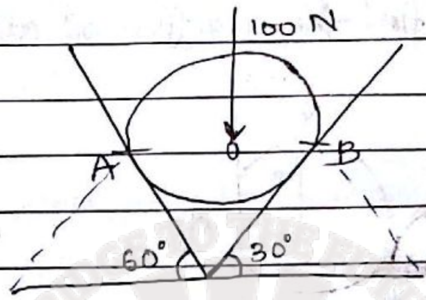
$T\sin\alpha + 27\sin 73.7 - 3 = 0$

$\Rightarrow T\sin 55.9 + 27(1.91) - 3 = 0$

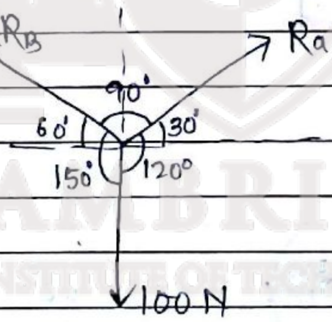
$\Rightarrow T = \dfrac{3}{\sin 55.9 + 1.91} = 1.09\,kN$ //

Q. A sphere of 100 N is fitted in a right angle ~~launch~~ notch as shown in fig.
Determine the reaction at A and B.

Ans-



FB at node O :-



By Lamis Theorem :-

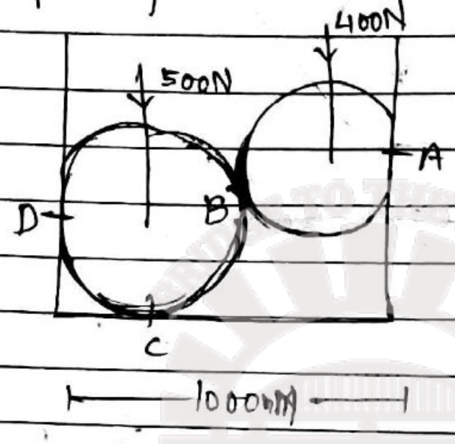
$$\frac{100}{\sin(90)} = \frac{R_a}{\sin 150} = \frac{R_B}{\sin 120}$$

$$\Rightarrow R_a = \frac{100 \sin 150}{\sin 90} = 50\ N$$

$$\Rightarrow R_B = \frac{100 \sin 120}{\sin 90} = 86.60\ N$$

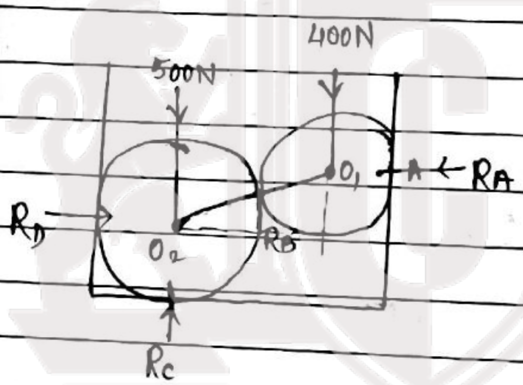
Q. (Imp) A horizontal channel of ~~interference~~ inner clearance of 1000 mm carries 2 spheres of radius 350 mm & 250 mm whose weight are 500 N and 400N respectively. Find the reaction at all the contact surfaces.

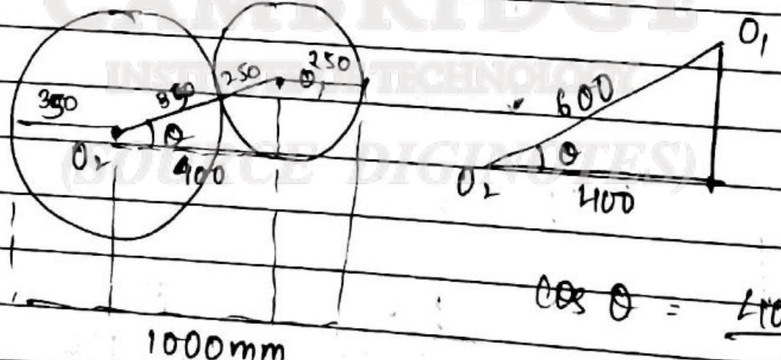
Ans-



500N → 350 mm

400N → 250 mm.



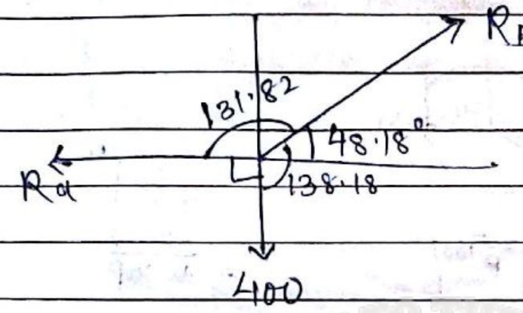


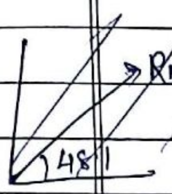
$$\cos\theta = \frac{400}{600}$$

$$\theta = \cos^{-1}\left(\frac{400}{600}\right)$$

$$= 48.18^{\circ}$$

FBD @ O_1 (min forces)





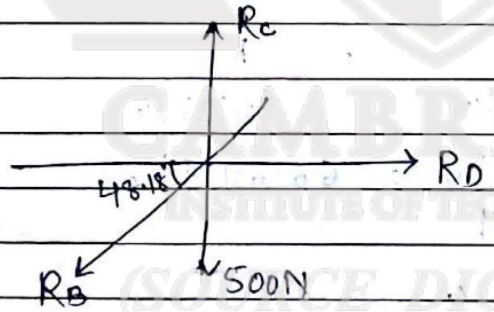
H = By Applying Lami's Theorem.

$$\frac{R_B}{\sin 90} = \frac{R_A}{\sin 138.18} = \frac{400}{\sin 131.82}$$

$\Rightarrow R_A = \frac{400 \sin 138.18}{\sin 131.82} = 357 N$ //

$\Rightarrow R_B = \frac{400 \sin 90}{\sin 131.82} = 536.73 N$ //

FBD at O_2



= 53673.

$\sum H = 0$

$\sum V = 0.$

$\sum H = 0$

$\Rightarrow R_D - 536.73 \cos 48.18 = 0$

$\Rightarrow R_D = 357.88 N.$

$\sum V = 0$

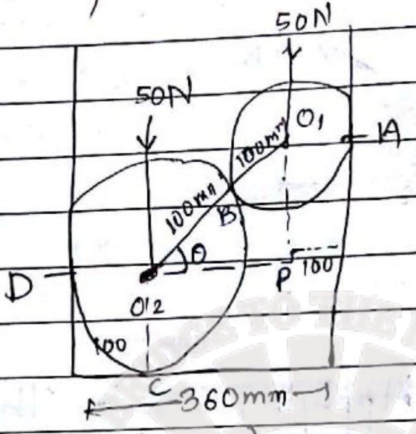
$\Rightarrow -500 + R_c - 536.73 \sin 48.18 = 0$

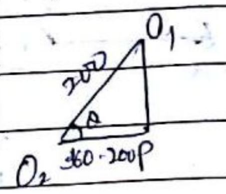
$\Rightarrow R_c = 500 + 536.73 \sin 48.18$

$\Rightarrow R_c = 899.99 N$ //

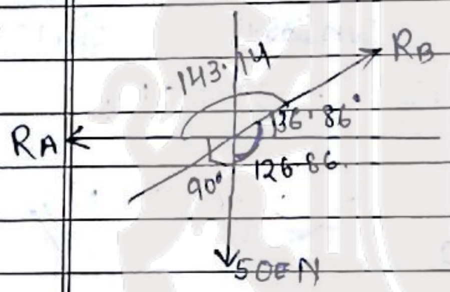
8/4/17

Q- Find the reactions at A, B, C as shown in figure.





$$\theta = \cos^{-1}\left(\frac{160}{200}\right) = 36.86^\circ$$



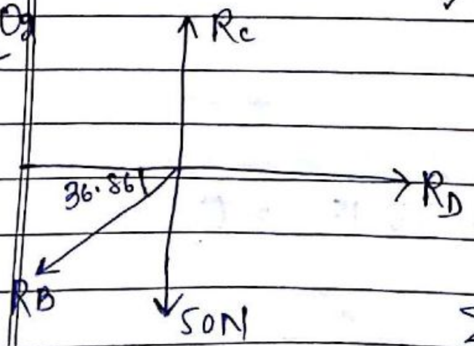
Lami's theorem

$$\frac{R_B}{\sin 90} = \frac{50}{\sin 143.14} = \frac{R_A}{\sin 126.86}$$

$\Rightarrow R_A = \dfrac{50 \sin 126.86}{\sin 143.14} = 66.69\ N$

$\Rightarrow R_B = 83.35\ N$

At O_2



$\Sigma H = 0$

$\Rightarrow R_D - 83.35 \cos 36.86 = 0$

$\Rightarrow R_D = 66.68\ N$

$\Sigma V = 0.$

$\Rightarrow R_C - 50 - 83.35 \sin 36.86 = 0$

$\Rightarrow R_C = 50 + 83.35 \sin 36.86 = 99.99\ N$

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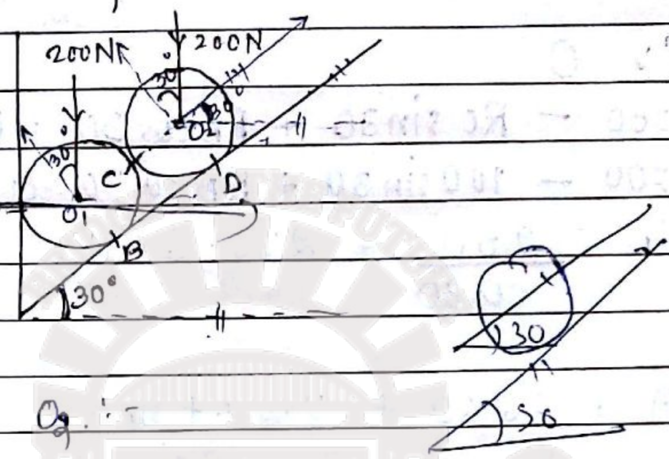
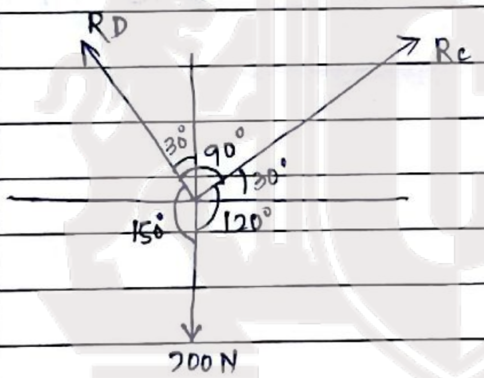
Q. Two identical rollers each of weight 200 N are placed in a notch as shown in figure with all contact surfaces as smooth. Determine the reaction developed at all the contact surfaces.

Ans-



FBD at O_2 :-



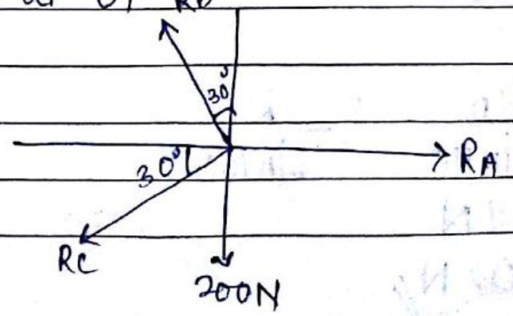
By Lami's Theorem:-

$$\frac{200}{\sin 90} = \frac{R_D}{\sin 120} = \frac{R_C}{\sin 150}$$

$\Rightarrow R_C = \frac{200 \sin 150}{\sin 90} = 100\ N$

$\Rightarrow R_D = \frac{200 \sin 120}{\sin 90} = 173.20\ N$

FBD at O_1



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$\Sigma H = 0$

$\Rightarrow RA - Rc \cos 30 - R_B \sin 30 = 0$

$\Rightarrow RA - 100 \cos 30 - R_B \sin 30 = 0$

$\Rightarrow RA - R_B \sin 30 = 86.60$

$\Sigma V = 0$

$\rightarrow -200 - Rc \sin 30 + R_B \cos 30 = 0$

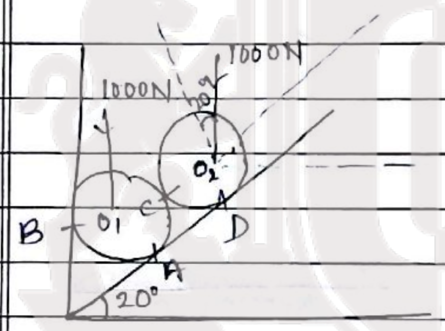
$\Rightarrow -200 - 100 \sin 30 + R_B \cos 30 = 0$

$\Rightarrow R_B = \frac{250}{\cos 30} = 288.67N$

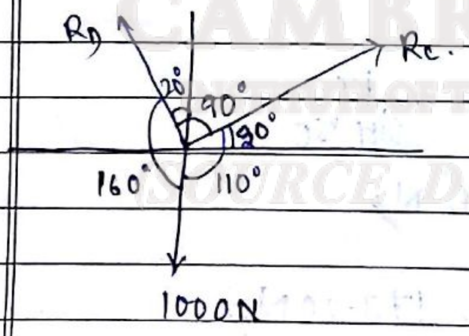
$\rightarrow RA = 86.60 + 288.67 \sin 30$

$= 86.60 + 288.67 \sin 30 = 230.935N$

Q.



FBD at O_2 :-



By lamis theo :-

$$\frac{1000}{\sin 90} = \frac{R_D}{\sin 110} = \frac{R_c}{\sin 160}$$

$RD = 939.69 N$

$Rc = 342.02 N$

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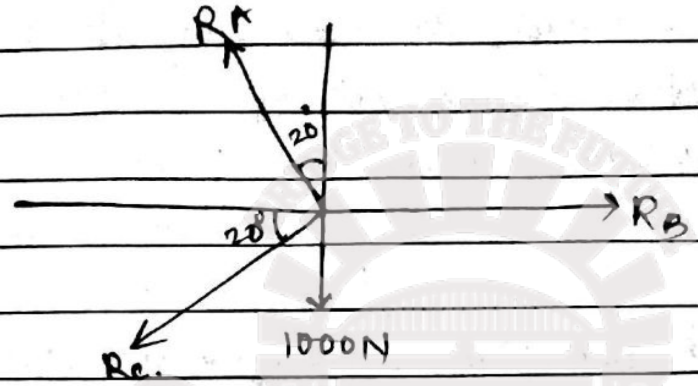
FBD at O_1 :-



$\Sigma V = 0$

$\Rightarrow -1000 - R_A \sin 20 - R_c \cos 20 = 0$

$\Rightarrow -1000 - R_A \sin 20 - 342.02 \cos 20 = 0$

$\Rightarrow R_A \sin 20 = -1321.39$

$\Rightarrow R_A = -3863.49N$

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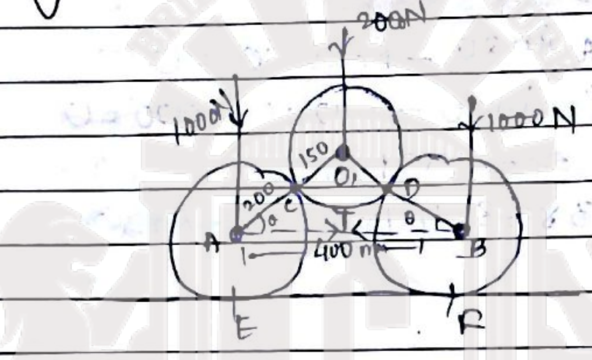
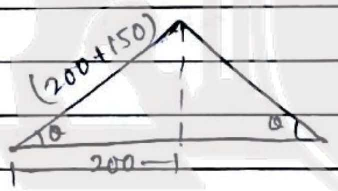
Imp Q - 2 smooth circular cylinder each of weight 1000N and radius 200 mm are ~~collected~~ connected at their centres by the ~~centres~~ string AB of length 400mm and rest upon the horizontal flow. Supporting above them in an another cylinder of wt 2000N and radius 150 mm. Determine the tension in string and reaction developed at contact surfaces

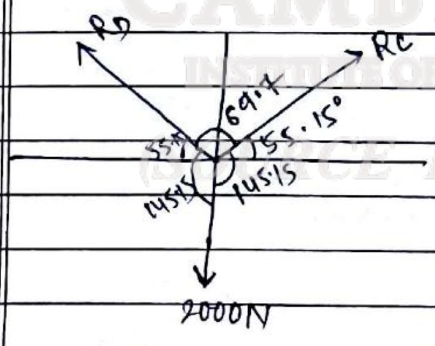
Ans-





$\theta = \cos^{-1}\left(\frac{200}{350}\right) = 55.15^\circ$

FBD at O_1 :-



By Lami's theorem:-

$$\frac{2000}{\sin 69.7} = \frac{R_C}{\sin 145.15} = \frac{R_D}{\sin 145.15}$$

$R_C = 1218.54$ N

$R_D = 1218.54$ N

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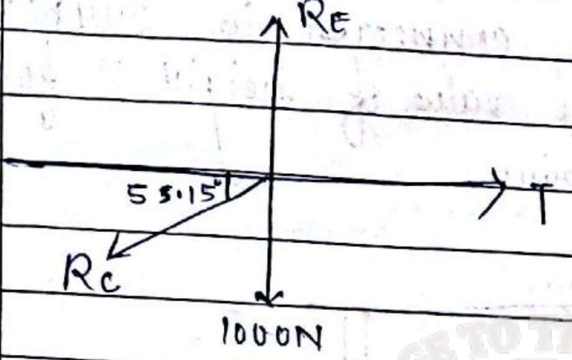
FBD at A :-



$\Sigma H = 0$

$\Rightarrow$ ~~100~~ $T - R_C \cos 55.15 = 0$.

$\Rightarrow T = 1218.54 \cos 55.15 = 696.31\ N$ //.

$\Sigma V = 0$.

$\Rightarrow -1000 + R_E - R_C \sin 55.15 = 0$

$\Rightarrow R_E = 1000 + 1218.54 \sin 55.15 = 0$

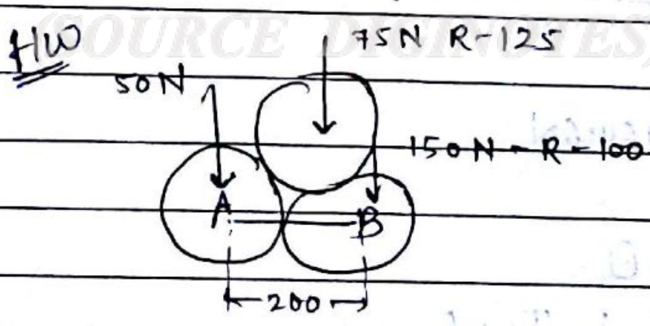
$= 1999.99\ N$ //.

$\therefore R_E = R_F = 1999.99\ N$.

$R_C = R_D = 1218.54\ N$.

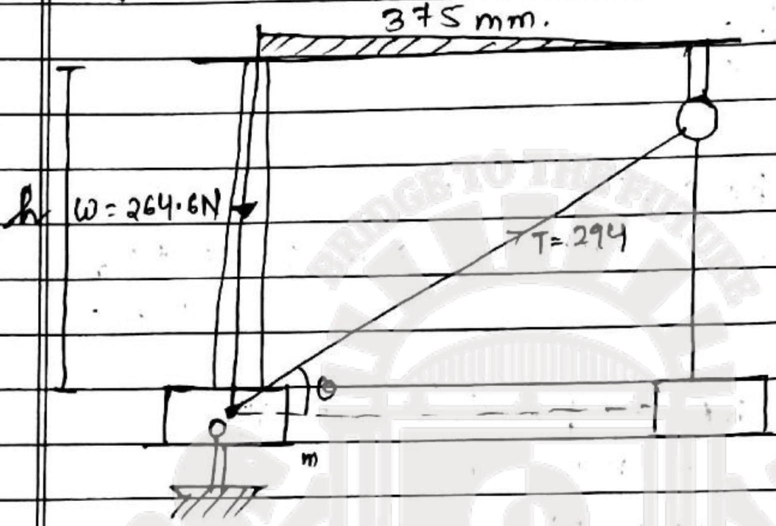
$T = 696.31\ N$ //.

HW

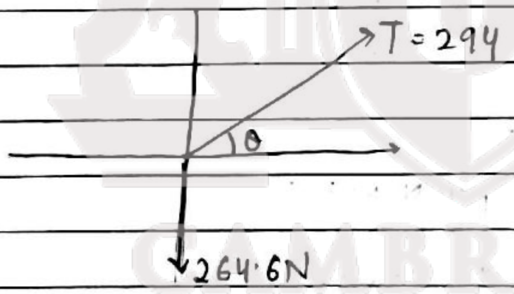


(crane)

Q- The collor of weight 264.4 N slides on a frictionless vertical rod and it's connected to 294N countor weight. Determine the value of height H for which system is in equilibrium.



FBD at O :-

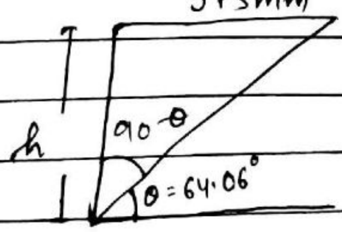


$\Sigma V = 0$

$\Rightarrow -264.6 + T\sin\theta = 0$

$\Rightarrow -264.6 + 294\sin\theta = 0.$

$\Rightarrow \theta = \sin^{-1}\left(\frac{264.6}{294}\right) = 64.06°$ //



$90 - \theta = 90 - 64.06 = 25.94$ //

$\tan\theta = \frac{375}{h}$ = ~~375~~

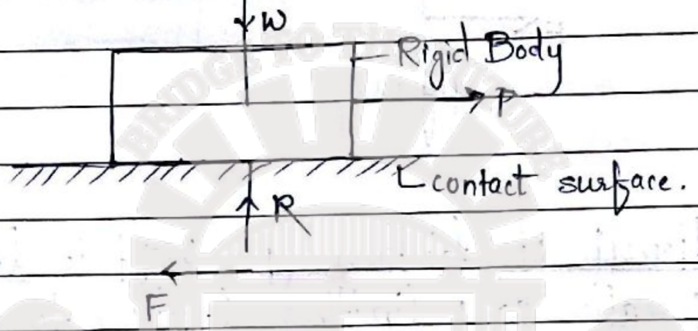
$h = \frac{375}{\tan 25.94} = 774$ m //

FRICTION

10/4/17

Define friction. explain the types of friction with a neat figure.

Ans. When one body tends to move in contact with another body a resistance to its movement is setup. This resistance to the movement is called as friction or force of friction.



where W is the self weight of the body.
R is reaction.
P is applied force.
F is frictional force.

The frictional force always acts opposite to the body in motion.

Types of friction :-

(1) Static friction.
(2) Limiting friction.
(3) Dynamic friction.
(4) fluid friction.
(5) dry friction.

- Static friction :- in rest
- Limiting friction :- about to move.
- dynamic friction :- in motion.
- fluid friction :- when body is in contact with fluid.
- dry friction :- when body is in contact with dry surface.

13/4/17

TM Define coefficient of friction and show that angle of friction is equal to coefficient of friction

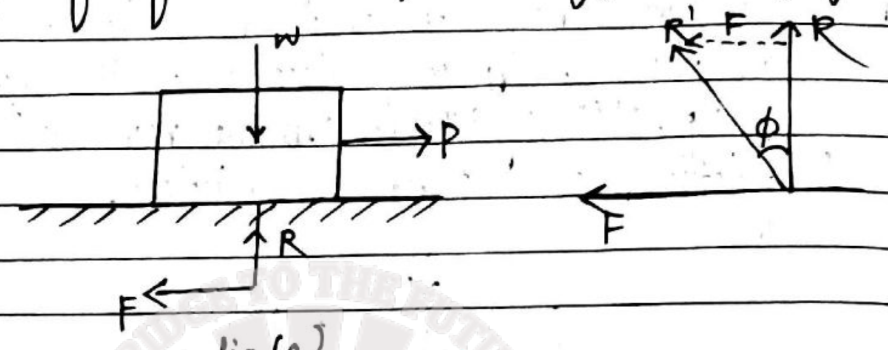


fig (a)

Coefficient of friction (μ) : It is defined as the ratio of limiting friction to the normal reaction b/w 2 surfaces and is given by

$$\mu = \frac{F}{R} \quad \text{--- (1)}$$

Angle of friction (ϕ) : The angle with the resultant reaction R' due to the normal reaction R makes with the normal surface is called as angle of friction ie ϕ.

Let a body of weight w subjected to in the figure(a)

R is the normal reaction which is ⊥ to the frictional force F acting opp to the applied force.

From the figure (b) we can see that the normal reaction R and the frictional force are ⊥ to each other and makes an angle ϕ with R'

ie $\tan\phi = \frac{F}{R}$ --- (2)

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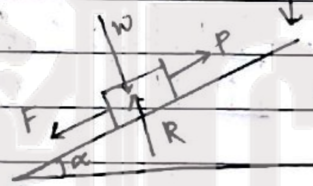
Comparing eq ① and ②

$$\boxed{\tan \phi = \frac{F}{R} = \mu}$$

- **Angle of Repose (α)** :- If a body is placed on a inclined plane then the angle at which body is sliding down is called as angle of repose α. ie

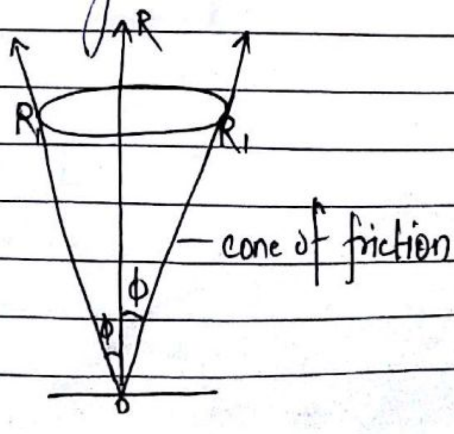
$$\boxed{\phi = \alpha}$$

Imp • **Cone of friction** :-



Whenever a body is in contact with other tend to move, then the normal reaction OR and friction comes into play. The normal reaction & friction can be replaced by resultant reaction OR_1. When this reaction OR_1 making angle ϕ is resolved around point O, will form a right circular cone.

This cone having the point of contact as the vertex O, the normal OR at the point of contact as its axis and ϕ as the semi-vertex angle is called the Cone of friction.



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• EXPLAIN THE LAWS OF DRY FRICTION :-

(1) The force of friction always act in the direction opposite to the body motion.

(2) The magnitude of limiting friction (F) bears a constant ratio R between the two surfaces ie

$$\mu = \frac{F}{R}$$

(3) The magnitude of the force of friction which will be equal to the force applied as long as body is at rest ie $P = F$

(4) The force of friction is independent of the area of contact between the two surfaces.

(5) The force of friction depends upon the roughness of surfaces.

