FLUIDS MECHANICS

Unit-I: PROPERTIES OF FLUIDS

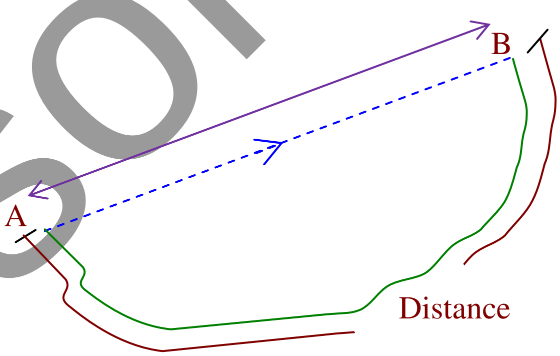
Fundamental Concepts:

- **Mechanics :** Deals with action of forces on bodies at rest or in motion.

- **State of rest and Motion:** They are relative and depend on the frame of reference. If the position with reference to frame of reference is fixed with time, then the body is said to be in a state of rest. Otherwise, it is said to be in a state of motion.

- **Scalar and heater quantities:** Quantities which require only magnitude to represent them are called scalar quantities. Quantities which acquire magnitudes and direction to represent them are called vector quantities.
  Eg: Mass, time internal, Distance traveled → Scalars
  Weight, Displacement, Velocity → Vectors

- **Displacement and Distance**



Unit: m

- **Velocity and Speed:** Rate of displacement is called velocity and Rate and distance traveled is called Speed.

Unit: m/s

- **Acceleration:** Rate of change of velocity is called acceleration. Negative acceleration is called retardation.

- **Momentum:** The capacity of a body to impart motion to other bodies is called momentum.

The momentum of a moving body is measured by the product of mass and velocity the moving body

Momentum = Mass x Velocity

Unit: Kgm/s

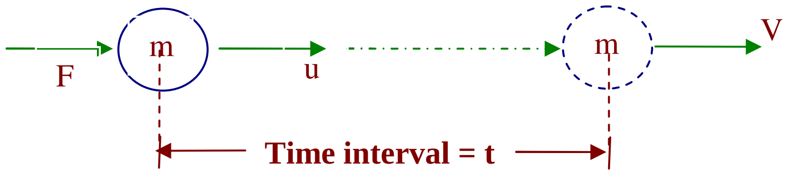
- **Newton's first law of motion:** Every body continues to be in its state of rest or uniform motion unless compelled by an external agency.

- **Inertia:** It is the inherent property the body to retain its state of rest or uniform motion.

- **Force:** It is an external agency which overcomes or tends to overcome the inertia of a body.

- **Newton's second law of motion:** The rate of change of momentum of a body is directly proportional to the magnitudes of the applied force and takes place in the direction of the applied force.

- **Measurement of force:**



Change in momentum in time 't' = mv – mu

Rate of change of momentum = $\frac{mv - mu}{t}$

$$F \,\alpha\, \frac{mv - mu}{t}$$

$$F \,\alpha\, m\left(\frac{v - u}{t}\right)$$

$$F \,\alpha\, ma$$

$$F = K\ ma$$

If F = 1 When m = 1 and u = 1

then K = 1

$\therefore$ F = ma.

Unit: newton (N)

- **Mass:** Measure of amount of matter contained by the body it is a scalar quantity.

  Unit: Kg.

- **Weight:** Gravitational force on the body. It is a vector quantity.

  F = ma

  W = mg

  Unit: newton (N) $g = 9.81\ m/s^2$

- **Volume:** Measure of space occupied by the body.

  Unit: m^3

  $1\ m^3 = 1000$ litres

- **Work:** Work done = Force x Displacement → Linear motion.

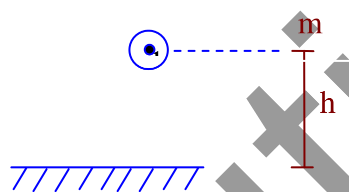
  Work done = Torque x Angular displacement → Rotatory motion.

Unit: Nm or J

- **Energy:** Capacity of doing work is called energy.

Unit: Nm or J

Potential energy = mgh



Kinetic energy = $\frac{1}{2} mv^2$ or $\frac{1}{2} mr\omega^2$ ω = Angular velocity

Power: Rate of doing work is called Power.

$$\text{Power:} = \frac{\text{Force x displacement}}{\text{time}}$$

= Force x Velocity → Linear Motion.

$$P = \frac{2\Pi N T}{60}$$ → Rotatory Motion.

- **Matter:** Anything which possess mass and requires space to occupy is called matter.
- **States of matter**:

  Matter can exist in the following states

  - Solid state.
  - Fluid state.

- **Solid state:** In case of solids intermolecular force is very large and hence molecules are not free to move. Solids exhibit definite shape and volume. Solids undergo certain amount of deformation and then attain state of equilibrium when subjected to tensile, compressive and shear forces.

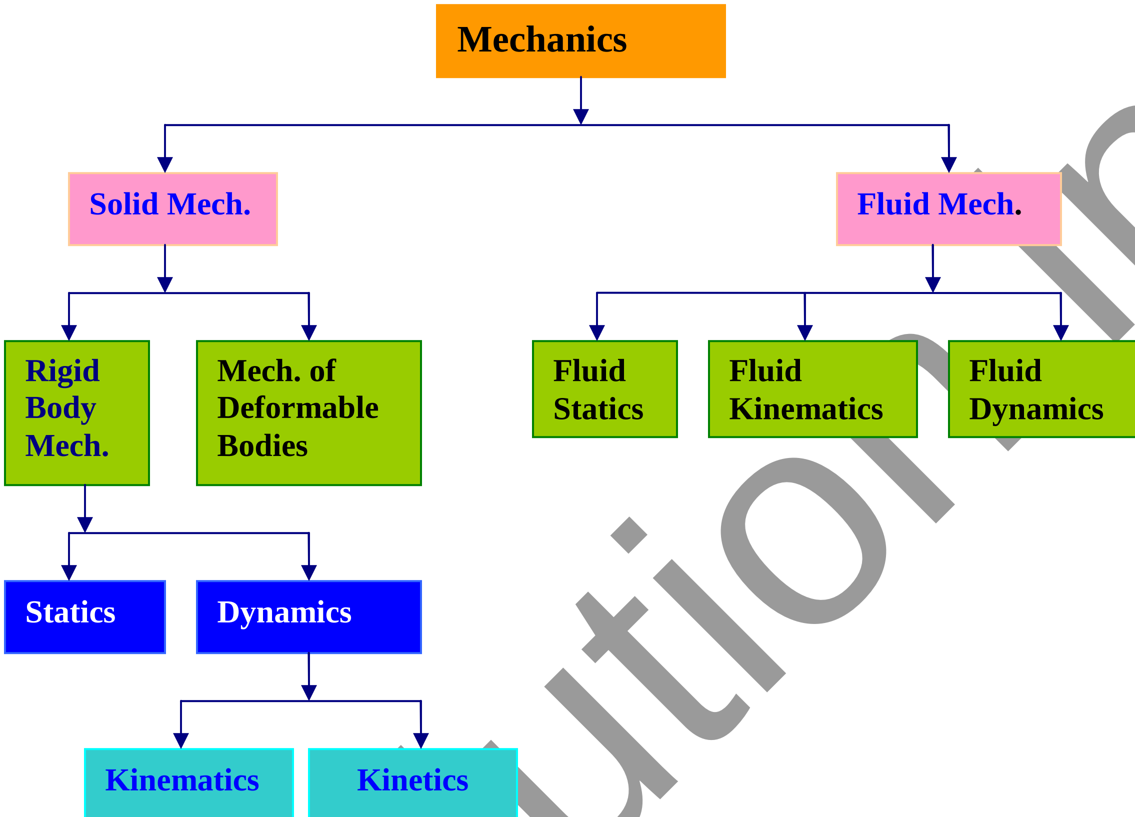
- **Fluid State:** Liquids and gases together are called fluids. Incase of liquids Intermolecular force is comparatively small. Therefore liquids exhibit definite volume. But they assume the shape of the container

Liquids offer very little resistance against tensile force. Liquids offer maximum resistance against compressive forces. Therefore, liquids are also called incompressible fluids. Liquids undergo continuous or prolonged angular deformation or shear strain when subjected to tangential force or shear force. This property of the liquid is called flow of liquid. Any substance which exhibits the property of flow is called fluid. Therefore liquids are considered as fluids.

In case of gases intermolecular force is very small. Therefore the molecules are free to move along any direction. Therefore gases will occupy or assume the shape as well as the volume of the container.

Gases offer little resistance against compressive forces. Therefore gases are called compressible fluids. When subjected to shear force gases undergo continuous or prolonged angular deformation or shear strain. This property of gas is called flow of gases. Any substance which exhibits the property of flow is called fluid. Therefore gases are also considered as fluids.

- **Branches of Mechanics:**



- Fluid Statics deals with action of forces on fluids at rest or in equilibrium.
- Fluid Kinematics deals with geometry of motion of fluids without considering the cause of motion.
- Fluid dynamics deals with the motion of fluids considering the cause of motion.

Properties of fluids:

1. Mass density or Specific mass (ρ):

Mass density or specific mass is the mass per unit volume of the fluid.

$$\therefore \quad \rho = \frac{\textit{Mass}}{\textit{Volume}}$$

$$\rho = \frac{M}{V} \text{ or } \frac{dM}{dV}$$

Unit: kg/m^3 or kgm^3

With the increase in temperature volume of fluid increases and hence mass density decreases.

In case of fluids as the pressure increases volume decreases and hence mass density increases.

2. Weight density or Specific weight (γ):

Weight density or Specific weight of a fluid is the weight per unit volume.

$$\therefore \quad \gamma = \frac{\textit{Weight}}{\textit{Volume}}$$

$$\gamma = \frac{W}{V} \text{ or } \frac{dW}{dV}$$

Unit: N/m^3 or Nm^{-3}.

With increase in temperature volume increases and hence specific weight decreases.

With increases in pressure volume decreases and hence specific weight increases.

Note: Relationship between mass density and weight density:

We have $\gamma = \dfrac{Weight}{Volume}$

$$\gamma = \frac{mass\ x\ g}{Volume}$$

$$\gamma = \rho \text{ x } g$$

3. Specific gravity or Relative density (S):

It is the ratio of specific weight of the fluid to the specific weight of a standard fluid.

$$S = \frac{\gamma \text{ of fluid}}{\gamma \text{ of standard fluid}}$$

Unit: It is a dimensionless quantity and has no unit.

In case of liquids water at 4^oC is considered as standard liquid.

γ (specific weight) of water at 4^oC (standard liquid) is 9.81 $\dfrac{kN}{m^3}$ or 9.81 x 10^3 $\dfrac{kN}{m^3}$

Note: We have

1. $S = \dfrac{\gamma}{\gamma_{standard}}$

   $\therefore \quad \gamma = S \text{ x } \gamma_{standard}$

2. $S = \dfrac{\gamma}{\gamma_{standard}}$

   $S = \dfrac{\rho \text{ x } g}{\rho_{standard} \text{ x } g}$

   $S = \dfrac{\rho}{\rho_{standard}}$

$\therefore$ Specific gravity or relative density of a fluid can also be defined as the ratio of mass density of the fluid to mass density of the standard fluid. Mass density of standard water is 1000 kg/m^3.

$$\rho = S \; x \; \gamma_{standard}$$

4. **Specific volume ($\forall$):** It is the volume per unit mass of the fluid.

$$\therefore \quad \forall = \frac{Volume}{mass}$$

$$\forall = \frac{V}{M} \; or \; \frac{dV}{dM}$$

Unit: m^3/kg

As the temperature increases volume increases and hence specific volume increases. As the pressure increases volume decreases and hence specific volume decreases.

Problems:

1. Calculate specific weight, mass density, specific volume and specific gravity of a liquid having a volume of $4m^3$ and weighing 29.43 kN. Assume missing data suitably.

$\gamma = ?$
$\rho = ?$
$\forall = ?$
$S = ?$
$V = 4\,m^3$
$W = 29.43\ kN$
$= 29.43 \times 10^3\ N$

$$\gamma = \frac{W}{V}$$
$$= \frac{29.43 \times 10^3}{4}$$
$$\gamma = 7357.58\ N/m^3$$

To find ρ - Method 1:

$$W = mg$$
$$29.43 \times 10^3 = m \times 9.81$$
$$m = 3000\,kg$$
$$\therefore \rho = \frac{m}{v} = \frac{3000}{4}$$
$$\rho = 750\,kg/m^3$$

Method 2:

$$\gamma = \rho\, g$$
$$7357.5 = \rho\ 9.81$$
$$\rho = 750\ kg/m^3$$

$$\rho = \frac{M}{V}$$

i) $\forall = \frac{V}{M}$

$$\forall = \frac{1}{\left(\frac{M}{V}\right)}$$

$$= \frac{4}{3000}$$

$$\forall = \frac{1}{\rho} = \frac{1}{750}$$

$$\forall = 1.33 \text{ x } 10^{-3} m^3 / kg$$

$$\forall = 1.33X10^{-3} m^3 / kg$$

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$S = \frac{\rho}{\rho_{Standaard}}$$

$$= \frac{7357.5}{9810}$$

or

$$S = \frac{750}{1000}$$

$$S = 0.75$$

$$S = 0.75$$

2. Calculate specific weight, density, specific volume and specific gravity and if one liter of Petrol weighs 6.867N.

$$\gamma = \frac{W}{V}$$

$$V = 1 \text{Litre} = 10^{-3} m^3$$

$$= \frac{6.867}{10^{-3}}$$

$$W = 6.867N$$

$$\gamma = 6867 N / m^3$$

$$S = \frac{\gamma}{\gamma_{S\tan dard}}$$

$$\rho = S\ g$$

$$= \frac{6867}{9810}$$

$$6867 = \rho \times 9.81$$

$$S = 0.7$$

$$\rho = 700 kg/m^3$$

$$\forall = \frac{V}{M}$$

$$M = W/g$$

$$= \frac{10^{-3}}{0.7}$$

$$M = 6.867 \div 9.81$$

$$\forall = 1.4 x 10^{-3} m^3/kg$$

$$M = 0.7\ kg$$

3. Specific gravity of a liquid is 0.7 Find i) Mass density ii) specific weight. Also find the mass and weight of 10 Liters of liquid.

$$S = \frac{\gamma}{\gamma_{S\tan dard}}$$

$$0.7 = \frac{\gamma}{9810}$$

$$\gamma = 6867 N/m^3$$

$$\gamma = \rho g$$

$$6867 = \rho \times 9.81$$

$$\rho = 700\ kg/m^3$$

$S = 0.7$

$V = ?$

$\rho = ?$

$M = ?$

$W = ?$

$V = 10$ litre

$= 10 x 10^{-3} m^3$

$$S = \frac{\rho}{\rho_{Standard}}$$

$$0.7 = \frac{\rho}{1000}$$

$$\rho = 700 kg/m^3$$

$$\rho = \frac{M}{V}$$

$$700 = \frac{M}{10x10^{-3}}$$

$$M = 7 kg$$

$$\gamma = \frac{W}{V}$$

$$6867 = \frac{W}{10^{-2}}$$

$$W = 68.67 N$$

or

$$W = m\ g$$

$$= 7\ x\ 9.81$$

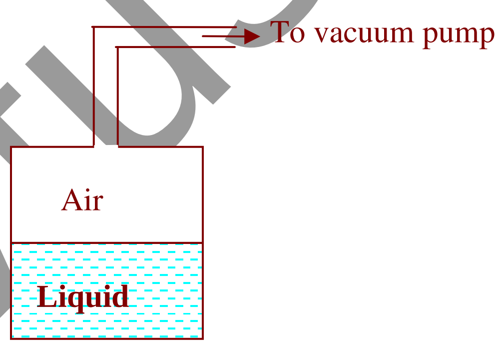
$$W = 68.67 N$$

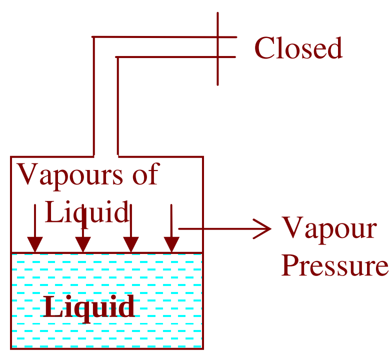
4. **Vapour Pressure:** The process by which the molecules of the liquid go out of its surface in the form of vapour is called Vapourisation.

There are two ways of causing Vapourisation.

a) By increasing the temperature of the liquid to its boiling point.

b) By reducing the pressure above the surface of the liquid to a value less than Vapour pressure of the liquid.





As the pressure above the surface of the liquid is reduced, at some point, there will be vapourisation of the liquid. If the reduction in pressure is continued vapourisation will also continue. If the reduction in pressure is stopped, vapourisation continues until vapours of the liquid exert certain pressure which will just stop the vapourisation. This minimum partial pressure exerted by the vapours of the liquid just to stop vapourisation is called Vapour Pressure of the liquid.

If the pressure over the surface goes below the vapour pressure, then, there will be vapourisation. But if the pressure above the surface is more than the vapour pressure then there will not be vapourisation unless there is heating.

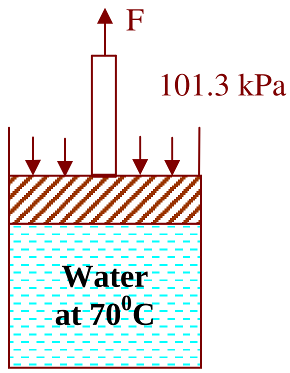
- **Importance of Vapour Pressure:**

1. In case of Hydraulic turbines sometimes pressure goes below the vapour pressure of the liquid. This leads to vapourisation and formation of bubbles of liquid. When bubbles are carried to high Pressure zone they get busted leaving partial vacuum. Surrounding liquid enters this space with very high velocity exerting large force on the part of the machinery. This shenornenon is called cavitation. Turbines are designed such that there is no cavitation.
2. In Carburetors and sprayers vapours of liquid are created by reducing the pressure below vapour pressure of the liquid.

Unit of Vapour Pressure: N/m^2 (Pascal - Pa)
Vapour Pressure of a fluid increases with increase in temperature.

Problem

1. A vertical cylinder 300mm in diameter is fitted at the top with a tight but frictionless piston and filled with water at 70^0 C. The outer portion of the piston is exposed to atmospheric pressure of 101.3 kPa. Calculate the minimum force applied on the piston that will cause water to boil at 70^0 C. Take Vapour pressure of water at 70^0 C as 32k Pa.



D = 300 mm

= 0.3 m

F Should be applied such that the Pressure is reduced from 101.3kPa to 32kPa. There fore reduction in pressure required

= 101.3 – 32

= 69.3 kPa

=69.3 x 10^3 N/m^2

$\therefore$ F / Area = 69.3 x 10^3

$$F / \frac{\Pi}{4} \times (0.3)^2 = 69.3 \times 10^3$$

F = 4.9 x 10^3N

F = 4.9 kN

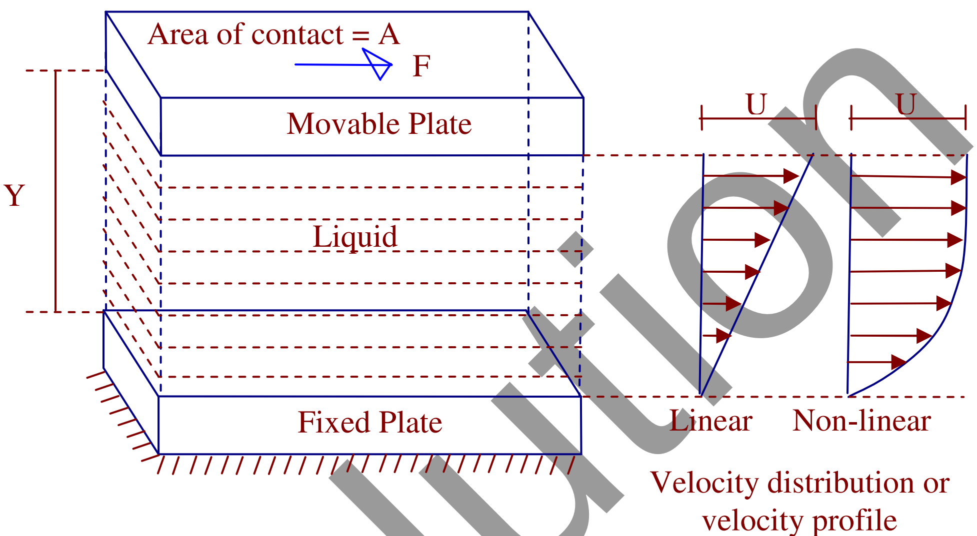
6. Viscosity:

Viscosity is the property by virtue of which fluid offers resistance against the flow or shear deformation. In other words, it is the reluctance of the fluid to flow. Viscous force is that force of resistance offered by a layer of fluid for the motion of another layer over it.

In case of liquids, viscosity is due to cohesive force between the molecules of adjacent layers of liquid. In case of gases, molecular activity between adjacent layers is the cause of viscosity.

- **Newton's law of viscosity:**

Let us consider a liquid between the fixed plate and the movable plate at a distance 'Y' apart , 'A' is the contact area (Wetted area) of the movable plate , 'F' is the force required to move the plate with a velocity 'U' According to Newton



- $F \alpha A$
- $F \alpha \frac{1}{Y}$
- $F \alpha U$

$\therefore F \alpha \frac{AU}{Y}$

$F = \mu . \frac{AU}{Y}$

'μ' is the constant of proportionality called Dynamic Viscosity or Absolute Viscosity or Coefficient of Viscosity or Viscosity of the fluid.

$$\frac{F}{A} = \mu . \frac{U}{Y}$$

$$\therefore \tau = \mu \frac{U}{Y}$$

'τ' is the force required; per unit area called 'Shear Stress'.

The above equation is called Newton's law of viscosity.

Velocity gradient or rate of shear strain:

It is the difference in velocity per unit distance between any two layers.

If the velocity profile is linear then velocity gradient is given by $\frac{U}{Y}$. If the velocity profile is non – linear then it is given by $\frac{du}{dy}$.

- Unit of force (F): N.
- Unit of distance between the tow plates (Y): m
- Unit of velocity (U): m/s
- Unit of velocity gradient : $\frac{U}{Y} = \frac{m/s}{m} = /s = s^{-1}$
- Unit of dynamic viscosity (τ): $\tau = \mu \cdot \frac{u}{y}$

$$\mu = \frac{\tau y}{U}$$

$$\Rightarrow \frac{N/m^2 \cdot m}{m/s}$$

$$\mu \Rightarrow \frac{Ns}{m^2} \text{ or } \mu \Rightarrow P_a s$$

NOTE:

In CGS system unit of dynamic viscosity is $\frac{\text{dyne . Sec}}{cm^2}$ and is called poise (P).

If the value of μ is given in poise, multiply it by 0.1 to get it in $\frac{NS}{m^2}$.

1 Centipoise = 10^{-2} Poise.

- **Effect of Pressure on Viscosity of fluids:**

  Pressure has very little or no effect on the viscosity of fluids.

- **Effect of Temperature on Viscosity of fluids:**

  1. ***Effect of temperature on viscosity of liquids:*** Viscosity of liquids is due to cohesive force between the molecules of adjacent layers. As the temperature increases cohesive force decreases and hence viscosity decreases.
  2. ***Effect of temperature on viscosity of gases:*** Viscosity of gases is due to molecular activity between adjacent layers. As the temperature increases molecular activity increases and hence viscosity increases.

- **Kinematics Viscosity:** It is the ratio of dynamic viscosity of the fluid to its mass density.

$$\therefore \text{KinematicViscosity} = \frac{\mu}{\rho}$$

Unit of KV:

$$KV \Rightarrow \frac{\mu}{\rho}$$

$$\Rightarrow \frac{NS/m^2}{kg/m^3}$$

$$= \frac{NS}{m^2} x \frac{m^3}{kg}$$

$$= \left(\frac{kg\,m}{s^2}\right) x \frac{s}{m^2} x \frac{m^3}{kg} = m^2/s$$

$F = ma$
$N = Kg.m/s^2$

$$\therefore \text{KinematicViscosity} = m^2/s$$

NOTE: Unit of kinematics viscosity in CGS system is cm^2/s and is called stoke (S)

If the value of KV is given in stoke, multiply it by 10^{-4} to convert it into m^2/s.

Problems:

1. Viscosity of water is 0.01 poise. Find its kinematics viscosity if specific gravity is 0.998.

Kinematics viscosity = ?

$S = 0.998$

$$S = \frac{\rho}{\rho_{s\tan drad}}$$

$$0.998 = \frac{\rho}{1000}$$

$$\rho = 998\ kg/m^3$$

$\mu = 0.01P$

$= 0.01x0.1$

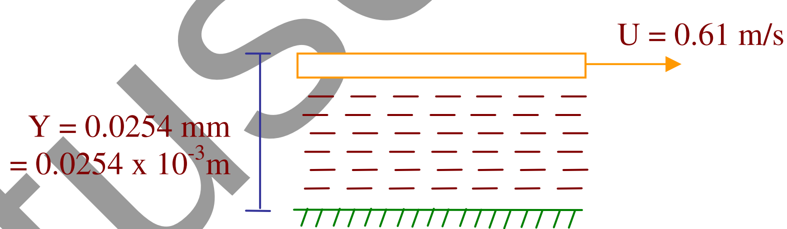
$$\mu = 0.001\ \frac{\boldsymbol{NS}}{\boldsymbol{m}^2}$$

$$\therefore KV = \frac{\mu}{\rho}$$

$$= \frac{0.001}{998}$$

$$KV = 1\ x\ 10^{-6}\ m^2/s$$

2. A Plate at a distance 0.0254mm from a fixed plate moves at 0.61m/s and requires a force of 1.962N/m^2 area of plate. Determine dynamic viscosity of liquid between the plates.



$$\tau = 1.962\ N/m^2$$

$$\mu = ?$$

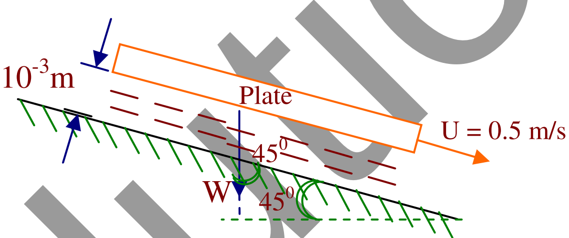
Assuming linear velocity distribution

$$\tau = \mu\ \frac{U}{Y}$$

$$1.962 = \mu \text{ x } \frac{0.61}{0.0254 \text{ x } 10^{-3}}$$

$$\mu = 8.17 \text{ x } 10^{-5} \frac{NS}{m^2}$$

3. A plate having an area of $1m^2$ is dragged down an inclined plane at 45^0 to horizontal with a velocity of 0.5m/s due to its own weight. Three is a cushion of liquid 1mm thick between the inclined plane and the plate. If viscosity of oil is 0.1 Pas find the weight of the plate.



A =$1m^2$

U = 0.5m/s

Y = $1x10^{-3}$m

μ = $0.1NS/m^2$

W = ?

F = W x cos 45^0

= W x 0.707

F = 0.707W

$$\tau = \frac{F}{A}$$

$$\tau = \frac{0.707W}{1}$$

$$\tau = 0.707 \text{ W } N/m^2$$

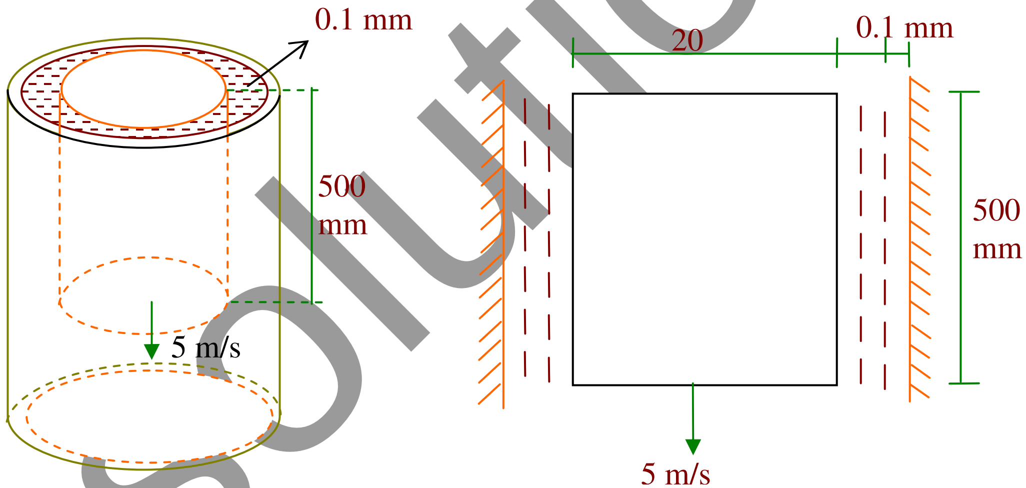
Assuming linear velocity distribution,

$$\tau = \mu.\frac{U}{Y}$$

$$0.707W = 0.1 \text{ x} \frac{0.5}{1 \text{ x } 10^{-3}}$$

$$W = 70.72\,N$$

4. A shaft of ϕ 20mm and mass 15kg slides vertically in a sleeve with a velocity of 5 m/s. The gap between the shaft and the sleeve is 0.1mm and is filled with oil. Calculate the viscosity of oil if the length of the shaft is 500mm.



D = 20mm = $20 \text{x} 10^{-3}$m

M = 15 kg

W = 15x 9.81

W = 147.15N

y = 0.1mm

y = 0.1 x 10^{-3}mm

U = 5m/s

F = W

F = 147.15N

μ= ?

A = Π D L

A = Π x 20 x 10^{-3} x 0.5

A = 0.031 m^2

$$\tau = \mu.\frac{U}{Y}$$

$$4746.7 = \mu x \frac{5}{0.1x10^{-3}}$$

$$\mu = 0.095 \ \frac{NS}{m^2}$$

$$\tau = \frac{F}{A}$$

$$= \frac{147.15}{0.031}$$

$$\tau = 4746.7 N/m^2$$

5. If the equation of velocity profile over 2 plate is V= $2y^{2/3}$. in which 'V' is the velocity in m/s and 'y' is the distance in 'm' . Determine shear stress at (i) y = 0 (ii) y = 75mm. Take μ = 8.35P.

(i) at y = 0

(ii) at y = 75mm

= 75 x 10^{-3}m

$$\tau = 8.35\ P$$

$$= 8.35 \text{ x } 0.1\ \frac{NS}{m^2}$$

$$= 0.835\ \frac{NS}{m^2}$$

$$V = 2y^{2/3}$$

$$\frac{dv}{dy} = 2x\frac{2}{3}y^{2/3-1}$$

$$= \frac{4}{3}y^{-1/3} = \frac{4}{3}\frac{1}{\sqrt[3]{y}}$$

$$\text{at, } y = 0, \frac{dv}{dy} = 3\frac{4}{\sqrt[3]{0}} = \infty$$

$$\text{at, } y = 75x10^{-3}\text{ m}, \frac{dv}{dy} = 3\frac{4}{\sqrt[3]{75x10^{-3}}}$$

$$\frac{dv}{dy} = 3.16/s$$

$$\tau = \mu.\frac{dv}{dy}$$

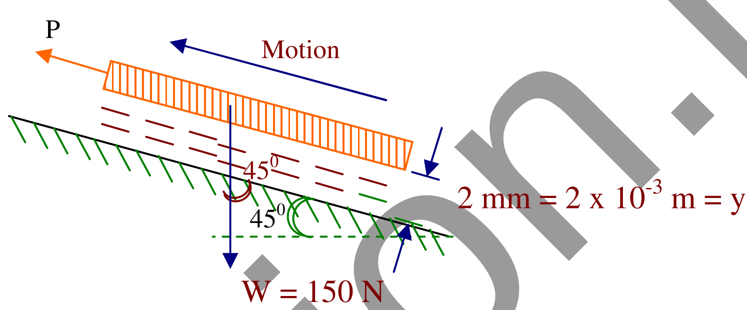
$$at, y = 0,\ \ \tau = 0.835x\infty$$

$$\tau = \infty$$

$$at, y = 75\,x10^{-3}\,m, \tau = 0.835\ x\ 3.16$$

$$\tau = 2.64\ N/m^2$$

6. A circular disc of 0.3m dia and weight 50 N is kept on an inclined surface with a slope of 45^0. The space between the disc and the surface is 2 mm and is filled with oil of dynamics viscosity $\frac{1NS}{m^2}$. What force will be required to pull the disc up the inclined plane with a velocity of 0.5m/s.



D = 0.3m

$$A = \frac{\prod x\, 0.3m^2}{4}$$

A = 0.07m²

W = 50N

$$\mu = 1\ \frac{NS}{m^2}$$

$$y = 2x10^{-3}\, m$$

$$U = 0.5 m/s$$

$$F = P - 50\cos 45$$

$$F = (P - 35,35)$$

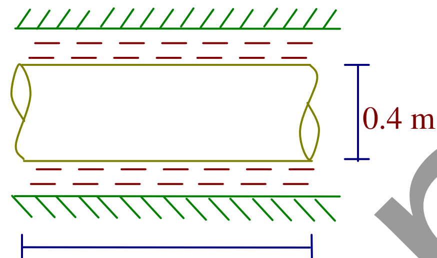
$$\upsilon = \frac{(P - 35.35)}{0.07} N/m^2$$

$$\tau = \mu.\frac{U}{Y}$$

$$\left(\frac{P - 35,35}{0.07}\right) = 1x\ \frac{0.5}{2x10^{-3}}$$

$$P = 52.85N$$

7. Dynamic viscosity of oil used for lubrication between a shaft and a sleeve is 6 P. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 0.09 m .Thickness of oil is 1.5 mm.



$\mu = 6P$

$= 0.6 \frac{Ns}{m^2}$

N = 190 rpm

Power lost = ?

$A = \Pi D L$

$= \Pi \times 0.4 \times 0.09 \qquad A = 0.11 m^2$

$Y = 1.5 \times 10^{-3}$ m

$$U = \frac{\Pi DN}{60}$$

$$= \frac{\Pi \text{ x } 0.4 \text{ x } 190}{60}$$

$$U = 3.979 \text{ m/s}$$

$$\tau = \mu.\frac{U}{Y}$$

$$= 0.6 \text{ x } \frac{3.979}{1.5 \text{ x } 10^{-3}}$$

$$\tau = 1.592 \text{ x } 10^{3} \text{ N/m}^{2}$$

$$\frac{F}{A} = 1.59 \text{ x } 10^{3}$$

$$F = 1.591 \text{x } 10^{3} \text{ x } 0.11$$

$$F = 175.01 \text{ N}$$

$$T = F \text{x} R$$

$$= 175.01 \text{x} 0.2$$

$$T = 35 \text{Nm}$$

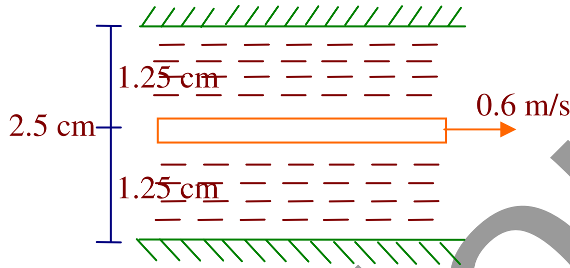
$$P = \frac{2\Pi NT}{60{,}000}$$

$$P = 0.6964 \text{KW}$$

$$P = 696.4 \text{W}$$

8. Two large surfaces are 2.5 cm apart. This space is filled with glycerin of absolute viscosity 0.82 NS/m^2. Find what force is required to drag a plate of area 0.5m^2 between the two surfaces at a speed of 0.6m/s. (i) When the plate is equidistant from the surfaces, (ii) when the plate is at 1cm from one of the surfaces.

Case (i)



Let F_1 be the force required to overcome viscosity resistance of liquid above the plate and F_2 be the force required to overcome viscous resistance of liquid below the plate. In this case $F_1 = F_2$. Since the liquid is same on either side or the plate is equidistant from the surfaces.

$$\tau_1 = \mu_1 \frac{U}{Y}$$

$$\tau_1 = 0.82x\frac{0.6}{0.0125}$$

$$\tau_1 = 39.36N/m^2$$

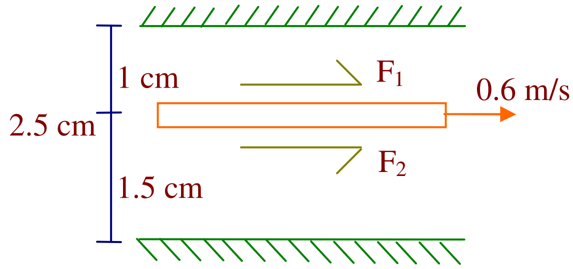
$$\frac{F_1}{A} = 39.36$$

$$F_1 = 19.68N$$

∴ Tatal force required to drag the plate $= F_1 + F_2 = 19.68 + 19.68$

F= 39.36N

Case (ii)



Here $F_1 \neq F_2$

$$\tau_1 = \mu_1 \frac{U}{y1}$$

$$= 0.82 \text{ x } \frac{0.62}{1 \text{ x } 10^{-2}}$$

$$\tau_1 = 49.2 \text{ N/m}^2$$

$$\tau_2 = \mu_2 \frac{U}{y_2}$$

$$= 0.82 \text{ x } \frac{0.6}{1.5 \text{ x } 10^{-2}}$$

$$\tau_1 = 32.8 \text{ N/m}^2$$

$$\frac{F_1}{A} = 49.2$$

$$F_1 = 49.2x0.5$$

$$F_1 = 24.6N$$

$$\frac{F_2}{A} = 32.8$$

$$F_2 = 32.8x0.5$$

$$F_2 = 16.4N$$

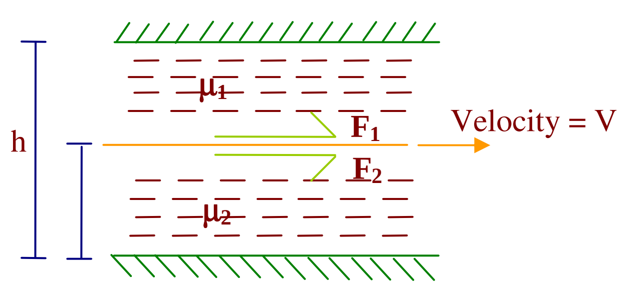
Total Force $F = F_1 + F_2 = 24.6 + 16.4$

$F = 41N$

9. Through a very narrow gap of height a thin plate of large extent is pulled at a velocity `V'. On one side of the plate is oil of viscosity μ_1 and on the other side there is oil of viscosity μ_2. Determine the position of the plate for the following conditions.

   i. Shear stress on the two sides of the plate is equal.
   ii. The pull required, to drag the plate is minimum.

Conditions 1



$y = ?\ \text{for}\ F_1 = F_2$

$\tau = \mu . \frac{U}{Y}$

$\frac{F}{A} = \mu . \frac{U}{Y}$

$F = A\mu . \frac{U}{Y}$

$F_1 = \frac{A\mu_1 V}{(h - y)}$

$F_2 = \frac{A\mu_2 V}{y}$

$F_1 = F_2$

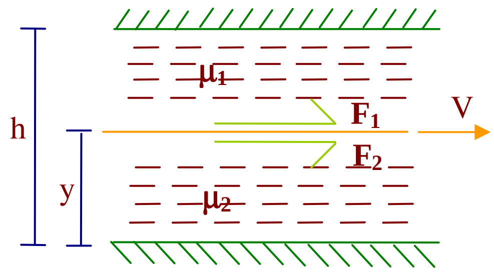
$\frac{A\mu_1 V}{h-y} = \frac{A\mu_2 V}{y}$

$\mu_1 y = \mu_2 (h-y)$

$\mu_1 y + \mu_2 y = \mu_2 h$

$y = \frac{\mu_2 h}{\mu_1 + \mu_2}\ \text{or}\ y = \frac{h}{\frac{\mu_1}{\mu_2} + 1}$

Conditions 2:



$y = ?$ if, $F_1 + F_2$ is to be minimum

$$F_1 = \frac{A\mu_1 V}{h-y}$$

$$F_2 = \frac{A\mu_2 V}{y}$$

$\therefore$ Total drag forced required

$$F = F_1 + F_2$$

$$F = \frac{A\mu_1 V}{h-y} + \frac{A\mu_2 V}{y}$$

$$\text{For F to be min.} \frac{dF}{dy} = 0$$

$$\frac{dF}{dy} = 0 = +A\mu_1 V \equiv (h-y)^{-2} - A\mu_2 V y^{-2}$$

$$= \frac{V\mu_1 A}{(h-y)^{+2}} - \frac{V\mu_2 A}{y^2}$$

$$\frac{(h-y)^2}{y^2} = \frac{\mu_1}{\mu_2}$$

$$\frac{h-y}{y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

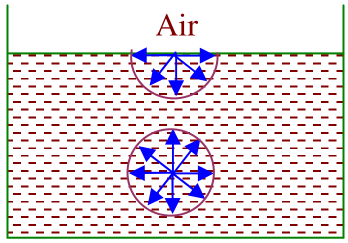
$$(h-y) = y\sqrt{\frac{\mu_1}{\mu_2}}$$

$$h = y\sqrt{\frac{\mu_1}{\mu_2}} + y$$

$$h = y\left(1 + \sqrt{\frac{\mu_1}{\mu_2}}\right)$$

$$\therefore y = \frac{h}{1 + \sqrt{\frac{\mu_1}{\mu_2}}}$$

(7) Surface Tension (σ)

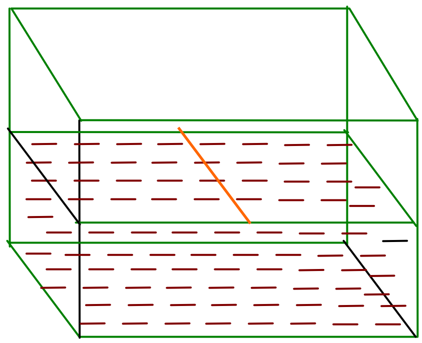


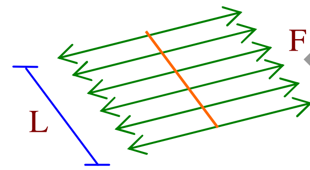
Surface tension is due to cohesion between the molecules of liquid and weak adhesion between the molecules on the exposed surface of the liquid and molecules of air.

A molecule inside the surface gets attracted by equal forces from the surrounding molecules whereas a molecule on the surface gets attracted by the molecule below it. Since there are no molecules above it, it experiences an unbalanced vertically downward force. Due to this entire surface of the liquid expose of to air will have a tendency to move in ward and hence the surface will be under tension. The property of the liquid surface to offer resistance against tension is called surface tension.

- **Consequences of Surface tension:**
  - Liquid surface supports small loads.
  - Formation of spherical droplets of liquid
  - Formation of spherical bubbles of liquid
  - Formation of cylindrical jet of liquids.

- **Measurement of surface tension:**





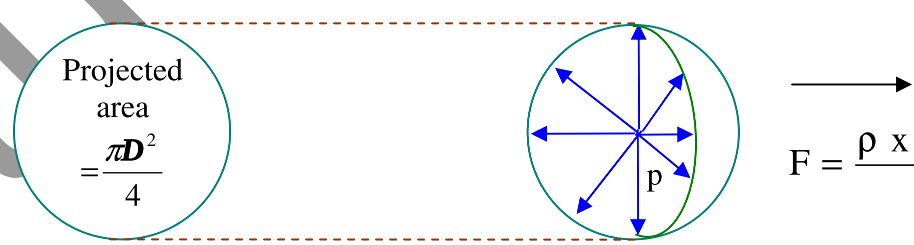
Surface tension is measured as the force exerted by the film on a line of unit length on the surface of the liquid. It can also be defined as the force required maintaining unit length of film in equilibrium.

$$\therefore \sigma = \frac{F}{L} \qquad \therefore F = \sigma L$$

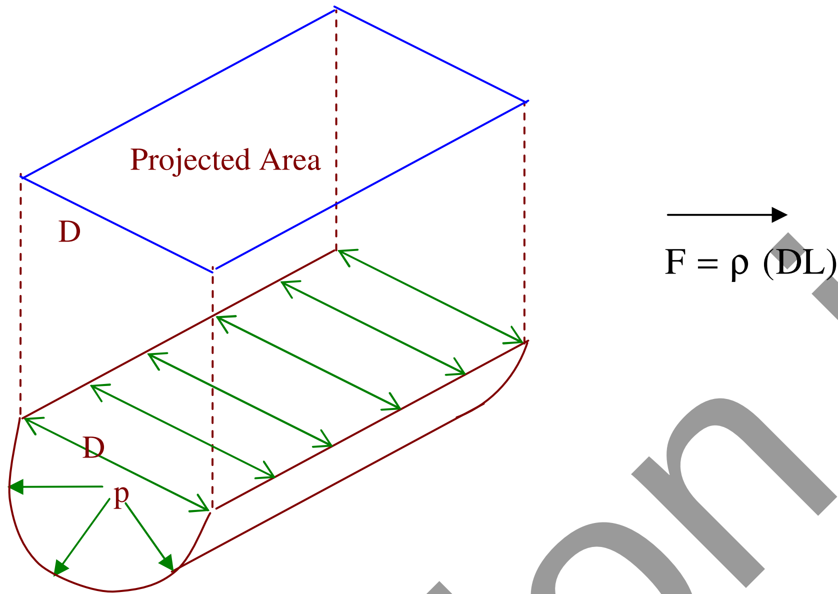
Unit: N/m

Force due to surface tension = σ x length of film

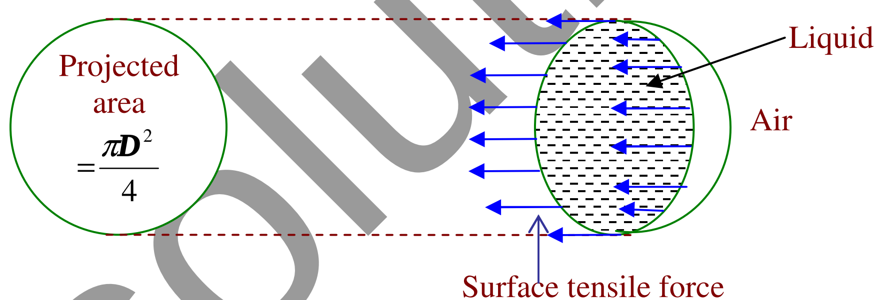
NOTE: Force experienced by a curved surface due to radial pressure is given by the product of intensity of pressure and projected area of the curved surface.



$$F = \frac{\rho \ x \ \pi D^2}{4}$$



- **To derive an expression for the pressure inside the droplet of a liquid.**



Let us consider droplet of liquid of surface tension 'σ'. 'D' is the diameter of the droplet. Let 'p' be the pressure inside the droplet in excess of outside pressure ($p = p_{inside} - p_{outside}$).

For the equilibrium of the part of the droplet,

Force due to surface tension = Force due to pressure

$$\sigma \times \Pi D = p \times \text{projected area}$$

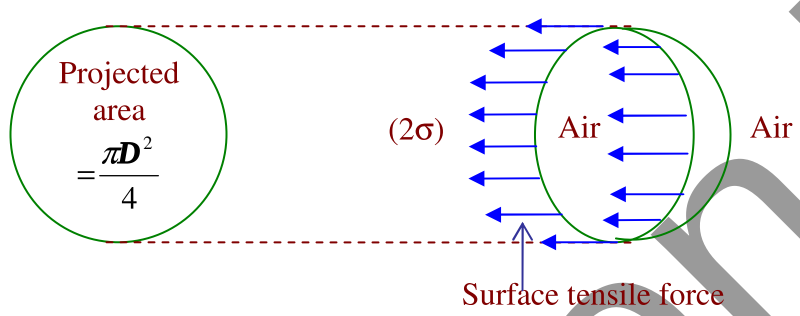
$$\sigma \times \Pi D = p \times \frac{\Pi D^2}{4}$$

$$p = \frac{4\sigma}{D}$$

As the diameter increases pressure decreases.

- **To derive an expression for the pressure inside the bubble of liquid:**

`D' is the diameter of bubble of liquid of surface tension σ. Let 'p' be the pressure inside the bubble which is in excess of outside pressure. In case of bubble the liquid layer will be in contact with air both inside and outside.



