

**MODULE - 1****OSCILLATIONS AND SHOCK WAVES**

Oscillations and vibrations are most frequently occurring phenomena. Oscillation is a repeating motion that occurs when a time varying force acts on the system. Oscillations are periodic motions. If the oscillations occur in the system without the action of an external force, then the oscillation is said to be “free oscillation”. Simple harmonic motion is a special kind of periodic motion.

**IMPORTANT TERMINOLOGIES TO BE KNOWN TO UNDERSTAND SHM OF A BODY:**

- **Displacement (x):** The distance of location of a body from its mean position at a particular instant of time.
- **Amplitude (a):** The maximum value of displacement that a body can undergo on either side from its mean position during the oscillation.
- **Frequency (ν):** The number of oscillations executed by an oscillating body in unit time  
 $(\nu) = \frac{1}{T}$
- **Angular frequency (ω) (or) angular velocity:** It is the angle covered in unit time by a body moving in circular motion at that instant of time.  $\omega = 2\pi\nu$
- **Period (T):** It is the time taken by the body to complete one oscillation.  $T = \frac{2\pi}{\omega}$

**Restoring force:** When a body is oscillating in a medium, the action of force whose magnitude is proportional to displacement and acting in a direction opposite to displacement w.r.t equilibrium position. This force is called restoring force and is basically responsible for the oscillation of the body.

If F is the restoring force, and “x” is the displacement then,

$$F \propto -x,$$

$$\mathbf{F} = -\mathbf{k x}$$

Where, k is force constant.

The above equation is called Hooke’s law. This law states that, “the restoring force in oscillating body is directly proportional to displacement and acting in a direction opposite to displacement”.

**Simple Harmonic Motion (SHM):** The motion of a body is said to be SHM if the (restoring force) acceleration is directly proportional to the displacement and acts in a direction opposite to that of motion from the equilibrium position”.

(OR)

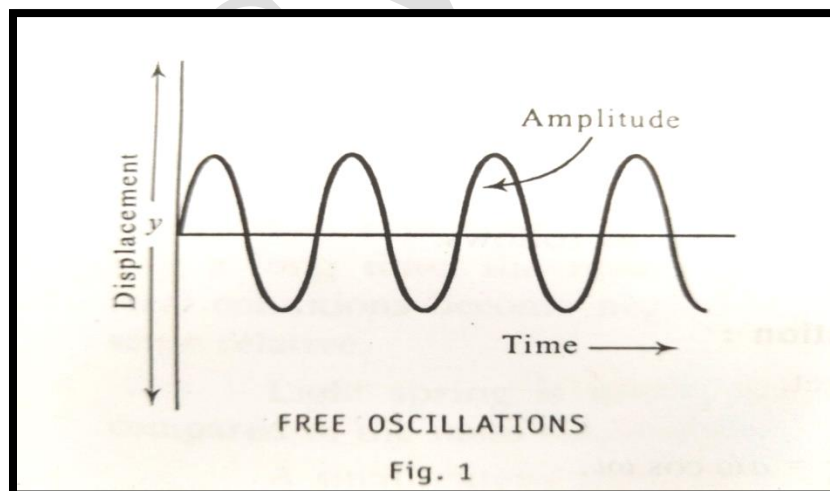
It is a periodic motion executed by a body such that its acceleration is proportional to its displacement from the equilibrium position and always directed towards it under the action of restoring force.

**Characteristics of SHM:**

- ❖ It is a particular type of periodic motion.
- ❖ There is a constant restoring force continuously acting on the body.
- ❖ The acceleration developed during the motion is directly proportional to displacement.
- ❖ Restoring force acts in a direction opposite to displacement. i.e.,  $F \propto -x$
- ❖ It is represented by sine or cosine functions. i.e.  $x = a \sin \omega t$

**Examples of SHM:**

1. A pendulum set for oscillation.
2. Excited tuning fork.
3. A shock absorber after being bumped.
4. A mass suspended to a spring and left free to oscillate.
5. Oscillations of LC circuits.

**Differential equation of motion of SHM:**

Consider a body of mass “m” executing SHM. Let “x” be the displacement of the body under the action of restoring force.

For an oscillating body, from Hooke's law

$$F = -k x \quad \dots (1)$$

From Newton's second law of motion, the force experienced by the body under motion is given by

$$F = m a = m \frac{d^2 x}{dt^2} \quad \dots (2) \quad \text{Where 'a' is acceleration}$$

Equating equation (1) and (2),

$$m \frac{d^2 x}{dt^2} = -k x$$

$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

$$\frac{d^2 x}{dt^2} + \frac{k}{m} x = 0$$

OR

$$\frac{d^2 x}{dt^2} + \omega^2 x = 0 \quad \dots (3) \quad \text{where } \omega = \sqrt{\frac{k}{m}} \text{ (angular frequency)}$$

This equation is called, equation of motion of SHM

The solution of equation (3) is given by

$$x = a \sin \omega t$$

Differentiate w.r.t "t" twice

$$\frac{dx}{dt} = \omega a \cos \omega t$$

$$\frac{d^2 x}{dt^2} = -\omega^2 a \sin \omega t$$

$$\frac{d^2 x}{dt^2} = -\omega^2 x$$

$$\frac{d^2 x}{dt^2} + \omega^2 x = 0 \quad \dots (4)$$

Comparing equations (3) and (4)

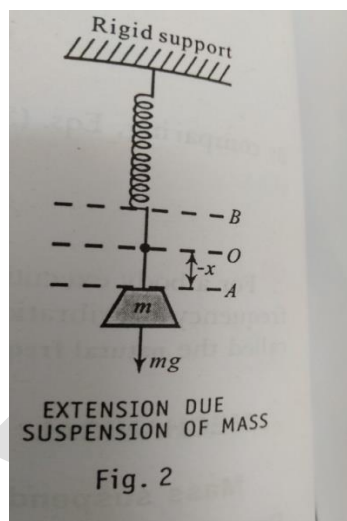
$$\omega^2 = \frac{k}{m}$$

The angular frequency is given by  $\omega = \sqrt{\frac{k}{m}}$

## Mechanical Simple Harmonic Oscillator:

### Mass suspended to a spring [vertical vibrations]

Consider a spring fixed at one end by the rigid support. Let a body of mass “m” is suspended at the lower end of the spring due to which force “mg” acts on the spring vertically downwards. Let “O” be equilibrium position of the spring with the application of mass “m”. The body of mass “m” is pulled down and left free to oscillate with respect to “O” and “x” be the displacement of the body under the action of restoring force.



Therefore, from Hooke's law,

$$F = -kx$$

$$k = -\frac{F}{x}$$

Consider the magnitude of force.  $|F| = |-F|$

Then for  $x = 1$ ,  $k = F$

Thus force constant  $k$  is defined as, “the magnitude of the applied force that produces unit extension in the spring while it is loaded within the elastic limit.”

### Physical significance of force constant (k):

Force constant is a measure of stiffness of the material. In case of spring it represents the amount of force required to stretch the spring by unit length. The springs with larger value of force constant will be stiffer. It is also called spring constant (or) stiffness factor.

**Period of oscillation (T):** The period of oscillation of mass spring system is given by

$$T = 2\pi \sqrt{\frac{m}{k}} \text{ second}$$

Where, m is suspended mass and k is force constant.

**Frequency of oscillation (v):** The frequency is the reciprocal of time period and is given by

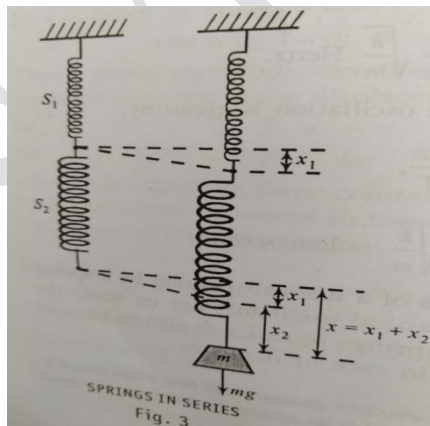
$$v = \frac{1}{T} \quad , \quad v = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \text{ Hz}$$

**Angular frequency ( $\omega$ ):** Angular frequency of oscillation is given by

$$\omega = 2\pi v = \frac{2\pi}{T}$$

$$\omega = \sqrt{\frac{k}{m}} \text{ radian/second}$$

**Expression for force constant (Spring constant) for series combination of springs:**



Consider the spring “S<sub>1</sub>” with force constant “k<sub>1</sub>” suspended by mass “m” and displaced through “x<sub>1</sub>” under the action of restoring force.

Therefore, from Hooke’s law,

$$F = -k_1 x_1$$

$$x_1 = - \frac{F}{k_1} \quad \dots\dots (1)$$

Similarly for the spring s<sub>2</sub>

$$F = -k_2 x_2$$

$$x_2 = -\frac{F}{k_2} \quad \dots\dots(2)$$

$x_2$  is the displacement of  $s_2$ ,  $k_2$  is the force constant of  $S_2$ . When the springs are connected in series,  $k_s$  is the force constant and displaced through “x” suspended by a same mass “m”.

From Hooke’s law,

$$F = -k_s x$$

$$x = -\frac{F}{k_s} \quad \dots\dots(3)$$

The displacement for series combination is  $x = x_1 + x_2 \quad \dots\dots\dots(4)$

Substituting eqn (1), (2) and (3) in equation (4)

$$-\frac{F}{k_s} = -\frac{F}{k_1} - \frac{F}{k_2}$$

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$

This is an expression for spring constant of two springs connected in series combination.

If “n” number of springs connected in series, then

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots\dots\dots + \frac{1}{k_n}$$

$$\text{Time period, } T = 2\pi \sqrt{\frac{m}{k_s}}$$

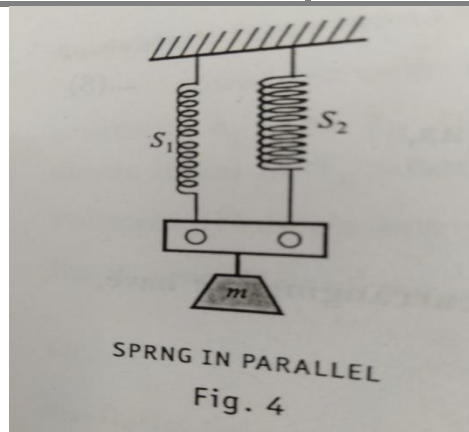
### Expression for force constant (spring constant) for parallel combination of two springs:

Consider two springs  $S_1$  and  $S_2$  with spring constants  $k_1$  and  $k_2$  respectively. Let  $x_1$  and  $x_2$  be displacement (extension) of the individual springs when they suspended by a mass “m”.

$$\text{Hence, } F_1 = -k_1 x_1 \quad \dots\dots(1)$$

$$\text{Similarly for spring } S_2 \quad F_2 = -k_2 x_2 \quad \dots\dots(2)$$

where  $F_1$  and  $F_2$  are restoring force of the springs  $S_1$  and  $S_2$  respectively.



When the springs are connected in parallel and suspended by the same load, with force constant ( $K_p$ ) and displaced through distance “x” under the action of restoring force ( $F_p$ ).

$$\text{From Hooke's law, } F_p = k_p x \quad \dots(3)$$

The restoring force ( $F_p$ ) is equally shared by two springs when they are connected in parallel with same extension  $x_1 = x_2 = x$ .

$$\text{Then } F_p = F_1 + F_2$$

From (1), (2) and (3)

$$- k_p x = -k_1 x_1 - k_2 x_2 \quad (\text{since } x_1 = x_2 = x)$$

$$k_p = k_1 + k_2$$

**This is the expression for equivalent force constant for springs in parallel combination.**

If 'n' number of spring are connected in parallel then,

$$K_p = k_1 + k_2 + k_3 + \dots k_n$$

The time period for parallel combination of springs is given by

$$T = 2\pi \sqrt{\frac{m}{k_p}}$$

**Free oscillations:** “The oscillatory body oscillates with undiminished amplitude with its own natural frequency of vibrations for infinite length of time under the action of restoring force, until an external force affects its motion” are called free oscillations.

Examples:

- Oscillation of mass suspended to spring with negligible damping and small displacement.
- Oscillations of simple pendulum.
- LC oscillations.

**Equation of Motion of Free Oscillations:**

If “m” is the mass of oscillating body with a force constant “k” and ‘x’ is the displacement at the instant “t” of the oscillating body, then

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Where  $\omega$  is the angular frequency,  $\omega = \sqrt{\frac{k}{m}}$

**Natural frequency of oscillations:** It is a characteristic with which a body (system) body oscillates under the action of restoring force after it gets displaced from its equilibrium position and left force.

**Damped oscillations:** An oscillatory body oscillates such that its amplitude gradually decreases and comes to rest at equilibrium position in a finite interval of time due to the action of resistive force.

Examples:

- Mechanical oscillations of simple pendulum.
- Electrical oscillations of LC circuit
- A swing left free to oscillate after being pushed once.



**Theory of Damped Oscillations:**

Consider a body of mass “m” executing damped oscillations in a resistive medium. The resistive force is proportional to velocity of the body and acting in opposite directions.

$$\text{Therefore, Resistive force} = -r \frac{dx}{dt} \quad \dots (1)$$

Where,  $r \rightarrow$  damping constant,  $\frac{dx}{dt} \rightarrow$  velocity.

The restoring force acting on the body set for oscillations given by

$$\text{Restoring force} = -k x \quad \dots (2)$$

Where,  $k \rightarrow$  force constant,  $x \rightarrow$  displacement

The resultant force acting on the body is,

$$\text{Resultant force} = \text{resistive force} + \text{restoring force}$$

From (1) and (2)

$$\text{Resultant force} = -r \frac{dx}{dt} - k x \quad \dots (3)$$

From Newton's second law of motion, the resultant force experienced by the body under motion is given by

$$\text{Resultant force} = m a = m \frac{d^2x}{dt^2} \quad \dots(4)$$

where  $a \rightarrow$  acceleration

Equating equations (3) and (4)

$$m \frac{d^2x}{dt^2} = -r \frac{dx}{dt} - kx$$

$$\frac{d^2x}{dt^2} = -\frac{r}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$$

$$\frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + \omega^2 x = 0 \quad \dots(5) \quad \text{where, } 2b = \frac{r}{m}, \omega = \sqrt{\frac{k}{m}}$$

**this is the equation of motion for damped oscillations**

the solution of equation (5) is given by

$$x = C e^{(-b + \sqrt{b^2 - \omega^2})t} + D e^{(-b - \sqrt{b^2 - \omega^2})t} \quad \dots(6)$$

where C and D are constants given by,

$$C = \frac{x_0}{2} \left[ 1 + \frac{b}{\sqrt{b^2 - \omega^2}} \right], \quad D = \frac{x_0}{2} \left[ 1 - \frac{b}{\sqrt{b^2 - \omega^2}} \right]$$

where, x is displacement at t = 0

the general solution of equation (5) is given by substituting the values of C and D in equation (6)

$$x = \frac{x_0}{2} \left\{ \left( 1 + \frac{b}{\sqrt{b^2 - \omega^2}} \right) e^{(-b + \sqrt{b^2 - \omega^2})t} + \left( 1 - \frac{b}{\sqrt{b^2 - \omega^2}} \right) e^{(-b - \sqrt{b^2 - \omega^2})t} \right\} \dots\dots(7)$$

**This is the General solution of damped oscillations.**

In the above equation, as “t” varies, x also varies. But the nature of variation depends on  $\sqrt{b^2 - \omega^2}$ .

**Case (i): If  $b^2 > \omega^2$ , over damping (or) dead beat case:**

When  $b^2 > \omega^2$ ,  $b^2 - \omega^2$  is positive, but  $\sqrt{b^2 - \omega^2} < b$

Therefore, the coefficient of “t” in both terms of equation (7) is negative, this indicates exponential decay of displacement W.r.t time, and the body after maximum displacement comes to rest at equilibrium position with the decay of time.

Over damping is the condition such that the body comes to rest at equilibrium position under the action of restoring force and resistive force with long interval.

Example: motion of pendulum in highly viscous liquid.

**Case (ii): when  $b^2 = \omega^2$ , critical damping:**

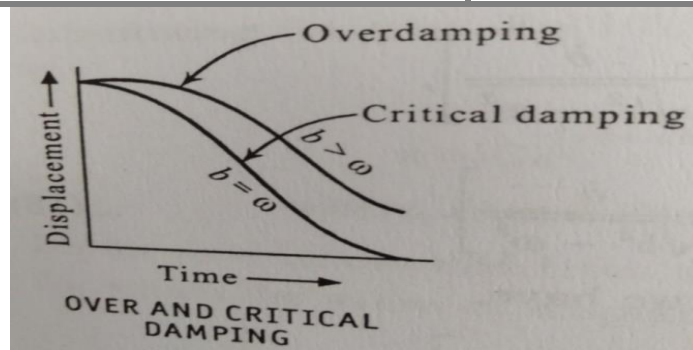
If  $b^2 = \omega^2$ , then  $\sqrt{b^2 - \omega^2} \approx \xi$  (small quantity),

Then equation (6) reduces to  $x = e^{-bt} [ (C+D) + (C-D) \xi t ]$

As “t” varies, the displacement decreases exponentially and become zero for short interval of time.

“It is a condition such that the body comes to half at equilibrium position under the action of restoring and resistive force in a short interval of time.”

**Example:** Pointer galvanometer, voltmeter, current meter. Shock absorbers designed with springs.



**Case(iii): when  $b^2 < \omega^2$ , under damping:**

When  $b^2 > \omega^2$ ,  $b^2 - \omega^2$  is negative. But  $\omega^2 - b^2$  is positive

Therefore,  $\sqrt{b^2 - \omega^2} = \sqrt{-(\omega^2 - b^2)} = i\sqrt{\omega^2 - b^2}$  ( $i = \sqrt{-1}$ )

Let  $\omega^2 - b^2 = n$

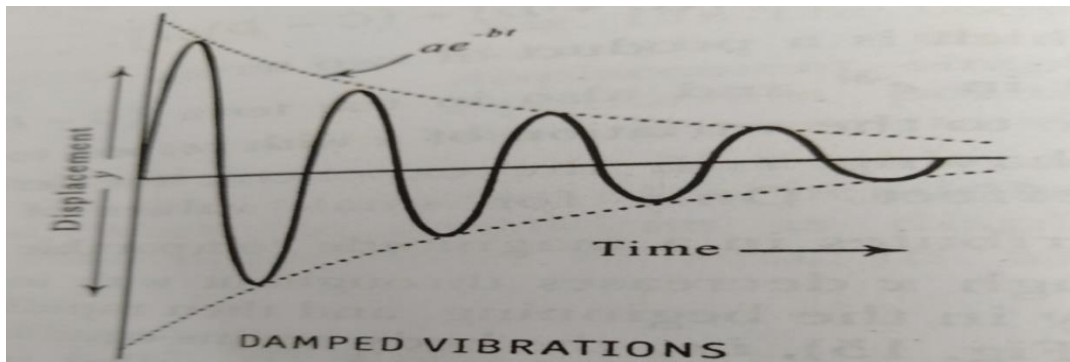
Then, equation (7) reduces to,

$$x = a e^{-bt} \sin(nt + \phi), \quad \text{where } a = x_0 (\alpha/n), \phi = \tan^{-1}(n/b)$$

As time “t” increases, the displacement ‘x’ decreases exponentially.

“It is a condition such that, the body vibrates with diminishing amplitude with the decay of time under the action of restoring and resistive forces and comes to rest at equilibrium position”.

**Examples:** Any real physical system like oscillations of simple pendulum, vibrations of tuning fork, mass suspended to spring.



**Quality factor (Q):**

The amount of damping is described by the quantity called quality factor and is given by,

$$Q = \frac{\omega}{2b}, \quad \omega \rightarrow \text{angular frequency}, \quad 2b \rightarrow \frac{r}{m}, \quad r \rightarrow \text{damping factor.}$$

“Quality factor is the number of cycles required for the energy to fall off by a factor  $e^{2\pi}$  ( $\approx 535$ ).

If Q value is more indicated the sustained oscillations overcoming the resistive forces.

Q factor describes how much under damped is the oscillatory system.

**Forced oscillations (or) forced vibrations:**

“It is a steady state sustained vibrations of a body vibrating in a resistive medium under the action of external periodic force which acts independently of the restoring force.”

Examples:

- Oscillations of swing pushed periodically by a person
- oscillations of LC circuit with the applied ac source
- Motion of diaphragm in a telephone receiver.

**Theory of forced oscillations (OR) Expression for amplitude and phase of the forced vibrations:**

Consider a body of mass “m” executing vibrations in a damping medium under the application of external periodic force.  $F \sin p t$ ,

If ‘x’ is the displacement of the body at any instant of time “t”.

$$\text{The restoring force} = -k x \quad \dots(1), \quad \text{where } k \rightarrow \text{force constant.}$$

The resistive force acts opposite to velocity given by

$$\text{Resistive force} = -r \frac{dx}{dt} \quad \dots(2), \quad \text{where } r \rightarrow \text{damping constant, } \frac{dx}{dt} \rightarrow \text{velocity}$$

$$\text{The applied periodic force} = F \sin p t \quad \dots(3), \quad \text{where } p \rightarrow \text{frequency of external force.}$$

Therefore, the resultant force experienced by the body is given by

From equations (1), (2) & (3)

$$\text{Resultant force} = -r \frac{dx}{dt} - kx + F \sin p t \quad \dots(4)$$

From Newton second law of motion, the resultant force experienced by moving body is given by

Resultant force =  $ma = m \frac{d^2x}{dt^2}$  .....(5), where  $a \rightarrow$  acceleration.

Equating (4) and (5)

$$m \frac{d^2x}{dt^2} = -r \frac{dx}{dt} - kx + F \sin pt$$

$$\frac{d^2x}{dt^2} + \frac{r}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{F}{m} \sin pt \quad \dots\dots(6)$$

**This is the equation of motion of forced oscillations.**

$$\frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + \omega^2 x = \frac{F}{m} \sin pt \quad \dots\dots(7) \quad \text{where, } 2b = \frac{r}{m}, \omega = \sqrt{\frac{k}{m}}$$

The solution of equation (7) is given by

$$x = a \sin(pt - \alpha)$$

Differentiate W.r.t "t" twice,

$$\frac{dx}{dt} = p a \cos(pt - \alpha)$$

$$\frac{d^2x}{dt^2} = -p^2 a \sin(pt - \alpha)$$

equation (7) becomes,

$$-p^2 a \sin(pt - \alpha) + 2b p a \cos(pt - \alpha) + \omega^2 a \sin(pt - \alpha) = \frac{F}{m} \sin[(pt - \alpha) + \alpha]$$

$$-p^2 a \sin(pt - \alpha) + 2 b p a \cos(pt - \alpha) + \omega^2 a \sin(pt - \alpha) = \frac{F}{m} [\sin(pt - \alpha) \cos \alpha + \cos(pt - \alpha) \sin \alpha]$$

Equating the coefficients of  $\cos(pt - \alpha)$  &  $\sin(pt - \alpha)$  on both sides of the equation

$$2 b p a = \frac{F}{m} \sin \alpha \quad \dots\dots(8)$$

$$a(\omega^2 - p^2) = \frac{F}{m} \cos \alpha \quad \dots\dots(9)$$

squaring and adding equations (8) and (9)

$$a^2 [4b^2 p^2 + (\omega^2 - p^2)^2] = \left(\frac{F}{m}\right)^2 (\sin^2 \alpha + \cos^2 \alpha)$$

$$a^2 = \frac{\left(\frac{F}{m}\right)^2}{(4b^2 p^2 + (\omega^2 - p^2)^2)}$$

$$a = \frac{\frac{F}{m}}{\sqrt{4b^2 p^2 - (\omega^2 - p^2)^2}}$$

**This is the equation for amplitude of the forced oscillations.**

Equation (8) divided by (9) gives,

$$\frac{2bp}{\omega^2 - b^2} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

$$\tan \alpha = \left( \frac{2bp}{\omega^2 - b^2} \right)$$

$$\alpha = \tan^{-1} \left( \frac{2bp}{\omega^2 - b^2} \right)$$

this is the equation for phase of forced oscillations.

### **Dependence of amplitude and phase on the frequency of the applied force:**

**Case (i):**  $p \ll \omega$ ,  $p \rightarrow$  angular frequency of applied force,  $\omega \rightarrow$  natural frequency of the vibrating body. When  $p \ll \omega$ ,  $p^2$  is very small.

Thus,  $\omega^2 - p^2 = \omega^2$  and  $2bp = 0$ . Since  $b$  is small,

$$\text{therefore the amplitude } a = \frac{F}{m\omega^2}$$

The phase,  $\alpha = \tan^{-1}\left(\frac{0}{\omega^2}\right) = 0$ . The displacement and force will be in the same phase.

#### **Case(ii): $p = \omega$ , (resonance)**

$$\omega^2 - p^2 = 0$$

Amplitude  $a = \frac{F/m}{2bp} = \frac{F/r\omega}{r}$  the amplitude have highest value. Since  $a \propto \left(\frac{1}{\omega}\right)$ , phase

$$a = \tan^{-1}(2bp / 0) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

The displacement has a phase lag of  $\left(\frac{\pi}{2}\right)$  w.r.t phase of the applied force.