$[\mu < 1]$

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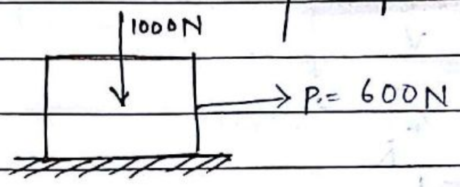
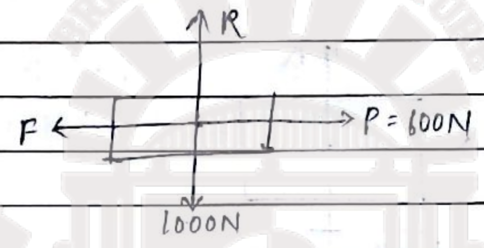
Q.1. A body of weight 1000 N is placed on the rough horizontal plane. Determine the coefficient of friction due to the force of 600 N in horizontal direction.

Ans.



$\mu = \frac{F}{R}$

FBD



$\Sigma H = 0.$

$P = F = 600N.$

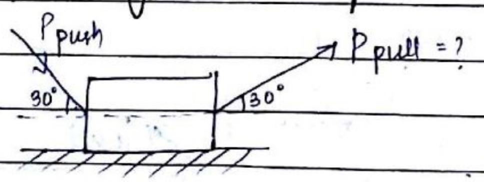
$\Sigma V = 0.$

$R = 1000N.$

$\therefore \mu = \frac{F}{R} = \frac{600}{1000} = 0.6$ // $\boxed{\mu = 0.6}$

Q.2. A block of weight 5kN rest on a horizontal rough surface and the coefficient of friction b/w them is 0.4. Show that the magnitude of force required to pull is less than magnitude of force reqd. to push. If the angle made by both forces pull and push is 30°.

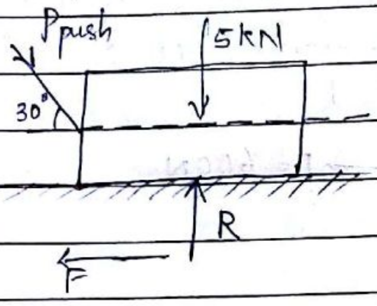
Ans.



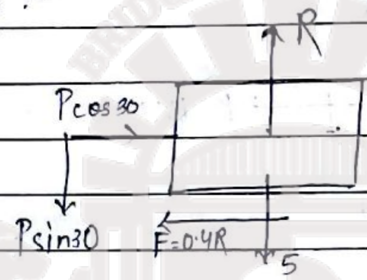
$\mu = 0.4$

$\frac{F}{R} = 0.4$ $\boxed{F = 0.4R}$

Case : I :- P_{push}



FBD :-



$\Sigma H = 0$

$P\cos 30 - 0.4R = 0$ —— ①

$\Sigma V = 0$

$R - 5 - P\sin\theta = 0$

$-P\sin\theta + R = 5$ —— ②

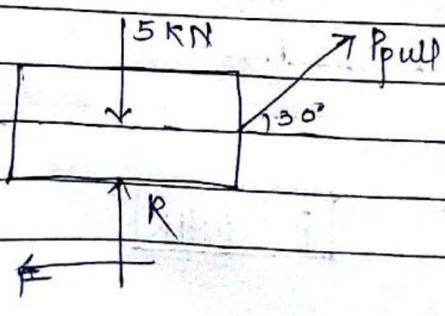
Solving eq ① and ② :-

$P = 3kN$

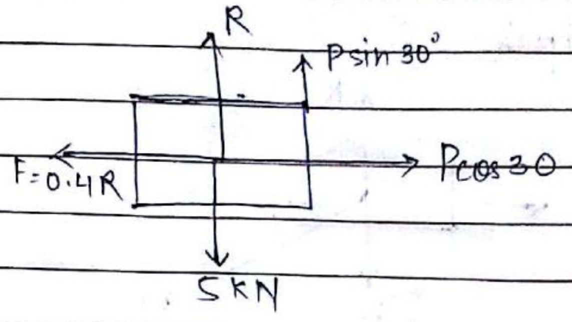
$R = 6.5kN$

$\boxed{P_{push} = 3kN}$

Case II :- $P_{pull} = ?$



FBD :-



$\Sigma H = 0$

$P\cos 30 - 0.4R = 0$ ——— (3).

$\Sigma V = 0$.

$R - 5 + P\sin 30 = 0$

$P\sin\theta + R = 5$ ——— (4).

Solving eq (3) and (4) :-

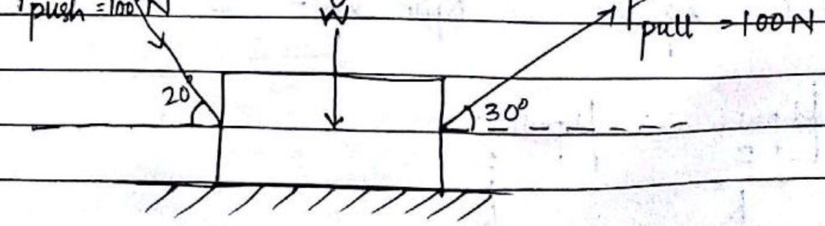
$P = 1.87\ kN$.

$R = 4.06\ kN$.

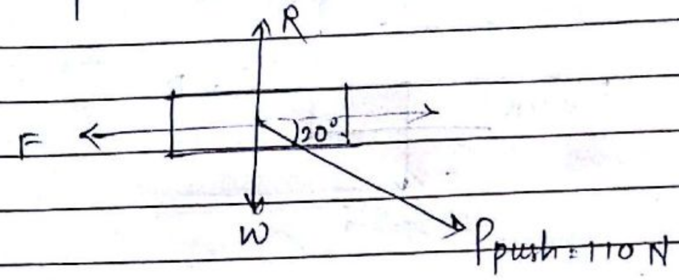
$\therefore \boxed{P_{pull} = 1.87\ kN}$

$\therefore P_{pull} < P_{push}$. Hence Proved.

Q - Imp: A body is resting on a horizontal plane reqd to pull of 100N make an angle of 30° to the horizontal just to move it. It was also found that a push of 110 N inclined at 20° to the plane just to move the body. Determine the weight of the body and also the coefficient of friction.



Ans- FBD :- P_{push}



$\Sigma H = 0.$

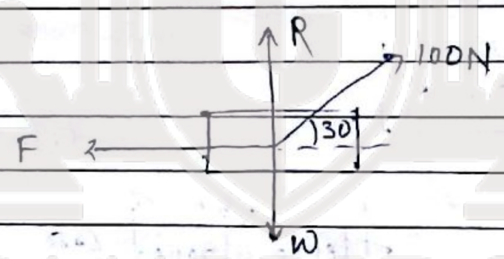
$\Rightarrow -F + 110\cos 20 = 0$

$\Rightarrow F = 103.36\ N$

$\Sigma V = 0.$

$\Rightarrow R - W - 110\sin 20 = 0$ ——— (1)

FBD :- P_{pull}



$\Sigma H = 0$

$-F + 100\cos 30 = 0$

$F = 86.60$

$\Sigma V = 0$

$R - W + 100\sin 30 = 0$ ←—— (2)

From eq (1) :- $R_{push} = W + 37.62$

From eq (2) :- $R_{pull} = W - 50$

$\mu_{push} = \mu_{pull}$

$\frac{F}{R} = \frac{F}{R}$

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$$\Rightarrow \frac{103.36}{W + 37.62} = \frac{86.6}{W - 50}$$

$\Rightarrow 103.36\,W - 5168 = 86.6\,W + 3257.89$

$\Rightarrow 16.76\,W = 8422.89$

$\Rightarrow \boxed{W = 504.36\ N}$ //

$R = 504.36 - 50 = 454.36$

$$\mu = \frac{F}{R} = \frac{86.6}{454.36} = 0.1905 \ //$$

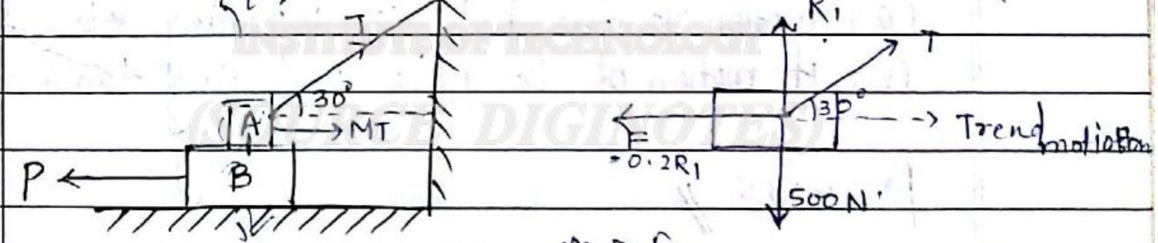
The weight of body is 504.36 N with the coefficient friction of 0.1905.

$$\mu = \frac{F}{W + 37.62} = \frac{103.36}{504.36 + 37.62} = 0.19.$$

Q. Find the force P just reqd. to slide the block B in the arrangement shown in the figure. Find also tension in the string, if the weight of block A is 500 N and weight of the block B is 1000 N. Take μ for all the contact surface as 0.2.

Ans.

FBD at Block A :-



$$m = \frac{F}{R_1}$$

$R_1 = F/\mu$

$F = R_1 \times \mu$

$\Sigma H = 0.$

$T\cos 30 - F = 0$

$T\cos 30 - 0.2R_1 = 0.$ —— (1)

$\Sigma V = 0$

$R_1 - 500 + T\sin 30 = 0$

$T\sin 30 + R_1 = 500$ —— (2)

$T = 103.51\ N.$

$R_1 = 448.24\ N$ //

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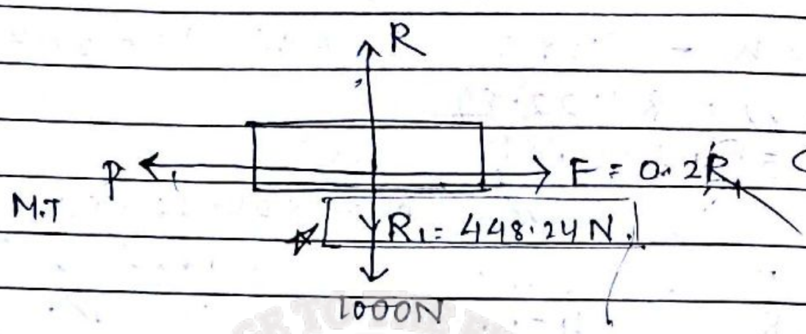
FBD at B :-



$\Sigma H = 0$.

$\Rightarrow -P + F$

$\Rightarrow -P + 0.2R = 0$ —— (1)

$\Sigma V = 0$.

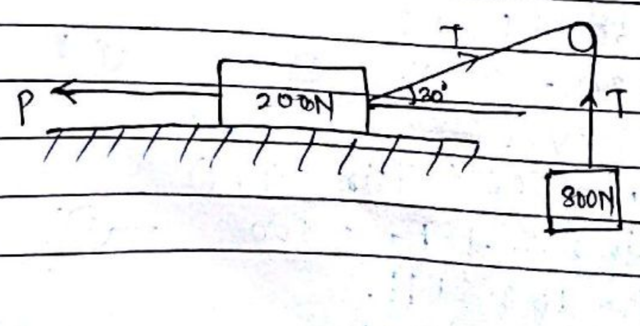
$\Rightarrow R - 448.24 - 1000 = 0$

$\Rightarrow R = 1448.24$ N.

$\Rightarrow P = 0.2R = 289.64$ N //.

Q. A block of wt. 2000N is attached to one end of the cord which passes round of frictionless pulley as shown in fig. Carries a load of 800N at the other end. Find the value of P if
(a) motion of the body is impending towards left.
(b) if motion of body is impending towards right.

$\mu = 0.35$ //



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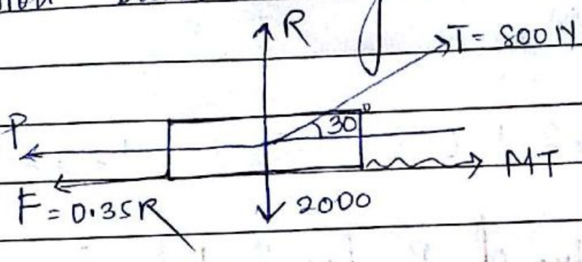
Case (i) :-

Motion trend → Right.



$\therefore \Sigma H = 0$

$-P + 800\cos 30 - 0.35R = 0$

$P + 0.35R = 800\cos 30$

$P + 0.35R = 692.82$ —— (1)

$\Sigma V = 0$

$R + T\sin 30 - 2000 = 0.$

$R = 2000 - 800\sin 30$

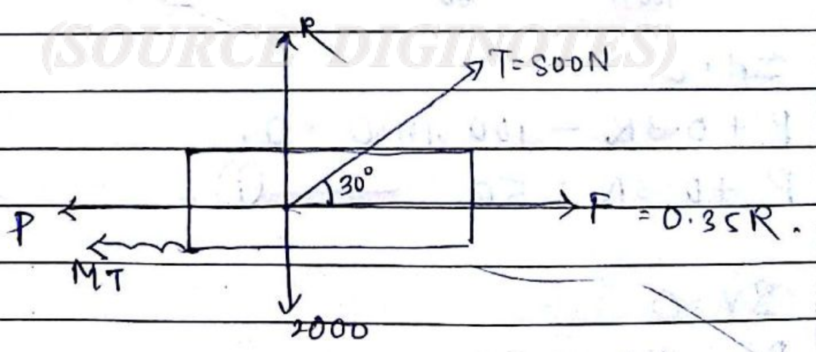
$R = 1600\ N$ //

$P = 692.82 - 0.35(1600)$

$= 692.82 - 560$

$= 132.82\ N$ //

Case (ii) :- Motion Trend → Left.



$\Sigma H = 0$

$-P + 0.35R + 800\cos 30 = 0$

$P - 0.35R = 692.82$ —— (1).

$\Sigma V = 0$

$R + 800 \sin 30 - 2000 = 0.$

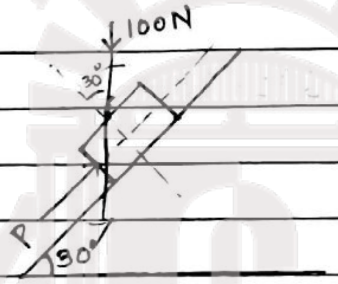
$R = 1600\ N$

$P = 692.82 + 0.35(1600) = 1252.82\ N$

Q - Find the value of P for the body impending down the plane and also find P for the body impending up the plane.

$\mu = 0.3$

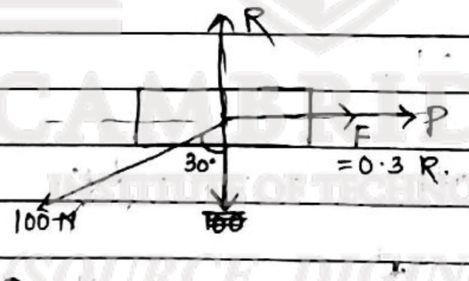
Ans-



Case (i) impending down :-

$\mu = 0.3$

$F = 0.3R$



$\Sigma H = 0.$

$\Rightarrow P + 0.3R - 100 \sin 30 = 0.$

$\Rightarrow P + 0.3R = 50$ —— ①.

$\Sigma V = 0$

$\Rightarrow R - 100 \cos 30 =$

$\Rightarrow R = 86.60\ N$

$\Rightarrow P = 50 - 0.3(86.60) = 24.02\ N$

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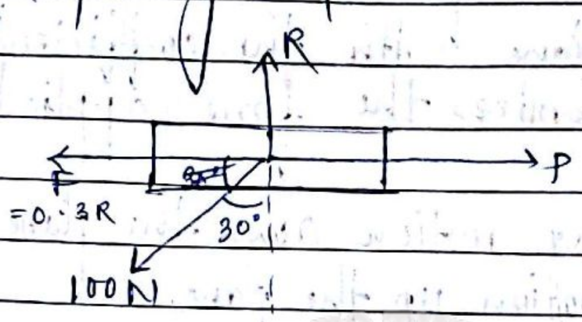
Case (ii) impending upwards :-



$\Sigma H = 0$

$\Rightarrow P - 0.3R - 100 \sin 30 = 0 \Rightarrow P = 0.3R + 50$.

$\Rightarrow$ ~~$P - 0.3R = 86.60 N$ — (1)~~

$\Sigma V = 0$

$\Rightarrow R - 100 \cos 30 = 0$

$\Rightarrow R =$ ~~$50 N$~~ 86.60

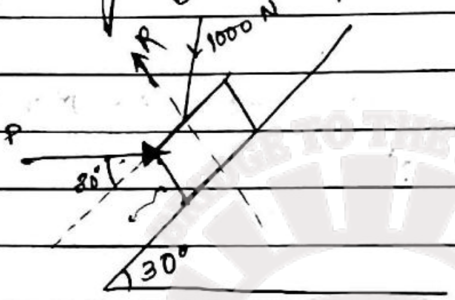
$\Rightarrow P =$ ~~86.60~~ $50 + 0.3$ ~~(50)~~ (86.60)

~~$= 86.60 + 15$~~ $25.98 + 50 = 75.98 N$

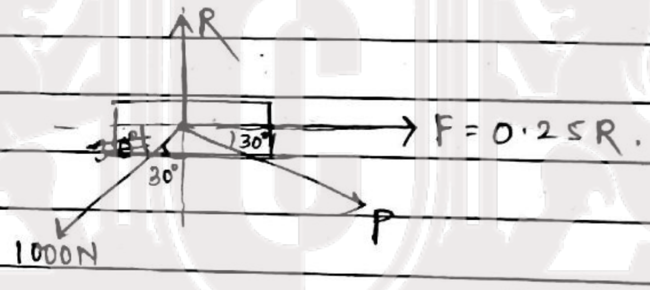
~~$= 101.6 N$~~

Q. A small block of 1000 N is placed on a 30° inclined plane with the coefficient friction of 0.25. Determine the force applied for.

(a) an impending motion down the plane.
(b) impending motion up the plane.



(a) Case (i) :-



$\Sigma H = 0.25R - 1000 \sin 30 + P\cos 30°$.

$0 \Rightarrow 0.25R + P\cos 30 - 500$

$\Rightarrow P\cos 30 + 0.25R = 500$ —— (1).

$\Sigma V = 0$

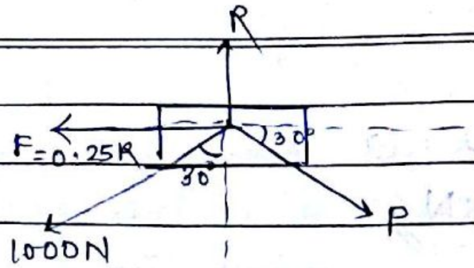
$\Rightarrow R - P\sin 30 - 1000\cos 30 = 0$

$\Rightarrow P\sin 30 - R = -866.02$ —— (2)

$P = 286.06$ N

$R = 1009.05$N

Case (ii)



$\Sigma H = -0.25R - 1000\sin 30 + P\cos 30°$

$0 = P\cos 30 - 0.25R - 500$

$P\cos 30 - 0.25R = 500$ —— (1).

$\Sigma V = R - 1000\cos 30 - P\sin 30$

$0 = -P\sin 30 + R - 866.06$

$\Rightarrow P\sin 30 + R = -866.06$ —— (2)

$P = 966.92N$

$R = 1349.52N$.

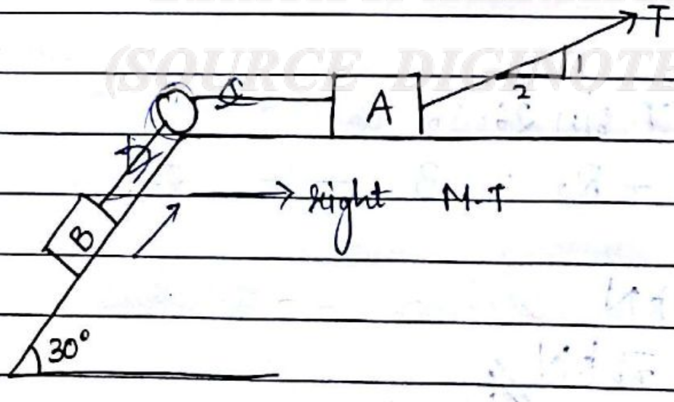
Imp

Q- 2 blocks A and B 3kN and 1.3kN respt. are connected by a string over a frictionless pulley as shown in the figure:-

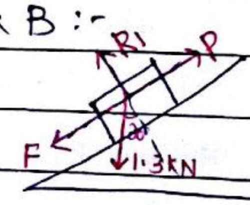
Find the min. value of T to generate an impending motion towards right.

Take $\mu = 0.2$ for block A and $\mu = 0.3$ for block B

Ans-



FBD of block B :-



$F = 0.3R_1$

$\Sigma V = 0$

$\Rightarrow R_1 - 1.3\cos 30 = 0$

$R_1 = 1.125\ kN$

$\Sigma H = 0$

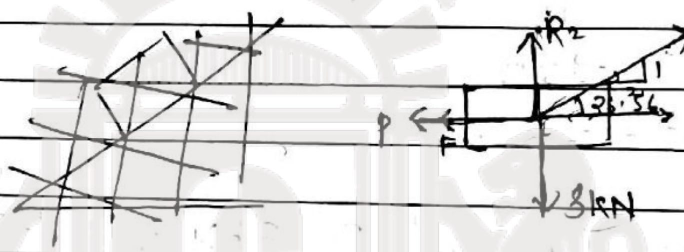
$\Rightarrow P - F - 1.3\sin 30 = 0$

$\Rightarrow P - 0.3R_1 = 1.3\sin 30$

$\Rightarrow P = 1.3\sin 30 + 0.3 \times 1.125$

$\Rightarrow P = 0.9875\ kN$

FBD at block A:



$\tan^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow 26.56$

$\Sigma H = 0$

$\Rightarrow T\cos 26.56 - P - 0.2R_2 = 0$

$\Rightarrow T\cos 26.56 - 0.9875 - 0.2R_2 = 0$

$\Rightarrow T\cos 26.56 - 0.2R_2 = 0.9875$ —— (1)

$\Sigma V = 0$

$\Rightarrow R_2 - 3 + T\sin 26.56 = 0$

$\Rightarrow T\sin 26.56 + R_2 = 3$ —— (2)

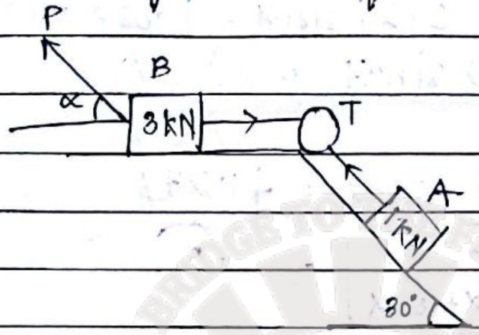
$T = 1.612\ kN$

$R_2 = 2.278\ kN$

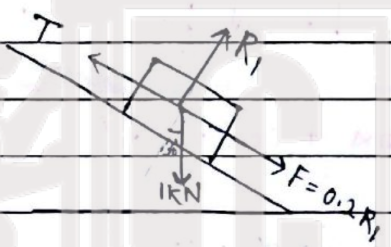
Imp

Q- Find the least value of P reqd. to ~~form~~ cause the ~~system~~ block as shown in figure to have impending motion to the left, $\mu = 0.2$ for all the contact surface.

Ans-



FBD at A



$\Sigma V = 0$

$\Sigma V = R_1 - 1\cos 30$

$0 = R_1 - \cos 30$

$R_1 = 0.866$ kN

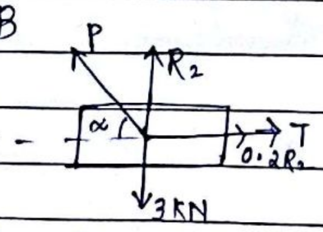
$\Sigma H = 0$

$\Sigma H = -T + 0.2R_1 + 1\sin 30$

$0 = -T + 0.2 \times 0.866 + 0.5$

$\Rightarrow T = 0.6732$ kN

FBD at B



$\Sigma V = R_2 - 3 + P\sin\alpha = 0$

$\Rightarrow R_2 + P\sin\alpha = 3$ —— (1)

$\Rightarrow R_2 = 3 - P\sin\alpha$

$\Sigma H = 0$

$0.673 + 0.2R_2 - P\cos\alpha = 0$ —— (2)

$\Rightarrow 0.673 + 0.2(3 - P\sin\alpha) - P\cos\alpha = 0$

$\Rightarrow 0.673 + 0.6 - 0.2P\sin\alpha - P\cos\alpha = 0$

$\Rightarrow 1.273 - P(0.2\sin\alpha + \cos\alpha) = 0.$

$\Rightarrow +P(0.2\sin\alpha + \cos\alpha) = +1.273$

$\Rightarrow \boxed{P = \dfrac{1.273}{0.2\sin\alpha + \cos\alpha}}$ —— (3)

Max means directly put $\alpha = 0$.

Since the value of P should be minimum, denominator should be maximum in eq (3).

$\therefore \dfrac{d}{d\alpha}(0.2\sin\alpha + \cos\alpha) = 0$

$\Rightarrow 0.2\cos\alpha - \sin\alpha = 0$

$\Rightarrow 0.2 = \dfrac{\sin\alpha}{\cos\alpha}$

$\Rightarrow \tan\alpha = 0.2$

$\Rightarrow \alpha = \tan^{-1}(0.2) = 11.309°$

$P = \dfrac{1.273}{0.2\sin 11.309 + \cos 11.309}$

$P = 1.24$ KN

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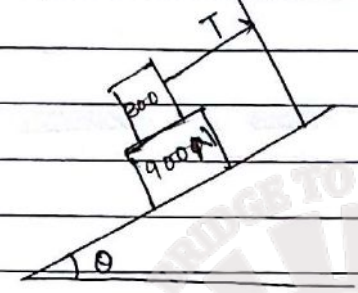
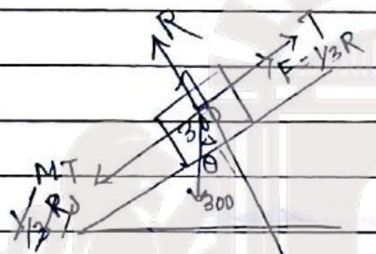
Q- What should be the value of θ which will make the motion of the block 900 N down the plane. Take $\mu = 1/3$ for all the contact surfaces.

Ans -





$\Sigma V = 0$

$\Rightarrow R - 300\cos\theta = 0$

$\Rightarrow R = 300\cos\theta$.

$\Sigma H = 0$

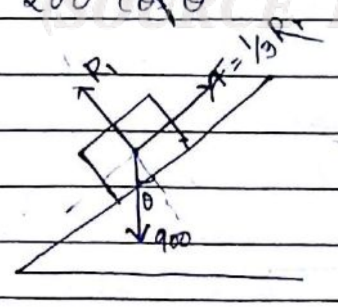
$\Rightarrow T + \frac{1}{3}R - 300\sin\theta = 0$.

$\Rightarrow T + \frac{1}{3}(300\cos\theta) - 300\sin\theta = 0$

$\Rightarrow 300\cos\theta\left(\frac{1}{3} - 1\right) + T = 0$

$\Rightarrow T = -300\cos\theta(-0.66)$.