For the equilibrium of the part of the bubble,

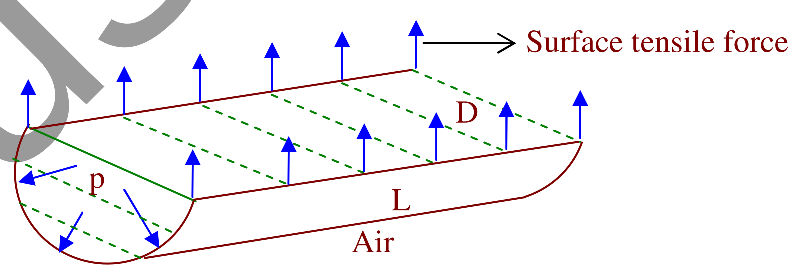
Force due to surface tension = Force due to pressure

(2 σ) x Π D = p x projected area

$$2[\sigma \, x \, \Pi D] = p \, x \frac{\Pi D^2}{4}$$

$$p = \frac{8\sigma}{D}$$

- **To derive an expression for the pressure inside the jet of liquid:**



Let us consider a jet of diameter D of liquid of surface tension σ and p is the intensity of pressure inside the jet in excess of outside atmospheric pressure. For the equilibrium of the part of the jet shown in fig, Force due to Radial pressure = Force due to surface tension

p x Projected area = σ x Length

p x D x L = σ x 2L

$$P = \frac{2\sigma}{D}$$

- **Effect of temperature on surface tension of liquids:**

In case of liquids, surface tension decreases with increase in temperature. Pressure has no or very little effect on surface tension of liquids.

Problems:

1. What is the pressure inside the droplet of water 0.05mm in diameter at 20^0C, if the pressure outside the droplet is 103 kPa Take σ = 0.0736 N/m at 20^0C

$$p = \frac{4\sigma}{D}$$

$$= \frac{4x0.0736}{0.05x10^{-3}}$$

$$p = 5.888x10^{+3} N/m^2$$

$$p = p_{inside} - p_{outside}$$

$$p_{inside} = (5.888 + 103)10^3$$

$$p_{inside} = 108.88x10^3 Pa$$

$p_{inside} = ?$

$D = 0.05x10^{-3} m$

$p_{outside} = 103kPa$

$= 103x10^3 N/m^2$

$\sigma = 0.0736 N/m$

2. A liquid bubble 2cm in radius has an internal pressure of 13Pa. Calculate the surface tension of liquid film.

$$p = \frac{8\sigma}{D}$$

$$\sigma = \frac{13x4x10^{-2}}{8}$$

$$\sigma = 0.065 N/m$$

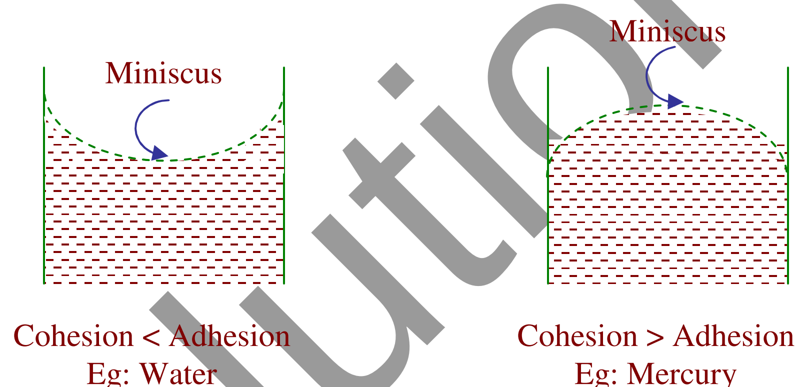
$R = 2cm$

$D = 4cm$

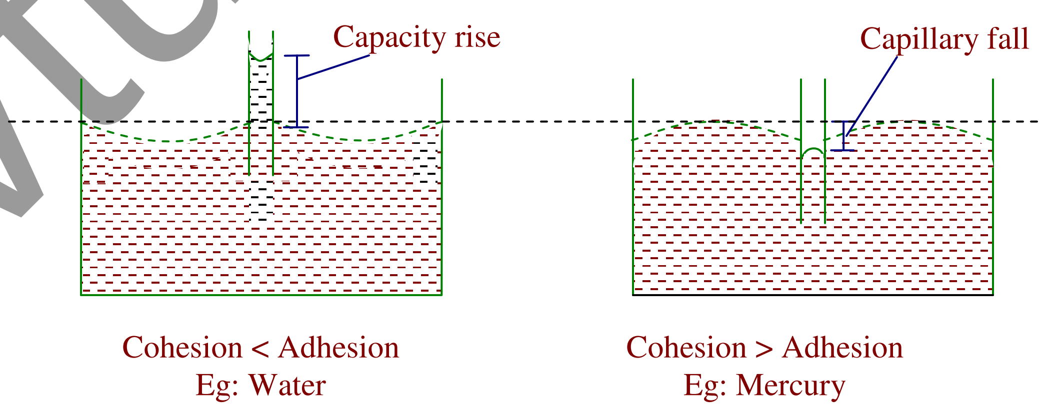
$= 4x10^{-2} m$

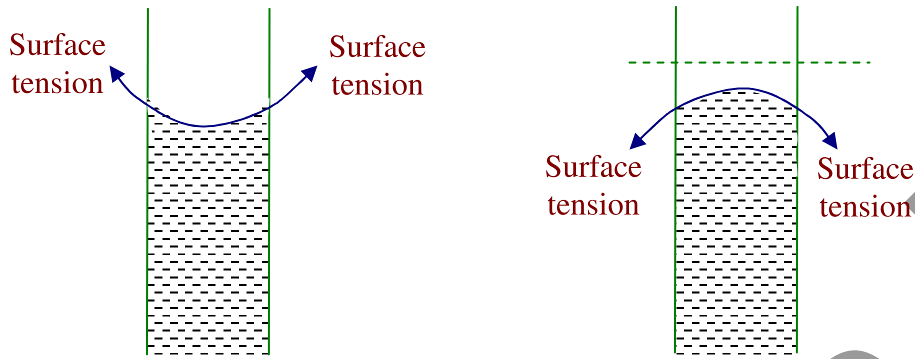
$p = 13Pa(N/m^2)$

8. Capillarity:



Any liquid between contact surfaces attains curved surface as shown in figure. The curved surface of the liquid is called Meniscus. If adhesion is more than cohesion then the meniscus will be concave. If cohesion is greater than adhesion meniscus will be convex.

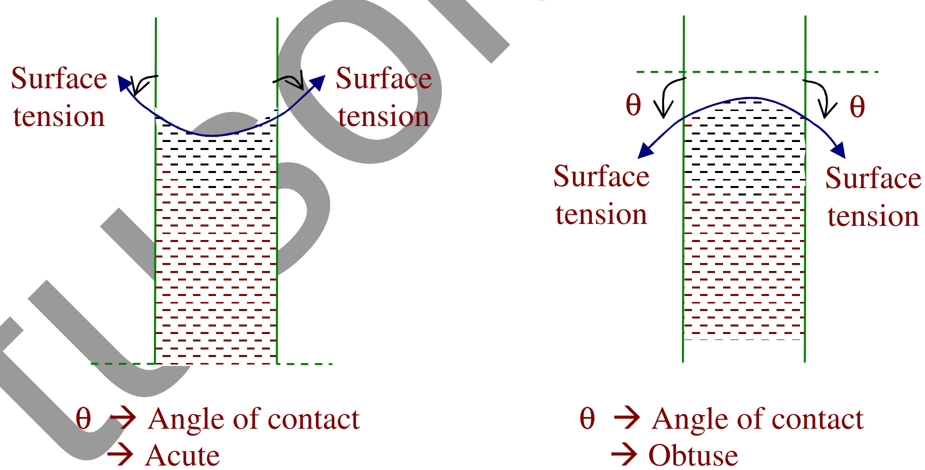




Capillarity is the phenomena by which liquids will rise or fall in a tube of small diameter dipped in them. Capillarity is due to cohesion / adhesion and surface tension of liquids. If adhesion is more than cohesion then there will be capillary rise. If cohesion is greater than adhesion then will be capillary fall or depression. The surface tensile force supports capillary rise or depression.

Note:

Angle of contact:

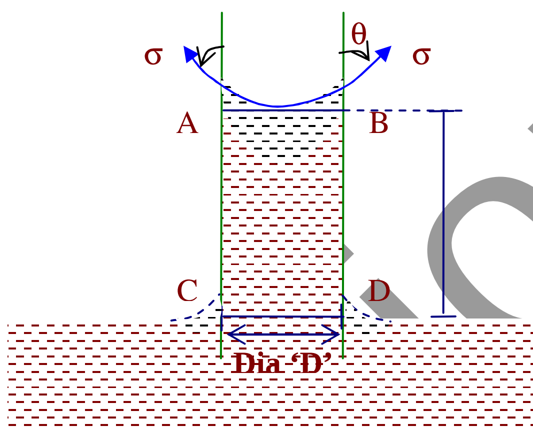


The angle between surface tensile force and the vertical is called angle of contact. If adhesion is more than cohesion then angle of contact is obtuse.

- **To derive an expression for the capillary rise of a liquid in small tube dipped in it:**

Let us consider a small tube of diameter 'D' dipped in a liquid of specific weight γ. 'h' is the capillary rise. For the equilibrium,

Vertical force due to surface tension = Weight of column of liquid ABCD



$$[\sigma(\Pi D)]\cos\theta = \gamma \text{ x volume}$$

$$[\sigma(\Pi D)]\cos\theta = \gamma \text{ x } \frac{\Pi D^2}{4} \text{ x h}$$

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

It can be observed that the capillary rise is inversely proportional to the diameter of the tube.

Note:

The same equation can be used to calculate capillary depression. In such cases ' θ ' will be obtuse 'h' works out to be –ve.

Problems:

1. Capillary tube having an inside diameter 5mm is dipped in water at 20^0. Determine the heat of water which will rise in tube. Take σ =0.0736N/m at 20^0 C.

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$\theta = 0^0 \text{ (assumed)}$$

$$= \frac{4 \text{ x } 0.0736 \text{ x } \cos\theta}{9810 \text{ x } 5 \text{ x } 10^{-3}}$$

$$\gamma = 9810 N/m^3$$

$$h = 6 \text{ x } 10^{-3} m$$

2. Calculate capillary rise in a glass tube when immersed in Hg at 20^0c. Assume σ for Hg at 20^0c as 0.51N/m. The diameter of the tube is 5mm. θ = 130^0c.

$$S = \frac{\gamma}{\gamma_{S\tan dard}}$$

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$13.6 = \frac{\gamma}{9810}$$

$$h = -1.965 \text{ x} 10^{-3} m$$

$$\gamma = 133.416 \text{ x } 10^3 N/m^3$$

-ve sign indicates capillary depression.

3. Determine the minimum size of the glass tubing that can be used to measure water level if capillary rise is not to exceed 2.5mm. Take $\sigma = 0.0736$ N/m.

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$D = \frac{4 \text{ x } 0.0736 \text{ x} \cos 0}{9810 \text{ x } 2.5 \text{ x } 10^{-3}}$$

$D = ?$

$h = 2.5x10^{-3}\,m$

$$D = 0.012\,m$$

$\sigma = 0.0736\ N/m$

$$D = 12\,mm$$

4. A glass tube 0.25mm in diameter contains Hg column with air above it. If σ = 0.51N/m, what will be the capillary depression? Take θ = - 40^0 or 140^0.

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$D = 0.25x10^{-3}\,m$

$$= \frac{4x0.51x\cos 140}{133.146x10^{-3}x0.25x10^{-3}}$$

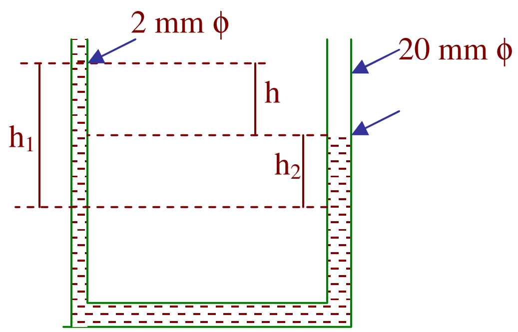
$\sigma = 0.51N/m$

$\theta = 140$

$$h = -46.851x10^{-3}\,m$$

$\gamma = 133.416 \text{ x } 10^3\ N/m^2$

5. If a tube is made so that one limb is 20mm in ϕ and the other 2mm in ϕ and water is poured in the tube, what is the difference in the level of surface of liquid in the two limbs. σ = 0.073 N/m for water.



$$h_1 = \frac{4\sigma\cos\theta}{\gamma D}$$

$$= \frac{4 \text{ x } 0.073 \text{ x } \cos o}{9810 \text{ x } (20 \text{x} 10^{-3})}$$

$$= 0.01488\text{m}$$

$$h_2 = \frac{4 \text{ x } 0.073 \text{x} \cos o}{9810 \text{ x } (20 \text{x} 10^{-3})}$$

$$= 1.488 \text{ x } 10^{-3}\text{ m}$$

$$h = h_1 - h_2$$

$$= 0.01339\text{m}$$

$$h = 13.39\text{mm}$$

6. Compressibility:

It is the property by virtue of which there will be change in volume of fluid due to change in pressure.

Let 'v' be the original volume and 'dv' be the change in volume due to change in pressure 'dp' , $\frac{dv}{v}$ i.e., the ratio of change in volume to original volume is called volumetric strain or bulk strain.

The ratio of change in pressure to the volumetric strain produced is called Bulk modulus of elasticity of the fluid and is denoted by 'K'

$$\therefore K = \frac{dp}{\left(\frac{dv}{v}\right)}.$$

Sometimes 'K' is written as $K = -\frac{dp}{\left(\frac{dv}{v}\right)}$. –ve sign indicates that as there is increase in pressure, there is decrease in volume. Reciprocal of Bulk modulus of elasticity is called Compressibility of the fluid.

$$\therefore \text{Compressibility} = \frac{1}{K} = \frac{\frac{dv}{v}}{dp}$$

Unit of Bulk modulus of elasticity is N/m^2 or Pa. Unit of compressibility is m^2/N.

Problem:

1. The change in volume of certain mass of liquids is observed to be $\frac{1}{500}$th of original volume when pressure on it is increased by 5Mpa. Determine the Bulk modulus and compressibility of the liquid.

$$dv = \frac{1}{500}V \qquad K = \frac{dp}{\frac{dv}{v}}$$

$$\frac{dv}{v} = \frac{1}{500} \qquad = 2.5 \text{ x } 10^9\,Pa$$

$$dp = 5 \text{ x } 10^6\,N/m^2 \qquad K = 2.5\,GPa$$

$$Compresibility = \frac{1}{K}$$

$$= \frac{1}{25 \text{ x } 10^8}$$

$$= 4 \text{ x } 10^{-10}\,m^2/N$$

2. Find the pressure that must be applied to water at atmospheric pressure to reduce its volume by 1% .Take K=2 GPa.

$$K = \frac{dp}{\frac{dv}{v}}$$

$$2 \text{ x } 10^9 = \frac{dP}{\frac{1}{100}}$$

$$dp = 20 \text{ x } 10^6\ N/m^2$$

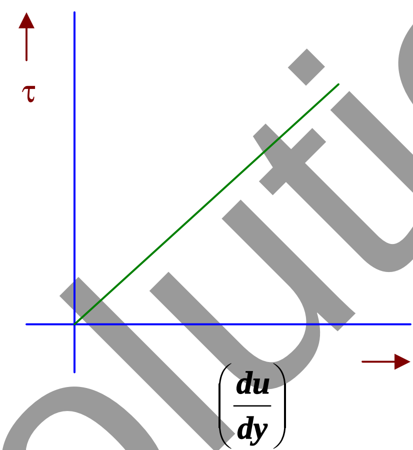
$$dp = 20\,MPa$$

- **Rheological classification of fluids: (Rheology → Study of stress – strain behavior).**

1. **Newtonian fluids:** A fluid which obeys Newton's law of viscosity i.e., $\tau = \mu . \frac{du}{dy}$ is called Newtonian fluid. In such fluids shear stress varies directly as shear strain.

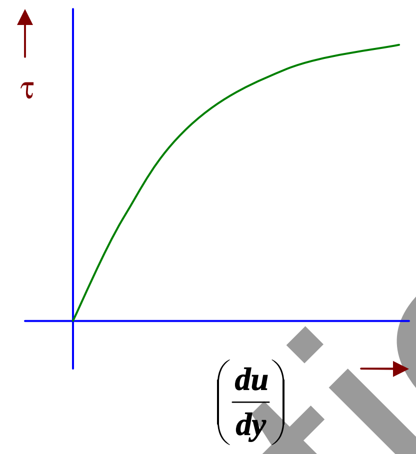
In this case the stress strain curve is a stress line passing through origin the slope of the line gives dynamic viscosity of the fluid.

Eg: Water, Kerosene.



2. **Non- Newtonian fluid:** A fluid which does not obey Newton's law of viscosity is called non-Newton fluid. For such fluids,

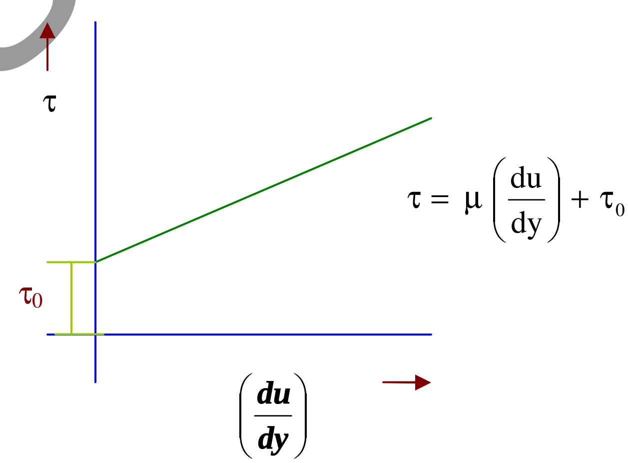
$$\tau = \mu . \left(\frac{du}{dy}\right)^n$$



3. **Ideal Plastic fluids:**

In this case the strain starts after certain initial stress (τ_0) and then the stress-strain relationship will be linear. τ_0 is called initial yield stress. Sometimes they are also called Bingham's Plastics:

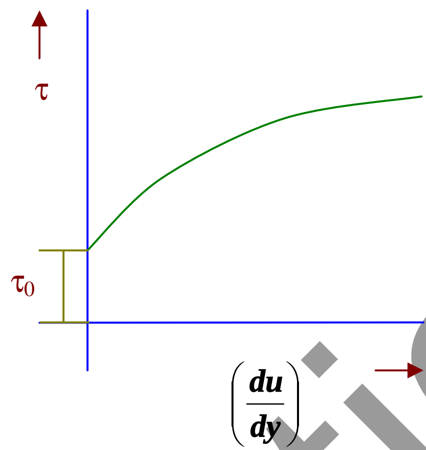
Eg: Industrial sludge.



4. **Thixotropic fluids:**

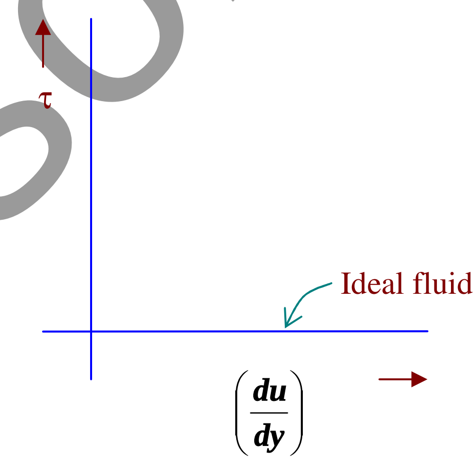
These require certain amount of yield stress to initiate shear strain. After wards stress-strain relationship will be non – linear.

Eg; Printers ink.



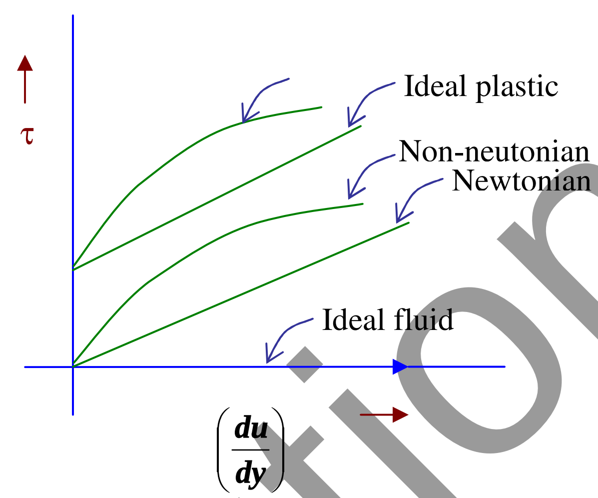
5. **Ideal fluid:**

Any fluid for which viscosity is assumed to be zero is called Ideal fluid. For ideal fluid $\tau = 0$ for all values of $\frac{du}{dy}$

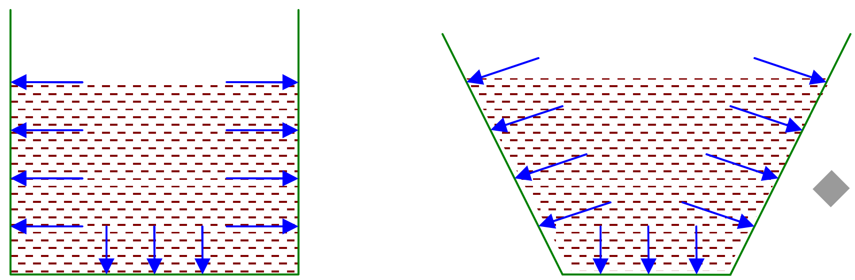


6. **Real fluid :**

Any fluid which posses certain viscosity is called real fluid. It can be Newtonian or non – Newtonian, thixotropic or ideal plastic.



Unit-II: PRESSURE AND ITS MEASUREMENTS



Fluid is a state of matter which exhibits the property of flow. When a certain mass of fluids is held in static equilibrium by confining it within solid boundaries, it exerts force along direction perpendicular to the boundary in contact. This force is called fluid pressure.

- **Pressure distribution:**

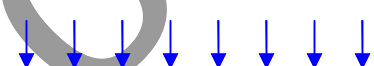
It is the variation of pressure over the boundary in contact with the fluid.

There are two types of pressure distribution.

a) Uniform Pressure distribution.

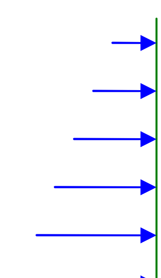
b) Non-Uniform Pressure distribution.

(a) Uniform Pressure distribution:



If the force exerted by the fluid is same at all the points of contact boundary then the pressure distribution is said to be uniform.

(b) Non –Uniform Pressure distribution:



If the force exerted by the fluid is not same at all the points then the pressure distribution is said to be non-uniform.

- **Intensity of pressure or unit pressure or Pressure:**

Intensity of pressure at a point is defined as the force exerted over unit area considered around that point. If the pressure distribution is uniform then intensity of pressure will be same at all the points.

- **Calculation of Intensity of Pressure:**

When the pressure distribution is uniform, intensity of pressure at any points is given by the ratio of total force to the total area of the boundary in contact.

$\therefore$ Intensity of Pressure 'p' = $\frac{\boldsymbol{F}}{\boldsymbol{A}}$

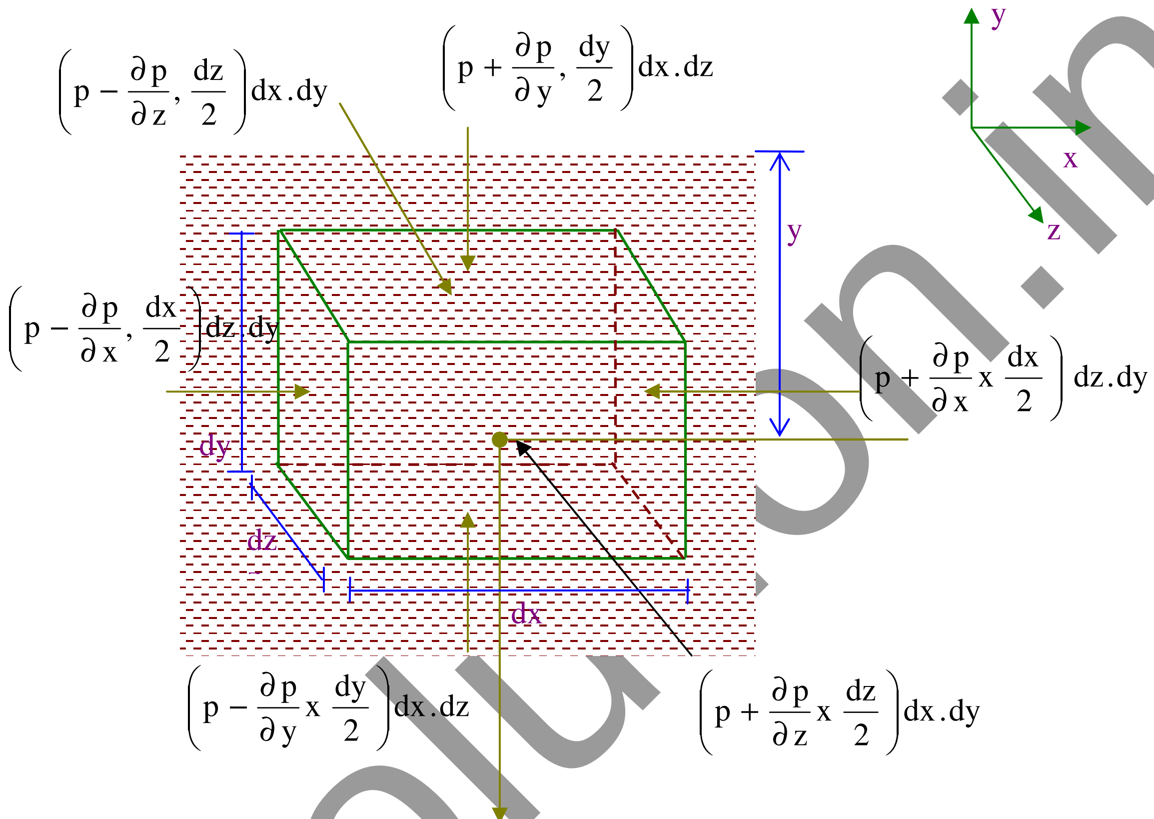
When the pressure distribution is non- uniform, then intensity of pressure at a point is given by $\frac{dF}{dA}$.

- **Unit of Intensity of Pressure:**

N/m^2 or pascal (Pa).

Note: 1 MPa =1N/mm^2

- **To study the variation of intensity of pressure in a static mass of fluid: or derive hydrostatic law of pressure.**



Self weight = γ x volume = γ x dx dy dz

Let us consider a point 'M' at a depth 'y' below the free surface of the liquid of specific weight 'γ'. (dx dy dz) is the elemental volume of the fluid considered around the point 'M'.

Fig shows forces acting on the element including self weight. The element of fluid is in equilibrium and hence we can apply conditions of equilibrium.

$\sum Fx=0$

$$+\left[p-\frac{\partial p}{\partial x}.\frac{dx}{2}\right]dy\ \ dz-\left[P+\frac{\partial p}{\partial x}.\frac{dx}{2}\right]dy\ dz=0$$

$$i,e\ \ p-\frac{\partial p}{\partial x}.\frac{dx}{2}-p-\frac{\partial p}{\partial x}\ .\ \frac{dx}{2}=0$$

$$-2.\frac{\partial P}{\partial x}.\frac{dx}{2}=0$$

$$\therefore\frac{\partial P}{\partial x}=0$$

$\therefore$ Rate of change of intensity of pressure along x – direction is zero. In other words there is no change in intensity of pressure along x – direction inside the fluid.

$\sum Fz=0$

$$+\left[p-\frac{\partial p}{\partial z}.\frac{dz}{2}\right]dx\ \ dy-\left[p+\frac{\partial p}{\partial z}.\frac{dz}{2}=0\right]dx\ \ dy=0$$

$$\therefore\frac{\partial p}{\partial z}=0$$

$\therefore$ The Rate of change of intensity of pressure along z direction is zero.

In other words there is no change in intensity of pressure along z – direction.

$$\sum Fy=0$$

$$+\left[p-\frac{\partial p}{\partial y}.\frac{dy}{2}\right]dx\ dz-\left[p+\frac{\partial p}{\partial y}.\frac{dy}{2}\right]dx\ dz-\gamma dx\ dy\, dz=0$$

$$\text{i.e. } p-\frac{\partial p}{\partial y}.\frac{dy}{2}-p-\frac{\partial P}{\partial y}.\frac{dy}{2}=\gamma dy$$

$$\text{i,e}-\frac{\partial p}{\partial y}dy=\gamma dy$$

$$\therefore \frac{\partial p}{\partial y}=-\gamma$$

-ve sign indicates that the pressure increases in the downward direction i.e., as the depth below the surface increases intensity of pressure increases.

$$\therefore \frac{\partial p}{\partial y}=\gamma$$

$$\therefore \partial p=\gamma\ .\ oy$$

integrating,

$$p=\gamma y+C$$

at $y = 0$; $p = p_{Atmospheric}$

$p_{atm} = \gamma x0 + C$

$\therefore\ C = p_{atm}$

$\therefore\ p = \gamma y + p_{atm}$

The above equation is called hydrostatic law of pressure.

- **Atmospheric pressure**

Air above the surface of liquids exerts pressure on the exposed surface of the liquid and normal to the surface.

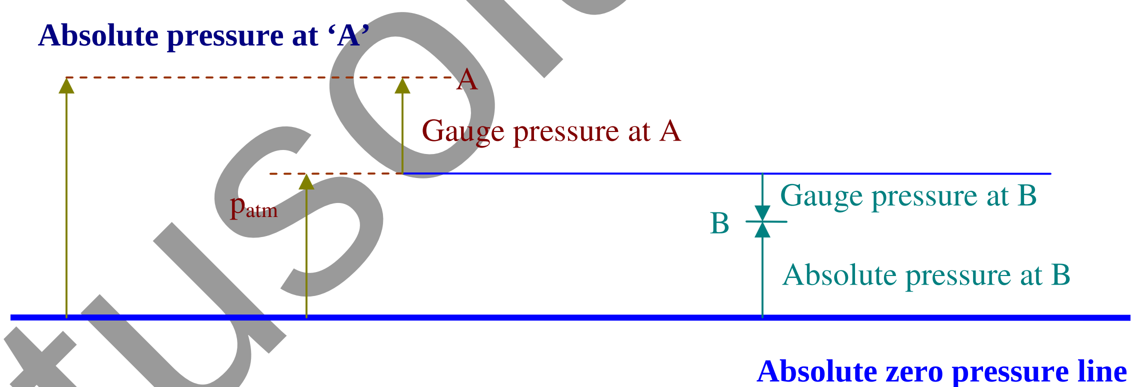
This pressure exerted by the atmosphere is called atmospheric pressure. Atmospheric pressure at a place depends on the elevation of the place and the temperature.

Atmospheric pressure is measured using an instrument called 'Barometer' and hence atmospheric pressure is also called Barometric pressure.

Unit: kPa .

'bar' is also a unit of atmospheric pressure 1bar = 100 kPa.

- **Absolute pressure and Gauge Pressure:**



Absolute pressure at a point is the intensity of pressure at that point measured with reference to absolute vacuum or absolute zero pressure.

Absolute pressure at a point can never be negative since there can be no pressure less than absolute zero pressure.

If the intensity of pressure at a point is measured with reference to atmospheric pressure, then it is called gauge pressure at that point.

Gauge pressure at a point may be more than the atmospheric pressure or less than the atmospheric pressure. Accordingly gauge pressure at the point may be positive or negative.

Negative gauge pressure is also called vacuum pressure.

From the figure, It is evident that, Absolute pressure at a point = Atmospheric pressure ± Gauge pressure.

NOTE: If we measure absolute pressure at a Point below the free surface of the liquid, then,

$$p = \gamma . Y + p_{atm}$$

If gauge pressure at a point is required, then atmospheric pressure is taken as zero, then,

$$p = \gamma . Y$$

♦ Pressure Head

It is the depth below the free surface of liquid at which the required pressure intensity is available.

$$P = \gamma h$$

$$\therefore \boldsymbol{h} = \frac{\boldsymbol{P}}{\gamma}$$

For a given pressure intensity 'h' will be different for different liquids since, 'γ' will be different for different liquids.

$\therefore$ Whenever pressure head is given, liquid or the property of liquid like specify gravity, specific weight, mass density should be given.

Eg:

(i) 3m of water

(ii) 10m of oil of S = 0.8.

(iii) 3m of liquid of $\gamma = 15$ kN/m^3

(iv) 760mm of Mercury.

(v) 10m → not correct.

NOTE:

1. To convert head of a liquid to head of another liquid.

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$S_1 = \frac{\gamma_1}{\gamma_{Standard}}$$

$$p = \gamma_1 h_1$$

$$\therefore \gamma_1 = S_1\, \gamma_{Standard}$$

$$p = \gamma_2 h_2$$

$$\gamma_{21} = S_2\, \gamma_{Standard}$$

$$\boxed{\gamma_1 h_1 = \gamma_2 h_2}$$

$$\therefore S_1\ \gamma_{Standard}\ h_1 = S_2\ \gamma_{Standard}\ h_2$$

$$\boxed{S_1 h_1 = S_2 h_2}$$

2. $S_{water} \times h_{water} = S_{liquid} \times h_{liquid}$

$1 \times h_{water} = S_{liquid} \times h_{liquid}$

$h_{water} = S_{liquid} \times h_{liquid}$

Pressure head in meters of water is given by the product of pressure head in meters of liquid and specific gravity of the liquid.

Eg: 10meters of oil of specific gravity 0.8 is equal to 10x0.8 = 8 meters of water.

Eg: Atmospheric pressure is 760mm of Mercury.

NOTE:

$P = \gamma \; h$

↓ ↓ ↓

kPa $\frac{kN}{m^3}$ m

Problem:

1. Calculate intensity of pressure due to a column of 0.3m of (a) water (b) Mercury (c) Oil of specific gravity-0.8.

a) h = 0.3m of water

$\gamma = 9.81 \frac{kN}{m^3}$

p = ?

$p = \gamma h$

p = 2.943 kPa

c) h = 0.3 of Hg

$\gamma = 13.6 \times 9.81$

$\gamma = 133.416 \text{ kN/m}^3$

$p = \gamma\ h$

$= 133.416 \times 0.3$

$p = 40.025 \text{ kPa}$

2. Intensity of pressure required at a points is 40kPa. Find corresponding head in (a) water (b) Mercury (c) oil of specific gravity-0.9.

(a) $p = 40 \text{ kPa}$

$\gamma = 9.81 \frac{kN}{m^3}$

$h = ?$

$h = \frac{p}{\gamma}$

$h = 4.077 \text{ m of water}$

(b) $p = 40 \text{ kPa}$

$\gamma = (13.6 \text{x} 9.81 \text{ N/m}^3$

$\gamma = 133.416 \frac{KN}{m^3}$

$h = \frac{p}{\gamma}$

$h = 0.299 \text{ m of Mercury}$

(c) $p = 40 \text{ kPa}$

$\gamma = 0.9 \times 9.81$

$\gamma = 8.829 \frac{KN}{m^3}$

$h = \frac{p}{\gamma}$

$h = 4.53 \text{ m of oil } S = 0.9$

3. Standard atmospheric pressure is 101.3 kPa Find the pressure head in (i) Meters of water (ii) mm of mercury (iii) m of oil of specific gravity 0.8.

(i) $p = \gamma h$

$101.3 = 9.81 \times h$

$h = 10.3$ m of water

(ii) $p = \gamma h$

$101.3 = (13.6 \times 9.81) \times h$

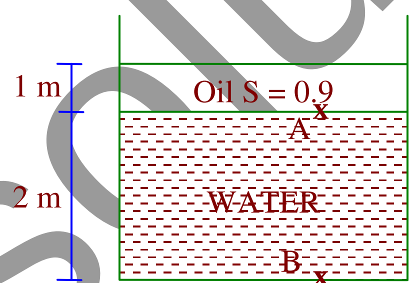
$h = 0.76$ m of mercury

(iii) $p = \gamma h$

$101.3 = (0.8 \times 9.81 \times h$

$h = 12.9$ m of oil of $S = 0.8$

4. An open container has water to a depth of 2m and above this an oil of S = 0.9 for a depth of 1m. Find the intensity of pressure at the interface of two liquids and at the bottom of the tank.



$$p_A = \gamma_{a1}\ h_{oil}$$

$$= (0.9\ x\ 9.81)\ x\ 1$$

$$p_A = 8.829\ kPa$$

$$p_B = \gamma_{oil}\ xh_{oil} + \gamma_{water} + h_{water}$$

$$p_A = 8.829\,kPa + 9.81\ x\ 2$$

$$p_B = 28.45\,kPa$$

5. Convert the following absolute pressure to gauge pressure (a) 120kPa (b) 3kPa (c) 15m of H_2o (d) 800mm of Hg.

(a) $p_{abs} = p_{atm} + p_{gauge}$

$\therefore\ p_{gauge} = p_{abs} - p_{atm} = 120 - 101.3 = 18.7$ kPa

(b) $p_{gauge} = 3\text{-}101.3 = \text{-}98.3$ kPa

$p_{gauge} = 98.3$ kPa (vacuum)

(c) $h_{abs} = h_{atm} + h_{gauge}$

$15 = 10.3 + h_{gauge}$

$h_{gauge} = 4.7$m of water

(d) $h_{abs} = h_{atm} + h_{gauge}$

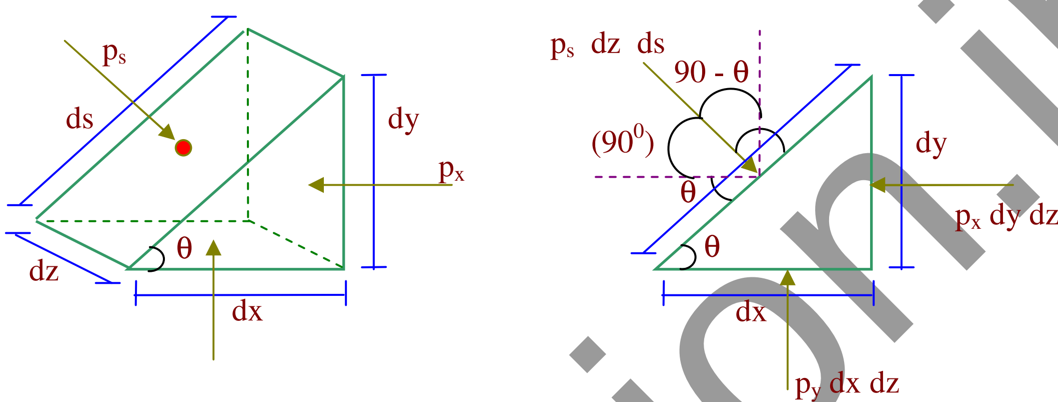
$800 = 760 + h_{gauge}$

$h_{gauge} = 40$ mm of mercury

♦ PASCAL 'S LAW

Statement: Intensity of pressure at a point in a static mass of fluid is same along the directions.

Proof:



Let us consider three planes around a point as shown in figure. Figure shows intensity of pressure and force along different directions. The system of forces should be in equilibrium.

$\therefore \sum Fx = 0$

$- p_x\, dy.dz + p_s\, ds\, dz\ \cos(90^0\ ) = 0$

$p_s\, ds\, \sin\theta = p_x\, dy$

$p_s\, dy\ = p_x\, dy$

$p_s = \ p_x$

$\therefore \sum Fy=0$

$- p_s\, ds.dz \cos\theta + p_y\, dx\, dz\ = 0$

$p_y\, dx = p_s\, ds \cos\theta$

$p_y\, dx\ = p_s\, dx$

$p_y = p_s$

$\therefore\ p_x = p_y = p_z$

$\therefore$ Intensity of pressure at a point is same along all the directions.

- **Measurement of Pressure**

Various devices used to measure fluid pressure can be classified into,

1. Manometers
2. Mechanical gauges.

Manometers are the pressure measuring devices which are based on the principal of balancing the column of the liquids whose pressure is to be measured by the same liquid or another liquid.

Mechanical gauges consist of an elastic element which deflects under the action of applied pressure and this movement will operate a pointer on a graduated scale.

Classification of Manometers:

Manometers are broadly classified into

a) Simple Manometers
b) Differential Manometers.

a) Simple Manometers

Simple monometers are used to measure intensity of pressure at a point. They are connected to the point at which the intensity of pressure is required. Such a point is called gauge point.

b) Differential Manometers

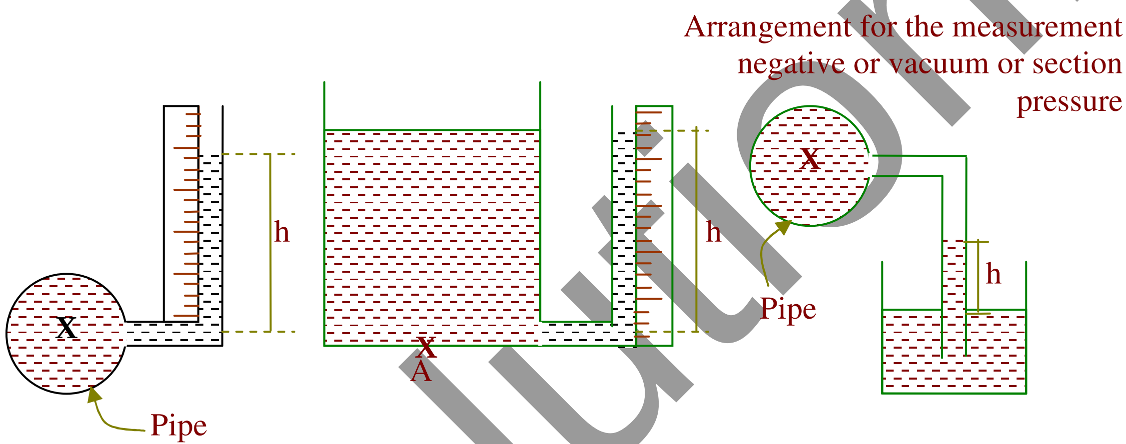
Differential manometers are used to measure the pressure difference between two points. They are connected to the two points between which the intensity of pressure is required.

♦ Types of Simple Manometers

Common types of simple manometers are

a) Piezometers
b) U-tube manometers
c) Single tube manometers
d) Inclined tube manometers

a) Piezometers



Piezometer consists of a glass tube inserted in the wall of the vessel or pipe at the level of point at which the intensity of pressure is to be measured. The other end of the piezometer is exposed to air. The height of the liquid in the piezometer gives the pressure head from which the intensity of pressure can be calculated.

To minimize capillary rise effects the diameters of the tube is kept more than 12mm.

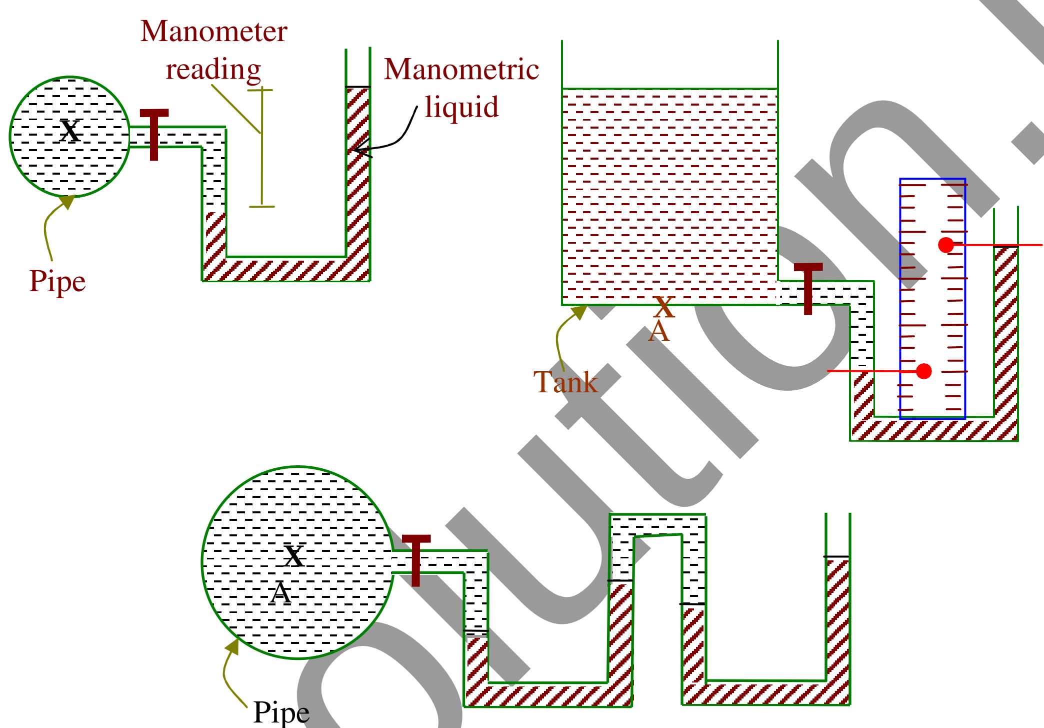
Merits

- Simple in construction
- Economical

Demerits

- Not suitable for high pressure intensity.
- Pressure of gases cannot be measured.

(b) U-tube Manometers:



A U-tube manometers consists of a glass tube bent in U-Shape, one end of which is connected to gauge point and the other end is exposed to atmosphere. U-tube consists of a liquid of specific of gravity other than that of fluid whose pressure intensity is to be measured and is called monometric liquid.

- **Manometric liquids**
  - Manometric liquids should neither mix nor have any chemical reaction with the fluid whose pressure intensity is to be measured.
  - It should not undergo any thermal variation.
  - Manometric liquid should have very low vapour pressure.
  - Manometric liquid should have pressure sensitivity depending upon the magnitude of pressure to be measured and accuracy requirement.

- **To write the gauge equation for manometers**

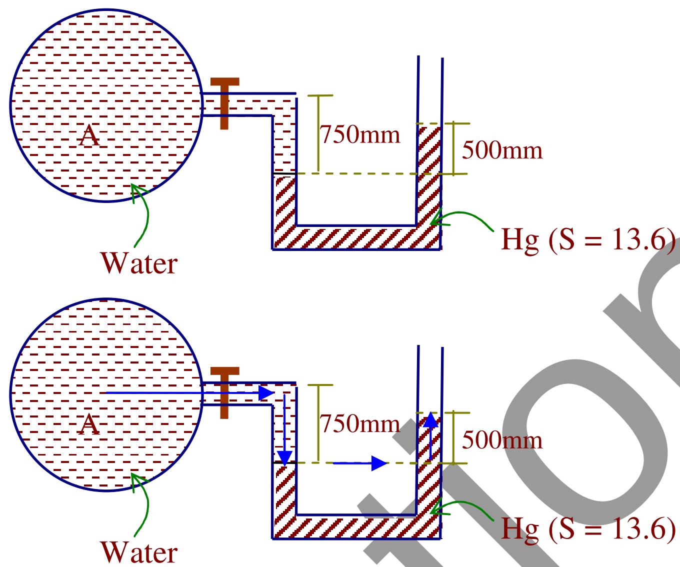
Gauge equations are written for the system to solve for unknown quantities.

Steps:

1. Convert all given pressure to meters of water and assume unknown pressure in meters of waters.
2. Starting from one end move towards the other observing the following points.
   - Any horizontal movement inside the same liquid will not cause change in pressure.
   - Vertically downward movement causes increase in pressure and upward motion causes decrease in pressure.
   - Convert all vertical columns of liquids to meters of water by multiplying them by corresponding specify gravity.
   - Take atmospheric pressure as zero (gauge pressure computation).
3. Solve for the unknown quantity and convert it into the required unit.
4. If required calculate absolute pressure.