**Case(iii):  $p \gg \omega$** , it is significant when damping forces are small.

$$\text{When } p \gg \omega, (\omega^2 - p^2) \approx p^4$$

$$\text{Amplitude } a = \frac{\frac{F}{m}}{\sqrt{4b^2 p^2 + p^4}}$$

As 'p' increases 'a' becomes smaller. Since b is small

$$4b^2 p^2 \ll p^4$$

$$\text{Amplitude } a = \frac{F/m}{p^4} = \frac{F/m}{p^2}$$

$$\text{Phase } \alpha = \tan^{-1} \left[ \frac{2bp}{\omega^2 - p^2} \right] = \tan^{-1} \left( \frac{2bp}{p^2} \right) = \tan^{-1} \left( \frac{2bp}{p} \right) = \pi$$

The displacement has a phase lag of ' $\pi$ ' w.r.t phase of the applied force.

**Resonance:** The expression for amplitude of the forced oscillations given by

$$a = \frac{F/m}{\sqrt{4b^2 p^2 + (\omega^2 - p^2)^2}}$$

the amplitude of vibration is maximum, this state of vibration called resonance.

**Condition for resonance:**

In the above equation, "a" is maximum when, "b" is minimum (or) damping caused by the medium is made minimum (since  $b = \frac{r}{2m}$ ),

$p = \omega$ , i.e., frequency of applied force (p) is equal to natural frequency of vibration ( $\omega$ ) of the body.

$$\text{Therefore, } a_{\max} = \frac{F/m}{2bp} = \frac{F/m}{2b\omega}$$

**Significance of resonance:**

At resonance, the vibrating body will have the ability to receive completely the energy delivered by the periodic force. Hence amplitude of vibrations are maximum. "When the frequency of a periodic force acting on a vibrating body is equal to natural frequency of vibrations of the body, the energy transfer from the periodic force to the body becomes maximum because of which the body vibrates with maximum amplitude", this phenomenon is called resonance.

**Sharpness of resonance:**

“sharpness of resonance is the rate at which the amplitude changes with respect to change in the frequency of the applied force at the stage of resonance”.

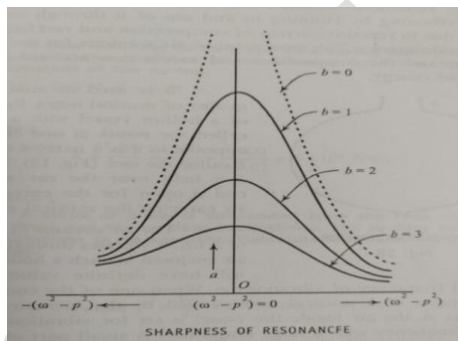
Therefore, sharpness of resonance =  $\frac{\text{change in amplitude}}{\text{change in frequency}}$

$$\text{Sharpness of resonance} = \frac{F/m}{2b\omega p}$$

**Effect of damping on sharpness of resonance:**

In the above equation, sharpness of resonance  $\propto \frac{1}{b}$

(since  $b = \frac{r}{2m}$ ,  $r \rightarrow$  damping constant)



From the graph, for small value of  $b$  exhibits higher peaks refer to sharpness of resonance, on the other for higher value of  $b$  the resonance is flat. For  $b = 0$ , the amplitude is  $\infty$ , can not exist in reality.

**Significance of sharpness of resonance:**

The rise of amplitude of amplitude will be very sharp when the damping is very small at resonance.

Examples: Helmholtz resonator, a radio receiver tuned to frequency of transmitting station, vibrations of excited tuning fork, LC oscillations with applied ac source.



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**SHOCK WAVES****Mach number:**

The ratio of speed of the object to the speed of sound in the given medium is called as a mach number.

$$\text{Mach number} = \frac{\text{Object speed}}{\text{Speed of sound in the medium}}$$

$$M = \frac{v}{a}$$

Mach number is a pure number.

**Distinctions between –acoustic, ultrasonic, subsonic & supersonic waves:****Acoustic waves:**

- An acoustic wave is simply a sound wave which travels with the speed of 333 m/s in air at STP.
- Amplitude of acoustic wave is very small.
- Acoustic waves are a type of longitudinal waves that propagate by means of adiabatic compression and decompression.
- Important quantities for describing acoustic waves are sound pressure, particle velocity, particle displacement and sound intensity.
- Acoustic waves travel with the speed of sound which depends on the medium they're passing through.

**Ultrasonic waves:**

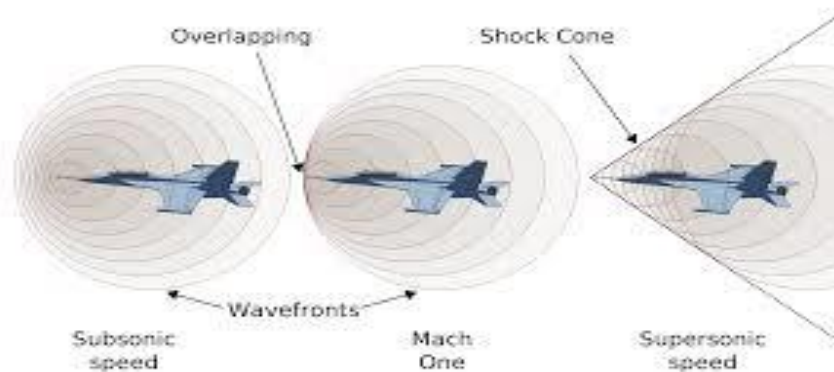
- Ultrasonic waves are pressure waves having frequencies beyond 20000 Hz.
- At higher power levels, ultrasonic's is useful for changing the chemical properties of substances.
- Amplitude of the ultrasonic wave is also small.

**Subsonic waves:**

- If the speed of mechanical wave or body moving in the fluid is lesser than that of sound, then such a speed is referred to as subsonic & the wave is a subsonic wave.
- All subsonic waves have a **mach number < 1**.
- E.g., Speed of flight of birds is subsonic, vehicles such as motor cars or trains move with subsonic speed.

**Supersonic waves:**

- Supersonic waves are mechanical waves which travel with speed greater than that of sound.
- They have a **mach number > 1**
- E.g.:- Fighter planes fly with supersonic speed.
- Amplitude of supersonic waves will be high & it affects the medium in which it is travelling.

**Wave fronts of Subsonic and Supersonic Waves :****SHOCK WAVES:**

A shock wave is a thin transitive surface of a fluid medium propagating with supersonic speed characterized by instantaneous changes in pressure, temperature and density of the medium.

Example:

- Shock waves are produced in nature during earth quakes ( as seismic waves which travel with speed ranging from 2 km/s to 8 km/s) & when lightning strikes.
- Shock waves are produced during the propagation of fighter planes moving at supersonic speed
- Shock waves are produced during Nuclear bomb explosion
- Shock waves are produced during bullet is fired from rifle

When the speed of a source exceeds the speed of sound ( $v > c$ ) the wave fronts lag behind the source in a cone –shaped region with the source at the vertex. The edge of the cone forms a supersonic wave front with unusually large amplitude called a “Shock wave”. When a shock wave reaches an observer a “sonic boom” is heard.

**Properties of shock waves:**

- Shock waves travel in a medium with Mach number exceeds 1.
  - Shock waves are produced by a sudden dissipation of mechanical energy in a medium enclosed in a small space.
  - Shock waves obey the laws of fluid dynamics.
  - The change in entropy of the medium due to the generation of shock waves.
  - Shock waves are confined in a medium of very thin space of thickness about  $1\mu\text{m}$ .
  - Shock waves are not electromagnetic in nature.
  - Shock waves are not physical waves.
- 
- Shock waves are produced in medium where the object speed is more than the speed of sound. When a shock wave passes through a matter, the total energy is preserved but the energy which can be extracted as work decreases & the entropy increases. This for example, creates additional drag force on aircraft with shocks.
  - Like an ordinary wave, it carries energy & can propagate through a medium (solid, liquid, gas or plasma) or in some cases in the absence of a material medium, through a field such as an electromagnetic field.
  - Contact front: in a shock wave caused by a driver gas (for example the “impact” of a high explosive on the surrounding air), the boundary between the driver (explosive products) & the driven (air) gases. The contact front trails the shock wave.
  - Measurements of the thickness of shock waves in air have resulted in values around 200nm (about  $10^{-5}$  in), which is on the same order of magnitude as the mean free gas molecule path.

**Strong and weak shock waves**

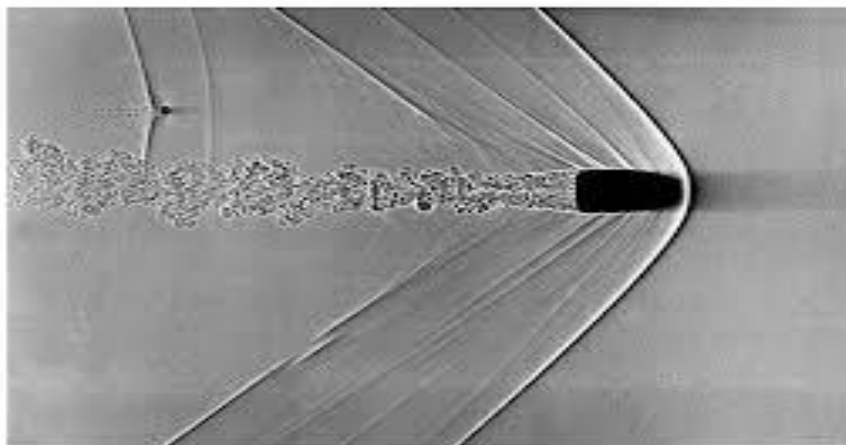
- Shock waves are identified as strong or weak depending on the magnitude of the instantaneous changes in pressure & temperature in the medium that is held pressed in the space bound within the thickness of the shock front.
- Shock wave created by the Bursting of tyre & crackers is weak. Whereas those created during lightning thunder or bombing are strong. The shock waves produced during nuclear explosion are strongest ever witnessed on earth.

**NOTE:** pictures of shockwaves formed

**1) When fighter plane moving at supersonic speed**



**2) When bullet moving at supersonic speed**



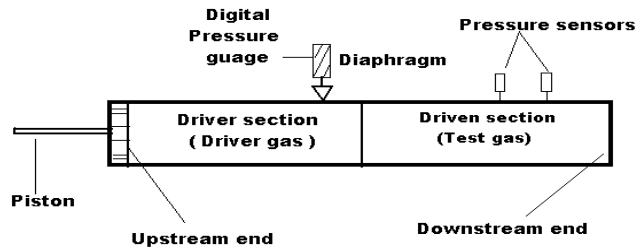
**3) When Atomic bomb explodes**



### Construction and working of Reddy Shock tube.

Reddy tube is a hand operated shock tube capable of producing shock waves by using human energy.

It is long cylindrical tube with two sections separated by a diaphragm. It's one end is fitted with a piston & the other end is closed or open to the surrounding.



#### Construction:

- Reddy tube consists of a cylindrical stainless steel tube of about 30mm diameter & of length nearly 1m.
- It is divided into two sections one is **driver section** & the other is **driven section** separated by a thick aluminium or Mylar or paper diaphragm of thickness 0.1mm.
- Far end of driver tube is fitted with a piston & the far end of driven tube is closed.
- A digital pressure gauge is mounted in the driver section next to the diaphragm.
- Two piezoelectric sensors  $S_1$  and  $S_2$  are mounted 70mm apart towards the closed end of the shock tube.
- A port is provided at the closed end of the driven section for filling the test gas to the required pressure.
- The driver section is filled with driver gas which is held at a relatively high pressure due to the compressing action of the piston. The gas in the driven section is called driven gas (test gas).

#### Working:

- The driver gas is compressed by pushing the piston hard into the driver tube until the diaphragm ruptures.
- Due to rupture of diaphragm the driver gas moves to the driven section, & pushes the driven gas towards the far downstream end. Hence there is sudden increase of pressure, temperature and density of driven section near the downstream end. This generates a moving shock wave that traverses the length of the driven section.

- The propagating primary shock wave is reflected back from the downstream end. After reflection, the test gas further undergoes compression produces the secondary shock waves, which increases the pressure & temperature to still higher values.
- This state of high values of pressure & temperature is sustained at the downstream end until an expansion waves reflected from the downstream end of the driver tube arrives there & neutralizes the compression partially
- The period over which the extreme temperature & pressure conditions at the downstream end is sustained, is typically in the order of milliseconds.
- The pressure rise caused by the primary & also the reflected shock wave are sensed as signals by the sensors  $S_1$  and  $S_2$  respectively & are recorded in a digital cathode ray oscilloscope.
- From the recording in the CRO, the shock arrival times are found out by the associated time base calculations. Using the data so obtained, Mach number, pressure & temperatures can be calculated.

#### Characteristics of Reddy tube:

- a. The Reddy tube operates on the principle of free piston driven shock tube.
- b. It is a hand operated shock producing device.
- c. It is capable of producing mach number exceeding 1.5
- d. The rupture pressure is a function of the thickness of the diaphragm.
- e. Temperature exceeding 900K can be easily obtained by the Reddy tube using helium as driver gas & argon as the driven gas. This temperature is useful in chemical kinetic studies.

#### Applications of shock waves:

##### Cell information (biological application) :

By passing shock wave of appropriate strength, DNA can be pushed inside a cell.  
(Functionality of DNA will not be affected by the impact of the shock wave)

##### Wood preservation:

By using shock waves, chemical preservatives in the form of solutions could be pushed into the interior of wood samples which helps in withstanding the microbial attacks. The actual set-up used for this purpose is named as “shockwave reactor”.  
This method helps in substantial increase in the life of ordinary bamboo & soft wood.

##### Use in pencil industry:

In the manufacture of pencil, the wood need to be softened by soaking it in a polymer for about 3 hours at  $70^{\circ}\text{c}$  & then dried which is time consuming.  
In modern technique, the wood is placed in the liquid & a shock wave is sent through. The liquid gets into the wood almost instantaneously & takes no longer time to dry. The wood treated with shock wave is now ready for the further processing.

**Kidney stone treatment:**

Shock wave is used in a therapy called ‘extra–corporal lithotripsy’ to shatter the kidney stones into smaller fragments after which they are passed out of the body smoothly through the urinary tracts.

**Gas dynamic studies:**

The extreme conditions of pressure & temperature that can be produced in the shock tube, enables the study of high temperature gas dynamics. This knowledge is crucial in the study of supersonic motion of bodies & hypersonic re-entry of space vehicles into the atmosphere.

**Shock wave assisted needleless drug delivery:**

By using shock waves, drugs can be injected into the body without using needles.

The drug is filled in the cartridge which is kept pressed on the skin & the shock wave is sent into the body using high pressure. The drug enters the body directly through the porosity of the skin. Typical depth of penetration is about 100 microns.

**Treatment of dry bore wells:**

Water will be available in the bore wells when water from the feeder sources accumulates in the bore well through a number of seepage points which are porous. Sometimes such seepage routes are blocked by sand particles clogging the pores. It has been observed that, a shockwave sent through such dry bore well, clears the blockages & rejuvenates the bore well into a water source.

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## MODULE - 2

### MODERN PHYSICS & QUANTUM MECHANICS

#### MODERN PHYSICS

##### Blackbody:

A Blackbody is one which absorbs the entire radiation incident on it and emits all the absorbed radiation when it is heated to a suitable high temperature. A true blackbody does not exist practically.

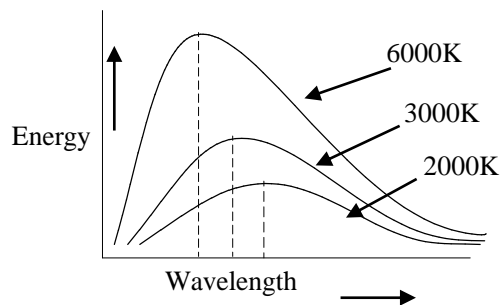
Ex: Lamp black body can be considered to be nearest natural black body

Ferry's Black body can be considered to be nearest man made black body

##### Blackbody Radiation spectrum

Since a perfect black body does not exist in nature, in 1895, Lumer and Preingshiem designed a special type of black body which has features very close to the black body. They studied the spectrum of black body.

A graph plotted energy radiated (Intensity) versus wavelength of emitted radiation is as called black body radiation spectrum and is shown below.



Salient features of the spectrum are

1. There are different curves for different temperatures of the black body.
2. At a given temperature the distribution of energy is not uniform over all the wave length.
3. At a given temperature intensity of radiation increases with increase in wavelength and reaches a maximum for a particular temperature, beyond which the intensity decreases.
4. With increase in temperature of the body the maximum intensity increases and the wavelength corresponding to maximum intensity  $\lambda_m$  shifts towards lower wavelength side.
5. The area under the curve gives the energy emitted per unit area of cross section of the black body.



The laws accounts for the black body radiation spectrum are as follows:

### **Wein's Displacement Law:**

The law states that “the wavelength corresponds to maximum intensity is inversely proportional to the absolute temperature of the black body, because of which the peaks of the energy curves for different temperatures get displaced towards the lower wavelength side”.

$$\text{i.e. } \lambda_m \propto \left(\frac{1}{T}\right) \text{ or } \lambda_m T = \text{constant} = 2.898 \times 10^{-3} \text{ mK}$$

Wein showed that the maximum energy of the peak emission is directly proportional to the fifth power of absolute temperature.

$$E_m \propto T^5 \quad \text{or} \quad E_m = \text{constant} \times T^5$$

Wein, Rayleigh-Jeans and Planck have given their explanations to account these observed experimental facts as follows:

#### **1. Wein's law:**

In 1896, Wein obtained the relation between the wavelength of emission and the temperature of the source as

$$U_\lambda d\lambda = C_1 \lambda^{-5} e^{-\left(\frac{C_2}{\lambda T}\right)} d\lambda$$

Where  $U_\lambda d\lambda$  is the energy / unit volume in the range of wavelength  $\lambda$  and  $\lambda+d\lambda$ ,  $C_1$  and  $C_2$  are constants.

This is called Wein's law of energy distribution in the black body radiation spectrum.

#### **Drawbacks of Wein's law:**

Wein's law holds good for the shorter wavelength region and high temperature of the source. It is failed to explain gradual drop in intensity of radiation corresponding to longer wavelength greater than the peak value.

#### **2. Rayleigh-Jeans Law:**

In 1900, Rayleigh derived an equation for the blackbody radiation on the basis of principle of equipartition of energy. The principle of equipartition of energy suggests that an average energy  $kT$  is assigned to each mode of vibration. The number of modes vibrations/unit volume whose wavelength is in the range of  $\lambda$  and  $\lambda+d\lambda$  is given by  $8\pi\lambda^{-4}d\lambda$ .

The energy/unit volume in the wavelength range  $\lambda$  and  $\lambda + d\lambda$  is

$$U_\lambda d\lambda = 8\pi k T \lambda^{-4} d\lambda$$

Where  $k$  is Boltzmann constant =  $1.38 \times 10^{-23} \text{ J/K}$ .

This is Rayleigh-Jeans equation. Accordingly, energy radiated by the blackbody decreases with increasing wavelength.

### Drawbacks of Rayleigh-Jeans Law: (or Ultra Violet Catastrophe)

Rayleigh-Jeans Law holds good only for longer wavelength region but failed to explain shorter wavelength region. According to Rayleigh-Jeans Law the energy increases with decrease in wavelength and is very large in the ultra-violet region. But such a large increase in energy emission does not occur experimentally. This discrepancy is known as ultra-violet catastrophe.

### 3. Stefan Boltzmann's law

Stefan's law states that the energy radiated by a perfect black body is directly proportional to fourth power of absolute temperature.

$$E = \sigma T^4$$

Where  $\sigma$  = Stefan Boltzmann's constant =  $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

### 4. Planck's Law:

- i) Planck assumed that walls of the experimental blackbody consists very large number of electrical oscillators. Each oscillator vibrates with its own frequency.
- ii) Each oscillator has an energy given by integral multiple of  $h\nu$  where  $h$  is Planck's constant &  $\nu$  is the frequency of vibration.  
 $E = nh\nu$  where  $n = 1, 2, 3 \dots$  etc.
- iii) An oscillator may lose or gain energy by emitting or absorbing respectively a radiation of frequency  $\nu$  where  $\nu = \Delta E/h$ ,  $\Delta E$  is difference in energies of the oscillator before and after the emission or absorption take place.

Planck derived the law which holds good for the entire spectrum of the blackbody radiation as

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \left[ \frac{1}{e^{h\nu/kt} - 1} \right] d\lambda \quad (\text{since } \nu = c/\lambda) \quad \longrightarrow \quad (1)$$

This is Planck's Radiation Law.

### Reduction of Planck's law to Wein's law:

For shorter wavelengths,  $\nu = c/\lambda$  is large.

When  $\nu$  is large,  $e^{h\nu/kT}$  is very large.

$$\therefore e^{h\nu/kT} \gg 1$$

$$\therefore (e^{h\nu/kT} - 1) \approx e^{h\nu/kT} = e^{hc/\lambda kT}$$

Substituting in eqn 1:

$$U_{\lambda} d\lambda = \frac{8\pi hc}{\lambda^5} \left[ \frac{1}{e^{hc/\lambda kT}} \right] d\lambda = C_1 \lambda^{-5} e^{\left(\frac{-C_2}{\lambda T}\right)} d\lambda$$

Where  $C_1 = 8\pi hc$  and  $C_2 = hc/k$

This is the Wein's law of radiation.

### Reduction of Planck's law to Rayleigh Jeans law:

For longer wavelengths  $\nu = c/\lambda$  is small.

When  $\nu$  is small  $h\nu/kT$  is very small.

Expanding  $e^{h\nu/kT}$  as power series:

$$e^{h\nu/kT} = 1 + h\nu/kT + (h\nu/kT)^2 + \dots$$

$$\approx 1 + h\nu/kT.$$

$\therefore$  If  $h\nu/kT$  is small, its higher powers are neglected.

$$\therefore e^{h\nu/kT} - 1 = \frac{h\nu}{kT} = \frac{hc}{\lambda kT}$$

Substituting in eqn 1:

$$U_{\lambda} d\lambda = \left[ \frac{8\pi hc}{\lambda^5 \left(\frac{hc}{\lambda kT}\right)} \right] d\lambda$$
$$= \left[ \frac{8\pi kT}{\lambda^4} \right] d\lambda$$

This is Rayleigh Jeans Law of Radiation.

### Dual nature of matter (de-Broglie Hypothesis)

#### Dual nature of light:

The concept of photoelectric effect and Compton Effect gives the evidence for particle nature of light. Where as in physical optics the phenomenon like interference, diffraction, superposition was explained by considering wave nature of light. This is wave particle duality of light.

### **Dual nature of matter:**

On the basis of above concept (dual nature of light), in 1923, Louis de Broglie gave a hypothesis

“Since nature loves symmetry, if the radiation behaves as particles under certain conditions and as waves under certain conditions, then one can expect that, the entities which ordinarily behaves as particles (ex. Like electrons, protons, neutrons) must also exhibit properties attributable to waves under appropriate circumstances” This is known as **deBroglie hypothesis**

Matter is made up of discrete constituent particles like atoms, molecules, protons, neutrons and electrons, hence matter has particle nature. Wave nature of matter is explained by Davisson and Germer experiment. Hence matter also exhibit wave particle duality.

The waves associated with the moving particles are called de Broglie waves or matter waves or pilot waves.

### **Characteristics of matter waves:**

1. Waves associated with moving particles are called matter waves. The wavelength ' $\lambda$ ' of a de-Broglie wave associated with particle of mass ' $m$ ' moving with velocity ' $v$ ' is  
$$\lambda = h/(mv)$$
2. Matter waves are not electromagnetic waves because the de Broglie wavelength is independent of charge of the moving particle.
3. The amplitude of the matter wave depends on the probability of finding the particle in that position.
4. The speed of matter waves depends on the mass and velocity of the particle associated with the wave.

### **Debroglie's Wavelength:**

A particle of mass ' $m$ ' moving with velocity ' $c$ ' possess energy given by

$$E = mc^2 \quad \rightarrow \text{(Einstein's Equation) (1)}$$

According to Planck's quantum theory the energy of quantum of frequency ' $\nu$ ' is

$$E = h\nu \rightarrow (2)$$

From (1) & (2)

$$mc^2 = h\nu = hc / \lambda \quad \text{since } \nu = c/\lambda$$

$$\lambda = hc / mc^2 = h/mc$$

$$\lambda = h/mv \quad \text{since } v \approx c$$

**de-Broglie wavelength of a free particle in terms of its kinetic energy**

Consider a particle, since the particle is free, the total energy is same as

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Where 'm' is the mass, 'v' is the velocity and 'p' is the momentum of the particle.

$$p = \sqrt{2mE}$$

The expression for de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

**Debroglie Wavelength of an Accelerated Electron:**

If an electron accelerated with potential difference 'V' the work done on the 'eV', which is converted to kinetic energy.

$$\text{Then } eV = \frac{1}{2}mv^2 \quad \rightarrow (1)$$

If 'p' is the momentum of the electron, then  $p=mv$

Squaring on both sides, we have

$$p^2 = m^2v^2$$

$$mv^2 = p^2/m$$

Using in equation (1) we have

$$eV = p^2/(2m)$$

$$\text{or } p = \sqrt{2meV}$$

According to de-Broglie  $\lambda = h/p$

$$\text{Therefore } \lambda = \left[ \frac{h}{\sqrt{2meV}} \right]$$

$$\lambda = \frac{1}{\sqrt{V}} \left[ \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.602 \times 10^{-19}}} \right] = \frac{1.226 \times 10^{-9}}{\sqrt{V}} \text{ m}, \quad \lambda = \frac{1.226}{\sqrt{V}}$$

## QUANTUM MECHANICS

### Heisenberg's Uncertainty Principle:

According to classical mechanics a particle occupies a definite place in space and possesses a definite momentum. If the position and momentum of a particle is known at any instant of time, it is possible to calculate its position and momentum at any later instant of time. The path of the particle could be traced. This concept breaks down in quantum mechanics leading to Heisenberg's Uncertainty Principle.