$= T = 200\cos\theta$



$\Sigma V = 0$

$R_1 - 900\cos\theta = 0$.

$\Sigma H = 0$.

$+\frac{1}{3}R_1$

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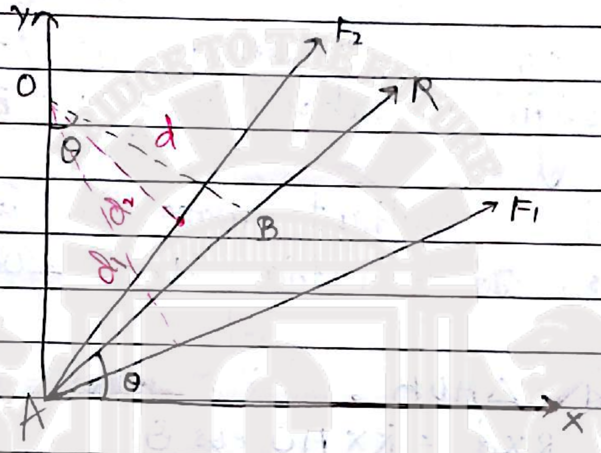
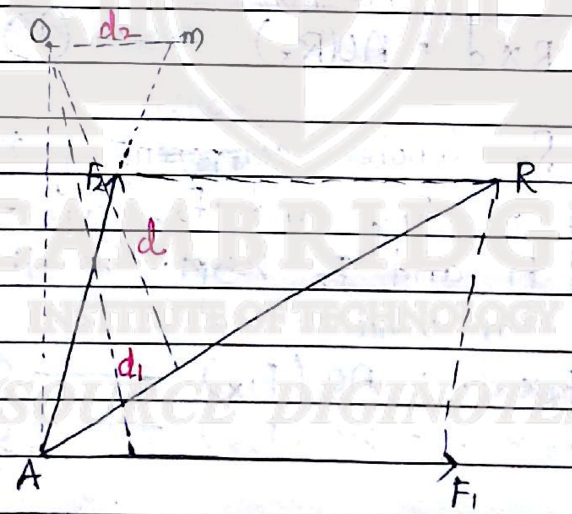
MODULE 3

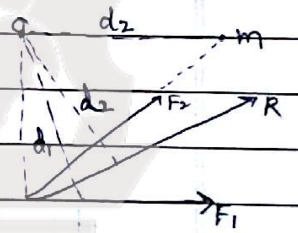
ANALYSIS OF NON-CONCURRENT FORCE SYSTEM

Imp. State and prove Varignon's theorem of moment :-

Statement : It states that the algebraic sum of moments of a system of co-planar forces about a moment centre is equal to the moment of their resultant forces about the same moment centre.







Correct Diagram

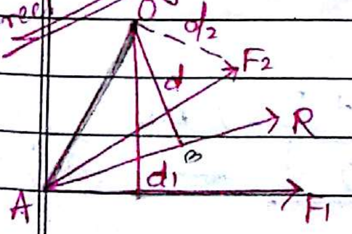


figure (a)

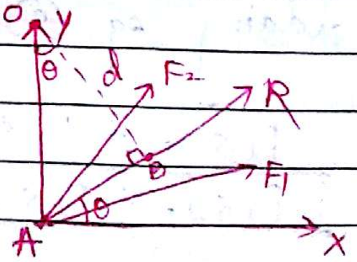


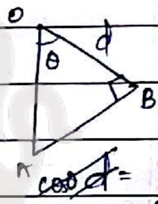
figure (b)

Refering to fig (a), let R be the resultant of 2 forces F_1 and F_2 and O be the moment centre. Let D, D_1 and D_2 are the moment arms of the forces R, F_1 and F_2 respt. Then according to the theorem, we have to prove

$$R \times d = F_1 \times d_1 + F_2 \times d_2$$

Join OA and Consider it as Y-axis being A as origin as shown in figure B.

Let the resultant make an angle θ wrt x-axis. It is nothing that $\angle AOB = \theta$.



From $\triangle AOB$,

$$R \times d = R \times AO \cos\theta$$
$$= R \times \cos\theta \times AO.$$
$$= AO \times (R_x)$$
$$R \times d = AO(R_x) \quad \text{——— (1)}$$

$\cos\theta = \frac{d}{OA}$

If R_x denotes component of R in x-direction.
Similarly if F_{1x} and F_{2x} are the component of F_1 and F_2 respt. in the x-direction then we can write:-

$$F_1 \times d_1 = AO(F_{1x}) \quad \text{——— (2)}$$
$$F_2 \times d_2 = AO(F_{2x}) \quad \text{——— (3)}$$

Adding eq (2) and (3):-

$$F_1 \times d_1 + F_2 \times d_2 = (F_{1x} + F_{2x}) AO$$
$$F_1 \times d_1 + F_2 \times d_2 = R_x AO \quad \text{——— (4)}$$

Since $F_{1x} + F_{2x} = R_x$
Comparing eq (1) and (4):-

$$R \times d = F_1 \times d_1 + F_2 \times d_2$$

(1) Locate the moment centre. (Given in ques.)

(2) Find the moment of all the forces about the moment centre.

$\Sigma H \neq 0$

$\Sigma V \neq 0$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}, \quad \theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

From Varigon's Theorem * $d = \frac{|m_c|}{R}$

(3) Find the distance of the point

$$d = \frac{|m_c|}{R}$$

(4) x-intercept $= \frac{|m_c|}{\Sigma V}$

(5) Y-intercept $= \frac{|m_c|}{\Sigma H}$

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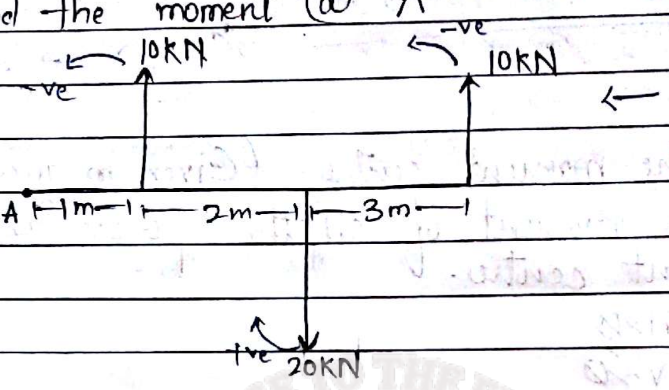
Q.1- Find the moment @ A :-

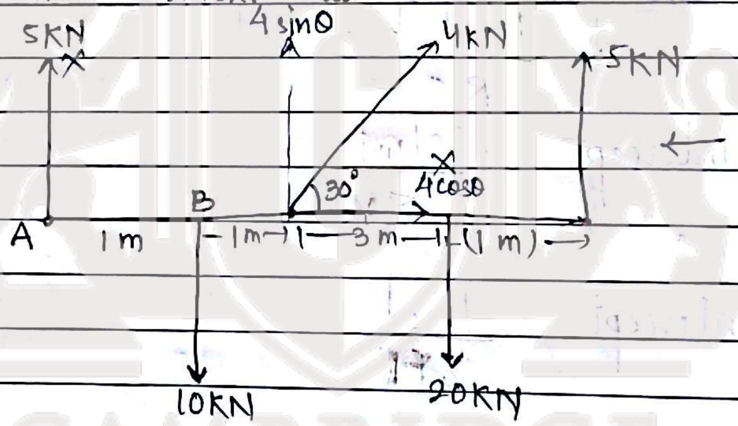
Ans-



$M_A = (-10) \times 6 + (20)(3) - (10)(1)$

$= -60 + 60 - 10 = -10\ KNm$ //.

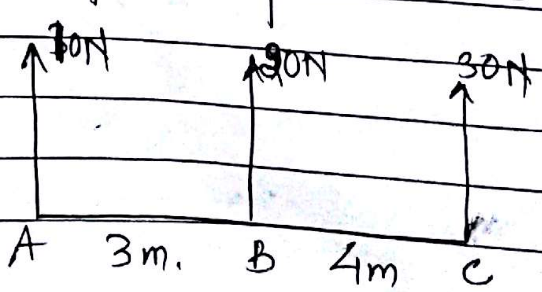
Q.2- Find the moment at A :-



Ans- $M_A = (-5)(6) + (20)(5) - (4\sin 30)(2) + (10)(1)$

$M_A = 76\ KN \cdot m$ //.

Q.3- 3 like parallel forces 10, 20 and 30 N are acting on line ABC respectively. Find the magnitude, direction and distance of resultant from the point A. AB = 3m BC = 4m.



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Ans- $\Sigma H = 0$

$\Sigma V = 10 + 20 + 30 = 60.$

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{(60)^2} = 60N$

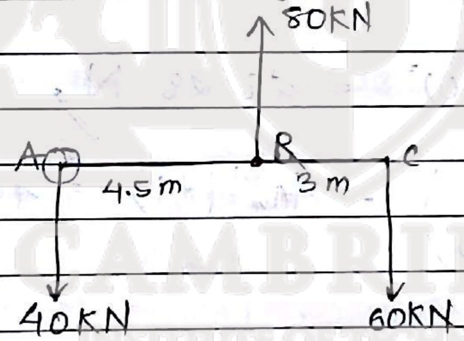
$d = \frac{|m_A|}{R}$

$\Sigma m_A = -(30 \times 7) - (20 \times 3)$
$= -210 - 60 = -270.$

$\therefore d = \frac{|-270|}{60} = \frac{270}{60} = 4.5\ m$

Q.4- A coplanar parallel force system consists of 3 forces acting on a rigid bar as shown in fig. Determine the single force (resultant) and its location from the point A.

Ans-



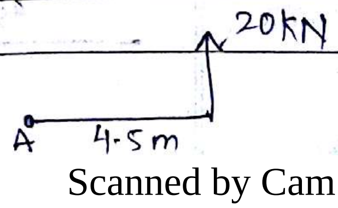
$\Sigma H = 0$

$\Sigma V = 80 - 40 - 60 = 80 - 100 = -20\ KN$

$R = \sqrt{0^2 + (-20)^2} = 20KN$

$M_A = (60 \times 7.5) - (80)(4.5) + 0$
$= 90\ Nm$

$\therefore d = \frac{|M_A|}{R} = \frac{90}{20} = 4.5\ m.$



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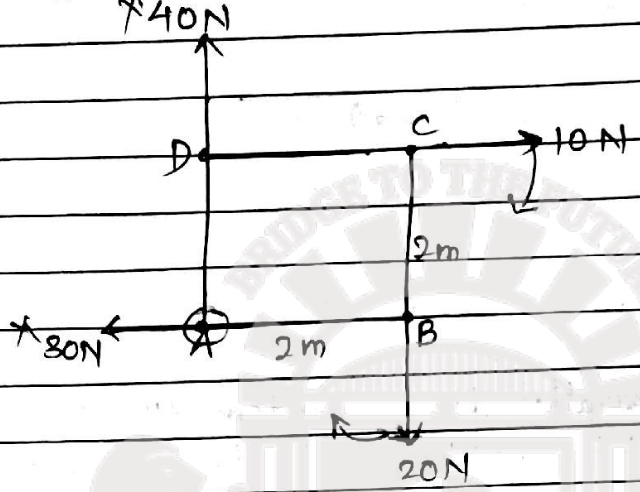
Q. 1 imp 5 - 4 forces are simultaneously acting of magnitude 10 N, 20N, 30N & 40 N along the 4 sides of the square of size (2×2) as shown in fig. Determine the magnitude and direction and its distance from A.



$\Sigma H = 10 - 30 = -20$ N.

$\Sigma V = 40 - 20 = 20$ N.

$R = \sqrt{(-20)^2 + (20)^2} = 28.28$ N

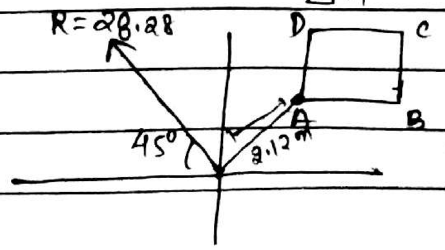
$\theta = \tan^{-1}\left(\frac{20}{-20}\right) = \tan^{-1}(+1) = 45°$

$m_A = 10 \times 2 + 20 \times 2 = 20 + 40 = 60$ Nm

$d = \frac{|m_A|}{R} = \frac{60}{28.28} = 2.1216$ m

x - intercept $= \frac{|m_A|}{\Sigma V} = \frac{60}{20} = 3$ m

y - intercept $= \frac{|m_A|}{\Sigma H} = \frac{60}{-20} = 3$ m



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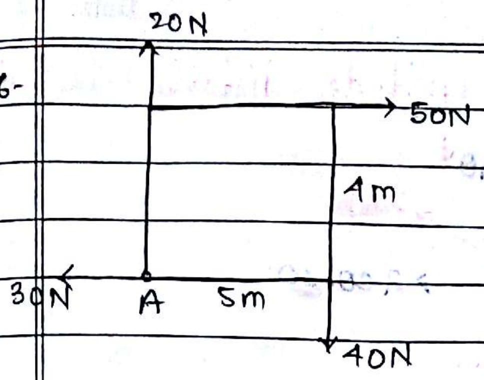
Q.6-



$\Sigma H = 50 - 30 = 20\ N$.

$\Sigma V = 20 - 40 = -20\ N$ //

$R = \sqrt{(20)^2 + (-20)^2} = 28.28\ N$.

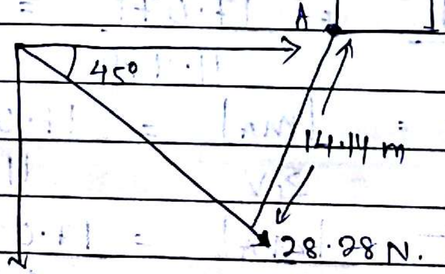
$\theta = \tan^{-1}\left(\frac{20}{20}\right) = 45^\circ$ //

$m_A = (50 \times 4) + (40) \times 5$
$= 200 + 200 = 400\ Nm$

$d = \frac{|m_A|}{R} = \frac{400}{28.28} = 14.144\ m$ //

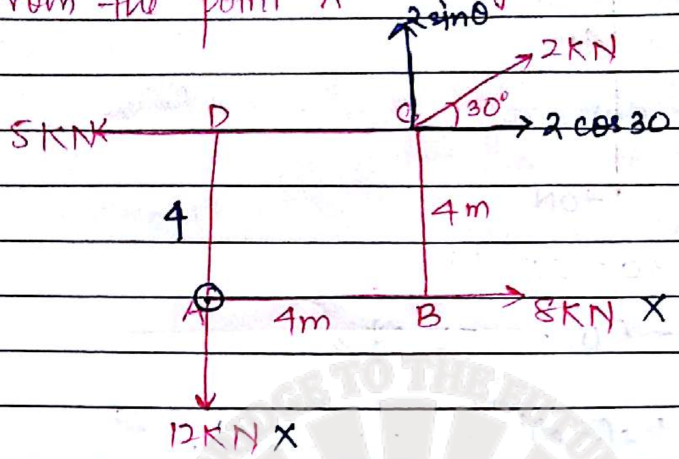
$x\text{-intercept} = \frac{|m_A|}{\Sigma V} = \frac{400}{-20} = 20\ m$ //

$y\text{-intercept} = \frac{|m_A|}{\Sigma H} = \frac{400}{20} = 20\ m$ //



Q.7. Find the resultant magnitude, distance and direction from the point A

Ans-



$M_A = ?$

$\Sigma H = 8 - 5 + 2\cos 30° = 4.732\ kN.$

$\Sigma V = -12 + 2\sin 30 = -11\ kN$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 11.974\ kN$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 66.723°$

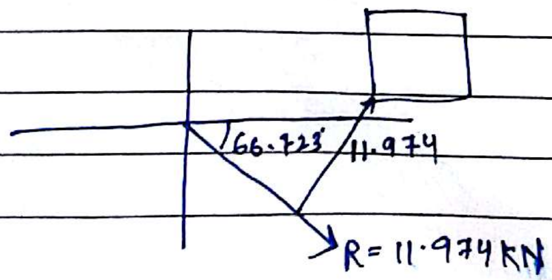
$M_A = -2\sin 30 \times 4 + 2\cos 30 \times 4 - 5 \times 4$

$= -17.071\ Nm$

$d = \frac{|M_A|}{R} = \frac{17.071}{11.974} = 1.425\ m$

$x\text{-intercept} = \frac{|M_A|}{\Sigma V} = \frac{17.071}{11} = 1.551\ m$

$y\text{-intercept} = \frac{|M_A|}{\Sigma H} = \frac{17.071}{4.732} = 3.607\ m$



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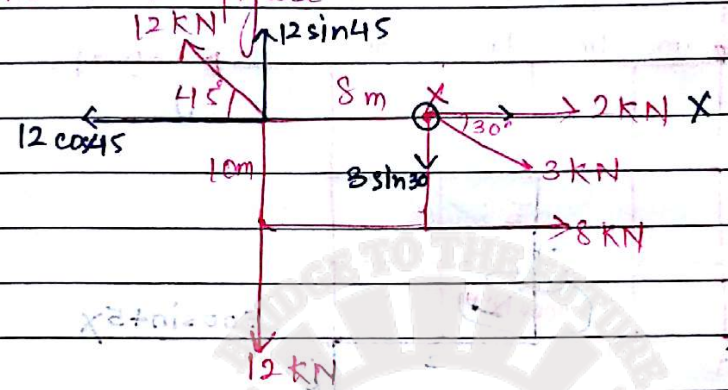
Q.8- Find the resultant magnitude and direction & the distance from x of the force system shown in figure :-

Ans-



$\Sigma H = 2 - 12\cos 45 + 8 + 3\cos 30 = 4.112$ KN

$\Sigma V = 12\sin 45 - 12 - 3\sin 30$
$= -5.014$ KN

$R = \sqrt{\Sigma V^2 + \Sigma H^2} = 6.4832$ KN.

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{5.014}{4.112}\right) = 50.64°$

$m_A = (12\sin 45 \times 8) - (12 \times 8) - (8 \times 10)$
$= -108.117$ KN-m.

$d = \frac{|m_A|}{R} = \frac{108.117}{6.4832} = 16.67$ m

x-intercept $= \frac{108.117}{5.014} = 21.56$ m

y-intercept $= \frac{108.117}{4.112} = 26.293$ m

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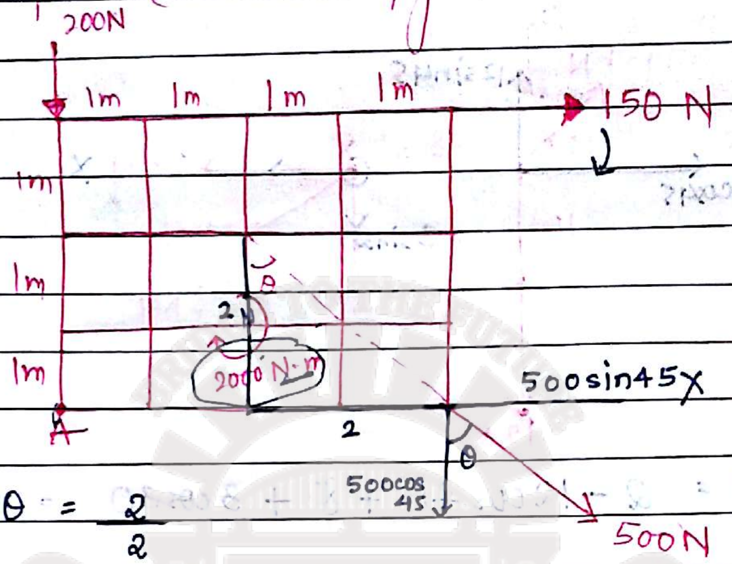
Imp. Find the equilibrium wrt A as origin of system of forces shown in figure :-



Ans- $\tan\theta = \frac{2}{2}$

$\theta = \tan^{-1}(1) = 45°$

$\Sigma H = 150 + 500\sin 45 = 503.55\ N$

$\Sigma V = -200 - 500\cos 45 = -553.55\ N$

$R = \sqrt{\Sigma V^2 + \Sigma H^2} = 748.31\ N$

$\theta = \tan^{-1}\left(\frac{553.55}{503.55}\right) = 47.708°$

$M_A = 2000 + 500\cos 45 \times 4 + 150 \times 3$
$= 3864.21\ N\text{-}m$

$d = \frac{|3864.21|}{748.31} = 5.16\ m$

$x\text{-intercept} = \frac{3864.21}{553.55} = 6.98\ m$

$y\text{-intercept} = \frac{3864.21}{503.55} = 7.67\ m$

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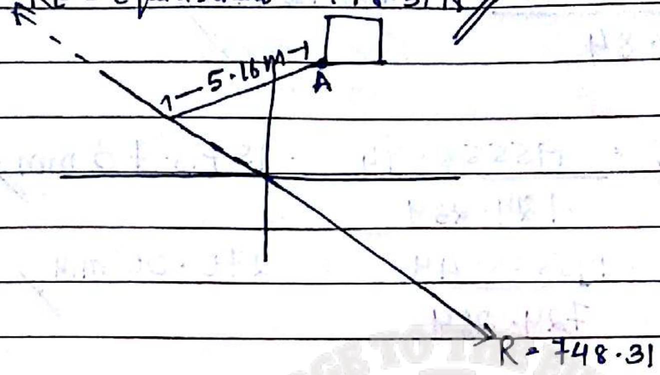
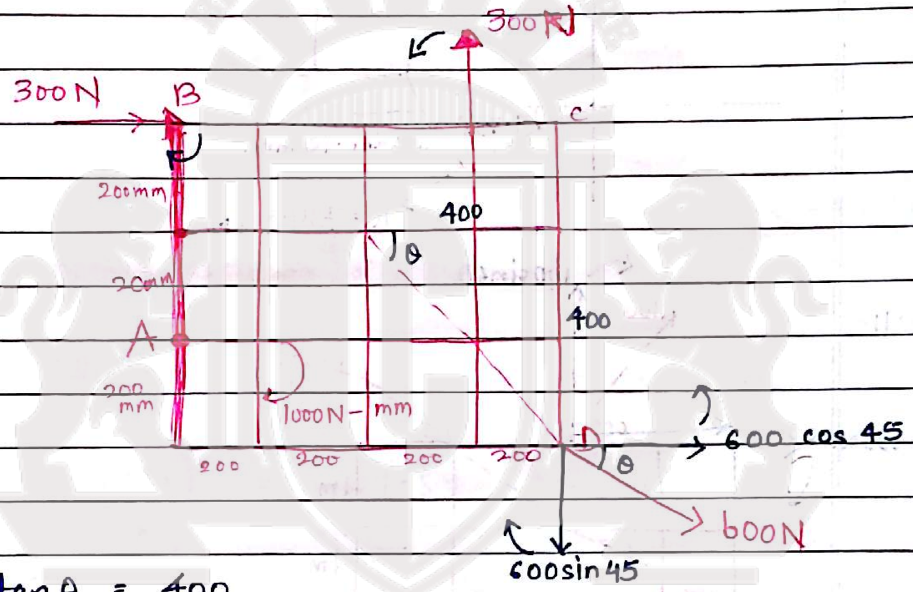
R_E = Equilibriant = 748.31 N



Q.10.