Problem:

1. Determine the pressure at A for the U- tube manometer shown in fig. Also calculate the absolute pressure at A in kPa.



Let ‘h_A’ be the pressure head at ‘A’ in ‘meters of water’.

$$h_A + 0.75 - 0.5 x 13.6 = 0$$

$$h_A = 6.05 \, m \ of \ water$$

$$p = \gamma h$$

$$= 9.81 x 6.05$$

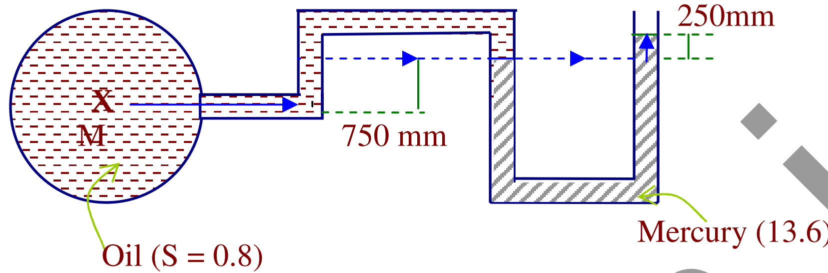
$$p = 59.35 \ kPa (gauge \ pressure)$$

$$p_{abs} = p_{atm} + p_{gauge}$$

$$= 101.3 + 59.35$$

$$p_{abc} = 160.65 \ kPa$$

2. For the arrangement shown in figure, determine gauge and absolute pressure at the point M.



Let 'h_M' be the pressure head at the point 'M' in m of water,

$h_M - 0.75 \times 0.8 - 0.25 \times 13.6 = 0$

$h_M = 4$ m of water

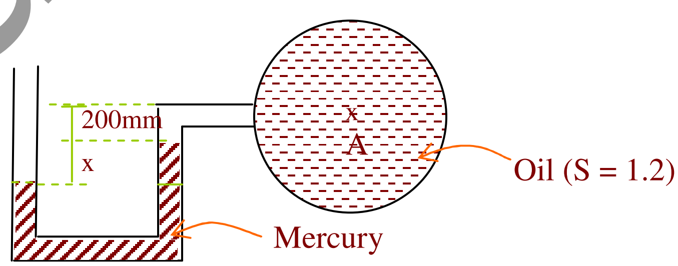
$p = \gamma h$

$p = 39.24 \text{ kPa}$

$p_{abs} = 101.3 + 39.24$

$p_{abs}\ 140.54 \text{ kPa}$

3. If the pressure at 'At' is 10 kPa (Vacuum) what is the value of 'x'?



$p_A = 10$ kPa (Vacuum)

$p_A = -10$ kPa

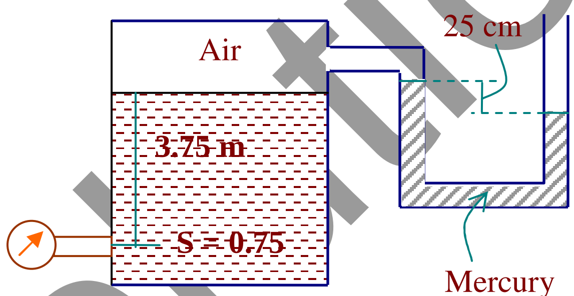
$$\frac{p_A}{\gamma} = \frac{-10}{9.81} = -1.019 \text{ m of water}$$

$$h_A = -1.019 \text{ m of water}$$

$$-1.019 + 0.2 \times 1.2 + x(13.6) = 0$$

$$x = 0.0572 \text{ m}$$

4. The tank in the accompanying figure consists of oil of S = 0.75. Determine the pressure gauge reading in $\frac{kN}{m^2}$.



Let the pressure gauge reading be 'h' m of water

h – 3.75 x 0.75 + 0.25 x 13.6 = 0

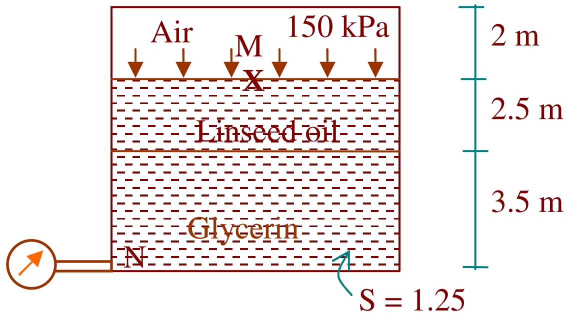
h = - 0.5875 m of water

$p = \gamma h$

p = -5.763 kPa

p = 5.763 kPa (Vacuum)

5. A closed tank is 8m high. It is filled with Glycerine up to a depth of 3.5m and linseed oil to another 2.5m. The remaining space is filled with air under a pressure of 150 kPa. If a pressure gauge is fixed at the bottom of the tank what will be its reading. Also calculate absolute pressure. Take relative density of Glycerine and Linseed oil as 1.25 and 0.93 respectively.



$$P_H = 150\,\text{kPa}$$

$$h_M = \frac{150}{9.81}$$

$$h_M = 15.29\,\text{m of water}$$

Let 'h_N' be the pressure gauge reading in m of water.

h_N -3.5 x 1.25 -2.5 x 0.93 =15.29

h_N = 21.99 m of water

p = 9.81 x 21.99

p = 215.72 kPa (gauge)

p_{abs} = 317.02 kPa

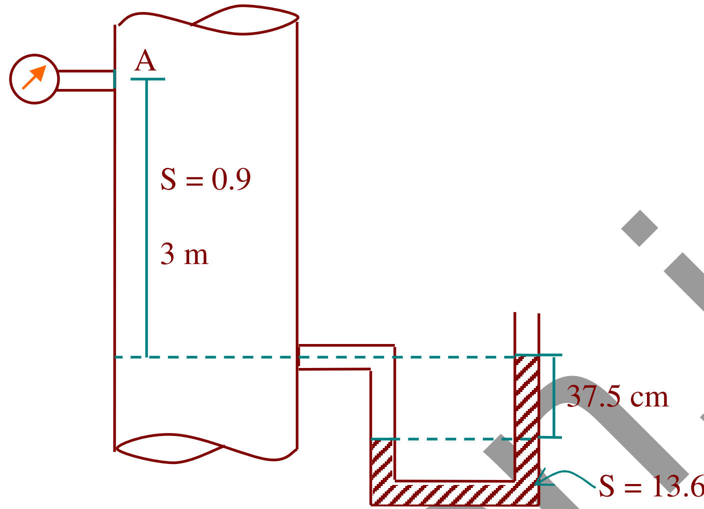
6. A vertical pipe line attached with a gauge and a manometer contains oil and Mercury as shown in figure. The manometer is opened to atmosphere. What is the gauge reading at 'A'? Assume no flow in the pipe.

h_A-3 x 0.9 + 0.375 x 0.9 - 0.375 x 13.6 = 0

h_A = 2.0625 m of water

p = γ x h

= 9.81 x 21.99



p = 20.23 kPa (gauge)

p_{abs} = 101.3 +20.23

p_{abs} = 121.53 kPa

- **DIFFERENTIAL MANOMETERS**

Differential manometers are used to measure pressure difference between any two points. Common varieties of differential manometers are:

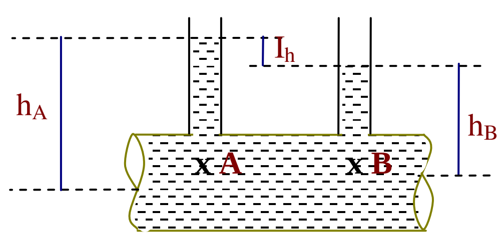
(a) Two piezometers.

(b) Inverted U-tube manometer.

(c) U-tube differential manometers.

(d) Micromanometers.

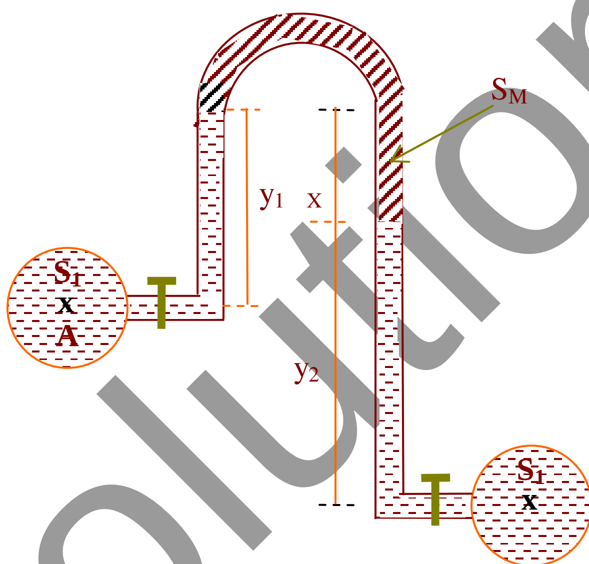
(a) Two Pizometers



The arrangement consists of two pizometers at the two points between which the pressure difference is required. The liquid will rise in both the piezometers. The difference in elevation of liquid levels can be recorded and the pressure difference can be calculated.

It has all the merits and demerits of piezometer.

(b) Inverted U-tube manometers



Inverted U-tube manometer is used to measure small difference in pressure between any two points. It consists of an inverted U-tube connecting the two points between which the pressure difference is required. In between there will be a lighter manometric liquid. Pressure difference between the two points can be calculated by writing the gauge equations for the system.

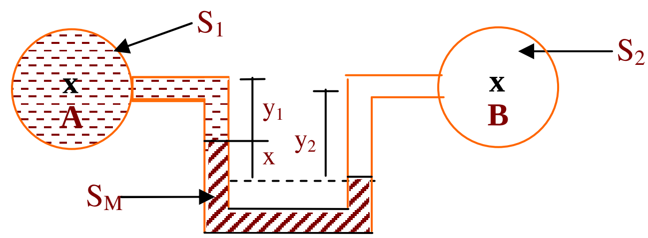
Let 'h_A' and 'h_B' be the pr head at 'A' and 'B' in meters of water

$h_A - (Y_1\ S_1) + (x\ S_M) + (y_2\ S_2) = h_B.$

$h_A - h_B = S_1\ y_1 - S_M\ x - S_2\ y_2,$

$p_A - p_B = \gamma\,(h_A - h_B)$

(c) U-tube Differential manometers



A differential U-tube manometer is used to measure pressure difference between any two points. It consists of a U-tube containing heavier manometric liquid, the two limbs of which are connected to the gauge points between which the pressure difference is required. U-tube differential manometers can also be used for gases. By writing the gauge equation for the system pressure difference can be determined.

Let 'h_A' and 'h_B' be the pressure head of 'A' and 'B' in meters of water

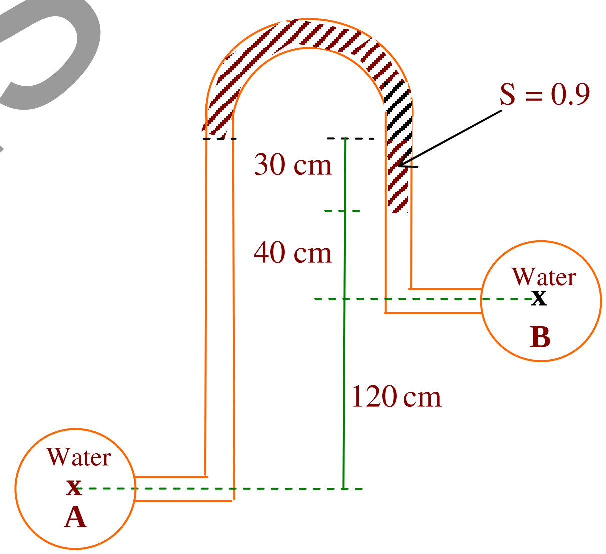
$h_A + S_1 Y_1 + x S_M - Y_2 S_2 = h_B$

$h_A - h_B = Y_2 S_2 - Y_1 S_1 - x S_M$

Problems

(1) An inverted U-tube manometer is shown in figure. Determine the pressure difference between A and B in N/M^2.

Let h_A and h_B be the pressure heads at A and B in meters of water.



$h_A - (190 \times 10^{-2}) + (0.3 \times 0.9) + (0.4)\ 0.9 = h_B$

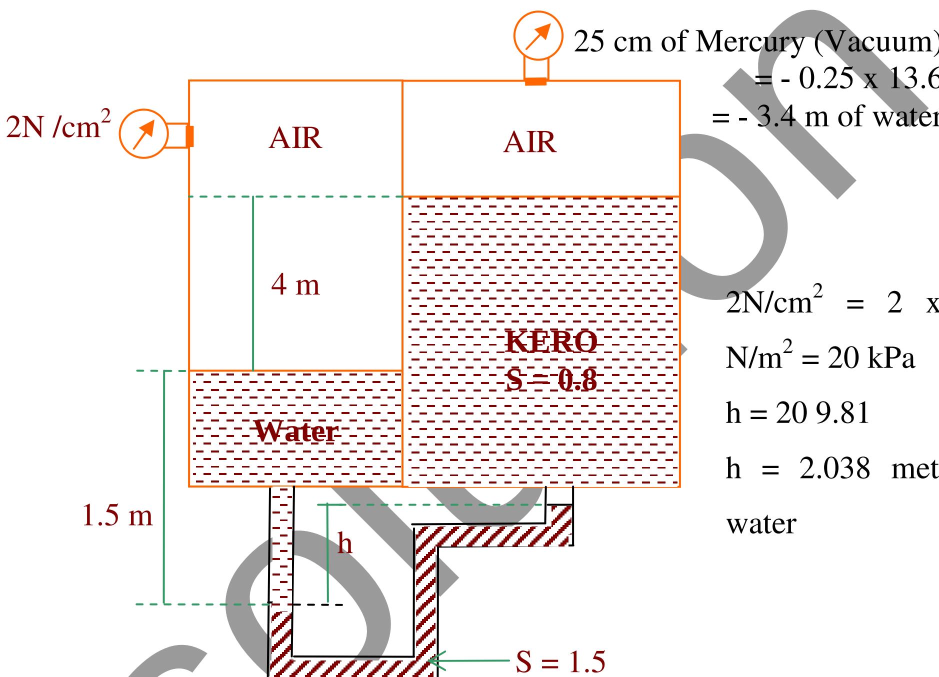
$h_A - h_B = 1.23$ meters of water

$p_A - p_B = \gamma\,(h_A - h_B) = 9.81 \times 1.23$

$p_A - p_B = 12.06$ kPa

$p_A - p_B = 12.06 \times 10^3\ N/m^2$

2. In the arrangements shown in figure. Determine the ho 'h'.



$2N/cm^2 = 2 \times 100^2\ N/m^2 = 20$ kPa

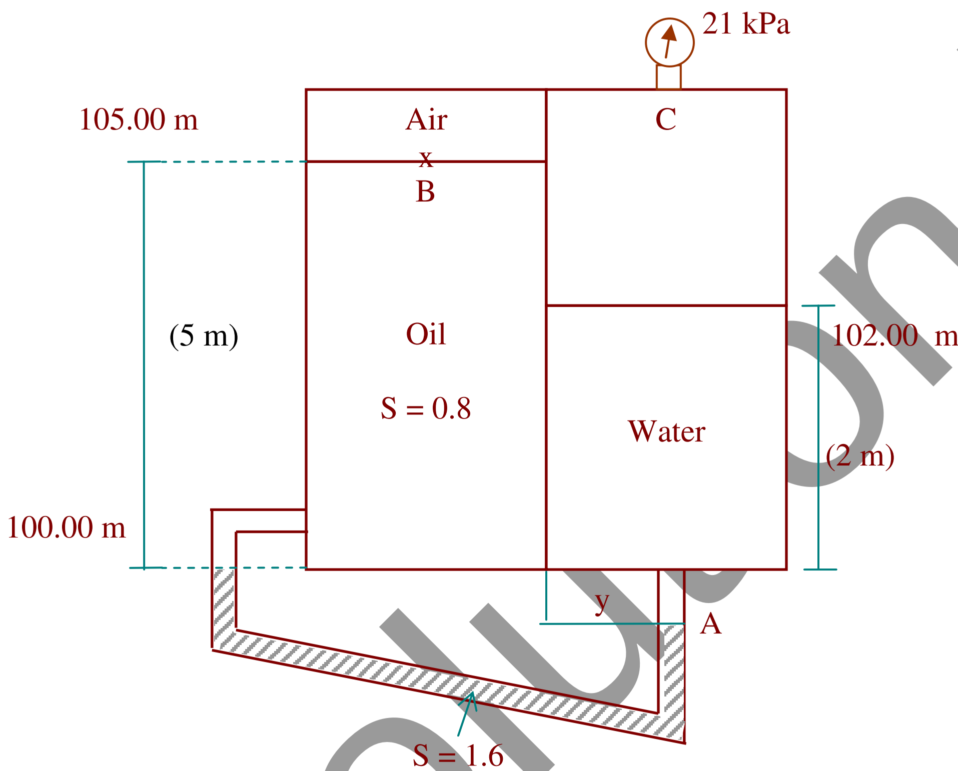
h = 20 9.81

h = 2.038 meters of water

$2.038 + 1.5 - (4 + 1.5 - h)\ 0.8 = -3.4$

h = 3.6 m

3. In the figure given, the air pressure in the left tank is 230 mm of Mercury (Vacuum). Determine the elevation of gauge liquid in the right limb at A. If the liquid in the right tank is water.



$$h_c = \frac{Pc}{\gamma}$$

$$\frac{21}{9.81}$$

$$h_c = 2.14 \, m \text{ of water}$$

h_B = 230mm of Hg

= 0.23 x 13.6

h_B = - 3.128 m of water

– 3.128 + 5 x 0.8 + y x 1.6 – (y + 2) = 2.14

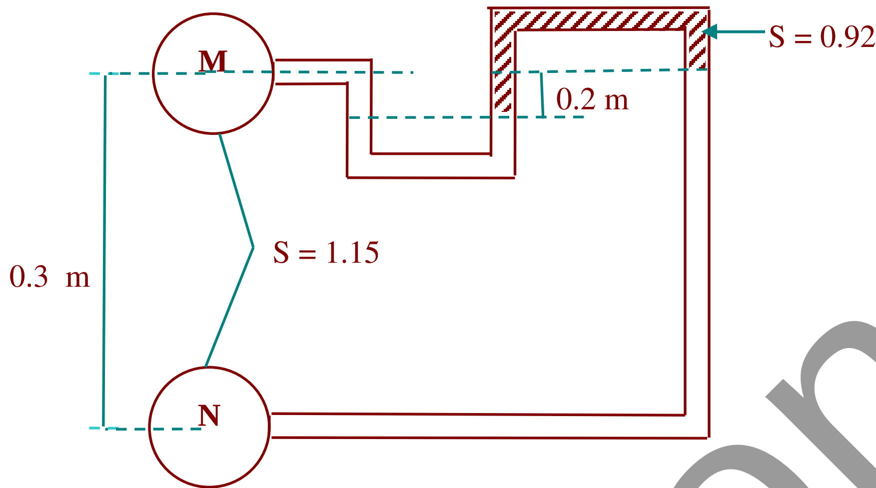
– 3.128 + 5 x 0.8 + y x 1.6 – y – 2 = 2.14

y = 5.446 m

∴ Elevation of A = 100 – 5.446

Elevation of A = 94.553m

4. Compute the pressure different between 'M' and 'N' for the system shown in figure.



Let 'h_M' and ' h_N' be the pressure heads at M and N in m of water.

hm + y x 1.15 – 0.2 x 0.92 + (0.3 – y + 0.2) 1.15 = hn

hm + 1.15 y – 0.184 + 0.3 x 1.15 – 1.15 y + 0.2 x 1.15 = hn

hm + 0.391 = hn

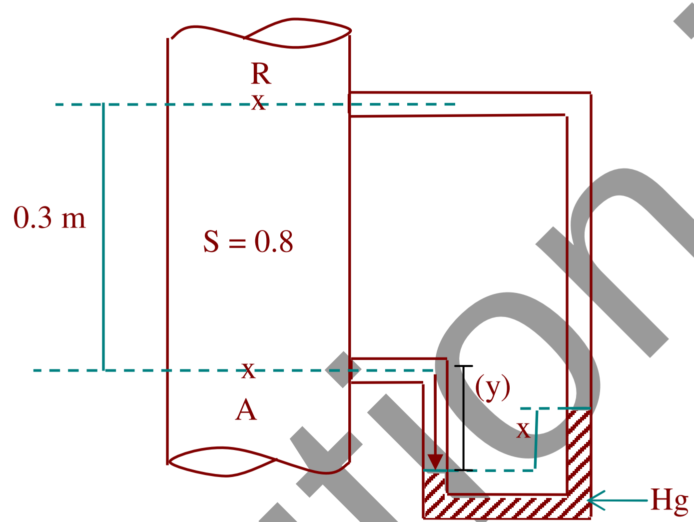
hn – hm = 0.391meters of water

$p_n - p_m = \gamma (h_N - h_m)$

$= 9.81 \times 0.391$

$p_n - p_m = 3.835$ kPa

5. Petrol of specify gravity 0.8 flows up through a vertical pipe. A and B are the two points in the pipe, B being 0.3 m higher than A. Connection are led from A and B to a U–tube containing Mercury. If the pressure difference between A and B is 18 kPa, find the reading of manometer.



$p_A - p_B = 18 kPa$

$$\frac{P_A - P_B}{\gamma}$$

$$h_A - h_B = \frac{18}{9.81}$$

$$h_A - h_B = 1.835 m \text{ of water}$$

$h_A + y \times 0.8 - x\ 13.6 - (0.3 + y - x)\ 0.8 = h_B$

$h_A - h_B = -0.8y + 13.66\ x + 0.24 + 0.8\ y - 0.8\ x$

$$h_A - h_B = 12.8x + 0.24$$

$1.835 = 12.8x + 0.24$

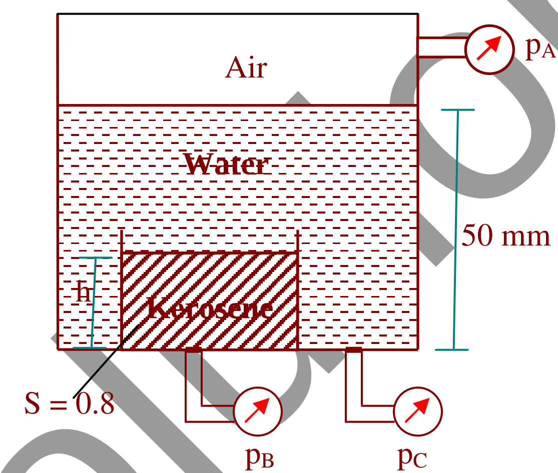
$x = 0.1246$ m

6. A cylindrical tank contains water to a height of 50mm. Inside is a small open cylindrical tank containing kerosene at a height specify gravity 0.8. The following pressures are known from indicated gauges.

p_B = 13.8 kPa (gauge)

p_C = 13.82 kPa (gauge)

Determine the gauge pressure p_A and height h. Assume that kerosene is prevented from moving to the top of the tank.



p_C = 13.82 kPa

h_C = 1.409 m of water

p_B = 13.8 kPa

h_B = 1.407 meters of water

$1.409 – 0.05 = h_A \quad \therefore \; h_A = 1.359$ meters of water

$\therefore \; p_A = 1.359 \times 9.81$

$\therefore \; p_A = 13.33$ kPa

$h_B – h \times 0.8 – (0.05 – h) = h_A$

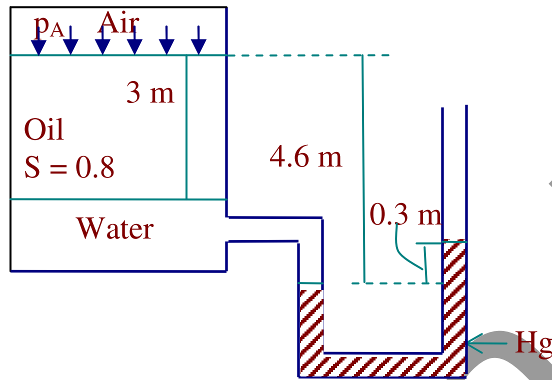
$1.407 – 0.8\,h – 0.05 + h = 1.359$

$0.2\,h = 1.359 – 1.407 + 0.05$

$0.2\,h = 0.002$

$h = 0.02$ m

7. What is the pressure p_A in the fig given below? Take specific gravity of oil as 0.8.



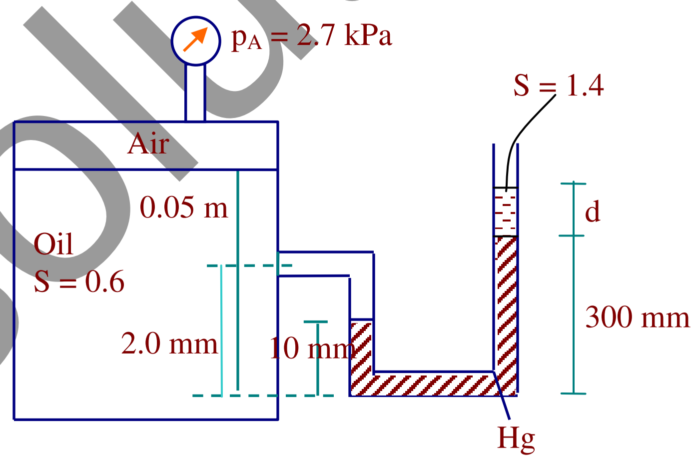
$h_A + (3 \times 0.8) + (4.6 - 0.3)(13.6) = 0$

$h_A = 2.24$ m of oil

$p_A = 9.81 \times 2.24$

$p_A = 21.97$ kPa

8. Find 'd' in the system shown in fig. If $p_A = 2.7$ kPa



$$h_A = \frac{p_A}{\gamma} = \frac{2.7}{9.81}$$

$$h_A = 0.2752 \, \textbf{\textit{m of water}}$$

$$h_A + (0.05 \, \boldsymbol{x} 0.6) + (0.05 + 0.02 - 0.01)0.6$$

$$+ (0.01 \, \boldsymbol{x} 13.6) - (0.03 \, \boldsymbol{x} 13.6) - \boldsymbol{d} \, \boldsymbol{x} 1.4) = 0$$

$$0.0692 - 1.4\boldsymbol{d} = 0$$

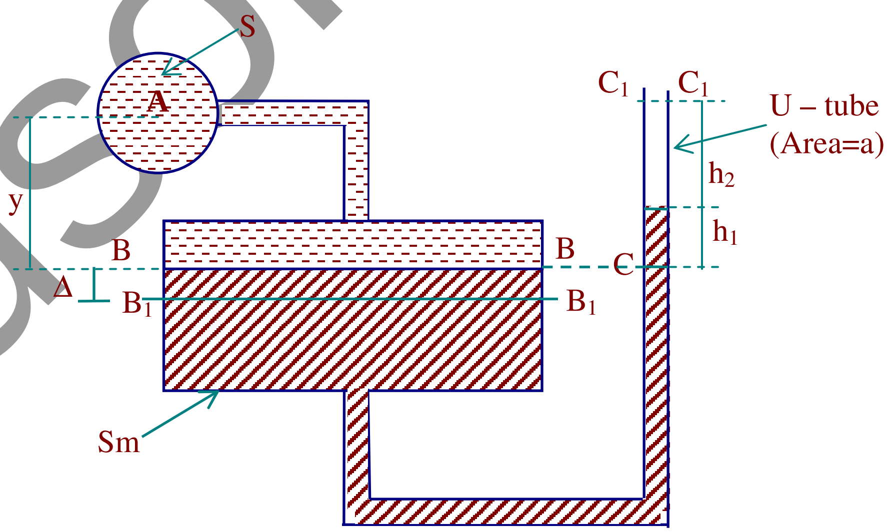
$$\boldsymbol{d} = 0.0494 \, \boldsymbol{m}$$

or

$$\boldsymbol{d} = 49.4 \, \boldsymbol{mm}$$

SINGLE COLUMN MANOMETER:

Single column manometer is used to measure small pressure intensities.



A single column manometer consists of a shallow reservoir having large cross sectional area when compared to cross sectional area of U – tube connected to it. For any change in pressure, change in the level of manometeric liquid in the reservoir is small and change in level of manometric liquid in the U- tube is large.

To derive expression for pressure head at A:

BB and CC are the levels of manometric liquid in the reservoir and U-tube before connecting the point A to the manometer, writing gauge equation for the system we have,

$+ y \times S - h_1 \times S_m = 0$

$\therefore Sy = S_m h_1$

Let the point A be connected to the manometer. B_1B_1 and C_1 C_1 are the levels of manometeric liquid. Volume of liquid between BBB_1B_1 = Volume of liquid between CCC_1C_1

$$A\Delta = a\, h_2$$

$$\Delta = \frac{ah_2}{A}$$

Let 'h_A' be the pressure head at A in m of water.

$h_A + (y + \Delta)\, S - (\Delta + h_1 + h_2)\, Sm = 0$

$h_A = (\Delta + h_1 + h_2)\, Sm - (y + \Delta)\, S$

$= \Delta\, Sm + \underline{h_1\, Sm} + h_2\, Sm - \underline{yS} - \Delta S$

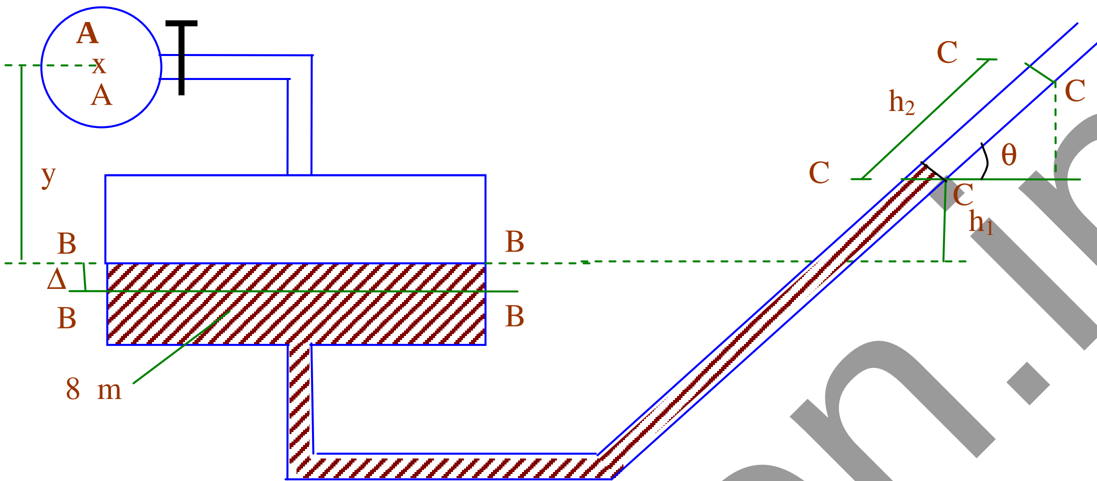
$h_A = \Delta\, (Sm - S) + h_2\, Sm$

$h_A = \frac{ah_2}{A}\, (Sm - S) + h_2\, Sm$

$\therefore$ It is enough if we take one reading to get 'h_2' If '$\frac{a}{A}$' is made very small (by increasing 'A') then the I term on the RHS will be negligible.

Then $h_A = h_2\, Sm$

INCLINED TUBE SINGLE COLUMN MANOMETER:



Inclined tube SCM is used to measure small intensity pressure. It consists of a large reservoir to which an inclined U – tube is connected as shown in fig. For small changes in pressure the reading ‘h_2’ in the inclined tube is more than that of SCM. Knowing the inclination of the tube the pressure intensity at the gauge point can be determined.

$$h_A = \frac{a}{A} h_2 \sin\theta (Sm - S) + h_2 \sin\theta . Sm$$

If ‘$\frac{a}{A}$’ is very small then $h_A = (h_2 = \text{Sin}\theta)$ Sm.

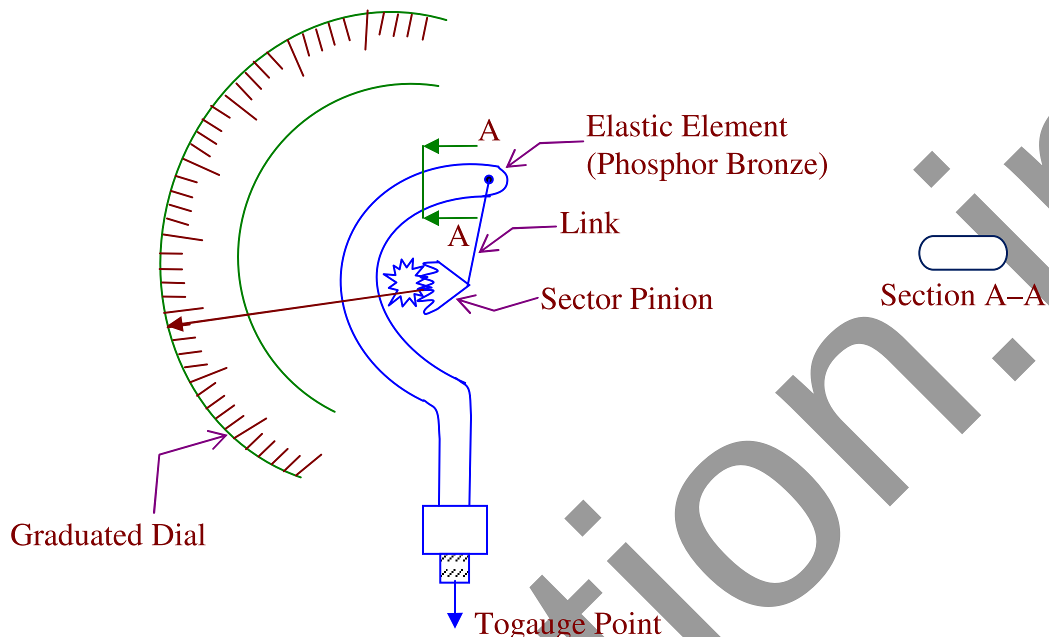
MECHANICAL GAUGES:

Pressure gauges are the devices used to measure pressure at a point.

They are used to measure high intensity pressures where accuracy requirement is less.

Pressure gauges are separate for positive pressure measurement and negative pressure measurement. Negative pressure gauges are called Vacuum gauges.

BASIC PRINCIPLE:



Mechanical gauge consists of an elastic element which deflects under the action of applied pressure and this deflection will move a pointer on a graduated dial leading to the measurement of pressure. Most popular pressure gauge used is Bordon pressure gauge.

The arrangement consists of a pressure responsive element made up of phosphor bronze or special steel having elliptical cross section. The element is curved into a circular arc, one end of the tube is closed and free to move and the other end is connected to gauge point. The changes in pressure cause change in section leading to the movement. The movement is transferred to a needle using sector pinion mechanism. The needle moves over a graduated dial.

DIMENSIONAL ANALYSIS

It is a mathematical technique which makes use of study of dynamics as an art to the solution of engineering problems.

- **Fundamental Dimensions**

All physical quantities are measured by comparison which is made with respect to a fixed value.

Length, Mass and Time are three fixed dimensions which are of importance in fluid mechanics and fluid machinery. In compressible flow problems, temperature is also considered as a fundamental dimensions.

- **Secondary Quantities or Derived Quantities**

Secondary quantities are derived quantities or quantities which can be expressed in terms of two or more fundamental quantities.

- **Dimensional Homogeneity**

In an equation if each and every term or unit has same dimensions, then it is said to have Dimensional Homogeneity.

$$V = u + at$$

$$m/s \quad m/s \quad m/s^2 \cdot s$$

$$LT^{-1} = (LT^{-1}) + (LT^{-2})(T)$$

- **Uses of Dimensional Analysis**

1. It is used to test the dimensional homogeneity of any derived equation.
2. It is used to derive equation.
3. Dimensional analysis helps in planning model tests.

- **Dimensions of quantities**

1.	Length	LM^oT^o
2.	Mass	L^oMT^o
3.	Time	L^oM^oT
4.	Area	$L^2M^oT^o$

5. Volume	$L^{3}M^{o}T^{o}$
6. Velocity	$LM^{o}T^{-1}$
7. Acceleration	$LM^{o}T^{-2}$
8. Momentum	LMT^{-1}
9. Force	LMT^{-2}
10. Moment or Torque	$L^{2}MT^{-2}$
11. Weight	LMT^{-2}
12. Mass density	$L^{-3}MT^{o}$
13. Weight density	$L^{-2}MT^{-2}$
14. Specific gravity	$L^{o}M^{o}T^{o}$
15. Specific volume	$L^{3}M^{-1}T^{o}$
16. Volume flow rate	$L^{3}M^{o}T^{-1}$
17. Mass flow rate	$L^{o}MT^{-1}$
18. Weight flow rate	LMT^{-3}
19. Work done	$L^{2}MT^{-2}$
20. Energy	$L^{2}MT^{-2}$
21. Power	$L^{2}MT^{-3}$
22. Surface tension	$L^{o}MT^{-2}$
23. Dynamic viscosity	$L^{-1}M^{+1}T^{-1}$
24. Kinematic viscosity	$L^{2}M^{o}T^{-1}$
25. Frequency	$L^{o}M^{o}T^{-1}$
26. Pressure	$L^{-1}MT^{-2}$
27. Stress	$L^{-1}MT^{-2}$
28. E, C, K	$L^{-1}MT^{-2}$
29. Compressibility	$LM^{-1}T^{2}$
30. Efficiency	$L^{o}M^{o}T^{o}$
31. Angular velocity	$L^{o}M^{o}T^{-1}$
32. Thrust	LMT^{-2}
33. Energy head (Energy/unit mass)	$L^{2}M^{o}T^{-2}$
34. Energy head (Energy/unit weight)	$LM^{o}T^{o}$

- **Methods of Dimensional Analysis**

  There are two methods of dimensional analysis.

  1. Rayleigh's method
  2. Buckingham's (Π – theorem) method

2. Rayleigh's method

Rayleigh's method of analysis is adopted when number of parameters or variables are less (3 or 4 or 5).

Methodology

X_1 is a function of

$X_2, X_3, X_4, \ldots\ldots, X_n$ then it can be written as

$X_1 = f(X_2, X_3, X_4, \ldots\ldots, X_n)$

$X_1 = K\,(X_2^a \,.\, X_3^b \,.\, X_4^c \,.\, \ldots\ldots)$

Taking dimensions for all the quantities

$$[X_1] = [X_2]^a\,[X_3]^b\,[X_4]^c \ldots\ldots$$

Dimensions for quantities on left hand side as well as on the right hand side are written and using the concept of Dimensional Homogeneity a, b, c …. can be determined.

Then,

$$X_1 = K \cdot X_2^a \cdot X_3^b \cdot X_4^c \cdot \ldots\ldots$$

- **Problems 1:** Velocity of sound in air varies as bulk modulus of elasticity K, Mass density ρ. Derive an expression for velocity in form $C = \sqrt{\frac{K}{\rho}}$

- **Solution:**

$C = f(K, \rho)$

$\mathbf{C = M \cdot K^a \cdot \rho^b}$

M – Constant of proportionality

C – Velocity	– LM^oT^{-1}
K – Bulk modulus	– $L^{-1}MT^{-2}$
ρ - Mass density	– $L^{-3}MT^o$

$[C] = [K]^a \cdot [\rho]^b$

$[LM^oT^{-1}] = [L^{-1}MT^{-2}]^a \ [L^{-3}MT^o]^b$

$[LM^oT^{-1}] = [L^{-a+(-36)} M^{a+b} T^{-2a}]$

$-a - 3b = 1$

$a + b = 0$

$-2a = -1$

$a = \frac{1}{2}$

$\mathbf{b} = -\frac{1}{2}$

$C = MK^{1/2} \rho^{-1/2}$

$C = M\sqrt{\frac{K}{\rho}}$

If, M = 1, $\mathbf{C = \sqrt{\frac{K}{\rho}}}$

- **Problem 2:** Find the equation for the power developed by a pump if it depends on head H discharge Q and specific weight γ of the fluid.

- **Solution**:

$P = f(H, Q, \gamma)$

$P = K \cdot H^a \cdot Q^b \cdot \gamma^c$

$[P] = [H]^a \cdot [Q]^b \cdot [\gamma]^c$

$[L^2MT^{-3}] = [LM^oT^o]^a \cdot [L^3M^0T^{-1}]^b \cdot [L^{-2}MT^{-2}]^c$

Power	$= L^2MT^{-3}$
Head	$= LM^oT^o$
Discharge	$= L^3M^oT^{-1}$
Specific Weight	$= L^{-2}MT^{-2}$

$2 = a + 3b - 2c$

$\mathbf{1 = c}$

$-3 = -b - 2c$

$-3 = -b - 2$

$b = -2 + 3$

$\mathbf{b = 1}$

$2 = a + 3 - 2$

$\mathbf{a = 1}$

$P = K \cdot H^1 \cdot Q^1 \cdot \gamma^1$

$P = K \cdot H \cdot Q \cdot \gamma$

When, $K = 1$

$\mathbf{P = H \cdot Q \cdot \gamma}$

- **Problem 3:** Find an expression for drag force R on a smooth sphere of diameter D moving with uniform velocity V in a fluid of density ρ and dynamic viscosity μ..

- **Solution:**

$R = f(D, V, \rho, \mu)$

$R = K \cdot D^a \cdot V^b \cdot \rho^c, \mu^d$

$[R] = [D]^a \cdot [V]^b \cdot [\rho]^c \cdot [\mu]^d$

$[LMT^{-2}] = [LM^oT^o]^a \cdot [LM^oT^{-1}]^b \cdot [L^{-3}MT^o]^c \cdot [L^{-1}MT^{-1}]^d$

Force	$= LMT^{-2}$
Diameter	$= LM^oT^o$
Velocity	$= LM^oT^{-1}$
Mass density	$= L^3MT^o$
Dynamic Viscosity	$= L^{-1}MT^{-1}$

$c + d = 1$

$\mathbf{c = 1 - d}$

$-b - d = -2$

$\mathbf{b = 2 - d}$

$1 = a + b - 3c - d$

$1 = a + 2 - d - 3(1 - d) - d$

$1 = a + 2 - d - 3 + 3d - d$

$\mathbf{a = 2 - d}$

$R = K \cdot D^{2-d} \cdot V^{2-d} \cdot \rho^{1-d}, \mu^d$

$$R = K\,\frac{D^2}{D^d} \cdot \frac{V^2}{V^d} \cdot \frac{\rho}{\rho^d} \cdot \mu^d$$

$$R = K \cdot \rho V^2 D^2 \left[\frac{\mu}{\rho V D}\right]^d$$

$$R = \rho V^2 D^2 \phi \left[\frac{\mu}{\rho V D}\right]$$

$$R = \rho V^2 D^2 \phi \left[\frac{\rho V D}{\mu}\right]$$

$\mathbf{R = \rho V^2 D^2 \phi [N_{Re}]}$

- **Problem 4:** The efficiency of a fan depends on the density ρ dynamic viscosity μ, angular velocity ω, diameter D, discharge Q. Express efficiency in terms of dimensionless parameters using Rayleigh's Method.

η - $L^oM^oT^o$
ρ - $L^{-3}MT^o$
μ - $L^{-1}MT^{-1}$
ω - $L^oM^oT^{-1}$
D - LM^oT^o
Q - $L^3M^oT^{-1}$

- **Solution:**

$$\eta = f(\rho, \mu, \omega, D, Q)$$

$$\eta = K \cdot \rho^a \cdot \mu^b \cdot \omega^c \cdot D^d \cdot Q^e$$

$$[\eta] = [\rho]^a \cdot [\mu]^b \cdot [\omega]^c \cdot [D]^d \cdot [Q]^e$$

$$[L^oM^oT^o] = [L^{-3}MT^o]^a \cdot [L^{-1}MT^{-1}]^b \cdot [L^oM^oT^{-1}]^c \cdot [LM^oT^o]^d \cdot [L^3M^oT^{-1}]^e$$

$$= [L^{-3a-b+d+3e}]\,[M^{a+b}]\,[T^{-b-c-e}]$$

$$a + b = 0$$

$$\mathbf{a = -\ b}$$

$$-\ b - c - e = 0$$

$$\mathbf{c = -b - e}$$

$$-3a - b + d + 3e = o$$

$$+3b - b + d + 3e = 0$$

$$\mathbf{d = -2b - 3e}$$

$$\therefore \eta = K \cdot \rho^{-b} \cdot \mu^b \cdot \omega^{-b-e} \cdot D^{-2b-3e} \cdot Q^e$$

$$\eta = K \cdot \frac{1}{\rho^b} \cdot \mu^b \frac{1}{\omega^b \cdot \omega^e} \cdot \frac{1}{(D^2)^b \cdot (D^3)^e} Q^2$$

$$\eta = K\left(\frac{\mu}{\rho\omega D^2}\right)^b \cdot \left(\frac{Q}{\omega D^3}\right)^e$$

$$\boldsymbol{\eta = \phi\left[\frac{\mu}{\rho\omega D^2}, \frac{Q}{\omega D^3}\right]}$$

- **Problem 5:** The capillary rise H of a fluid in a tube depends on its specific weight γ and surface tension σ and radius of the tube R prove that $\frac{\mathbf{H}}{\mathbf{R}} = \phi\left[\frac{\boldsymbol{\sigma}}{\boldsymbol{\gamma}\mathbf{R}^2}\right]$.

- **Solution:**

$H = f(\gamma, \sigma, R)$

$H = K \cdot \gamma^a \cdot \sigma^b \cdot R^c$

$[H] = [\gamma]^a \cdot [\sigma]^b \cdot [R]^c$

$[LM^oT^o] = [L^{-2}MT^{-2}]^a \cdot [L^oMT^{-2}]^b \cdot [LM^oT^o]^c$

$[LM^oT^o] = \cdot [L^{-2a+c} \cdot M^{a+b} \cdot T^{-2a-2b}]$

H - LM^oT^o
γ - $L^{-2}MT^{-2}$
σ - L^oMT^{-2}
R – LM^oT^o

$-2a + c = 1$

$a + b = 0$

$\mathbf{-2a - 2b = 0}$

$a = -b$

$c = 1 - 2b$

$H = K \cdot \gamma^{-b} \cdot \sigma^b \cdot R^{1-2b}$

$$H = K \frac{\sigma^b}{\gamma^b} \cdot \frac{R}{(R^2)^b}$$

$$\mathbf{H = K \left[\frac{\sigma}{\gamma R^2}\right]^b}$$

$$\frac{\mathbf{H}}{\mathbf{R}} = \phi\left[\frac{\boldsymbol{\sigma}}{\boldsymbol{\gamma}\mathbf{R}^2}\right].$$

3. Buckingham's Π Method

This method of analysis is used when number of variables are more.

Buckingham's Π Theorem

If there are n – variables in a physical phenomenon and those n-variables contain 'm' dimensions, then the variables can be arranged into (n-m) dimensionless groups called Π terms.

Explanation:

If f (X_1, X_2, X_3, ……… X_n) = 0 and variables can be expressed using m dimensions then.

$$f(\Pi_1, \Pi_2, \Pi_3, \ldots\ldots\ldots \Pi_{n-m}) = 0$$

Where, Π_1, Π_2, Π_3, ……… are dimensionless groups.

Each Π term contains (m + 1) variables out of which m are of repeating type and one is of non-repeating type.

Each Π term being dimensionless, the dimensional homogeneity can be used to get each Π term.

- **Selecting Repeating Variables**
  1. Avoid taking the quantity required as the repeating variable.
  2. Repeating variables put together should not form dimensionless group.
  3. No two repeating variables should have same dimensions.
  4. Repeating variables can be selected from each of the following properties.
     a. Geometric property → Length, height, width, area
     b. Flow property → Velocity, Acceleration, Discharge
     c. Fluid property → Mass density, Viscosity, Surface tension

Problem 1: Find an expression for drag force R on a smooth sphere of diameter D moving with uniform velocity V in a fluid of density ρ and dynamic viscosity μ..

- **Solution:**

$R = LMT^{-2}$
$D = LM^{o}T^{o}$
$V = LT^{-1}$
$\rho = ML^{-3}$
$\mu = L^{-1}MT^{-1}$

$f(R, D, V, \rho, \mu) = 0$

Here, $n = 5, m = 3$

$\therefore$ Number of Π terms = $(n - m) = 5 - 3 = 2$

$\therefore f(\Pi_1, \Pi_2) = 0$

Let D, V, ρ be the repeating variables.

$\Pi_1 = D^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot R$

$[L^oM^oT^o] = [L]^{a_1} [LT^{-1}]^{b_1} [ML^{-3}]^{c_1} [LMT^{-2}]$

$L^oM^oT^o = [L]^{a_1+b_1-3c_1+1} [M]^{c_1+1} [T]^{-b_1-2}$

$- b_1 = 2$

$\mathbf{b_1 = -2}$

$c_1 + 1 = 0$

$\mathbf{c_1 = -1}$

$a_1 + b_1 - 3c_1 + 1 = 0$

$a_1 + 2 + 3 + 1 = 0$

$\mathbf{a_1 = -2}$

$\Pi_1 = D^{-2} \cdot V^{-2} \cdot \rho^{-1} \cdot R$

$\Pi_1 = \dfrac{\mathbf{R}}{\mathbf{D^2V^2\rho}}$

$\Pi_2 = D^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot \mu$

$[L^oM^oT^o] = [L]^{a_2} [LT^{-1}]^{b_2} [ML^{-3}]^{c_2} [L^{-1}MT^{-1}]$

$[L^oM^oT^o] = [L]^{a_2+b_2-3c_2-1} [M]^{c_2+1} [T]^{-b_2-1}$

$-b_2 - 1 = 0$

$\mathbf{b_2 = -1}$

$\mathbf{c_2 = -1}$

$a_2 + b_2 - 3c_2 - 1 = 0$

$a_2 - 1 + 3 - 1 = 0$

$\mathbf{a_2 = -1}$

$\Pi_2 = D^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu$

$$\boldsymbol{\Pi_2 = \frac{\mu}{\rho VD}}$$

$f(\Pi_1, \Pi_2) = 0$

$$f\left(\frac{R}{D^2V^2\rho}, \frac{\mu}{\rho VD}\right) = 0$$

$$\frac{R}{D^2V^2\rho} = \phi\left(\frac{\mu}{\rho VD}\right)$$

$$R = \rho V^2 D^2 \phi\left(\frac{\mu}{\rho VD}\right)$$

- **Problem 2:** The efficiency of a fan depends on density ρ, dynamic viscosity μ, angular velocity ω, diameter D and discharge Q. Express efficiency in terms of dimensionless parameters.