Heisenberg's Uncertainty Principle states that "It is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa".

If  $\Delta x$  and  $\Delta P_x$  are the uncertainties in the measurement of position and momentum of the particle then the uncertainty can be written as

$$\Delta x \cdot \Delta P_x \geq (h/4\pi)$$

In any simultaneous determination of the position and momentum of the particle, the product of the corresponding uncertainties inherently present in the measurement is equal to or greater than  $h/4\pi$ .

Similarly      1)  $\Delta E \cdot \Delta t \geq h/4\pi$                       2)  $\Delta L \cdot \Delta \theta \geq h/4\pi$

### Significance of Heisenberg's Uncertainty Principle:

Heisenberg's Uncertainty Principle asserts that it is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa.

versa. Therefore one should think only of the probability of finding the particle at a certain position or of the probable value for the momentum of the particle.

**Application of Uncertainty Principle:**

**Impossibility of existence of electrons in the atomic nucleus:**

According to the theory of relativity, the energy E of a particle is:

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - (v^2 / c^2)}}$$

Where 'm<sub>0</sub>' is the rest mass of the particle and 'm' is the mass when its velocity is 'v'.

$$\text{i.e. } E^2 = \frac{m_0^2 c^4}{1 - (v^2 / c^2)} = \frac{m_0^2 c^6}{c^2 - v^2} \rightarrow (1)$$

If 'p' is the momentum of the particle:

$$\text{i.e. } p = mv = \frac{m_0 v}{\sqrt{1 - (v^2 / c^2)}}$$

$$p^2 = \frac{m_0^2 v^2 c^2}{c^2 - v^2}$$

Multiply by c<sup>2</sup>

$$p^2 c^2 = \frac{m_0^2 v^2 c^4}{c^2 - v^2} \rightarrow (2)$$

Subtracting (2) by (1) we have

$$E^2 - p^2 c^2 = \frac{m_0^2 c^4 (c^2 - v^2)}{c^2 - v^2}$$

$$E^2 = p^2 c^2 + m_0^2 c^4 \rightarrow (3)$$

Heisenberg's uncertainty principle states that

$$\Delta x \cdot \Delta P_x \geq \frac{h}{4\pi} \rightarrow (4)$$

The diameter of the nucleus is of the order  $10^{-14}$ m. If an electron is to exist inside the nucleus, the uncertainty in its position  $\Delta x$  must not exceed  $10^{-14}$ m.

i.e.  $\Delta x \leq 10^{-14}$ m

The minimum uncertainty in the momentum

$$(\Delta P_x)_{\min} \geq \frac{h}{4\pi (\Delta x)_{\max}} \geq \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}} \geq 0.5 \times 10^{-20} \text{ kg. m/s} \rightarrow (5)$$

By considering minimum uncertainty in the momentum of the electron

i.e.,  $(\Delta P_x)_{\min} \geq 0.5 \times 10^{-20} \text{ kg.m/s} = p \rightarrow (6)$

Consider eqn (3)

$$E^2 = p^2c^2 + m_0^2c^4 = c^2(p^2 + m_0^2c^2)$$

Where  $m_0 = 9.11 \times 10^{-31}$  kg

If the electron exists in the nucleus its energy must be

$$E^2 \geq (3 \times 10^8)^2 [(0.5 \times 10^{-20})^2 + (9.11 \times 10^{-31})^2 (3 \times 10^8)^2]$$

i.e.  $E^2 \geq (3 \times 10^8)^2 [0.25 \times 10^{-40} + 7.4629 \times 10^{-44}]$

Neglecting the second term as it is smaller by more than the 3 orders of the magnitude compared to first term.

Taking square roots on both sides and simplifying

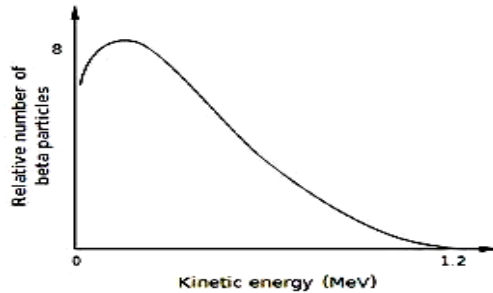
$$E \geq 1.5 \times 10^{-12} \text{ J} \geq \frac{1.5 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} \geq 9.4 \text{ MeV}$$

If an electron exists in the nucleus its energy must be greater than or equal to 9.4MeV. It is experimentally measured that the beta particles ejected from the nucleus during beta decay have energies of about 3 to 4 MeV. This shows that electrons cannot exist in the nucleus.

**[Beta decay:** In beta decay process, from the nucleus of an atom, when neutrons are converting into protons in releasing an electron (beta particle) and an antineutrino. When proton is converted into a neutron in releasing a positron (beta particle) and a neutrino. In both the processes energy



sharing is statistical in nature. When beta particles carry maximum energy neutrino's carries minimum energy and vice-versa. In all other processes energy sharing is in between maximum and minimum energies. The maximum energy carried by the beta particle is called as the end point energy ( $E_{\max}$ ).

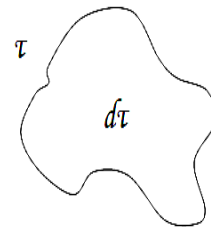


### Wave Function:

A physical situation in quantum mechanics is represented by a function called wave function. It is denoted by ' $\psi$ '. It accounts for the wave like properties of particles. Wave function is obtained by solving Schrodinger equation.

### Physical significance of wave function:

**Probability density:** If  $\psi$  is the wave function associated with a particle, then  $|\psi|^2$  is the probability of finding a particle in unit volume. If ' $\tau$ ' is the volume in which the particle is present but where it is exactly present is not known. Then the probability of finding a particle in certain elemental volume  $d\tau$  is given by  $|\psi|^2 d\tau$ . Thus  $|\psi|^2$  is called probability density. The probability of finding an event is real and positive quantity. In the case of complex wave functions, the probability density is  $|\psi|^2 = \psi^* \psi$  where  $\psi^*$  is Complex conjugate of  $\psi$ .



### Normalization:

The probability of finding a particle having wave function ' $\psi$ ' in a volume ' $d\tau$ ' is ' $|\psi|^2 d\tau$ '. If it is certain that the particle is present in finite volume ' $\tau$ ', then

$$\int_0^{\tau} |\psi|^2 d\tau = 1$$

If we are not certain that the particle is present in finite volume, then

$$\int_{-\infty}^{\infty} |\psi|^2 d\tau = 1$$

In some cases  $\int |\psi|^2 d\tau \neq 1$  and involves constant.

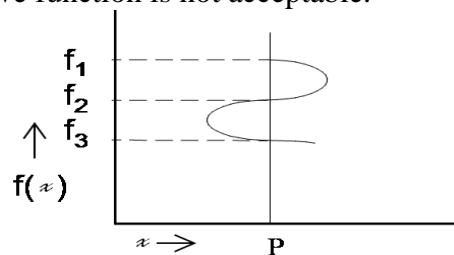
The process of integrating the square of the wave function within a suitable limits and equating it to unity the value of the constant involved in the wave function is estimated. The constant value is substituted in the wave function. This process is called as normalization. The wave function with constant value included is called as the normalized wave function and the value of constant is called normalization factor.

**Properties of the wave function:**

A system or state of the particle is defined by its energy, momentum, position etc. If the wave function ‘ $\psi$ ’ of the system is known, the system can be defined. The wave function ‘ $\psi$ ’ of the system changes with its state. To find ‘ $\psi$ ’ Schrodinger equation has to be solved. As it is a second order differential equation, there are several solutions. All the solutions may not be correct. We have to select those wave functions which are suitable to the system. The acceptable wave function has to possess the following properties:

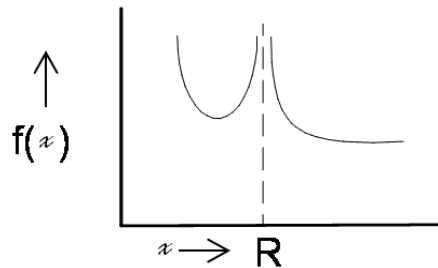
- 1) ‘ $\psi$ ’ is single valued everywhere:

Consider the function  $f(x)$  which varies with position as represented in the graph. The function  $f(x)$  has three values  $f_1$ ,  $f_2$  and  $f_3$  at  $x = p$ . Since  $f_1 \neq f_2 \neq f_3$  it is to state that if  $f(x)$  were to be the wave function. The probability of finding the particle has three different values at the same location which is not true. Thus the wave function is not acceptable.



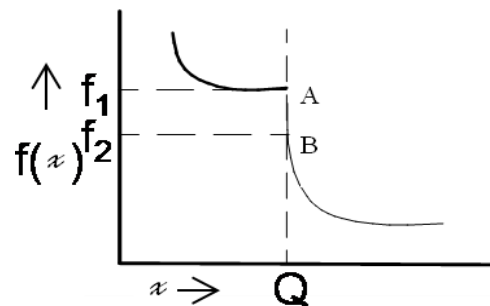
- 2) ‘ $\psi$ ’ is finite everywhere:

Consider the function  $f(x)$  which varies with position as represented in the graph. The function  $f(x)$  is not finite at  $x = R$  but  $f(x) = \infty$ . Thus it indicates large probability of finding the particle at a location. It violates uncertainty principle. Thus the wave function is not acceptable.



- 3) ' $\psi$ ' and its first derivatives with respect to its variables are continuous everywhere:

Consider the function  $f(x)$  which varies with position as represented in the graph. The function  $f(x)$  is truncated at  $x=Q$  between the points A & B, the state of the system is not defined. To obtain the wave function associated with the system, we have to solve Schrodinger wave equation. Since it is a second order differential wave equation, the wave function and its first derivative must be continuous at  $x=Q$ . As it is a discontinuous wave function, the wave function is not acceptable.



- 4) For bound states ' $\psi$ ' must vanish at potential boundary and outside. If ' $\psi^*$ ' is a complex function, then  $\psi^* \psi$  must also vanish at potential boundary and outside.

The wave function which satisfies the above 4 properties are called *Eigen functions*.

### Time independent Schrodinger wave equation

Consider a particle of mass ' $m$ ' moving with velocity ' $v$ '. The de-Broglie wavelength ' $\lambda$ ' is

$$\lambda = \frac{h}{mv} = \frac{h}{P} \rightarrow (1) \quad \text{Where 'mv' is the momentum of the particle.}$$

The wave eqn is

$$\psi = A e^{i(kx - \omega t)} \rightarrow (2)$$

Where ' $A$ ' is a constant and ' $\omega$ ' is the angular frequency of the wave.

Differentiating equation (2) with respect to 't' twice

$$\frac{d^2\psi}{dt^2} = -A\omega^2 e^{i(kx-\omega t)} = -\omega^2\psi \rightarrow (3)$$

The equation of a travelling wave is

$$\frac{d^2y}{dx^2} = \frac{1}{v^2} \frac{d^2y}{dt^2}$$

Where 'y' is the displacement and 'v' is the velocity.

Similarly for the de-Broglie wave associated with the particle

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2} \frac{d^2\psi}{dt^2} \rightarrow (4)$$

where 'ψ' is the displacement at time 't'.

From eqns (3) & (4)

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2}\psi$$

But  $\omega = 2\pi\nu$  and  $v = \nu\lambda$  where 'ν' is the frequency and 'λ' is the wavelength.

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi \text{ or } \frac{1}{\lambda^2} = -\frac{1}{4\pi^2\nu^2} \frac{d^2\psi}{dx^2} \rightarrow (5)$$

$$K.E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{P^2}{2m} \rightarrow (6)$$

$$= \frac{h^2}{2m\lambda^2} \rightarrow (7)$$

Using eqn (5)

$$K.E = \frac{h^2}{2m} \left( -\frac{1}{4\pi^2\psi} \right) \frac{d^2\psi}{dx^2} = -\frac{h^2}{8\pi^2m\psi} \frac{d^2\psi}{dx^2} \rightarrow (8)$$

Total Energy  $E = K.E + P.E$

$$E = -\frac{h^2}{8\pi^2 m \psi} \frac{d^2 \psi}{dx^2} + V$$

$$E - V = -\frac{h^2}{8\pi^2 m \psi} \frac{d^2 \psi}{dx^2}$$

$$\frac{d^2 \psi}{dx^2} = -\frac{8\pi^2 m}{h^2} (E - V) \psi$$

$$\boxed{\frac{d^2 \psi}{dx^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0}$$

This is the time independent Schrodinger wave equation.

### What are Eigen functions and Eigen values?

Eigen functions: Eigen functions are those wave functions of quantum mechanics which possess the properties that they are single valued & finite everywhere, and also their first derivatives with respect to their variables are continuous everywhere.

Eigen values: Eigen values are the possible values of a physical quantity like momentum, energy, etc., of a system that are observed when eigen functions are operated by quantum mechanical operators.

i.e., If an operator  $\hat{A}$  operating on the well – behaved function  $\Psi$  produces back the wave function multiplied by a constant  $\lambda$  i.e.,

$$\hat{A}\Psi = \lambda\Psi$$

then the function  $\Psi$  is called as the **eigen function** and the value  $\lambda$  is called the **eigen value** of the operator  $\hat{A}$  associated with eigenfunction  $\Psi$ .

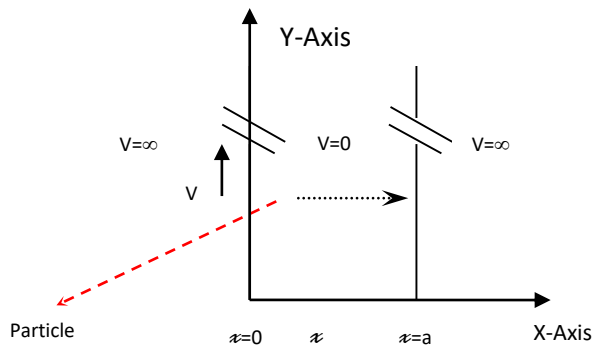
Example:

$$\frac{d}{dx} e^{ax} = a e^{ax}$$

Here,  $\frac{d}{dx}$  is the operator,  $e^{ax}$  is the eigen function and  $a$  is the eigen value.

**Application of Schrodinger wave equation:**

**Energy Eigen values of a particle in one dimensional, infinite potential well (potential well of infinite depth) or of a particle in a box**



Consider a particle of a mass 'm' free to move in one dimension along positive  $x$  -direction between  $x=0$  to  $x=a$ . The potential energy outside this region is infinite and within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box. The Schrödinger equation outside the well is,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}(E - \infty)\psi = 0 \rightarrow (1) \quad \because V = \infty$$

For outside, the equation holds good if  $\psi = 0$  &  $|\psi|^2 = 0$ . That is particle cannot be found outside the well and also at the walls

The Schrodinger's equation inside the well is:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}E\psi = 0 \rightarrow (2) \quad \because V = 0$$

$$\text{Let } \frac{8\pi^2m}{h^2}E = k^2 \rightarrow (3)$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The solution of this equation is:

$$\psi = C \cos kx + D \sin kx \rightarrow (4)$$

$$\text{at } x = 0 \rightarrow \psi = 0$$

$$0 = C \cos 0 + D \sin 0$$

$$\therefore C = 0$$

Also  $x = a \rightarrow \psi = 0$

$$0 = C \cos ka + D \sin ka$$

But  $C = 0$

$$\therefore D \sin ka = 0 \quad \longrightarrow \quad (5)$$

$D \neq 0$  (because the wave concept vanishes)

i.e.  $ka = n\pi$  where  $n = 0, 1, 2, 3, 4, \dots$  (Quantum number)

$$k = \frac{n\pi}{a} \rightarrow (6)$$

sub eqn (5) and (6) in (4)

$$\psi_n = D \sin \frac{n\pi}{a} x \rightarrow (7)$$

This gives permitted wave functions.

The Energy Eigen value given by

Substitute equation (6) in (3)

$$\frac{8\pi^2 m}{h^2} E = k^2 = \frac{n^2 \pi^2}{a^2}$$

For  $n = 0$   $E = \frac{n^2 h^2}{8ma^2} = 0$  is not acceptable inside the well because  $\psi_n = 0$ . It means that the electron is not present inside the well which is not true. Thus the lowest energy value for  $n = 1$  is called zero point energy value or ground state energy.

$$\text{i.e. } E_{\text{zero-point}} = \frac{h^2}{8ma^2}$$

The states for which  $n > 1$  are called excited states.

**To find out the value of D, normalization of the wave function is to be done.**

$$\text{i.e. } \int_0^a |\psi_n|^2 dx = 1 \rightarrow (8)$$

using the values of  $\psi_n$  from eqn (7)

$$\int_0^a D^2 \sin^2 \frac{n\pi}{a} x dx = 1$$

$$D^2 \int_0^a \left[ \frac{1 - \cos(2n\pi/a)x}{2} \right] dx = 1$$

$$\frac{D^2}{2} \left[ \int_0^a dx - \int_0^a \cos \frac{2n\pi}{a} x dx \right] = 1$$

$$\frac{D^2}{2} \left[ x - \frac{a}{2n\pi} \sin \frac{2n\pi}{a} x \right]_0^a = 1$$

$$\frac{D^2}{2} [a - 0] = 1$$

$$\frac{D^2}{2} a = 1$$

$$D = \sqrt{\frac{2}{a}}$$

$$\therefore \sin^2 \theta = \left( \frac{1 - \cos 2\theta}{2} \right)$$

Substitute D in equation (7)

the normalized wave functions of a particle in one dimensional infinite potential well is:

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x \rightarrow (9)$$



**Wave functions, probability densities and energy levels for particle in an infinite potential well:**

Let us consider the most probable location of the particle in the well and its energies for first three cases.

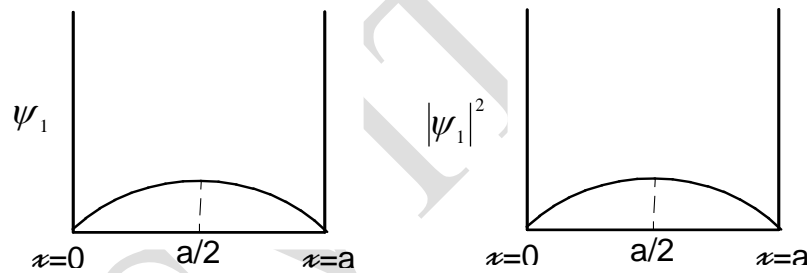
**Case I → n=1** It is the ground state and the particle is normally present in this state.

The Eigen function is

$$\psi_1 = \sqrt{\frac{2}{a}} \sin \frac{\pi}{a} x \quad \because \text{from eqn (7)}$$

$$\psi_1 = 0 \text{ for } x = 0 \text{ and } x = a$$

But  $\psi_1$  is maximum when  $x = a/2$ .



The plots of  $\psi_1$  versus  $x$  and  $|\psi_1|^2$  versus  $x$  are shown in the above figure.

$|\psi_1|^2 = 0$  for  $x = 0$  and  $x = a$  and it is maximum for  $x = a/2$ . i.e. in ground state the particle cannot be found at the walls, but the probability of finding it is maximum in the middle.

The energy of the particle at the ground state is

$$E_1 = \frac{h^2}{8ma^2} = E_0$$

**Case II → n=2**

In the first excited state the Eigen function of this state is

$$\psi_2 = \sqrt{\frac{2}{a}} \sin \frac{2\pi}{a} x$$

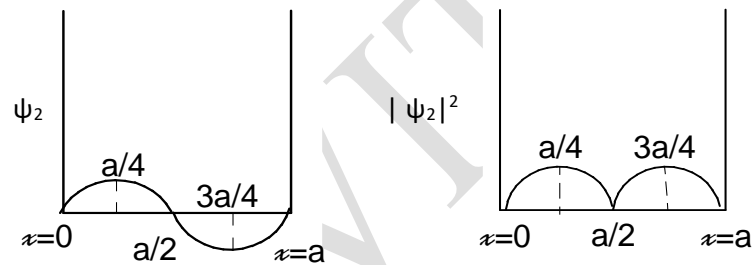
$\psi_2 = 0$  for the values  $x = 0, a/2, a$ .

Also  $\psi_2$  is maximum for the values  $x = a/4$  and  $3a/4$ .

These are represented in the graphs.

$|\psi_2|^2 = 0$  at  $x = 0, a/2, a$ , i.e. particle cannot be found either at the walls or at the centre.

$|\psi_2|^2 = \text{maximum}$  for  $x = \frac{a}{4}, x = \frac{3a}{4}$



The energy of the particle in the first excited state is  $E_2 = 4E_0$ .

**Case III → n=3**

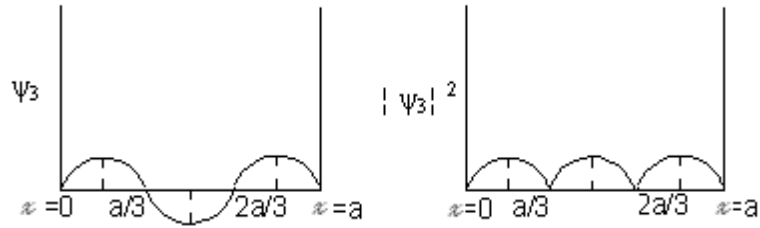
In the second excited state,

$$\psi_3 = \sqrt{\frac{2}{a}} \sin \frac{3\pi}{a} x$$

$\psi_3 = 0$ , for  $x = 0, a/3, 2a/3$  and  $a$ .

$\psi_3$  is maximum for  $x = a/6, a/2, 5a/6$ .

These are represented in the graphs.



$|\psi_3|^2 = 0$  for  $x = 0, a/3, 2a/3$  and  $a$ .  $|\psi_3|^2 = \text{maximum}$  for  $x = \frac{a}{6}, x = \frac{a}{2}, x = \frac{5a}{6}$

The energy of the particle in the second excited state is  $E_3=9 E_0$ .

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SVI

**MODULE 3- LASERS AND OPTICAL FIBERS****LASERS**

The word Laser stands for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation. It is a device which amplifies light. It has properties like Coherence, Unidirectional, Monochromatic, Focus ability, etc.

**Interaction of an electromagnetic wave with matter** leads to transition of an atom or a molecule from one energy state to another. If the transition is from lower state to higher state it absorbs the incident energy. If the transition is from higher state to lower state it emits a part of its energy.

Emission or Absorption takes through quantum of energy called photons.  $h\nu$  is called quantum energy or photon energy.

$h = 6.626 \times 10^{-34}$  Joules Second is Planck's constant and ' $\nu$ ' is the frequency.

If  $\Delta E$  is the difference between the two energy levels,

$$\text{Then } \Delta E = (E_2 - E_1) \text{ Joule}$$

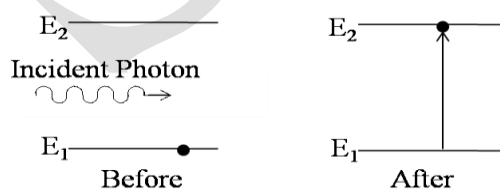
According to Max Planck,  $\Delta E = h\nu = (E_2 - E_1)$

$$\nu = (E_2 - E_1)/h \quad \text{Hz.}$$

Three types of interactions, which are possible, are as follows:

**1) Induced Absorption:**

Induced absorption is the absorption of an incident photon by system as a result of which the system is elevated from a lower energy state to a higher state, wherein the difference in energy of the two states is the energy of the photon.



Consider the system having two energy states  $E_1$  and  $E_2$ ,  $E_2 > E_1$ . When a photon of energy  $h\nu$  is incident on an atom at level  $E_1$ , the atom goes to a higher energy level by absorbing the energy.

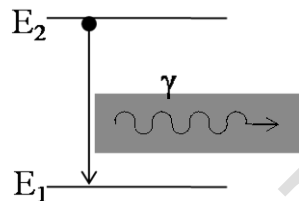
CBCS-2021 Scheme

When an atom is at ground level ( $E_1$ ), if an electromagnetic wave of frequency  $\nu$  is applied to the atom, there is possibility of getting excited to higher level ( $E_2$ ). The incident photon is absorbed. It is represented as



### **2) Spontaneous Emission:**

The emission of a photon by the transition of a system from a higher energy state to a lower energy state without the aid of an external energy is called spontaneous emission.