$\tan\theta = \frac{400}{400}$

$\theta = \tan^{-1}(1) = 45°$

$\Sigma H = 300 + 600\cos 45 = 724.264$ N

$\Sigma V = 300 - 600\sin 45 = -124.264$ N

$R = \sqrt{(724.264)^2 + (124.264)^2} = 734.84$ N

$\theta = \tan^{-1}\left(\frac{124.264}{724.264}\right) = 9.735°$

$M_A = (300 \times 400) - 300(600) - (600\cos 45)200 + (600\sin 45)(800)$

$= 194558.4412$ N-mm $+1000$

$= 195558.4412$ N-mm

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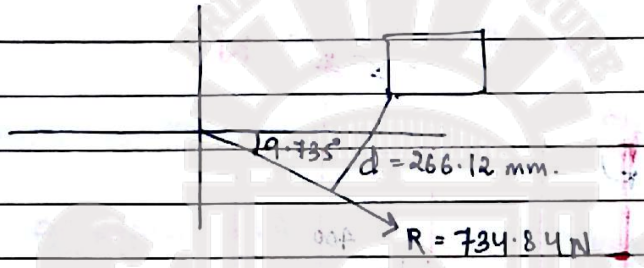
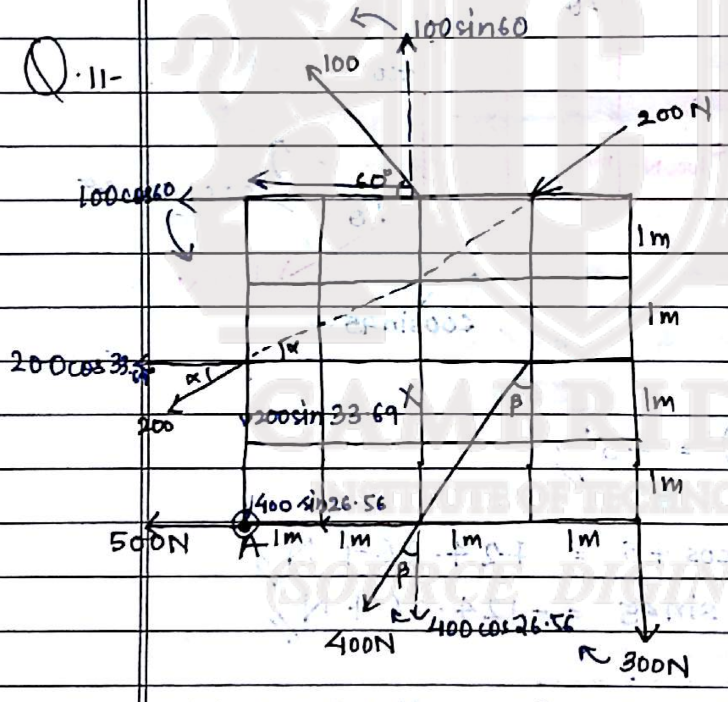
$d = \frac{|195558.4412|}{734.84} = 266.12\ mm$

x - intercept $= \frac{195558.44}{124.264} = 1573.73\ mm$

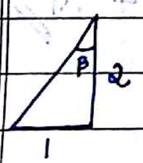
y - intercept $= \frac{195558.44}{724.264} = 270.00\ mm$



Q.11-

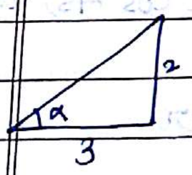


Ans-



$\tan\beta = \frac{1}{2}$

$\beta = \tan^{-1}(1/2) = 26.56°$



$\tan\alpha = \frac{2}{3}$

$\alpha = \tan^{-1}\left(\frac{2}{3}\right) = 33.69°$

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$\Sigma H = -500 - 200\cos 33.69 - 100\cos 60 - 400\sin 26.56$

~~= -850.76 N~~ $= -895.26$ N

$\Sigma V = 100\sin 60 - 200\sin 33.69 - 400\cos 26.56 - 300$

$= -682.12$ N

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 1125.5124$ N

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 37.30°$

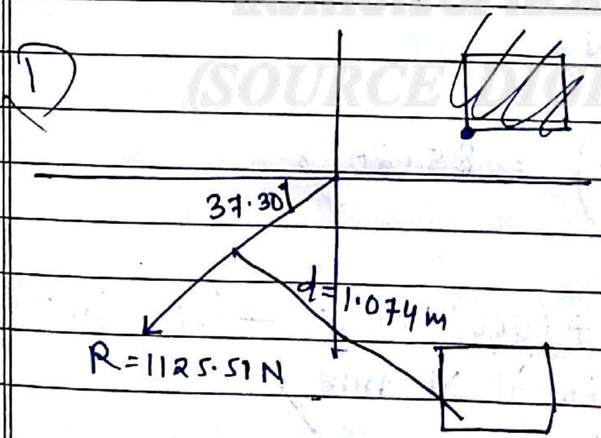
$m_A = (-100\sin 60)2 - (100\cos 60)4 - (200\cos 33.69)2 + (400\cos 26.56)2 + (300)4$

$= 1209.54$ Nm

$d = \frac{|m_A|}{R} = \frac{1209.54}{1125.5124} = 1.074$ m

x-intercept $= \frac{|m_A|}{\Sigma V} = \frac{1209.54}{682.12} = 1.773$ m

y-intercept $= \frac{|m_A|}{\Sigma H} = 1.3510$ m



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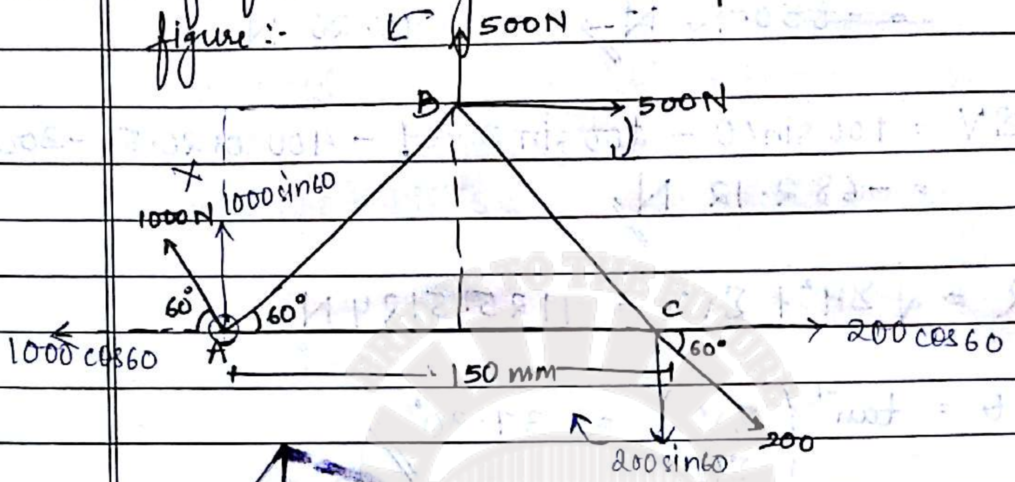
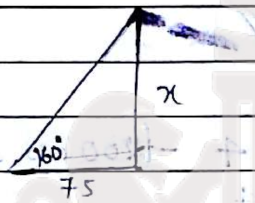
Q- Determine the resultant @ equilibrium for a system of forces acting on an equilateral Δ as shown in figure :-





$\tan 60° = \frac{x}{75}$

$x = 75 \times \sqrt{3} = 129.90$ mm

$\Sigma H = 500 + 200 \cos 60 - 1000 \cos 60$
$= 100$ N

$\Sigma V = 500 - 200 \sin 60 + 1000 \sin 60$
$= 1192.82$ N

$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

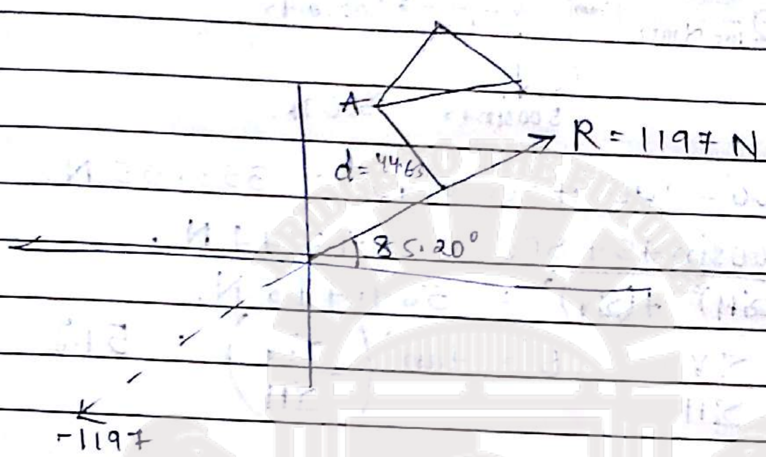
$= 1197.00$ N

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 85.20°$

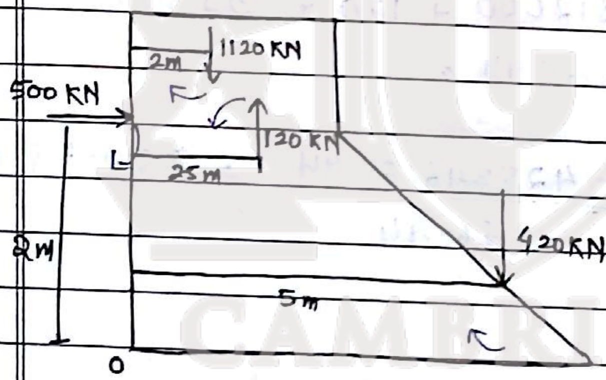
$M_A = -(500) 75 + (500) 129.90 + (200 \sin 60) 150$
$= 53430.76211$ N-mm

$d = \frac{53430.76211}{1197.0} = 44.63$ mm x - intercept $= \frac{|M_A|}{\Sigma V} = \frac{5343.76211}{1192.82} = 44.79$ mm.

y - intercept $= \frac{|M_A|}{\Sigma H} = 534.3076$ mm //.



Q-

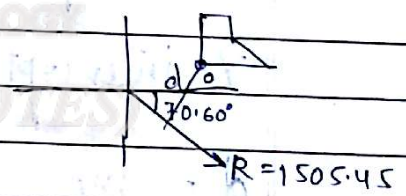


x - intercept $= \frac{|M_A|}{\Sigma V}$

$= 3.54$ m //

y - intercept $= \frac{|M_A|}{\Sigma H}$

$= 10.08$ m.



$\Sigma H = 500$ KN

$\Sigma V = -1120 + 120 - 420$

$= -1420$ KN //

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 1505.45$ KN //

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 70.60°$ //

$M_A = (1120) \times 2 - (120)\, 2.5 + (420) \times 5 + (500)\, 2$

$= 5040$ KN - m //.

$d = \frac{|M_A|}{R} = \frac{5040}{1505.45} = 3.347$ m //

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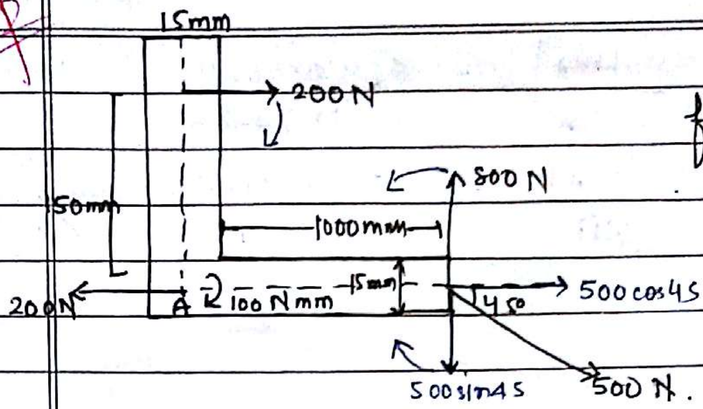
Q-



find moment @ 'A'

Ans-

$\Sigma H = 200 - 200 + 500\cos 45 = 353.55\ N.$

$\Sigma V = -500\sin 45 + 800 = 446.44\ N.$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 569.478\ N.$

$\tan\theta = \frac{\Sigma V}{\Sigma H} \quad \theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 51.6^\circ$

$M_A = 200 \times (150+15) - 800 \times (1000+15) + 100 + (500\sin 45)\,1000$

$= 33000 - 812000 + 100 + 353553.3906.$

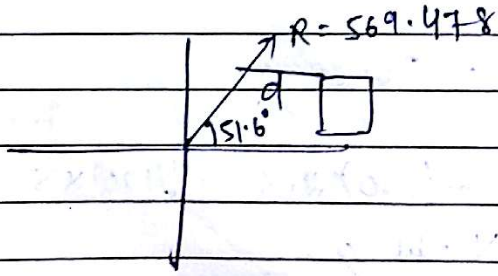
$= -425346.6094$

~~$d = \frac{M_A}{\Sigma V} = \frac{-425346.6094}{446.44} = 952.75\ mm.$~~

X-intercept =

$Y\text{-intercept} = \frac{-425346.6094}{353.55} = +1203.073\ mm$

$d = \frac{M_A}{R} = \frac{+425346.6094}{569.478} = 746.906\ mm$



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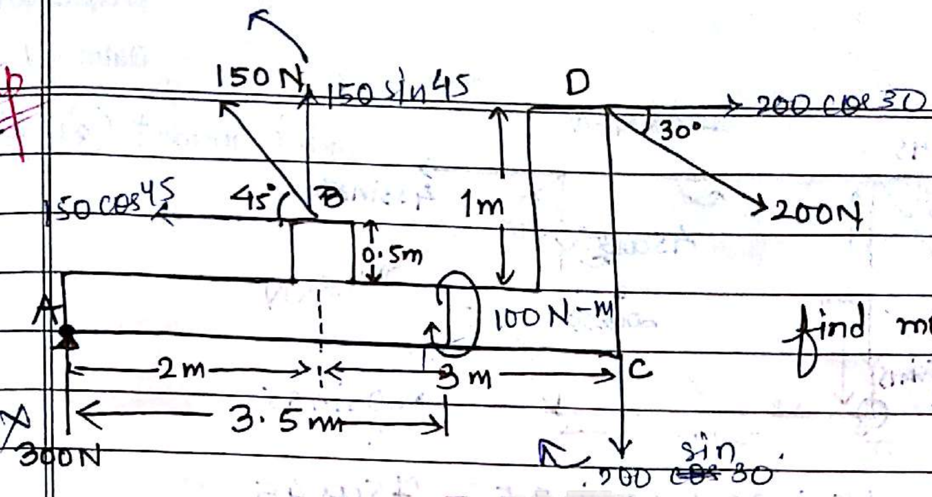
Q-



find moment @ A.

Ans –

$\Sigma H = -150\cos 45 + 200\cos 30 = 67.13\ N.$

$\Sigma V = 300 + 150\sin 45 + 200\sin 30 = 306.06\ N$

$R = \sqrt{\Sigma V^2 + \Sigma H^2} = 313.335\ N$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 77.62°$ //

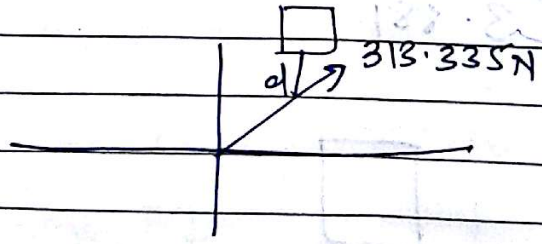
$M_A = -(150\sin 45)(2) + (200\sin 30)(3+2) - (150)\cos 45 \times 0.5 + 100 + 200\cos 30 \times (1 + \cancel{0.5})$

$= 508.040\ Nm$ //

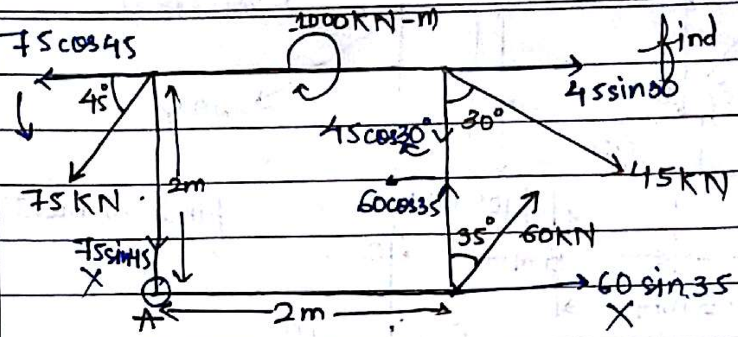
$d = \frac{508.04}{313.335} = 1.621\ m$ //

$x\text{-intercept} = \frac{508.04}{306.06} = 1.660\ m$ //

$y\text{-intercept} = \frac{508.40}{67.13} = 7.573\ m$ //



Q- find moment @ 'A'.



Ans -
$\Sigma H = 45\sin 30 + 60\sin 35 - 75\cos 45$
$= 3.881$ KN

$\Sigma V = -75\sin 45 + 60\cos 35 - 45\cos 30 = -42.855$ KN

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{(42.85)^2 + (3.88)^2} = 43.030$ KN

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{42.855}{3.881}\right) = 84.82°$

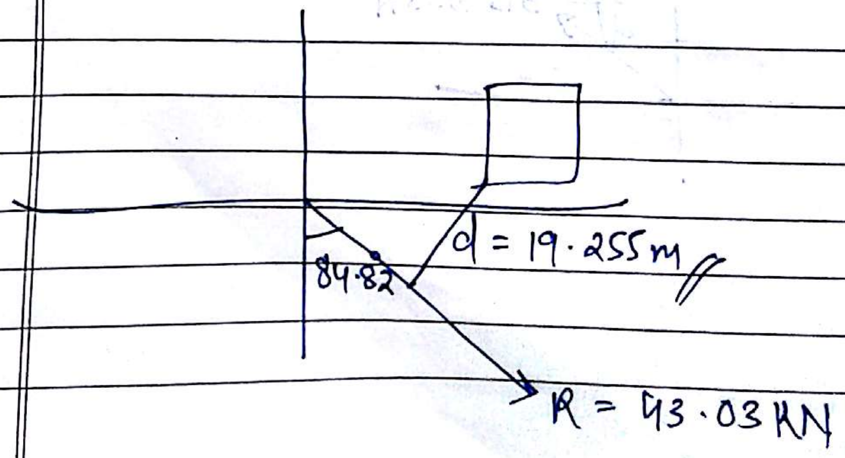
$M_A = -(75\cos 45)2 - (60\cos 35)2 + (45\cos 30)2 - (45\sin 30)2 + 1000$

$\Rightarrow 828.578$ Nm

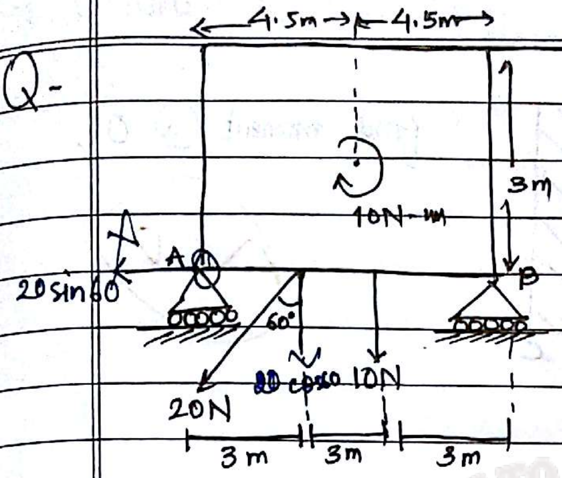
$d = \frac{828.578}{43.03} = 19.255$ m

x-intercept $= \frac{828.578}{42.855} = 19.3344$ m

y-intercept $= \frac{828.578}{3.881} = 213.496$ m



Q- find moment at 'A'.



$\Sigma H = 150 - 20\sin 60 = 132.679\ N$

$\Sigma V = -10 - 20\cos 60 = -20\ N$

$R = \sqrt{\Sigma V^2 + \Sigma H^2} = 134.1779\ N$

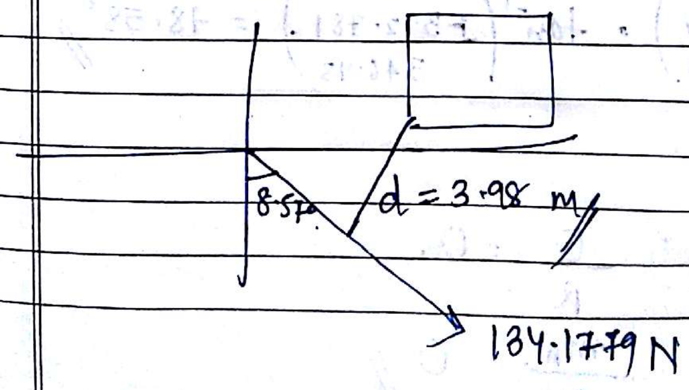
$\theta = \tan^{-1}\left(\frac{20}{132.679}\right) = 8.57°$

$M_A = 150 \times 3 + (10 \times 4.5) + 10 + (20\cos 60)3$
$= 535\ Nm$

$d = \frac{535}{134.17} = 3.98\ m$

$x\text{-intercept} = \frac{535}{20} = 26.75\ m$

$y\text{-intercept} = \frac{535}{132.679} = 4.032\ m$



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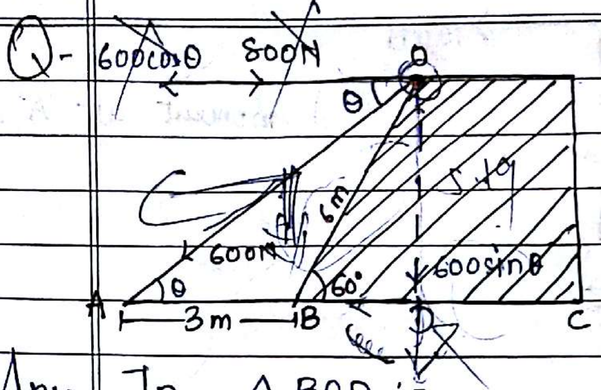
Q- find moment @ O.



Ans- In ΔBOD :-

$$\sin 60 = \frac{DO}{6}$$

$$OD = 5.196\ m$$

$$\cos 60 = \frac{BD}{6}$$

$$BD = 3\ m$$

In ΔADO :-

$$\tan\theta = \frac{OD}{AD} = \frac{5.196}{3+3} = \frac{5.196}{6}$$

$$\theta = 40.892^\circ$$

$$\Sigma H = 800 - 600\cos(40.892)$$
$$= 346.43\ N$$

$$\Sigma V = -600\sin(40.892) = -392.781\ N.$$

$$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 523.727\ N$$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{-392.781}{346.43}\right) = 48.58^\circ$$

$$M_O = 0$$

$$d = \frac{M_A}{R} = \frac{0}{R} = 0.$$

$$x\text{-intercept} = \frac{M_A}{\Sigma V} = 0$$

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Y - intercept $\Rightarrow \frac{MA}{\Sigma H} = 0$

MODULE 3.

Support And Support Reaction

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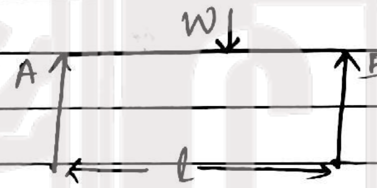
- **Define beam on an horizontal member and what are the different types of beam.**

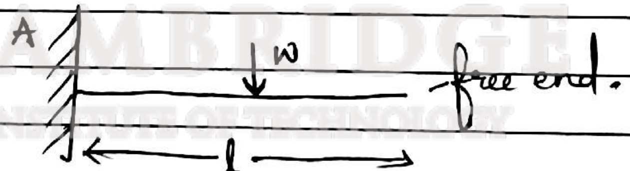
A beam is a structural member which is having longer length compared to its width & depth. It is also called as horizontal member.

Different Types of Beam :-

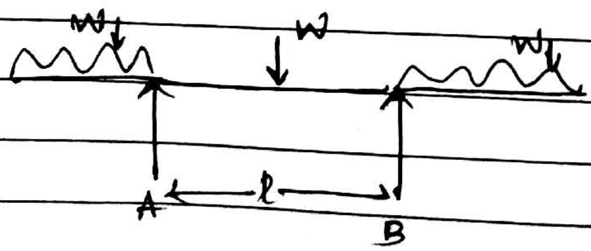
(1) Simply Supported Beam : In this beam both ends of the beam are simply supported



(2) Cantilever Beam :- In this beam, one end of the beam is supported and the other end is left free.



(3) Over hanging Beam :- In this beam, a part of the beam overhangs at one end or both the ends.



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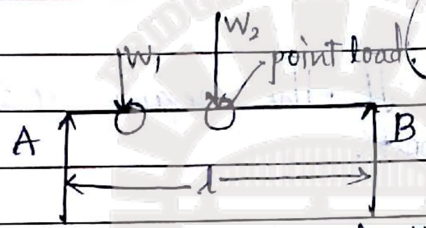
• DIFFERENT TYPES OF LOADINGS :-

(1) Point load
(2) Uniformly Distributed load.
(3) Uniformly Varying Load.

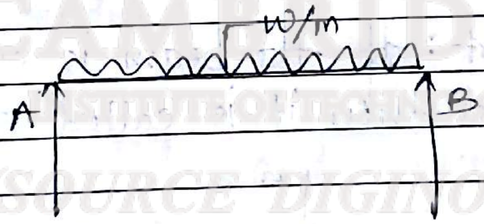
POINT LOAD :- Load is acting at a point on a beam.



Example :- two person standing on the beam.

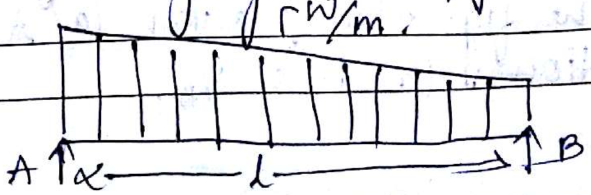
UNIFORMLY DISTRIBUTED LOAD :- Load which has got same intensity over a considerable length is called as uniformly distributed load. (UDL).

Example :- Brick wall constructed on a beam.



UNIFORMLY VARYING LOAD :- Intensity of the load increases lineare along the length is called as uniformly varying load. (UVL).

Example :- Brick Wall constructed on a beam carrying a slopy roof.



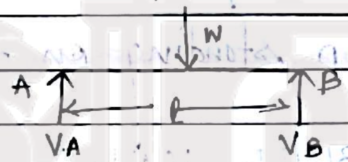
• SUPPORTS :-

The diff types of supports are :-

(1) Simply Support (2) Roller Support (3) Hinged Support

(4) Fixed Support (5) Continous Support.

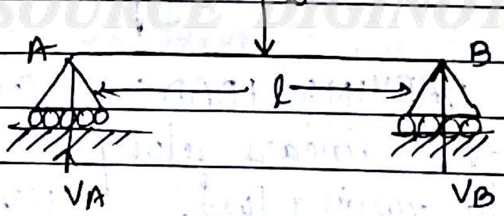
(1) **Simply Support** : If the beam rest simply on a support.



For a simply supported there will be only one reaction which is acting vertically upwards

From the fig. the reaction @ A is V_A and reaction @ B is V_B.

(2) **Roller Support** : If the beam is supported on the roller, then it is called as roller support



For roller support there will be only one reaction which is acting vertically upward

From the fig. the reaction @ A is V_A and reation @ B is V_B.

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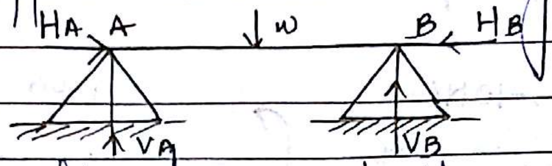
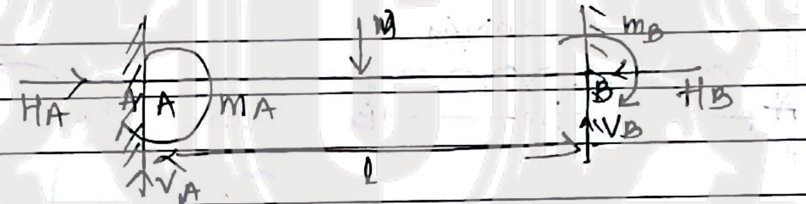
(3) Hinged Support :- If the beam is supported on the hinge or a pin then that type of support is called as hinged support.



For the hinged support there will be 2 reaction ie vertically upwards and horizontally inwards.

From the fig. the reaction @ A is VA and HA and reaction @ B is VB or HB.

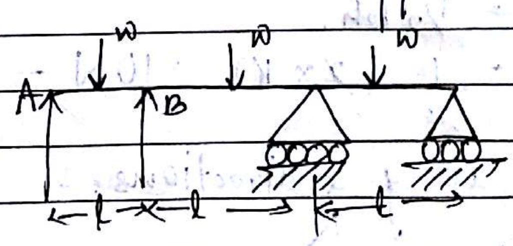
(4) Fixed Support :- If the beam is supported on one end or both the ends.



For the fixed support there will be 3 reaction ie vertically upwards and horizontally inwards and moment.

From the fig. the reaction @ A are VA, HA and MA and reactions @ B are VB, HB and MB.

(5) CONTINOUS SUPPORT :- If the beam is having 3 or more support.



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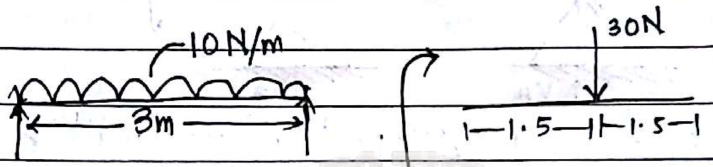
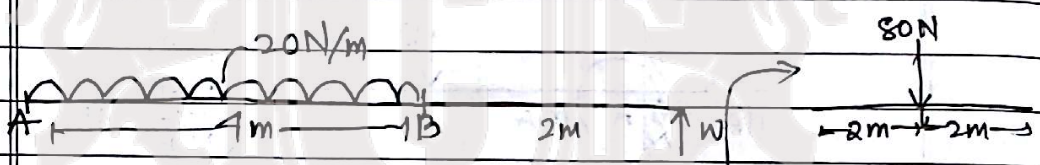
NOTE:

(1) CONVERTING UDL TO POINT LOAD :-



$10 N/m \times 3m$

$\Rightarrow 30N$
Point Load.

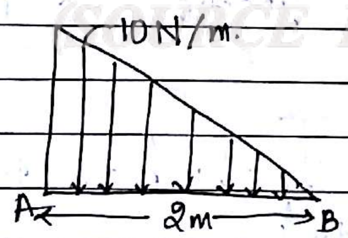


$20 N/m \times 4m$

80N - Point Load.

(2) CONVERTING UVL TO POINT LOAD :-

(I)

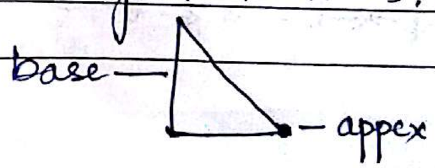


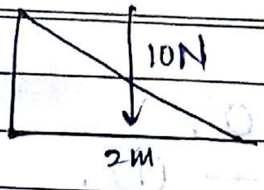
Area of $\Delta = \frac{1}{2} bh$

$= \frac{1}{2} \times 2 \times 10 = 10N$ — Point Load.