- **Solution:**

$f(\eta, \rho, \mu\ \omega, D, Q) = 0$

Here, $n = 6, m = 3$

$\therefore$ Number of Π terms = 3

$\therefore f(\Pi_1, \Pi_2, \Pi_3) = 0$

$\eta = L^o M^o T^o$
$\rho = ML^{-3}$
$\mu = L^{-1}MT^{-1}$
$\omega = T^{-1}$
$D = L$
$Q = L^3T^{-1}$

Let, D, ω, ρ be the repeating variables.

$\Pi_1 = D^{a_1} \cdot \omega^{b_1} \cdot \rho^{c_1} \cdot \mu$

$[L^oM^oT^o] = [L]^{a_1} [T^{-1}]^{b_1} [ML^{-3}]^{c_1} [L^{-1}MT^{-1}]$

$[L^oM^oT^o] = [L]^{a_1-3c_1-1} [M]^{c_1+1} [T]^{-b_1-1}$

$\mathbf{b_1 = -1}$

$\mathbf{c_1 = -1}$

$\mathbf{a_1 - 3c_1 - 1 = 0}$

$\mathbf{a_1 = -2}$

$\Pi_1 = D^{-2} \cdot \omega^{-1} \cdot \rho^{-1} \cdot \mu$

$$\Pi_1 = \frac{\mu}{D^2 . \omega . \rho}$$

$\Pi_2 = D^{a_2} \cdot \omega^{b_2} \cdot \rho^{c_2} \cdot Q$

$[L^oM^oT^o] = [L]^{a_2} [T^{-1}]^{b_2} [ML^{-3}]^{c_2} [L^3T^{-1}]$

$[L^oM^oT^o] = [L]^{a_2+3-3C_2} [M]^{c_2} [T]^{-b_2-1}$

$\mathbf{c_2 = 0}$

$\mathbf{-b_2 - 1 = 0}$

$\mathbf{b_2 = -1}$

$\mathbf{a_2 + 3 - 3c_2 = 0}$

$\mathbf{a_2 + 3 = 0}$

$\mathbf{a_2 = -3}$

$\Pi_2 = D^{-3} \cdot \omega^{-1} \cdot \rho^{o} \cdot Q$

$$\Pi_2 = \frac{Q}{\omega D^3}$$

$\Pi_3 = D^{a_3} \cdot \omega^{b_3} \cdot \rho^{c_3} \cdot \eta$

$[L^oM^oT^o] = [L]^{a_3} [T^{-1}]^{b_3} [ML^{-3}]^{c_3} [M^oL^oT^o]$

$[L^oM^oT^o] = [L]^{a_3-3c_3} [M]^{c_3} [T]^{-b_3}$

$\mathbf{b_3 = 0}$

$\mathbf{c_3 = 0}$

$\mathbf{a_3 = 0}$

$\mathbf{\Pi_3 = \eta}$

$f(\Pi_1, \Pi_2, \Pi_3) = 0$

$$f\left(\frac{\mu}{D^2 \cdot \omega \cdot \rho}, \frac{\mu}{\omega D^3}, \eta\right) = 0$$

$$\eta = \phi\left(\frac{\mu}{D^2 \omega \rho}, \frac{\mu}{\omega D^3}\right)$$

- **Problem 3:** The resisting force of a supersonic plane during flight can be considered as dependent on the length of the aircraft L, velocity V, viscosity μ, mass density ρ, Bulk modulus K. Express the fundamental relationship between resisting force and these variables.

- **Solution:**

$f(R, L, K, \mu, \rho, V) = 0$

$n = 6$

$\therefore$ Number of Π terms = 6 – 3 = 3

$\therefore f(\Pi_1, \Pi_2, \Pi_3) = 0$

Let, L, V, ρ be the repeating variables.

$\Pi_1 = L^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot K$

$L^o M^o T^o = [L]^{a_1} [LT^{-1}]^{b_1} [ML^{-3}]^{c_1} [L^{-1}MT^{-2}]$

$L^o M^o T^o = [L]^{a_1 + b_1 - 3c_1 - 1} [M]^{c_1 + 1} [T]^{-b_1 - 2}$

$\mathbf{b_1 = -2}$

$\mathbf{c_1 = -1}$

$a_1 + b_1 - 3c_1 - 1 = 0$

$a_1 - 2 + 3 - 1 = 0$

$a_1 = 0$

$\Pi_1 = L^o \cdot V^{-1} \cdot \rho^{-1} \cdot K$

$\Pi_1 = \dfrac{K}{V^2 \cdot \rho}$

$\Pi_2 = L^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot R$

$L^oM^oT^o = [L]^{a_2} [LT^{-1}]^{b_2} [ML^{-3}]^{c_2} [LMT^{-2}]$

$L^oM^oT^o = [L]^{a_2+b_2-3c_2+1} [M]^{c_2+1} [T]^{-b_2-2}$

$-b_2 - 2 = 0$

$\mathbf{b_2 = -2}$

$\mathbf{c_2 = -1}$

$a_2 + b_2 - 3c_2 + 1 = 0$

$a_1 + 2 + 3 + 1 = 0$

$a_2 = -2$

$\Pi_2 = L^{-2} \cdot V^{-2} \cdot \rho^{-1} \cdot R$

$\Pi_2 = \dfrac{R}{L^2V^2\rho}$

$\Pi_3 = L^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot \mu$

$L^oM^oT^o = [L]^{a_3} [LT^{-1}]^{b_3} [ML^{-3}]^{c_3} [L^{-1}MT^{-1}]$

$L^oM^oT^o = [L]^{a_3+b_3-3c_3-1} [M]^{c_3+1} [T]^{-b_3-1}$

$-b_3 - 1 = 0$

$\mathbf{b_3 = -1}$

$\mathbf{c_3 + 1 = 0}$

$\mathbf{c_3 = -1}$

$a_3 + b_3 - 3c_3 - 1 = 0$

$a_1 - 1 + 3 - 1 = 0$

$a_3 = -1$

$\Pi_3 = L^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu$

$\Pi_2 = \dfrac{\mu}{LV\rho}$

$f\left(\dfrac{K}{V^2\rho}, \dfrac{R}{L^2V^2\rho}, \dfrac{\mu}{LV\rho}\right) = 0$

$\dfrac{R}{L^2V^2\rho} = \phi\left(\dfrac{K}{V^2\rho}, \dfrac{\mu}{LV\rho}\right)$

$R = L^2V^2\rho\ \phi\left(\dfrac{K}{V^2\rho}, \dfrac{\mu}{LV\rho}\right)$

- **Problem 4:** Using Buckingham Π - theorem, show that velocity of fluid through a circular orifice is given by $V = \sqrt{2gH}\ \phi\left(\dfrac{D}{H}, \dfrac{\mu}{\rho VH}\right)$.

- **Solution:**

We have, $f(V, D, H, \mu, \rho, g) = 0$

$V = LT^{-1}$, $D = L$, $H = L$, $\mu = L^{-1}MT^{-1}$, $\rho = ML^{-3}$, $g = LT^{-2}$

$n = 6$

$m = 3$

$\therefore$ Number of Π terms $= (6 - 3) = 3$

Let H, g and ρ be repeating variables

$\Pi_1 = H^{a1} \cdot g^{b_1} \cdot \rho^{c_1} \cdot V$

$[L^oM^oT^o] = [L]^{a_1} [LT^{-2}]^{b_1} [ML^{-3}]^{c_1} [LT^{-1}]$

$[L^oM^oT^o] = [L]^{c_1+b1-3c_1+1} [M]^{c_1} [T]^{-2b_1-1}$

$\mathbf{-2b_1 = 1}$

$\mathbf{b_1} = -\frac{1}{2}$

$\mathbf{c_1 = 0}$

$a_1 + b_1 - 3c_1 + 1 = 0$

$a_1 - \frac{1}{2} - 0 + 1 = 0$

$a_1 = -\frac{1}{2}$

$\Pi_1 = H^{-1/2} \cdot g^{-1/2} \cdot \rho^o \cdot V$

$\Pi_1 = \frac{V}{\sqrt{gH}}$

$\Pi_2 = H^{a_2} \cdot g^{b_2} \cdot \rho^{c_2} \cdot D$

$[L^oM^oT^o] = [L]^{a_2} [LT^{-2}]^{b_2} [ML^{-3}]^{c_2} [L]$

$[L^oM^oT^o] = [L]^{a_2+b_2-3c_2+1} [M]^{c_2} [T]^{-2b_2}$

$-2b_2 = 0$

$\mathbf{b_2 = 0}$

$\mathbf{c_2 = 0}$

$a_2 + b_2 - 3c_2 + 1 = 0$

$a_2 = -1$

$\Pi_2 = H^{-1} \cdot g^o \cdot \rho^o \cdot D$

$\Pi_2 = \frac{H}{D}$

$\Pi_3 = H^{a_3} \cdot g^{b_3} \cdot \rho^{c_3} \cdot \mu$

$[M^oL^oT^o] = [L]^{a_3} [LT^{-2}]^{b_3} [ML^{-3}]^{c_3} [L^{-1}MT^{-1}]$

$[M^oL^oT^o] = [M]^{c_3+1}\,[L]^{a_3+b_3-3c_3-1}\,[T]^{-2b_3-1}$

$c_3 = -1$

$\mathbf{b_3 = -\frac{1}{2}}$

$\mathbf{a_3 - \frac{1}{2} + 3 - 1 = 0}$

$\mathbf{c_3 = -\frac{3}{2}}$

$\Pi_3 = H^{-3/2} \cdot g^{-1/2} \cdot \rho^{-1} \cdot \mu$

$\Pi_3 = \frac{\mu}{\rho\sqrt{gH^3}}$

$\Pi_3 = \frac{\mu}{\rho\sqrt{gH}.H}$

$f(\Pi_1, \Pi_2, \Pi_3) = 0$

$f\left(\frac{V}{\sqrt{gH}}, \frac{H}{D}, \frac{\mu}{\rho\sqrt{gH}\cdot H}\right) = 0$

$\frac{V}{\sqrt{gH}} = \phi\left(\frac{H}{D}, \frac{\mu}{\rho\sqrt{gH}\cdot H}\right)$

$v = \sqrt{2gH}\,\phi\left(\frac{D}{H}, \frac{\mu}{\rho VH}\right)$

- **Problem 5**: Using dimensional analysis, derive an expression for thrust P developed by a propeller assuming that it depends on angular velocity ω, speed of advance V, diameter D, dynamic viscosity μ, mass density ρ, elasticity of the fluid medium which can be denoted by speed of sound in the medium C.

- **Solution:**

$f(P, \omega, V, D, \mu, \rho, C) = 0$

$n = 7$

$m = 3$

Taking D, V and ρ as repeating variables.

$P = LMT^{-2}$

$\omega = L^oM^oT^{-1}$

$V = LM^oT^{-1}$

$D = L$

$\mu = L^{-1}MT^{-1}$

$\rho = ML^{-3}$

$C = LT^{-1}$

$\Pi_1 = D^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot P$

$[L^oM^oT^o] = [L]^{a_1} [LT^{-1}]^{b_1} [ML^{-3}]^{c_1} [LMT^{-2}]$

$[L^oM^oT^o] = [L]^{c_1+b_1-3c_1+1} [M]^{c_1+1} [T]^{-b_1-2}$

$\mathbf{-b_1 - 2 = 0}$

$\mathbf{b_1 = -2}$

$\mathbf{c_1 + 1 = 0}$

$\mathbf{c_1 = -1}$

$a_1 + b_1 - 3c_1 + 1 = 0$

$a_1 - 2 + 3 + 1 = 0$

$\mathbf{a_1 = -2}$

$\Pi_1 = D^{-2} \cdot V^{-2} \cdot \rho^{-1} \cdot P$

$$\Pi_1 = \frac{P}{D^2V^2\rho}$$

$\Pi_2 = D^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot \omega$

$[L^oM^oT^o] = [L]^{a_2} [LT^{-2}]^{b_2} [ML^{-3}]^{c_2} [T^{-1}]$

$[L^oM^oT^o] = [L]^{a_2+b_2-3c_2} [M]^{c_2} [T]^{-b_2-1}$

$-b_2 - 1 = 0$

$b_2 = -1$

$c_2 = 0$

$a_2 - 1 + 0 = 0$

$a_2 = -1$

$\Pi_2 = D^1 \cdot V^{-1} \cdot \rho^o \cdot \omega$

$$\Pi_2 = \frac{D\omega}{V}$$

$\Pi_3 = D^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot \mu$

$[M^oL^oT^o] = [L]^{a_3} [LT^{-1}]^{b_3} [ML^{-3}]^{c_3} [L^{-1}MT^{-1}]$

$[M^oL^oT^o] = [L]^{a_3+b_3-3c_3-1} [M]^{c_3+1} [T]^{-b_3-1}$

$b_3 = -1$

$c_3 - 1 + 3 - 1 = 0$

$c_3 = -1$

$\Pi_3 = D^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu$

$$\Pi_3 = \frac{\mu}{\rho VD}$$

$\Pi_4 = D^{a_4} \cdot V^{b_4} \cdot \rho^{c_4} \cdot C$

$M^oL^oT^o = [L]^{a_4} [LT^{-1}]^{b_4} [ML^{-3}]^{c_4} [LT^{-1}]$

$M^oL^oT^o = [L]^{a_4 \cdot b_4-3c_4+1} [M]^{c_4} [T]^{-b_4-1}$

$b_4 = -1$

$c_4 = 0$

$a_4 - 1 + 0 + 1 = 0$

$a_4 = 0$

$\Pi_4 = D^o \cdot V^{-1} \cdot \rho^o \cdot C$

$\Pi_4 = \frac{C}{V}$

$f(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0$

$$f\left(\frac{P}{D^2V^2\rho}, \frac{D\omega}{V}, \frac{\mu}{\rho VD}, \frac{C}{V}\right) = 0$$

$$P = D^2V^2\rho\ \phi\left(\frac{D\omega}{V}, \frac{\mu}{\rho VD}, \frac{C}{V}\right)$$

- **Problem 6**: The pressure drop ΔP in a pipe depends on mean velocity of flow V, length of pipe l, viscosity of the fluid μ, diameter D, height of roughness projection K and mass density of the liquid ρ. Using Buckingham's method obtain an expression for ΔP.

- **Solution:**

$f(\Delta P, V, l, \mu, D, K, \rho) = 0$

$n = 7$

$\therefore$ number of Π terms $= 7 - 3 = 4$

$\Delta P = L^{-1}MT^{-2}$

$V = LM^oT^{-1}$

$l = LM^oT^o$

$\mu = L^{-1}MT^{-1}$

$K = LM^oT^o$

$\rho = ML^{-3}$

Let D, V, ρ be the repeating variables

$\Pi_1 = D^{a1} \cdot V^{b1} \cdot \rho^{c1} \cdot \Delta P$

$[L^oM^oT^o] = [L]^{a1}[LT^{-1}]^{b1}[ML^{-3}]^{c1}[L^{-1}MT^{-2}]$

$[L^oM^oT^o] = [L]^{a1+b1-3c1-1}[M^{c1+1}][T^{-b1-2}]$

$-b_1 - 2 = 0$

$b_1 = -2$

$c_1 = -1$

$a_1 + b_1 - 3c_1 - 1 = 0$

$a_1 - 2 + 3 - 1 = 0$

$a_1 = 0$

$\Pi_1 = D^o \cdot V^{-2} \cdot \rho^{-1} \cdot \Delta P$

$\Pi_1 = \dfrac{\Delta P}{V^2 \cdot \rho}$

$\Pi_2 = D^{a2} \cdot V^{b2} \cdot \rho^{c2} \cdot l$

$[L^oM^oT^o] = [L]^{a2}\,[LT^{-1}]^{b2}\,[ML^{-3}]^{c2}\,[L]$

$[L^oM^oT^o] = [L]^{a2+b2-3c2+1}\,[M]^{c2}\,[T]^{-b2}$

$b_2 = 0$

$c_2 = 0$

$a_2 + 0 + 0 + 1 = 0$

$a_2 = -1$

$\Pi_2 = D^{-1} \cdot V^o \cdot \rho^o \cdot L$

$\Pi_2 = \dfrac{l}{D}$

$\Pi_3 = D^{a3} \cdot V^{b3} \cdot \rho^{c3} \cdot K$

$[L^oM^oT^o] = [L]^{a3}\,[LT^{-1}]^{b3}\,[ML^{-3}]^{c3}\,[L]$

$[L^oM^oT^o] = [L]^{a3+b3-3c3+1}\,[M]^{c3}\,[T]^{-b3}$

$b_3 = 0$

$c_3 = 0$

$a_3 + b_3 - 3c_3 + 1 = 0$

$a_3 = -1$

$\Pi_3 = \dfrac{K}{D}$

$\Pi_4 = D^{a4} \cdot V^{b4} \cdot \rho^{c4} \cdot \mu$

$[L^oM^oT^o] = [L]^{a4}\,[LT^{-1}]^{b4}\,[ML^{-3}]^{c4}\,[L^{-1}MT^{-1}]$

$[L^oM^oT^o] = [L]^{a_4+b_4-3c_4-1}\,[M]^{c_4+1}\,[T]^{-b_4-1}$

$b_4 = -1$

$c_4 = -1$

$a_4 + b_4 - 3c_4 - 1 = 0$

$a_4 - 1 + 3 - 1 = 0$

$a_4 = -1$

$\Pi_4 = D^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu$

$$\Pi_4 = \frac{\mu}{\rho VD}$$

$$f\left(\frac{\Delta P}{V^2\rho}, \frac{l}{D}, \frac{K}{D}, \frac{\mu}{\rho VD}\right) = 0$$

$$\Delta P = \rho V^2 \phi\left(\frac{L}{D}, \frac{K}{D}, \frac{\mu}{\rho VD}\right)$$

MODEL ANALYSIS

Before constructing or manufacturing hydraulics structures or hydraulics machines tests are performed on their models to obtain desired information about their performance. Models are small scale replica of actual structure or machine. The actual structure is called prototype.

- **Similitude / Similarity**

  It is defined as the similarity between the prototype and it's model.

- **Types of Similarity**

  There are three types of similarity.
  - Geometric similarity
  - Kinematic similarity
  - Dynamic similarity

- **Geometrical Similarity**

  Geometric similarity is said to exist between the model and prototype if the ratio of corresponding linear dimensions between model and prototype are equal.

i.e. $\frac{L_p}{L_m} = \frac{h_p}{h_m} = \frac{H_p}{H_m} .... = L_r$

$L_r \rightarrow$ scale ratio / linear ratio

$\frac{A_p}{A_m} = (L_r)^2 \quad \frac{V_p}{V_m} = (L_r)^3$

- **Kinematic Similarity**

  Kinematic similarity exists between prototype and model if quantities such at velocity and acceleration at corresponding points on model and prototype are same.

$$\frac{(V_1)_p}{(V_1)_m} = \frac{(V_2)_p}{(V_2)_m} = \frac{(V_3)_p}{(V_3)_m} .... = V_r$$

$V_r \rightarrow$ Velocity ratio

- **Dynamic Similarity**

  Dynamic similarity is said to exist between model and prototype if ratio of forces at corresponding points of model and prototype is constant.

$$\frac{(F_1)_p}{(F_1)_m} = \frac{(F_2)_p}{(F_2)_m} = \frac{(F_3)_p}{(F_3)_m} ....= F_R$$

F_R → Force ratio

- **Dimensionless Numbers**

  Following dimensionless numbers are used in fluid mechanics.

  1. Reynold's number
  2. Froude's number
  3. Euler's number
  4. Weber's number
  5. Mach number

1. Reynold's number

It is defined as the ratio of inertia force of the fluid to viscous force.

$$\therefore\ N_{Re} = \frac{F_i}{F_v}$$

Expression for N_{Re}

F_i = Mass x Acceleration

$F_i = \rho$ x Volume x Acceleration

$$F_i = \rho \text{ x Volume x } \frac{\text{Change in velocity}}{\text{Time}}$$

$F_i = \rho$ x Q x V

$F_i = \rho AV^2$

F_V → Viscous force

$F_V = \tau$ x A

$$F_V = \mu \frac{V}{y} A$$

$$F_V = \mu \frac{V}{L} A$$

$$N_{Re} = \frac{\rho AV^2}{\mu \frac{V}{L} A}$$

$$N_{Re} = \frac{\rho VL}{\mu}$$

In case of pipeline diameter is the linear dimension.

$$N_{Re} = \frac{\rho VD}{\mu}$$

2. Froude's Number (F_r)

It is defined as the ratio of square root of inertia force to gravity force.

$$F_r = \sqrt{\frac{F_i}{F_g}}$$

F_i = m x a

F_i = ρ x Volume x Acceleration

$F_i = \rho AV^2$

F_g = m x g

F_g = ρ x Volume x g

F_g = ρ x A x L x g

$$F_\gamma = \sqrt{\frac{\rho AV^2}{\rho \text{ x } A \text{ x } L \text{ x } g}}$$

$$F_\gamma = \sqrt{\frac{V^2}{Lg}}$$

$$F_\gamma = \frac{V}{\sqrt{Lg}}$$

3. Euler's Number (ε_u)

It is defined as the square root of ratio of inertia force to pressure force.

$$\varepsilon_u = \sqrt{\frac{F_i}{F_p}}$$

F_i = Mass x Acceleration

$$F_i = \rho \times \text{Volume} \times \frac{\text{Velocity}}{\text{Time}}$$

$F_i = \rho \times Q \times V$

$F_i = \rho A V^2$

$F_p = p \times A$

$$\varepsilon_u = \sqrt{\frac{\rho A V^2}{pA}} = V\sqrt{\frac{\rho}{p}}$$

$$\boldsymbol{\varepsilon_u} = \frac{V}{\sqrt{\frac{p}{\rho}}}$$

4. Weber's Number (W_b)

It is defined as the square root of ratio of inertia force to surface tensile force.

$$W_b = \sqrt{\frac{F_i}{F_p}}$$

$F_b = \rho A V^2$

$F_s = \sigma \times L$

$$W_b = \sqrt{\frac{\rho A V^2}{\sigma L}} = V\sqrt{\frac{\rho L}{\sigma}}$$

$$\mathbf{W_b} = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$

5. Mach Number (M)

It is defined as the square root of ratio of inertia force to elastic force.

$$M = \sqrt{\frac{F_i}{F_e}}$$

$F_i = \rho A V^2$

$F_e = K \times A$

K → Bulk modulus of elasticity

A → Area

$$M = \sqrt{\frac{\rho A V^2}{K A}}$$

$$M = \frac{V}{\sqrt{K/\rho}}$$

$$M = \frac{V}{C}$$

C → Velocity of sound in fluid.

MODEL LAWS (SIMILARITY LAWS)

1. Reynold's Model Law

For the flows where in addition to inertia force, similarity of flow in model and predominant force, similarity of flow in model and prototype can be established if Re is same for both the system.

This is known as Reynold's Model Law.

Re for model = Re for prototype

$(N_{Re})_m = (N_{Re})_p$

$$\left(\frac{\rho VD}{\mu}\right)_m = \left(\frac{\rho VD}{\mu}\right)_p$$

$$\frac{\dfrac{\rho_m \cdot V_m \cdot D_m}{\rho_p \cdot V_p \cdot D_p}}{\dfrac{\mu_m}{\mu_p}} = 1$$

$$\frac{\rho_r \cdot V_r \cdot D_r}{\mu_r} = 1$$

Applications:

i) In flow of incompressible fluids in closed pipes.

ii) Motion of submarine completely under water.

iii) Motion of air-planes.

2. Froude's Model Law

When the force of gravity is predominant in addition to inertia force then similarity can be established by Froude's number. This is known as Froude's model law.

$(F_r)_m = (F_r)_p$

$$\left(\frac{V}{\sqrt{gL}}\right)_m = \left(\frac{V}{\sqrt{gL}}\right)_p$$

$$\left(\frac{V}{\sqrt{gL}}\right)_r = 1$$

<u>Applications:</u>

i) Flow over spillways.

ii) Channels, rivers (free surface flows).

iii) Waves on surface.

iv) Flow of different density fluids one above the other.

3. Euler's Model Law

When pressure force is predominant in addition to inertia force, similarity can be established by equating Euler number of model and prototype. This is called Euler's model law.

$$(\varepsilon_u)_m = (\varepsilon_u)_p$$

$$\left(\frac{V_m}{\sqrt{p_m/\rho_m}}\right) = \left(\frac{V_p}{\sqrt{p_p/\rho_p}}\right)$$

<u>Application:</u>

Turbulent flow in pipeline where viscous force and surface tensile forces are entirely absent.

4. Mach Model Law

In places where elastic forces are significant in addition to inertia, similarity can be achieved by equating Mach numbers for both the system.

This is known as Mach model law.

$$M_m = M_\gamma$$

$$\left(\frac{V_m}{\sqrt{K_m/\rho_m}}\right) = \left(\frac{V_p}{\sqrt{K_p/\rho_p}}\right)$$

$$\left(\frac{V_\gamma}{\sqrt{K_\gamma/\rho_\gamma}}\right) = 1$$

Applications:

i) Aerodynamic testing where velocity exceeds speed of sound.

Eg: Flow of airplane at supersonic speed.

ii) Water hammer problems.

5. Weber's Model Law:

If surface tension forces are predominant with inertia force, similarity can be established by equating Weber number of model and prototype.

$$W_m = W_\gamma$$

$$\left(\frac{V}{\sqrt{\sigma/\rho L}}\right)_m = \left(\frac{V}{\sqrt{\sigma/\rho L}}\right)_p$$

$$\left(\frac{V}{\sqrt{\omega/\rho L}}\right)_r = 1$$

Applications:

i) Flow over wires with low heads.

ii) Flow of very thin sheet of liquid over a surface.

iii) Capillary flows.

- **Problem 1**: A pipe of diameter 1.5 m is required to transmit an oil of S = 0.9 and viscosity 3 x 10^{-2} poise at 3000 lps. Tests were conducted on 15 cm diameter pipe using water at 20^oC. Find velocity and rate of flow of model if μ water at 20^oC is 0.01 poise.

- **Solution**

$D_p = 1.5$ m
$S_p = 0.9$
$\mu_p = 3 \times 10^{-2}$ poise $= 3 \times 10^{-3}$ Ns/m^2
$Q_p = 3000$ lps $= 3000 \times 10^{-3}$ m^3/s $= 3$ m^3/s
$D_m = 0.15$ m
$S_m = 1$
$V_m = ?$
$Q_m = ?$
$A_p V_p = Q_p$
$V_p = 1.698$ m/s
$\mu_m = 0.01$ poise
$= 0.001$ poise
$\rho_m = 1000$ kg/m^3
$\rho_p = 0.9 \times 1000 = 900$ kg/m^3

$(R_e)_m = (R_e)_p$

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$$

$$\frac{1000 \times V_m \times 0.15}{0.001} = \frac{900 \times 1.698 \times 1.5}{3 \times 10^{-3}}$$

$V_m = 5.094$ m/s

$Q = A_m V_m$

$$Q = \frac{\Pi}{4}(0.15)^2 (5.094)$$

$Q = 0.09$ m^3/s

Q = 90 lps.

- **Problem 2**: In a 1 in 40 model of spillway velocity and discharge are 2 m/s and 2.5 m^3/s. Find the corresponding velocity and discharge in prototype.

- **Solution**

$$L_\gamma = \frac{L_m}{L_p} = \frac{1}{40}$$
$V_m = 2$ m/s
$Q_m = 2.5\ m^3/s$

Since it is a spillway problem, Froude's law of similarity is used.

$$(F_\gamma)_m = (F_\gamma)_p$$

$$\left(\frac{V}{\sqrt{gL}}\right)_m = \left(\frac{V}{\sqrt{gL}}\right)_p$$

$$\frac{2}{\sqrt{9.81 \text{ x } 1}} = \frac{V_p}{\sqrt{9.81 \text{ x } 40}}$$

$V_p = 12.65$ m/s

For a spillway,

$$Q \alpha L^{2.5}$$

$$\frac{Q_p}{Q_m} = \frac{L_p^{\ 2.5}}{L_m^{\ 2.5}}$$

$$\frac{Q_p}{2.5} = (40)^{2.5}$$

$Q_p = 25298.22\ m^3/s$

- **Problem 3**: Experiments area to be conducted on a model ball which is twice as large as actual golf ball. For dynamic similarity, find ratio of initial velocity of model to that of actual ball. Take fluid in both cases as air at STP.

It is a case of motion of fully submerged body.

∴ Reynolds's number of flow determines dynamic similarity.

- **Solution**

$\therefore\ (R_e)_m = (R_e)_p$

$\rho_m = \rho_p$

$\mu_m = \mu_p$

$$\left(\frac{\rho V d}{\mu}\right)_m = \left(\frac{\rho V d}{\mu}\right)_p$$

$$\frac{d_m}{d_p} = 2$$

$V_m \cdot d_m = V_p \cdot d_p$

$$\frac{V_m}{V_p} = \frac{d_p}{d_m}$$

$$\frac{V_m}{V_p} = \frac{1}{2}$$

$\mathbf{V_m = 0.5\ V_p}$

- **Problem 4**: Water at 15°C flows at 4 m/s in a 150 mm diameter pipe. At what velocity oil at 30°C must flow in a 75 mm diameter pipe for the flows to be dynamically similar? Take kinematic viscosity of water at 15°C as $1.145 \times 10^{-6}\ m^2/s$ and that for oil at 30°C as $3 \times 10^{-6}\ m^2/s$.

- **Solution**

$V_p = 4$ m/s

$d_p = 0.15$ m

$V_m = ?$

$D_m = 0.075$ m

$$\left(\frac{\mu}{\rho}\right)_p = 1.145 \text{ x } 10^{-6}\ m^2/s$$

$$\left(\frac{\mu}{\rho}\right)_m = 3 \text{ x } 10^{-6}\ m^2/s$$

$$\left(\frac{\rho V d}{\mu}\right)_m = \left(\frac{\rho V d}{\mu}\right)_p$$

$$\frac{V_m \text{ x } 0.075}{3 \text{ x } 10^{-6}} = \frac{4 \text{ x } 0.15}{1.145 \text{ x } 10^{-6}}$$

$\mathbf{V_m = 20.96 \text{ m/s}}$

- **Problem 5**: A model with linear scale ratio (model to prototype) x, of a mach 2 supersonic aircraft is tested in a wind tunnel where in pressure is y times the atmospheric pressure. Determine the speed of model in tunnel given that velocity of sound m atmospheric air is Z.

- **Solution:**

$$\frac{L_m}{L_p} = x$$

$M = 2$

$P_m = y\ p_{atm}$

$\rho_m = y\ \rho_{atm}$

$C = Z$

$$\frac{V}{C} = 2$$

$$\frac{V}{Z} = 2$$

$\mathbf{V_p = 2Z}$

Dynamic similarity in this case is established by Reynold's Model law.

$(R_e)_m = (R_e)_p$

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p$$

$$y\ \frac{\rho_{atm} \text{ x } V_m \text{ x } L_m}{\mu_m} = \frac{\rho_p \text{ x } V_p \text{ x } L_p}{\mu_p}$$

$$y\ \frac{\rho_{atm} \cdot V_m \cdot x}{\mu_m / \tfrac{1}{\mu_p}} = \rho_{atm} \cdot 2Z$$

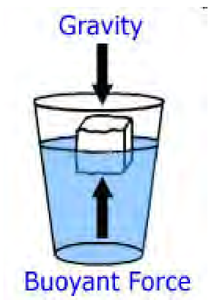
$$\mathbf{V_m} = \frac{2Z}{xy}$$

FLUID MECHANICS

Fluid Statics

BUOYANCY

When a body is either wholly or partially immersed in a fluid, the hydrostatic lift due to the net vertical component of the hydrostatic pressure forces experienced by the body is called the "Buoyant Force" and the phenomenon is called "Buoyancy".



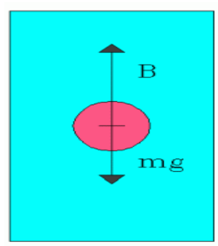
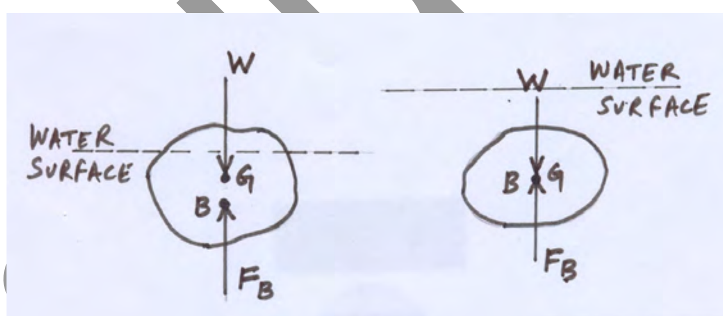


Fig. Buoyancy

The Buoyancy is an upward force exerted by the fluid on the body when the body is immersed in a fluid or floating on a fluid. This upward force is equal to the weight of the fluid displaced by the body.

CENTER OF BUOYANCY



(a) Floating Body (b) Submerged Body

Fig. Center of Buoyancy

Center of Buoyancy is a point through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the Center of Buoyancy will be the center of the fluid displaced.

LOCATION OF CENTER OF BUOYANCY

Consider a solid body of arbitrary shape immersed in a homogeneous fluid. Hydrostatic pressure forces act on the entire surface of the body. Resultant horizontal forces for a closed surface are zero.

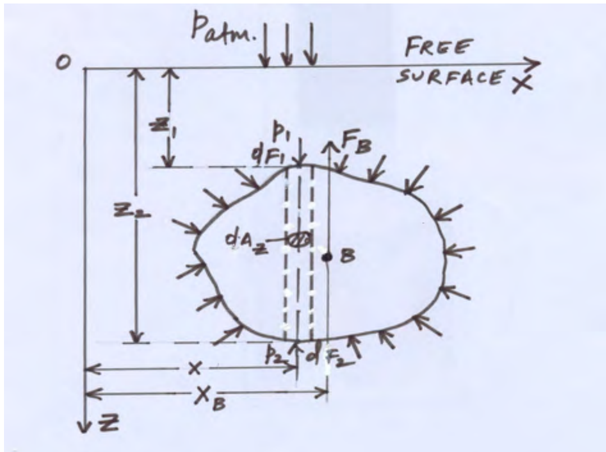


Fig. Location of Center of Buoyancy

The body is considered to be divided into a number of vertical elementary prisms of cross section $d(A_Z)$. Consider vertical forces dF_1 and dF_2 acting on the two ends of the prism.

$dF_1 = (P_{atm} + \rho g z_1) d(A_Z)$

$dF_2 = (P_{atm} + \rho g z_2)\ d(A_Z)$

The buoyant force acting on the element:

$dF_B = dF_2 - dF_1 = \rho g(z_2 - z_1) d(A_Z) = \rho g(dv)$ where dv = volume of the element.

The buoyant force on the entire submerged body $(F_B) = {}_V\int\int\int \rho g(dv) = \rho g V$;

Where V = Total volume of the submerged body or the volume of the displaced liquid.

LINE OF ACTION OF BUOYANT FORCE

To find the line of action of the Buoyant Force F_B, take moments about z-axis,

$X_B F_B = \int x dF_B$

But $F_B = \rho g V$ and $dF_B = \rho g(dv)$

Substituting we get, $X_B = (1/V)\ {}_V\int\int\int x dv$ where X_B = Centroid of the Displaced Volume.

ARCHIMEDES PRINCIPLE

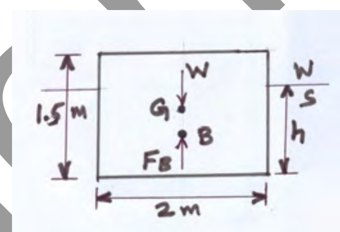
The Buoyant Force (F_B) is equal to the weight of the liquid displaced by the submerged body and acts vertically upwards through the centroid of the displaced volume.

Net weight of the submerged body = Actual weight – Buoyant force.

The buoyant force on a partially immersed body is also equal to the weight of the displaced liquid. The buoyant force depends upon the density of the fluid and submerged volume of the body. For a floating body in static equilibrium, the buoyant force is equal to the weight of the body.

Problem –1

Find the volume of the water displaced and the position of Center of Buoyancy for a wooden block of width 2.0m and depth 1.5m when it floats horizontally in water. Density of wooden block is 650kg/m^3 and its length is 4.0m.



Volume of the block, V = 12m^3

Weight of the block, ρgV= 76518N

Volume of water displaced =

76,518 / (1000 × 9.81)= 7.8m^3

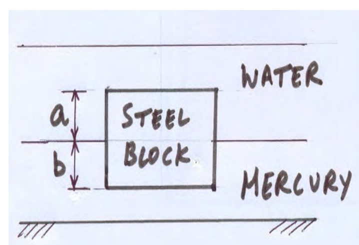
Depth of immersion, h =7.8 / (2×4) = 0.975m.

The Center of Buoyancy is at 0.4875m from the base.

Problem-2

A block of steel (specific gravity= 7.85) floats at the mercury-water interface as shown in figure. What is the ratio (a / b) for this condition?

(Specific gravity of Mercury = 13.57)



Let A = Cross sectional area of the block

Weight of the body = Total buoyancy forces

A(a+b)× 7850× g

=A(b × 13.57 +a) × g × 1000

7.85 (a+b) = 13.57 × b + a

(a / b) = 0.835

Problem – 3

A body having the dimensions of 1.5m ×1.0m× 3.0m weighs 1962N in water. Find its weight in air. What will be its specific gravity?

Volume of the body = 4.5m^3 = Volume of water displaced.

Weight of water displaced = 1000 × 9.81× 4.5 = 44145N

For equilibrium,

Weight of body in air – Weight of water displaced = Weight in water

W_{air} = 44145N + 1962N = 46107N

Mass of body = (46107 / 9.81) = 4700kg.

Density = (4700 / 4.5) = 1044.4 kg/m^3

Specific gravity = 1.044

STABILITY OF UN-CONSTRAINED SUBMERGED BODIES IN A FLUID

When a body is submerged in a liquid (or a fluid), the equilibrium requires that the weight of the body acting through its Center of Gravity should be co-linear with the Buoyancy Force acting through the Center of Buoyancy. If the Body is Not Homogeneous in its distribution of mass over the entire volume, the location of Center of Gravity (G) does not coincide with the Center of Volume (B). Depending upon the relative locations of (G) and (B), the submerged body attains different states of equilibrium: Stable, Unstable and Neutral.

STABLE, UNSTABLE AND NEUTRAL EQUILIBRIUM

Stable Equilibrium: (G) is located below (B). A body being given a small angular displacement and then released, returns to its original position by retaining the original vertical axis as vertical because of the restoring couple produced by the action of the Buoyant Force and the Weight.

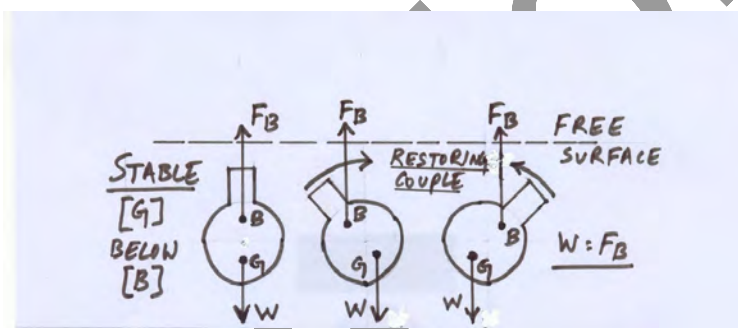


Fig. Stable Equilibrium

Unstable Equilibrium: (G) is located above (B). Any disturbance from the equilibrium position will create a destroying couple that will turn the body away from the original position

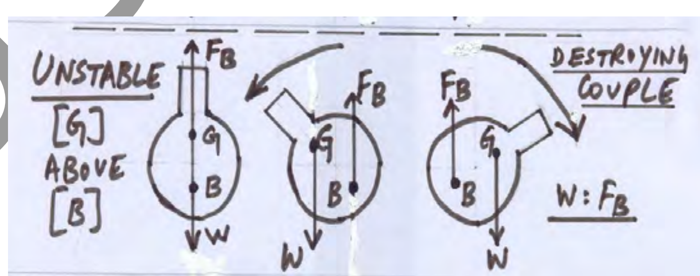


Fig. Unstable Equilibrium

Neutral Equilibrium: (G) and (B) coincide. The body will always assume the same position in which it is placed. A body having a small displacement and then released, neither returns to the original position nor increases it's displacement- It will simply adapt to the new position.

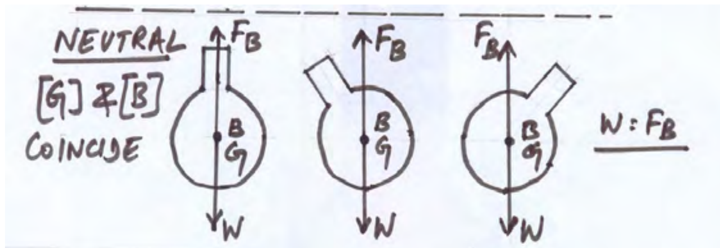


Fig. Neutral Equilibrium

A submerged body will be in stable, unstable or neutral equilibrium if the Center of Gravity (G) is below, above or coincident with the Center of Buoyancy (B) respectively.

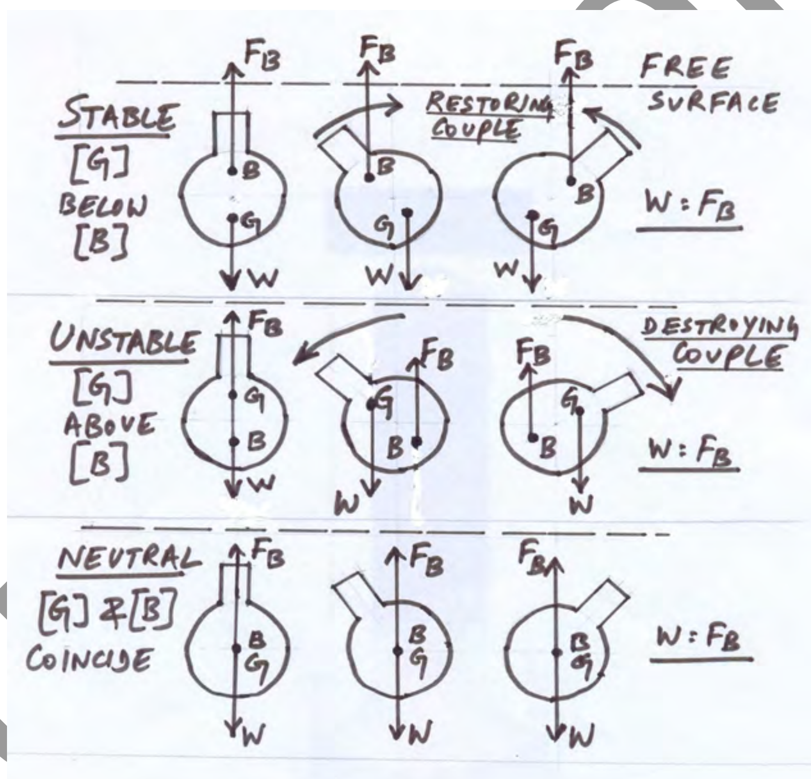


Fig. Stable, Unstable and Neutral Equilibrium

STABILITY OF FLOATING BODIES

Stable conditions of the floating body can be achieved, under certain conditions even though (G) is above (B). When a floating body undergoes angular displacement about the horizontal position, the shape of the immersed volume changes and so, the Center of Buoyancy moves relative to the body.

META CENTER

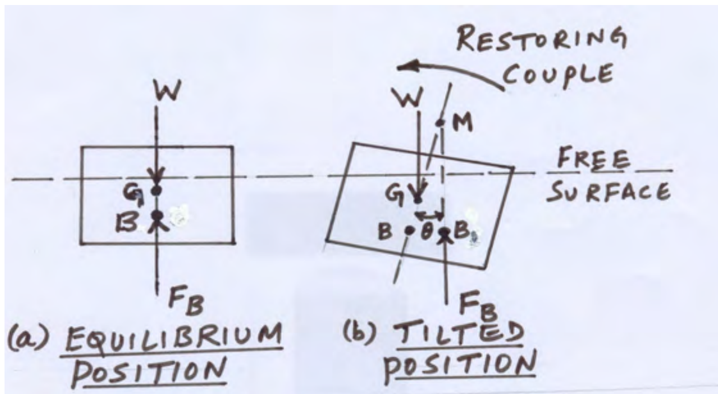


Fig. Meta Center

Fig. (a) shows equilibrium position; (G) is above (B), F_B and W are co-linear. Fig. (b) shows the situation after the body has undergone a small angular displacement (θ) with respect to the vertical axis. (G) remains unchanged relative to the body. (B) is the Center of Buoyancy (Centroid of the Immersed Volume) and it moves towards the right to the new position [B_1]. The new line of action of the buoyant force through [B_1] which is always vertical intersects the axis BG (old vertical line through [B] and [G]) at [M]. For small angles of (θ), point [M] is practically constant and is known as Meta Center.

Meta Center [M] is a point of intersection of the lines of action of Buoyant Force before and after heel. The distance between Center of Gravity and Meta Center (GM) is called Meta-Centric Height. The distance [BM] is known as Meta-Centric Radius.

In Fig. (b), [M] is above [G], the Restoring Couple acts on the body in its displaced position and tends to turn the body to the original position - Floating body is in stable equilibrium.

If [M] were below [G], the couple would be an Over-turning Couple and the body would be in Unstable Equilibrium.

If [M] coincides with [G], the body will assume a new position without any further movement and thus will be in Neutral Equilibrium.

For a floating body, stability is determined not simply by the relative positions of [B] and [G]. The stability is determined by the relative positions of [M] and [G]. The distance of the Meta-Center [M] above [G] along the line [BG] is known as the Meta-Centric height (GM).

GM=BM-BG

GM>0, [M] above [G]------- Stable Equilibrium
GM=0, [M] coinciding with [G]------Neutral Equilibrium
GM<0, [M] below [G]------- Unstable Equilibrium.

DETERMINATION OF META-CENTRIC HEIGHT

Consider a floating object as shown. It is given a small tilt angle(θ) from the initial state. Increase in the volume of displacement on the right hand side displaces the Center of Buoyancy from (B) to (B_1)

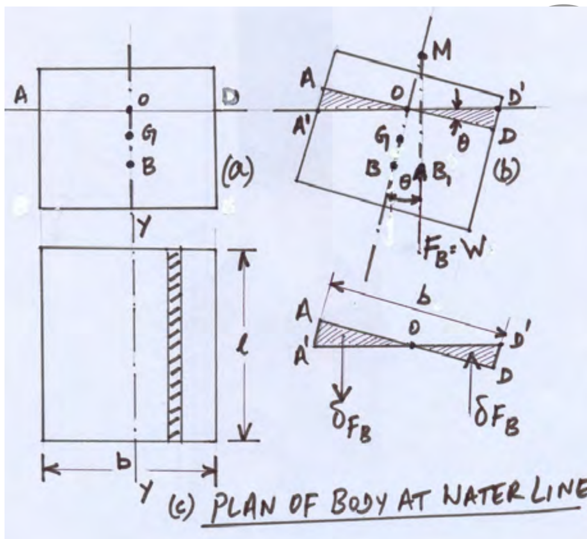


Fig. Determination of Meta-centric Height.

The shift in the center of Buoyancy results in the Restoring Couple = W (BM tan θ);
Since F_B= W; W=Weight of the body= Buoyant force= F_B
This is the moment caused by the movement of Center of Buoyancy from (B) to (B_1)
Volume of the liquid displaced by the object remains same.
Area AOA^1=Area DOD^1
Weight of the wedge AOA^1(which emerges out)=Weight of the wedge DOD^1(that was submerged)
Let (l) and (b) be the length and breadth of the object. .
Weight of each wedge shaped portion of the liquid
= dF_B = (w/2)(b/2)(b/2)(tanθ)(l) =[(wb^2 l tan θ)/8]
w=ρg=specific weight of the liquid.