Let ' $E_1$ ' and ' $E_2$ ' be two energy levels in a material, such that  $E_2 > E_1$ .  $E_1$  is ground level and  $E_2$  is the higher level.  $h\nu = E_2 - E_1$  is the difference in the energy. The atom at higher level ( $E_2$ ) is more unstable as compared to that at lower level ( $E_1$ ).

The life time of an atom is less in the excited state, In spontaneous emission atom emits the photon without the aid of any external energy. It is called spontaneous emission. The process is represented as



The photons emitted in spontaneous emission may not have same direction and phase similarities. It is incoherent.

Ex: Glowing electric bulbs, Candle flame etc.

### **3) Stimulated Emission:**

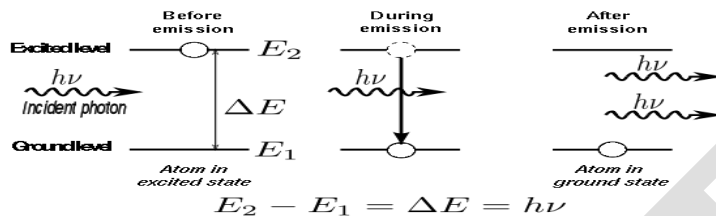
Stimulated emission is the emission of a photon by a system under the influence of a passing photon of just the right energy due to which the system transits from a higher energy state to a lower energy state.

The photon thus emitted is called stimulated photon and will have the same phase, energy and direction of movement as that of the passing photon called the stimulation photon.

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Initially the atom is at higher level  $E_2$ . The incident photon of energy  $h\nu$  forces the atom to get de-excited from higher level  $E_2$  to lower level  $E_1$ .

i.e.  $h\nu = E_2 - E_1$  is the change in energy.



The incident photon stimulates the excited atom to emit a photon of exactly the same energy as that of the incident photons. The emitted two photons have same phase, frequency, direction and polarization with the incident photon and results in coherent beam of radiation. This kind of action is responsible for lasing action.



### Expression for energy density in terms of Einstein's Coefficients

Consider two energy levels  $E_1$  and  $E_2$  of a system of atoms with  $N_1$  and  $N_2$  are population of energy levels respectively.

Let  $U_\nu$  be the energy density of incident beam of radiation of frequency  $\nu$ . Let us consider the absorption and two emission process

#### **1) Induced absorption:**

Induced absorption is the absorption of an incident photon by system as a result of which the system is elevated from a lower energy state to a higher state.

The rate of absorption is proportional to  $N_1 U_\nu$

$$\text{Rate of absorption} = B_{12} N_1 U_\nu \dots\dots\dots (1)$$

Where ' $B_{12}$ ' is the proportionality constant called Einstein Coefficient of induced absorption.

#### **2) Spontaneous emission:**

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The emission of a photon by the transition of a system from a higher energy state to a lower energy state without the aid of an external energy is called spontaneous emission.

Spontaneous emission depends on  $N_2$  and independent of energy density.

The rate of spontaneous emission =  $A_{21}N_2$  .....(2)

Where ' $A_{21}$ ' is called proportionality constant called Einstein coefficient of spontaneous emission.

### 3) Stimulated emission:

Stimulated emission is the emission of a photon by a system under the influence of a passing photon of just the right energy due to which the system transits from a higher energy state to a lower energy state

The rate of stimulated emission is directly proportional to  $N_2U_\nu$ .

The rate of stimulated emission =  $B_{21}N_2U_\nu$  ..... (3)

Where ' $B_{21}$ ' is the proportionality constant called Einstein's Coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = (Rate of spontaneous emission + Rate of stimulated emission)

$$B_{12}N_1U_\nu = A_{21}N_2 + B_{21}N_2U_\nu$$

$$U_\nu (B_{12}N_1 - B_{21}N_2) = A_{21}N_2$$

$$U_\nu = \frac{A_{21}N_2}{(B_{12}N_1 - B_{21}N_2)}$$

$$\text{i.e. } U_\nu = \frac{A_{21}}{B_{21}} \left[ \frac{N_2}{\left(\frac{B_{12}}{B_{21}}N_1 - N_2\right)} \right]$$

$$U_\nu = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\left(\frac{B_{12}N_1}{B_{21}N_2} - 1\right)} \right] \rightarrow (4)$$

By Boltzmann's law,  $N_2 = N_1 e^{-\left(\frac{E_2 - E_1}{KT}\right)} = N_1 e^{-h\nu/KT}$

CBCS-2021 Scheme

$$\text{i.e., } N_1/N_2 = e^{h\nu/KT}$$

$$\text{Eqn. (4) becomes } U_\nu = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\left( \frac{B_{12}}{B_{21}} e^{\left(\frac{h\nu}{kT}\right)} - 1 \right)} \right] \rightarrow (5)$$

$$\text{By Planck's law, } U_\nu = \frac{8\pi h \nu^3}{c^3} \left[ \frac{1}{\left( e^{\left(\frac{h\nu}{kT}\right)} - 1 \right)} \right] \rightarrow (6)$$

Comparing equation (5) & (6)

$$\frac{A_{21}}{B_{21}} = 8\pi h \nu^3 / c^3 \quad \& \quad \frac{B_{12}}{B_{21}} = 1 \quad \text{i.e. } B_{12} = B_{21}$$

The probability of induced adsorption is equal to the stimulated emission.

Therefore,  $A_{12}$  is written as  $A$  and  $B_{12}$ ,  $B_{21}$  written as  $B$ .

Equation (5) becomes

$$U_\nu = \frac{A}{B} \left[ \frac{1}{\left[ e^{\frac{h\nu}{kT}} - 1 \right]} \right]$$

Above equation is the expression for energy density

### **Condition for laser action:**

#### **1) Meta Stable State:**

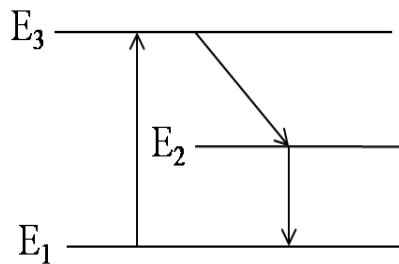
It is the special type of excited state where in the life time of atom is more than the normal excited state.

This state plays an important role in lasing action. In metastable state, atoms stay of the order of  $10^{-3}$  to  $10^{-2}$  second. In normal excited state other than metastable atom stay of order of  $10^{-8}$  to  $10^{-9}$  seconds. It is possible to achieve population inversion condition in certain system which possesses a metastable state.



## 2) Population Inversion:

It is the state of the system at which the population of a higher energy level is greater than that of the lower energy level.



Let  $E_1$ ,  $E_2$ ,  $E_3$  be the energy levels of the system  $E_3 > E_2 > E_1$ .  $E_2$  is the metastable state of the system. Atoms get excited from the state  $E_1$  to  $E_3$  by means of external source and stay there for short time. These atoms undergo spontaneous transitions to  $E_2$  and  $E_1$ . The atoms at the state  $E_2$  stay for longer time. A stage is reached in which the number of atoms at state  $E_2$  is more than the number of atoms at  $E_1$  which is known as population inversion.

### Requisites of a Laser System:

#### 1) **The pumping process:**

It is the process of supplying energy to the medium in order to transfer it to the state of population inversion is known as pumping process

**Optical Pumping:** It is the process of exciting atoms from lower energy level to higher energy level by using high intensity light or by operating flash tube as an external source called optical pumping.

**Electrical pumping:** It is the process of exciting atoms from lower energy level to higher energy level by using dc power supply as an external source called electrical pumping.

#### 2) **Active medium:**

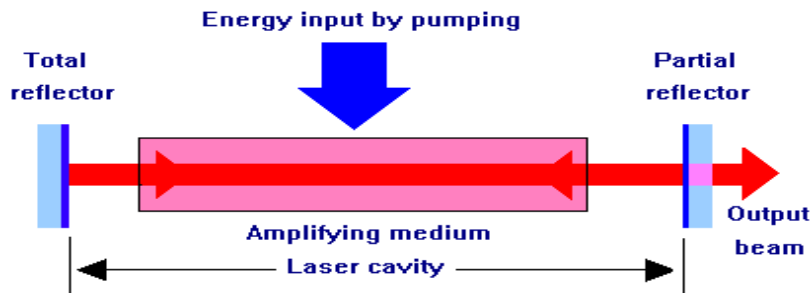
It is a medium which supports population inversion and promotes stimulated emission leading to light amplification

**Active centers:** In a medium consisting of different species of atoms only small fraction of the atoms of a particular type are responsible for stimulated emission and consequent light amplification they are known as Active centers

#### 3) **Laser cavity.**

An active medium bounded between two mirrors is called as a laser cavity.

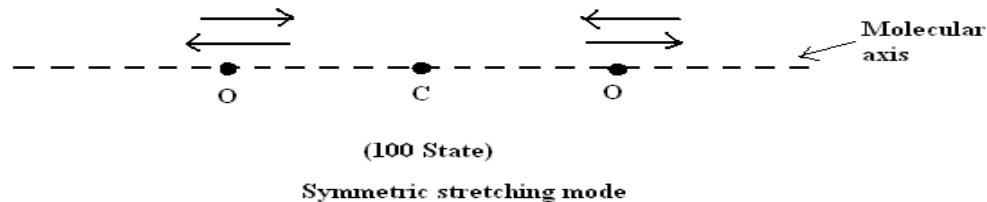
CBCS-2021 Scheme



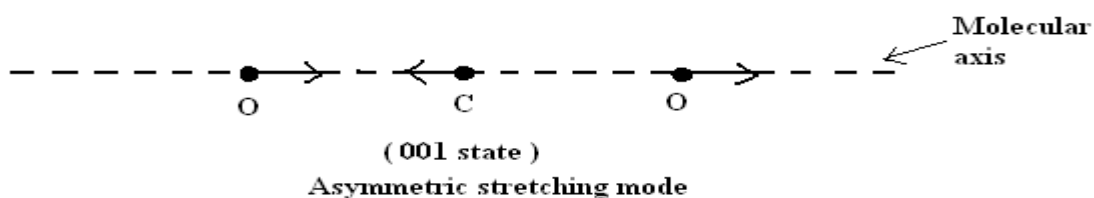
### Vibrational modes of CO<sub>2</sub> molecule:

A carbon dioxide molecule has two oxygen atoms between which there is a carbon atom. It has 3 different modes of vibration.

1. **Symmetric stretching mode** : In this mode, carbon atom is stationary and the oxygen atoms oscillate to and fro along the molecular axis. This state is referred as (100) state.

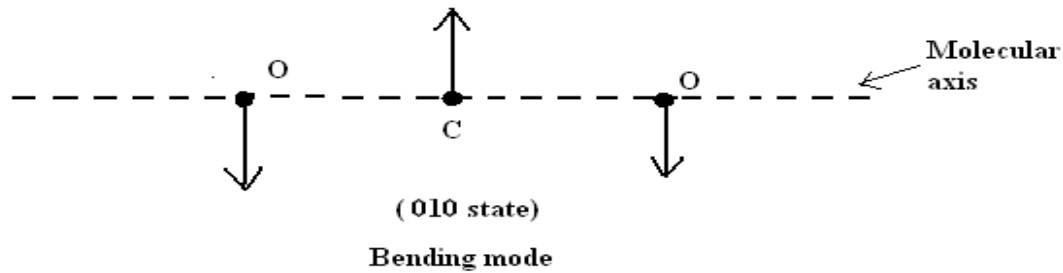


2. **Asymmetric stretching mode**: In this mode, both the oxygen atoms moves in one direction while the carbon atom moves in opposite direction along the molecular axis. This state is referred as (001) state.

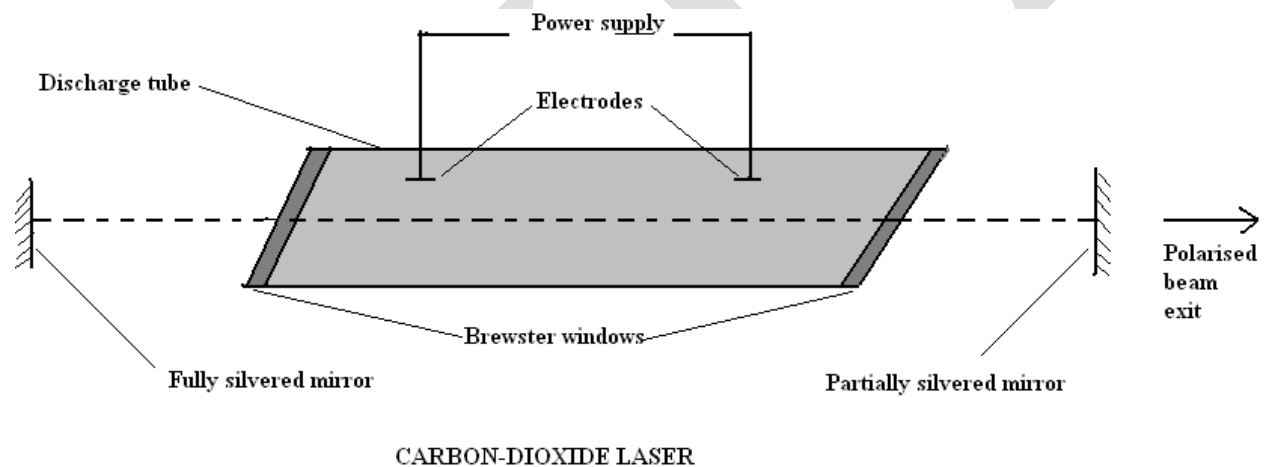


3. **Bending mode**: In this mode, the carbon atom and oxygen atoms moves perpendicular to molecular axis in the opposite direction. This state is referred to as (010)state.

CBCS-2021 Scheme

**CO<sub>2</sub> LASER:**

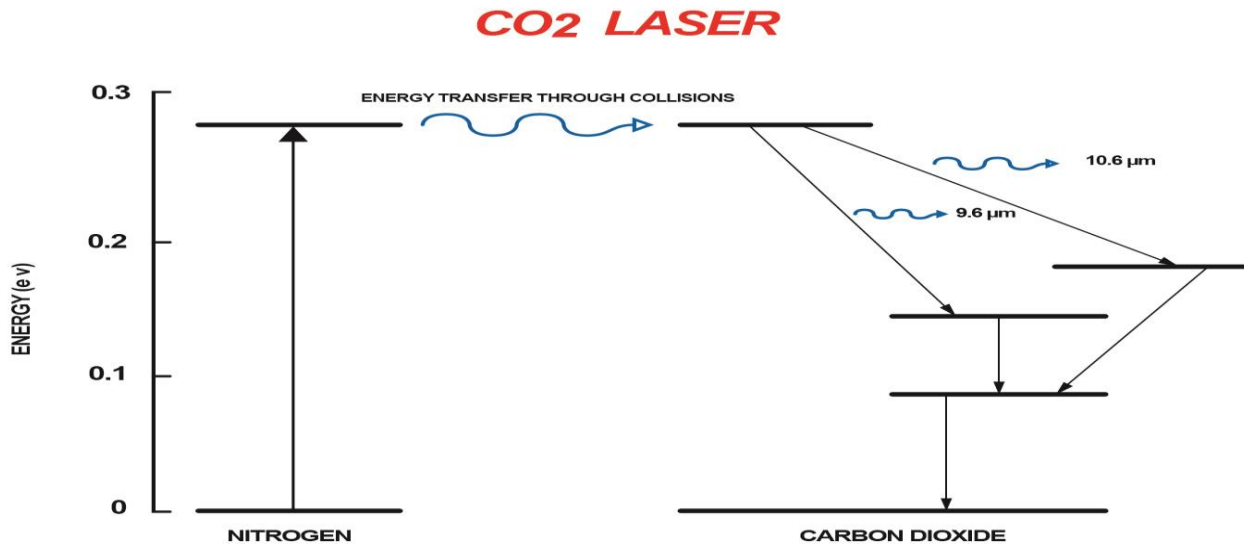
It was devised by C.K.N Patel in 1924. CO<sub>2</sub> laser is molecular gas laser which operates in the IR region involving a set of vibrational – rotational transitions. It is a four level laser producing both continuous and pulsed laser.

**Construction:**

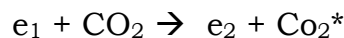
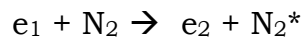
CARBON-DIOXIDE LASER

- It consists of discharge tube (quartz) of diameter 2.5cm and length of 5m.
- Either side of the discharge tube fitted with quartz plates function as Brewster's window.
- The tube has got two parallel mirrors. One is partially silvered and the other is fully silvered to function as laser cavity
- External source is connected to discharge tube using the electrodes.
- Sometimes water vapour is added because during discharge CO<sub>2</sub> molecule breaks up into CO and O. The water vapour additives help in deoxidizing CO to CO<sub>2</sub>.
- The tube is filled with a mixture of CO<sub>2</sub>, N<sub>2</sub> and He gas in the ratio 1:2:3.

CBCS-2021 Scheme

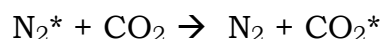
**Working:****CO<sub>2</sub> laser Energy level diagram**

- When an electric field is applied electrons rendered free from atoms, collide with N<sub>2</sub> & CO<sub>2</sub> molecules in their path towards positive electrodes due to which N<sub>2</sub> atoms are excited to the higher energy level v<sub>1</sub>.
- Likewise, it happens to the CO<sub>2</sub> molecule. This is called collision of first kind



Where, e<sub>1</sub> and e<sub>2</sub> are the energies of electrons before and after collisions.

- Let the ground state, (010) state, (020) state, (100) state and (001) are represented as E<sub>1</sub>, E<sub>2</sub>, E<sub>3</sub>, E<sub>4</sub> and E<sub>5</sub> levels respectively
- Because of matching energy levels, v = 1 state of N<sub>2</sub> is equal to (001) state of CO<sub>2</sub>, N<sub>2</sub> molecule in the metastable state collide with the CO<sub>2</sub> in the ground state and transfer of energy takes place from N<sub>2</sub> to CO<sub>2</sub>. As a result of which CO<sub>2</sub> molecule moved to (001) state where as the N<sub>2</sub> molecule moved to ground state. This is the collision of second kind.



CBCS-2021 Scheme

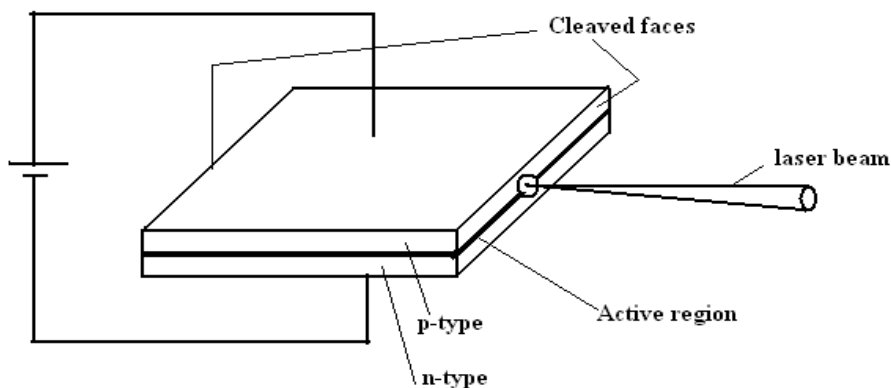
Where,  $CO_2$  and  $CO_2^*$  are the energies of  $CO_2$  in ground state and excited states.

- Because of the resonant transfer of energy, the population inversion is achieved in (001) state with respect to (100) and (020)
- The transition from  $E_5$  to  $E_4$  levels gives wavelength of  $10.6\mu\text{m}$  (in far IR region)
- The transitions from  $E_5$  to  $E_3$  level gives wavelength of  $9.6\mu\text{m}$  (in far IR region)
- Following these transitions the  $CO_2$  molecules in  $E_4$  and  $E_3$  collide with the ground state  $CO_2$  molecules (because of the matching energy levels) and arrive at  $E_2$  state.
- The molecules in the  $E_2$  state collide with He and water vapour molecules, so that come down to the ground state.
- The cycle of operation gives both continuous and pulsed laser.

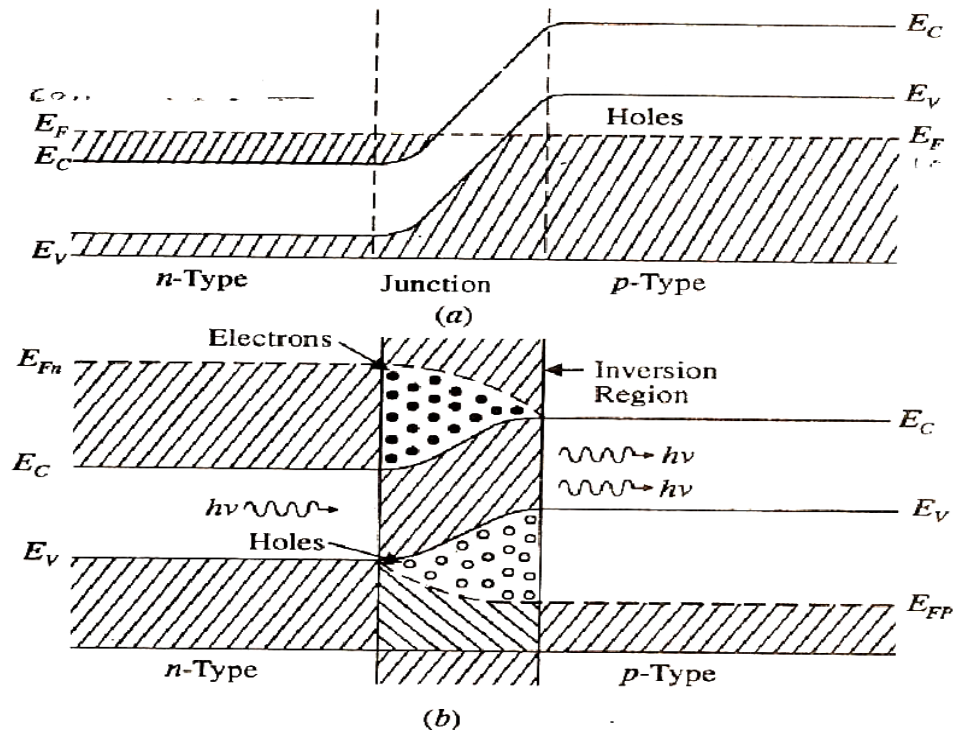
### **Gallium-Arsenide Laser Semiconductor laser:**

A Semiconductor diode laser is one in which the active medium is formulated by semiconducting materials.

**Construction:** Gallium-Arsenide Laser is a single crystal of GaAs consists of heavily doped n-type and p-type. The diode is very small size with sides of the order of 1mm. The width of the junction varies from 1-100 $\mu\text{m}$ . The top and bottom surfaces are metalized and Ohmic contacts are provided for external connection. The front and rear faces are polished. The polished faces functions as the resonant cavity. The other two faces are roughened to prevent lasing action in that direction.



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**Working:**

**Energy level diagram of p-n junction Ga-As semiconductor diode laser**

**(a) Before biasing      (b) After biasing**

- The energy band diagram of heavily doped p-n junction is as shown. At thermal equilibrium the Fermi level is uniform.
- Because of very high doping on **n-side**, the Fermi level is pushed in to the conduction band and electrons occupy the portions of the conduction band that lies below the Fermi level and on **p-side**, the Fermi level lies within the valence band and holes occupy the portions of the valence band that lies above the Fermi level.
- A suitable forward bias is applied to overcome the potential barrier. As a result electrons from n-region and holes from p-region injected into the junction.
- The current begins to flow following which there will be a region in junction in which the population inversion can be achieved.
- Initially concentration of electrons in the energy levels at the bottom of the conduction band will be less than that of energy levels at top of valence

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band. So that the recombination of electrons and holes result only in spontaneous emission then junction works as LED.

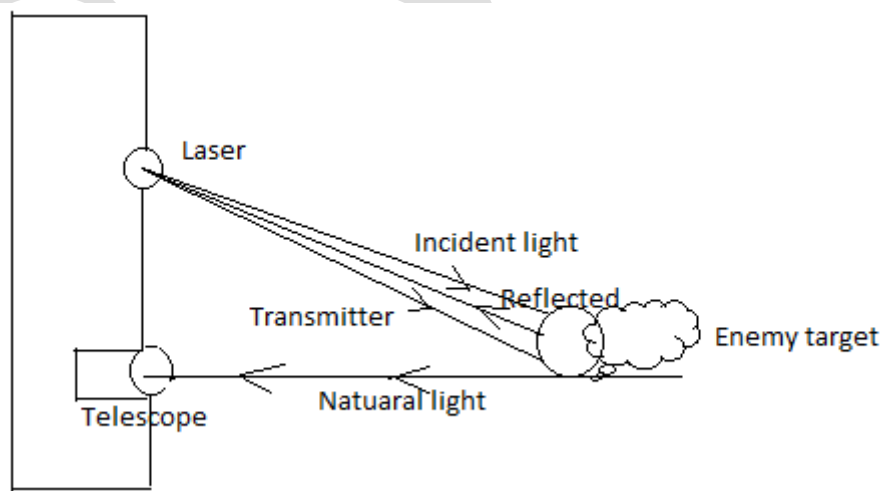
- When the current exceeds the threshold value, population inversion is achieved in the active region which is formulated in the junction.
- At this stage the photons emitted by spontaneous emission triggers stimulated emission, over a large number of recombination leading to build up laser.
- Since the energy gap of GaAs is 1.4eV, the wavelength of emitted light is 8400 Å.