It will act along 2 directions :-

(i) from base





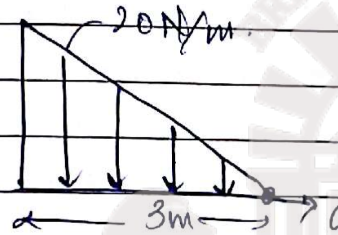
from base $= \frac{1}{3} d$

$= \frac{1}{3} \times 2 = 0.66m$

from appex $= \frac{2}{3} d$

$= \frac{2}{3} \times 2 = \frac{4}{3} = 1.33\ m$

(II)

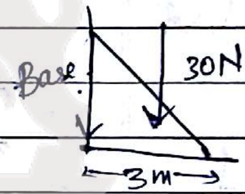


Area $= \frac{1}{2} bh$

$= \frac{1}{2} \times 3 \times 20\ 10 = 30\ N$

↓

Point Load.



from base :- $\frac{1}{3} d$

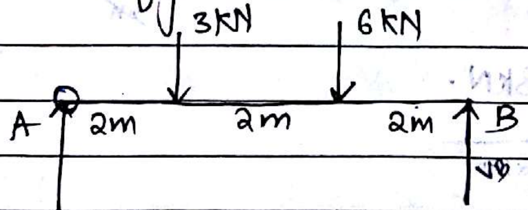
$\Rightarrow \frac{1}{3} \times 3 = 1\ m$

from appex :- $\frac{2}{3} d \Rightarrow \frac{2}{3} \times 3 = 2m$.

distance from base + distance from appex = span

Q- Find the support and support reaction for the beam shown in figure :-

Ans-



Support @ A – simply support → vertically ↑wards. $= V_A$

@ B – simply support → vertically ↑wards V_B

$\Sigma V = 0$

$V_A + V_B - 3 - 6 = 0$

$\boxed{V_A + V_B = 9}$ —— (1)

$\Sigma M_A = 0$

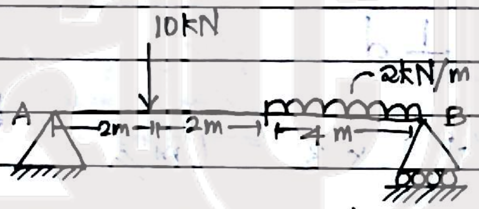
$(-V_B \times 6) + 6 \times 4 + 3 \times 2 = 0$

$6 V_B = 24 + 6$

$V_B = \frac{30}{6} = 5\ kN$

$V_A = 9 - 5 = 4\ kN$

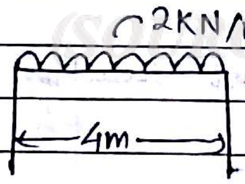
Q2.



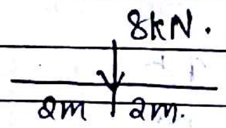
Ans. Support @ A – hinged support → $\uparrow V_A$, $\rightarrow H_A$

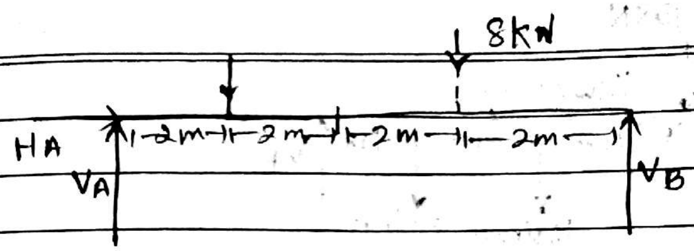
@ B – roller support → $V_B \uparrow$

$H_A = ?$ $V_A = ?$ $V_B = ?$



Point Load = $\frac{2kN}{m} \times 4m = 8\ kN$





$\Sigma H = 0.$

$\boxed{H_A = 0 \text{ kN.}}$

$\Sigma V = 0.$

$\boxed{V_A + V_B - 10 - 8 = 0}$

$\Rightarrow V_A + V_B = 18 \text{ kN}$

$\Sigma M_A = 0.$

$- V_B \times 8 + 8 \times 6 + 10 \times 2 = 0.$

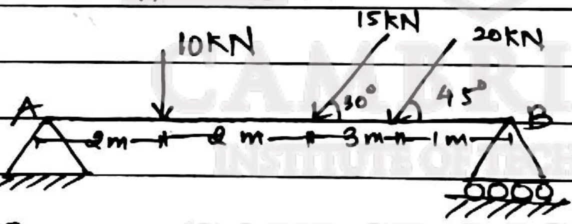
$V_B \times 8 = 68$

$V_B = \frac{68}{8} = 8.5 \text{ kN.}$

Substituting the value.

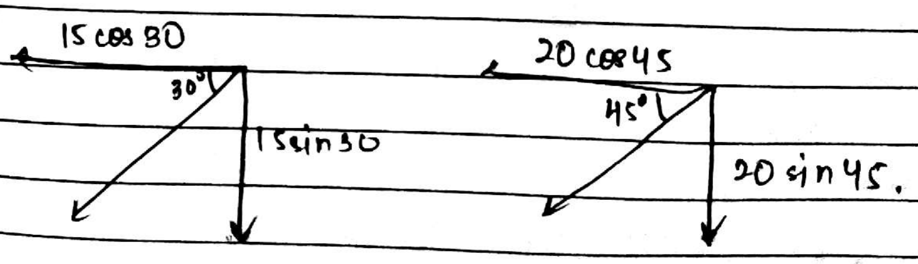
$V_A = 18 - 8.5 = 9.5 \text{ kN} //.$

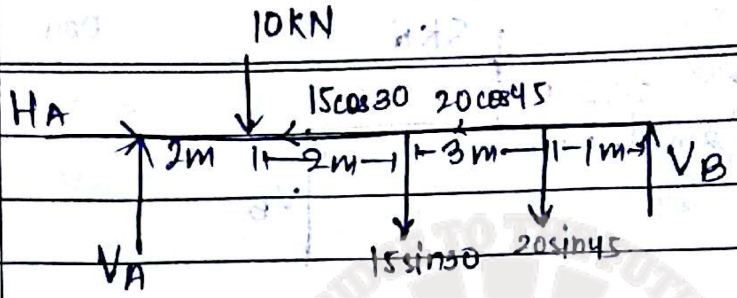
Q.3.



Ans- Support @ A - hinged support → $\uparrow V_A$, $\rightarrow H_A$.
@ B - roller support → $V_B \uparrow$.

$H_A = ?, V_A = ?, V_B = ?$





$\Sigma H = 0.$

$H_A - 15\cos 30 - 20\cos 45 = 0$

$H_A = 27.13$ KN

$\Sigma V = 0.$

$\Rightarrow V_B + V_A - 10 - 15\sin 30 - 20\sin 45 = 0$

$\Rightarrow V_A + V_B = +31.64$ KN —— ①.

$\Sigma M_A = 0.$

$\Rightarrow -V_B \times 8 + (20\sin 45)7 + (15\sin 30)4 + 10 \times 2 = 0$

$\Rightarrow V_B \times 8 = 148.99$

$\Rightarrow V_B = \frac{148.99}{8} = 18.62$ KN.

$\therefore V_A = 31.64 - 18.62 = 13.02$ KN

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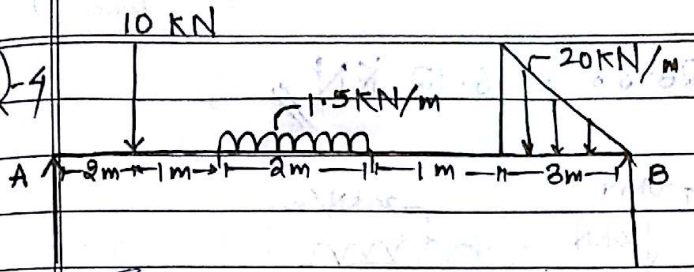
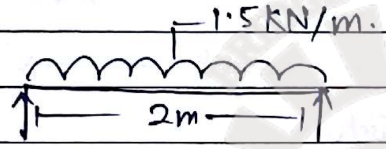
Q-4

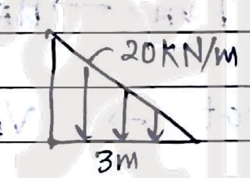


Ans- Support @ A - simply support → $V_A \uparrow$

@ B - simply support → $V_B \uparrow$.



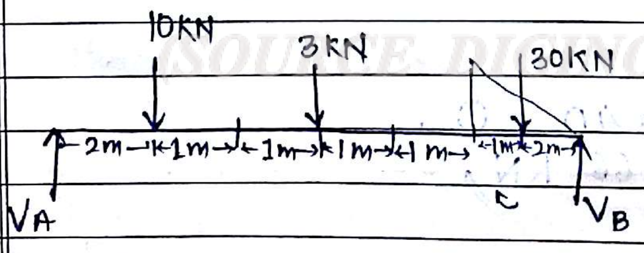
Point Load $= 1.5 \frac{KN}{m} \times 2m = 3\ KN$ //.



Point Load $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 3 \times 20 = 30\ KN$ //

from base $= \frac{1}{3} \times d = \frac{1}{3} \times 3 = 1\ m$.

from apex $= \frac{2}{3} \times d = \frac{2}{3} \times 3 = 2m$.



$\Sigma V = V_A + V_B - 10 - 3 - 30$

$\Rightarrow 0 = V_A + V_B - 43$.

$\Rightarrow V_A + V_B = 43\ KN$ ——— (1).

$\Sigma M_A = 0$

$\Rightarrow -V_B \times 9 + 30 \times 7 + 3 \times 4 + 10 \times 2 = 0$

$\Rightarrow V_B \times 9 = 242 \Rightarrow V_B = \frac{242}{9} = 26.88\ KN$ //

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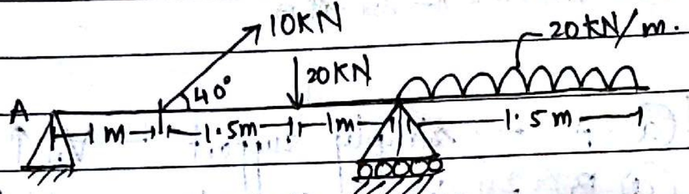
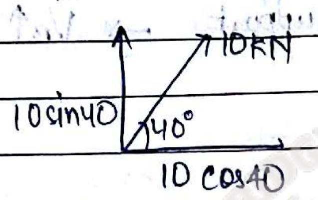
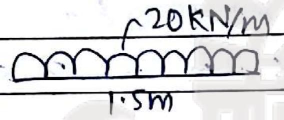
$\therefore \quad V_A = 43 - 26.88 = 16.12$ KN

Q.5 -



Ans -

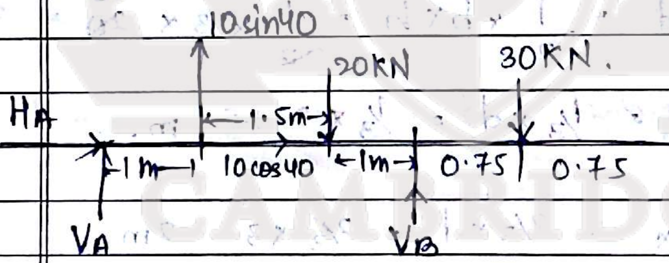




$\rightarrow 20\frac{KN}{m} \times 1.5m = 30$ KN.

Support @ A is hinged $\rightarrow \uparrow V_A \longrightarrow H_A$

@B is roller support $\rightarrow V_B \uparrow$.



$\Sigma H = 0.$

$H_A + 10\cos 40 = 0$

$H_A = -7.66$ KN

$\Sigma V = 0.$

$\Rightarrow V_A + V_B + 10\sin 40 - 30 - 20 = 0.$

$\Rightarrow V_A + V_B = 43.572$ KN $\longleftarrow$ ①

$\Sigma M_A = 0$

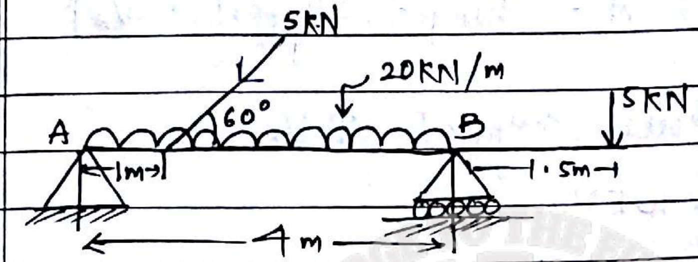
~~$\Rightarrow (30) \times 10.41 - V_B \times 9.66 + 20(10\cos 40 + 1)$~~

$\Rightarrow 30 \times 4.25 - V_B \times 3.5 + 20 \times 2.5 - 10 \sin 40 \times 1 = 0$

$\Rightarrow V_B = \frac{171.072}{3.5} = 48.87\ KN$

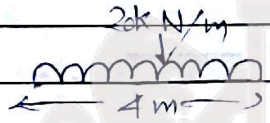
$V_A = 43.57 - 48.87 = -5.3\ KN$

Q.6-

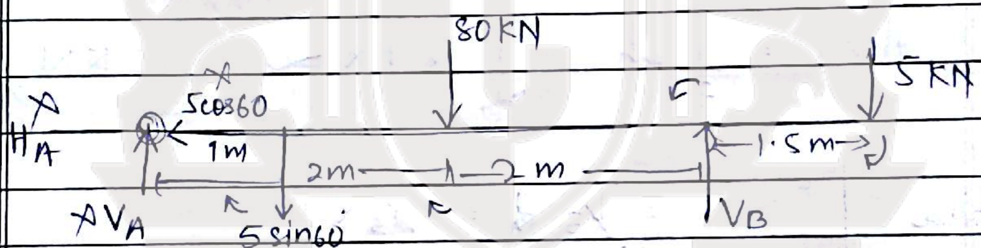


Support @ A is hinged → $V_A \uparrow$ → H_A

@ B is roller support → $V_B \uparrow$



→ $\frac{20\ kN}{m} \times 4m = 80\ KN$



$\Sigma H = 0$

$\Rightarrow H_A - 5\cos 60 = 0$

$\Rightarrow H_A = 2.5\ KN$

$\Sigma V = 0$

$\Rightarrow V_A + V_B - 5\sin 60 - 80 - 5 = 0$

$\Rightarrow V_A + V_B = 89.3301$ —— (1)

$\Sigma M_A = 0.$

$\Rightarrow (5\sin 60)1 + (80)2 - (V_B)4 + (5)(5.5) = 0$

$\Rightarrow V_B = 47.957\ KN$

$\therefore V_A = 89.3301 - 47.957 = 41.3731\ KN$

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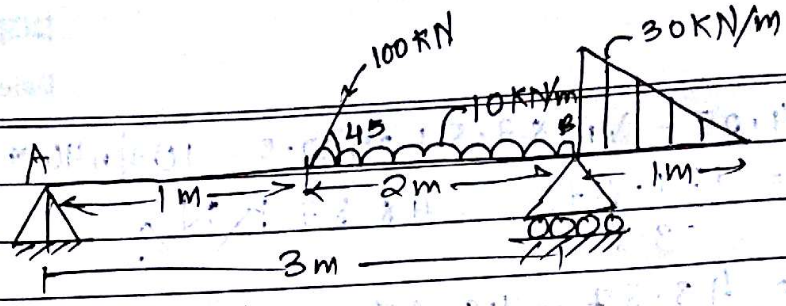
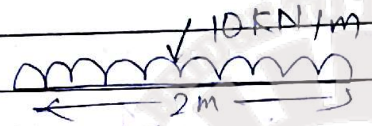
Q:7-



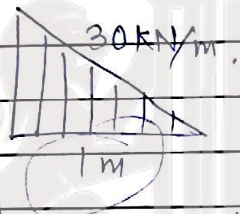
Ans-

Support @ A - hinged support → $V_A \uparrow$ $H_A \rightarrow$

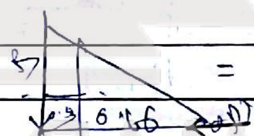
@ B - roller support $\uparrow V_B$.



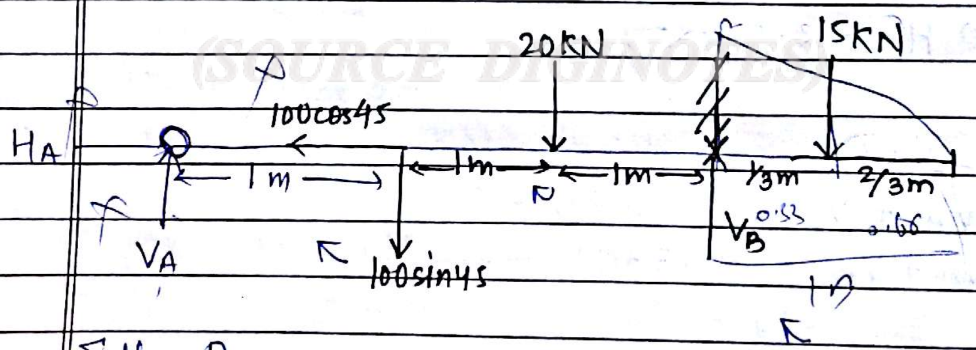
Point Load = $\frac{10 KN}{m} \times 2m = 20\ KN$ //.



Area = $\frac{1}{2} bh = \frac{1}{2} \times 1 \times 30 = 15\ KN$ //

from base  = $\frac{1}{3} d = \frac{1}{3} \times 1 = 0.33$

from appex = $\frac{2}{3} d = \frac{2}{3} \times 1 = 0.66$



$\Sigma H = 0$

$\Rightarrow H_A - 100 \cos 45 = 0$.

$\Rightarrow H_A = 70.71\ KN$ //

$\Sigma V = 0$

$\Sigma V = 0$

$\Rightarrow V_A - 100\sin 45 - 20 + V_B - 15 = 0$

$\Rightarrow V_A + V_B = 105.710$ —— (1).

$\Sigma M_A = 0$

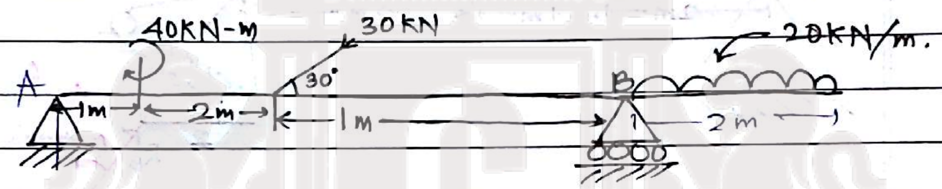
$\Rightarrow (100\sin 45)1 + (20)2 - 3V_B + (15)(3.33) = 0$

$\Rightarrow 3V_B = 160.66$

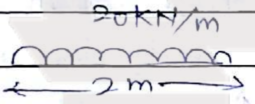
$\Rightarrow V_B = \frac{160.66}{3} = 53.55\,KN.$

$V_A = 105.710 - 53.55 = 52.16\,KN$ //

Q.8 -

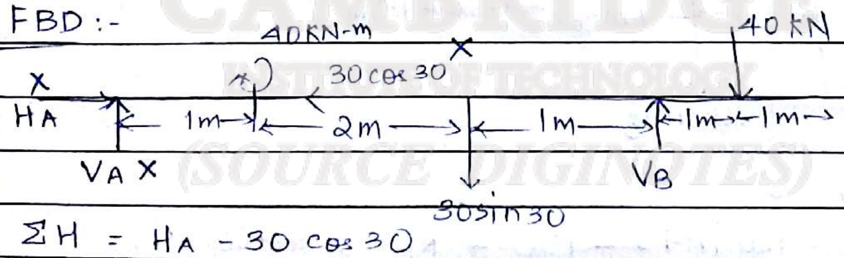


Ans - Support @ A is hinged support $\uparrow V_A$ $\rightarrow H_A$.
@ B is roller support $\uparrow V_B$.



Point Load $\Rightarrow \frac{20\,KN}{m} \times 2m = 40\,KN$

FBD :-



$\Sigma H = H_A - 30\cos 30$

$\Rightarrow 0 = H_A - 25.98$

$\Rightarrow H_A = 25.98\,KN$ //.

$\Sigma V = 0$

$\Rightarrow V_A - 30\sin 30 + V_B - 40 = 0$

$\Rightarrow V_A + V_B = +55$ —— (1).

$\Sigma M_A = 0.$

$\Rightarrow 40 + (30\sin 30)3 - (V_B)4 + (40)5 = 0$

$\Rightarrow 4V_B = 285 \Rightarrow V_B = 71.25\,KN$ //

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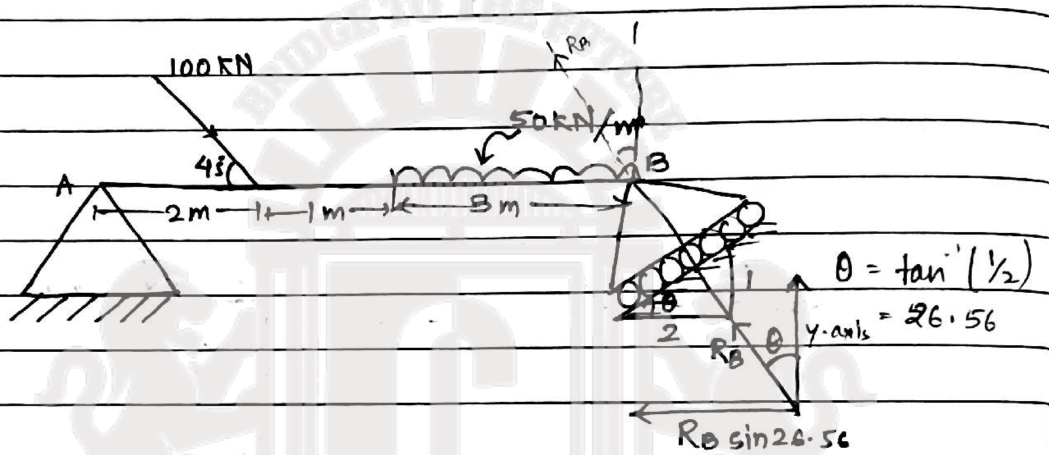
$\Rightarrow V_A + V_B = 55$

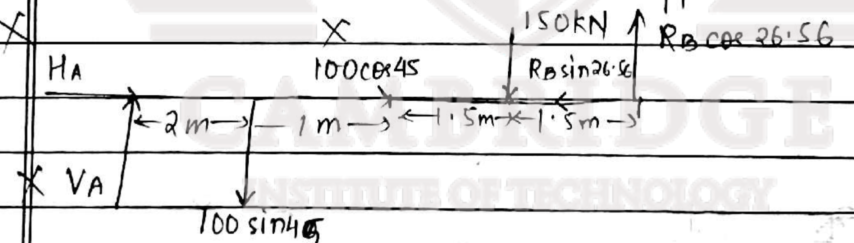
$\Rightarrow V_A = 55 - V_B = 55 - 71.25 = -16.25$ //

Imp Q-9

Ans.



Support @ A is hinged support ↑ V_A H_A →
@ B is roller support R_B inclined @ 26.56° to y-axis



$\Sigma V = 0$

$\Rightarrow V_A - 100\sin 45 - 150 + R_B \cos 26.56 = 0$

$\Rightarrow V_A + R_B \cos 26.56 = 220.710$ —— (1)

$\Sigma H = 0$

$H_A + 100\cos 45 - R_B \sin 26.56 = 0$

$\Rightarrow H_A - R_B \sin 26.56 = -70.71$ —— (2)

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$\Sigma m_A = (100 \sin 45) 2 + (150)(4.5) - (R_B \cos 26.56) 6 = 0$

$\Rightarrow (R_B \cos 26.56) \cdot 6 = 816.421$

$\Rightarrow R_B = \frac{816.421}{6 \cos 26.56} = 152.124 \text{ KN}$

$\Rightarrow V_A = 220.710 - R_B \cos 26.56$

$= 220.710 - (152.124) \cos 26.56$

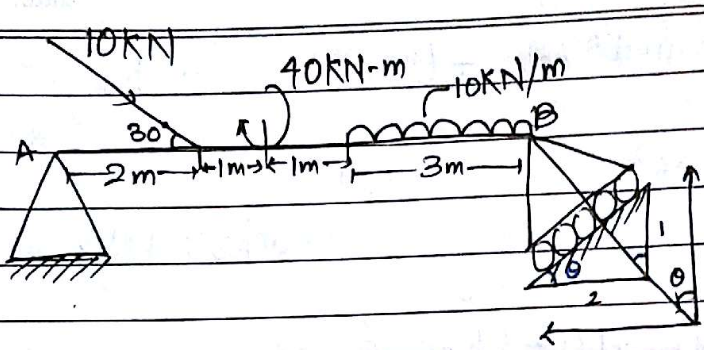
$= 84.64 \text{ KN}$

$\Rightarrow H_A = 152.124 \sin 26.56 - 70.71$

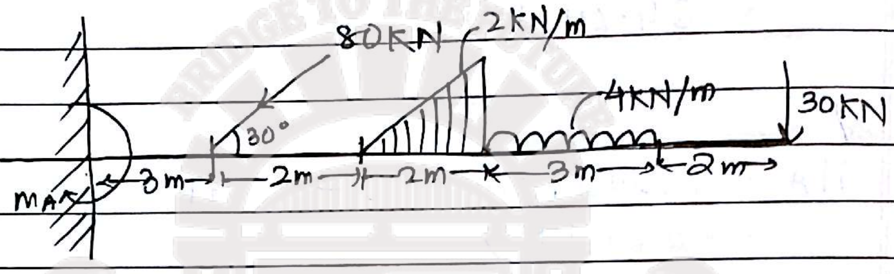
$= -2.69 \text{ KN}$

—— x —— x ——

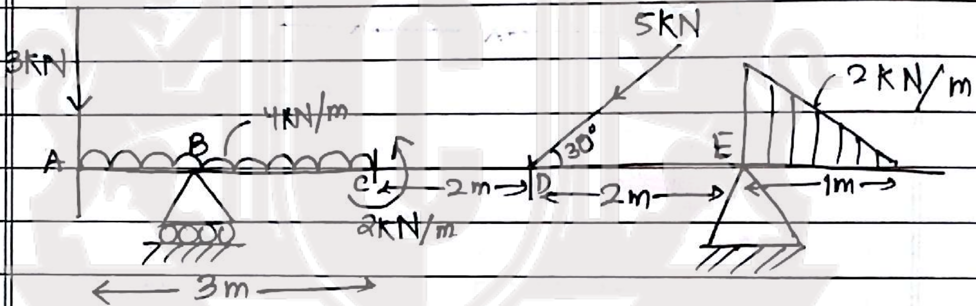
Q.1-



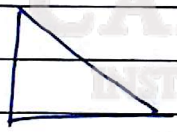
Q.2-



Q.3-



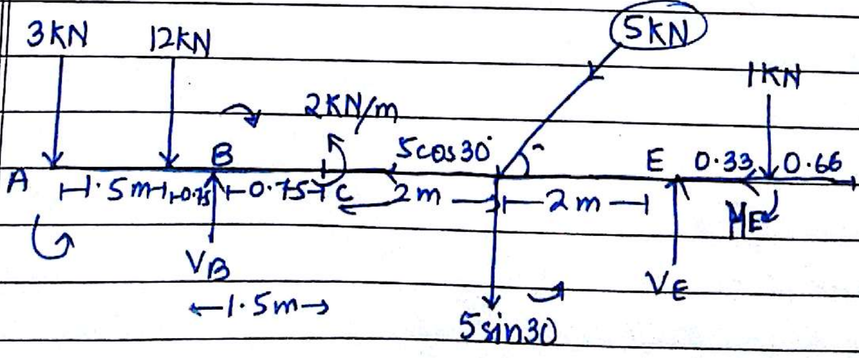
Ans 3 Point load = 4 × 3 = 12 KN.