There exists a buoyant force dF_B upwards on the wedge (ODD^1) and dF_B downwards on the wedge (OAA^1) each at a distance of (2/3)(b/2)=(b/3) from the center.
The two forces are equal and opposite and constitute a couple of magnitude,
$dM= dF_B\ (2/3)b\ =[(wb^2\ l\ \tan\theta)/8](2/3)b = w(lb^3/12)\tan\theta = wI_{YY}\tan\theta$ Where, I_{YY} is the moment of inertia of the floating object about the longitudinal axis.
This moment is equal to the moment caused by the movement of buoyant force from (B) to (B_1).
$W(BM)\tan\theta = w(I_{YY})\tan\theta$; Since W=wV, where V=volume of liquid displaced by the object, $wV(BM)\tan\theta = w\ I_{YY}\tan\theta$
Therefore,
$BM= (I_{YY} / V)$ and $GM = BM-BG= (I_{YY} / V) - BG$
Where BM = [Second moment of the area of the plane of flotation about the centroidal axis perpendicular to the plane of rotation / Immersed Volume]

EXPERIMENTAL METHOD OF DETERMINATION OF META-CENTRIC HEIGHT

Let w_1= known weight placed over the center of the vessel as shown in Fig. (a) and vessel is floating. Let W=Weight of the vessel including (w_1)
G=Center of gravity of the vessel
B=Center of buoyancy of the vessel

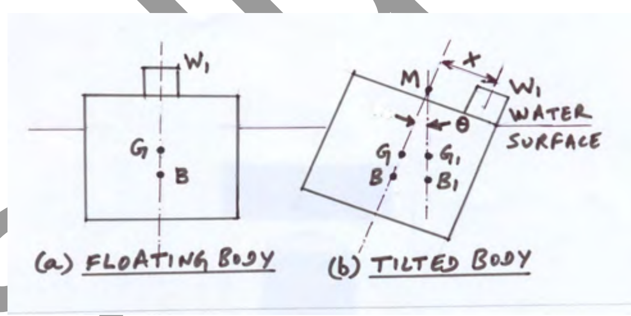
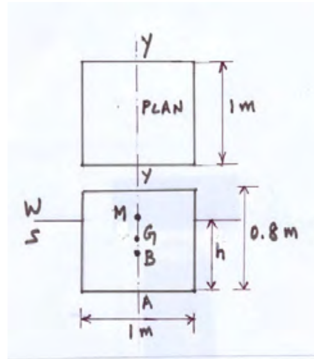


Fig. Experimental method for determination of Meta-centric height.

Move weight (w_1) across the vessel towards right by a distance (x) as shown in Fig. (b). The angle of heel can be measured by means of a plumb line. The new Center of Gravity of the vessel will shift to (G_1) and the Center of Buoyancy will change to B_1. Under equilibrium, the moment caused by the movement of the load (w_1) through a distance (x)=Moment caused by the shift of center of gravity from (G) to (G_1).
Moment due to the change of G = W (GG_1) = W (GM $\tan\theta$)
Moment due to movement of $w_1 = w_1(x)$ = W (GM. $\tan\theta$)
Therefore, $GM= [(w_1\ x) / (W\tan\theta)]$

Problem - 1

A block of wood (specific gravity=0.7) floats in water. Determine the meta-centric height if it's size is $1m \times 1m \times 0.8m$.



Let the depth of immersion=h

Weight of the wooden block = 5494 N

Weight of water displaced = $1000 \times 9.81 \times 1 \times 1 \times h$= 5494N

Therefore, h=0.56 m; AB =0.28 m; AG=0.4 m; BG=0.12 m;

$GM=(I_{yy}/V) - BG$

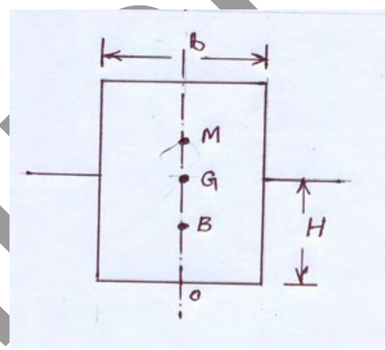
$I_{yy}=(1 / 12)\ m^4$, $V=0.56m^3$

$GM = 1 / (12 \times 0.56) - 0.12 = 0.0288m$

(The body is Stable)

Problem - 2

A rectangular barge of width 'b' and a submerged depth 'H' has its center of gravity at the water line. Find the meta-centric height in terms of (b/H) and show that for stable equilibrium of the barge, $(b/ H) > \sqrt{6}$



OB =(H / 2) and OG = H

L= Length of the barge

BG = (H / 2);

$BM = (I_{yy} / V)= [(Lb^3 / 12) /(LbH)] = (b^2 /12H)$

$GM=BM-BG= (b^2 / 12H) - (H/ 2)$

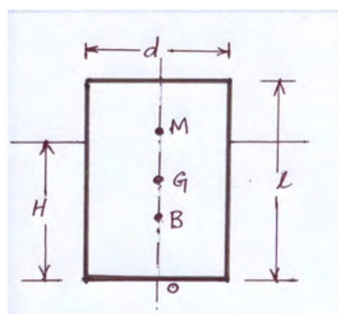
$=(H/2)[\{(b/H)^2 / 6\}- 1]$;

For stable equilibrium, GM > 0;

Therefore, $[b /H] > \sqrt{6}$

Problem 3

A wooden cylinder having a specific gravity of 0.6 is required to float in an oil of specific gravity 0.8. If the diameter of cylinder is 'd' and length is 'L', show that 'L' cannot exceed 0.817d for the cylinder to float with its longitudinal axis vertical.



Weight of the cylinder = Weight of oil displaced

$(\pi d^2 /4) \times L \times 600 \times g = (\pi d^2/4) \times H \times 800 \times g$;

Therefore, H=0.75L

OG=(L/2); OB=(H/2) = (3L/8)

BG= OG - OB = (L/8)

$BM=(I_{yy} /V)$; $I_{yy}= (\pi d^4/64)$; $V=(\pi d^2 H /4)$

$BM= (d^2 / 12L)$

$GM=BM -BG = (d^2 / 12L) - (L /8)$;

For Stable equilibrium,

GM > 0; 0.817d > L or L< 0.817d

FLUID MECHANICS

FLUID KINEMATICS

Fluid Kinematics gives the geometry of fluid motion. It is a branch of fluid mechanics, which describes the fluid motion, and it's consequences without consideration of the nature of forces causing the motion. Fluid kinematics is the study of velocity as a function of space and time in the flow field. From velocity, pressure variations and hence, forces acting on the fluid can be determined.

VELOCITY FIELD

Velocity at a given point is defined as the instantaneous velocity of the fluid particle, which at a given instant is passing through the point. It is represented by V=V(x,y,z,t). Vectorially, V=ui+vj+wk where u,v,w are three scalar components of velocity in x,y and z directions and (t) is the time. Velocity is a vector quantity and velocity field is a vector field.

FLOW PATTERNS

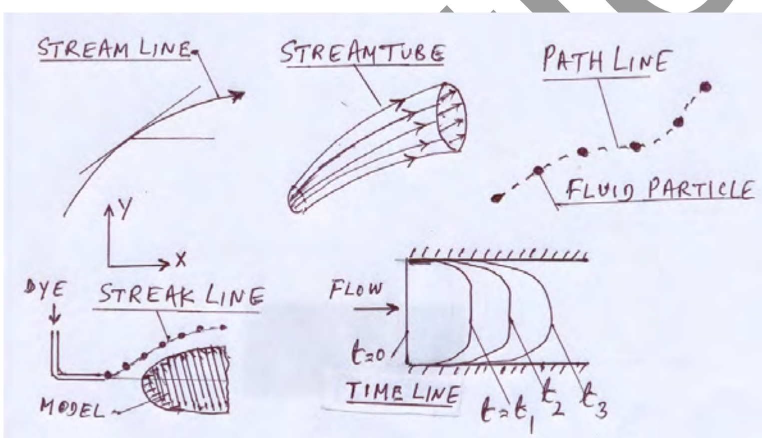


Fig. Flow Patterns

Fluid Mechanics is a visual subject. Patterns of flow can be visualized in several ways. Basic types of line patterns used to visualize flow are streamline, path line, streak line and time line.

(a) Stream line is a line, which is everywhere tangent to the velocity vector at a given instant.
(b) Path line is the actual path traversed by a given particle.
(c) Streak line is the locus of particles that have earlier passed through a prescribed point.
(d) Time line is a set of fluid particles that form a line at a given instant.

Streamline is convenient to calculate mathematically. Other three lines are easier to obtain experimentally. Streamlines are difficult to generate experimentally. Streamlines and Time lines are instantaneous lines. Path lines and streak lines are generated by passage of time. In a steady flow situation, streamlines, path lines and streak lines are identical. In Fluid Mechanics, the most common mathematical result for flow visualization is the streamline pattern – It is a common method of flow pattern presentation.

Streamlines are everywhere tangent to the local velocity vector. For a stream line, (dx/u) = (dy/v) = (dz/w). Stream tube is formed by a closed collection of streamlines. Fluid within the stream tube is confined there because flow cannot cross streamlines. Stream tube walls need not be solid, but may be fluid surfaces

METHOD OF DESCRIBING FLUID MOTION

Two methods of describing the fluid motion are: (a) Lagrangian method and (b) Eularian method.

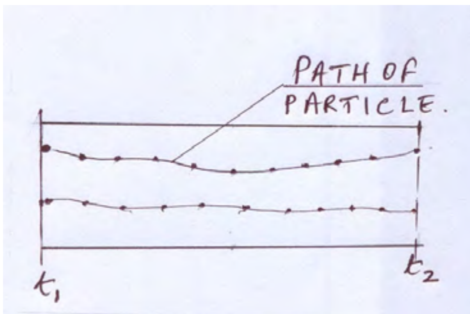


Fig. Lagrangian method

A single fluid particle is followed during its motion and its velocity, acceleration etc. are described with respect to time. Fluid motion is described by tracing the kinematics behavior of each and every individual particle constituting the flow. We follow individual fluid particle as it moves through the flow. The particle is identified by its position at some instant and the time elapsed since that instant. We identify and follow small, fixed masses of fluid. To describe the fluid flow where there is a relative motion, we need to follow many particles and to resolve details of the flow; we need a large number of particles. Therefore, Langrangian method is very difficult and not widely used in Fluid Mechanics.

EULARIAN METHOD

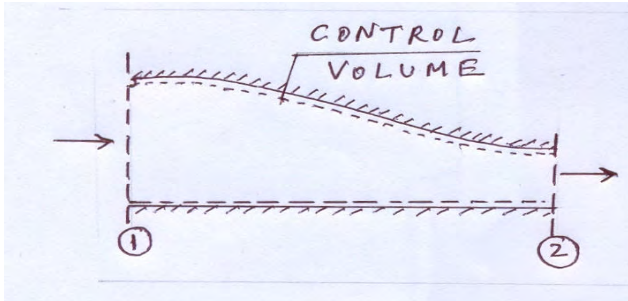


Fig. Eulerian Method

The velocity, acceleration, pressure etc. are described at a point or at a section as a function of time. This method commonly used in Fluid Mechanics. We look for field description, for Ex.; seek the velocity and its variation with time at each and every location in a flow field. Ex., V=V(x,y,z,t). This is also called control volume approach. We draw an imaginary box around a fluid system. The box can be large or small, and it can be stationary or in motion.

TYPES OF FLUID FLOW

1. Steady and Un-steady flows
2. Uniform and Non-uniform flows
3. Laminar and Turbulent flows
4. Compressible and Incompressible flows
5. Rotational and Irrotational flows
6. One, Two and Three dimensional flows

STEADY AND UNSTEADY FLOW

Steady flow is the type of flow in which the various flow parameters and fluid properties at any point do not change with time. In a steady flow, any property may vary from point to point in the field, but all properties remain constant with time at every point. $[\partial V/\partial t]_{x,y,z} = 0$; $[\partial p/\partial t]_{x,y,z} = 0$. Ex.: V=V(x,y,z); p=p(x,y,z) . Time is a criterion.

Unsteady flow is the type of flow in which the various flow parameters and fluid properties at any point change with time. $[\partial V/\partial t]_{x,y,z} \neq 0$; $[\partial p/\partial t]_{x,y,z} \neq 0$, Eg.:V=V(x,y,z,t), p=p(x,y,z,t) or V=V(t), p=p(t) . Time is a criterion

UNIFORM AND NON-UNIFORM FLOWS

Uniform Flow is the type of flow in which velocity and other flow parameters at any instant of time do not change with respect to space. Eg., V=V(x) indicates that the flow is uniform in 'y' and 'z' axis. V=V (t) indicates that the flow is uniform in 'x', 'y' and 'z' directions. Space is a criterion.

Uniform flow field is used to describe a flow in which the magnitude and direction of the velocity vector are constant, i.e., independent of all space coordinates throughout the entire flow field (as opposed to uniform flow at a cross section). That is, $[\partial V/ \partial s]_{t=constant}$ =0, that is 'V' has unique value in entire flow field

Non-uniform flow is the type of flow in which velocity and other flow parameters at any instant change with respect to space.
$[\partial V/ \partial s]_{t=constant}$ is not equal to zero. Distance or space is a criterion

LAMINAR AND TURBULANT FLOWS

Laminar Flow is a type of flow in which the fluid particles move along well-defined paths or stream-lines. The fluid particles move in laminas or layers gliding smoothly over one another. The behavior of fluid particles in motion is a criterion.

Turbulent Flow is a type of flow in which the fluid particles move in zigzag way in the flow field. Fluid particles move randomly from one layer to another. Reynolds number is a criterion. We can assume that for a flow in pipe, for Reynolds No. less than 2000, the flow is laminar; between 2000-4000, the flow is transitional; and greater than 4000, the flow is turbulent.

COMPRESSIBLE AND INCOMPRESSIBLE FLOWS

Incompressible Flow is a type of flow in which the density (ρ) is constant in the flow field. This assumption is valid for flow Mach numbers with in 0.25. Mach number is used as a criterion. Mach Number is the ratio of flow velocity to velocity of sound waves in the fluid medium

Compressible Flow is the type of flow in which the density of the fluid changes in the flow field. Density is not constant in the flow field. Classification of flow based on Mach number is given below:

M < 0.25 – Low speed
M < unity – Subsonic
M around unity – Transonic
M > unity – Supersonic
M > > unity, (say 7) – Hypersonic

ROTATIONAL AND IRROTATIONAL FLOWS

Rotational flow is the type of flow in which the fluid particles while flowing along stream-lines also rotate about their own axis.

Ir-rotational flow is the type of flow in which the fluid particles while flowing along stream-lines do not rotate about their own axis.

ONE,TWO AND THREE DIMENSIONAL FLOWS

The number of space dimensions needed to define the flow field completely governs dimensionality of flow field. Flow is classified as one, two and three-dimensional depending upon the number of space co-ordinates required to specify the velocity fields.

One-dimensional flow is the type of flow in which flow parameters such as velocity is a function of time and one space coordinate only.
For Ex., V=V(x,t) – 1-D, unsteady ; V=V(x) – 1-D, steady

Two-dimensional flow is the type of flow in which flow parameters describing the flow vary in two space coordinates and time.
For Ex., V=V(x,y,t) – 2-D, unsteady; V=V(x,y) – 2-D, steady

Three-dimensional flow is the type of flow in which the flow parameters describing the flow vary in three space coordinates and time.
For Ex., V=V(x,y,z,t) – 3-D, unsteady ; V=V(x,y,z) – 3D, steady

CONTINUITY EQUATION

Rate of flow or discharge (Q) is the volume of fluid flowing per second. For incompressible fluids flowing across a section,
Volume flow rate, $Q = A \times V$ m^3/s where A=cross sectional area and V= average velocity.
For compressible fluids, rate of flow is expressed as mass of fluid flowing across a section per second.
Mass flow rate (m) =(ρAV) kg/s where ρ = density.

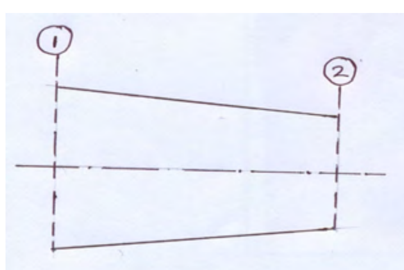


Fig. Continuity Equation

Continuity equation is based on Law of Conservation of Mass. For a fluid flowing through a pipe, in a steady flow, the quantity of fluid flowing per second at all cross-sections is a constant.

Let v_1=average velocity at section [1], ρ_1=density of fluid at [1], A_1=area of flow at [1]; Let v_2, ρ_2, A_2 be corresponding values at section [2].

Rate of flow at section [1]= $\rho_1 A_1 v_1$

Rate of flow at section [2]= $\rho_2 A_2 v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

This equation is applicable to steady compressible or incompressible fluid flows and is called Continuity Equation. If the fluid is incompressible, $\rho_1 = \rho_2$ and the continuity equation reduces to $A_1 v_1 = A_2 v_2$

For steady, one dimensional flow with one inlet and one outlet,

$\rho_1 A_1 v_1 - \rho_2 A_2 v_2 = 0$

For control volume with N inlets and outlets

$\sum_{i=1}^{N} (\rho_i A_i v_i) = 0$ where inflows are positive and outflows are negative .

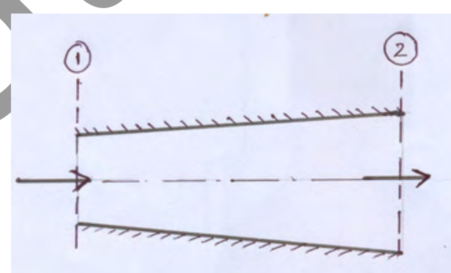
Velocities are normal to the areas. This is the continuity equation for steady one dimensional flow through a fixed control volume

When density is constant, $\sum_{i=1}^{N} (A_i v_i) = 0$

Problem 1

The diameters of the pipe at sections (1) and (2) are 15cm and 20cm respectively. Find the discharge through the pipe if the velocity of water at section (1) is 4m/s. Determine also the velocity at section (2)

(Answers: 0.0706m^3/s, 2.25m/s)



Problem-2

A 40cm diameter pipe conveying water branches into two pipes of diameters 30cm and 20cm respectively. If the average velocity in the 40cm diameter pipe is 3m/s., find the discharge in this pipe. Also, determine the velocity in 20cm diameter pipe if the average velocity in 30cm diameter pipe is 2m/s.

(Answers: 0.3769m^3/s., 7.5m/s.)

CONTINUITY EQUATION IN 3-DIMENSIONS

(Differential form, Cartesian co-ordinates)

Consider infinitesimal control volume as shown of dimensions dx, dy and dz in x,y,and z directions

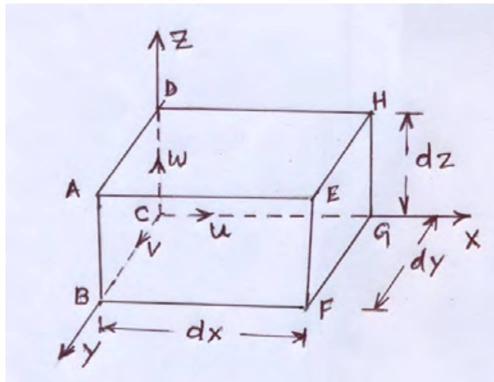


Fig. Continuity Equation in Three Dimensions

u,v,w are the velocities in x,y,z directions.
Mass of fluid entering the face ABCD =
Density ×velocity in x-direction ×Area ABCD = ρudy.dz
Mass of fluid leaving the face EFGH= ρudy.dz+ [∂ (ρudy.dz)/ ∂x](dx)
Therefore, net rate of mass efflux in x-direction= –[∂ (ρudy.dz)/ ∂x](dx)
= - [∂ (ρu)/ ∂ x](dxdydz)
Similarly, the net rate of mass efflux in
y-direction= - [∂ (ρv)/ ∂y](dxdydz)
z-direction= - [∂ (ρw)/ ∂z](dxdydz)
The rate of accumulation of mass within the control volume =∂ (ρdV)/ ∂t = ρ∂/∂t (dV)
where dV=Volume of the element=dxdydz and dV is invarient with time.
From conservation of mass, the net rate of efflux = Rate of accumulation of mass within the control volume.
- [∂(ρu)/ ∂ x + ∂ (ρv)/ ∂ y + ∂ (ρw)/ ∂ z](dxdydz) = ρ∂/∂t(dxdydz) OR
ρ∂/∂t + ∂ (ρu)/ ∂ x +∂ (ρv)/ ∂ y + ∂ (ρw)/ ∂ z =0

This is the continuity equation applicable for
(a) Steady and unsteady flows
(b) Uniform and non-uniform flows
(c) Compressible and incompressible flows.
For steady flows, (∂/∂t) = 0 and [∂ (ρu) /∂x + ∂ (ρv)/ ∂y + ∂ (ρw)/ ∂z] =0
If the fluid is incompressible, ρ= constant [∂u/∂x + ∂v/∂y + ∂w/∂z] =0
This is the continuity equation for 3-D flows.
For 2-D flows, w=0 and [∂u/∂x + ∂v/∂y] = 0

VELOCITY AND ACCELERATION

Let V= Resultant velocity at any point in a fluid flow. (u,v,w) are the velocity components in x,y and z directions which are functions of space coordinates and time.

$u=u(x,y,z,t)$; $v=v(x,y,z,t)$; $w=w(x,y,z,t)$.

Resultant velocity= $V=ui+vj+wk$

$|V|= (u^2+v^2+w^2)^{1/2}$

Let a_x, a_y, a_z are the total accelerations in the x,y,z directions respectively

$a_x= [du/ dt]= [\partial u/ \partial x] [\partial x/ \partial t]+ [\partial u/ \partial y][\partial y/ \partial t]+ [\partial u/ \partial z][\partial z/ \partial t]+ [\partial u/ \partial t]$

$a_x= [du/dt] =u[\partial u/ \partial x]+v[\partial u/ \partial y]+w[\partial u/ \partial z]+[\partial u/\partial t]$

Similarly,

$a_y= [dv/ dt] = u[\partial v/ \partial x]+ v[\partial v/ \partial y]+w[\partial v/ \partial z]+[\partial v/\partial t]$

$a_z= [dw/dt] = u[\partial w/ \partial x]+ v[\partial w/ \partial y]+w[\partial w/ \partial z]+[\partial w/ \partial t]$

1. Convective Acceleration Terms – The first three terms in the expressions for a_x, a_y, a_z. Convective acceleration is defined as the rate of change of velocity due to change of position of the fluid particles in a flow field

2. Local Acceleration Terms- The 4th term, $[\partial (\ )/ \partial t]$ in the expressions for a_x, a_y, a_z. Local or temporal acceleration is the rate of change of velocity with respect to time at a given point in a flow field.

Material or Substantial Acceleration = Convective Acceleration + Local or Temporal Acceleration.
In a steady flow, temporal or local acceleration is zero.
In uniform flow, convective acceleration is zero.

For steady flow, $[\partial u/ \partial t]= [\partial v/ \partial t]= [\partial w/ \partial t]= 0$

$a_x= [du/dt]= u[\partial u/\partial x]+v[\partial u/\partial y]+w[\partial u/\partial z]$

$a_y= [dv/dt]= u[\partial v/\partial x]+v[\partial v/\partial y]+w[\partial v/\partial z]$

$a_z= [dw/dt]= u[\partial w/\partial x]+v[\partial w/\partial y]+w[\partial w/\partial z]$

Acceleration Vector= $a_x i+a_y j+a_z k$; $|A|=[a_x^2+a_Y^2+a_z^2]^{1/2}$

Problem 1

The fluid flow field is given by $V=x^2yi+y^2zj-(2xyz+yz^2)k$.
Prove that this is a case of a possible steady incompressible flow field.
$u=x^2y$; $v=y^2z$; $w= -2xyz -yz^2$
$(\partial u/\partial x) = 2xy$; $(\partial v/\partial y)=2yz$; $(\partial w/\partial z)= -2xy -2yz$
For steady incompressible flow, the continuity equation is
$[\partial u/\partial x + \partial v/\partial y + \partial w/\partial z] =0$
$2xy + 2yz -2xy -2yz =0$
Therefore, the given flow field is a possible case of steady incompressible fluid flow.

Problem-2.

Given $v=2y^2$ and $w=2xyz$, the two velocity components. Determine the third component such that it satisfies the continuity equation.

$v=2y^2$; $w=2xyz$; $(\partial v/\partial y)=4y$; $(\partial w/\partial z)=2xy$

$(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z) = 0$

$(\partial u/\partial x) = -4y - 2xy$; $\partial u = (-4y - 2xy)\, \partial x$

$u = -4xy - x^2y + f(y,z)$; $f(y,z)$ can not be the function of (x)

Problem-3.

Find the acceleration components at a point (1,1,1) for the following flow field:

$u=2x^2+3y$; $v= -2xy+3y^2+3zy$; $w= -(3/2)z^2+2xz - 9y^2z$

$a_x = [\partial u/\partial t] + u[\partial u/\partial x] + v[\partial u/\partial y] + w[\partial u/\partial z]$

$0+(2x^2+3y)4x+(-2xy+3y^2+3zy)3+0$; $[a_x]_{1,1,1} = 32$ units

Similarly, $a_y = [\partial v/\partial t] + u[\partial v/\partial x] + v[\partial v/\partial y] + w[\partial v/\partial z]$

$a_y = 0+(2x^2+3y)(-2x)+(-2xy+3y^2+3zy)(-2x+6y+3z)+\{-(3/2)z^2+2xz - 9y^2z\}3y$

$[a_y]_{1,1,1} = -7.5$ units

Similarly, $a_z = [\partial w/\partial t] + u[\partial w/\partial x] + v[\partial w/\partial y] + w[\partial w/\partial z]$

Substituting, $a_z = 23$ units

Resultant $|a| = (a_x^2 + a_y^2 + a_z^2)^{1/2}$

Problem-4.

Given the velocity field $V = (4+xy+2t)i + 6x^3j + (3xt^2+z)k$. Find acceleration of a fluid particle at (2,4,-4) at t=3.

$[dV/dt] = [\partial V/\partial t] + u[\partial V/\partial x] + v[\partial V/\partial y] + w[\partial V/\partial z]$

$u = (4+xy+2t)$; $v=6x^3$; $w = (3xt^2+z)$

$[\partial V/\partial x] = (yi+18x^2j+3t^2k)$; $[\partial V/\partial y] = xi$; $[\partial V/\partial z] = k$; $[\partial V/\partial t] = 2i+6xtk$. Substituting,

$[dV/dt] = (2+4y+xy^2+2ty+6x^4)i + (72x^2+18x^3y+36tx^2)j +$
$(6xt+12t^2+3xyt^2+6t^3+z+3xt^2)k$

The acceleration vector at the point (2,4,-4) and time t=3 is obtained by substitution,

$a = 170i+1296j+572k$; Therefore, $a_x=170$, $a_y=1296$, $a_z=572$

Resultant $|a| = [170^2+1296^2+572^2]^{1/2}$ units = 1426.8 units.

VELOCITY POTENTIAL AND STREAM FUNCTION

Velocity Potential Function is a Scalar Function of space and time co-ordinates such that its negative derivatives with respect to any direction give the fluid velocity in that direction.

$\Phi = \Phi(x,y,z)$ for steady flow.

$u = -(\partial\Phi/\partial x)$; $v = -(\partial\Phi/\partial y)$; $w = -(\partial\Phi/\partial z)$ where u,v,w are the components of velocity in x,y and z directions.

In cylindrical co-ordinates, the velocity potential function is given by $u_r = (\partial\Phi/\partial r)$, $u_\theta = (1/r)(\partial\Phi/\partial\theta)$

The continuity equation for an incompressible flow in steady state is

$(\partial u/\partial x + \partial v/\partial y + \partial w/\partial z) = 0$

Substituting for u, v and w and simplifying,

$(\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2 + \partial^2\Phi/\partial z^2) = 0$

Which is a Laplace Equation. For 2-D Flow, $(\partial^2\Phi/\partial x^2 + \partial^2\Phi/\partial y^2) = 0$

If any function satisfies Laplace equation, it corresponds to some case of steady incompressible fluid flow.

IRROTATIONAL FLOW AND VELOCITY POTENTIAL

Assumption of Ir-rotational flow leads to the existence of velocity potential. Consider the rotation of the fluid particle about an axis parallel to z-axis. The rotation component is defined as the average angular velocity of two infinitesimal linear segments that are mutually perpendicular to each other and to the axis of rotation.

Consider two-line segments δx, δy. The particle at P(x,y) has velocity components u,v in the x-y plane.

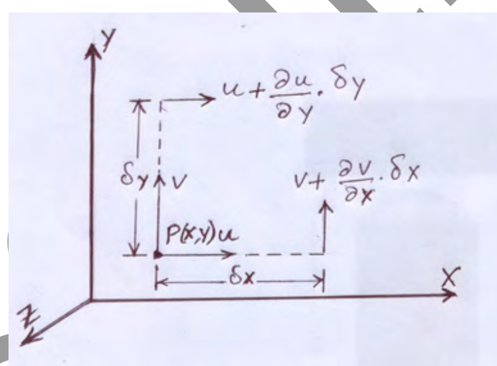


Fig. Rotation of a fluid partical.

The angular velocities of δx and δy are sought.

The angular velocity of (δx) is $\{[v + (\partial v/\partial x)\,\delta x - v] / \delta x\} = (\partial v/\partial x)$ rad/sec

The angular velocity of (δy) is $-\{[u+ (\partial u/\partial y)\ \delta y - u] / \delta y\} = -(\partial u/\partial y)$ rad/sec
Counter clockwise direction is taken positive. Hence, by definition, rotation component (ω_z) is $\omega_z = 1/2\ \{(\partial v/\partial x) - (\partial u/\partial y)\}$. The other two components are

$\omega_x = 1/2\ \{(\partial w/\partial y) - (\partial v/\partial z)\}$

$\omega_y = 1/2\ \{(\partial u/\partial z) - (\partial w/\partial x)\}$

The rotation vector = $\omega = i\omega_x + j\omega_y + k\omega_z$.

The vorticity vector(Ω) is defined as twice the rotation vector = 2ω

PROPERTIES OF POTENTIAL FUNCTION

$\omega_z = 1/2\ \{(\partial v/\partial x) - (\partial u/\partial y)\}$

$\omega_x = 1/2\ \{(\partial w/\partial y) - (\partial v/\partial z)\}$

$\omega_y = 1/2\ \{(\partial u/\partial z) - (\partial w/\partial x)\}$;

Substituting $u = -(\partial \Phi/\partial x)$; $v = -(\partial \Phi/\partial y)$; $w = -(\partial \Phi/\partial z)$; we get

$\omega_z = 1/2\ \{(\partial/\partial x)(-\partial \Phi/\partial y) - (\partial/\partial y)(-\partial \Phi/\partial x)\}$

$= ½\{-(\partial^2 \Phi/\partial x \partial y) + (\partial^2 \Phi/\partial y \partial x)\} = 0$ since Φ is a continuous function.

Similarly, $\omega_x = 0$ and $\omega_y = 0$

All rotational components are zero and the flow is irrotational.– Therefore, irrotational flow is also called as Potential Flow.

If the velocity potential (Φ) exists, the flow should be irrotational. If velocity potential function satisfies Laplace Equation, It represents the possible case of steady, incompressible, irrotational flow. Assumption of a velocity potential is equivalent to the assumption of irrotational flow.

Laplace equation has several solutions depending upon boundary conditions.
If Φ_1 and Φ_2 are both solutions, $\Phi_1 + \Phi_2$ is also a solution

$\nabla^2(\Phi_1)=0$, $\nabla^2(\Phi_2)=0$, $\nabla^2(\Phi_1 + \Phi_2)=0$

Also if Φ_1 is a solution, $C\Phi_1$ is also a solution (where C=Constant)

STREAM FUNCTION (ψ)

Stream Function is defined as the scalar function of space and time such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. Stream function is defined only for two dimensional flows and 3-D flows with axial symmetry.

$(\partial \psi/\partial x) = v$; $(\partial \psi/\partial y) = -u$

In Cylindrical coordinates, $u_r = (1/r)\ (\partial \psi/\partial \theta)$ and $u_\theta = (\partial \psi/\partial r)$

Continuity equation for 2-D flow is $(\partial u/\partial x) + (\partial v/\partial y) = 0$

$(\partial/\partial x)\ (-\partial \psi/\partial y) + (\partial/\partial y)\ (\partial \psi/\partial x) = 0$

$-(\partial^2\psi/\partial x\partial y) + (\partial^2\psi/\partial y\partial x) = 0$; Therefore, continuity equation is satisfied. Hence, the existence of (ψ) means a possible case of fluid flow. The flow may be rotational or irrotational. The rotational component are:

$\omega_z = 1/2 \{(\partial v/\partial x) - (\partial u/\partial y)\}$

$\omega_z = 1/2 \{(\partial/\partial x)(\partial\psi/\partial x) - (\partial/\partial y)(-\partial\psi/\partial y)\}$

$\omega_z = ½\{(\partial^2 \psi/\partial x^2) + (\partial^2\psi/\partial y^2)\}$

For irrotational flow, $\omega_z = 0$. Hence for 2-D flow, $(\partial^2 \psi/\partial x^2) + (\partial^2\psi/\partial y^2) = 0$ which is a Laplace equation.

PROPERTIES OF STREAM FUNCTION

1.If the Stream Function (ψ) exists, it is a possible case of fluid flow, which may be rotational or irrotational.
2.If Stream Function satisfies Laplace Equation, it is a possible case of an irrotational flow.

EQUI-POTENTIAL & CONSTANT STREAM FUNCTION LINES

On an equi-potential line, the velocity potential is constant, Φ=constant or d(Φ)=0.
$\Phi = \Phi(x,y)$ for steady flow.
$d(\Phi) = (\partial\Phi/\partial x)\,dx + (\partial\Phi/\partial y)\,dy$.
$d(\Phi) = -u\,dx - v\,dy = -(u\,dx + v\,dy) = 0$.
For equi-potential line, $u\,dx + v\,dy = 0$
Therefore, $(dy/dx) = -(u/v)$ which is a slope of equi-potential lines
For lines of constant stream Function,
ψ = Constant or $d(\psi)=0$, $\psi = \psi(x,y)$
$d(\psi) = (\partial\psi/\partial x)\,dx + (\partial\psi/\partial y)\,dy = vdx - udy$
Since $(\partial\psi/\partial x) = v$; $(\partial\psi/\partial y) = -u$
Therefore, $(dy/dx) = (v/u)$ = slope of the constant stream function line. This is the slope of the stream line.

The product of the slope of the equi-potential line and the slope of the constant stream function line (or stream Line) at the point of intersection = -1.
Thus, equi-potential lines and streamlines are orthogonal at all points of intersection.

FLOW NET

A grid obtained by drawing a series of equi-potential lines and streamlines is called a Flow Net. A Flow Net is an important tool in analyzing two-dimensional ir-rotational flow problems.

EXAMPLES OF FLOW NETS

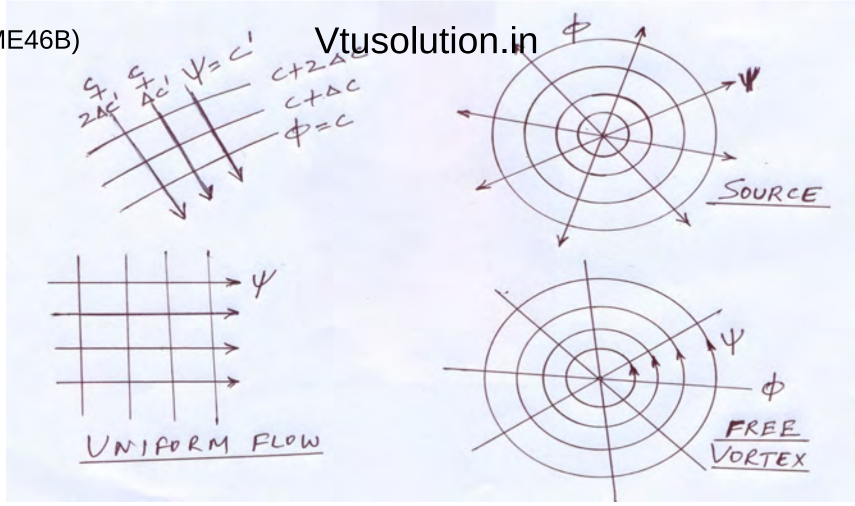


Fig. Flow Nets

Examples: Uniform flow, Line source and sink, Line vortex
Two-dimensional doublet – a limiting case of a line source approaching a line sink

RELATIONSHIP BETWEEN STREAM FUNCTION AND VELOCITY POTENTIAL

$u = -(\partial\Phi/\partial x)$, $v = -(\partial\Phi/\partial y)$
$u = -(\partial\psi/\partial y)$, $v = (\partial\psi/\partial x)$; Therefore,
$-(\partial\Phi/\partial x) = -(\partial\psi/\partial y)$ and $-(\partial\Phi/\partial y) = (\partial\psi/\partial x)$
Hence, $(\partial\Phi/\partial x) = (\partial\psi/\partial y)$ and $(\partial\Phi/\partial y) = -(\partial\psi/\partial x)$

Problem-1

The velocity potential function for a flow is given by $\Phi = (x^2 - y^2)$. Verify that the flow is incompressible and determine the stream function for the flow.
$u = -(\partial\Phi/\partial x) = -2x$, $v = -(\partial\Phi/\partial y) = 2y$
For incompressible flow, $(\partial u/\partial x) + (\partial v/\partial y) = 0$
Continuity equation is satisfied. The flow is 2-D and incompressible and exists.

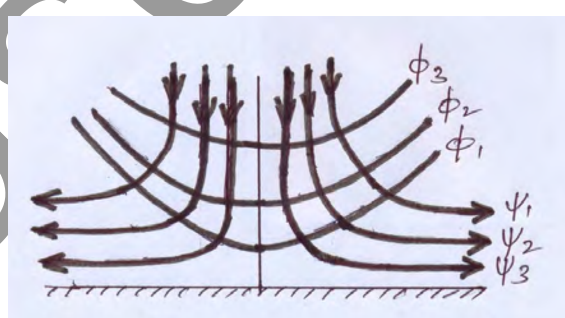


Fig. Flow net for flow around 90^0 bend

$u = -(\partial\psi/\partial y)$; $v = (\partial\psi/\partial x)$; $(\partial\psi/\partial y) = -u = 2x$;
$\psi = 2xy + F(x) + C$; C=Constant

$(\partial\psi/\partial x)$= v = 2y; ψ = 2xy+ F(y)+C
Comparing we get, ψ= 2xy+C

Problem-2.
The stream function for a 2-D flow is given by ψ = 2xy. Calculate the velocity at the point P (2,3) and velocity function (Φ).
Given ψ = 2xy; u= –$(\partial\psi/\partial y)$ = -2x; v= $(\partial\psi/\partial x)$=2y
Therefore, u= –4 units/sec. and v= 6 units/sec.
Resultant=$\sqrt{(u^2+v^2)}$ = 7.21 units/sec.
$(\partial\Phi/\partial x)$= –u = 2x; Φ= x^2+F(y)+C; C=Constant.
$(\partial\Phi/\partial y)$ = –v= –2y; Φ= – y^2+F(x)+C,
Therefore, we get, Φ= $(x^2 - y^2)$ +C

TYPES OF MOTION

A Fluid particle while moving in a fluid may undergo any one or a combination of the following four types of displacements:

1. Linear or pure translation
2. Linear deformation
3. Angular deformation
4. Rotation.

(1) Linear Translation is defined as the movement of fluid element in which fluid element moves from one position to another bodily – Two axes ab & cd and a'b'& c'd' are parallel (Fig. 8a)

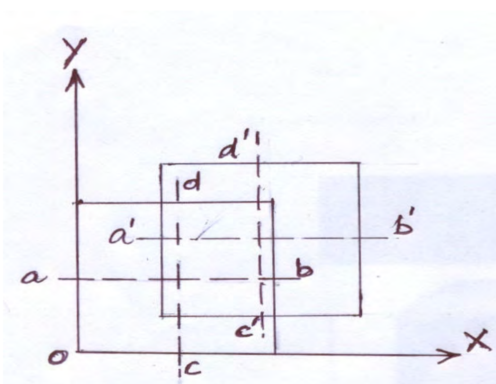


Fig. Linear translation.

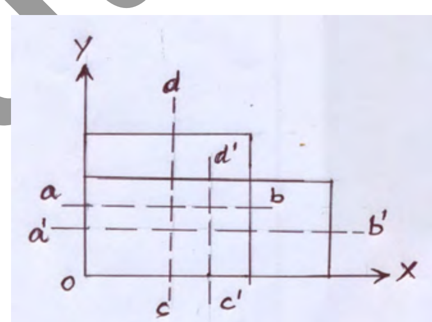


Fig. Linear deformation.

(2) Linear deformation is defined as deformation of fluid element in linear direction – axes are parallel, but length changes.

(3) Angular deformation, also called shear deformation is defined as the average change in the angle contained by two adjacent sides. The angular deformation or shear strain rate = ½$(\Delta\theta_1 + \Delta\theta_2)$ = ½$(\partial v/\partial x + \partial u/\partial y)$

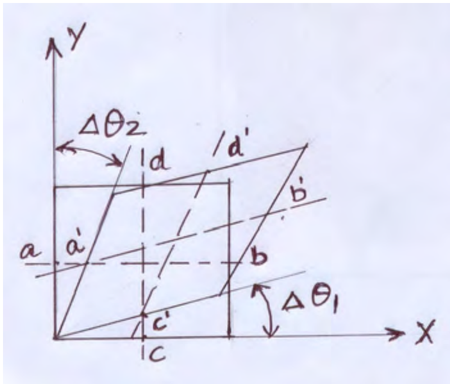


Fig Angular deformation

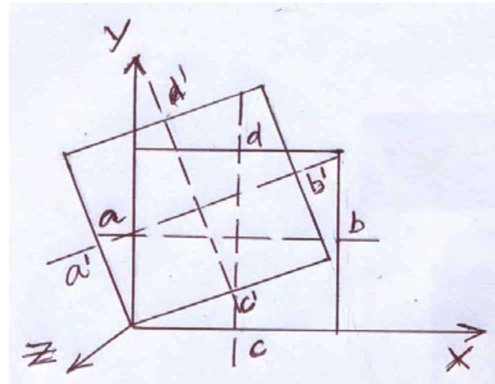


Fig. Rotation

(4) Rotation is defined as the movement of the fluid element in such a way that both its axes (horizontal as well as vertical) rotate in the same direction. Rotational components are:

ω_z= 1/2 $\{(\partial v/\partial x)- (\partial u/\partial y)\}$

ω_x= 1/2 $\{(\partial w/\partial y)- (\partial v/\partial z)\}$

ω_y= 1/2 $\{(\partial u/\partial z)- (\partial w/\partial x)\}$. Vorticity ($\Omega$) is defined as the value twice of the rotation and is given as 2ω

Problem-1.

Find the vorticity components at the point (1,1,1) for the following flow field;

$u=2x^2+3y$, $v= -2xy+3y^2+3zy$, $w= -(3z^2/2) +2xz - 9y^2z$

$\Omega=2\omega$ where Ω= Vorticity and ω= component of rotation.

$\Omega_x= \{(\partial w/\partial y)- (\partial v/\partial z)\}= -18yz-3y= -21$ units

$\Omega_y= \{(\partial u/\partial z)- (\partial w/\partial x)\}= 0-2z= -2$ units

$\Omega_z= \{(\partial v/\partial x)- (\partial u/\partial y)\}= -2y -3 = -5$ units

Problem-2.

The x-component of velocity in a two dimensional incompressible flow over a solid surface is given by $u=1.5y -0.5y^2$, y is measured from the solid surface in the direction perpendicular to it. Verify whether the flow is ir-rotational; if not, estimate the rotational velocity at (3,2).

Using continuity equation, we get

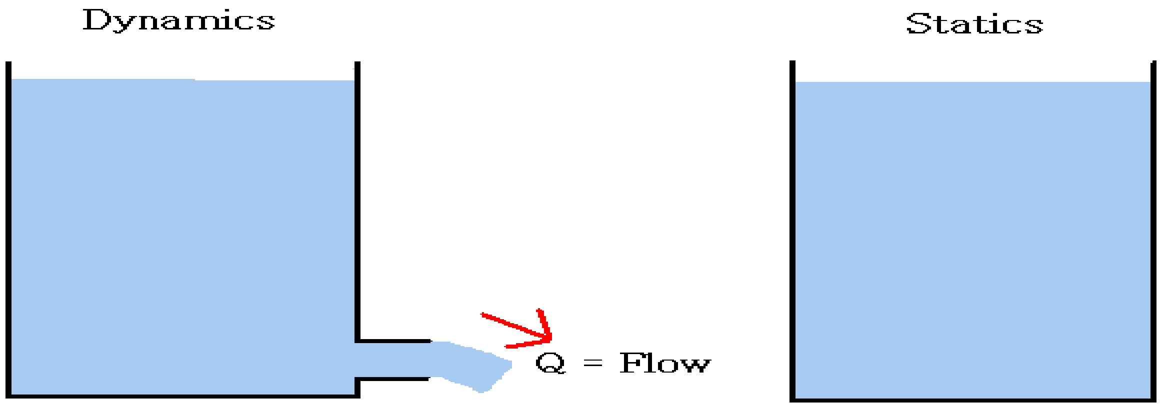
$(\partial v/\partial y) = -(\partial u/\partial x) = 0$ or v=f(x) only.

Since v=0 at y=0 (Solid surface), f(x)=0, and therefore, v=0 everywhere in the flow field. ω_z= 1/2 $\{(\partial v/\partial x) - (\partial u/\partial y)\}$ = – ½(1.5 –y)

ω_z at (3,2) = 0.25 units. Therefore, the rotation about the z-axis is counter clockwise at (y)=2 and is independent of the value of (x)

Fluid dynamics:

Introduction:



The laws of Statics that we have learned cannot solve Dynamic Problems. There is no way to solve for the flow rate, or Q. Therefore, we need a new dynamic approach to Fluid Mechanics.

Equations of Motion

The dynamics of fluid flow is the study of fluid motion with forces causing flow. The dynamic behaviors of the fluid flow is analyzed by the **Newton's law of motion (F=ma),** which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

Mathematically, $F_x = m.a_x$

In the fluid flow, following forces are present:

- **pressure force 'F_p'**
- **gravity force 'F_g'**
- **viscous force 'F_v'**
- **turbulent flow 'F_t'**
- **surface tension force 'F_s'**
- **compressibility force 'F_e'**

- The **pressure force 'F_p'** is exerted on the fluid mass, if there exists a pressure gradient between the 2 parts in the direction of flow.

- The **gravity force 'F_g'** is due to the weight of the fluid and it is equal to 'M_g'. The gravity force for unit volume is equal to 'ρg'.
- The **viscous force 'F_v'** is due to the viscosity of the flowing fluid and thus exists in the case of all real fluid.
- The **turbulent flow 'F_t'** is due to the turbulence of the flow. In the turbulent flow, the fluid particles move from one layer to other and therefore, there is a continuous momentum transfer between adjacent layer, which results in developing additional stresses(called Reynolds stresses) for the flowing fluid.
- The **surface tension force 'F_s'** is due to the cohesive property of the fluid mass. It is, however, important only when the depth of flow is extremely small.
- The **compressibility force 'F_e'** is due to elastic property of fluid and it is important only either for compressible fluids or in the cases of flowing fluids in which the elastic properties of fluids are significant.
- If a certain mass of fluid in the motion is influenced by all the above mentioned forces, thus according to Newton's law of motion, the following equation of motion may be written as

$$M\,a = F_g + F_p + F_v + F_t + F_s + F_e = \text{net force } F_x \qquad \text{---- (1)}$$

Further by resolving the various forces and the acceleration along the x, y and z directions, the following equation of motion may be obtained.

$$Ma_x = F_{gx} + F_{px} + F_{vx} + F_{tx} + F_{sx} + F_{ex}$$

$$Ma_y = F_{gy} + F_{py} + F_{vy} + F_{ty} + F_{sy} + F_{ey} \qquad \text{---- (1a)}$$

$$Ma_z = F_{gz} + F_{pz} + F_{vz} + F_{tz} + F_{sz} + F_{ez}$$

The subscripts x, y and z are introduced to represent the component of each of the forces and the acceleration in the respective directions.