### **Properties of laser:**

1. **Coherence:** The emitted radiation after getting triggered is in phase with the incident radiation.
2. **Monochromaticity:** The laser beam is highly monochromatic than any other radiations.
3. **Unidirectionality:** Laser beam travels in only one direction. It can travel long distance without spreading.
4. **Focusability:** A laser beam can be focused to an extremely fine spot.

### **Applications of laser:**

#### **1) Lasers in Defense - Laser range finder in defense**



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A high power pulsed laser (Nd-YAG) beam is directed towards the enemy target from the transmitter. The beam bounces back from the surface of the target as a reflection. A part of the reflected beam is received as a signal by the receiver. The unwanted noise signal will be filtered by the optical filter and pure signal is amplified by the photomultiplier in the receiver. The range finders high speed clock measures the exact time of incident and reflection of the pulse and then convert it in to distance.

## **2. Medical Applications- Eye Surgery and Skin Treatment**

### **a) Eye Surgery:**

LASIK, which stands for laser in-situ keratomileusis, is a popular surgery that can correct vision in people who are nearsighted or farsighted, or who have astigmatism.

It's one of many vision correction surgeries that work by reshaping your cornea, the clear front part of your eye, so that light focuses on the retina in the back of your eye.

### **The LASIK procedure:**

There are three essential steps to a LASIK procedure:

1. A mechanical surgical tool called a microkeratome or a femtosecond laser is used to create a thin, circular flap in the cornea. The surgeon then folds back this hinged flap to access the underlying cornea (called the stroma).
2. An excimer laser is used to reshape the corneal stroma. This highly specialized laser uses a cool ultraviolet light beam to remove ("ablate") microscopic amounts of tissue from the cornea to reshape it so it more accurately focuses light on the retina for improved vision.
3. The corneal flap is laid back in place, where it adheres to the corneal stroma without stitches.

### **b) Skin Treatment:**

Laser therapy is also used cosmetically to:

- remove warts, moles, birthmarks, and sun spots
- remove hair
- lessen the appearance of wrinkles, blemishes, or scars
- remove tattoos



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Laser toning and tightening is a non-surgical cosmetic correction method that uses lasers to penetrate the deeper layers of the skin.

Skin damage due to sun burning or aged skin is removed by directing an intense wavelength of light. Most of the times CO<sub>2</sub>, Yttrium Aluminium-Garnet lasers are used.

Carbon dioxide (CO<sub>2</sub>) lasers make shallow cuts. They're often used for superficial cancers, such as skin cancer.

## **OPTICAL FIBERS**

An optical fiber is a cylindrical wave guide made of transparent dielectric material (glass or plastic) which guides light waves along its length by total internal reflection.

### **Principle**

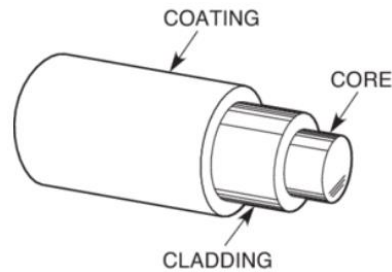
The propagation of light in an optical fiber from one end to the other end is based on the principle of Total internal reflection (TIR). They are used in optical communication.

When a light enters one end of the fiber, it undergoes successive total internal reflections from side walls and travels down the length of the fiber along zigzag path.

### **Construction**

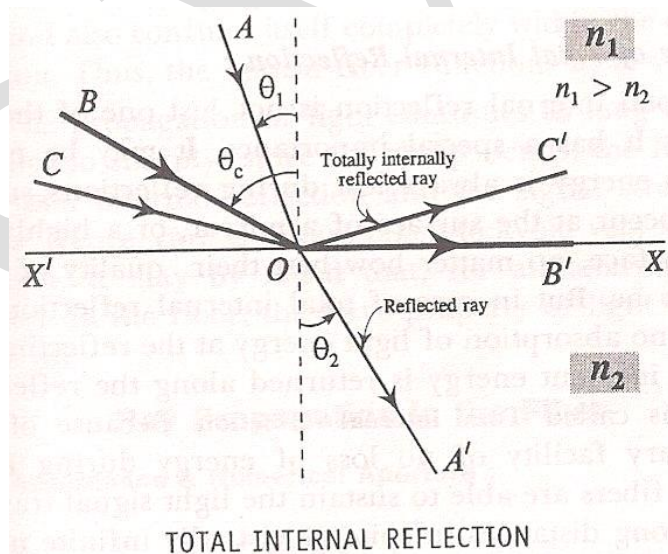
- A practical optical fiber is cylindrical in shape and has three regions.
- The innermost cylindrical region is the light guiding region called as core which is usually made up of glass or plastic.
- The outer part which is a concentric cylinder surrounding the core is called as cladding and is also made up of similar material but of lesser refractive index.
- The outermost region is called a Sheath or Protective buffer coating, nothing but the plastic coating providing a physical and environmental protection for the fiber. Number of such fibers is grouped to form a cable.

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### Total Internal Reflection

- When a ray of light travels from denser to rarer medium it bends away from the normal.
- As the angle of incidence increases in the denser medium, the angle of refraction also increases. For a particular angle of incidence called the “critical angle” ( $\theta_c$ ), the refracted ray grazes the surface separating the media or the angle of refraction is equal to  $90^\circ$ .
- If the angle of incidence is further increased beyond the critical angle, the light ray is reflected back to the same medium. This is called “Total Internal Reflection”.
- In total internal reflection, there is no loss of energy. The entire incident ray is reflected back.



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Let  $XX^1$  is the surface separating medium of refractive index  $n_1$  and medium of refractive index  $n_2$ ,  $n_1 > n_2$ . AO and  $OA^1$  are incident and refracted rays.  $\theta_1$  and  $\theta_2$  are angle of incidence and angle of refraction,  $\theta_2 > \theta_1$ . For the ray BO,  $\theta_c$  is the critical angle.  $OB^1$  is the refracted ray which grazes the interface. The ray CO incident with an angle greater than  $\theta_c$  is totally reflected back along  $OC^1$ .

From Snells law,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

For total internal reflection,

$$\theta_1 = \theta_c \text{ and } \theta_2 = 90^\circ$$

$$n_1 \sin \theta_c = n_2 (\text{because } \sin 90^\circ = 1)$$

$$\therefore \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$

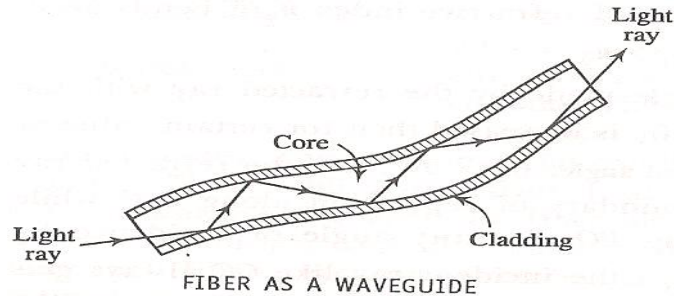
In total internal reflection there is no loss or absorption of light energy. The entire energy is returned along the reflected light. Thus is called Total internal reflection.

### **Propagation mechanism**

- The cladding in an optical fiber always has a lower refractive index than that of the core.
- The light signal which enters into the core and strikes the interface of the core and cladding with an angle greater than the critical angle will undergo total internal reflection.
- Thus the light signal undergoes multiple reflections within the core and propagates through the fiber.
- Since each reflection is a total internal reflection, there is no absorption of light energy at the reflecting surface.

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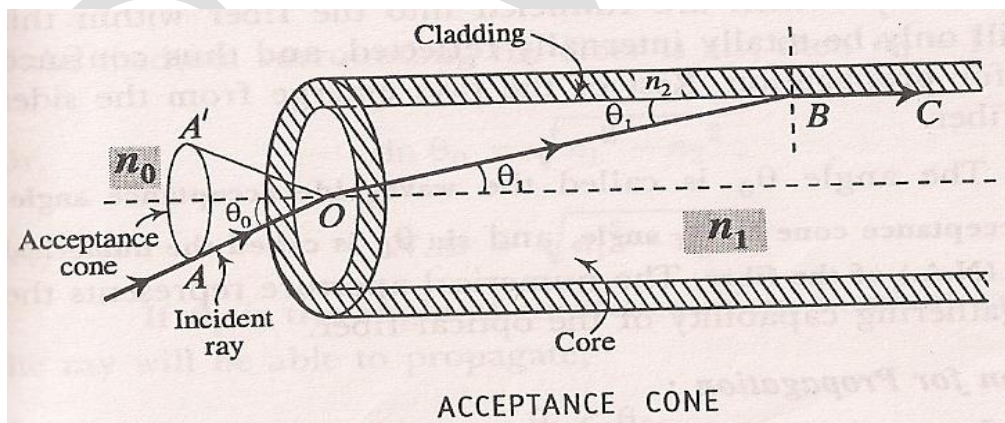
- Therefore the signal sustains its strength and also confines itself completely within the core during the propagation.



- After series of such total internal reflection, it emerges out of the core. Thus the optical fiber works as a waveguide. Care must be taken to avoid very sharp bends in the fiber because at sharp bends, the light ray fails to undergo total internal reflection.

### **Expression for Numerical aperture and condition for propagation**

Consider a light ray AO incident at an angle ' $\theta_0$ ' enters into the fiber. Let ' $\theta_1$ ' be the angle of refraction for the ray OB. The refracted ray OB incident at a critical angle ( $90^\circ - \theta_1$ ) at B grazes the interface between core and cladding along BC. If the angle of incidence is greater than critical angle, it undergoes total internal reflection. Thus  $\theta_0$  is called the waveguide acceptance angle and  $\sin\theta_0$  is called the numerical aperture.



Let  $n_0$ ,  $n_1$  and  $n_2$  be the refractive indices of the surrounding medium, core and cladding respectively.

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Applying Snell's law at O,  $n_0 \sin \theta_0 = n_1 \sin \theta_1$ 

$$\sin \theta_0 = \frac{n_1}{n_0} \sin \theta_1 \dots \dots \dots (1)$$

Applying Snell's law at B,

$$n_1 \sin (90 - \theta_1) = n_2 \sin 90$$

$$n_1 \cos \theta_1 = n_2$$

$$\Rightarrow \cos \theta_1 = \frac{n_2}{n_1} \dots \dots \dots (2)$$

From expression (1)

$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{1 - \cos^2 \theta_1}$$

Substituting for  $\cos \theta_1$  from (2)

$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin \theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

If  $n_0 = 1$  i.e., surrounding medium if it is air

$$\sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

Where,  $\sin \theta_0$  is called numerical aperture

$$N.A. = \sqrt{n_1^2 - n_2^2}$$

**Condition for propagation:**

If  $\theta_i$  is the angle of incidence of the incident ray, then the ray will be able to propagate,

$$\text{if } \theta_i < \theta_0$$

$$\Rightarrow \text{if } \sin \theta_i < \sin \theta_0$$

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$$\text{or } \sin \theta_i < \sqrt{n_1^2 - n_2^2}$$

$$\text{i. e., } \sin \theta_i < N.A.$$

**Acceptance angle** is defined as the maximum angle that a light ray can have relative to the axis of the fiber and propagate through the fiber.

**Numerical aperture** indicates the ability of the optical fiber to accept light i.e the light gathering capability of the optical fiber. The sign of the acceptance angle also called numerical aperture.

**Fractional index change ( $\Delta$ )**

The ratio of the difference in refractive index of core and cladding to the refractive index of core of an optical fiber.

$$\text{i. e., } \Delta = \frac{n_1 - n_2}{n_1}$$

**Mode of Propagation:**

The number of light signals passing through an optical fiber is called mode of propagation. If only one light wave passing through an optical fiber then it is single mode fiber, if more than one light wave passing through an optical fiber then it is call multimode fiber.

**V- number**

The number of modes supported for propagation in the fiber is determined by a parameter called V-number.

If the surrounding medium is air, then V-number is given by,

$$V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2}$$

Where, d is the core diameter,

$n_1$  and  $n_2$  are refractive indices of core and cladding respectively,

$\lambda$  is the wavelength of light propagating in the fiber.

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If the fiber is surrounded by a medium of refractive index  $n_0$ , then,

$$V = \frac{\pi d}{\lambda} \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

For  $V \gg 1$ , the number of modes supported by the fiber is given by, number of

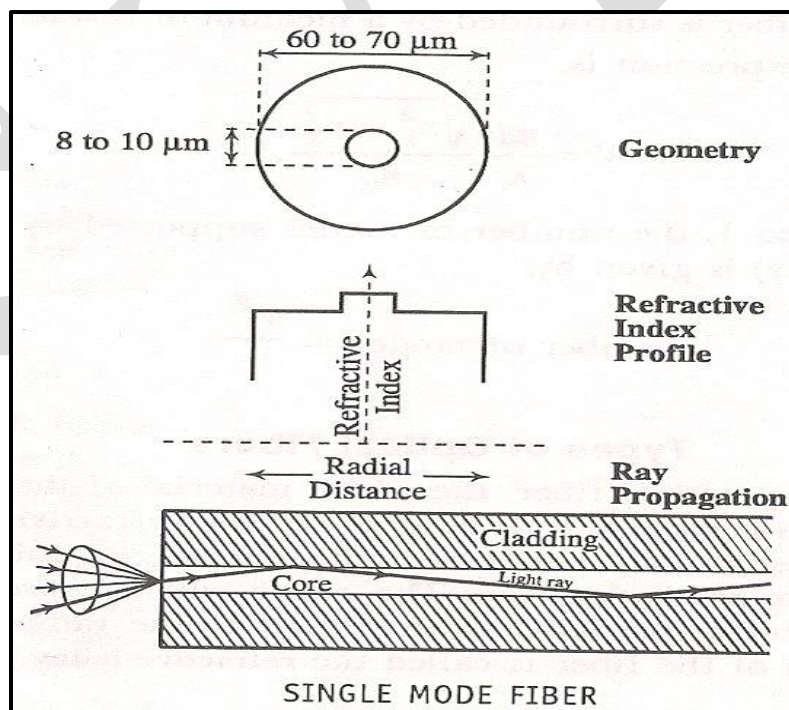
$$\text{modes} \cong \frac{V^2}{2}$$

### **Types of optical fibers**

Based on the refractive index profile and mode of propagation, There are three types of optical fibers,

1. Single mode fiber
2. Step index multimode fiber
3. Graded index multimode fiber

#### **(i) Single mode fiber**

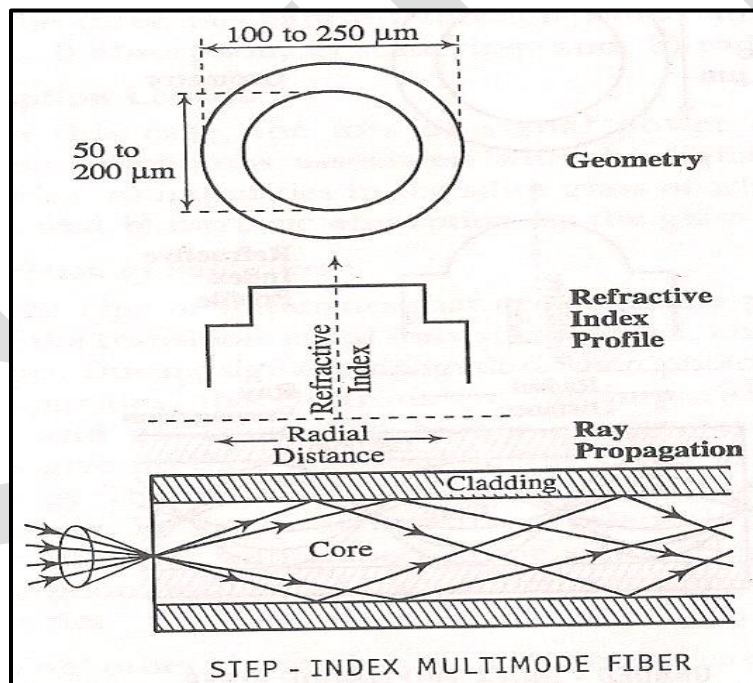




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- Single mode fibers have a core material of uniform refractive index value.
- Cladding material also has a uniform refractive index but of lesser value than that of core.
- Thus its refractive index profile takes a shape of a step. The diameter of the core is about 8-10  $\mu\text{m}$  and the diameter of the cladding is about 60-70  $\mu\text{m}$ .
- Because of its narrow core, it can guide just a single mode as shown in above figure.
- Single mode fibers are the extensively used ones and they are less expensive. They need LASERS as the source of light.

**(ii) Step index multimode fiber**



- A step index multimode fiber is very much similar to the single mode fiber except that its core is of large diameter. A typical fiber has a core diameter 50 to 200  $\mu\text{m}$  and a cladding about 100 to 250  $\mu\text{m}$  outer diameter.
- Its refractive index profile is also similar to that of a single mode fiber but with a larger plane region for the core.
- Due to the large core diameter it can transmit a number of modes of wave



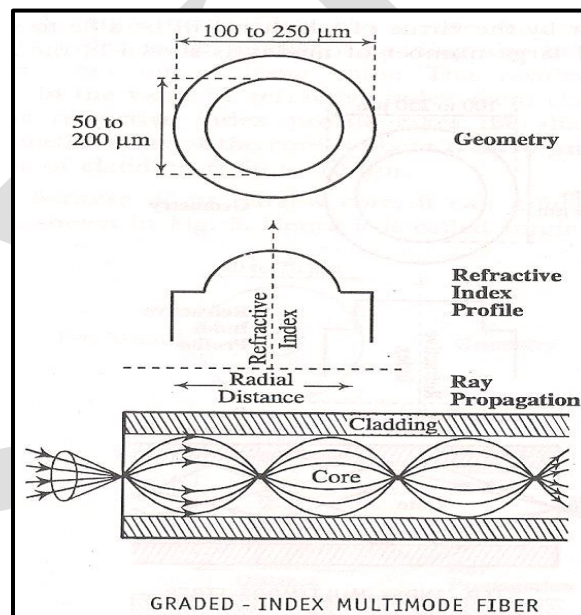
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propagation.

- The step index multimode fiber can accept either a LASER or an LED as source of light.
- It is the least expensive of all and its typical application is in data links which has lower bandwidth requirements.

**(iii) Graded index multimode fiber**

- It is also called GRIN..
- The refractive index of core decreases in the radially outward direction from the axis of the fiber and becomes equal to that of cladding at the interface but the refractive index of the cladding remains uniform.
- Laser or LED is used as a source of light.
- It is the expensive of all. It is used in telephone trunk between central offices.



**Signal attenuation in optical fibers**

- Attenuation is the loss of optical power suffered by the optical signal as it propagates through a fiber also called as the fiber loss.
- There are three mechanisms through which attenuation takes place.

Attenuation can be caused by three mechanisms.

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**(i) Absorption losses**

- Absorption of photons by impurities like metal ions such as iron, chromium, cobalt and copper in the silica glass of which the fiber is made of.
- During signal propagation photons interact with electrons of impurity atoms and the electrons are excited to higher energy levels.
- Then the electrons give up their absorbed energy either in the form of heat or light energy.
- The re-emission of light energy will usually be in a different wavelength; hence it is referred as loss of energy.
- The other impurity such as hydroxyl (OH) ions which enters into the fiber at the time of fabrication causes significant absorption loss.
- The absorption of photons by fiber itself assuming that there are no impurities and in-homogeneities in it is called as *intrinsic absorption*.

**(ii) Scattering losses**

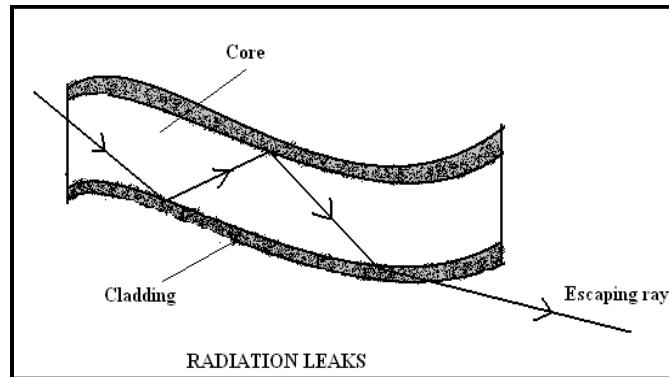
- Scattering of light waves occurs whenever a light wave travels through a medium having scattering objects whose dimensions are smaller than the wavelength of light.
- Similarly when a light signal travels in the fiber, the photons may be scattered due to the sharp changes in refractive index values inside the core over distances and also due to the structural impurities present in the fiber material.
- This type of scattering is called as Rayleigh scattering. Scattering of photons also takes place due to trapped gas bubbles which are not dissolved at the time of manufacturing.
- A scattered photon moves in random direction and leaves the fiber.

**(iii) Radiation losses**

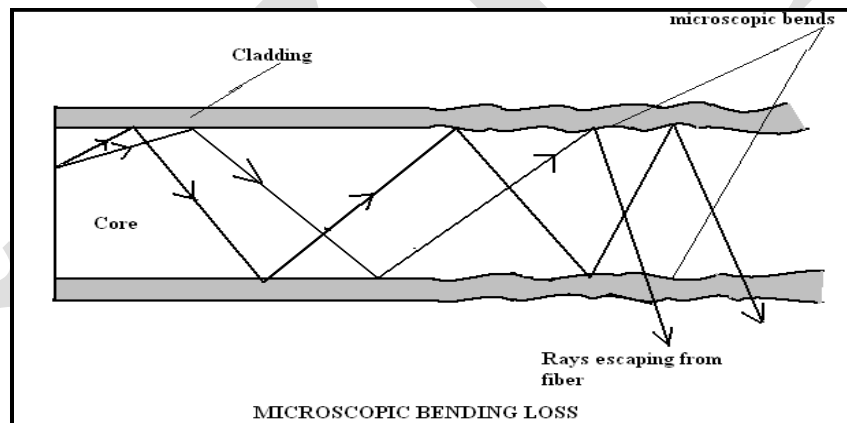
Radiation losses occur due to macroscopic bends and microscopic bends.

- ***Macroscopic bending:*** All optical fibers are having critical radius of curvature provided by the manufacturer. If the fiber is bent below that specification of radius of curvature, the light ray incident on the core cladding interface will not satisfy the condition of total internal reflection. This causes loss of optical power.

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- **Microscopic bending:** Microscopic bends are repetitive small scale fluctuations in the linearity of the fiber axis. They occur due to non-uniformities in the manufacturing and also lateral pressure built up on the fiber. They cause irregular reflections and some of them leak through the fiber. The defect due to non-uniformity (micro-bending) can be overcome by introducing optical fiber inside a good strengthened polyurethane jacket.



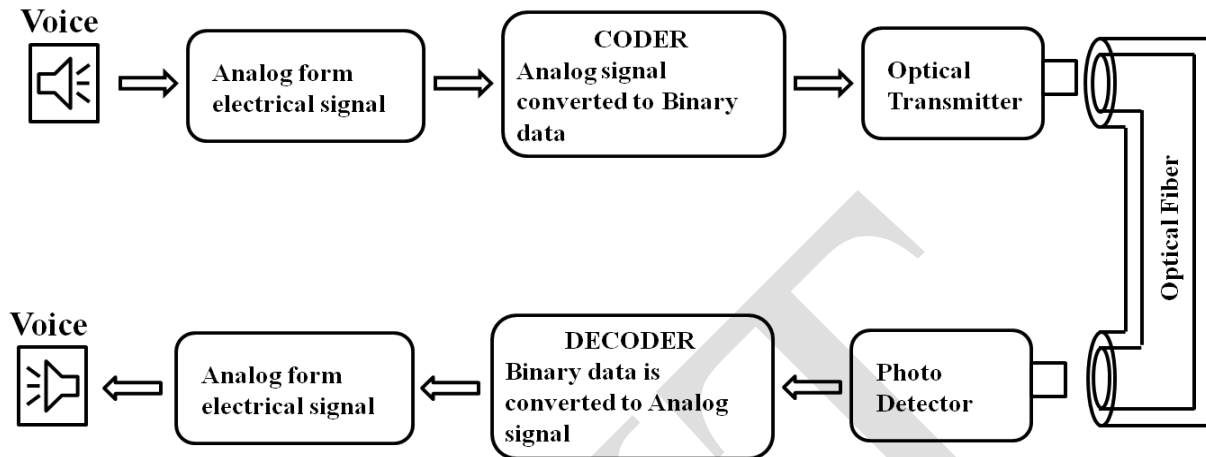
### **Attenuation co-efficient**

- The attenuation of a fiber optic cable is expressed in decibels.

$$\text{i.e., } \alpha = -\frac{10}{L} \log \left[ \frac{P_{out}}{P_{in}} \right] \frac{dB}{km}$$

- The main reasons for the loss in light intensity over the length of the cable is due to light absorption, scattering and due to bending losses.