$\frac{1}{2} bh = \frac{1}{2} \times 1 \times 2 = 1$ KN

from apex = $\frac{2}{3}d = \frac{2}{3} \times 1 = 0.66$

from base = $\frac{1}{3}d = \frac{1}{3} \times 1 = 0.33$.



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$\Sigma H = 0.$

$\Rightarrow -H_E - 5\cos 30 = 0$

$\Rightarrow H_E = -4.330\ KN$ //

$\Rightarrow \Sigma V = 0$

$\Rightarrow -3 - 12 + V_B - 5\sin 30 + V_E - 1 = 0$

$\Rightarrow V_B + V_E = 18.5\ KN$

$\Rightarrow \Sigma M_E = 0$

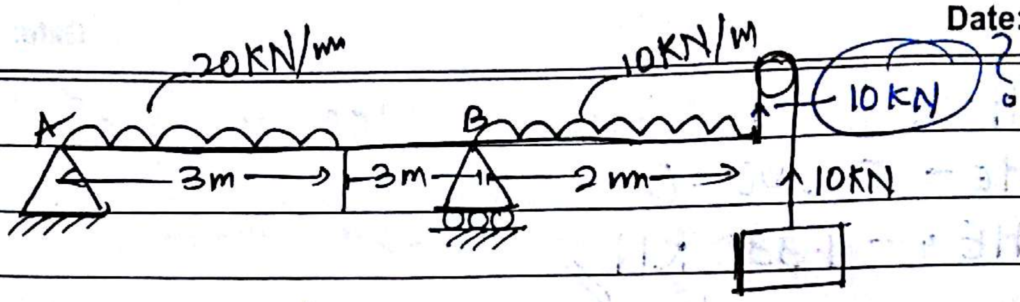
$\Rightarrow -(3)7 - (12)5.5 + (V_B)4.75 - 2 - (5\sin 30)2 + (1)0.33 = 0$

$\Rightarrow -21 - 66 + 4.75 V_B - 2 - 5 + 0.33 = 0$

$\Rightarrow V_B = 19.72\ KN$ //

$\therefore V_E = 18.5 - 19.72 = -1.22\ KN$ //

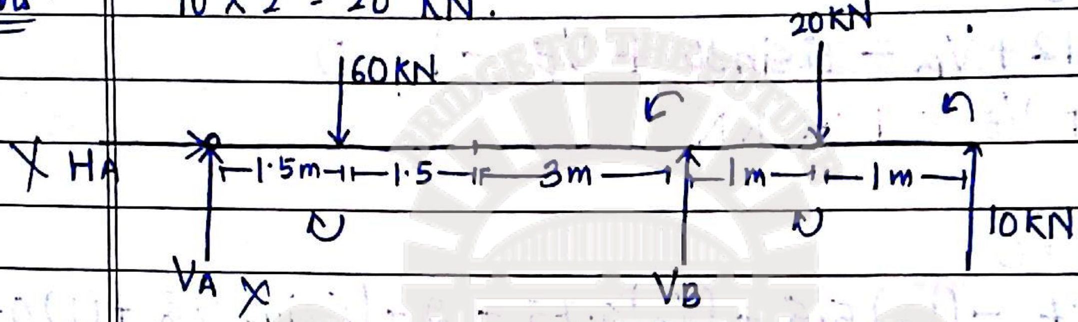
Q.



Ans -

$20 \times 3 = 60\ KN.$

$10 \times 2 = 20\ KN.$



$\Sigma H = 0.$

$\Rightarrow H_A = 0$

$\Sigma V = 0.$

$\Rightarrow V_A - 60 + V_B - 20 + 10 = 0$

$\Rightarrow V_A + V_B = 60 + 10 \rightarrow V_A + V_B = 70$ —— ①.

$\Sigma m_A = 0$

$\Rightarrow (60)\,1.5 - (V_B)(6) + (20)(7) - (10)\,8 = 0$

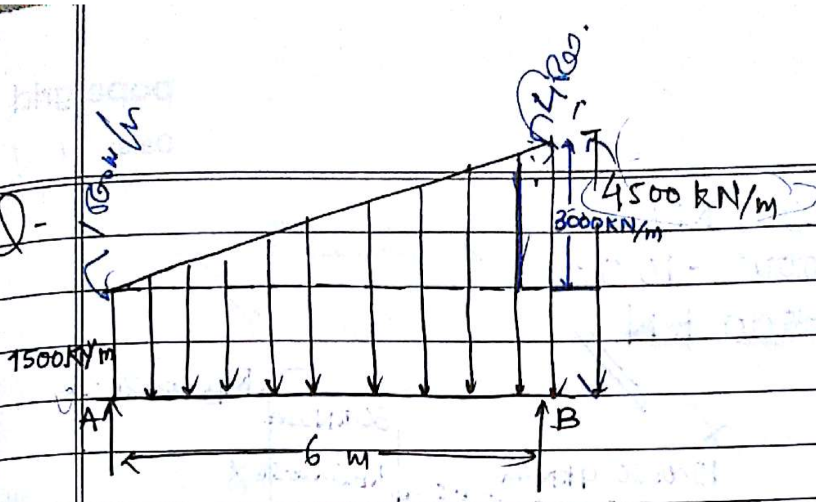
$\Rightarrow \dfrac{150}{6} = V_B$

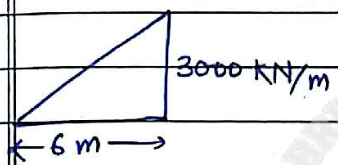
$\Rightarrow V_B = 25\ KN$ //

$V_A = 70 - 25 = 45\ KN$ //

Answer

Q-



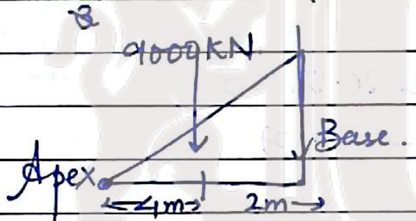


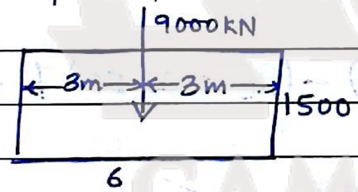
Area of $\Delta = \frac{1}{2} bh$

$= \frac{1}{2} \times 6 \times 3000 = 9000 \text{ kN}$

from base $= \frac{1}{3} d$, from apex $= \frac{2}{3} d = \frac{2}{3} \times 6 = 4 \text{ m}$

$= \frac{1}{3} \times 6 = 2 \text{ m}$

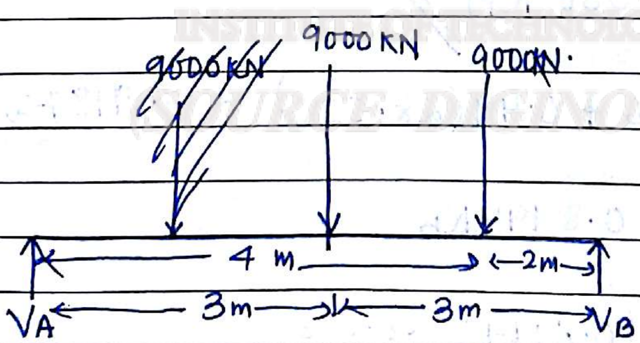




Area of rec. $= bh$

$= 1500 \times 6$

$= 9000 \text{ kN}.$



$\Sigma V = 0$

$V_A - 9000 - 9000 + V_B = 0$

$V_A + V_B = 18000$ ——— (1)

$\Sigma M_A = 0$

$\Rightarrow (9000)3 + (9000)4 - (V_B)6 = 0$

$\Rightarrow V_B = 10500 \text{ kN}$ //

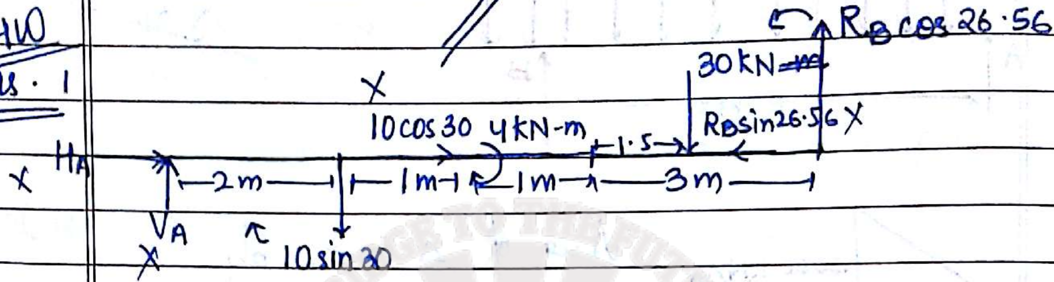
$\therefore V_B = 10500\ kN$

$V_A = 18000 - 10500$

$= 7500\ kN$

HW

Ans. 1



$\tan\theta = \frac{1}{2} \Rightarrow \theta = 26.56$

$\Sigma H = 0$

$\Rightarrow H_A + 10\cos 30 - R_B \sin 26.56 = 0$

$\Rightarrow H_A - 0.44 R_B = -8.66$ ——— (1)

$\Sigma V = 0$

$\Rightarrow V_A - 10\sin 30 - 30 + R_B \cos 26.56 = 0$

$\Rightarrow V_A + 0.8944 R_B = 35$ ——— (2)

$\Sigma M_A = 0.$

$\Rightarrow (+10\sin 30)2 + 4 + 30(5.5) - (R_B \cos 26.56)7 = 0$

$\Rightarrow \frac{179}{7} = R_B \cos 26.56$

$\Rightarrow R_B = 28.588\ kN$

$\Rightarrow H_A = -8.66 + 0.44(28.588) = 3.91872\ kN$

$\Rightarrow V_A = 35 - 0.8944 R_B$

$\Rightarrow V_A = 35 - (0.8944)(28.588)$

$\Rightarrow V_A = 9.43\ kN$

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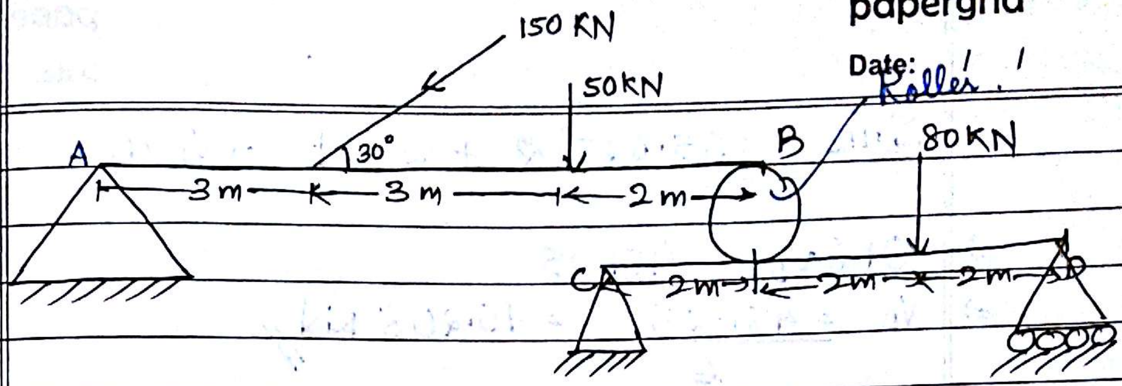
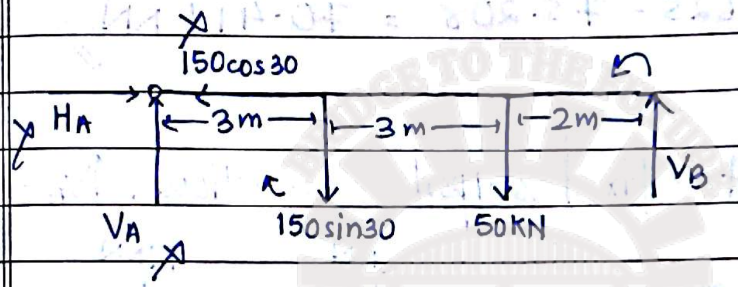
Q.



Ans- FBD @ AB line :-



$\Sigma H = 0$

$\Rightarrow H_A - 150\cos 30 = 0$

$\Rightarrow H_A = 150\cos 30 = 129.90\ KN$

$\Sigma V = 0$

$\Rightarrow V_A - 150\sin 30 - 50 + V_B = 0$

$\Rightarrow V_A + V_B = 125 \longrightarrow (1)$

$\Sigma m_A = 0$

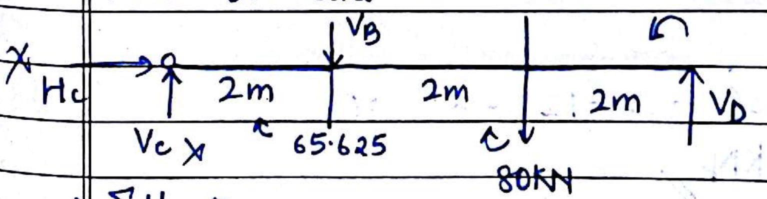
$\Rightarrow (150\sin 30)\,3 + (50)\,6 - (V_B)(8) = 0$

$\Rightarrow 8V_B = 525$

$\Rightarrow V_B = 65.625\ KN$

$\therefore V_A + V_B = 125 \Rightarrow V_A = 59.375\ KN$

FBD on line CD :-



$\Rightarrow \Sigma H = 0$

$\Rightarrow H_C = 0.$

$\Rightarrow \Sigma V = 0$

$\Rightarrow V_C - 65.625 - 80 + V_D = 0$

$\Rightarrow V_C + V_D = 145.625 \longrightarrow (1)$

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Date: / /

$\Sigma m_c = (65.625)2 + (80)4 - (V_D)6$

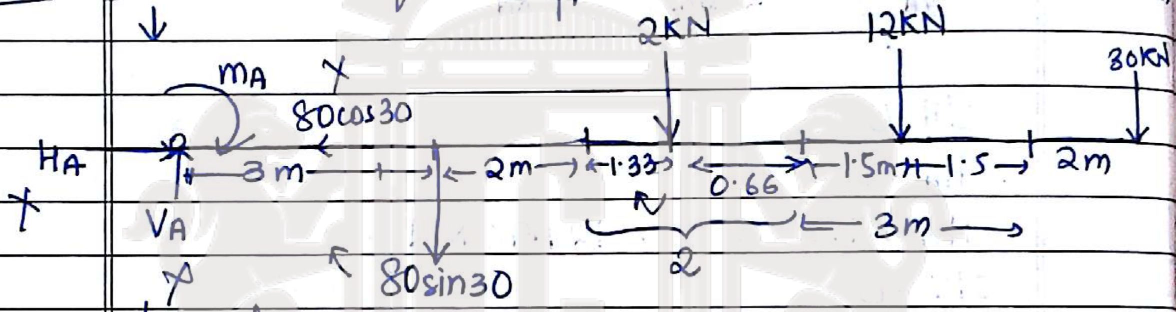
$\Rightarrow 0 + 6V_D = 451.25$

$\Rightarrow V_D = \frac{451.25}{6} = 75.208\ KN$ //

$V_c = 145.625 - 75.208 = 70.417\ KN$ //

(HW)

Ans. 2 FBD @ A → fixed support - 3 reactions. ↑V_A → H_A ↺m_A



Area of $\Delta = \frac{1}{2}bh = \frac{1}{2} \times 2 \times 2 = 2\ KN$.

from apex $= \frac{2}{3}d = \frac{2 \times 2}{3} = \frac{4}{3} = 1.33\ m$.

from base $= \frac{1}{3}d = \frac{2}{3} = 0.66\ m$.

$\Sigma H = 0$

$\Rightarrow H_A - 80\cos 30 = 0$

$\Rightarrow H_A = 69.282\ KN$ //

$\Sigma V = 0$

$\Rightarrow V_A - 80\sin 30 - 2 - 12 - 30 = 0$

$\Rightarrow V_A = 84\ KN$ //

$\Sigma m_A = ?$

$\Rightarrow (80\sin 30)3 + 2(6.33) + 12(8.5) + (30)(12)$

$\Rightarrow 594.66$ // Clockwise ☺

MODULE 4

Date: 13/5/17

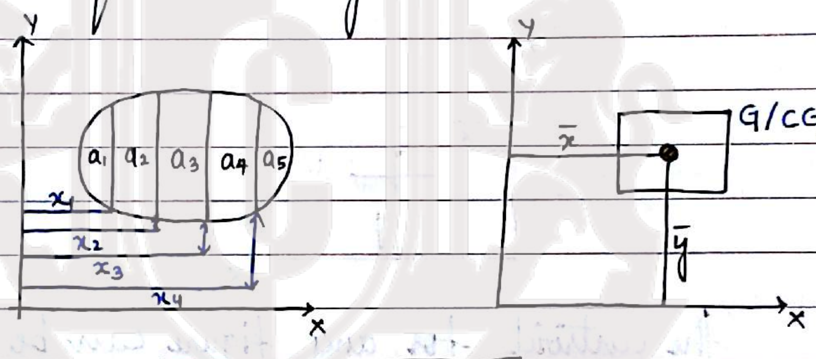
CENTROID AND MOMENT OF INERTIA OF ENGINEERING SECTION...

- **CENTRE OF GRAVITY**: Centre of gravity of a body is a point through which the whole weight of the body acts at the centre. It is denoted by centre of gravity (CG) or G.

- **CENTROID**: The point at which the total area of the plane figures is assumed to be concentrated at the centre.

Centroid and centre of gravity are at same point.

- Determination of centroid by the moment :-



$$\bar{x} = \frac{\Sigma ax}{\Sigma a} \qquad \bar{y} = \frac{\Sigma ay}{\Sigma a}$$

Let us consider a body of weight of w as shown in figure. The centre of gravity of whole figure is located at a distance $\bar{x}$ from y-axis and $\bar{y}$ from $\bar{x}$-axis.

Let us divide the whole figure into no. of elemental stripe of weight a_1, a_2, a_3, a_4 and whose centroid are located by the distance x_1, x_2, x_3, x_4 from y-axis and y_1, y_2, y_3, y_4 from x-axis.

Now by applying the theorem of moment we have

$$\Sigma ax = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4$$

$$\Sigma ay = a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4$$

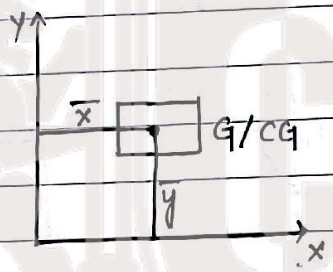
$$\Sigma a = a_1 + a_2 + a_3 + a_4$$

From y-axis, $\bar{x}$ is given by :-

$$\bar{x} = \frac{\Sigma ax}{\Sigma a}$$

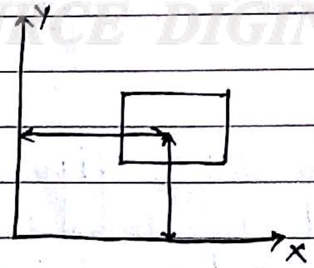
From x-axis, $\bar{y}$ is given by :-

$$\bar{y} = \frac{\Sigma ay}{\Sigma a}$$

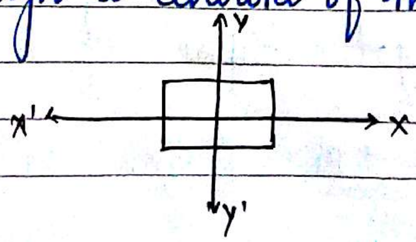


The centroid for any figure can be determined by calculating $\bar{x}$ and $\bar{y}$.

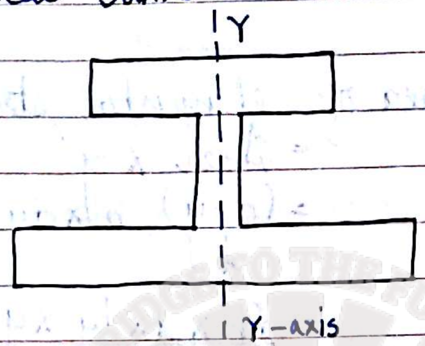
- AXIS REFERENCE : These are the axis with respect to which the centroid of the given figure is determined.



- CENTROIDAL AXIS : This is the axis which passes through a centroid of the given figure.



• SYMMETRICAL AXIS:- The axis which divides the whole figure into equal parts is known as symmetrical axis.

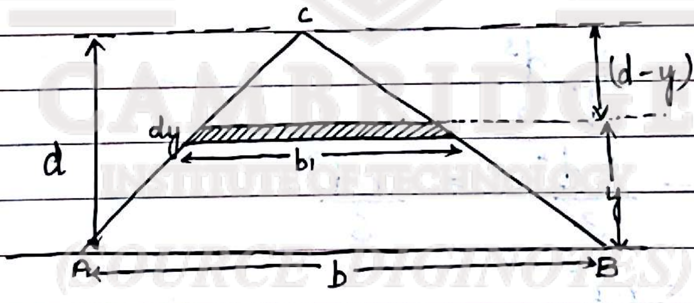


If the section is symmetrical about x-axis then $\bar{y} = 0$ and $\bar{x} = \frac{\Sigma ax}{\Sigma a}$

If the section is symmetrical about y-axis then $\bar{x} = 0$ and $\bar{y} = \frac{\Sigma ay}{\Sigma a}$

If the section is symmetrical about both axes then $\bar{x} = 0$ and $\bar{y} = 0$

• DERIVATION OF CENTROID OF TRIANGLE :



Consider a triangular lamina ABC of area ½bh as shown in figure:-

Now consider an elemental stripe of area of $(b_1 \times dy)$ at a distance of y from AB. Using property of similar Δ, $\frac{b_1}{b} = \frac{d-y}{d}$

$$\Rightarrow b_1 = \frac{(d-y)b}{d}$$

Area of elemental strip is given by $b_1 \times dy$

$$= \frac{(d-y)b}{d} dy$$

Moment of Area of elemental strip about AB =

$$= \text{Area} \times y$$

$$= \frac{(d-y)}{d} b \times dy \times y$$

$$= \frac{\cancel{d} b \times \cancel{d} \times dy}{\cancel{d}} \times y - \frac{y \times b \times dy \times y}{d}$$

$$= b \times y \times dy - \frac{y^2 \times b \times dy}{d}$$

Now area of the whole Δ is

$$\int_0^d b \times y \times dy - \int_0^d \frac{y^2 b}{d} dy$$

$$= b\left[\frac{y^2}{2}\right]_0^d - \frac{b}{d}\left[\frac{y^3}{3}\right]_0^d$$

$$= \frac{bd^2}{2} - \frac{b}{d}\frac{d^3}{3}$$

$$= \frac{bd^2}{2} - \frac{bd^2}{3} = \frac{bd^2}{6}$$

$$\therefore \Sigma ay = \frac{bd^2}{6}$$

$$\Sigma a = \frac{1}{2} \times b \times d$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{bd^2}{6_3} \times \frac{2}{bd} = \frac{d}{3} \Rightarrow \bar{y} = \frac{d}{3}$$

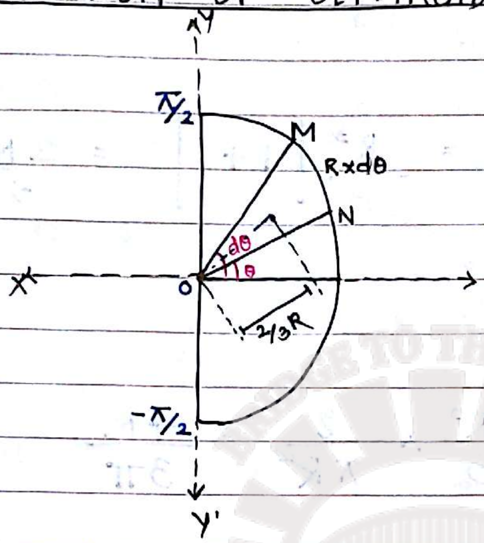
from the base $\bar{y} = d/3$

from the apex its $\bar{y} = 2d/3$

//

Date 13/5/17

• DERIVATION OF CENTROID OF SEMICIRCLE :-

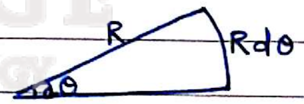


Consider a semicircular lamina of area $\frac{\pi R^2}{2}$ as shown in figure.

Now consider a triangular elemental strip OMN of area $\left(\frac{1}{2} \times R \times R d\theta\right)$ at an angle of ~~dθ~~ θ from x-axis and whose centre of gravity is @t a distance of $\frac{2}{3}R$ from the point O.

$$CG = \frac{2}{3}R$$

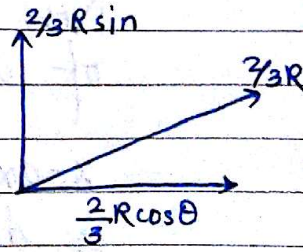
⇒ Area of OMN $= \frac{1}{2} \times R \times R d\theta$



⇒ $CG = \frac{2}{3}R$

The projection of centre of gravity on x-axis is given by $\frac{2}{3}R\cos\theta$.

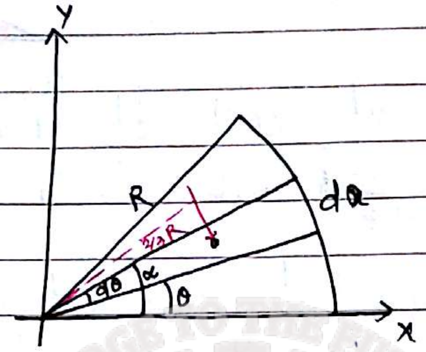
Moment of area of elemental strip about y-axis is given by :-



$$A \times \frac{2}{3}R \times \cos\theta$$

$$\Rightarrow \frac{1}{2} \times R^2 d\theta \, \frac{2}{3} R\cos = \frac{R^3}{3}\cos\theta \, d\theta$$

• DERIVATION OF CENTROID OF SECTOR OF A CIRCLE:



Consider a sector of circular lamina as shown in figure.

Consider a Δangular elementary strip ($\frac{1}{2}R^2 d\theta$) at an angle of θ from x-axis whose centre of gravity is $\frac{2}{3}R$ from point O & Projection is $\frac{2}{3}R\cos\theta$.