In most of the problems of the fluids in motion, the **tension forces** and the **compressibility forces** are not significant. Hence, the forces may be neglected, thus equations (1) and (1a) became.

$$Ma = F_g + F_p + F_v + F_t \qquad \text{--- (2)}$$

And

$$Ma_x = F_{gx} + F_{px} + F_{vx} + F_{tx}$$

$$Ma_y = F_{gy} + F_{py} + F_{vy} + F_{ty} \qquad \text{--- (2a)}$$

$$Ma_z = F_{gz} + F_{pz} + F_{vz} + F_{tz}$$

Equations (2a) are known as **Reynolds's equations of motion which are** useful in the analysis of the turbulent flows. Further, for laminar or viscous flows the turbulent forces are less significant and hence they may be neglected. The eqns.(2) & (2a) may then be modified as,

$$M_a = F_g + F_p + F_v$$

And

$$M_{ax} = F_{gx} + F_{px} + F_{vx}$$

$$M_{ay} = F_{gy} + F_{py} + F_{vy} \qquad \text{---- (3a)}$$

$$M_{az} = F_{gz} + F_{pz} + F_{vz}$$

Equations (3a) are known as **Navier-stokes equations which are** useful in the analysis of viscous flow. Further, if the viscous forces are also of less significance in the problems of fluid flows, then these force may also neglected. The viscous forces will become insignificant if the flowing fluid is an ideal fluid. However, in case of real fluids, the viscous forces may be considered insignificant if the viscosity of flowing fluid is small. In such cases the eqn.(3)&(3a) may be further modified as

$$M_a = F_g + F_p \qquad \text{------ (4)}$$

And

$$M_{ax} = F_{gx} + F_{px}$$

$$M_{ay} = F_{gy} + F_{py} \qquad \text{------- (4a)}$$

$M_{az} = F_{gz} + F_{pz}$

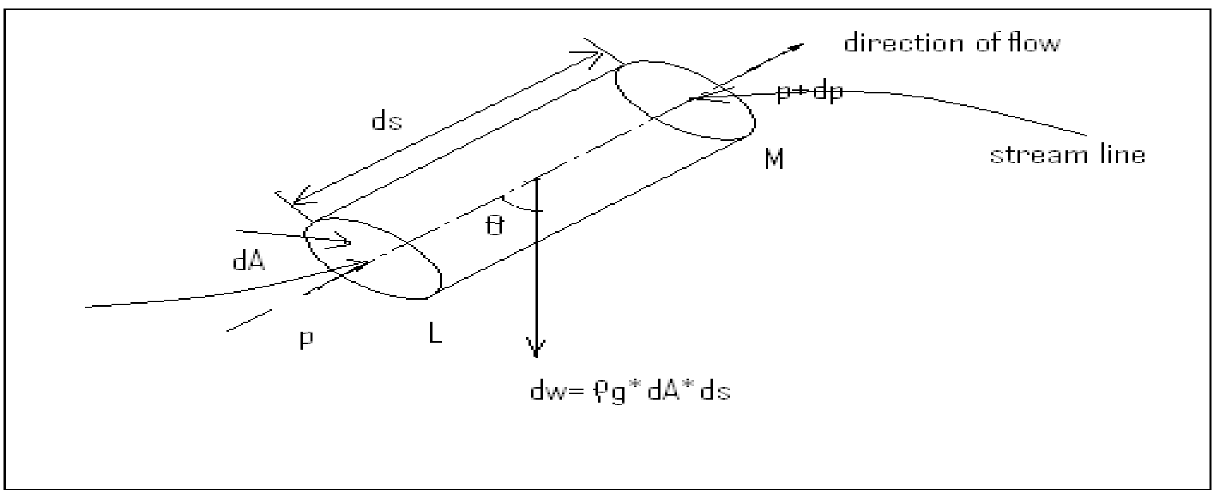
Equation (4a) is known as **Euler's equation of motion**.

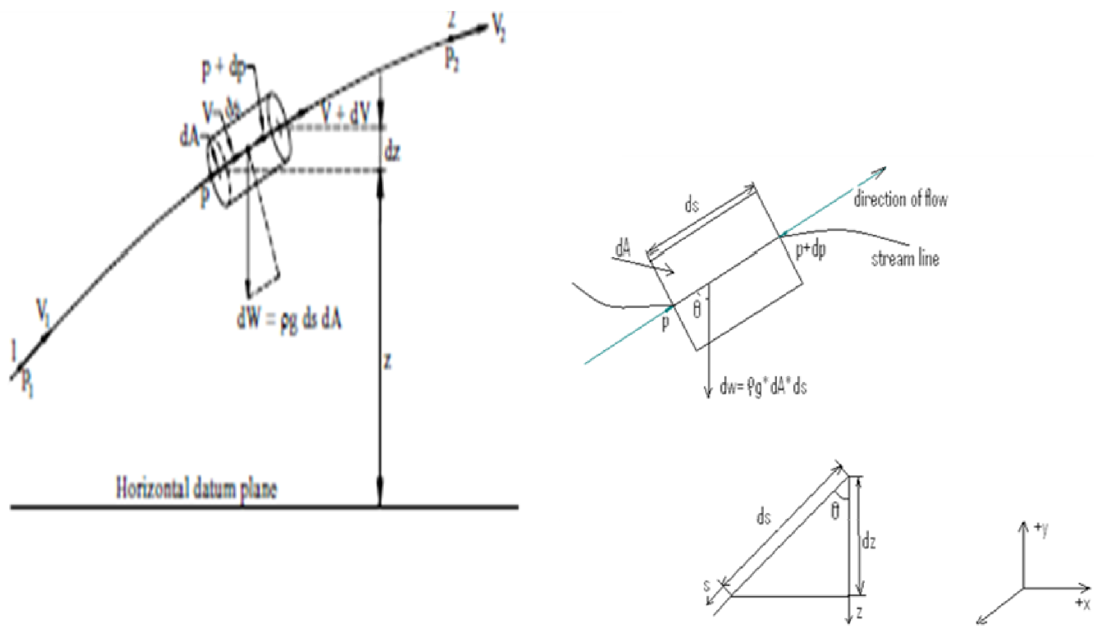
Euler's equation of motion:

Statement: **In an ideal incompressible fluid, when the flow is steady and continuous, sum of the velocity head, pressure head and datum head along a stream line is constant.**

Assumptions:

- The fluid is ideal and incompressible.
- Flow is steady and continuous.
- Flow is along streamline and it is 1-D.
- The velocity is uniform across the section and is equal to the mean velocity.
- Flow is Irrotational.
- The only forces acting on the fluid are gravity and the pressure forces.





Figures 1(b):Forces on a fluid element

Consider a streamline and select a small cylindrical fluid system for analysis as shown in Figs. 1(a) & (b) of length ‘ds’ and c/s area ‘dA’ as a free body from the moving fluid,

Let, p = pressure on the element at ‘L’

p+dp = pressure on the element at M and

v= velocity of the fluid element.

The forces acting (tending to accelerate) the fluid element in the direction of stream line are as follows,

1) Net pressure force in the direction of flow is

$p.dA-(p+dp)\ dA= -\ dp.dA$ ------- (1)

2) Component of the weight of the fluid element in the direction of flow is

$= -\ \rho g.dA.ds.\cos\theta$

$= -\ \rho g.dA.ds\ (dz/ds)$ (because $\cos\theta=dz/ds$)

$= -\ \rho g.dA.dz$ -- (2)

Mass of the fluid element = $\rho.dA.ds$ -- (3)

The acceleration of the fluid element

$a= dv/dt = (dv/ds).(ds/dt)= v(dv/ds)$

Now according to Newton law of motion

Force = mass * acceleration

Therefore $-dp.dA - \rho g.dA.dz= (\rho.dA.ds) (v.dv/ds)$ --- (4)

Dividing both sides by ρdA we get

$-dp/\rho - gdz = vdv$ (divide by -1)

$(dp/\rho)+vdv+gdz=0$ ------ (A)

This is the required **Euler's equation for motion**,

Bernoulli's Equation from Euler's equation for motion:

By Integrating **Euler's equation for motion,** we get

$1/\rho \int dp + \int vdv + \int gdz=$ constant

$p/\rho + v^2/2 + gz =$constant dividing by 'g' we get

$p/\rho g + v^2/2g + z=$ constant

$p/w+v^2/2g+z =$constant

In other words,

$p_1/w + v_1^2/2g + z_1 = p_2/w_2 + v_2^2/2g + z_2$

As points 1 and 2 are any two arbitrary points on the streamline, the quantity

$P/w + v^2/2g + z = H =$ constant ------ B

Applies to all points on the streamline and thus provides a useful relationship between *pressure p*, the *magnitude V of the velocity*, and the *height z* above datum. Eqn. B is known as the Bernoulli equation and the Bernoulli constant *H is also termed the total head*.

Bernoulli's equation from energy principle:

Statement: **In an ideal, incompressible fluid, when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential energy (or datum) energy is constant along a stream line.**

Mathematically, $p/w + v^2/2g + z = \text{constant}$

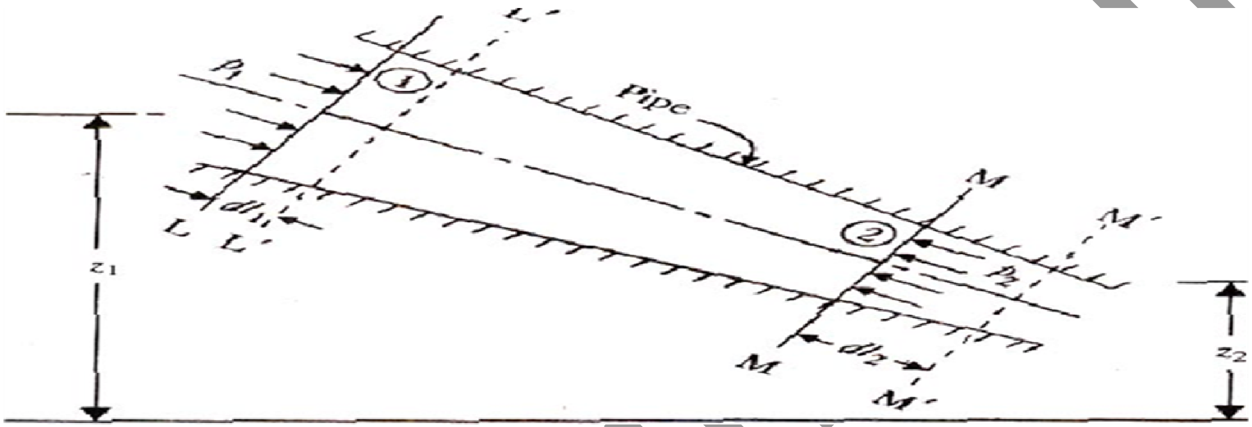


Fig. 2 : Liquid flowing through a non-uniform pipe

Proof: Consider an ideal & incompressible fluid flowing through a non-uniform pipe as shown in fig. 2. Let us consider 2 sections LL&MM and assume that the pipe is running full and there is continuity of flow between the two sections.

Let p_1=pressure at LL

V_1=velocity of liquid at LL

Z_1=height of LL above the datum

A_1=area of pipe at LL and

Similarly, P_2, v_2, z_2, A_2 are the corresponding values at MM

Let the liquid b/w 2sections LL&MM move to L^1L^1&M^1M^1 through very small length dl_1&dl_2 as shown in figure 2. This movement of liquid b/w LL&MM is equivalent to the movement of liquid b/w L^1L^1&M^1M^1 being unaffected

Let W = wt of liquid b/w LL&L^1L^1 as the flow is continuous

$W = wA_1dl_1 = wA_2dl_2$ … Volume of fluid

Or $A_1dl_1 = W/w$ and $A_2dl_2 = W/w$

Therefore $A_1dl_1 = A_2dl_2$

Work done by press at LL in moving the liquid to L^1L^1= force * distance$=p_1A_1dl_1$

Similarly, work done by press at MM in moving the liquid to $M_1M_1= P_2A_2dl_2$ (negative sign indicates that direction of p_2 is opposite to that of p_1)

Therefore, work done by the pressure

$=p_1A_1dl_1 - p_2A_2dl_2$

$=p_1A_1dl_1 - p_2A_2dl_2$ (because $A_1dl_1=A_2dl_2$)

$=A_1dl_1 (p_1-p_2)$

$=W/w (p_1-p_2)$ (because $A_1dl_1=W/w$)

Loss of potential energy (PE) = $W (Z_1-Z_2)$

Gain of kinetic energy (KE) = $W (v_2^2/2g-v_1^2/2g) = W/2g(v_2^2-v_1^2)$

Also, **loss of P.E+ work done by pressure = gain in K.E**

Therefore $W (z_1-z_2) + W/w (p_1-p_2) = W/2g (v_2^2-v_1^2)$

or $(z_1-z_2) + (p_1/w-p_2/w) = (v_2^2/2g-v_1^2/2g)$

$P_1/w+v_1^2/2g+z_1=p_2/w+v_2^2/2g+z_2$

Which prove ***Bernoulli's equation***

P/w = pressure energy per unit weight

= pressure head

$v^2/2g$ = Kinetic energy per unit weight

= kinetic head

Z = datum energy per unit weight

= datum head

Bernoulli's equation for real fluid:

Bernoulli's equation earlier derived was based on the assumption that fluid is non viscous and therefore frictionless. Practically, all fluids are real (and not ideal) and therefore are viscous and as such always some losses in fluid flow. These losses have, therefore, to be taken into consideration in the application of Bernoulli's equation which gets modified (between sections 1 & 2) for real fluids as follows:

$p_1/w + v_1^2/2g + z_1 = p_2/w + v_2^2/2g + z_2 + \mathbf{h_L}$

Where

$\mathbf{h_L}$ = loss of head/energy between sections1 & 2

26/27-3-2010

Problems on Bernoulli's Equation:

1. Water is flowing through a pipe of diameter 5cm under a pressure of $29.43 N/cm^2$ (gauge) and with mean velocity of 2 m/s. Find the total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

Solution

Diameter of pipe = 5cm = 0.5 m

Pressure $p = 29.43 N/cm^2 = 29.43 \times 10^4 N/m^2$

Datum head z =5m

Total head = pressure head + kinetic head+ datum head

Pressure head= $p/\rho g = 29.43 \times 10^4 /1000 \times 9.81 = 30m$

Kinetic head=$v^2/2g$ = 2x2/2x9.81= 0.204m

Total head= $p/\rho g + v^2/2g + z$ = 30+0.204+5 = **35.204 m**

2). A pipe through which the water is flowing, is having diameters 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 40m/s. find the velocity head at sections 1 and 2, and also rate of discharge.

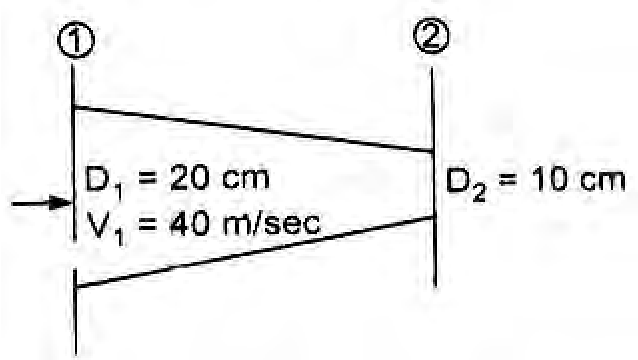


Fig. 2

Solution

D_1=20 cm =0.2m,

$A_1=\pi/4 \times D_1^2 = 0.0314\ m^2$,

v_1=4 m/s

D_2=0.1m,

$A_2=\pi/4 \times D_2^2 = 0.00785\ m^2$

Velocity head at section 1

$V_1^2/2g$ = 4 x4 /2x9.81=**0.815 m**

Velocity head at section 2 = $V_2^2/2g$

To find V_2, apply continuity equation at sections 1&2

$A_1V_1=A_2V_2$

$V_2= A_2V_2/A_2$= 0.0314 x4/0.00785 =16.0 m/s

Velocity head at sec.2 = $V_2^2/2g$ = 16 x16 /2 x9.81

V_2 = 83.047 m

Rate of discharge = A_1V_1 or A_2V_2= 0.0314 x 4

=0.1256 m3/s

= **125.6 lit/s** [1 m^3 = 1000litres]

3) The water is flowing through a tapering pipe having diameter 300mm and 150mm at section 1 & 2 respectively. The discharge through the pipe is 40lit/sec. the section 1 is 10m above datum and section 2 is 6m above datum. Find the intensity of pressure at section 2, if that at section 1 is 400kN/m^2

Solution:

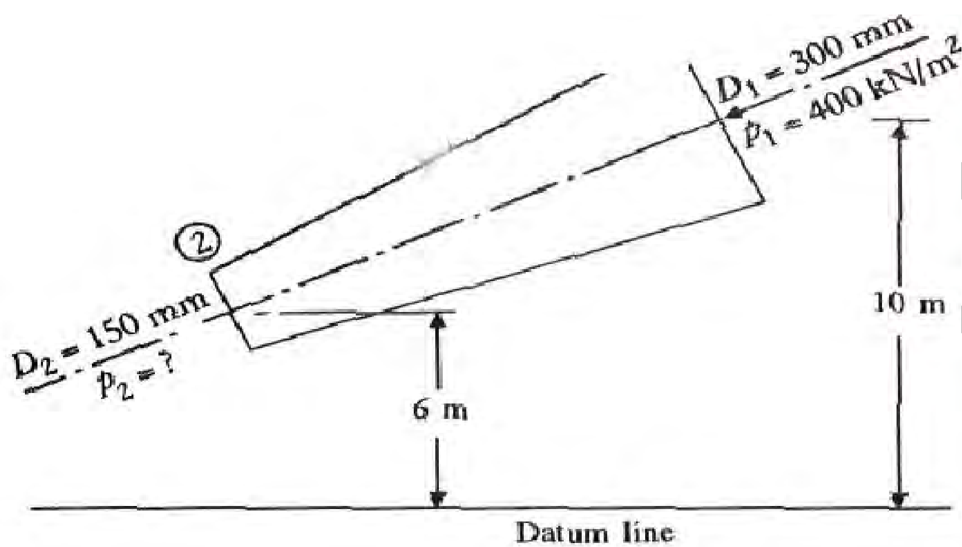


Fig. 3

At section 1

D_1=300mm=0.3m, Area $a_1 = \pi/4*0.3^2 = 0.0707m^2$

Pressure p_1=400kN/m^2

Height of upper end above the datum, z_1=10m

At section 2

D_2=150mm=0.15m,

Area $A_2 = (\pi/4)*0.15^2 = 0.01767m^2$

Height of lower end above the datum, z_2=6m

Rate of flow (that is discharge)

Q=40lit/sec=40/1000 (1litre = 1 m^3/sec) =0.04m^3/sec

Intensity of pressure at section 2, p_2

As the flow is continuous, $Q = A_1V_1 = A_2V_2$ (Continuity equation)

Therefore, $V_1 = Q/A_1 = 0.04/0.0707 = 0.566$m/sec

And $V_2 = Q/A_2 = 0.04/0.01767 = 2.264$m/sec

Apply **Bernoulli's equation** at sections 1 & 2,

We get, $\mathbf{p_1/w+v_1^2/2g+z_1=p_2/w+v_2^2/2g+z_2}$

And $p_2/w = p_1/w + (v_1^2-v_2^2/2g) + z_1-z_2$

$= (400/9.81) + 1/(2*9.81)*(0.566^2-2.264^2)+(10-6)$

$= 40.77- 0.245+4$ (as $w = \rho.g = 1000 \times 9.81$ N/m^3)

$= 44.525$m $= 9.81$ kN/m^3

$P_2 = 44.525 * w = 44.525 * 9.81 =$ **436.8 kN/m^2**

4) Water is flowing through a taper pipe of length 100 m, having diameter 600mm and 300mm at the upper end and lower end respectively, at the rate of 50 lit/s. the pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm^2.

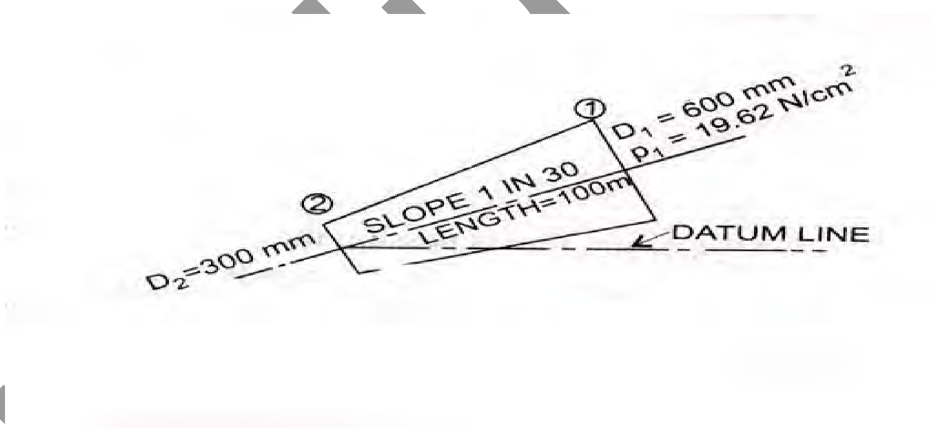


Fig. 4

Solution:

Pipe length L= 100 m

Dia. At the upper end, D_1=600mm =0.6m

$A_1 = \pi/4 \times D_1^2 = 0.2827$ m^2

P_1= pr. At the upper end =19.62 N/ cm^2 =19.62 x10^4 N/m^2

Dia. At the lower end, D_2=300mm =0.3m

$A_2=\pi/4 \times D_2^2 = 0.07068\ m^2$

Rate of flow, Q =50 lit/s, Q=50/1000=0.05 m^3/s

Let the datum line is passing through the centre of the lower end, Then $z_2=0$

As slope is 1 in 30 means $z_1=1/30 \times 100= 10/3$ m

$Q= A_1V_1=A_2V_2$

$V_1=0.05/A_1 =0.1768=0.177$ m/s

$V_2=0.05/A_2=0.7074 =0.707$ m/s

Applying Bernoulli's equation at (1) and (2) we get

$P_1/ \rho g + V_1^2/ 2g +z_1 = P_2/ \rho g + V_2^2/ 2g +z_2$

$19.62 \times 10^4/1000 \times 9.81 + 0.177^2 /2 \times 9.81 + 3.334= P_2/ \rho g +0.707^2/2 \times 9.81+ 0$

$20 + 0.001596 + 3.334= P_2/ \rho g +0.0254$

$23.335 - 0.0254 = P_2/ 1000 \times 9.81$

P_2 =228573 N/m^2

= **22.857 N/cm²**

5) A pipe 200m long slopes down at 1 in 100 and tapers from 600mm diameter at the higher end to 300mm diameter at the lower end, and carries 100 lit/sec of oil (specific gravity 0.8). If the pressure gauge at the higher end reads 60 kN/m^2. Determine,

i. Velocities at both ends.
ii. Pressure at the lower end. Neglect the losses

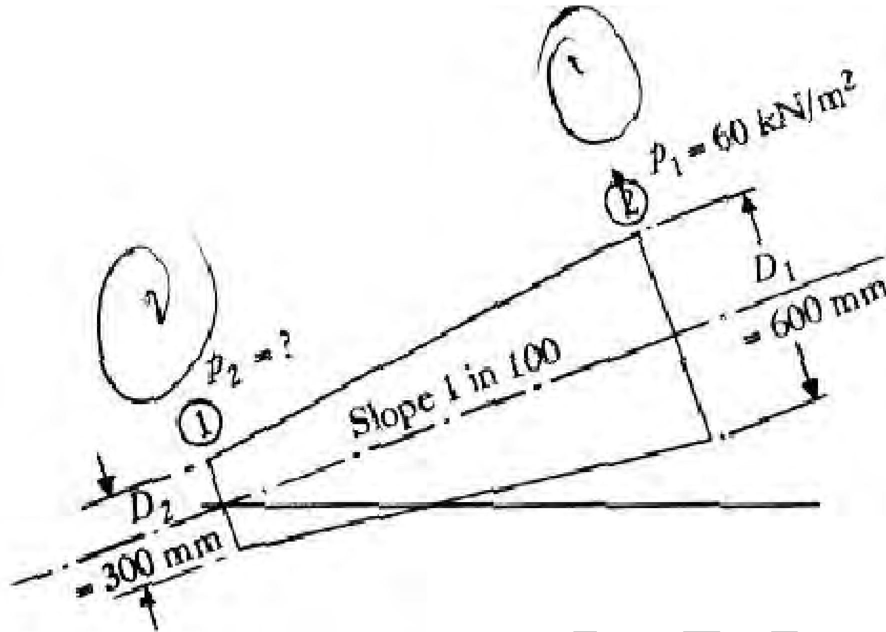


Fig. 5

Solution:

Given: Length of the pipe, l= 200mm

Diameter of the pipe at the higher end,

D_1 =600mm= 0.6m, Area, $A_1 = (\pi/4)*0.6^2 = 0.283\ m^2$

Diameter of the pipe at the lower end,

D_2 =300mm= 0.3m, Area, $A_2 = (\pi/4)*0.3^2 = 0.0707\ m^2$

Height of the lower end, above datum $Z_2 = 0$

Rate of oil flow, Q=100lit/sec= 0.1 m^3/sec

Pressure at the higher end, p_1 =60kN/m^2

(i) Velocities, V_1 & V_2

Now $Q=A_1V_1 = A_2V_2$

Where V_1 & V_2 are the velocities at the higher and lower side respectively.

$V_1 = Q/A_1 = 0.1/0.283 = 0.353$m/sec

$V_2 = Q/A_2 = 0.1/0.0707 = 1.414$m/sec, and

(ii) Pressure at the lower end, p_2

Using Bernoulli's equation for both ends of pipe, we have

$p_1/w+v_1^2/2g+z_1=p_2/w+v_2^2/2g+z_2$

$60/(0.8*9.81)+0.353^2/(2*9.81)+2=p_2/(0.8*9.81)+(1.414^2/2*9.81)+0$

$p_2/(0.8*9.81) = 9.54,$

Pressure at lower end, $p_2 = 74.8$ kN/m^2

6) Water is flowing through a pipe having diameter 300mm and 200mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and at upper end is 9.81 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.

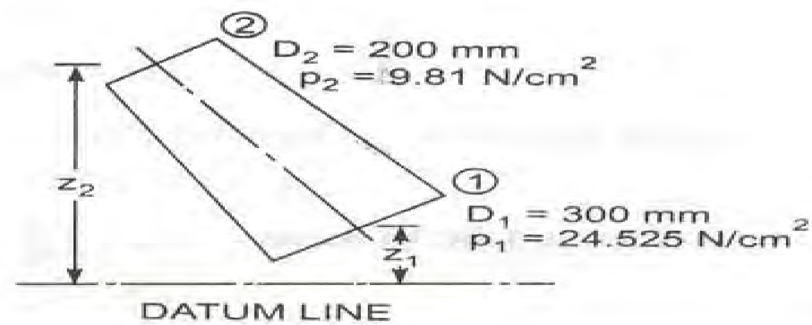


Fig. 6

Solution:

At Section (1), D_1=300mm =0.3m

P_1=24.525 N/cm^2 =24.525 x 10^4 N/m^2

Rate of flow = 40 lit/sec, Q=40/1000=0.04 m^3/s

Now $A_1V_1=A_2V_2=0.04$

$V_1=0.04/A_1$ =0.5658 m/s; $V_2=0.04/A_2$=1.274 m/s

Applying Bernoulli's eqn. at (1) and (2) we get

$P_1/\rho g + V_1^2/2g + z_1 = P_2/\rho g + V_2^2/2g + z_2$

$24.525 \times 10^4/1000 \times 9.81 + 0.566 \times 0.566 / 2 \times 9.81 + z_1$

$= 9.81 \times 10^4/1000 \times 9.81 + 1.274^2/2 \times 9.81 + z_2$

$25 + 0.32 + z_1 = 10 + 1.623 + z_2$

$z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70$ m,

Difference in head, $z_2 - z_1$ = **13.70 m**

7) A non-uniform part of a pipe line 5 m long is laid at a slope of 2 in 5. Two pressure gauges each fitted at upper and lower ends read 20 N/cm^2 and 12.5 N/ cm^2. If the diameters at the upper end and lower end are 15 cm 10 cm respectively. Determine the quantity of water flowing per second.

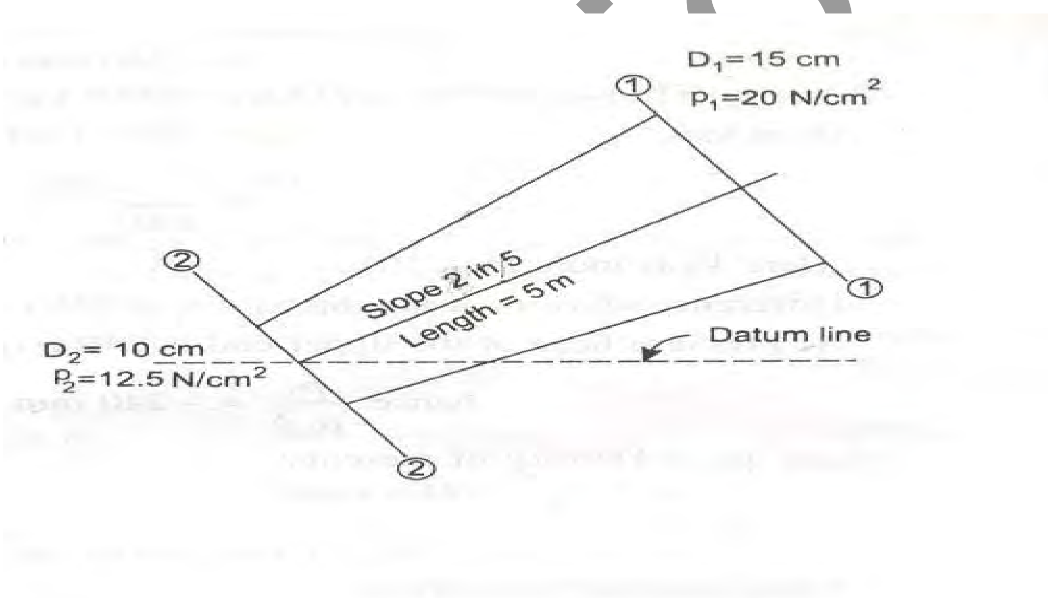


Fig.7

Solution:

L= 5 m, D_1=15cm =0.15m

$A_1 = \pi/4 \times D_1^2 = 0.01767$ m^2

$P_1 = 20$ N/ cm^2 $= 20 \times 10^4$ N/m^2 ,

$P_2 = 12.5$N/ cm^2 $= 12.5 \times 10^4$ N/m^2

Dia. At the lower end, D_2=300mm =0.3m

$A_2=\pi/4 \times D_2^2 = 0.00785\ m^2$

Let the datum line is passing through the centre of the lower end

Then $z_2=0$

As slope is 2 in 5 hence, $z_1=2/5 \times 5= 2$ m

$Q= A_1V_1=A_2V_2$

$V_1 = A_2V_2/A_1 =0.00785 \times V_2 /0.01767$

$V_1 = 0.444\ V_2$

Applying Bernoulli's eqn. at (1) and (2), we get

$P_1/ \rho g + V_1^2/ 2g +z_1 = P_2/ \rho g + V_2^2/ 2g +z_2$

$7.645 + 2 = V_2^2 /2g \times 0.8028$

V_2=15.35 m/s

Discharge, $Q= A_2V_2 = 0.00785 \times 15.35 = 0.1205\ m^3/s$

Q = **120.5 lit/s**

Problems on Bernoulli's Eqn. for real fluid:

1) A pipe line carrying oil (specific gravity of 0.8) changes in diameter from 300 mm at position 1 to 600 mm diameter at poison 2, which is 5m at a higher level. If the pressure at position 1 and 2 are 100 kN/m^2 and 60 kN/m^2 respectively and the discharge is 300 lit/sec, determine,

(a) Loss of head, and

(b) Direction of flow.

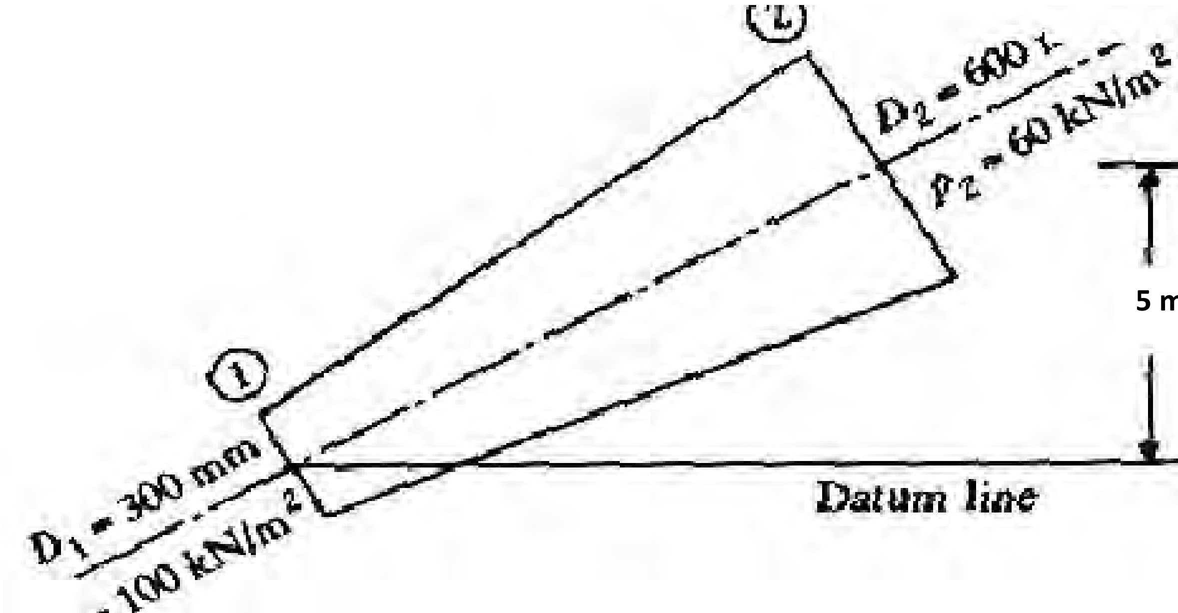


Fig.1

Solution:

Discharge Q= 300 lit/sec = 300/1000= 0.3 m^3/sec

Specific gravity of oil= 0.8

Weight of oil, W_{oil} = 0.8*9.81= 7.85 kN/m^3

At position 1:

Dia of pipe, D_1= 300 mm= 0.3 m

Therefore area of pipe, A_1= (π/4)*$(0.3)^2$ = 0.0707 m^2

Pressure at 1, p_1= 100 kN/m^3

If the datum line passes through section 1,

Then Z_1=0

Velocity, V_1= (Q/A_1) = (0.3/0.0707)

V_1= 4.24 m/sec

At position 2

Dia of pipe, D_2= 600 mm= 0.6 m

Therefore area of pipe, A_2= (π/4)*$(0.6)^2$ = 0.2828 m^2

Pressure, $p_2 = 60\ kN/m^2$

Datum, Z_2=5m

Velocity, V_2= (Q/A_2) = (0.3/0.2828)= **1.06 m/sec**

(a) Loss of head, h_L

Total energy at position 1,

$E_1 = (p_1/W) + (V_1^2/2g) + Z_1$

$E_1 = (100/7.85) + (4.24^2/2*9.81)+0$

$\mathbf{E_1 = 12.74+0.92 = 13.66m}$

Total energy at position 2,

$E_2 = (p_2/W) + (V_2^2/2g) + Z_2$

$E_2 = (60/7.85) + (1.06^2/2*9.81) + 5 = 7.64+0.06+5$

$\mathbf{E_2 = 12.76m}$

Therefore **loss of head,**

$\mathbf{h_L = E_1 - E_2} = 13.66-12.76 = 0.9m$

(b) Direction of flow

Since $E_1 > E_2$, therefore flow taken place from position 1 to position 2

2) A conical tube length 3m is fixed vertically with its small end upwards. The velocity of flow at the smaller end is 10 m/sec. The pressure head at the smaller end is 4m of liquid. The loss of head in the fluid in the tube is $0.4(V_1-V_2)^2/2g$, where V_1 is the velocity at the smaller end and V_2 at the lower/larger end respectively. Determine the pressure head at lower (larger) end.

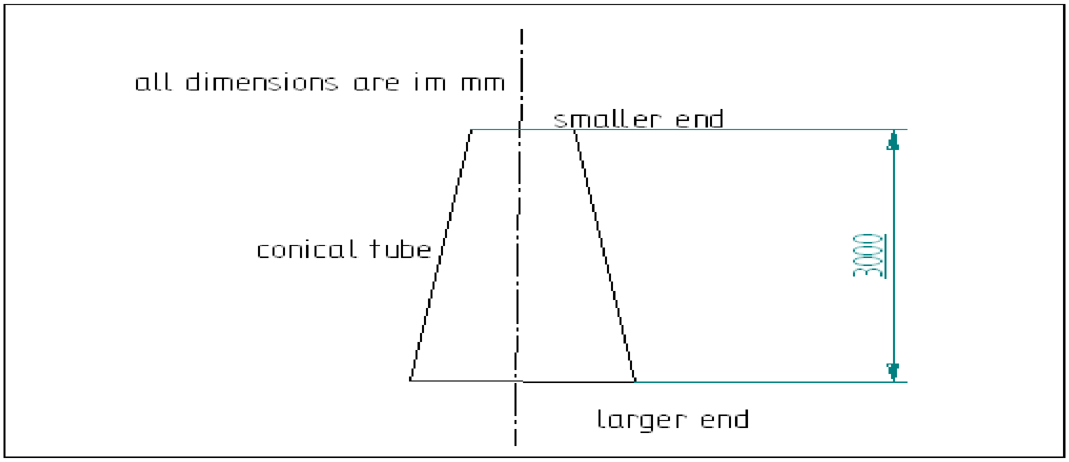


Fig. 2

Given: Length of the tube, $l = 3m$

Velocity, $V_1 = 10$ m/sec

Pressure head, $p_1/w = 4m$ of fluid

Velocity, $V_2 = 4$ m/sec

Solution: Loss of head, $h_L = 0.4(V_1-V_2)^2/2g$

$= 0.4(10-4)^2/2*9.81$

$\mathbf{h_L = 0.73\ m}$

<u>Pressure head at the larger end</u>, (p_2/w)

Applying Bernoulli's equation at sections 1 & 2 we get

$(p_1/w) + (V_1^2/2g) + (Z_1) = (p_2/w) + (V_2^2/2g) + (Z_2) + h_2$

Let the datum line through section 2

Then $Z_2=0$, $Z_1=3m$

$4 + (10^2/2g) + 3 = (p_2/w) + 0 + 0.73 + 0.815$

$4 + 5.09 + 3 = (p_2/w) + 0.815 + 0.73$

Pressure head $(p_2/w) = 10.55$ m of fluid

3) A conical tube of length 2 m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 5 m/s while at the lower end it is 2m/s. the pressure head at the smaller end is 2.5 m of liquid. The loss of head in the tube is $0.35(v_1 - v_2)^2/2g$, where V_1 is the velocity at smaller end and V_2 at the lower end respectively. Determine the pressure head at the lower end. Flow takes place in the downward direction.

Solution:

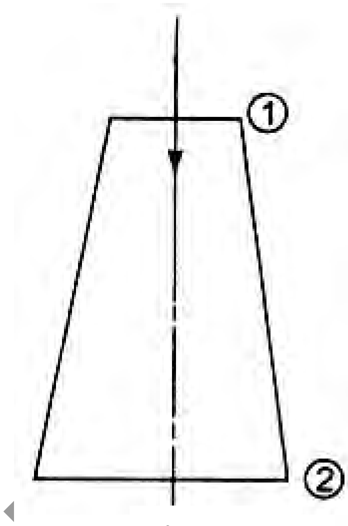


Fig. 3

Length of the tube, L=2m

V_1= 5 m/s

$P_1/\rho g$ = 2.5 m of liquid

V_2= 2 m/s

Loss of head = $h_L = 0.35(v_1 - v_2)^2/2g$

$= 0.35(5 - 2)^2/2g = 0.35 \times 9/ 2 \times 9.81$

=0.16 m

Pressure head = $P_2/\rho g$

Applying Bernoulli's equation at (1) and (2) we get

$P_1/\rho g + V_1^2/2g + z_1 = P_2/\rho g + V_2^2/2g + h_L$

Let the datum line passes through section (2). Then

$z_1 = 2$, $z_2 = 0$

$2.5 + 5^2/2 \times 9.81 + 2 = P_2/\rho g + 2^2/2 \times 9.81 + 0.16 + 0$

$2.5 + 1.27 + 2 = P_2/\rho g + 0.203 + 0.16$

$P_2/\rho g = 5.77 - 0.363$

= 5.047 m of fluid

4) A pipe line carrying oil of specific gravity 0.87, changes in diameter from 200 mm diameter at a position A to 500 mm diameter at a position B which is 4m at a higher level. If the pressures at A and B are 9.81 N/cm^2 and 5.886 N/cm^2 respectively and the discharge is 200 liters/s, determine the loss of the head and the direction of the flow.

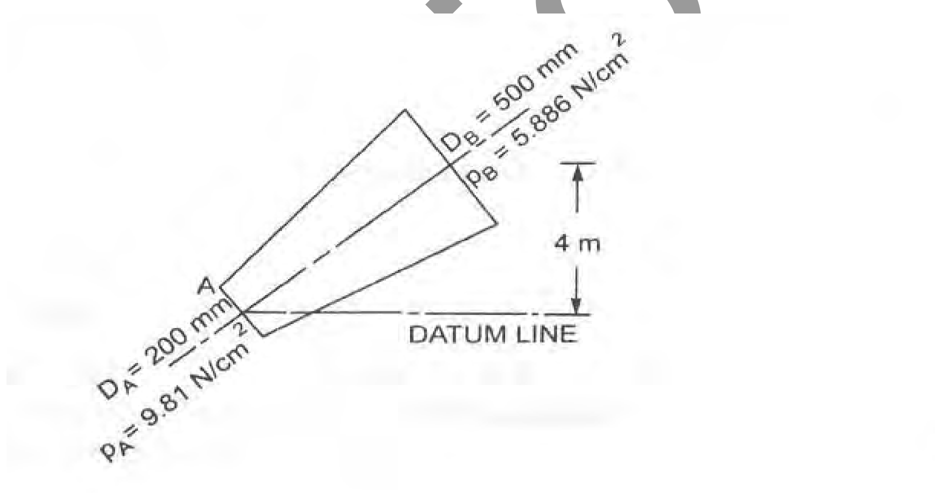


Fig. 4

Solution:

Given, Discharge, $Q = 200$ liters/s $= 0.2\ m^3/s$

Specific gravity of oil = 0.87

ρg for oil = 0.87 x 1000 = 870 kg/m^3

At Section A, $D_A = 200$ mm = 0.2 m

Area, $A_A = \pi/4\,(0.2)^2 = 0.0314\ m^2$

$P_A = 9.81\ N/cm^2 = 9.81 \times 10^4\ N/m^2$ If the Datum line is passing through A, then $Z_A = 0$

$V_A = Q/A_A = 0.2/0.0314 = 6.369$ m/s

At section B, $D_B = 500$ mm = 0.5 m

Area, $A_B = \pi/4\ (0.5)^2 = 0.1963\ m^2$

$P_B = 5.886\ N/cm^2 = 5.886 \times 10^4\ N/m^2$

$Z_B = 4$ m

$V_B = Q/$ Area = 0.2/0.1963 = 1.018 m/s

Total energy at A, $E_A = P_A/\rho g + V_A^2/2g + z_A$

= 11.49+2.067+0

= 13.557 m

Total energy at B, $E_B = P_B/\rho g + V_B^2/2g + z_B$

= 6.896+0.052+4

= 10.948 m

Direction of flow. As E_A is more than E_B and hence flow is taking place from A to B.

Loss of head = $h_L = E_A - E_B =$ **2.609 m**

5) A pump has a tapering pipe running full of water. The pipe is placed vertically with the diameters at the base and the base and top being 1.2 m and 0.6 m respectively. The pressure at the upper end is 240 mm of Hg vacuum, while the pressure at the lower end is $15 kN/m^2$. Assume the head loss to be 20 percent of difference of velocity head. Calculate the discharge, the flow is vertically upwards and difference of elevation is 3.9 m.

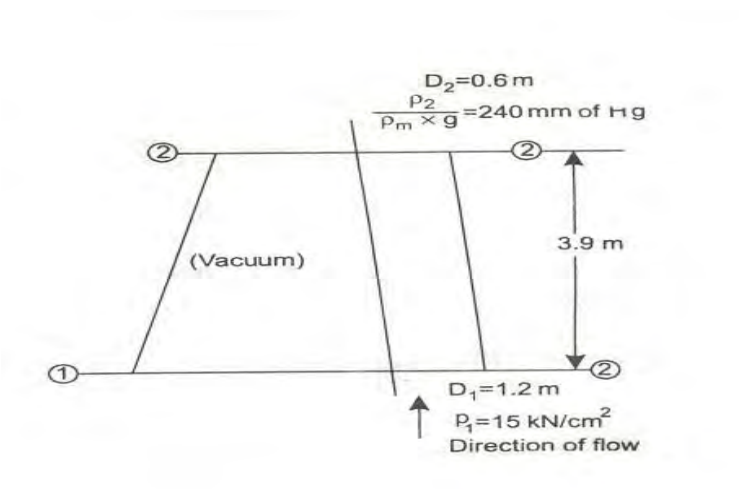


Fig.5

D_1=1.2m, D_2=0.6m

P_1 =15 kN/m^2 =15 x 1000 N/m^2,

P_2/ ρg=240 mm of Hg =0.24 m of Hg

ρ_m=density of Hg =(13.6 x 1000) kg/m^3

Head loss

h_L =20/100 of difference of velocity head,

$= 0.2(V_2^2 - V_1^2) /2g$

Difference of vertical height $z_2 - z_1$=3.9 m

Pressure head at upper end is 240 mm of Hg

Hence $P_2/ \rho_m g$ = -0.24 m of Hg

P_2= -0.24 x 13.6 x 1000 x 9.81

= -32019.8 N/m^2

Using *continuity equation*

$$A_1V_1=A_2V_2$$

$$V_2= A_2V_2/A_{2,} = (D_1 /D_2)^2 \times V_1 = 4\ V_1$$

Applying Bernoulli's eqn. at (1) and (2) we get

$P_1/\rho g + V_1^2/2g + z_1 = P_2/\rho g + V_2^2/2g + h_L$

$V_2^2/2g - V_1^2/2g + 3.9 + 0.2(V_2^2 - V_1^2/2g$

$1.529+3.264=1.2(V_2^2 - V_1^2)/2g + 3.9$

$4.793 = 1.2((4V_1)^2 - V_1^2)/2g + 3.9$

$0.893 = 9V_1^2/g$

$V_1 = 0.9865$ m/s

Discharge $Q=A_1V_1$ = **1.1157 m^3/s**

Practical applications of Bernoulli's equation:

Although Bernoulli's equation is applicable in all problems of incompressible flow where there is involvement of energy considerations. But we shall consider its application to the following measuring devices.

1) Venturimeter
2) Orifice meter
3) Pitot tube

Differential Pressure Flow Meters

Differential pressure flow meters all infer the flow rate from a pressure drop across a restriction in the pipe. For many years, they were the only reliable methods available, and they remain popular despite the development of higher performance modern devices, mostly on account of exceptionally well researched and documented standards.

The analysis of the flow through a restriction (Fig.1) begins with assuming straight, parallel stream lines at cross sections 1 and 2, and the absence of energy losses along the streamline from point 1 to point 2.

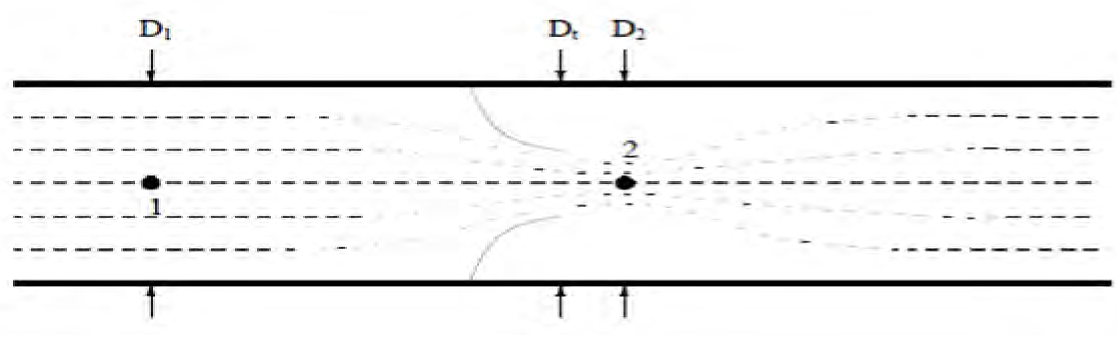


Fig. 1: A generalized restriction/differential pressure flow meter

The objective is to measure the mass flow rate, ***m˙ By eqn. continuity***

$\dot{m} = \rho v_1 a_1 = \rho v_2 a_2.$

Bernoulli's equation may now be applied to a streamline down the centre of the pipe from a point 1 well upstream of the restriction to point 2 in the ***vena contracta of the jet immediately*** downstream of the restriction where the streamlines are parallel and the pressure across the duct may therefore be taken to be uniform:

Unit 5: Fluid Flow Measurements

Introduction:

Fluid flow measurements means the measuring the rate of flow of a fluid flowing through a pipe or through an open channel. The rate of flow of a fluid through a pipe is measured by four main restriction devices are.

- Venturimeter
- Orifice meter
- Pitot tube
- flow nozzle

Whereas through an open channel the rate of flow is measured by

- Notches
- weirs

The **Venturi effect** is the reduction in fluid pressure that results when a fluid flows through a constricted section of pipe. The Venturi effect is named after Giovanni Battista Venturi (1746–1822), an Italian physicist.