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**Point to point optical fiber communication System**

- In point to point communication analog information such as voice of a telephone coming out of the transmitter section of the telephone are fed to the coder.
- The coder converts analog information into binary data which comes out as electrical pulses.
- The electrical pulses from the coder are fed to optical transmitter which converts signals into pulses of optical power.
- These optical pulses are fed into the fiber. The incident light which is funneled into the core within the acceptance angle propagate within the fiber by means of total internal reflection.
- The photo detector converts optical signals into electrical pulses in binary form and the decoder converts the binary data into analogue signal which will be the same information such as voice.

**Optical Fiber Sensors**

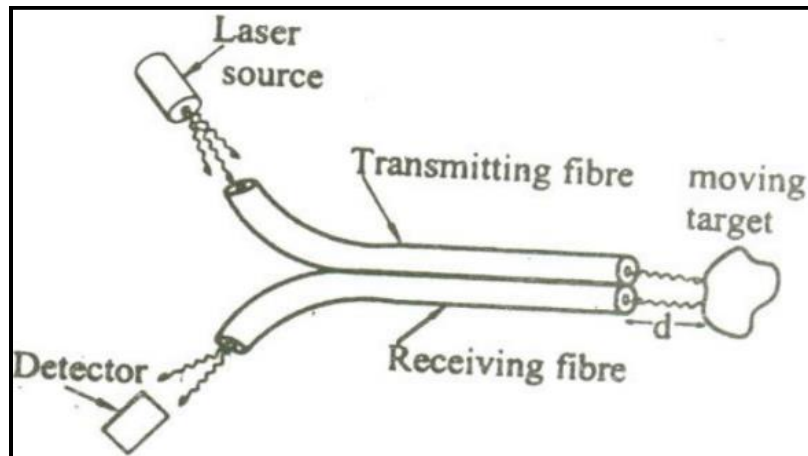
Optical Fiber sensors are meant for measuring and sensing the rate of data transmission, change in phase, intensity and wavelength and in the case of incentive conditions as noise, unstable environment conditions, high vibration and extreme heat etc.

On the basis of operating principle, Optical Fiber sensors are classified into

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1. Intensity based Displacement Sensor.
2. Temperature Sensor based on Phase Modulation.

### **1. Intensity based Displacement Sensor**



#### **Principle:**

Light is sent through a transmitting fiber and is made to fall on a moving target. The reflected light from the target is sensed by a detector. With respect to intensity of the reflected light from the target, displacement of the target is measured.

#### **Description:**

It consists of a bundle of transmitting fibers coupled to the laser source and a bundle of receiving fibers coupled to the detector as shown in the figure.

The axis of the transmitting fiber and the receiving fiber with respect to the moving target can be adjusted to increase the sensitivity of the sensor.

#### **Working:**

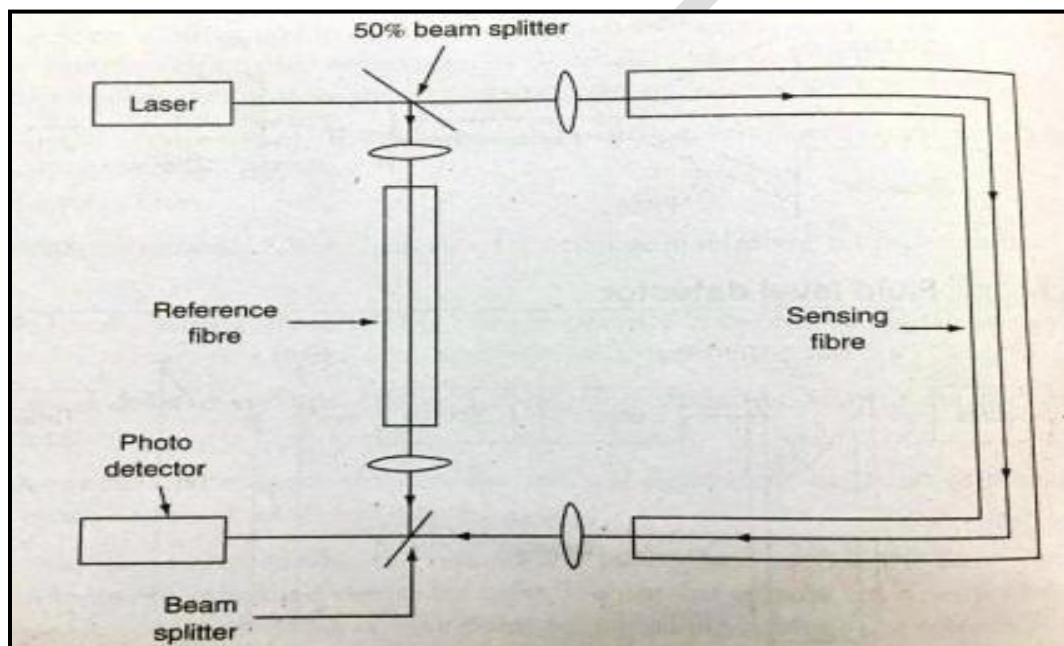
Light from the source is transmitted through the transmitting fiber and is made to fall on the moving target. The light reflected from the target is made to pass through the receiving fiber and the same is detected by the detector.

Based on the intensity of the light received, the displacement of the target can be measured, (i.e.) if the received intensity is more than we can say that the target is moving towards the sensor and if the intensity is less, we can say that the target is moving away from the sensor.

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**2. Temperature Sensor based on Phase Modulation**

- When a single optical fiber is subjected to temperature or pressure variations, then its length and refractive index changes. This causes change in phase of light at the end of the fiber.
- The change in phase of light is proportional to the magnitude of the change in temperature or pressure. The phase changes can be measured by an interferometer method as shown in fig.



- The light from the laser source splits into two beams approximately equal in amplitude by a 50% beam splitter.
- One beam is passed through sensing fiber, which is subjected to temperature variations and the other beam through reference fiber, which is not subjected to any changes and is used for comparison.
- Light from these two fiber is superimposed using another beam splitter. Interference of these two waves gives fringes.
- The intensity of the fringe depends on the phase relation between two waves.
- If the waves are in phase, then the intensity is maximum. This happens when the sensing fiber is not disturbed.
- The intensity is minimum if the waves are out of phase due to  $\lambda/2$  change in the length of the sensing fiber.

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- The intensity of interference fringes can be measured with a photodetector and pressure or temperature changes can be measured.

**Merits of optical communication system:**

- It carries very large amount of information in either digital or analog form due to its large bandwidth.
- The materials used for making optical fiber are dielectric nature. So, it doesn't produce or receive any electromagnetic and R-F interferences.
- Fibers are much easier to transport because of their compactness and lightweight.
- It is easily compatible with electronic system.
- It can be operated in high temperature range.
- It does not pick up any conducted noise.
- Not affected by corrosion and moisture.
- It does not get affected by nuclear radiations.
- No sparks are generated because the signal is optical signal.

**Demerits of optical communication system:**

- **Low power** — Light emitting sources are limited to low power. Although high power emitters are available to improve power supply, it would add extra cost.
- **Fragility** — Fiber optic cable is made of glass, which is more fragile than electrical wires such as copper cabling. Not only that, but glass can be damaged by chemicals such as hydrogen gas that can affect transmission. Particular care has to be taken with laying undersea fiber cabling because of its fragility.
- **Attenuation and Dispersion** ---- The distance between the transmitter and receiver should keep short. With long distance transmission, light will attenuate and disperse, which means additional components such as EDFA (Erbium-doped fiber amplifier – an optical repeater device that is used to boost the intensity of optical signals being carried through a fiber optic communications system) are required.

\*\*\*\*\*ALL THE BEST\*\*\*\*\*



**MODULE 4****MATERIAL SCIENCE****Classical free electron theory****Assumptions of classical free electron theory:**

- The metal consists of free electrons; the current flowing through a metal is due drift velocity of electrons.
- The free electrons are treated as equivalent to gas molecules and they are assumed to obey the laws of kinetic theory of gases. In the absence of the field, the energy associated with each electron at a temperature T is given by  $3/2 kT$ , where k is a Boltzmann constant. It is related to the kinetic energy.

$$3/2 kT = \frac{1}{2} m v_{th}^2$$

Where  $v_{th}$  is the thermal velocity same as root mean square velocity.

- The electric potential due to the ionic cores is assumed to be constant throughout the metal
- The attraction between the lattice ions and electrons, the repulsion between the electrons are neglected.

**Expression for electrical conductivity of conductor according to classical free electron theory**

According to classical free electron theory the expression for electrical conductivity is given by

$$\sigma_{CFET} = \frac{ne^2 \tau}{m}$$

Where  $\sigma$  - Electrical conductivity  
n- Electron density  
 $\tau$  - mean collision time  
m- mass of electron

**Failures of classical free electron theory:**

Electrical and thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.



**i) Specific heat:** The molar specific heat of a gas at constant volume is

$$C_v = \frac{3}{2} R$$

As per the classical free electron theory, free electrons in a metal are expected to behave just as gas molecules. Thus the above equation holds good equally well for the free electrons also.

But experimentally it was found that, the contribution to the specific heat of a metal by its conduction electrons was

$$C_v = 10^{-4} RT$$

which is lower than the expected value. Also according to the theory the specific heat is independent of temperature whereas experimentally specific heat is proportional to temperature.

**ii) Temperature dependence of electrical conductivity:**

Experimentally, electrical conductivity  $\sigma$  is inversely proportional to the temperature  $T$ .

i.e. 
$$\sigma_{\text{exp}} \propto \frac{1}{T} \rightarrow (1)$$

According to the assumptions of classical free electron theory

Since 
$$v_{\text{th}} \propto \sqrt{T}$$

But 
$$\tau \propto \frac{1}{v_{\text{th}}}, \quad \tau \propto \frac{1}{\sqrt{T}},$$

substituting in conductivity equation we get

$$\sigma_{CFET} = \frac{ne^2 \tau}{m} = \frac{ne^2}{m \sqrt{T}}$$

or 
$$\sigma_{CFET} \propto \frac{1}{\sqrt{T}} \rightarrow (2)$$

From equations (1) & (2) it is clear that the experimental value is not agreeing with the theory.

**iii) Dependence of electrical conductivity on electron concentration:**

According to classical free electron theory

$$\sigma = \frac{ne^2 \tau}{m} \quad \text{i.e., } \sigma \propto n, \quad \text{where } n \text{ is the electron concentration,}$$

Consider copper and aluminum. Their electrical conductivities are  $5.88 \times 10^7 / \Omega\text{m}$  and  $3.65 \times 10^7 / \Omega\text{m}$ . The electron concentrations for copper and aluminum are  $8.45 \times 10^{28} / \text{m}^3$  and  $18.06 \times 10^{28} / \text{m}^3$ . Hence the classical free electron theory fails to explain the dependence of  $\sigma$  on electron concentration.

Experimental results:

Metals	Electron concentration(n)	conductivity ( $\sigma$ )
Copper	$8.45 \times 10^{28} / \text{m}^3$	$5.88 \times 10^7 / \Omega\text{m}$
Aluminium	$18.06 \times 10^{28} / \text{m}^3$	$3.65 \times 10^7 / \Omega\text{m}$

### Quantum free electron theory:

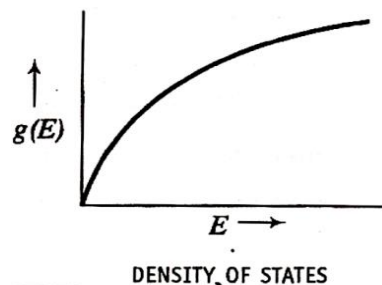
#### Assumptions of quantum free electron theory:

- The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
- The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.
- The electrons travel with a constant potential inside the metal but confined within its boundaries.
- The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

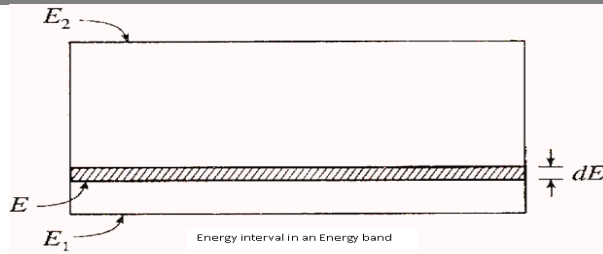
#### Density of states $g(E)$ :

Density of states is defined as the number of allowed energy states per unit energy range per unit volume in the valance band of a material. It is denoted as  $g(E)$ .

A graph of  $g(E)$  verses  $E$  is shown below.



Consider an energy band spread in an energy interval between  $E_1$  and  $E_2$ . Below  $E_1$  and above  $E_2$  there are energy gaps.  $g(E)$  represents the density of states at  $E$ . As  $dE$  is small, it is assumed that  $g(E)$  is constant between  $E$  and  $E+dE$ . The density of states in range  $E$  and  $(E+dE)$  is denoted by  $g(E)dE$ .

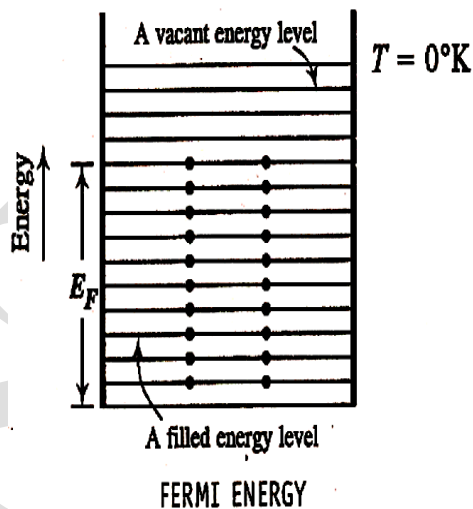


$$\text{i.e. } g(E)dE = \left[ \frac{8\sqrt{2\pi m^{\frac{3}{2}}}}{h^3} \right] E^{\frac{1}{2}} dE$$

It is clear  $g(E)$  is proportional to  $\sqrt{E}$  in the interval  $dE$

### **Fermi energy and Fermi level:**

The energy of electrons corresponding to the highest occupied energy level at absolute  $0^\circ\text{K}$  is called Fermi energy and the energy level is called Fermi level.



### **Fermi-Dirac statistics:**

- At thermal equilibrium the free electrons acquire energy obeying a statistical rule called Fermi-Dirac statistics.
- Fermi-Dirac statistics is applicable to the assembly of particles which obeys Pauli's exclusion principle; they must also be identical particles of spin  $\frac{1}{2}$  and are indistinguishable.
- Fermi-Dirac statistics permits to evaluation of probability of finding the electrons occupying energy levels in a certain energy range and is given by a function called Fermi factor  $f(E)$ .

**Fermi factor:**

Fermi factor is the probability of occupation of a given energy state by the electrons in a material at thermal equilibrium.

The probability  $f(E)$  that a given energy state with energy  $E$  is occupied by the electrons at a steady temperature  $T$  is given by

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

$f(E)$  is called the Fermi

factor.

**Dependence of Fermi factor with temperature and energy:**

The dependence of Fermi factor on temperature and energy is as shown in the figure.

**i) Probability of occupation for  $E < E_F$  at  $T=0K$ :**

When  $T=0K$  and  $E < E_F$

$$f(E) = \frac{1}{e^{-\infty+1}} = \frac{1}{0+1} = 1$$

The probability of occupation of energy state is 100%

**$f(E)=1$  for  $E < E_F$ .**

$f(E)=1$  means the energy level is certainly occupied and  $E < E_F$  applies to all energy levels below  $E_F$ . Therefore at  $T=0K$  all the energy levels below the Fermi level are occupied.

**ii) Probability of occupation for  $E > E_F$  at  $T=0K$ :**

When  $T=0K$  and  $E > E_F$

$$f(E) = \frac{1}{e^{\infty+1}} = \frac{1}{\infty} = 0$$

The probability of occupation of energy state is 0%

$f(E)=0$  for  $E > E_F$

∴ At  $T=0K$ , all the energy levels above Fermi levels are unoccupied. Hence at  $T=0K$  the variation of  $f(E)$  for different energy values, becomes a step function as shown in the above figure.

### ii) The probability of occupation at ordinary temperature (for $E \approx E_F$ at $T > 0K$ )

At ordinary temperatures though the value of probability remains 1, for  $E < E_F$  it starts reducing from 1 for values of  $E$  close to but lesser than  $E_F$  as in the figure.

The values of  $f(E)$  becomes  $\frac{1}{2}$  at  $E = E_F$

This is because for  $E = E_F$

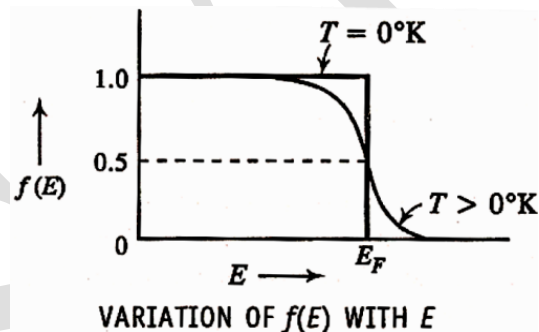
$$e^{(E-E_F)/kT} = e^0 = 1$$

$$\therefore f(E) = \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{1+1} = \frac{1}{2}$$

The probability of occupation of energy state is 50%

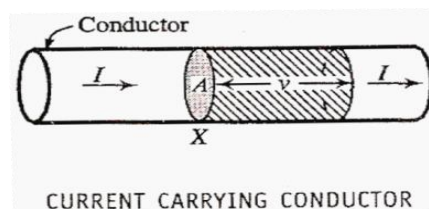
Further for  $E > E_F$  the probability value falls off to zero rapidly.

Hence, the Fermi energy is the most probable or the average energy of the electrons across which the energy transitions occur at temperature above zero degree absolute.



### Expression for electrical conductivity for conductor according to quantum free electron theory

Consider the motion of an electron in a metal in an influence of an electric field  $E$ . Let  $e$  is the charge;  $m$  is the mass of an electron and  $A$  is the area of cross section. Let  $v_d$  (drift velocity) is the distance travelled in unit time



According to de Broglie hypothesis,  $\lambda = \frac{h}{p}$

$$\rightarrow p = \frac{h}{\lambda} = \frac{hk}{2\pi} \quad \text{----- (1)} \quad \left(k = \frac{2\pi}{\lambda}\right)$$

Since  $p = mv = mv_d$

By substituting for p in equation (1)

$$mv_d = \frac{hk}{2\pi}$$

$$v_d = \frac{hk}{2\pi m^*} \quad \text{----- (2)}$$

Where  $m = m^*$ , called effective mass of electrons.

Integrate eqn (1) w.r.t "t"

$$\frac{dp}{dt} = \frac{h}{2\pi} \frac{dk}{dt} \quad \text{----- (3)}$$

By applying electric field E, the force experience by the electrons is eE and is equal to rate of change of momentum of the electrons.

$$\text{i.e., } F = eE = \frac{dp}{dt}$$

Substitute  $\frac{dp}{dt}$  in equation (3)

$$eE = \frac{h}{2\pi} \frac{dk}{dt}$$

$$dk = \frac{2\pi}{h} eE dt$$

Integrating on both sides

$$\int dk = \frac{2\pi}{h} eE \int dt$$

$$k = \frac{2\pi}{h} eE\tau \quad \text{----- (4)} \quad (\text{since } t = \tau \text{ mean collision time})$$

Substituting eqn (4) in eqn (2) we have

$$v_d = \frac{h}{2\pi m^*} \frac{2\pi}{h} eE\tau$$

$$v_d = \frac{eE\tau}{m^*} \quad \text{----- (5)}$$

Current density is  $J = neVd$

Substituting eqn (5) in the above equation

$$J = ne \frac{eE\tau}{m^*}$$

As we know,  $J = \sigma E$ ,

$$\sigma = \frac{J}{E}$$

$$\sigma = \frac{ne^2\tau}{m^*}$$

According to quantum free electron theory mean collision time  $\tau = \frac{\lambda}{v_f}$

Where  $v_f$  is the Fermi velocity

The expression for conductivity becomes

$$\sigma = \frac{ne^2\lambda}{m^*v_f} \quad \text{Where } m^* \text{ is the effective mass of electrons}$$

### **Merits of Quantum free electron theory:**

Quantum free electron theory has successfully explained following observed experimental facts whereas the classical free electron theory failed.

#### **i) Specific heat:**

According to quantum free electron theory, it is only those electron that are occupying energy levels close to  $E_F$ , which are capable of absorbing the heat energy to get excited to higher energy levels.

Hence only a small percentage of the conduction electrons are capable of receiving the thermal energy input, thus the specific heat value becomes very small for the metal.

According to quantum free electron theory, it can be shown

$$C_v = \left( \frac{2k}{E_F} \right) RT$$

Considering  $E_F = 5 \text{ eV} \left( \frac{2k}{E_F} \right) \approx 10^{-4}$

$\therefore C_v = 10^{-4} RT$  which is close to experimental value.

#### **ii) Temperature depends on electrical conductivity:**

Experimentally electrical conductivity  $\sigma_{\text{expt}} \propto \frac{1}{T}$

According to quantum free electron theory electrical conductivity is given by

$$\sigma_{QFT} = \frac{ne^2 \lambda}{m^* v_F} \quad \text{----- (1)}$$

Where  $m^*$  is called effective mass of an electron.

According to quantum free electron theory  $E_F$  and  $v_F$  are independent of temperature. The dependence of  $\lambda$  &  $T$  is as follows

Conduction electrons are scattered by the vibrating ions of the lattice. As the temperature increases the vibrational cross sectional areas ( $\pi r^2$ ) increases and hence mean free path decreases.

The mean free path of the electrons is given by

$$\therefore \lambda \propto \frac{1}{\pi r^2} \quad \text{Where 'r' is the amplitude of vibrations of lattice ions}$$

Considering the facts the energy of vibrating body is proportional to the square of amplitude and the energy of ions is due to the thermal energy.

The thermal energy is proportional to the temperature 'T'.

We can write  $r^2 \propto T$

$\therefore \lambda \propto 1/T$  substituting for  $\lambda$  in eqn (1)

Hence  $\sigma_{QFT} \propto \frac{1}{T}$

Thus  $\sigma_{QFT} \propto \frac{1}{T}$  is correctly explained by quantum free electron theory.

### iii) Electrical conductivity and electron concentration:

Aluminium and gallium which have three free electrons per atom have lower electrical conductivity than that of copper and silver, which have only one free electron per atom.

As per quantum free electron the electrical conductivity is

$$\sigma_{QFT} = \frac{ne^2 \lambda}{m^* v_F}$$

i.e  $\sigma \propto (n)$  and  $\sigma \propto (\lambda/v_F)$



The value of  $(n)$  for aluminium is 2.13 times higher than that of copper. But the value of  $(\lambda/v_f)$  for copper is about 3.73 times higher than that of aluminium. Thus the conductivity of copper exceeds that of aluminium.

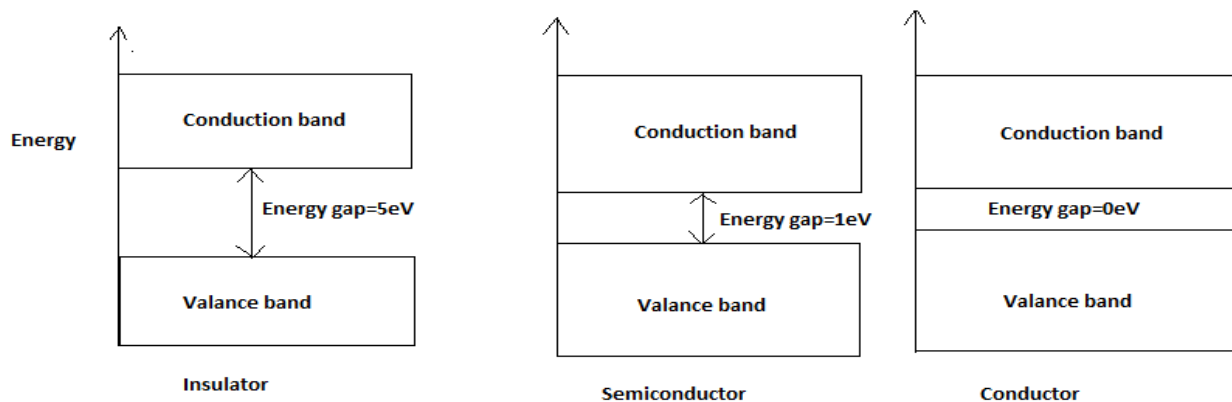
### **Physics of Semiconductors:**

Semiconductors are materials whose electrical conductivity lies between conductors and insulators. The conductivity of semiconductors is in the order of  $10^4$  to  $10^{-4}$  mho/m.

Valance band: Energy level occupied by valance electrons is called valance band. Valance band may be fully filled or partially filled at  $0\text{ K}$

Conduction band: Energy level occupied by conduction electrons (free electrons) is called conduction band. The conduction band is an empty band at  $0\text{ K}$

Forbidden energy gap: The separation between valance band and conduction band is known as forbidden energy gap. The energy gap is measured in electron volt (eV).



### **Fermi level in intrinsic semiconductor**

Fermi level represent the average energy of conduction electrons.