Area of strip $= \frac{1}{2} \times R \times R \times d\theta$

Area of sector $= \int_0^{\alpha} \frac{1}{2} R^2 \, d\theta = \frac{1}{2} R^2 \alpha \quad \left\{ \left[\frac{1}{2} R^2 \theta \right]_0^{\alpha} \right\}$

Moment of area of elementary strip $= \frac{2}{3} R\cos\theta \times \frac{1}{2} R^2 d\theta$

$= \frac{R^3 \cos\theta \, d\theta}{3}$

Sum of moment of all strips about y-axis

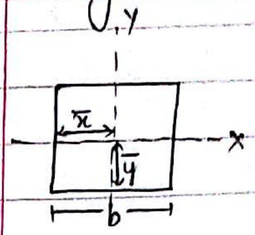
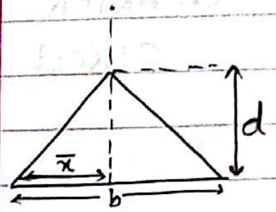
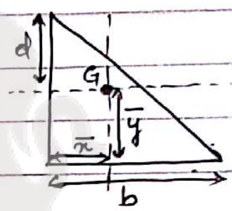
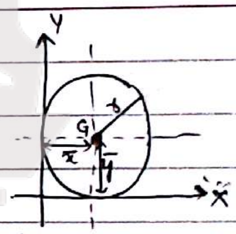
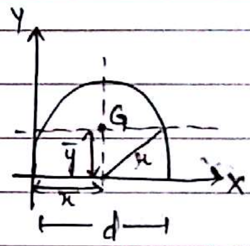
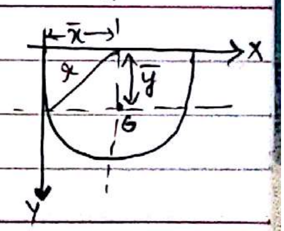
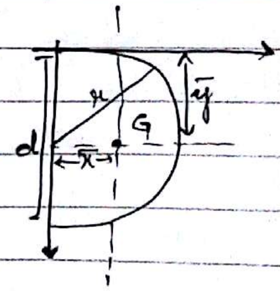
$= \int_0^{\alpha} \frac{R^3 \cos\theta}{3} \, d\theta = \frac{R^3}{3} \left[\sin\theta \right]_0^{\alpha}$

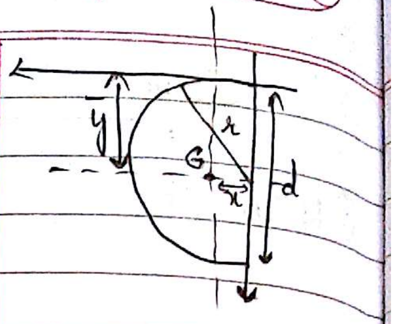
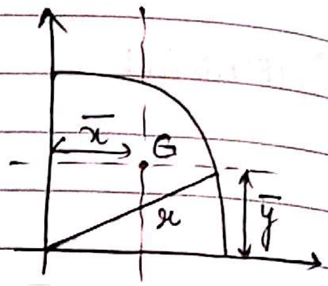
$\Sigma ax = \frac{R^3}{3} \left\{ [\sin\alpha] - \sin 0 \right\} = \frac{R^3 \sin\alpha}{3}$

$\Sigma a = \frac{R^2 \alpha}{2}$

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{R^3 \sin\alpha}{3} \times \frac{2}{R^2 \alpha} = \frac{2R}{3\alpha} \sin\alpha$

FORMULAE

Shape	Area	$\bar{x}$	$\bar{y}$	Figure
① RECTANGLE	$b \times d$	$\frac{b}{2}$	$\frac{d}{2}$	
② TRIANGLE	$\frac{1}{2} \times b \times d$	$\frac{b}{2}$	$\frac{d}{3}$	
③ RIGHT-ANGLED TRIANGLE	$\frac{1}{2} \times b \times d$	$\frac{b}{3}$	$\frac{d}{3}$	
④ CIRCLE	πr^2	$\bar{x} = r$	$\bar{y} = r$	
⑤ SEMI-CIRCLE	$\frac{\pi r^2}{2}$	$\frac{d}{2}$	$\frac{4r}{3\pi}$	
		$\frac{d}{2}$	$\frac{-4r}{3\pi}$	
		$\frac{4r}{3\pi}$	$\frac{d}{2}$	

		$-\frac{4r}{3\pi}$	$\frac{d}{2}$	
QUARTER CIRCLE	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	

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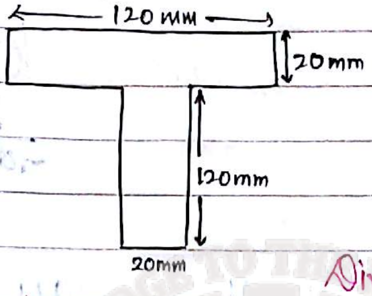
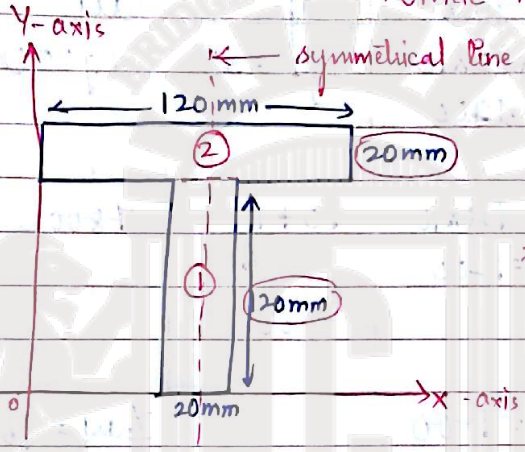
Date ______
Page ______

Q.1- Find the centroid for the given figure shown :-



Ans-



Divide the figure into possible no. of units.

← symmetrical line about y-axis so $\bar{y}$ we have to calculate.

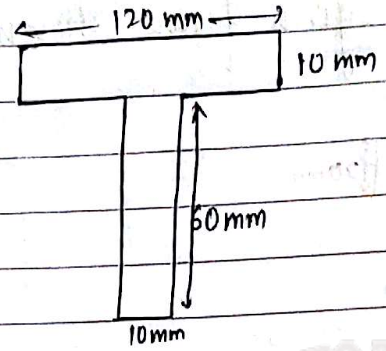
for calculating y, set the reference axis as (x)
$\bar{x}$ value will be direct.

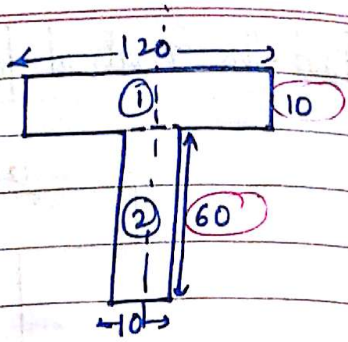
Component	Area (mm²)	y (mm)	ay (mm³)
(1) Rectangle	120 x 20 = 2400 mm²	$\frac{120}{2}$ = 60 mm	144000
(2) Rectangle	120 x 20 = 2400 mm²	$120 + \frac{20}{2}$ = 130 mm	312000
	$\Sigma a = 4800$		$\Sigma ay = 456000$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{456000}{4800} = 95 \text{ mm}$$

$$\bar{x} = (\text{Reference is } y\text{-axis}) = \frac{120}{2} = 60 \text{ mm}$$

Q.2-





Ans-

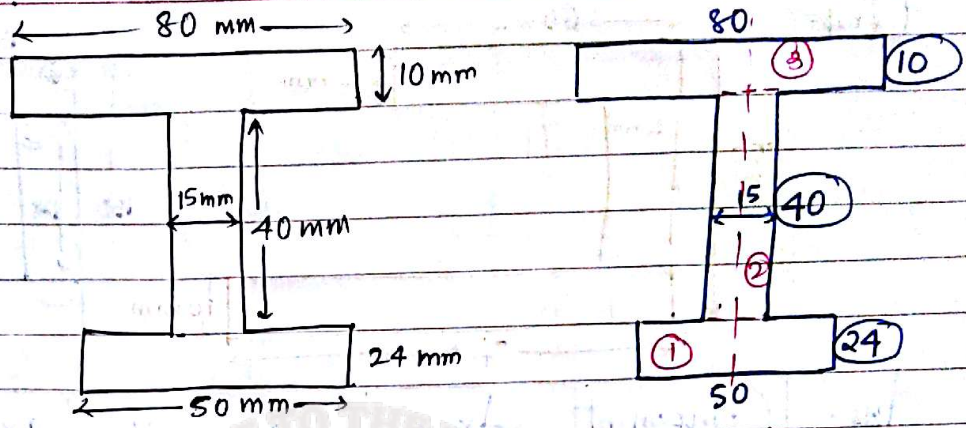
Component	Area	y	ay
② Rectangle	60×10 = 600	$\frac{60}{2} = 30$	18000
① Rectangle	120×10 = 1200	$60 + \frac{10}{2} = 65$	78000

$\Sigma a = 1800$ $\Sigma ay = 96000$

$$\therefore \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{96000}{1800} = 53.33 \text{ mm.}$$

$$\bar{x} = \frac{120}{2} = 60 \text{ mm}$$

Q.3-



Ans-

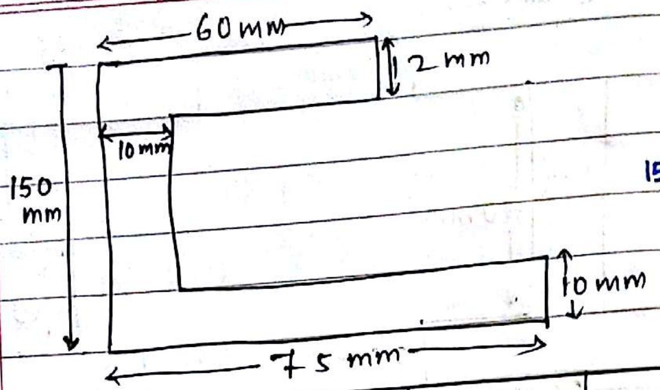
Component	Area	y	ay
(1) Rectangle	24x50 = 1200	$\frac{24}{2} = 12$	14400
(2) Rectangle	15x40 = 600	$24 + \frac{40}{2} = 44$	26400
(3) Rectangle	80x10 = 800	$24+40+\frac{10}{2} = 69$	55200
	$\Sigma a = 2600$		$\Sigma ay = 96000$

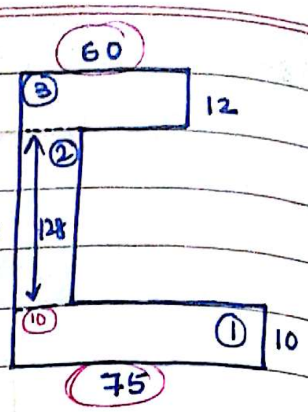
∴ Since the figure is symmetrical about y-axis :-

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{96000}{2600} = 36.923 \text{ mm}$$

$$\bar{x} = \frac{80}{2} = 40 \text{ mm}$$

Q.4-





Ans-

Component	Area	x	y	ax	ay
① Rectangle	75×10 = 750	$\frac{75}{2}$ = 37.5	$\frac{10}{2}$ = 5	28125	3750
② Rectangle	128×10 = 1280	$\frac{10}{2}$ = 5	$10+\frac{128}{2}$ = 74	6400	94720
③ Rectangle	60×12 = 720	$\frac{60}{2}$ = 30	$10+128+\frac{12}{2}$ = 144	21600	103680

$\Sigma a = 2750$

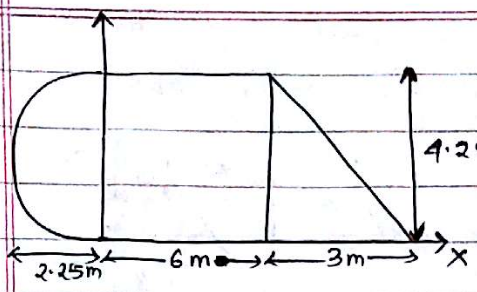
$\Sigma ax = 56125$

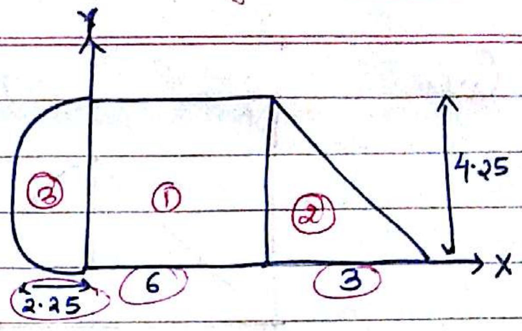
$\Sigma ay = 202150$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{202150}{2750} = 73.5 \text{ mm}$$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{56125}{2750} = 20.409 \text{ mm}$$

Imp Q.15-





Ans-

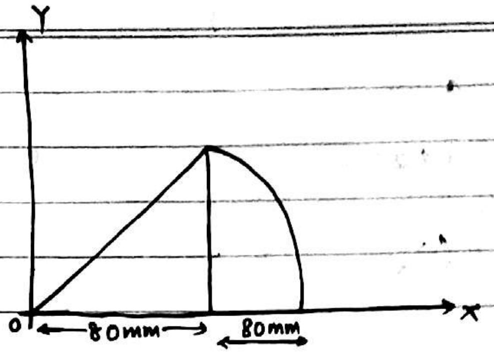
Component	Area	X	Y	ax	ay
① Rectangle	6×4.25 = 25.5	$\frac{6}{2} = 3$	$\frac{4.25}{2}$ = 2.125	76.5	54.1875
② Triangle	$\frac{1}{2} \times 3 \times 4.25$ = 6.375	$\frac{3}{3} = 1$	$\frac{4.25}{3}$ = 1.416	6.375	9.027
③ Semicircle	$\frac{\pi \times (2.25)^2}{2}$ $\frac{3.14 \times (2.25)^2}{2}$ = 7.9481	$\frac{-4 \times 2.25}{3 \times 3.14}$ = -0.95	$\frac{4.25}{2}$ = 2.125	-7.55	16.889

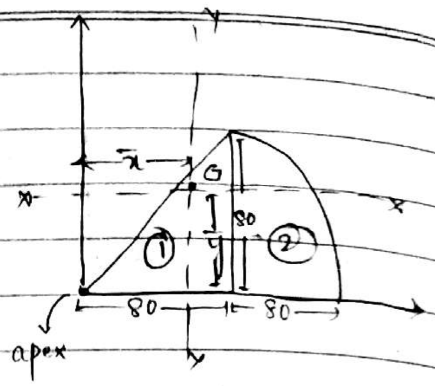
$\Sigma a = 39.8231$
$\Sigma ax = 75.325$
$\Sigma ay = 80.1035$

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{75.325}{39.8231} = 1.8914$ mm

$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{80.1035}{39.8231} = 2.0114$ mm

Q.6-





Ans-

	Component	Area (a)	x	y	ax	ay
1	Triangle	$\frac{1}{2} \times 80 \times 80$ = 3200	$\frac{2}{3} \times 80$ = 53.33	$\frac{80}{3}$ = 26.66	170656	85312
2	Quarter Circle	$\frac{3.14 \times (80)^2}{4}$ = 5024	$80 + \frac{4 \times 80}{3 \times 3.14}$ = 113.970	$\frac{4 \times 80}{3 \times 3.14}$ = 33.97	572585.28	170666.6

$\Sigma a = 8224$

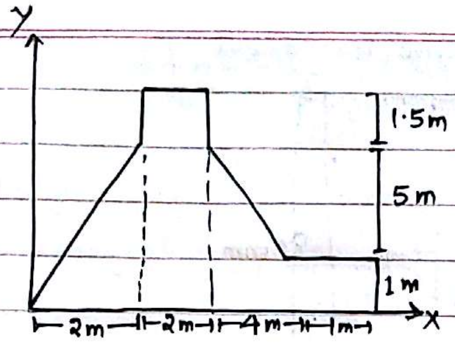
$\Sigma ay = 255978.66$

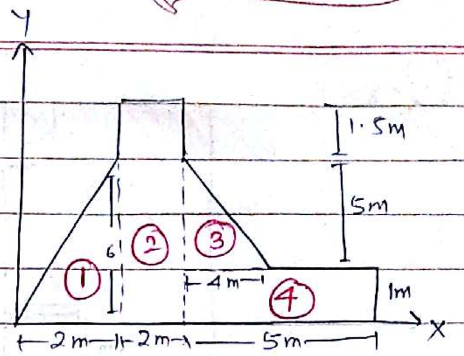
$\Sigma ax = 743241.28$

$\therefore \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{255978.66}{8224} = 31.1258 mm$

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{743241.28}{8224} = 90.374 mm$

Q.7-





Ans. Not symmetrical about any of the axis :-

Component	Area (a)	x	y	ax	ay
① Triangle	$\frac{1}{2} \times 2 \times 6$ = 6	$\frac{2}{3} \times 2$ = 1.33	$\frac{6}{3} = 2$	7.98	12
② Rectangle	7.5×2 = 15	$2 + \frac{2}{2}$ = 3	$\frac{7.5}{2}$ = 3.75	45	56.25
③ Triangle	$\frac{1}{2} \times 4 \times 5$ = 10	$2 + 2 + \frac{4}{3}$ = 5.33	$1 + \frac{5}{3}$ = 2.66	53.3	26.6
④ Rectangle	5×1 = 5	$2 + 2 + \frac{5}{2}$ = 6.5	$\frac{1}{2}$ = 0.5	32.5	2.5

$\Sigma a = 36$

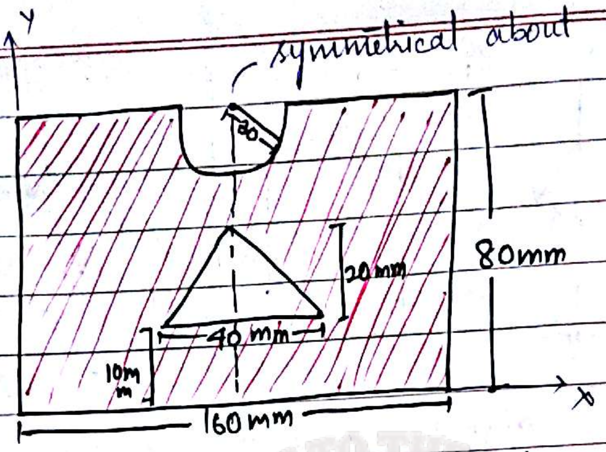
$\Sigma ax = 138.78$

$\Sigma ay = 97.35$

$$\therefore \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{138.78}{36} = 3.855 \text{ m}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{97.35}{36} = 2.704 \text{ m}$$

Q.8-



Determine the Centroid for the shaded area.

Ans-

Component	Area (mm^2)	y (mm)	Σay
① Rectangle.	$160 \times 80 = 12800$	$\frac{80}{2} = 40$	512000
② Triangle.	$-\frac{1}{2} \times 20 \times 40 = -400$	$10 + \frac{20}{3} = 16.66$	-6664
③ Semi-circle.	$-\frac{3.14 \times (30)^2}{2} = -1413$	$80 - \frac{4 \times 30}{3 \times 3.14} = 67.261$	-95039.793

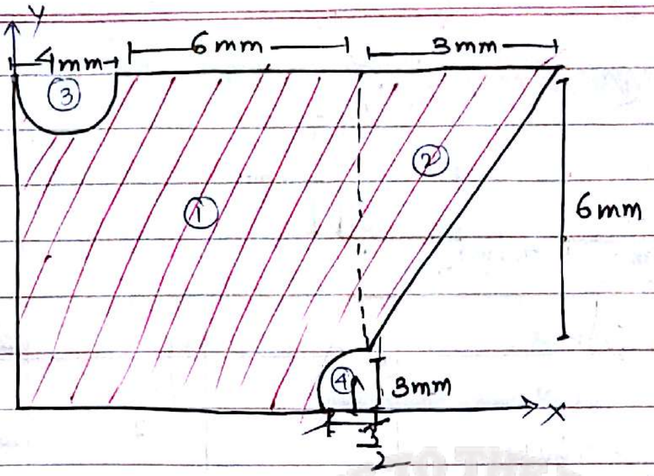
$\therefore$ $\Sigma a = 10987$

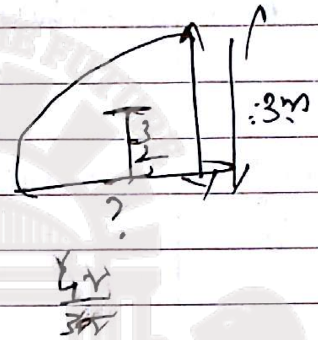
$\Sigma ay = 410296.207$

$\therefore \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{410296.207}{10987} = 37.34$ mm

$\bar{x} = \frac{160}{2} = 80$ mm

Q.9-





Ans-

Component	Area	x	y	Σax	Σay
① Rectangle	$10 \times 9 = 90$	$\frac{10}{2} = 5$	$\frac{9}{2} = 4.5$	450	405
② Triangle	$\frac{1}{2} \times 3 \times 6 = 9$	$10 + \frac{3}{3} = 11$	$3 + \frac{2}{3} \times 6 = 7$	99	63
③ Semi-circle	$-\frac{3.14 \times (2)^2}{2} = -6.28$	$\frac{4}{2} = 2$	$\frac{4 \times 2}{3 \times 3.14} = 0.84$	-12.56	-5.2752
④ Quater circle	$-\frac{3.14 \times (3)^2}{4} = -7.065$	$10 - \frac{4 \times 3}{3 \times 3.14} = 8.726$	$\frac{3}{2} = 1.5$	-61.649	-10.5975

$\Sigma a = 85.655$

$\Sigma ax = 474.791$

$\Sigma ay = 452.1273$

$\therefore \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{474.791}{85.655} = 5.543\ mm$

$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{452.1273}{85.655} = 5.278\ mm$

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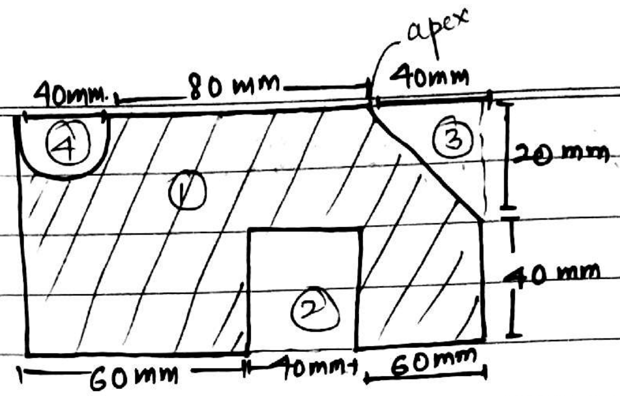
Q. 10-



Ans-

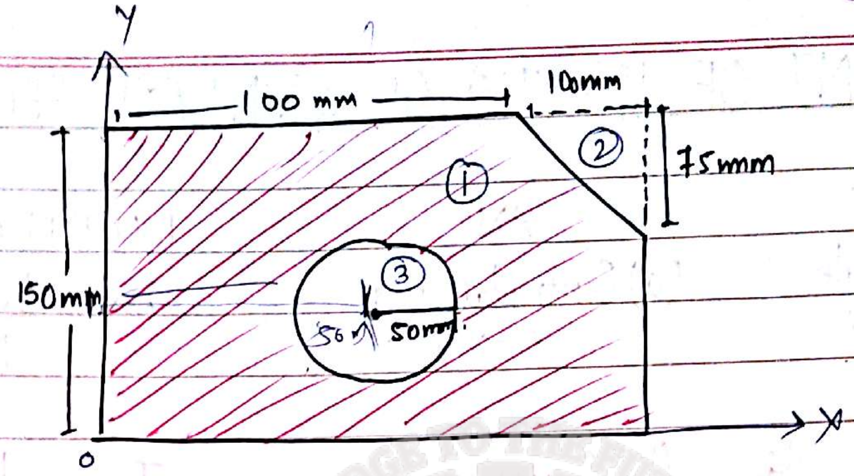
Component	Area	x	y	Σax	Σa…
① Rectangle	160×60 = 9600	$\frac{160}{2}$ = 80	$\frac{60}{2}$ = 30	768000	288 00
② Rectangle	−40×40 = 1600	$\frac{40}{2}$+60 = 80	$\frac{40}{2}$ = 20	(−128000)	(−3200
③ Triangle	$-\frac{1}{2}\times 20\times 40$ (10) = −400	$120+\frac{2}{3}\times 40$ = 146.66	40+ $\frac{2}{3}\times 20$ = 53.33	(−58664)	−2133
④ Semi-circle	$-\frac{3.14\times(20)^2}{2}$ = −628	$\frac{40}{2}$ = 20	$60-\frac{4\times 20}{3\times 3.14}$ = 51.507	−12560	−3234 / 396

Σa = 10172.
Σax = 593896.
Σay = 202321.604

$$\therefore \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{202321.604}{10172} = 19.89 \text{ mm}$$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = 58.385 \text{ mm}$$

Q.11-

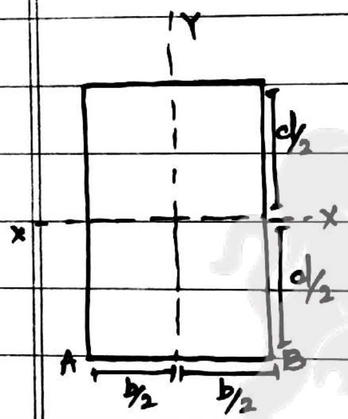


Ans- Component	Area (mm²)	x (mm)	y (mm)	Σax	ay
① Rectangle	150 × 200 = 30000	$\frac{200}{2}$ = 100	$\frac{150}{2}$ = 75	3000000	2250000
② Triangle	$\frac{1}{2}$ × 75 × 100 (50) = 3750	100 + $\frac{2}{3}$ × 100 = 166.66	75 + $\frac{2}{3}$ × 75 (25) = 125	624975	468750
③ Circle	3.14 × (50)² = 7850	50 + 50 = 100			

Σa = 41600
Σax
Σay =

MOMENT OF INERTIA OF STANDARD SECTIONS:

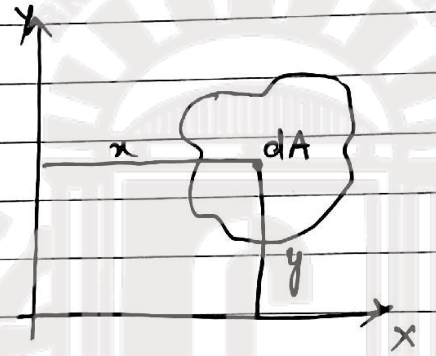
SHAPE	AXIS	MOMENT OF INERTIA
(1) Rectangle	(a) Centroidal axis X – X	$I_{xx} = \frac{bd^3}{12}$
	(b) Centroidal axis Y – Y	$I_{yy} = \frac{db^3}{12}$
	(c) A – B	



Difference b/w centroid & moment of inertia

* Moment of Inertia signifies shape and distribution of area.

The word inertia represent the property of materials by virtue of which it resists any change in the state of rest or uniform motion or rotational motion.



Consider a smaller area dA as shown in the figure of the Lamina with x and y as reference axis. The product of dA × y represent the moment

If dA × y × y it becomes moment of inertia.

dA × y represent 1st moment of area.
dA × y × y represent 2nd moment of area.