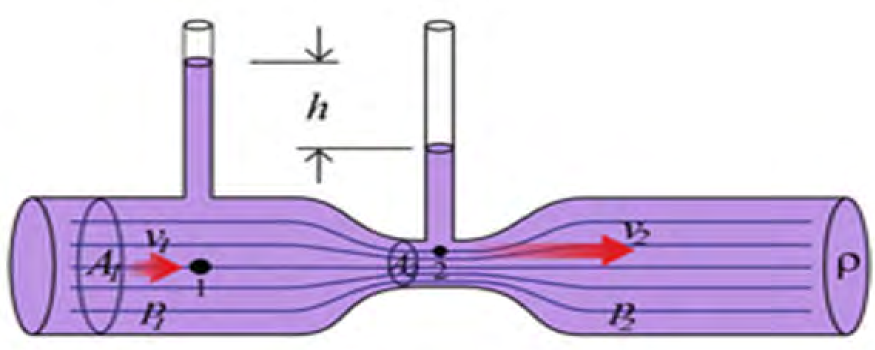


Fig. 1: Venturi effect

The pressure at "1" is higher than at "2" because the fluid speed at "1" is lower than at "2".The **Venturi effect may be observed or used in the following:**

- Inspirators that mix air and flammable gas in grills, gas stoves, Bunsen burners and airbrushes
- Atomizers that disperse perfume or spray paint (i.e. from a spray gun).
- Carburetors that use the effect to suck gasoline into an engine's intake air stream
- The capillaries of the human circulatory system, where it indicates aortic regurgitation
- Aortic insufficiency is a chronic heart condition that occurs when the aortic valve's initial large stroke volume is released and the Venturi effect draws the walls together, which obstructs blood flow, which leads to a Pulsus Bisferiens.
- Cargo Eductors on Oil, Product and Chemical ship tankers
- Protein skimmers (filtration devices for saltwater aquaria)
- Compressed air operated industrial vacuum cleaners
- Venturi scrubbers used to clean flue gas emissions
- Injectors (also called ejectors) used to add chlorine gas to water treatment chlorination systems
- Sand blasters used to draw fine sand in and mix it with air
- A scuba diving regulator to assist the flow of air once it starts flowing
- In Venturi masks used in medical oxygen therapy
- In recoilless rifles to decrease the recoil of firing
- Wine aerators, to aerate wine, putatively improving the taste.
- Ventilators
- The diffuser on an automobile

The **main advantages of the Venturimeter over the orifice plate** are:

• Low head loss

• Less affected by upstream flow disturbance

• Good performance at higher β

• Even more robust

• Self-cleaning

• Less affected by erosion

The **disadvantages compared to the orifice** are

• Occupies longer length of pipe

• More expensive (manufacture and installation)

The simplest apparatus, built out of PVC pipe as shown in the photograph is a tubular setup known as a Venturi tube or simply a venturi. Fluid flows through a length of pipe of varying diameter.

Fig. 2: Venturimeter - Experimental apparatus

To avoid undue drag, a Venturi tube typically has an entry cone of 21 to 30 degrees and an exit cone of 5 to 15 degrees. To account for the assumption of an in viscid fluid a coefficient of discharge is often introduced, which generally has a value of 0.98. A venturi can be used to measure the volumetric flow rate *Q*.

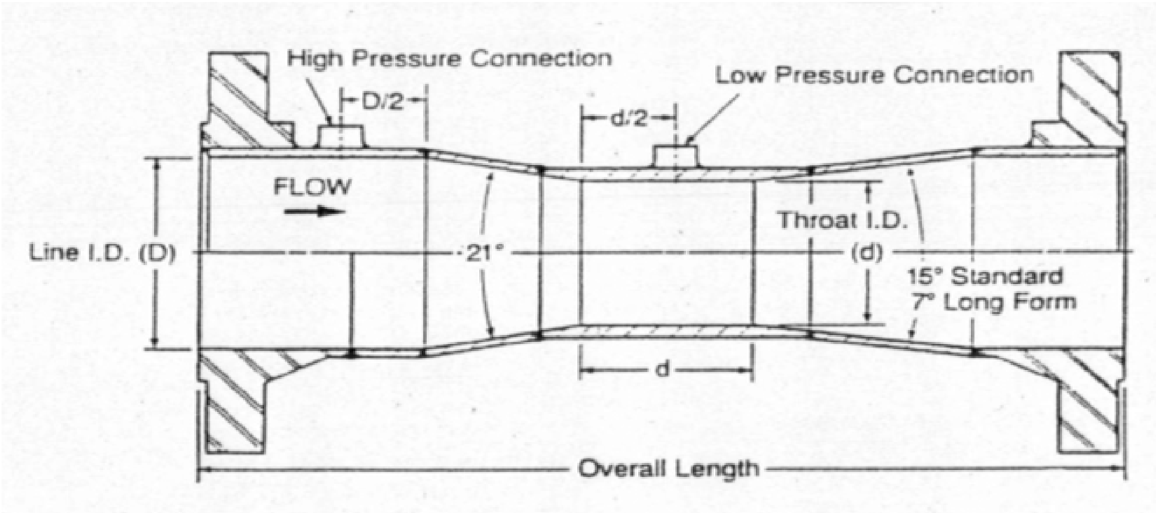


Fig 3 : Venturimeter (Furness 1989)

The fluid velocity must increase through the constriction to satisfy the equation of continuity, while its pressure must decrease due to conservation of energy: the gain in kinetic energy is balanced by a drop in pressure or a pressure gradient force. An equation for the drop in pressure due to the Venturi effect may be derived from a combination of Bernoulli's principle and the equation of continuity.

Expression for rate of flow through venturimeter :

Consider a venturimeter fixed in a horizontal pipe through

which a fluid is flowing (say water) as shown in figure 4.

Let d_1 = diameter at inlet or at section 1

p_1 =Pressure at section 1

v_1 = velocity of fluid at section 1

a_1 = area at section 1= $(\pi/4)*d_1^2$

And d_2, p_2, v_2, a_2 are corresponding values at section 2

Applying Bernoulli's equation at section 1 & 2 we get

$(p_1/\rho g) + (v_1^2/2g) + (z_1) = (p_2/\rho g) + (v_2^2/2g) + (z_2)$

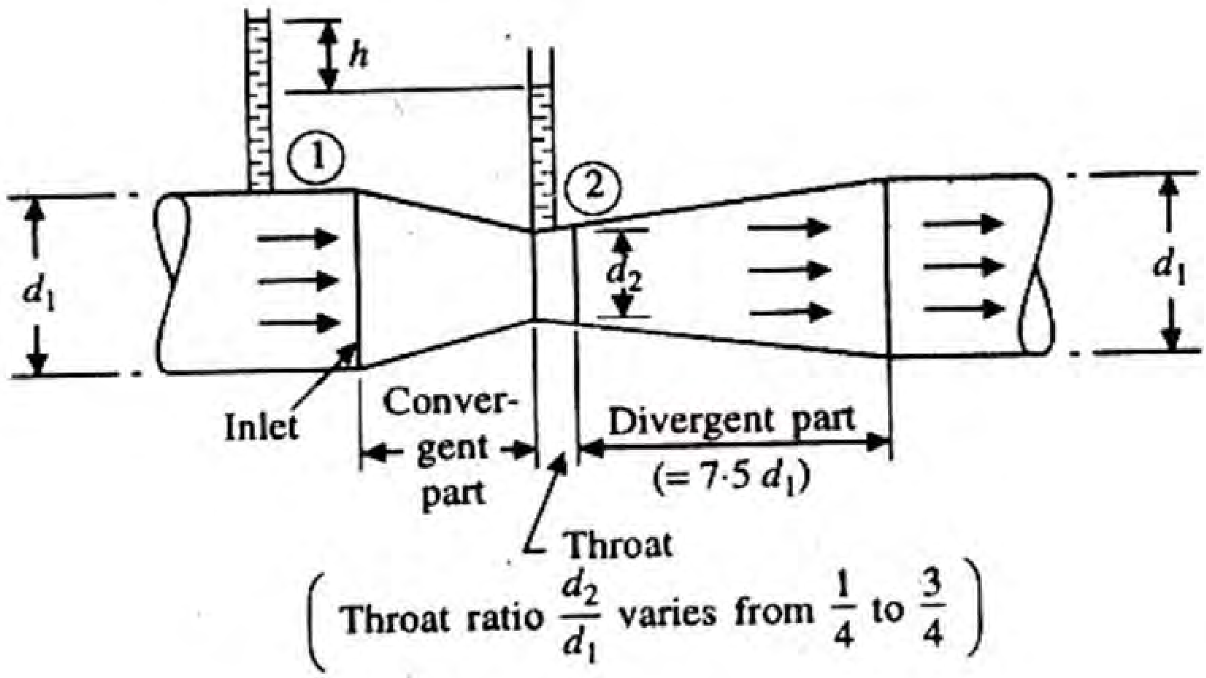


Fig. 4 Typical venturimeter

As pipe is horizontal, hence $z_1=z_2$

$(p_1/\rho g) + (v_1^2/2g) = (p_2/\rho g) + (v_2^2/2g)$ or

$(p_1-p_2)/\rho g = (v_2^2/2g) - (v_1^2/2g)$ ---- (1)

But $(p_1-p_2)/\rho g$, is the difference of pressure head at sections 1 & 2 and it is equal to 'h' or

$(p_1-p_2)/\rho g = h$

Substituting the value of $(p_1-p_2)/\rho g$ in the above eqn. (1) we

Get, $h = (v_2^2/2g) - (v_1^2/2g)$ ---- (2)

now applying **continuous equation at sections 1 & 2**

$a_1v_1= a_2v_2$ or $v_1 = (a_2v_2)/ a_1$

substitute the value of v_1 in equation (2)

$h= (v_2^2/2g) - [(a_2v_2/ a_1)^2/2g] = (v_2^2/2g)[1-(a_2^2/a_1^2)]$

$= (v_2^2/2g)[(a_1^2- a_2^2)/ a_1^2]$

$v_2^2 =2gh [a_1^2/(a_1^2- a_2^2)]$

Therefore $v_2 = \sqrt{[2gh \{a_1^2 /(a_1^2 - a_2^2)\}]}$

$$v_2 = [a_1 /\sqrt{(a_1^2 - a_2^2)}] * \sqrt{(2gh)}$$

Discharge $Q = a_2 v_2$

$Q_{th} = a_2 * [a_1 /\sqrt{(a_1^2 - a_2^2)}] * \sqrt{(2gh)}$ ---- (3)

Equation (3) gives the discharge under ideal conditions and is called theoretical discharge. Actual discharge will be less than theoretical discharge.

$$Q_{act} = C_d [a_1 a_2 /\sqrt{(a_1^2 - a_2^2)}] * \sqrt{(2gh)}]$$

Where C_d is coefficient of venturimeter and its value is less than 1.

<u>Value of ‘h’ by differential ‘U’ tube manometer</u>

Case 1:

Let the differential manometer contains a liquid which is **heavier than** the liquid flowing through a pipe.

Let

s_h=specific gravity of the heavier liquid.

s_p=specific gravity of the liquid flowing through pipe.

x=difference of the heavier liquid column in U-tube.

$$h = x[(s_h/s_p) - 1]$$

Case 2:

If the differential manometer contains a liquid which is **lighter than** the liquid flowing through the pipe, the value of ‘h’ is given by

$$h = x[1 - (s_L/s_p)]$$

Where

s_L=sp gravity of lighter liquid in U-tube.

s_p=sp gravity of the liquid flowing through pipe

x= difference of the lighter liquid column in U-tube.

Case 3:

Inclined venturimeter with differential U-tube manometer **(heavier liquid)**

$h = (p_1/\rho g + z_1) - (p_2/\rho g + z_2) = x[(s_h/s_p)-1]$

Case 4:

Inclined venturimeter with differential U-tube manometer**(lighter liquid)**

$h = (p_1/\rho g + z_1) - (p_2/\rho g + z_2) = x[1-(s_L/s_p)]$

Problems on Horizontal Venturimeter:

1) A horizontal venturimeter with inlet and throat diameters 30cm and 15cm respectively is used to measure the flow of water. The reading of differential manometer connected to the throat and inlet is 20cm of mercury. Determine the rate of flow. Take C_d=0.98.

Solution:

Given:

Dia at inlet, d_1 =30cm, Area at inlet, $a_1 = (\pi d_1^2)/4$

$= (\pi 30^2)/4 = 706.85 cm^2$

Dia at throat, d_2 =15cm, Area at throat, $a_2 = (\pi 15^2)/4 = 176.7 cm^2$

$C_d = 0.98$

Reading of differential manometer x = 20cm of mercury

Therefore difference of pressure head is given by

$h = x\ [(s_h/s_w)-1]$

Where s_h=specific gravity of mercury =13.6,

s_w=specific gravity of water(assumed) =1

h =20[(13.6/1)-1] =20*12.6cm =252.0 cm of water.

The discharge through venturimeter is given by

$Q = C_d*(a_1a_2/(\sqrt{a_1^2-a_2^2}))*(\sqrt{2gh})$

$= 0.98*(706.85*176.7/(\sqrt{706.85^2 176.7^2}))*(\sqrt{2*9.81*25})$

$= 86067593.36/(\sqrt{499636.9-31222.9})$

= 86067593.36/684.4

= 125756cm^3/s=125756lit/s

Q = 125.756 lit./s

2) An oil of specific gravity 0.8 is flowing through a venturimeter having inlet diameter 20cm and throat diameter 10cm. The oil(s_o = 0.8)-mercury differential manometer shows a reading of 25cm. Calculate the discharge of oil through the horizontal venturimeter. Take C_d=0.98.

Solution:

Given:

Specific gravity of oil, s_o =0.8

Specific gravity of mercury s_h =13.6

Reading of differential manometer x =25cm

Therefore difference of pressure head, h =x [(s_h/s_o) -1]

=25[(13.6/0.8) -1] cm of oil = 25[17-1] =400 cm of oil.

Dia at inlet, d_1=20cm

Area at inlet, $a_1 = (\pi d_1^2)/4 = (\pi 20^2)/4$ =314.16cm^2

Similarly at throat, d_2 =10cm

$a_2 = (\pi 10^2)/4$ =78.54cm^2

C_d = 0.98 (given)

Therefore **discharge Q** is given by

$Q = C_d*(a_1a_2/(\sqrt{a_1^2-a_2^2}))*(\sqrt{2gh})$

$=0.98*(314.16*78.54/(\sqrt{314.16^2-78.54^2}))*(\sqrt{2*9.81*400})$

=21421375.68/ ($\sqrt{98696-6168}$)

=21421375.68/304 cm^3/s

=70465cm^3/s

Q =**70.465 lit/s**

3) A venturimeter is to be fitted in a pipe of 0.25 m dia. where the pressure head is 7.6 m of flowing liquid and max. flow is 8.1 m^3/min. Find the least dia. of the throat to ensure that the pressure head does not become negative, $c_d = 0.96$.

Solution:

$Q= c_d((a_1a_2)/(\sqrt{a_1^2-a_2^2}))*\sqrt{(2gh)}$

$Q = (8.1/60) = 0.135\ m^3/s$, $c_d = 0.96$

$a_1 = (\pi/4)*(0.25)^2 = 0.049\ m^2$

h = 7.6 m

$0.135 = 0.96*((0.049*a_2)/(\sqrt{0.049^2-a_2^2}))*\sqrt{(2*9.81*7.6)}$

$a_2 = \mathbf{0.0112\ m^2}$

4) A venturimeter is used for measurement of discharge of water in a horizontal pipe line, if the ratio of upstream pipe diameter to that of throat is 2:1, upstream diameter is 300mm, the difference of pressure between the throat is equal to 3m head of water and loss of head through meter is one eighth of the throat velocity head, calculate discharge in pipe

Solution:

Given:

Ratio of inlet dia to throat i.e., $d_1/d_2=2$

d_1=300mm=0.3m

d_2=300/2=150mm=0.15m

$(p_1/\rho g - p_2/\rho g)$=3m of water ,

loss of head, h_f=1/8 of throat velocity head =$1/8*v_2^2/2g$

Using continuity equation

Using **Bernoulli's equation** at inlet and throat, we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+\frac{1}{8}\times\frac{v_2^2}{2g}$$

$$\frac{p_1}{\rho g}-\frac{p_2}{\rho g}=\frac{v_2^2}{2g}-\frac{v_1^2}{2g}+\frac{1}{8}\times\frac{v_2^2}{2g}$$

$$\frac{p_1}{\rho g}-\frac{p_2}{\rho g}=3\ m$$

$$3=\frac{v_2^2}{2g}-\frac{v_1^2}{2g}+\frac{1}{8}\frac{v_2^2}{2g}$$

$$3=\frac{16v_1^2}{2g}-\frac{v_1^2}{2g}+\frac{2v_1^2}{2g}=\frac{17v_1^2}{2g}$$

$$v_1=\sqrt{\frac{3\times2\times9.81}{17}}=1.86m/s$$

$$Q=(\frac{\pi}{4}d_1^2)\times v_1$$

$$Q=\left(\frac{\pi}{4}\times[.3]^2\right)\times1.86=.0.1315m^3/sec$$

$$Q=131.5lit/sec$$

5) A horizontal venturimeter with inlet diameter 200mm and throat diameter 100mm is employed to measure the flow of water. The reading of the differential manometer connected to the inlet is 180mm of Hg. If the coefficient of discharge is 0.98, determine the rate of flow.

Solution:

Inlet dia of venturimeter, D_1= 200mm= 0.2m

Therefore, area of inlet, A_1= (π/4)*0.2^2

Throat dia D_2= 100mm= 0.1m

Area of throat, A_2= (π/4)*0.1^2

Reading of differential manometer, x = 180mm

= 0.18m of Hg

Coefficient of discharge, C_d =0.98

Rate of flow, Q

To find the difference of pressure head (h), we have

$$h = x[(s_h/s_p)-1]$$

h = 0.18[(13.6/1) -1]= 2.268 m

To find 'Q' using this relation

$Q = C_d[a_1a_2 /\sqrt{(a_1^2- a_2^2)}]* \sqrt{(2gh)}$

$Q = 0.98[0.0314*0.00785 /\sqrt{(0.0314^2-0.00785^2)}]* \sqrt{(2*9.81*2.268)}$

Q= (0.000241*6.67)/0.0304

Q = **0.0528 m^3/sec**

6) A venturimeter having a diameter of 75mm at throat and 150mm dia at the enlarged end is installed in a horizontal pipeline 150mm in dia carrying an oil of specific gravity 0.9. The difference of pressure head between the enlarged end and the throat recorded by a U-tube is 175mm of mercury. Determine the discharge through the pipe. Assume the coefficient of discharge of the water as 0.97.

Solution:

The discharge through the venturimeter is given by

$Q = C_d[a_1a_2 /\sqrt{(a_1^2- a_2^2)}]* \sqrt{(2gh)}$

C_d =0.97

d_1 =150mm= 0.15m

$a_1 =(\pi/4)*0.15^2 = 0.0177m^2$

d_2 =75mm= 0.075m

$a_2 = (\pi/4)*0.075^2 = 0.0044m^2$

x= 175mm = 0.175m

$h =x[(s_h/s_p) -1] = 0.175[(13.6/0.9)-1]$

= 2.469m

by substitution, we get

$Q=0.97[0.0177*0.0044 /\sqrt{(0.0177^2- 0.0044^2)}]*\sqrt{(2*9.81*2.469)}$

$Q= 0.03067\ m^3/sec$

= **30.67 lit/sec**

7) A horizontal venturimeter with inlet diameter 20cm and throat dia 10cm is used to measure the flow of oil of specific gravity 0.8. The discharge of the oil through venturimeter is 60 lit/sec. Find the reading of the oil-Hg differential manometer, take C_d= 0.98.

Solution:

At entry, d_1=20cm

$a_1= (\pi/4)*20^2 = 314.16\ cm^2$

At throat, d_2 = 10cm

$a_2 = (\pi/4)*10^2 = 78.54\ cm^2$

$C_d = 0.98$

$Q = 60$ lit/sec= $60*1000\ cm^3/sec$

$Q = C_d[a_1a_2 /\sqrt{(a_1^2- a_2^2)}]* \sqrt{(2gh)}$

$60*1000 = 0.98[314.16*78.54/\sqrt{(314.16^2-78.54^2)}]*\sqrt{(2*9.81*h)}$

$\sqrt{h}$= 17.029, **h= 289.98 cm of oil**

To calculate reading of the oil-Hg differential manometer

we have

$h =x[(s_h/s_p)-1]$

Where s_h = 13.6 –> sp. gravity of the mercury

s_p = 0.8 --> sp. gravity of the oil

x = ?

x= **18.12 cm**

8) A horizontal venturimeter with inlet*throat diameter 300mm and 100mm respectively is used to measure the flow of water. The pressure intensity at inlet is 130 kN/m^2, while the vacuum pressure head at the throat is 350mm of Hg. Assume that 3% of head is lost in between the inlet and throat, find

1) The value of C_d for the venturimeter
2) Rate of flow.

Solution:

Inlet dia of the venturimeter, D_1 = 300mm = 0.3m

Area of inlet, $A_1 = (\pi/4)*0.3^2 = 0.07m^2$

Throat dia, D_2 = 100mm= 0.1m

Area of throat, $A_2 = (\pi/4)*0.1^2 = 0.00785m^2$

Pressure at inlet, p_1 =130KN/m^2

Pressure head, $p_1/w=130/9.81= 13.25m$

Similarly, pressure head at throat

p_2/w = - 350mm of Hg**(vacuum pr. Head)**

= - 0.35*13.6

= - 4.76 m of water

(a) coefficient of discharge, C_d

Differential head, $h = (p_1/w) - (p_2/w) = 13.25 -(- 4.76)$

$h = 18.01m$

head lost, h_f = 3% of h= (3/100)*18.01 = 0.54m

$C_d = \sqrt{[(h- h_f)/h]} = \sqrt{[(18.01-0.54)/18.01]} = 0.985$

(b)Rate of flow, Q

$Q = C_d[a_1a_2 /\sqrt{(a_1^2- a_2^2)}]* \sqrt{(2gh)}$

$Q = 0.985[0.07*0.00785 /\sqrt{(0.07^2- 000785^2)}]*\sqrt{(2*9.81*18.01)}$

Q= **0.146 m^3/sec**

9) The inlet and throat diameter of a horizontal venturimeter are 30cm and 10cm respectively. The liquid flowing through the meter is water. The pressure intensity at inlet is 13.734N/cm^3 while the vacuum pressure head at the throat is 37cm of mercury. Find the rate of flow. Assume that 4% of the differential head is lost between the inlet and throat. Find also the value of C_d for the venturimeter.

Solution:

Given:

Dia at inlet, d_1=30cm

Area at inlet, $a_1=(\pi d_1^2)/4=(\pi 30^2)/4=706.85cm^2$

Dia at throat, d_2=10cm

Area at throat, $a_2=(\pi 10^2)/4=78.54cm^2$

Pressure at entry, $p_1=13.734N/cm^2=13.734*10^4N/m^2$

Therefore pressure head, $p_1/\rho g=13.734*10^4/9.81*1000$

=14m of water

$p_2/\rho g$= -37cm of mercury

= (-37*13.6/100) m of water

=-5.032 m of water

Differential head, $h = p_1/\rho g- p_2/\rho g$=14-(-5.032) =14+5.032

= 19.032 m of water

=1903.2 cm

Head lost, h_f= 4% of h=0.04*19.032=0.7613 m

$C_d=\sqrt{((h- h_f)/h)} =\sqrt{(19.032-0.7613)/19.032}$

=0.98

Therefore discharge

$Q= C_d*(a_1a_2/(\sqrt{a_1^2-a_2^2}))*(\sqrt{2gh})$

$= 0.98*(706.85*78.54/(\sqrt{706.85^2 - 78.54^2}))*(\sqrt{2*9.81*1903.2})$

= (105132247.8)/√(499636.9-6168)=149692.8cm^3/s

Q = **0.14969m^3/s**

10) A horizontal venturimeter with inlet diameter 20cm and throat diameter 10cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm^2 and the vacuum pressure at the throat is 30cm of mercury. Find the discharge of water through venturimeter. Take C_d=0.98

Solution:

Dia at inlet, d_1=20cm

$a_1=(\pi d_1^2)/4=(\pi 20^2)/4=314.16cm^2$

Dia at throat,d_2=10cm

$a_2=(\pi 10^2)/4=78.54cm^2$

p_1=17.658 N/cm^2=17.658*10^{4}N/m^2

density of water = 1000kg/m^3 and

Therefore, $p_1/\rho g$=17.658*10^4/9.81*1000=18m of water

$p_2/\rho g$= -30cm of mercury **(vacuum pr. Head)**

= - 0.30m of mercury

= - 0.30*13.6

= - 4.08 m of water

Therefore **differential head**

h= $p_1/\rho g$ - $p_2/\rho g$ =18-(-4.08)

= 18+4.08=22.08 m of water

= 2208 cm of water

The **discharge Q** is given by equation

Q= C_d*(a_1a_2/(√a_1^2-a_2^2))*(√2gh)

=0.98*(314.16*78.54/(√314.16^{2}78.54^2))*(√2*9.81*2208)

=50328837.21/304

$=1655555\ cm^3/s$

Q =165.55 lit/s

Problems on venturimeter axis vertical/inclined

1) A venturimeter has its axis vertical , the inlet & throat diameter being 150mm & 75mm respectively. The throat is 225mm above inlet and $C_d = 0.96$. Petrol of specific gravity 0.78 flows up through the meter at a rate of 0.029 m^3 /sec. find the pressure difference between the inlet and throat.

Solution:

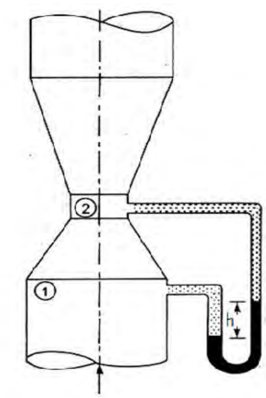


Fig. 1: venturimeter with its axis vertical

The discharge through a venturimeter is given by

$Q = C_d\,[a_1a_2/\sqrt{(a_1^2 - a_2^2)}] * \sqrt{(2gh)}$

Given:

$C_d = 0.96$

d_1 =150mm= 0.015m

d_2 =75mm= 0.0075m

$a_1 =(\pi/4)*0.015^2 =0.0177m^2$

$a_2 =(\pi/4)*0.0075^2 =0.0044m^2$

$Q = 0.029\ m^3/sec$

By substitution, we have

$0.029 = 0.96[0.0177*0.0044 /\sqrt{(0.0177^2- 0.0044^2)}]*\sqrt{(2*9.81*h)}$

$h= 2.254$m of oil

$h= (p_1/w+z_1) - (p_2/w + z_2)$

$2.254= [(p_1/w)- (p_2/w)]-[z_2-z_1]$

$2.254= [(p_1/w)- (p_2/w)]-[0.225]$

$p_1/\rho g - p_2/\rho g =2.479$

Therefore, $p_1 - p_2= 2.479*0.78*9810$

$= 18969$ N/m^2 $=18.969$k N/m^2 $=18969$ Pa

Pr. Difference, $p_1 - p_2 = 18.96$ kPa

2)Determine the rate of flow of water through a pipe of 300mm dia placed in an inclined position where a venturimeter is inserted, having a throat dia of 150mm.The difference of pressure between the main throat is measured by a liquid of specific gravity 0.7 in an inverted u-tube which gives a reading of 260mm.The loss of head the main and throat is 0.3 times the kinetic head of the pipe.

Solution:

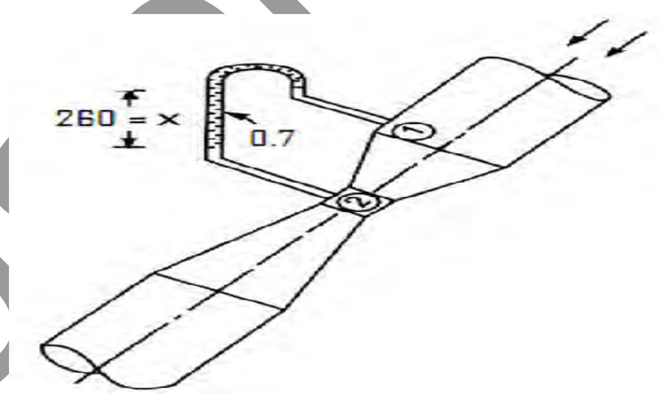


Fig. 2

Given:

Dia of inlet,

D_1=300mm=0.3m

Therefore area of inlet, $A_1= \pi/4*(0.3)^2=0.07m^2$

Throat dia, D_2=150mm=0.15m

Therefore area of throat, $A_2= \pi/4*(0.15)^2=0.01767m^2$

Specific gravity of lighter liquid (u-tube) s_l =0.7

Specific gravity of liquid (water) flowing through pipe,

Reading of differential manometer, x=260mm=0.26m

Difference of pressure head, h is given by

$((p_1/\rho g) +z_1) - (p_2/\rho g) +z_2) =h$

Also, $h= x (1- s_l/s_w) =0.26(1-0.7/1.0)$

$=0.078$m of H_2O

Loss of head, $h_L=0.3*$kinetic head of pipe = $0.3 * v_1^2/2g$

Now applying Bernoulli's equation at section '1' and '2',

We get, $(p_1/\rho g) + z_1 + (v_1^2/2g) = (p_2/\rho g) +z_2 + (v_2^2/2g) + h_L$

$[(p_1/\rho g) +z_1) - (p_2/\rho g) +z_2]+ [(v_1^2/2g) - (v_2^2/2g)] = h_L$

But $[(p_1/\rho g) +z_1) - (p_2/\rho g) +z_2] =0.078$ m of H_2O

And $h_L =0.3*(v_1^2/2g)$

therefore, $0.078 + [(v_1^2/2g) - (v_2^2/2g)] = 0.3*(v_1^2/2g)$

$0.078 +0.7(v_1^2/2g) - (v_2^2/2g) = 0$ ---- (1)

Applying **continuity equation on section (1) and (2)** ,

we get $A_1v_1=A_2v_2$

$v_1= A_2v_2/A_1 = v_2/4$

Substitute 'v_1' in equation (1), we get

$0.078 + (0.7(v_2^2/4))/2g - (v_2^2/2g) = 0$

$0.078 + (v_2^2/2g) ((0.7/16)-1) = 0$

$(v_2^2/2g) * (-0.956) = - 0.078$

$v_2^2=0.078*2*9.81/0.956=1.6$

v_2 =1.26m/s

Rate of flow Q= A_2v_2=0.01767*1.26

Q=0.0222m^3/s

3) 215 litres of gasoline (specific gravity 0.82) flow per second through an inclined venturimeter fitted to a 300 mm dia pipe. The venturimeter is inclined at an angle of 60^0 to the vertical and its 150 mm dia. throat is 1.2 m from the entrance along its length. Pressure at throat =0.077 N/mm^2, calculate C_d.

If instead of pressure gauges the entrance and throat of the venturimeter are connected to the two limbs of a U=tube manometer. Determine its reading in mm of differential mercury column.

Solution:

Discharge, $Q=C_d((a_1a_2)/(\sqrt{a_1^2-a_2^2}))*\sqrt{(2gh)} = 215*10^{-3} = 0.215\ m^3/s$

$a_1 = (\pi/4)*(300/1000)^2 = 0.0707\ m^2$

$a_2 = (\pi/4)*(150/1000)^2 = 0.0177\ m^2$

$h = ((p_1/w) +z_1) - ((p_2/w) +z_2)$

$p_1/w = (0.141 *10^6)/(9810*0.82) = 17.528$ m of gasoline

$p_2/w = (0.077 *10^6)/(9810*0.82) = 9.572$ m of gasoline

$z_1 = 0,\ z_2 = (1.2 \sin 30) = 0.66$m

$h = (17.528 + 0) – (9.572 + 0.66) = 7.356$ m

$0.215 = C_d((0.0707*0.0177)/(\sqrt{0.0707^2-0.0177^2}))*\sqrt{(2*9.81*7.356)}$

$C_d = 0.979$

When a U-tube manometer is connected,

$h = x((s_m/s_o)-1)$

$7.356 = x((13.6/0.82)-1)$

$x = 0.472$ m

x = 472 mm

4) The following data relate to an inclined venturimeter:

Diameter of the pipe line = 400 mm

Inclination of the pipe line with the horizontal =30^0

Throat diameter = 200 mm

The distance between the mouth and throat of the meter = 600 mm

Specific gravity of the oil flowing through the pipe line = 0.7

Specific gravity of the heavy liquid (U-tube) = 13.6

Reading of the differential manometer =50 mm

The co-efficient of the meter = 0.98

Determine the rate of flow in the pipe line.

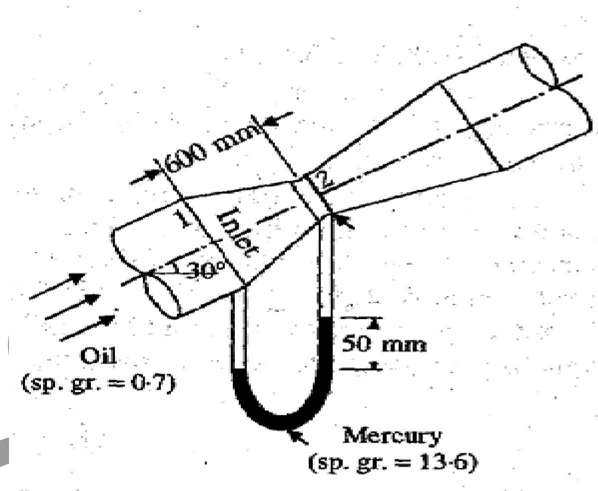


Fig. 4

Difference of pressure head h is given by :

$h = x[(s_h/s_p)-1]$

where s_{hl} = specific gravity of heavy liquid (i.e. mercury) in U-tube = 13.6

s_p = specific gravity of liquid (i.e. oil) flowing (sp. gr. = 0.7) trough the pipe = 0.7

Therefore h = 0.05[(13.6/0.7)-1] = 0.92 m of oil

Now applying Bernoulli's equation at section '1' and '2', we get,

$(p_1/w) + (v_1^2/2g) + z_1 = (p_2/w) + (v_2^2/2g) + z_2$ ……. (i)

$((p_1/w) + z_1)) – ((p_2/w) + z_2)) + (v_1^2/2g) - (v_2^2/2g) = 0$

$((p_1/w) + z_1)) – ((p_2/w) + z_2) = h$

$(p_1/w) - (p_2/w) + (z_1 - z_2) = h$

It may be noted that differential gauge reading will include in itself the difference of pressure head and the difference of datum head

Thus equation (i) reduces to :

$h + (v_1^2/2g) - (v_2^2/2g) = 0$ …… (ii)

applying continuity equation at section '1' and '2' we get, $A_1V_1 = A_2V_2$

or $V_1 = (A_2V_2)/ A_1$

$=(0.0314* V_2)/0.1257$

$= V_2/4$

Substituting the value of V_1 and h in eq. (ii) we get,

$0.92 + (v_2^2/16*2g) - (v_2^2/2g) = 0$

$(v_2^2/2g) (1-(1/16)) = 0.92$ or $v_2^2* (15/16)$

$= 0.92$

or $v_2^2 = (0.92*2*9.81*16)/15$

$= 19.52$

or $v_2 = 4.38$ m

rate of flow of oil, $Q = A_2V_2 = 0.0314*4.38$

Q = **0.1375 m³/s**

5) A vertical venturimeter has an area ratio 5. It has a throat diameter of 10 cm. when oil of specific gravity 0.8, flows through it the mercury in the differential gauge indicates a difference in height of 12 cm. Find the discharge through the venturimeter. Take C_d=0.98.

Solution:

Area ratio, $k=a_1/a_2=5$,

throat diameter, d_2=10cm=0.1m and area $a_2=\pi/4*0.12^2$

Specific gravity of oil, $s_0=0.8$,

difference of Hg level, x=12cm=0.12m

Now differential head (ii) is given by,

$$h = x\left(\frac{s_h}{s_0} - 1\right)$$

$$h = 0.12\left[\frac{13.6}{0.8}\right] = 1.92\,m$$

The discharge is given by,

$$Q = \frac{C_d.a_1.a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

The discharge can be expressed in terms of area, Ratio (k) as

$Q= C_d\ ((a_1a_2)/(\sqrt{}(a_1^2-a_2^2))*\sqrt{}(2gh)$

$= C_d\ ((a_1/a_2)*a_2)/(\sqrt{}((a_1^2/a_2^2)-a_2^2/a_2^2))*\sqrt{}(2gh)$

$= C_d\ ((k*a_2)/(\sqrt{}k^2-1))*\sqrt{}(2gh)$

$= 0.98*((5*\pi(0.1)^2/4)/(\sqrt{}(5)^2-1))*\sqrt{}(2*9.81*1.92)$

Q = 0.0482 m^3/s

6) In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16cm and 18cm respectively. A is 2 meters above B. The pressure gauge readings have shown that the pressure at B is greater than at A by 0.981N/cm^2. Neglecting all losses, calculate the flow rate. If the gauges at A and B is replaced by tubes filled with same liquid and connected to a U-tube containing mercury, calculate the difference of level of mercury in the two limbs of the U-tube.

Solution:

Specific gravity of oil s_0=0.8

Density, ρ =0.8*1000=800kg/m^3

Dia at A D_A=16cm =0.16m

area at A, A_1 =0.02 m^2

Dia at B, D_B =18cm =0.18m

area at B,

i)Difference of pressure,

p_B - p_A= 0.981 N/m^2= 9810 N/m^2

Difference of pressure head

p_B-p_A=9810/(800*9.81) = 1.25

Applying Bernoulli's theorem at A and B, we get

$$\frac{p_A}{\rho g} + \frac{p_B}{\rho g} + Z_A - Z_B = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

$$\left(\frac{p_A - p_B}{\rho g}\right) + 2.0 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

$$0.75 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

Now applying continuity equation at A and B, we get

V_A*A_1=V_B*A_2

$$V_B = \frac{V_A \times A_1}{A_2} = 4V_A$$

Substituting the value of V_B , we get

$$0.75 = \frac{16V_A^2}{2g} - \frac{V_A^2}{2g} = \frac{15V_A^2}{2g}$$

$$V_A = \sqrt{\frac{0.75 \times 2 \times 9.81}{15}} = 0.99\,m/sec$$

$$Q = V_A \times A_1$$

$$Q = \mathbf{0.01989\,m^3/sec}$$

Difference level of mercury in the U-tube

Let h=Difference of mercury level

Then

$$h = x\left(\frac{s_h}{s_0} - 1\right)$$

$$0.75 = x\left[\frac{13.6}{0.8} - 1\right] = x \times 16$$

$$x = \frac{0.75}{16} = 0.04687m = 4.687cm$$

7)Estimate the discharge of kerosene (sp gravity=0.8) through the given venturimeter shown in **Fig. 5**. specific gravity of mercury(Hg) is 13.55.

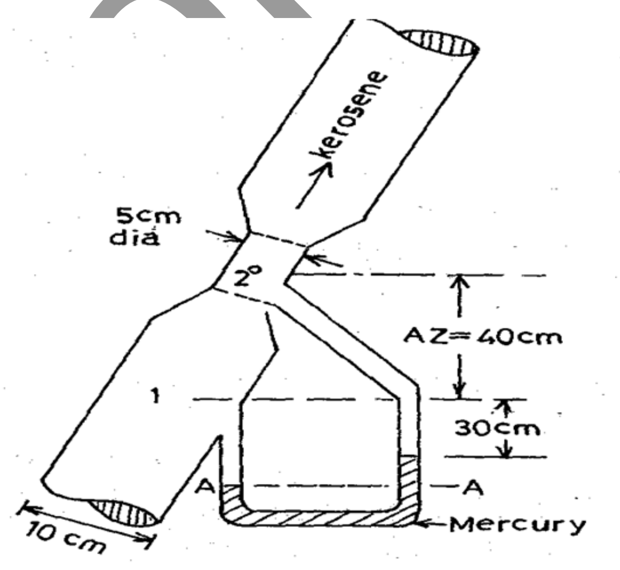


Fig. 5

Solution:

Applying Bernoulli's equation to section 1 and 2

$(p_1/\gamma) + (v_1^2/2g) = (p_2/\gamma) + (v_2^2/2g) + \Delta z$

or $(p_1/\gamma) - ((p_2/\gamma) + \Delta z) = (v_2^2/2g) - (v_1^2/2g)$

Hence $((p_1/\gamma) - (p_2/\gamma) + \Delta z) = 15\ v_1^2/2g$

Equating the pressures at section AA in the two limbs of the manometer

$(p_1/\gamma) + (x + 0.3) = ((p_2/\gamma) + 0.4) + x + (0.3 * (13.85/0.8))$

$(p_1/\gamma) - (p_2/\gamma) + 0.4) = 5.19 - 0.30 = 4.78$ m

$15\ v_1^2/2g = 5.29$ or $v_1 = 2.63$ m/s

Hence Q = 0.785*0.01*2.5 = **0.0196 m³/s = 19.6 l/s**

Orifice meter or orifice plate

Orifice Flow Measurement – History:

- **The first record** of the use of orifices for the measurement of fluids was by Giovanni B. Venturi, an Italian Physicist, who in 1797 did some work that led to the development of the modern Venturi Meter by Clemons Herschel in 1886.
- It has been reported that an orifice meter, designed by Professor Robinson of Ohio State University was used to measure gas near Columbus, Ohio, about 1890.
- About 1903 Mr. T.B. Weymouth began a series of tests in Pennsylvania leading to the publication of coefficients for orifice meters with flange taps.
- At the same time Mr. E.O. Hickstein made a similar series of tests at Joplin, Missouri, from which he developed data for orifice meters with pipe taps.
- A great deal of research and experimental work was conducted by the American Gas Association and the American Society of Mechanical Engineers between 1924 and 1935 in developing orifice meter coefficients and standards of construction for orifice meters

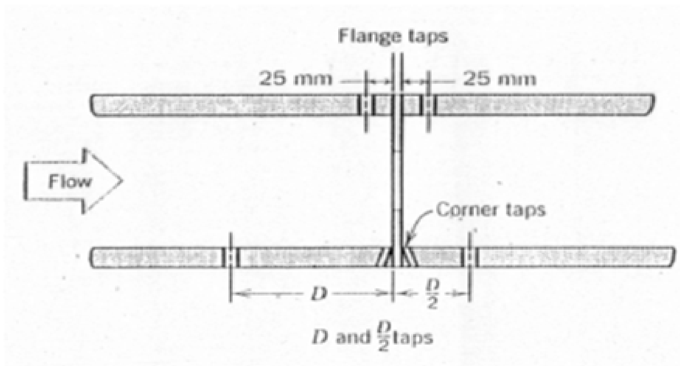


Fig. 1: Tapping arrangements

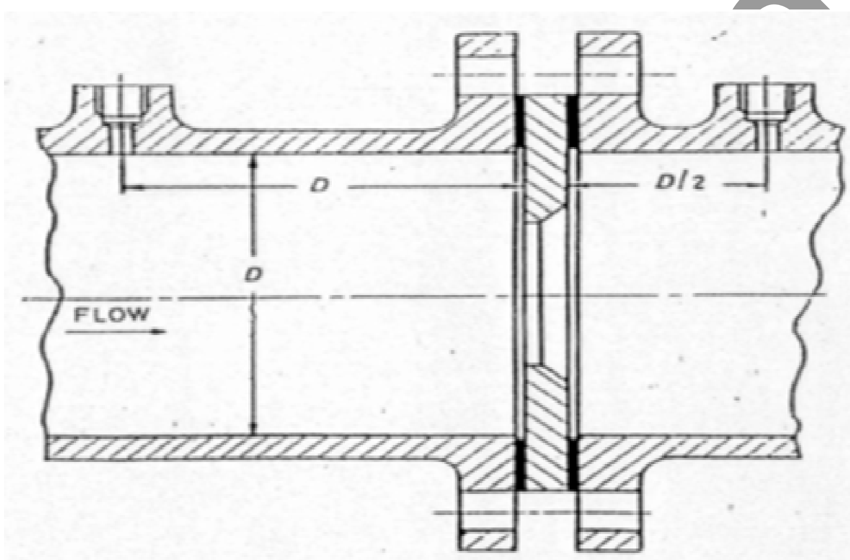


Fig. 2: Orifice profile

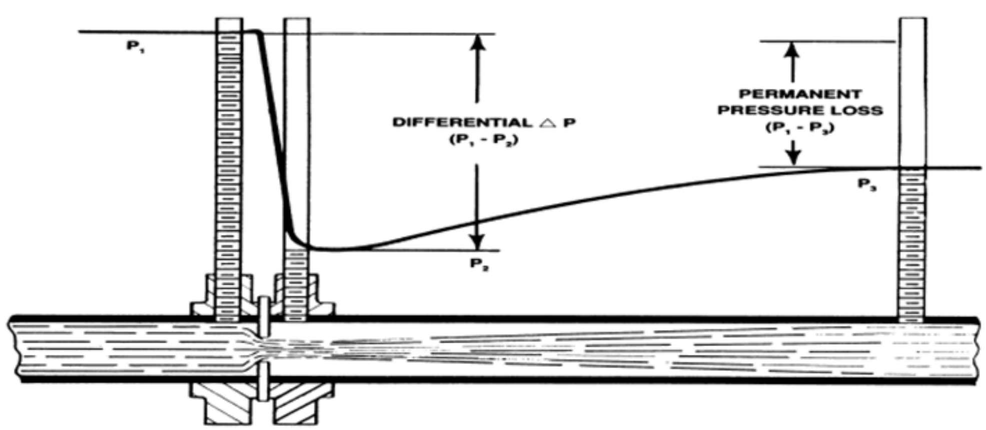


Fig. 3 Typical orifice flow pattern-Flanj taps shown

An orifice in a pipeline is shown in figure 3 with a manometer for measuring the drop in pressure (differential) as the fluid passes thru the orifice. **The minimum cross sectional area of the jet is known as the "vena contracta."**

What is an Orifice Meter?

- An orifice meter is a conduit and a restriction to create a pressure drop. An hour glass is a form of orifice.
- A nozzle, venturi or thin sharp edged orifice can be used as the flow restriction. In order to use any of these devices for measurement it is necessary to empirically calibrate them. That is, pass a known volume through the meter and note the reading in order to provide a standard for measuring other quantities.
- Due to the ease of duplicating and the simple construction, the thin sharp edged orifice has been adopted as a standard and extensive calibration work has been done so that it is widely accepted as a standard means of measuring fluids.
- Provided the standard mechanics of construction are followed no further calibration is required.

Major Advantages of Orifice Meter Measurement

- Flow can be accurately determined without the need for actual fluid flow calibration. Well established procedures convert the differential pressure into flow rate, using empirically derived coefficients.
- These coefficients are based on accurately measurable dimensions of the orifice plate and pipe diameters as defined in standards, combined with easily measurable characteristics of the fluid, rather than on fluid flow calibrations.
- With the exception of the orifice meter, almost all flow meters require a fluid flow calibration at flow and temperature conditions closely approximating service operation in order to establish accuracy.
- In addition to not requiring direct fluid flow calibration, orifice meters are simple, rugged, widely accepted, reliable and relatively inexpensive and no moving parts.

Expression for rate of flow through orifice meter:

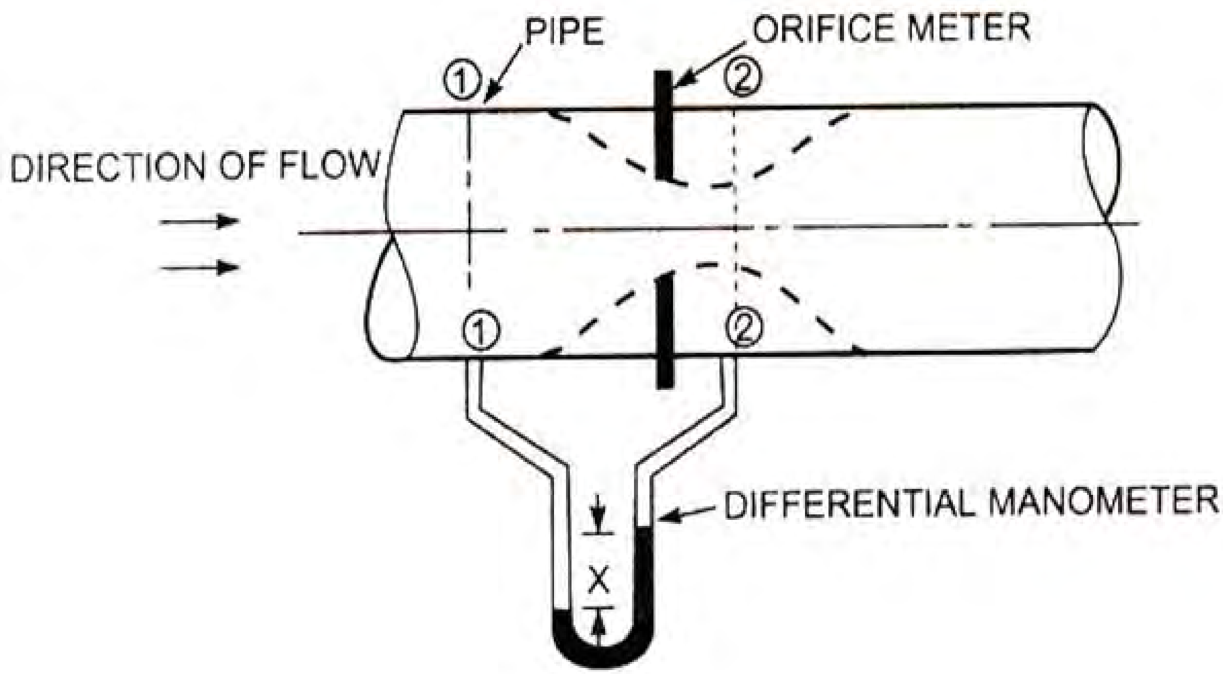


Fig. 4

Orifice meter or orifice plate is a device (cheaper than a venture meter) employed for measuring the discharge of fluid through a pipe. It works on the same principle of a venture meter. It consists of a flat circular plate which has a circular sharp edge hole called orifice, which is concentric with the pipe. The orifice dia is kept generally 0.5 times the dia of the pipe, though it may vary from 0.4 to 0.8 times the pipe dia.