For a semiconductor at  $T=0\text{K}$  all the energy levels in the valance band are completely filled and all energy levels in conduction band are empty. At room temperature due to thermal excitation some of the electron will jump to bottom of conduction and return soon to the energy levels in top of valance band. Therefore conduction electrons are distributed between top of valance band and bottom of conduction band. Hence the average energy of electrons will be equal to  $(1/2) E_g$ . Thus Fermi level lies at the middle of the forbidden energy gap for an intrinsic semiconductor.

**Concentration of electrons and holes in intrinsic semiconductors:**

The number of electrons in conduction band per unit volume of a material is called electron concentration and is given by

$$N_e = \frac{4\sqrt{2}}{h^3} (\pi m_e^* kT)^{3/2} e^{\frac{E_f - E_g}{kT}}$$

Where,  $m^*$  is the effective mass of electrons

$k$  is the Boltzmann constant

$E_f$  is the Fermi level

$E_g$  is the energy gap

$h$  is Planck's constant

Similarly, the number of holes in valance band per unit volume of a material is called holes concentration and is given by

$$N_h = \frac{4\sqrt{2}}{h^3} (\pi m_h^* kT)^{3/2} e^{\frac{-E_f}{kT}}$$

$m_h^*$  is the effective mass of holes

**Relation between fermi energy and energy gap in an intrinsic semiconductor**

For intrinsic semiconductor the number of electrons in conduction band is equal to the number of holes in valance band

$$\text{i.e } N_e = N_h$$

$$N_e = \frac{4\sqrt{2}}{h^3} (\pi m_e^* kT)^{3/2} e^{\frac{E_f - E_g}{kT}} = N_h = \frac{4\sqrt{2}}{h^3} (\pi m_h^* kT)^{3/2} e^{\frac{-E_f}{kT}}$$

$$\left( (m_e^*)^{3/2} \right) e^{\frac{E_f - E_g}{kT}} = \left( (m_h^*)^{3/2} \right) e^{\frac{-E_f}{kT}}$$

$$e^{\frac{2E_f - E_g}{kT}} = \left( \frac{m_e^*}{m_h^*} \right)^{3/2}$$

By taking natural log on both sides

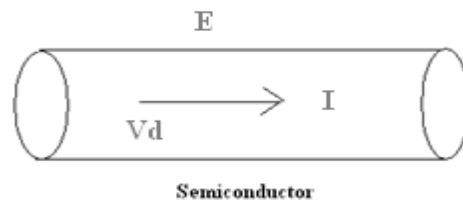
$$\frac{2E_F - E_g}{KT} = \frac{3}{2} \ln\left(\frac{m_e^*}{m_h^*}\right)$$

$$\text{Or } E_F = \frac{3}{4} KT \ln\left(\frac{m_e^*}{m_h^*} + \frac{E_g}{2}\right)$$

Since  $m_e^* = m_h^*$  and  $\ln 1 = 0$

$$E_F = \frac{E_g}{2}$$

### Expression for electrical conductivity of semiconducting materials:



Consider a semiconductor of area of cross section  $A$ , carrying a current  $I$ . Let  $V_d$  be the drift velocity of the charge carriers.

In the presence of electric field, both electrons and the holes contribute to the conductivity of the semiconductor. Let us consider first the conductivity in a semiconductor is due to the flow of electrons only.

The current due to flow of electrons is  $I = N e A V_d$  ..... (1)

$N = N_e$  = concentration of charge carriers (electrons)

The current density of electrons is given by  $J_e = N_e e V_d$  .....(2) [  $J = I/A$  ]

The quantity  $\frac{V_d}{E}$  is a constant, specific for a particular type of charge carriers and is called mobility of charge carriers. i.e.,  $\frac{V_d}{E} = \mu$

$V_d = \mu_e E$  in the above equation

$$J_e = N_e e \mu_e E \quad \text{.....(3)}$$

But current density is given by  $J_e = \sigma_e E$  .....(4)

$$\sigma_e = \frac{J}{E}$$

$$\sigma_e = N_e e \mu_e \quad \text{..... (5)}$$

eqn(5) represents the conductivity due to electrons in conduction band.

similarly the conductivity due holes in valance band is given by

$$\sigma_h = N_h e \mu_h \dots \dots \dots (6)$$

Total conductivity

$$\sigma = \sigma_e + \sigma_h = e (N_e \mu_e + N_h \mu_h)$$

$$\sigma = e (N_e \mu_n + N_h \mu_h)$$

For intrinsic semiconductors

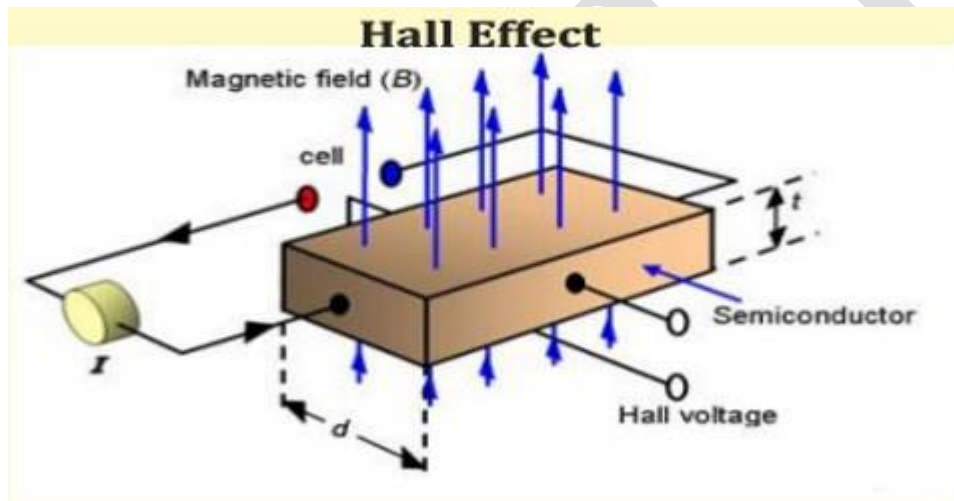
$$N_e = N_h = n_i$$

Hence the expression for conductivity in intrinsic semiconductor is given by

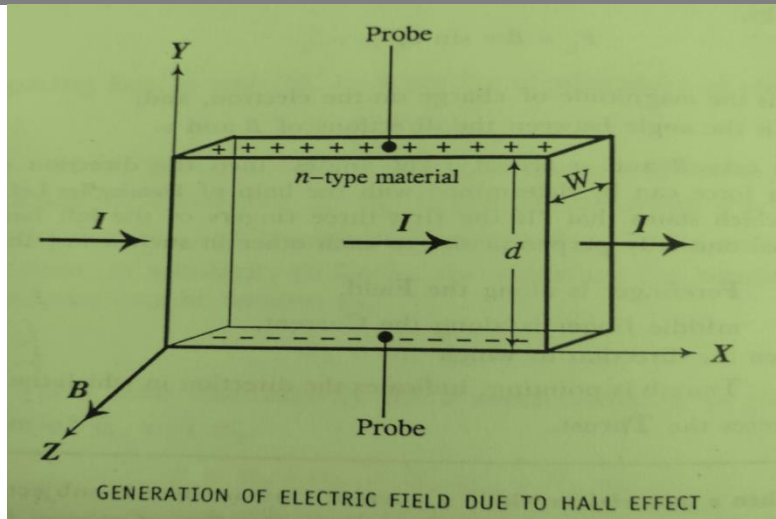
$$\sigma = n_i e (\mu_e + \mu_h)$$

### Hall Effect:

“When a current carrying conductor / semiconductor subjected to external magnetic field perpendicular to the direction of current. The electric field induced (produced) across the material in a direction perpendicular to both the magnetic field and direction of current flow”. This phenomenon is called Hall Effect.



### **Expression for Hall coefficient and Hall voltage:**



Consider a rectangular n -type semiconductor material in which current is flowing in +ve x -direction.

Let the magnetic field (B) applied along z-direction.

Therefore, the electrons experience the Lorentz force ( $F_L$ ) along -ve y-direction given by,

$$F_L = -Bev \quad \dots\dots(1)$$

Therefore, the electrons density increases in lower surface and in the upper surface become positively charged due to deficiency of electrons. Hence the potential developed between two surfaces called Hall voltage, which produces an electric field  $E_H$  called Hall field ( $E_H$ ).

The electrons exerts an upward force  $F_H$  due to the Hall field and is given by,

$$F_H = -eE_H \quad \dots\dots(2)$$

Equating equations (1) and (2)

$$-eE_H = -BeV_d$$

$$E_H = BV_d \quad \dots\dots(3) \quad V_d \rightarrow \text{drift velocity.}$$

If "d" is the distance between upper and lower surface then

$$E_H = V_H/d$$

$$V_H = E_H d \quad (\text{from 3})$$

$$V_H = BV_d d \quad \dots\dots(4)$$

$$\text{The current density } J = I/A = neV_d = \rho V_d \quad \dots\dots(5)$$

Therefore,  $\rho = ne$

For the given semiconductor,  $J = I/A = \frac{I}{\omega d}$  ..... (6)

$\omega \rightarrow$  thickness of the material . therefore ( $A = \omega d$ )

equating (5) & (6)  $\rho V_d = \frac{I}{\omega d}$

$$V_d = \frac{I}{\rho \omega d} \quad \dots\dots (7)$$

Substituting eqn (7) in (4)

$$V_H = \frac{B d I}{\rho \omega d} = \frac{B I}{\rho \omega} \quad \dots\dots (8)$$

$$\rho = \frac{B I}{V_H \omega}$$

### Expression for Hall coefficient:

Since,  $E_H \propto J B$

$$E_H = R_H J B \quad (R_H \rightarrow \text{Hall coefficient})$$

$$R_H = E_H / J B \quad [\text{From eqn (3) and (5)}]$$

$$R_H = B V_d / \rho V_d B = 1 / \rho = 1 / ne$$

$$\mathbf{R_H = 1 / ne} \quad \dots\dots(9)$$

The above expression represents **Hall coefficient**.

### Expression for Hall Voltage:

$$\text{From (8)} \quad V_H = \frac{B I}{\rho \omega} = \frac{R_H B I}{\omega} \quad (\text{since, } R_H = \frac{1}{\rho} = \frac{1}{ne})$$

$$V_H = \frac{R_H B I}{\omega}$$

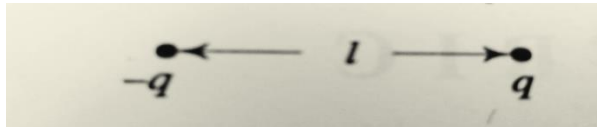
The above expression represents **Hall voltage**.

## DIELECTRIC MATERIALS

Dielectric materials are nothing but insulators which have the ability to get electrically polarized with the application of external electric field.

**Examples:** Glass, rubber, plastic, mica.

**Electric dipole:**



Two equal and opposite charges separated by a distance is called dipole.

The product of one of the charge and distance of separation between them is called dipole moment ( $\mu$ ).

$$\mu = q l$$

**Polarization:** The displacement of charges in the atoms or molecules of a dielectric under the action of an applied electric field leads to the development of dipole moment is called polarization.

**Types of dielectric materials:**

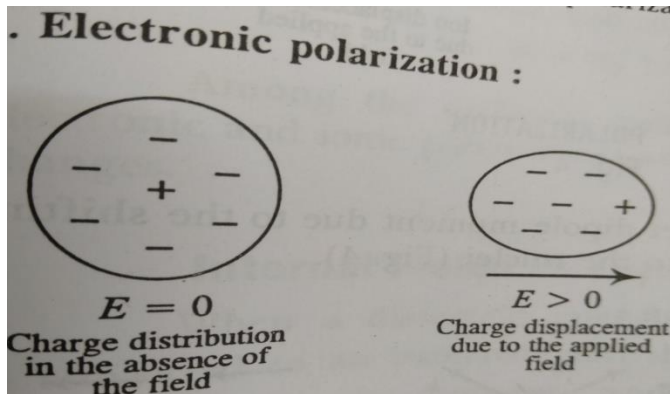
**Polar dielectrics:** In some dielectric materials, the effective centers of the negative and positive charges in the molecules do not coincide with each other and exhibit permanent dipoles in the absence of electric field are called polar dielectrics. **Example:** H<sub>2</sub>O.

**Non polar dielectrics:** In some dielectric materials, the effective centers of the negative and positive charges in the molecules do not coincide with each other and do not exhibit permanent dipoles in the absence of electric field are called non polar dielectrics. **Examples:** paper, wood, glass.

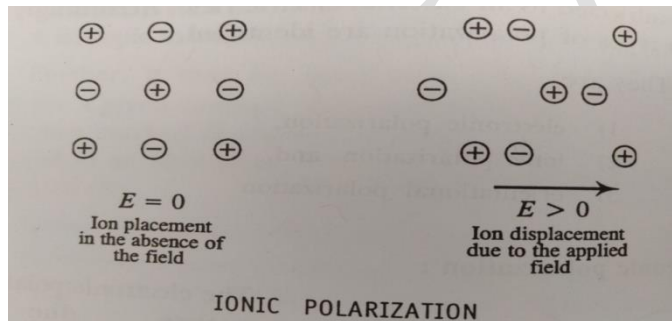
**TYPES OF POLARIZATION MECHANISMS:**

**Electronic Polarization:** The electronic polarization occurs due to displacement of the positive and negative charges in a dielectric material with the application of external electric field, which leads to development of dipole moment. The electronic polarizability is given by,

$$\alpha_e = \epsilon_0 (\epsilon_r - 1) / N .$$

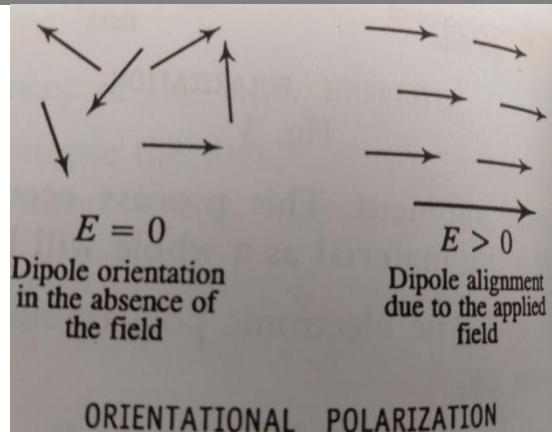


- **Ionic Polarization:** This is kind of polarization occurs in some dielectric materials which have ionic bonds like NaCl. When ionic solids are subjected to an external electric field, the adjacent ions of opposite sign undergo displacement. The distance of separation between the ion pair depends on the location of atoms in the lattice results in the development of dipole moment



- **Oriental Polarization:** This type of polarization occurs in polar dielectric material. (Permanent dipoles).





The orientation of molecules is random in the absence of electric field; therefore net dipole moment in the material is zero. But under the influence of an applied electric field each dipole reorient along the field direction. Thus the material develops an electrical polarization. This type of polarization is temperature dependent. In polar dielectrics the orientation polarizability  $\alpha_0$  is given by  $\alpha_0 = \mu^2 / 3kT$

- **Space charge Polarization:** It occurs in multiphase dielectric materials in which change in resistivity between different phases. When such materials subjected to external electric field at high temperatures, the charges get accumulated at the interface leads to change in conductivity across the material.

#### Relation between Polarization and Dielectric constant:

For isotropic materials, the applied electric field (E), flux density (D) are related by

$$D = \epsilon_0 \epsilon_r E$$

$\epsilon_0 \rightarrow$  dielectric constant of vacuum

$\epsilon \rightarrow$  relative dielectric constant of the material

The relation between dielectric  $\epsilon_r$  and polarization  $p$  is given by  $P = \epsilon_0(\epsilon_r - 1) E$

**Internal fields in solids:** When dielectric material subjected to an external electric field, each atom develops a dipole moment and acts as electric dipole. Hence the field at any given atom will be the applied electric field and the electric field due to the surrounding dipoles. "The internal field is the electric

field that acts at the site of any atom of a solid subjected to an external electric field and is the resultant of the applied field and the field due to all the surrounding dipoles”.

In one dimension the internal field is given by

$$E_i = E_{\text{applied}} + E_{\text{dipoles}}$$

$$E_i = E + \left(\frac{\nu}{\epsilon_0}\right)P$$

where  $\nu \rightarrow$  internal field constant,  $p \rightarrow$  polarization

For, 3D the internal field is **called Lorentz field** and is given by

$$\mathbf{E}_{\text{Lorentz}} = \mathbf{E} + \frac{P}{3\epsilon_0}$$

### Clausius – Mossotti Equation:

Consider a solid dielectric material with dielectric constant  $\epsilon_r$ . If ‘n’ is the number of atoms/unit volume and ‘ $\mu$ ’ is the dipole moment of the atoms in the material.

$$\text{Therefore, the dipole moment / unit volume} = N \mu \quad \dots\dots\dots(1)$$

The field experienced by the atom is an internal field  $E_i$ .  $\alpha_e$  is the electronic polarizability of the atoms.

$$\text{Then dipole moment / unit volume} = N \alpha_e E_i$$

$$\text{The dipole moment } \mu = \alpha_e E_i \quad \dots\dots\dots(2)$$

Dipole moment / unit volume is nothing but polarization (P)

$$P = N \alpha_e E_i$$

$$E_i = \frac{P}{N \alpha_e} \quad \dots\dots\dots(3)$$

$$\text{we have } P = \epsilon_0 (\epsilon_r - 1) E$$

$$E = \frac{P}{\epsilon_0 (\epsilon_r - 1)} \quad \dots\dots\dots(4)$$

The expression for internal field for 3D material is given by  $E_i = E + \frac{P}{3\epsilon_0}$

Substituting (3) and (4) in the above equation

$$\frac{P}{Nae} = \frac{P}{\epsilon_0(\epsilon_r - 1)} + \frac{P}{3\epsilon_0}$$

$$\frac{1}{Nae} = \frac{1}{\epsilon_0(\epsilon_r - 1)} + \frac{1}{3\epsilon_0} = \frac{1}{\epsilon_0} \left[ \frac{1}{(\epsilon_r - 1)} + \frac{1}{3} \right]$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{Nae}{3 \epsilon_0}$$

**This is called Clausius Mossotti equation.**

SVIT

**MODULE - 05****Material Characterization Techniques and Instrumentation****Nano materials**

Matter arranged by exercising control over lengths of one to hundred nano meter and the formulating structures exhibit characteristics that are specific to their size and dimensions, the resulting materials are called nano materials.

**Nanocomposites**

- A nanocomposite is as a multiphase solid material where one of the phases has one, two or three dimensions of less than 100 nm. These materials show differing in properties due to dissimilarities in structure and chemistry.
- The mechanical, electrical, thermal, optical, electrochemical, catalytic properties of the nanocomposite will differ noticeably from that of the component materials.
- Size limits for these effects is <5 nm for catalytic activity, <20 nm for making a hard and soft magnetic material, <50 nm for refractive index changes, and <100 nm for achieving mechanical strengthening.

The properties of materials can be different at the Nanoscale for two main reasons:

- Nanomaterials have a relatively larger surface to volume ratio when compared to bulk materials.
- Quantum effects can begin to dominate the behavior of matter at the nanoscale.

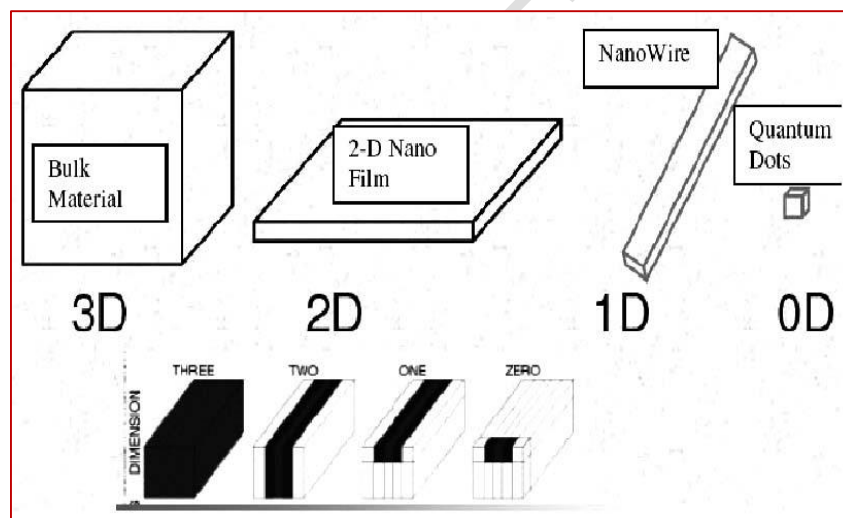
**Types of nano materials based on dimensions**

Based on dimensions the nano materials are classified into four types vary in size from 1 nm to 100 nm.

- **3D Bulk materials:** -The charge carriers can move in all the three directions and it has all parameter of length, breadth and height. (for example, Nano Particles).

CBCSE 2021 Scheme

- **2D films**- The movement of the charge carriers are confined to two directions and it has only length and breadth (for example, nano films and nanotubes)
- **1D quantum wire** –The movement of the charge carriers are restricted to one directions and it has only one parameter either length (or) breadth (or) height. (example: nano wires)
- **0D quantum dot** – The movement of the charge carriers are confined in all the three directions then the resulting structure is called quantum dot or nano particle and its length, breadth and heights are confined at single point. (for example, Quantum dots)



Charge carriers are able to move in all directions in a 3-D material, confined to a plane in a film in only one direction in a quantum wire but in a 0-D structure they will remain confined to a very small space. The film, wire and dots have certain thickness for the material along the direction where we say the corresponding dimension is absent. The thickness is less than the mean free path for the electron in the material; this thickness will be in nanometer range. The material along these directions exhibits mesoscopic properties.

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### **Properties of nanomaterials**

Due to the particle size in nano regime it affects many properties when compared to their bulk counterparts such as

- Melting point
- Boiling point
- Band gap
- Optical properties
- Electrical properties
- Magnetic properties

### **Applications of nanomaterials**

- Nanotechnology Applications in Medicine
  - Detect disease and the deliver treatment.
  - Nano shells as Cancer Therapy
  - Nanowires – used as medical sensor
- Nano Computing Technology
- Sunscreens and Cosmetics
- Sunscreens and Cosmetics
- Displays
- Batteries
- Catalysts
- Magnetic Nano Materials applications
- Medical Implantation
- Water purification

### **Principle, construction and working of X-ray Diffractometer**

#### **PRINCIPLE:**

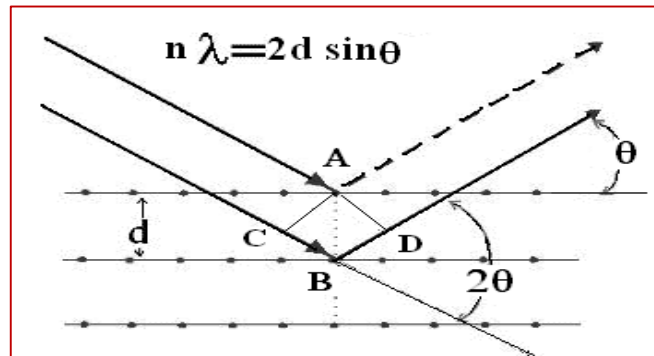
The Bragg's X-ray Diffractometer works on the principle of Bragg's law of diffraction  $2d\sin\theta = n\lambda$ .

where

d - interplanar distance

$\theta$  -glancing angle

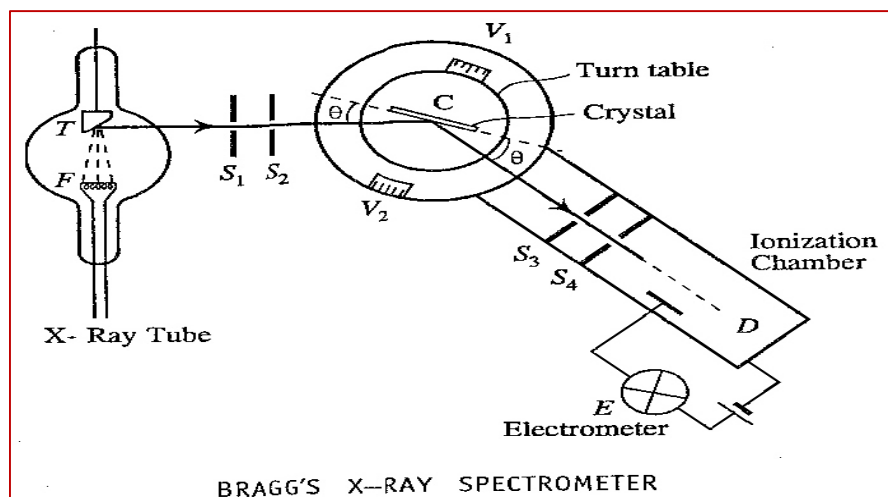
$\lambda$  -wavelength of X-ray



X-ray diffraction is based on constructive interference of monochromatic X-rays and a crystalline sample. The interaction of the incident rays with the sample produces constructive interference (and a diffracted ray) when conditions satisfy Bragg's Law ( $n\lambda = 2d \sin \theta$ ). These diffracted X-rays are then detected, processed and counted. By scanning the sample through a range of  $2\theta$  angles, all possible diffraction directions of the lattice should be attained due to the random orientation of the powdered material.