The moment of inertia of entire lamina along x-axis is given by

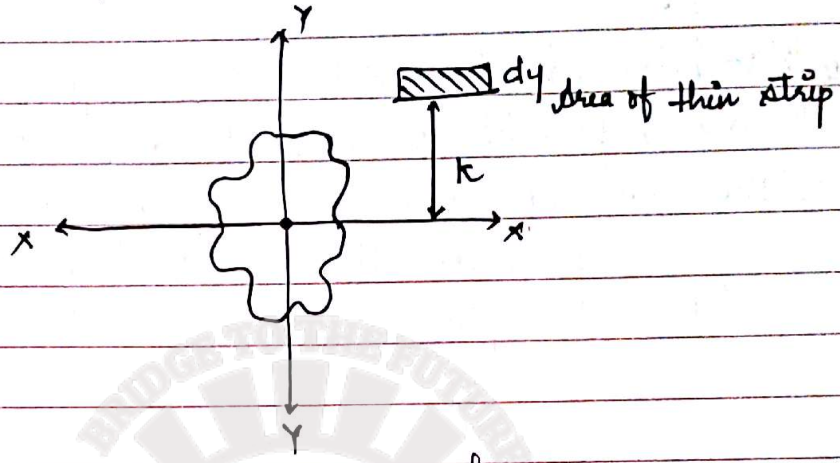
$$I_{xx} = \int dA \times y^2 \quad (mm)^4$$

Similarly moment of inertia along y-axis is given by

$$I_{yy} = \int dA \times x^2 \quad (mm)^4$$

The SI unit of moment of inertia is $(mm)^4$

- **RADIUS OF GYRATION :**



Consider the area as shown in figure whose moment of inertia along x and y - axis are I_{xx} and I_{yy}.
In order to maintain the same moment of inertia along the reference axis, the area must be thrown to a new location such that the elementary strips maintains the constant distance k.
Then the distance k can be defined as the distance of area @ which if it squeez and placed so that there is no change in moment of inertia is called as the radius of gyration k ie

$$I_{xx} = \int y^2 \, dA$$

$$I_{yy} = \int x^2 \, dA.$$

$$\therefore I_{xx} = k_{xx}^2 \, dA$$

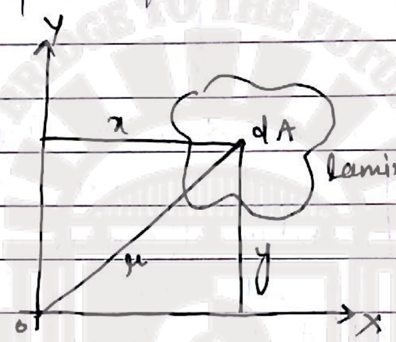
$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} \text{ mm}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{A}} \text{ mm.}$$

Imp

• PERPENDICULAR AXIS THEOREM :-

Statement :- Moment of inertia of a given area about an axis ⊥ to the plane through a point is equal to the sum of moment of inertia of that area about any 2 ⊥ axis through that point. The axis will be in the plane of area.



Let x is the distance of dA from y-axis and y be the distance of dA from x-axis.
Then from fig.

$$r^2 = x^2 + y^2 \quad \text{(Pythagoras)}$$

If z is the axis through a coordinate system at O, then according to the theorem we need to prove

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Proof : Let dA be an elemental area @ a distance r from O.

Then moment of inertia along z-axis is given by

$$I_{zz} = \Sigma dA \times r^2$$

We know that $r^2 = x^2 + y^2$

$$I_{zz} = \Sigma (x^2 + y^2)\, dA$$

$$= \Sigma x^2 dA + \Sigma y^2 dA$$

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$ // Hence proved.

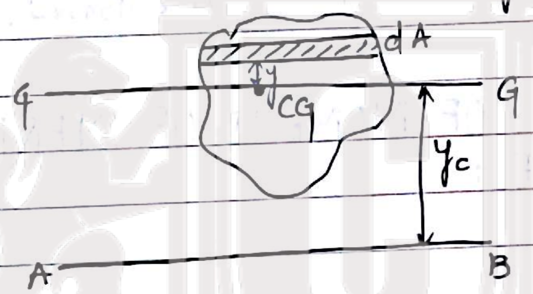
Page

Imp. • PARALLEL - AXIS THEOREM :-

Statement :- Moment of inertia of an area about any axis in a plane of area dA is equal to sum of moment of inertia about the centroidal parallel axis of the area and the product of the area and the square of distance of centroid of the area from the axis.

Then according to the theorem wrt to figure we need to prove:-

$$I_{AB} = I_{GG} + Ay_c^2$$



Proof

I_{AB} = moment of inertia of the area about the axis AB.

I_{GG} = moment of inertia of the area about the axis GG which is ∥ to AB.

y_c = distance of centroid G from the axis GG

Consider an elementary strip of area dA ∥ to AB and it is at a distance of y from GG axis.

Then acc. to theorem then $I_{AB} = \Sigma (y+y_c)^2 \cdot dA$

$$\Rightarrow I_{AB} = \Sigma (y^2 + 2yy_c + y_c^2)\, dA$$

$$\Rightarrow I_{AB} = \Sigma y^2 dA + \Sigma 2yy_c dA + \Sigma y_c^2 \quad \text{——} (1)$$

Now, $\Sigma y^2 dA$ = moment of inertia along the centroidal axis GG = I_{GG}.

$$\therefore \Sigma y^2 dA = I_{GG} \quad \text{——} (2)$$

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- ## PROCEDURE TO SOLVE THE PROBLEMS :-

Component	Area	x	y	Ax	Ay	Ax^2	Ay^2	I_x	I_y
	ΣA			ΣAx	ΣAy	ΣAx^2	ΣAy^2	ΣI_x	ΣI_y

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} \qquad \bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

$$I_{1-1} = I_x + Ay^2$$

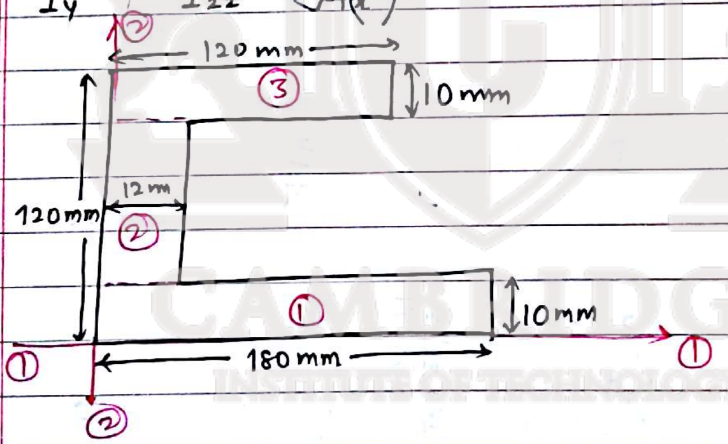
$$I_{1-1} = \Sigma I_x + \Sigma Ay^2$$

$$\bar{I}_x = I_{1-1} - \Sigma A\bar{y}^2 \Rightarrow I_{1-1} - \Sigma A \left(\frac{\Sigma Ay}{\Sigma A}\right)^2$$

$$I_{22} = \Sigma I_x + \Sigma Ax^2$$

$$\bar{I}_y = I_{22} - \Sigma A\bar{x}^2$$

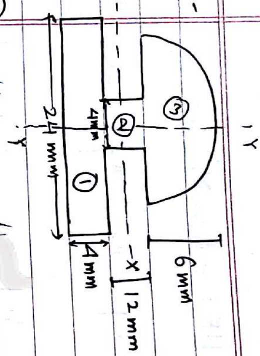
Q.1-



Ans.	Area (mm^2)	x	y	Ax	Ay	Ay^2	Ax^2	I_x	I_y
①	$180 \times 10 = 1800$	$\frac{180}{2} = 90$	$\frac{10}{2} = 5$	162000	9000	45000	1458×10^4	$\frac{180 \times 10^3}{12}$	$\frac{10 \times 180^3}{2}$
②	$100 \times 12 = 1200$	$\frac{12}{2} = 6$	$10 + \frac{100}{2} = 60$	7200	72000	4320000	432×10^3	$\frac{12 \times 100^3}{12}$	$\frac{100 \times 12^3}{12}$
③	$120 \times 10 = 1200$	$\frac{120}{2} = 60$	$10 + 100 + \frac{10}{2} = 115$	72000	138000	15870000	432×10^4	$\frac{120 \times 10^3}{12}$	$\frac{10 \times 120^3}{12}$

Imp

Q.1-



Component	Area	~~x~~	y	~~Ax~~	Ay	Ay^2	I_x (mm)	I_y (mm^4)
Rectangle	$24 \times 4 = 96$		$\frac{4}{2} = 2$.		192.	384	$\frac{24 \times (4)^3}{12} = 128$	$\frac{(24)^3 \times 4}{12} = 4608$
Rectangle.	$4 \times 12 = 48$		$4 + \frac{12}{2} = 10$		480	4800	$\frac{(4)(12)^3}{12} = 576$	$\frac{(4)^3\, 12}{12} = 64$
Semicircle.	$\frac{3.14 \times (6)^2}{2} = 56.52$		$4+12+\frac{4 \times 6}{3 \times 3.14} = 16 + \frac{24}{9.42} = 18.547$		1048.276	19442.383	$0.11 \times (6)^4 = 142.56$	$\frac{3.14 \times 6^4}{8} = 508.68$.
	$\Sigma A = 200.52$.				$\Sigma Ay = 1720.276$	$\Sigma Ay^2 = 24626.383$	$\Sigma I_x = 846.56$	$\Sigma I_y = 5180.68$.

$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = 8.579$

$I_{xx} = \Sigma \bar{I}_x + \Sigma A y^2$

$\frac{2 \to L}{I_y}$

MODULE - 5

KINEMATICS

classmate Date Page

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→ Kinematics : without considering forces.
→ Kinetics : considering the forces.
→ Displacement : rate of change of position.
→ Linear Velocity
→ Angular Velocity
→ Uniform velocity
→ Non-uniform velocity
→ Average Velocity

$$V_{av} = \frac{\Delta s}{\Delta t}$$

→ Instantaneous Velocity

$$V_{in} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

→ Deaccelaration / Retardation : Acceleration with decreasing velocity.

Imp → Derivation of equations of motions:-

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2.$$

$$v^2 - u^2 = 2as.$$

→ Acceleration due to gravity (g) -

$$a = \frac{dv}{dt} \qquad v = \frac{dx}{dt}$$

$\int a\,dx = \int v\,dv.$

$dv = a\,dt.$ $\qquad$ $dx = v\,dt$

$\Rightarrow ax = \frac{v^2}{2}$ $\quad \int_u^v dv = a\int_0^t dt$ $\qquad$ $dx = (u+at)\,dt$

$$\int_0^s dx = \int_0^t (u+at)\,dt$$

$\Rightarrow 2as = v^2$ $\quad (v-u) = a(t-0)$

$\Rightarrow 2as = v^2 - u^2$ $\quad v = u + at.$ $\qquad$ $s - 0 = \left(ut + \frac{1}{2}at^2\right)_0^t$

$$a = v \cdot \frac{dv}{dx} \qquad S = ut + \frac{at^2}{2}.$$

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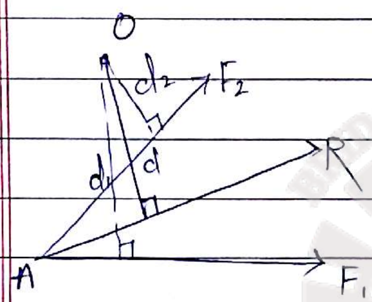
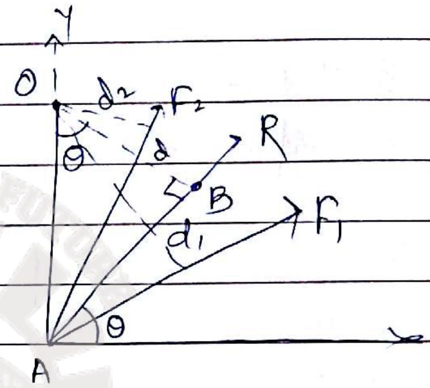
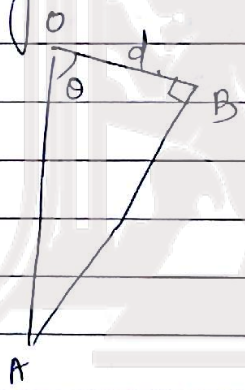
Varignon's Theorem :-

Statement :- The algebraic sum of all the moments of a system of co-planar forces about a moment centre is equal to the moment of the resultant force about the same moment centre.





$\angle AOB$ = angle made by resultant with axis = θ.



from ΔAOB :-

$$\cos\theta = \frac{d}{OA}$$

$\Rightarrow d = OA\cos\theta$.

$\Rightarrow R \times d = R \times OA\cos\theta$.

$\Rightarrow R \times d = R_x\, OA$. — ③(i)

R_x - Component of R along x-axis.

Similarly $F_1 \times d_1 = F_{1x}\, OA$. — ①

$F_2 \times d_2 = F_{2x} \times OA$ — ②.

∴ Adding eq ① and ②.

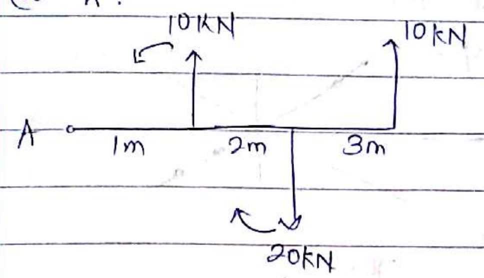
$$(F_1 \times d_1) + (F_2 \times d_2) = (F_{1x} + F_{2x})\, OA.$$

$$(F_1 \times d_1) + (F_2 \times d_2) = (R_x)\, OA. \quad \text{— ③(ii)}$$

$$(F_1 \times d_1) + (F_2 \times d_2) = R \times d$$

Hence proved ☺

@ A.

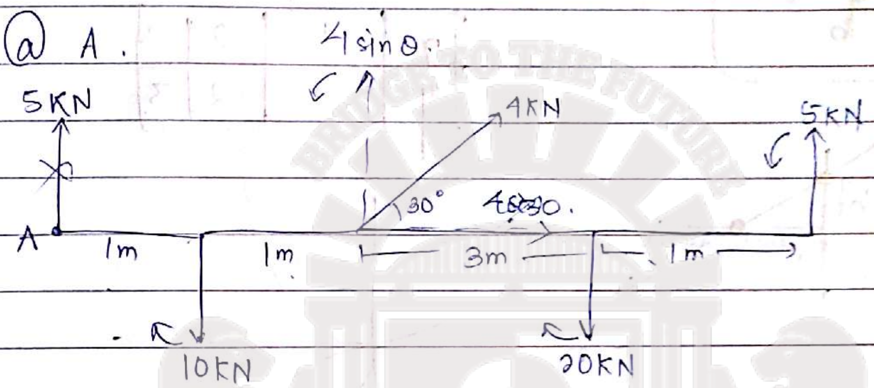


$m_A = (-10)1 + (20 \times 3) - (10)5.$

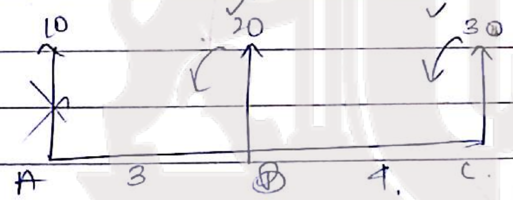
$= -10 + 60 - 50$

$= 0$

@ A.



$\therefore\ m_A = (-5)(6) + (20)(5) - (4\sin\theta)(2) + 10(1)$



@ A.

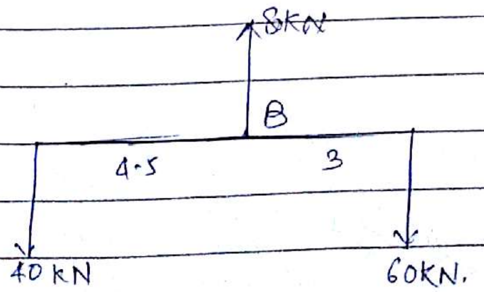
$m_A = (-20)3 - (30)7$

$= -60 - 210 = -270.$

$d = \frac{|m_A|}{R}.$

$x \text{ intercept} = \frac{|m_A|}{\Sigma V} = \frac{|M_A|}{\Sigma V} =$

@ A :-



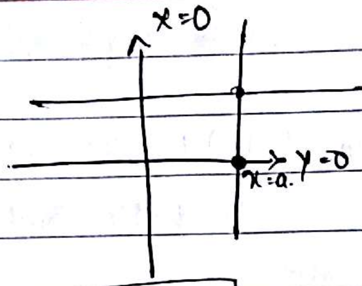
$\Sigma H = 0$

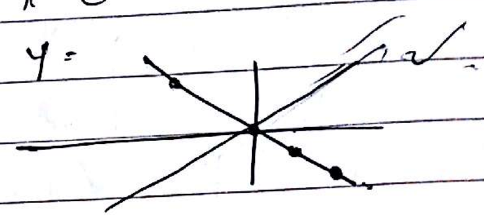
$\Sigma V = 80 - 40 - 60$

$= -20.$

$R = \sqrt{0^2 + (-20)^2} = \sqrt{400} = 20KN.$

$m_A =$

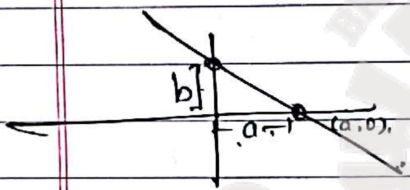


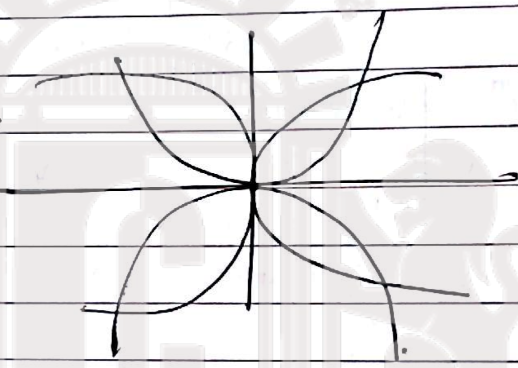


$$\frac{x}{a} + \frac{y}{b} = 1.$$

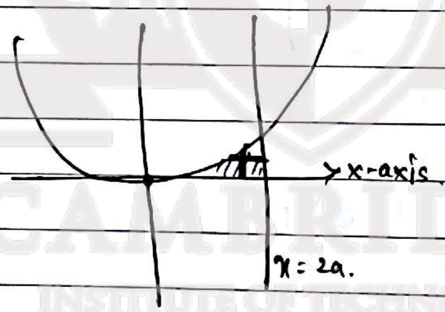
$x = -y.$

x	1	2	-3
y	-1	-2	3





$$\iint_A xy \, dx\,dy.$$



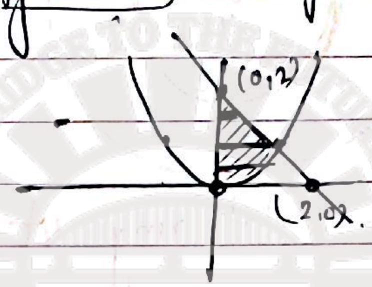
$$\iint.$$

$$\int_0^1 \int_{x^2}^{2x} x\,y \, dx\,dy.$$

$\Rightarrow$ $y = x^2$ to $2x$

$y = x^2$ $\qquad$ $y = 2x.$ $\qquad$ $y + x = 2.$ $\qquad$ $\frac{y}{2} + \frac{x}{2} = 1.$

x	0	1	-1
y	0	1	1



$x = r\sin\theta.$

$y = r\cos\theta.$

$r = \sqrt{\;}$

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx.$$

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\,dx.$$

$$\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$$

$$\Gamma m = \int_0^\infty e^{-t}\, t^{m-1}\,dt \qquad t = x^2.$$

$$= \int_0^\infty e^{-x^2}\, x^2$$

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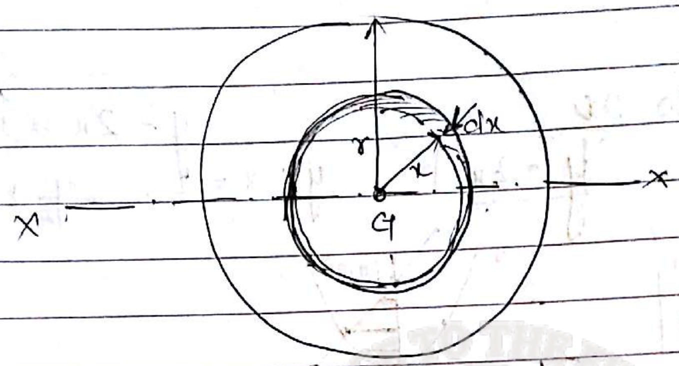
MI of Circle



Area of elemental ring $dA = (2\pi x)dx$.

MI of this element from centre $= \Sigma x^2 dA$.

$= (2\pi x dx) x^2$

$= 2\pi x^3 dx$

MI of whole cle from centre $I_{zz} = \int_0^r 2\pi x^3 dx$

$= 2\pi \left(\frac{x^4}{4}\right)_0^r$

$= \frac{2\pi r^4}{4}$

$I_{zz} = \frac{\pi r^4}{2}$ —— (1)

$I_{zz} = I_{xx} + I_{yy}$.

Circular lamina $I_{xx} = I_{yy}$.

$\therefore I_{zz} = 2 I_{xx}$ —— (2)

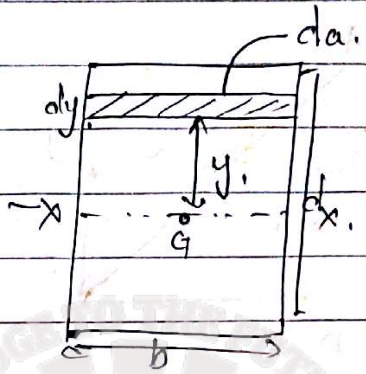
$2 I_{xx} = \frac{\pi r^4}{2}$

$$I_{xx} = I_{yy} = \frac{\pi r^4}{4}$$

If $d = 2r$

$$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$$

MI for rectangle :-



Area of elemental strip = $da = b \times dy$.

MI of this element area about $xx = (da\,y^2)$

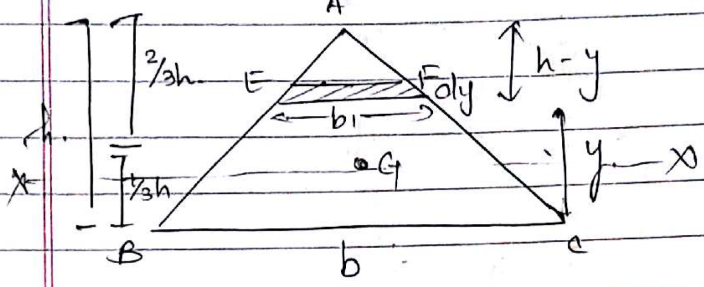
$= b \times dy\, y^2$

$= b y^2 dy$.

MI of whole rectangle $I_{xx} = \int_{-d/2}^{d/2} y^2\, b\, dy$.

$$I_{xx} = b\left(\frac{y^3}{3}\right) = \frac{bd^3}{12}$$

MI of Δ :-



ΔABC and ΔAEF are similar Δ.

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = b\left(\frac{h}{h} - \frac{y}{h}\right)$$

$$b_1 = b\left(1 - \frac{y}{h}\right).$$

Area of element EF $= da = b_1 dy$.

$$da = b\left(1 - \frac{y}{h}\right)dy.$$

MI of elemental strip about base BC $= da\, y^2$.

$$= b\left(1 - \frac{y}{h}\right) dy\, y^2.$$

$$= \left(y^2 - \frac{y^3}{h}\right) b\,dy.$$

MI of whole Δ :-

AB $= I_{BC} = b\int_0^h y^2 - \frac{y^3}{h}\, dy.$

$$\Rightarrow \left(\frac{y^3}{3} - \frac{y^4}{4h}\right)_0^h b$$

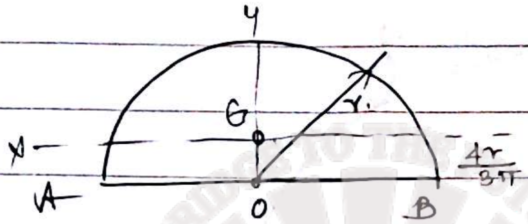
$$= b\left[\frac{h^3}{3} - \frac{h^{4}}{4h}\right]$$

$$= \left(\frac{b h^3}{12}\right)$$

MI of Centroid $\Rightarrow I_{BC} = I_{xx} + A\bar{y}^2$ $= \frac{bh^3}{12} = I_{xx} + \left(\frac{1}{2}bh\right)\left(\frac{h}{3}\right)^2$

$$\boxed{I_x = \frac{bh^3}{36}}$$

MI of semi-circle :-



MI of the semicircle about A $= \frac{1}{2}$ MI of circle.

$$= \frac{1}{2} \times \frac{\pi r^4}{4}$$

$$= \frac{\pi r^4}{8}$$

MI about centroidal axis

$$I_{AB} = I_{xx} + A\bar{y}^2$$

$$\frac{\pi r^4}{8} = I_{xx} + \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)^2$$

$$\frac{\pi r^4}{8} - \frac{\pi r^2}{2}\left(\frac{16 r^2}{9\pi^2}\right) = I_{xx}$$

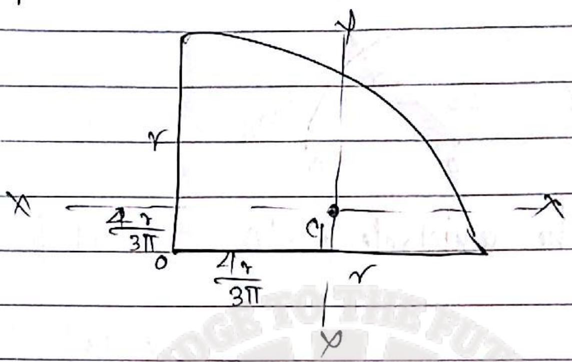
$$\frac{\pi r^4}{8} - \frac{8 r^4}{9\pi} = I_{xx}$$

$$\Rightarrow \boxed{I_{xx} = 0.11 r^4}$$

$$\boxed{I_{yy} = \frac{1}{2} \times \text{MI of circle} = \frac{\pi r^4}{8}}$$

$r^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ 0.3925 − 27[illegible]

MI of Quarter Circle.



MI of quarter circle = $\frac{1}{4}$ of MI of circle.

$$= \frac{1}{4} \times \frac{\pi r^4}{4}$$

$$I_{AB} = \frac{\pi r^4}{16}$$

MI about the centroidal axis :-

$$I_{xx} = I_1$$

$$I_{AB} = I_{xx} + A\bar{y}^2$$

$$I_{xx} = I_{AB} - A\bar{y}^2$$

$$= \frac{\pi r^4}{16} - \left(\frac{\pi r^2}{4}\right)\left(\frac{4r}{3\pi}\right)^2$$

$$= \frac{\pi r^4}{16} - \frac{\pi r^2}{4}\left(\frac{16 r^2}{9\pi^2}\right)$$

$$= \frac{\pi r^4}{16} - \frac{4 r^4}{9\pi}$$

$$= \frac{9\pi r^4 - 64 r^4}{16 \times 9\pi}$$

$$= \frac{r^4 (35.74)}{452.16}$$

$$= r^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$$

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