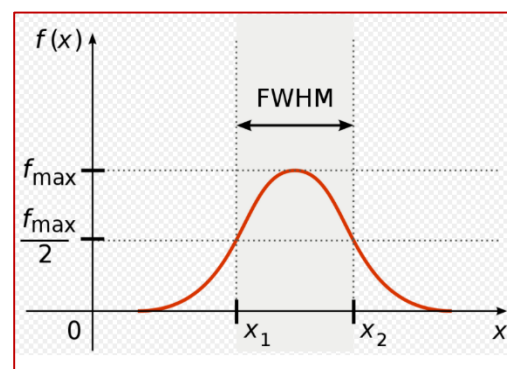
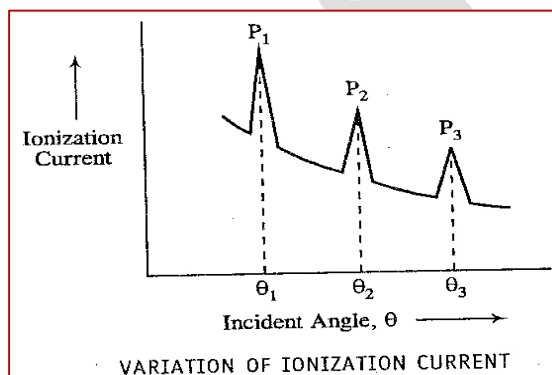
### Bragg's X-Ray Diffractometer

The schematic diagram of Bragg's x-ray spectrometer is shown in fig. It has 1) x-ray source 2) A Crystal fixed on a circular table provided with scale and Vernier. 3) Ionization chamber.



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- X-rays from the X-ray tube are allowed to pass through the slits  $S_1$  and  $S_2$ , then it is made to fall on a crystal  $C$  mounted at the center of rotating turn table provided with a graduated scale  $V_1$  to measure the angular displacement of crystal.
- X-ray after reflection enters into ionization chamber fixed to the turn table and the position can be noted on the scale  $V_2$ .
- The X-ray enters into ionization chamber ionizes the gas and produces ionization current which is measured by electrometer connected to it.
- For every rotation of turn table by an angle ' $\theta$ ' on the crystal, the ionization chamber must rotate by an angle  $2\theta$  to satisfy Bragg's law.
- While the experiment is carrying out by rotating the turn table at different reflected rays the sudden rise in current is observed.
- A plot of ionization current for different incident angles to study the x-ray diffraction spectrum is shown in fig.
- Let Peaks are observed at angles  $\theta_1, \theta_2, \theta_3$  etc. for  $n=1,2,3$ , etc. for a set of parallel planes in the crystal, when the x-ray beams satisfy Bragg's law of diffraction
- By knowing, wavelength of X-ray, interplanar spacing ( $d$ ) and order of diffraction ( $n$ ), the diffraction angle  $\theta$  can be calculated using Bragg's Law  **$2d\sin\theta = n\lambda$** .



### Crystallite size determination using Debye Scherrer equation

Scherrer equation is the oldest form of X-ray diffraction method used to determine the size of the crystals in the form of powder. The Scherrer equation relates the crystallite size in a solid with the broadening of the peak in a



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diffraction pattern. By knowing the values of  $\theta$ ,  $\lambda$ ,  $\beta$  and  $K$  the mean crystallite size  $D$  can be determined by using equation.

$$D = \frac{K \lambda}{\beta \cos \theta} \quad m$$

Where

$D$  - the mean crystallite size

$\beta$  - Full width at half maxima (FWHM)

$\theta$  - Bragg's angle

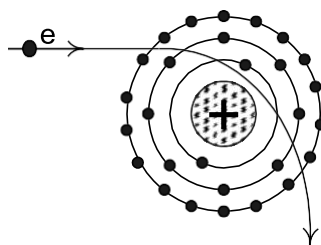
$K$ - Scherer's Constant  $\rightarrow 0.9$  for Cu - $K\alpha$  target

## Prerequisites to understand the electron microscopes

### Scattering of electrons

Scattering usually implies that the electron changed the path after 'hitting' some other small particle like the nucleus of an atom, or another electron. As the electrons come very close to other particles, their masses and all of their related fields, interact on a quantum level. Due to this the deviation occurs in the path of the electron.

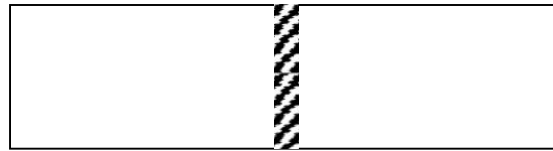
For example, when high speed electrons penetrate into the atom, they are attracted by a positively charged nucleus. Due to this, they deviate from their original path. This is equivalent to collision with nucleus. Electrons slow down and lose their energy in this process.



### Tunneling Effect

Consider a thin layer of insulator sandwiched between two metal layers as shown below-

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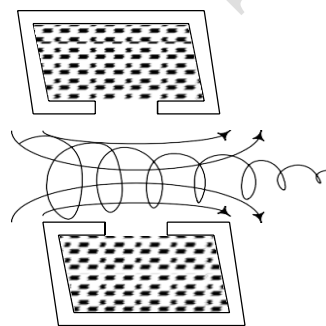
**Metal      Insulator      Metal****10-20 Å**

Insulator layer sandwiched between two metals

The insulator works as a barrier to the flow of conduction electrons travelling from one metal to the other. However, when the size of the barrier is extremely small of the order of 10-20 Å.

**Magnetostatic Focusing (Magnetic lenses)**

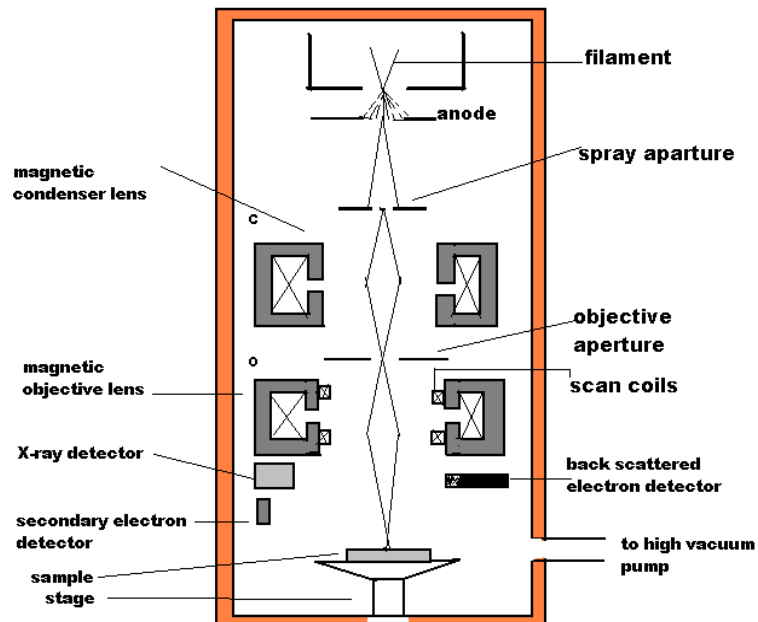
Magnetic fields, which are axially symmetric, can be used for focusing an electron beam passing through them. The axially symmetric magnetic fields are produced by short solenoids. The electron traveling in such field describes a helical path and can be focused to a particular point. Such solenoids are called as thin magnetic lenses. These magnetic lenses are used in electron microscopes.

Spiralling and focusing of electrons  
in a magnetic field**Magnetic lens**

**SCANNING ELECTRON MICROSCOPE (SEM)****PRINCIPLE:**

The basic principle involved in the working of all kinds of electron microscope is the wave nature of electrons. An electron accelerated under a potential difference of  $V$  volts behave like a wave of wavelength.

$$\lambda = \frac{h}{\sqrt{meV}} = \frac{1.226}{\sqrt{V}} \text{ nm}$$

**CONSTRUCTION:**

The apparatus consists of a highly evacuated chamber inside which there is an electron gun. There are 2 magnetic lenses one is the condensing lens C and the other one is the objective lens O. A scan coil accompanies the lens O. there is a spray aperture using which, spherical aberration during focusing will be minimized. A flat surface called stage is provided at the bottom portion of the apparatus to place the sample under study.

There are 3 types of detectors – back scattered electron detector, secondary electron detector, and an X-ray detector

**WORKING:**

- The sample to be investigated is placed on the specimen stage after which, inside of the chamber is evacuated by connecting it to a high vacuum pump.
- Electrons are emitted by the filament by thermionic emission. A suitable positive potential is applied to the anode with respect to the filament.
- The accelerated electrons from the electron gun pass through the spray aperture from where the electron beam emerges.
- The condensing lens C converges the beam & eliminates some high angle electrons.
- The beam then passes through the objective aperture where the size of the beam can be controlled. A thinner beam then enters into the field of objective lens O. the objective lens focuses the beam onto the desired part of the specimen.
- A set of coils called scan coils placed along with the objective lens enable the beam to scan the specimen in the particular way called raster. The scan coils are connected to the raster scan generator which directs the beam onto the spot on the specimen & dwells on it momentarily.
- The electrons incident on the sample are called primary electrons. Upon incidence some of the electrons are knocked out from the atoms in the specimen due to the impact of the beam which are called secondary electrons. Some of them will be scattered by the specimen called back scattered electrons. X- Rays are emitted when electrons from a higher shell in the atom transit to a vacant position created in its lower shell from where an electron has been knocked off.
- Back scattered electrons, secondary electrons & the X-rays emitted are detected by the respective detectors & a corresponding signal is produced. This signal is converted into a micro spot of corresponding brightness on a screen. The beam focus is then shifted to the next adjacent spot in order, where it again dwells momentarily. This way the image is built on the screen point by point.
- Image produced on the screen will be in grey scale. For aesthetic purposes, these are colorized by using feature-detection software.

**APPLICATIONS:**

SEM is used to study

1. External morphology of biological organisms.
2. Chemical composition.
3. Crystalline structure.
4. Forensic investigations.

**TRANSMISSION ELECTRON MICROSCOPE (TEM)**

**Principle:** Transmission Electron Microscope (TEM) works on the principle of wave nature and tunneling effect of electrons.

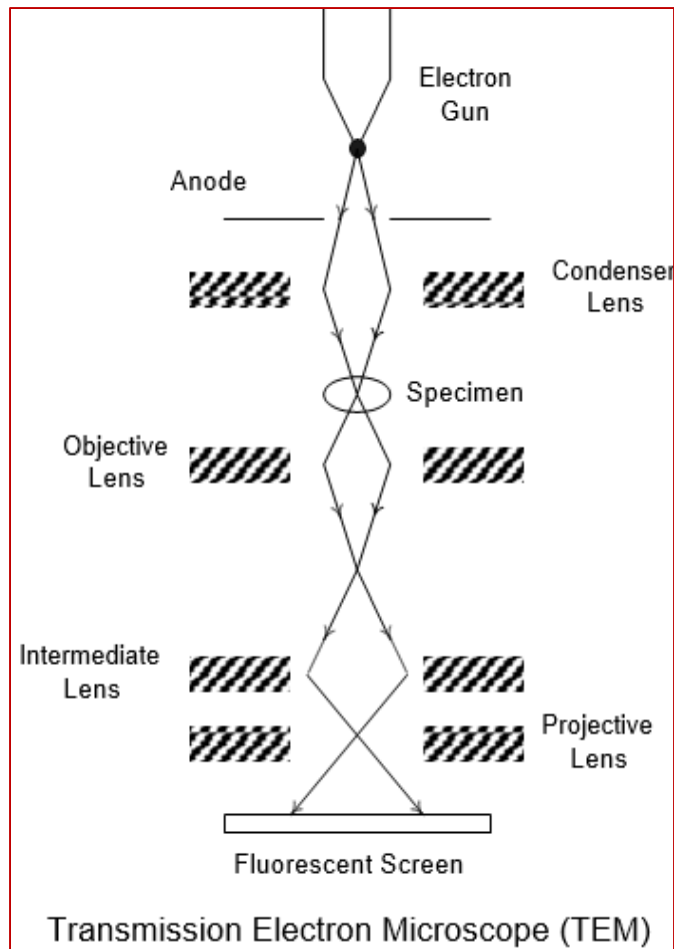
**Construction:**

Transmission Electron Microscope (TEM) is an electron microscope that has four main systems -

- (1) **An electron gun:** It produces the electron beam
- (2) **The condenser system:** It focuses the electron beam on the sample.
- (3) **The image-producing system:** It consists of the objective lens, movable specimen stage and intermediate and projector lenses to form a real and highly magnified image
- (4) **The image-recording system:** This converts the electron image into the format that can be seen by the human eye.

**Working:**

- The electron beams from electron gun are produced from the heated tungsten filament. Anode with an aperture is kept at positive potential. Electrons accelerate toward the anode and pass through the central aperture.
- These electron beam focused on the specimen by the condenser system. The intensity and angular aperture of the electron beam are also controlled by these lens system, between the electron gun and the specimen.



- The electron beams then pass through the specimen mounted on the specimen stage which can be adjustable. Then the beam passes through the objective lens, it is usually of short focal length (1–5 mm) and produces a real intermediate image that is further magnified by the projector lenses.
- Modern instruments employ two projector lenses (one of which is called the intermediate lens) to permit a greater range of magnification without increase in the physical length of the microscope.
- The intermediate electron image that is formed at the projector lenses are converted into the format that can be seen by the human eye by the image-recording system: This consists of a fluorescent screen for viewing and focusing the image, the higher magnification may be obtained by photographic or digital enlargement. Computerized images are stored in a

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format such as TIFF or JPEG and can be analysed and processed to get the final image.

**Advantages:**

- a) Three dimensional image obtained gives more information about the specimen.
- b) Very small amount of specimen is required for analysis.

**Disadvantage:**

- a) High vacuum is required to maintain.

**ATOMIC FORCE MICROSCOPE (AFM)**

Atomic Force Microscope is a high resolution scanning probe type microscope. It is a tool for imaging, measuring and manipulating matter at the Nano scale. It can image all type of surfaces including polymers, ceramics, composites, glass and biological samples.

The atomic force microscope was invented by Gerd Binnig et al. in 1986 at IBM Zurich based on the STM (Scanning Tunneling Microscope) already presented in 1981. In 1987, the inventors were awarded the Nobel Prize in Physics for the achievements.

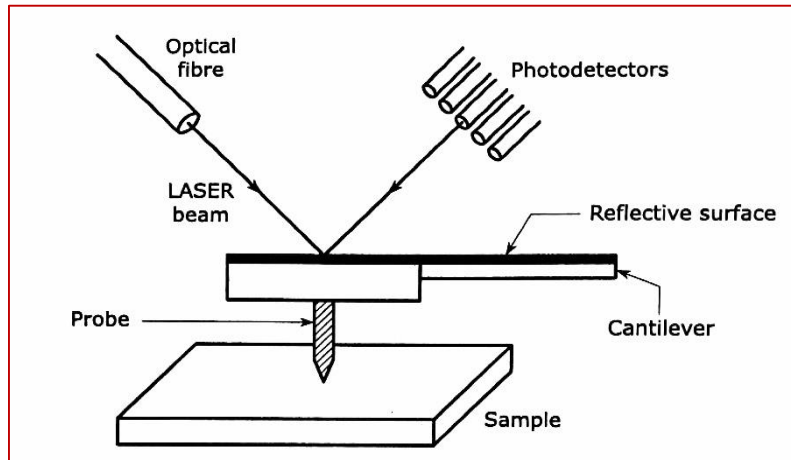
**PRINCIPLE:**

AFM produces image by physically pushing a cantilever probe against the sample. The probe movement is analyzed and converted into a three dimensional image of the sample surface.

**CONSTRUCTION:**

- A typical AFM consists of a cantilever around 100-500 microns in length with a small tip/probe of radius of 3-15 microns at the free end.
- A laser source, 4-quadrant photodetectors and deflection sensor.
- Sample stage attached to piezo electric sensor.
- Analyzer: The light collected by photo detectors is analyzed with computer controlled devices and a 3D image of the sample surface is constructed.

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### WORKING

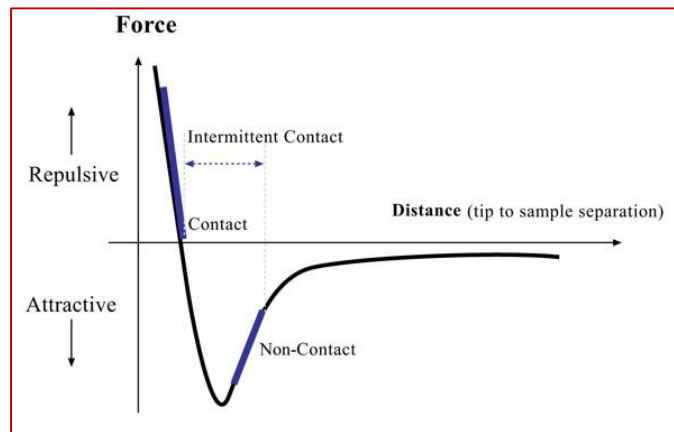
- When a cantilever probe is brought into proximity of the sample surface, the forces such as Vander Waal's forces, electrostatic forces, magnetic forces and the other forces which arise due to the physical interaction between the surface atoms, cause the cantilever tip to deflect.
- The cantilever can be thought of as a spring. The quantity of the generated force between the probe and the surface depends on the spring constant (stiffness) of the cantilever and the distance between the probe and the surface.
- This force can be characterized with Hooke's Law.  $F = -k.x$
- The deflection of the cantilever is detected by the help of a laser beam and deflection sensor.
- The displacement of the probe is measured and a topographical image is obtained.
- In AFM both conducting and non-conductive samples can be analyzed.

### Operation Modes of AFM

**Contact mode-** In contact mode, the tip is in a soft physical contact with the surface. The tip is able to move above the surface with a specific height or under a constant force. the force between the probe and the sample remains constant and an image of the surface is obtained.

The movement is strongly influenced by frictional and adhesive forces that can cause damage to the sample.





The relationship between force and distance is shown in Figure

**Dynamic (Tapping) Mode**-This mode eliminates the frictional force by intermittently contacting the surface and oscillating with sufficient amplitude to prevent it from being trapped in by adhesive forces. This mode of operation is less destructive than contact mode. The cantilever oscillates nearby its resonance frequency.

**Non-contact mode**- In this mode tip does not touch the sample, however it oscillates above the surface during scan. It uses feedback loop to monitor changes in the amplitude due to attractive Vander Waals forces so the surface topography can be monitored. It is better for soft samples and biological samples.

### Advantages

1. Easy to prepare samples for observation
2. It can be used in vacuums, air, and liquids.
3. Measurement of sample sizes is accurate
4. It has a 3D imaging
5. It can be used to study living and non-living elements
6. It can be used to quantify the roughness of surfaces
7. It is used in dynamic environments.

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**Disadvantages**

1. It can only scan a single nano sized image at a time of about 150x150nm.
2. They have a low scanning time which might cause thermal drift on the sample.
3. The tip and the sample can be damaged during detection.
4. It has a limited magnification and vertical range.

**X-ray Photoelectron Spectroscopy (XPS)****PRINCIPLE:**

X-ray photoelectron spectroscopy (XPS) involves irradiating the sample with low energy (1.5 Kev) X-rays such that photoelectric effect is induced. The kinetic energy of emitted electrons is given by

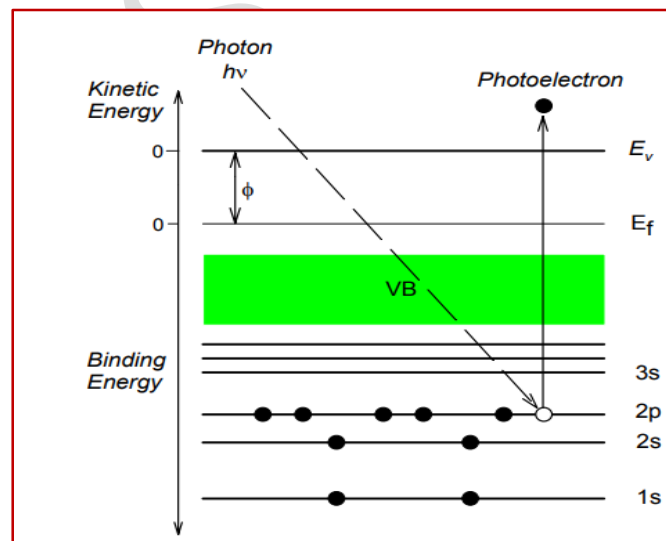
$$\text{K.E.} = h\nu - \text{BE} - \Phi$$

where

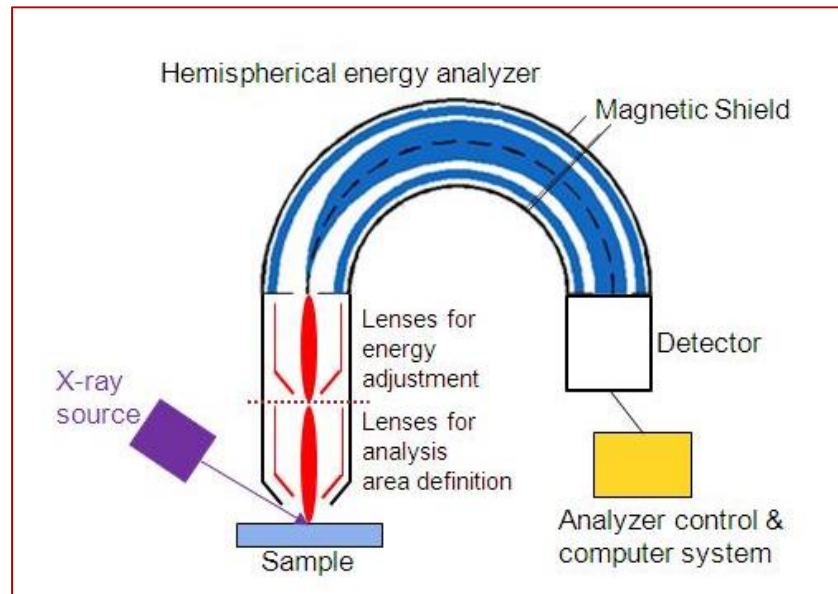
K.E- Kinetic energy

BE - Binding energy of the electrons

$\phi$  - work function of the sample.



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**CONSTRUCTION:**

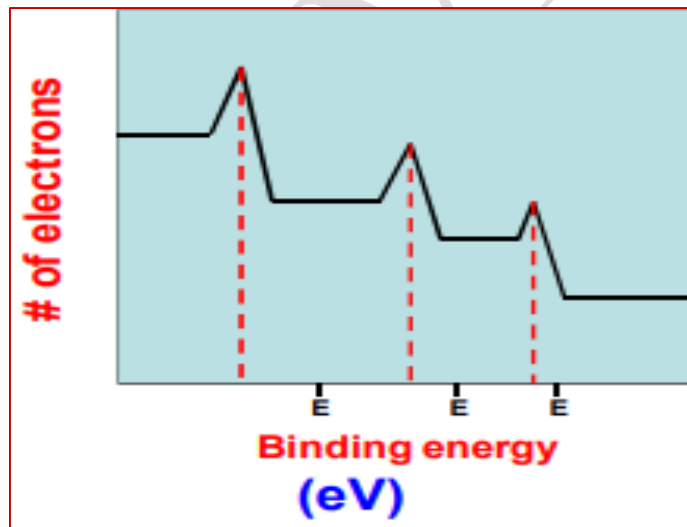
- X-ray source; Al K $\alpha$  or Mg $\alpha$  X-rays are typically used to excite the sample; Sample holder is present between the X-ray source and the entrance slit
- XPS mainly has lens system to collect the photo electrons.
- An electron energy analyzer is used to analyze to filter the electron energy of the photoelectrons that are produced. This is typically a Concentric Hemispherical Analyzer (CHA).
- The detector is provided to detect to count the electrons. It has electron channel multiplier tube where in single electron pass through this channel plates and emerge as pulse of electrons.
- Analyzer control and computer system contains computer and data reduction software. XPS data are compared to inventories or archives of experimentally determined XPS data of standard reference material.
- Ultrahigh vacuum system; typically operating conditions are at  $<10^{-9}$  Torr. This is required because the emitted photoelectrons have a

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relatively low energy and are readily absorbed by ambient atmosphere or the gas molecules in the chamber.

**Working:**

- The sample is kept in ultra-high vacuum is illuminated by the photons with energy  $h\nu$ ; low energy X-rays,
- A bound electron absorbs a photon, resulting in the emission of Photo electrons and converts part of the energy to kinetic energy
- XPS spectra is the plot of number of photo electrons emitted versus the KE/ BE. Each element has unique XPS spectra
- The identification of the elements in sample is made directly from the K.E of these ejected photo electrons. Each peak represents the number of electrons at a certain energy that is characteristic of some element. (Binding energy increases from left to right and kinetic energy increase from right to left)
- The relative concentration is made on the photo electron intensities
- XPS is used to determine the elemental composition, stoichiometry and examine surface contamination.



\*\*\*\*\*ALL THE BEST \*\*\*\*\*