Let p_1, v_1, a_1 at section (1)

p_2, v_2, a_2 at section (2)

Applying Bernoulli's equation at section (1) and (2)

$(p_1/\rho g) + z_1 + (v_1^2/2g) = (p_2/\rho g) + z_2 + (v_2^2/2g)$

$h = ((p_1/\rho g) + z_1) - ((p_2/\rho g) + z_2)$

$h = (v_2^2/2g) - (v_1^2/2g)$

$2gh = v_2^2 - v_1^2$

$v_2 = \sqrt{(2gh) + v_1^2}$ ---------(1)

Now section (2) is at the vena contracta and a_2 represents the area at the vena contracta, if a_0 is the area of the orifice,

we have, $C_c= a_2/a_0$

where **C_c=coefficient of contraction**

$a_2= C_c a_0$ --------(2)

by continuity equation, we have

$v_1a_1= v_2a_2$

$v_1= a_2/a_1 * v_2 = C_c a_0 /a_1 * v_2$ [as $a_2= C_c a_0$] -------(3)

Substitute the value of v_1 in eqn.(1)

$v_2 =\sqrt{(2gh) + (C_c a_0 /a_1 * v_2)^2}$

$v_2^2= (2gh) + (a_0/a_1)^2 C_c^2 v_2^2$

$v_2=\sqrt{(2gh)}/ \sqrt{1-(a_0/a_1)^2 C_c^2}$

Or $h = (v_2^2/2g) - (v_1^2/2g) \Rightarrow 2gh = (v_2^2 - v_1^2)$

$2gh= v_2^2 -(C_c a_0 /a_1 * v_2)^2$

$= v_2^2[1-C_c^2 (a_0 /a_1)^2]$

$v_2=\sqrt{(2gh)}/ \sqrt{1-(a_0/a_1)^2 C_c^2}$ 3a

Discharge, Q= $v_2a_2= v_2$ **$C_c a_0$** (since $a_2= C_c a_0$)

Substitute for V_2

Q = **$C_c a_0$** $\sqrt{(2gh)}/ \sqrt{1-(a_0/a_1)^2 C_c^2}$ -------(4)

$Q_{th} = a_0 a_1\sqrt{(2gh)}/ \sqrt{(a_1^2- a_0^2)}$

$Q_{act}= C_d a_0 a_1\sqrt{(2gh)}/ \sqrt{(a_1^2- a_0^2)}$

Where, C_d=coefficient of discharge of orifice meter. The coefficient of discharge for orifice meter is much smaller than for a venture meter.

Problems on orifice meter:

1)The following data refers to an orifice meter

Dia of the pipe= 240mm

Dia of the orifice=120mm

Specific gravity of oil=0.88

Reading of differential manometer

x = 400 mm of Hg

Coefficient of discharge of the meter,C_d= 0.65

Determine the rate of flow, Q, of oil

Solution:

Dia of the pipe D_1=240mm= 0.24m, A_1=0.0452m^2

Dia of the orifice D_0=120mm= 0.12m,A_2=0.0113 m^2

Coefficient of discharge, C_d= 0.65

Specific gravity of oil, s_0= 0.88

Reading of differential manometer, hg=400mm of Hg = 0.4 m 0f Hg

Therefore differential head, $h=x\,[(s_h/ s_0)-1]$

$=0.4[(13.6/0.88)-1]= 5.78$ m of oil

Discharge, $Q=C_d\, a_0\, a_1\sqrt{(2gh)}/ \sqrt{(a_1^2- a_0^2)}$

$Q = 0.65*0.0113*0.0452*\sqrt{(2*9.81*5.78)}/\sqrt{(0.0452^2- 0.0113^2)}$

=0.08 m^3/s

2) An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter give reading of 19.26 N/cm^2 and 9.81N/cm^2 respectively. Co-efficient of discharge for the meter is given as 0.6. Find the discharge of water through pipe.

Solution.

Given:

Dia. Of orifice. d_0 =10 cm

Therefore area, $a_0 = (\pi 10^2)/4 = 78.54\ cm^2$

Dia. Of pipe, d_1=20 cm

Therefore area, $a_0 = (\pi 20^2)/4 = 314.16\ cm^2$

$p_1 = 19.62\ N/cm^2 = 19.62*10^4 N/m^2$

$(p_1/\rho g) = (19.62*10^4)/(1000*9.81)$

=20 m of water

Similarly $(p_2/\rho g) = (9.81*10^4)/(1000*9.81)$

=10 m of water

Therefore $h = (p_1/\rho g) - (p_2/\rho g) = 20.0 - 10.0$

=10.0 m of water=1000 cm of water

$C_d = 0.6$

The discharge, Q is given by

$Q = C_d*(a_0 a_1/\sqrt{a_1^2 - a_0^2})*\sqrt{2gh}$

$= 0.6*(78.54*314.16/\sqrt{314.16^2 - 78.54^2})*\sqrt{2*981*1000}$

$= 68213.28\ cm^3/s$

=**68.21 lit./s**

3) An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter, the pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm mercury. Find the rate of oil of specific gravity 0.9 when the co-efficient of discharge of the meter = 0.64.

Solution:

Given:

Dia. of orifice, $d_0 = 15$ cm

Therefore Area, $a_0=(\pi 15^2)/4 =176.7cm^2$

Dia. of pipe, $d_1 = 30$ cm

Therefore Area, $a_1=(\pi 30^2)/4 =706.85cm^2$

Specific gravity of oil, $S_0 = 0.9$

Reading of diff. manometer, x= 50 cm of mercury

Differential head, h= $x(s_h/s_0-1)=50(13.6/0.9-1)$

$= 50*14.11=705.5$ cm of oil

Co-efficient of discharge, $C_d = 0.64$

Therefore the rate of the flow, Q is given by

$Q = C_d*(a_0a_1/\sqrt{a_1^2-a_0^2})*\sqrt{2gh}$

$=0.64*(176.7*706.85/\sqrt{706.85^2-176.7^2})*\sqrt{2*981*1000}$

$=137414.25\ cm^3/s$

= **137.414 lit./s**

Pitot tube

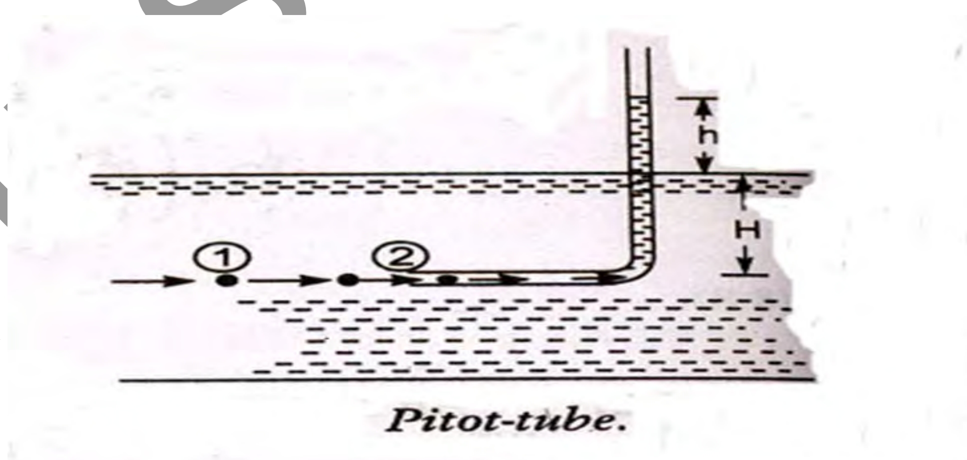


Fig. 1

H= depth of tube in liquid

h=rise of liquid in the tube above the free surface

The Pitot tube (named after the **French scientist Pitot**) is one of the simplest and most useful instruments ever devised. the tube is a small glass tube bent at **right angles** and is placed in flow such that lower end, which is **bent through 90^0 is directed in the upstream direction** as shown in figure. The liquid rises in the tube due to conversion of **kinetic energy** into **potential energy**. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) & (2) at the same level in such a way that the point (2) is just at the inlet of the pitot tube and point (1) is far away from the tube

Let p_1, v_1 & p_2, v_2 are pressure and velocities at point (1) & (2) respectively

H= depth of tube in liquid

h=rise of liquid in the tube above the free surface

Applying Bernoulli's equation at point (1) & (2) we get

$(p_1/\rho g) + z_1 + (v_1^2/2g) = (p_2/\rho g) + z_2 + (v_2^2/2g)$

But $z_1 = z_2$ as point 1 & are on the same line and $v_2=0$

$p_1/\rho g$= pressure head at (1) =H

$p_2/\rho g$ = pressure head at (2) =h+H

Substituting these values, we get

$H + v_1^2/2g = h + H$

$h = v_1^2/2g$

or

$v_1 = \sqrt{2gh}$ (theoretical velocity)

Therefore the actual velocity $(v_1)_{act} = C_v\sqrt{2gh})$

C_v =coefficient of pitot tube

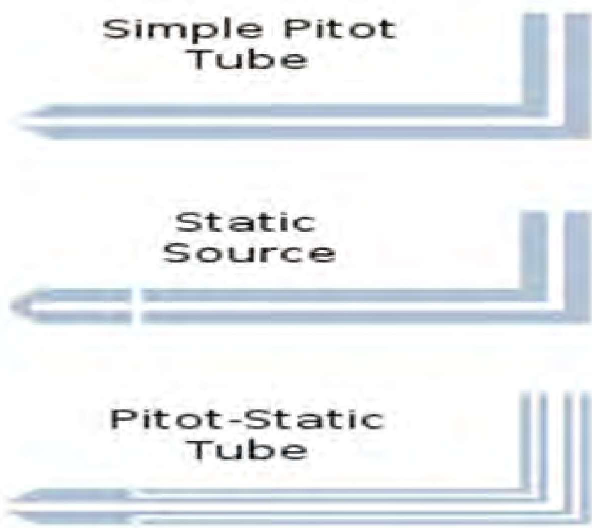


Fig.2: Types of pitot tubes

Stagnation pressure and dynamic pressure

Bernoulli's equation leads to some interesting conclusions regarding the variation of pressure along a streamline. Consider a steady flow impinging on a perpendicular plate (figure 3).

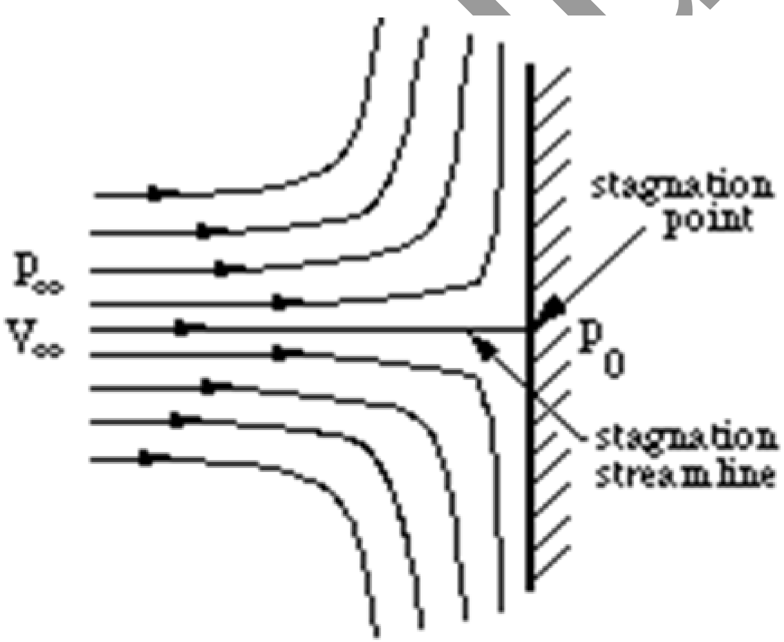


Fig.3: Stagnation point flow.

There is one streamline that divides the flow in half: above this streamline all the flow goes over the plate, and below this streamline all the flow goes under the plate. Along this dividing streamline, the fluid moves towards the plate. Since the flow cannot pass through the plate, the fluid must come to rest at the point where it meets the plate. In other words, it **``stagnates.''** The fluid along the dividing, or **``stagnation streamline''** slows down and eventually comes to rest without deflection at the **stagnation point.**

Bernoulli's equation along the stagnation streamline gives

$$p_e + \frac{1}{2}\rho V_e^2 = p_0 + \frac{1}{2}\rho V_0^2$$

where the **point e** is for upstream and **point 0** is at the stagnation point. Since the velocity at the stagnation point is zero,

$$p_e + \frac{1}{2}\rho V_e^2 = p_0$$

static pressure + dynamic pressure = stagnation pressure

Pitot-Static Tubes

The devices for measuring flow velocity directly is the Pitot-static tube. Figure 4 shows the principle of operation

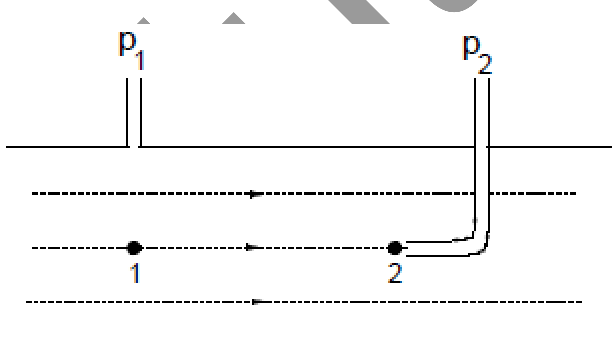


Fig.4: Principle of Pitot-Static tube

By applying Bernoulli's equation to a streamline which meets the tip of the tube. The flow is steady, so there is no flow in the tube. Thus there is a stagnation point, so $u_2 = 0$. *The pressure difference $p_2 - p_1$ is the difference between the impact or stagnation pressure at the tip of the tube, p_2, and* the *static pressure in the body of the fluid, p_1. From Bernoulli,*

$$u_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

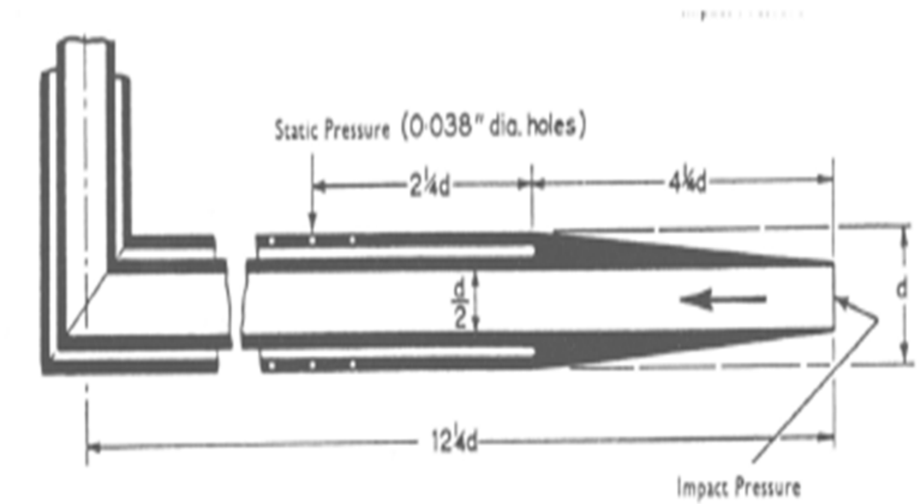


Fig.5- Pitot-Static tube; detail

The most common practical design based upon the above is shown in Figure 5. A pair of concentric tubes is used: the inner tube measured the impact pressure, the outer tube has a number of tiny tappings, flush with the tube, to measure the static pressure. Accuracy is crude, but these devices do provide a very simple and fast estimate of flow velocity.

They are clearly not well suited to dirty flows in which their tappings may become blocked.

Problems on Pitot tube:

1) A pitot-static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60 mm of water. Take the co-efficient of Pitot tube as $C_v = 0.98$.

Solution.

Given:

Dia. of pipe, $d = 30$ mm $= 0.30$ m

Diff. of pressure head, $h = 60$ mm of water

$= 0.06$ m of water

coefficient of pitot tube,$C_v = 0.98$

Mean velocity, $V = 0.80*$ central velocity

Central velocity,V, is given by

$=C_v\sqrt{2gh}=0.98*\sqrt{2*9.81*0.06}$

V $=1.063$ m/s

Mean velocity, V = 0.80 * 1.063 = 0.8504 m/s

Discharge, Q =area of pipe * V

$=(\pi d^2)/4*V$

$=(\pi*.30^2)/4*0.8504$

= **0.06 m³/s**

2) A pitot tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6 m and static pressure head is 5 m. calculate the velocity of flow assuming the co-efficient of tube equal to 0.98.

Solution:

Given: Stagnation pressure head, h_g= 6 m

Static pressure head, h_f = 5 m

h = 6-5 =1 m

Velocity of flow, $V=C_v\sqrt{2gh}=0.98\sqrt{2*9.81*1}$

= **4.34 m/s**

3) A pitot-tube is inserted in a pipe of 300 mm diameter. The static pressure in pipe is 100 mm of mercury (vacuum).The stagnation pressure at the centre of the pipe, recorded by the pitot-tube is 0.981 N/cm^2.Calculate the rate of flow of water through pipe, if the mean velocity of flow is 0.85 times the central velocity. Take C_v = 0.98.

Solution: *Given:* dia of pipe, d= 300 mm = 0.3 m

Area, $a=(\pi d^2)/4=\pi(0.3^2)/4=0.07068\ m^2$

Static pressure head = 100 mm of mercury (vacuum)

= -100/1000*13.6 = -1.36 m of water

Stagnation pressure = 0.981 N/Cm^2=0.981*10^4 N/m^2

Stagnation pressure head = (0.984*10^4)/ρg

=(0.984*10^4)/1000*9.81= 1 m

h =Stagnation pressure head-Stagnation pressure head

= 1.0 - (-1.36) = 1+1.36 = 2.36 m of water

Velocity at centre = C_v √2gh

= 0.98*√2*9.81*2.36 = 6.668 m/s

Mean velocity, = 0.85*6.668 = 5.6678 m/s

Rate of flow of water = mean velocity * area of pipe

= 5.6678*0.07068 m^3/s

= **0.4006 m^3/s**

4) A **submarine** moves horizontally in sea and has its axis 15m below the surface of water. A pitot tube properly placed just in front of the submarine and along its axis is connected to the 2 limbs of U-tube containing mercury. The difference in mercury level is found to be 170mm. Find the speed of the submarine knowing that the specific gravity of mercury is 13.6 and that of sea water is 1.026 with respect of fresh water.

Solution:

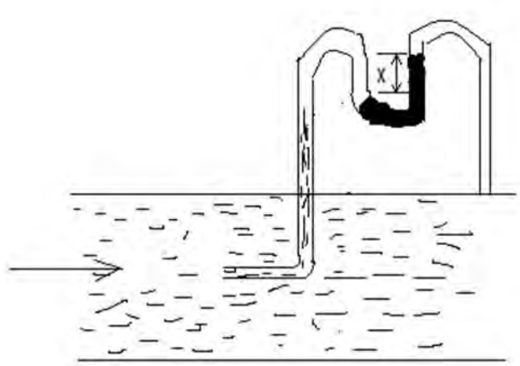


Fig.6: pitot tube

Difference of Hg level, x=170mm=0,17m

Specific gravity of Hg, s_h=13.6

Specific gravity of sea water (in pipe) s_p=1.026

$h=x\ [(s_h/\ s_p)-1]=[(13.6/1.026)-1]=2.0834m$

$v=\sqrt{2gh}=\sqrt{2*9.81*2.0834}=6.393$ m/s

speed of submarine,v =6.393*60*60/1000 km/hr

v =**23.01 km/hr**

Notches and weirs:

A **notch** is a device used for measuring the **rate of flow of liquid** through a **small channel or a tank.** The notch is defined as an opening in the side of the tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A **weir** is a **concrete or masonry structure**, placed in an

open channel over which the flow occurs.

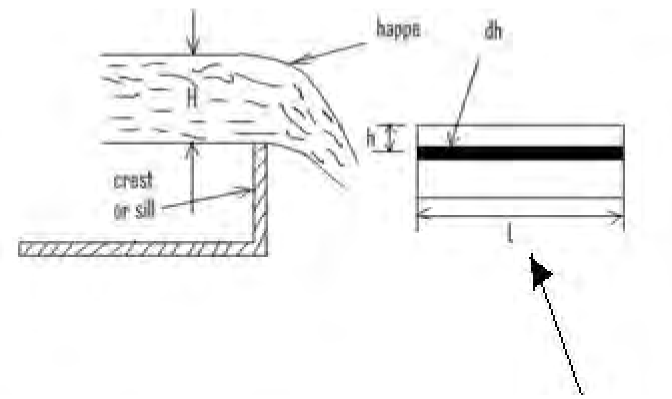
Nappe or Vein: The sheet of water flowing through a notch or over a weir is called Nappe or Vein

Crest or sill: the bottom edge of a notch or a top of a weir over which the water flows, is known as sill or crest

Classification of notches and weirs

- Rectangular notch/weir
- Triangular notch/weir
- Trapezoidal notch
- Stepped notch

Rectangular notch



Section at crest

Fig.1: rectangular notch

Discharge,Q =2/3 $C_d L\sqrt{2g}H^{3/2}$

where H= head of water

L=length of notch

Triangular notch(V-notch)

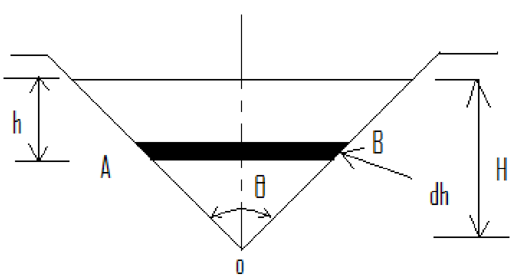


Fig. 2: V-notch

Discharge, Q=(8/15)* C_d, tan(θ/2)* $\sqrt{2g}H^{5/2}$

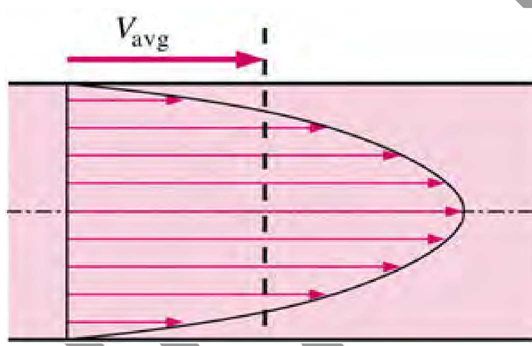
For right angled V-notch, if C_d=0.6, θ=90^0, tan(θ/2)=1,

Q=1.417 $H^{5/2}$

Unit 5: Flow through pipes

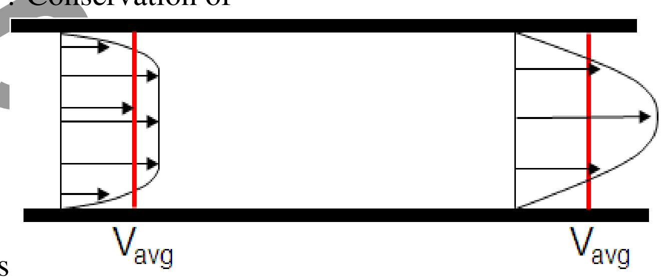
Introduction

- Average velocity in a pipe
  - Recall - because of the no-slip condition, the velocity at the walls of a pipe or duct flow is zero
  - We are often interested only in V_{avg}, which we usually call just V (drop the subscript for convenience)
  - Keep in mind that the no-slip condition causes shear stress and friction along the pipe walls



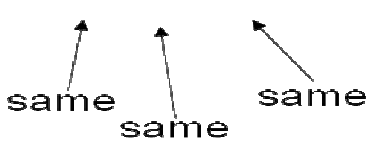
Friction force of wall on fluid

- For pipes of constant diameter and incompressible flow
  - V_{avg} stays the same down the pipe, even if the velocity profile changes
    - Why? Conservation of

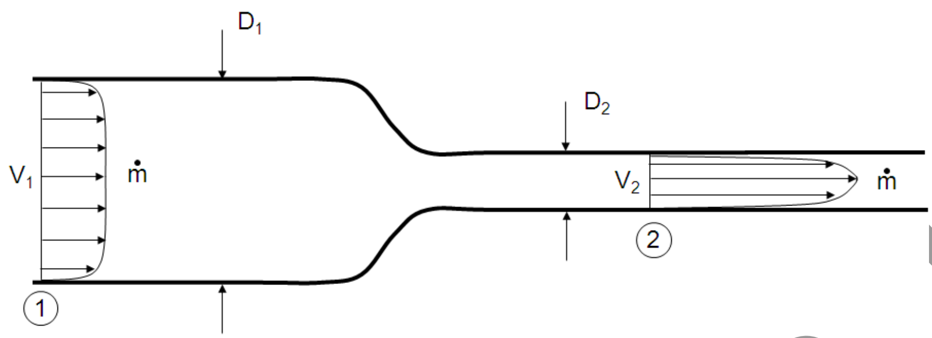


Mass

$$\dot{m} = \rho V_{avg} A = constant$$



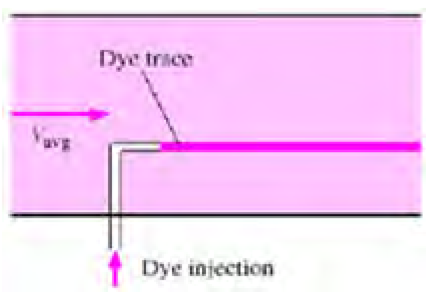
- For pipes with variable diameter, m is still the same due to conservation of mass, but $V_1 \neq V_2$



Laminar and Turbulent Flows

Laminar flow:

- Can be steady or unsteady (steady means the flow field at any instant of time is the same as at any other instant of time)
- Can be one-, two- or three dimensional
- Has regular, predictable behaviour



- Analytical solutions are possible
- Occurs at low Reynold's number

Turbulent flow:

- Is always unsteady.

Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow

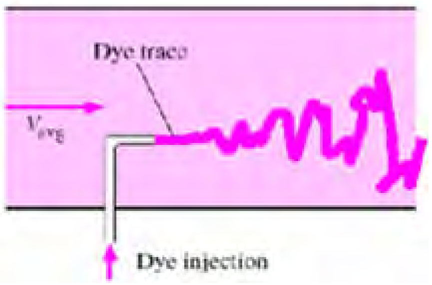
Note: however a turbulent flow can be steady in the mean. We call this a stationary turbulent flow.

- Is always three-dimensional.

Why? Again because of random, swirling eddies, which are in all directions.

Note: however, a turbulent flow can be 1-D or 2-D in the mean.

- Has irregular or chaotic behaviour (cannot predict exactly there is some randomness associated with any turbulent flow.



- No analytical solutions exist! (it is too complicated again because of the 3-D, unsteady, chaotic swirling eddies.)
- Occurs at high Reynold's number.

Definition of Reynolds number

R_e = (inertial force)/(viscous force)

$= (\rho V_{avg}^2 L^2)/(\mu V_{avg} L)$

$= (\rho V_{avg} L)/(\mu)$

$= (V_{avg} L)/(\nu)$

- Critical Reynolds number (Re_{cr}) for flow in a round pipe

$Re < 2300 \Rightarrow$ laminar

$2300 \leq Re \leq 4000 \Rightarrow$ transitional

$Re > 4000 \Rightarrow$ turbulent

- Note that these values are approximate.
- For a given application, Re_{cr} depends upon
  - Pipe roughness
  - Vibrations
  - Upstream fluctuations, disturbances (valves, elbows, etc. that may disturb the flow)

– Kin. vis. ν(nu) = μ/ρ =viscosity/ density

Loss of energy (or head) in pipe: When a fluid is flowing through a pipe, the fluid experiences some resistance to its motion due to which its **velocity** and ultimately the **head of water available** are reduced. This loss of energy or head is classified as follows

Major energy loss:

This is due to **friction** and it is calculated by the following formula

- Darcy-weisbach equation
- Chezy's equation

Minor energy loss:

This is due to:

- Sudden enlargement of pipe
- Sudden contraction of pipe
- Bend in pipe
- Pipe fittings
- An obstruction in pipe

Darcy-Weisbach equation for loss of head due to friction in pipes

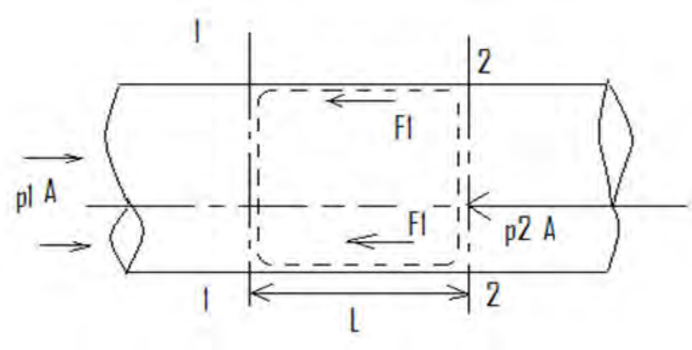


Fig 1: Uniform horozontal pipe

Let, p_1= pressure intensity at section 1-1

v_1= velocity of flow at section 1-1

L= length of the pipe between section 1-1 & 2-2

d= diameter of circular pipe

f^1= frictional resistance per unit wetted area/unit velocity

h_f= loss of head due to friction,

and, p_2 and v_2= are values of **pressure intensity** and **velocity** at section 2-2

Applying **Bernoulli's equation** between sections 1-1 & 2-2

Total head at 1-1 = total head at 2-2 + loss of head due to friction between 1-1 & 2-2

$(p_1/\rho g) + z_1 + (v_1^2/2g) = (p_2/\rho g) + z_2 + (v_2^2/2g)$

But, $Z_1 = Z_2$ as pipe is horizontal

$v_1 = v_2$ as diameter of pipe is same at 1-1 and 2-2

Therefore $(p_1/\rho g) = \{(p_2/\rho g)+h_f\}$ -----(1)

or $h_f = \{(p_1/w)-(p_2/w)\}$

But h_f is the head lost due to friction and hence intensity of pressure will be reduced in the direction of flow by **frictional resistance.**

Now frictional resistance = frictional resistance/unit wetted area/ unit

velocity * wetted area * velocity^2

$F_1 = f^1 * (\pi dL) * V^2$ [because wetted area= $(\pi d*L)$, Velocity=$V=V_1=V_2$]

$= f^1 * (P*L) * v^2$ ----(2) [since πd=perimeter=P]

The **forces acting on the fluid between sections 1-1 and 2-2** are:

- Pressure forces at section 1-1 = p_1A [A=area of pressure]
- Pressure forces at section 2-2 = p_2A
- Frictional force F_1 as shown in Fig. 1.

Resolving these forces in horizontal direction, we have, $p_1A- p_2 A - F_1=0$

$(p_1- p_2) = F_1= [f^1*(P*L)*v^2]/A$ [from equation (2)]

$F_1 = f^1 * P* L* V^2]$

But from equation (1)

$p_1 – p_2 = \rho gh_f$

equating the value of **$(p_1 – p_2)$**, we get

$\rho gh_f = f^1 * P*L* V^2 / A$ or

$h_f = f^1 /\rho g *P/A * L *V^2$ -------(3)

In equation(3) (P/A)= wetted perimeter(πd)/area $(\pi d_2)/4$ = (4/d)

$h_f = f^1 /\rho g * 4/d* * L *V^2$ -------(4)

Putting $f^1/\rho g = f/2$ where **f** is known as **co efficient of friction**

Equation (4) becomes $h_f = 4*f/2g *L V^2/d$ -----(5)

$h_f =4fLV^2/2gd$ ….. Darcy-Weisbach equation

Some times (5) is written as **$h_f =(f^* L V^2)/2gd$**

Then **f*** is known as **friction factor** **[as f* = 4f]**

co efficient of friction **f** which is function of Reynolds number is given by $f = 16/R_e$ for Re < 2000(viscous flow)

$= 0.079/R_e^{1/4}$ for R_e varying from 4000 to 10^6

Chezy's formula for loss of head due to friction:

An equilibrium between the **propelling force** due to pressure difference and the frictional difference gives

$(P_1-P_2)A=f^1PLV^2$ ÷ through out by **w**

$(P_1-P_2)A/w = f^1PLV^2/w$

Therefore, **Mean velocity**, $V=\sqrt{(w/f^1)} *\sqrt{(A/P * h_f/L)}$

Where the factor $\sqrt{(w/f)}$ is called the Chezy's constant 'c' is the ratio(A/P=area of flow /wetted perimeter.) is called the **hydraulic mean depth or hydraulic radius** and denoted by **m** (or R).

The ratio h_f/L is the loss of head/unit length and is denoted by 'i' or s (slope).

Therefore,

Mean velocity, $\mathbf{v=c\sqrt{(mi)}}$ → **Chezy's formula**

Darcy-Weisbach formula(for loss of head) is generally used for the flow through pipes.

Chezy's formula (for loss of head) is generally used for the flow through open channels.

Problems:

1) In a pipe of diameter 350 mm and length 75 m water is flowing at a velocity of 2.8 m/s. Find the head lost due to friction using :

(i) Darcy-Weisbach formula;

(ii) Chezy's formula for which C = 55

Assume kinematic viscosity of water as 0.012 stoke .

Solution:

Diameter of the pipe, D = 350mm = 0.35 m

Length of the pipe, L = 75 m

Velocity of flow, V = 2.8 m/s

Chezy's constant C = 55

Kinematic viscosity of water, ν = 0.012 stoke

$= 0.012*10^{-4}\ m^2/s$

Head lost due to friction, h_f :

(i)Darcy-Weisbach formula:

Darcy-Weisbach formula is given by , $h_f=4fLV^2/2gD$

where, f = co-efficeint of friction(a function of Reynolds number R_e)

$R_e = (v*D) / \nu = (2.8*0.35) / 0.012*10^{-4} = 8.167*10^5$

Therefore $f = 0.0719/ (R_e)^{0.25}$ [use when Re > 4000]

$= 0.0719/ (8.167*10^5)^{0.25}$

$= 0.00263$

Therefore head lost due to friction,

$h_f=(4* 0.00263*75*(2.8)^2) /2*9.81*0.35$

$\mathbf{h_f = 0.9\ m}$

(ii) Chezy's formula:

mean velocity $V = C\sqrt{(mi)}$

Where C = 55, $m=A /P = (\pi*D^2/4) /(\pi*D) = D/4 = 0.35/4$

$= 0.0875$ m

Therefore $2.8 = 55\sqrt{(0.0875*i)}$

or $0.0875*i = (2.8/55)^2 = 0.00259$

$i = 0.00296$

But $i = h_f/L = 0.0296$

Therefore $h_f/75 = 0.0296$

$h_f = 0.0296*75$

$h_f =$ **2.22 m**

2) water flows through a pipe of diameter 300 mm with a velocity of 5 m/s. If the co-efficient of friction is given by $f = 0.015 + (0.08/ (R_e)^{0.3})$ where R_e is the Reynolds number, find the head lost due to friction for a length of 10 m. Take kinematic viscosity of water as 0.01 stoke.

Solution:

Diameter of the pipe, D = 300mm = 0.30 m

Length of the pipe, L = 10 m

Velocity of flow, V = 5 m/s

Kinematic viscosity of water, $\nu = 0.01$ stoke

$= 0.01*10^{-4}\ m^2/s$

Head lost due to friction, h_f :

Co-efficient of friction, $f = 0.015 + (0.08/ (R_e)^{0.3})$

But Reynolds number, $R_e = \rho VD/\mu = VD/\nu$

$= 5*0.3/ 0.01*10^{-4} = 1.5*10^6$

$f = 0.015 + (0.08/ (1.5*10^6)^{0.3})$

$= 0.0161$

Therefore head lost due to friction,

$h_f = 4fLV^2/2gD = 4*0.0161*10*5^2/(0.3*2*9.81)$

$\mathbf{h_f = 2.735\ m}$

3) Water is to be supplied to the inhabitants of a college campus through a supply pipe. The following data is given:

Distance of the reservoir from the campus = 3000 m

Number of inhabitants = 4000

Consumption of water per day of each inhabitant = 180 liters

Loss of head due to friction = 18 m

Co-efficient of friction for the pipe, f = 0.007

If the half of the daily supply is pumped in 8 hours, determine the size of the supply main.

Solution:

Distance of the reservoir from the campus = 3000 m

Number of inhabitants = 4000

Consumption of water per day of each inhabitant

$= 180$ liters $= 0.18\ m^3$

Therefore total supply per day = 4000*0.18 = 720 m^3

Since half of the daily supply is pumped in 8 hours, therefore maximum flow for which the pipe is to be designed,

$$Q = 720/\ (2*8*3600)$$

$$= 0.0125\ m^3/s$$

Loss of head due to friction,

h_f = 18 m

and Co-efficient of friction, f = 0.007

Diameter of the supply line, D:

Using the relation:

$$h_f = 4fLV^2/2gD$$

velocity of flow, $V = Q/A = 0.0125/\ (\pi*D^2/4) = 0.0159/D^2$

By substitution for loss of head due to friction

$$18=4*0.007*3000*(0.0159/D^2)^2/D*2*9.81$$

or $D^5 = (4*0.007*3000*0.0159^2)/(18*2*9.81)$

$$= 6.013*10^{-5}$$

Size of the supply main, D = 0.143 m = **143 mm**

4) In a pipe of 300mm dia and 800m length oil of specific gravity 0.8 is flowing at the rate of 0.45 m^3/s. find Head lost due to friction, and Power required to maintain the flow. Take kinematic viscosity of oil as 0.3 stokes.

Solution:

dia of the pipe, D=0.3m

Length of the pipe, L=800m

Specific gravity of oil =0.8

Kinematic viscosity of oil ν=0.3 stokes=$0.3*10^{-4}\ m^2/s$

Discharge $Q=0.45\ m^3/s$

Head lost due to friction, h_f

Velocity, $v= Q/area =0.45/(\pi*0.3^2/4) =6.366m$

Reynolds number, $R_e=v*D/\nu =6.366*0.3/0.3*10^{-4} =6.366*10^4$

Coefficient of friction, $f = 0.0791/(Re)^{0.25} = 0.0791/(6.366*10^4)^{0.25}$

$= 0.00498$

$h_f = 4fLv^2/2gD=4* 0.00498*800*6.366^2/(0.3*2*9.81) = 109.72\ m$

Power required 'P' $=wQh_f$

$w=0.8*9.81=7.848kN/m^3$

$h_f=109.72m$ and $Q=0.43\ m^3/s$

$P=7.848*0.45*109.72$

P = **387.48kW**

5) A pipe conveys 0.25 kg/sec of air at 300K under an absolute pressure of 2.25bar. Calculate minimum diameter of the pipe required if the fluid velocity is limited to 7.5 m/sec.

Solution:

Density of air $\rho= P/RT= (2.25*10^5)/(287*300)= 2.61kg/m^3$

Mass flow of air, $m = \rho AV$

$0.25 =2.61*A*7.5$

Min. area (A) $= (0.25*1) / (2.61*7.5) = 0.01277\ m^2$

Min. dia. $=\sqrt{(0.01277*4)/\pi} = 0.1275m$

= 12.75 mm

6) A closed tank of a fire engine is partly filled with water, the air space above being under pressure. A 5 cm hose connected to the tank discharges on the roof of building 2 m above the level of water in tank, the friction losses are 50 cm of water. What air pressure must be maintained in the tank to deliver 15 lit/sec on a roof.

Solution:

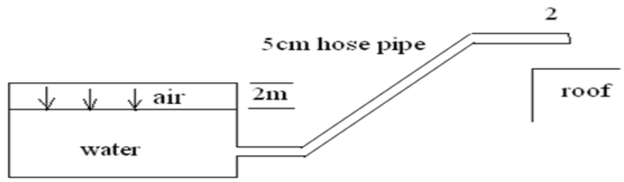


Fig.1

Discharge, $Q= 15$ lit/sec= 0.015 m^3/sec

Velocity in 5cm hose pipe= $0.015/[(\pi/4)*(0.05)^2$

= 7.64 m/sec

Applying Bernoulli's theorem to section 1 and 2, taking water surface level in the tank as datum

$$(v_1^2/2g)+(p_1/w)+y_1 = (v_2^2/2g)+(p_2/w)+y_2+h_f$$

The velocity v_1 at the surface is zero.

$$0+(p_1/w)+0 = [(7.64)^2/(2*9.81)]+0+2.05$$

$$(p_1/w)= 5.48 \text{ m of water}$$

p_1 = **0.548 kg f/cm²** (air pressure in tank)

7) Find the head lost due to friction in a pipe of diameter 300mm and length 50m, through which water is flowing at a velocity of 3m/s

(i) Darcy formula

(ii) Chezy's formula for which C = 60

Take ν for water = 0.01 stoke.

Solution:

Given

Dia. of pipe, d = 300mm = 0.30 m

Length of pipe, L = 50 m

Velocity of pipe, V = 3 m/s

Chezy's constant, C = 60

Kinematic viscosity, $\nu = 0.01$ stoke $= 0.01 cm^2/sec$

$$= 0.01 \times 10^{-4} m^2/sec$$

(i) Darcy Formula is given by equation as

$$h_f = \frac{4fLV^2}{d \times 2g}$$

Where 'f' = co-efficient of friction is a function of Reynolds number, R_e

But R_e is given by

$$R_e = \frac{V \times d}{\nu} = \frac{3.0 \times 0.30}{0.01 \times 10^{-4}} = 9 \times 10^5$$

Value of

$$f = \frac{0.079}{R_e^{\frac{1}{4}}} = \frac{0.079}{9 \times 10^5} = 0.00256$$

Head lost, $h_f = \frac{4 \times 0.00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81} = 0.7828 \text{ m}$

(ii) **Chezy's formula.** Using equation (4)

$$V = C\sqrt{mi}$$

where c = 60, $m = \frac{d}{4} = \frac{0.3}{4} = 0.075 \text{ m}$

Therefore $3 = 60\sqrt{0.075 \times i}$ or $i = \left(\frac{3}{60}\right)^2$

But $i = \frac{h_f}{L} = \frac{h_f}{50}$

Equating the two values of i , we have

$$\frac{h_f}{50} = 0.0333$$

$$h_f = 50 \times 0.0333 = 1.665 \text{ m}$$

8)Find the diameter of a pipe of length 2000m when the rate of flow of water through the pipe is 200 liters/s and the head lost due to friction is 4m.Take the value of c = 50 in chezy's formulae.

Solution:

Length of pipe,

$$L = 2000\, m$$

Discharge,

$$Q = 200 \frac{litres}{s} = 0.2\, m^3/s$$

Head lost due to friction,

$$h_f = 4\, m$$

Value of chezy's constant,

$$c = 50$$

Let the diameter of pipe = d

Velocity of flow,

$$V = \frac{Discharge}{Area} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.2 \times 4}{\pi d^2}$$

Hydraulic mean depth

$$m = \frac{d}{4}$$

Loss of head per unit length.

$$i = \frac{h_f}{L} = \frac{4}{2000} = .002$$

Chezy's formula is given by equation (4) as

$$V = C\sqrt{mi}$$

Substituting the values of V, m, i and C we get

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times 0.002}$$

or $$\sqrt{\frac{d}{4} \times 0.002} = \frac{0.2 \times 4}{\pi d^2 \times 50} = \frac{0.00509}{d^2}$$

Squaring both sides,

$$\frac{d}{4} \times 0.002 = \frac{0.00509^2}{d^4} = \frac{0.0000259}{d^4}$$

$$or \quad d^5 = \frac{4 \times 0.0000259}{0.002} = 0.0518$$

= 0.553 m Or

= 553 mm

9) A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300mm at the rate of 300mm litres/s. find the head lost due to friction for a length of 50m of the pipe.

Solution:

given

Kinematic viscosity, v = 0.4 stoke = 0.4 cm^2/s = 0.4 x 10^{-4} m^2/s

Dia. of pipe, d = 300mm = 0.30 m

Discharge, Q = 300 liters/s = 0.3m^3/s

Length of pipe, L = 50 m

Velocity of flow,

$$V = \frac{Q}{Area} = \frac{0.3}{\frac{\pi}{4}(0.3)^2} = 4.24 m/s$$

Reynolds number,

$$R_e = \frac{V \times d}{\nu} = \frac{4.24 \times 0.30}{0.4 \times 10^{-4}} = 3.18 \times 10^4$$

As R_e lies between 4000 and 100,000 the value of

f is given by,

$$f = \frac{0.079}{R_e^{\frac{1}{4}}} = \frac{.079}{(3.18 \times 10^4)^{1/4}} = .00591$$

Head lost due to friction,

$$h_f = \frac{4.f.L.V^2}{d \times 2g} = \frac{4 \times .00591 \times 50 \times 4.24^2}{0.3 \times 2 \times 9.81} = 3.61\ m.$$

10) An oil of sp.gr.0.7 is flowing through a pipe of diameter 300mm at the rate of 500 litres/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000m.Take ν = .29 stokes

Solution:

Given

Sp.gr. of oil, s = 0.7

Dia. of pipe, d = 300mm = 0.30 m

Discharge, Q = 500 liters/s = 0.5m^3/s

Length of pipe, L = 1000m

Velocity of flow,

$$V = \frac{Q}{Area} = \frac{0.5}{\frac{\pi}{4}d^2} = \frac{0.5 \times 4}{\pi \times 0.3^2} = 7.073 m/s$$

Reynolds number,

$$R_e = \frac{V \times d}{\nu} = \frac{7.073 \times 0.30}{0.29 \times 10^{-4}} = 7.316 \times 10^4$$

Co-efficient of friction,

$$f = \frac{0.079}{R_e^{\frac{1}{4}}} = \frac{.079}{(7.316 \times 10^4)^{1/4}} = .0048$$

Co-efficient of friction,

$$f = \frac{0.079}{R_e^{\frac{1}{4}}} = \frac{.079}{(7.316 \times 10^4)^{1/4}} = .0048$$

Head lost due to friction,

$$h_f = \frac{4.f.L.V^2}{d \times 2g} =$$
$$\frac{4 \times 0.0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81} = 163.18m$$

$$power\ required = \frac{\rho g.Q.h_f}{1000}\ kW$$

Where,

$$\rho = density\ of\ oil = 0.7 \times 1000$$
$$= 700 kg/m^3$$

Power required P= (700*9.81*0.5*163.18/1000)**=560.28 kW**

6)Calculate the discharge through a pipe of diameter 200mm when the difference of pressure head between the two ends of a pipe 500 m apart is 4m of water. Take the value of 'f'=0.009 in the formula,

$$h_f = \frac{4.f.L.V^2}{d \times 2g}.$$

Solution:

Dia. of pipe, d = 200mm = 0.20 m

Length of pipe, L =500 mm

Difference of pressure head,

$h_f = 4\, m\, of\, water$

Co efficient of friction

$f = .009$

Using equation,we have

$$h_f = \frac{4.f.L.V^2}{d \times 2g}$$

$$4.0 = \frac{4 \times 0.009 \times 500 \times V^2}{0.2 \times 2 \times 9.81} \quad or$$

$$V^2 = \frac{4 \times 0.2 \times 2 \times 9.81}{4.0 \times .009 \times 500} = 0.872$$

$$V = \sqrt{.0872} = .9338 \cong .934 m/s$$

Discharge $Q = velocity \times area$

$$= .0934 \times \frac{\pi}{4} d^2 = 0.934 \times \frac{\pi}{4} 0.2^2 = 0.0293 m^3/s$$

=29.3 litres/s.

7)Water is flowing through a pipe of diameter 200mm with a velocity of 3m/s. Find the head lost due to friction for a length of 5m if the co-efficient of friction is given by

$$f = .002 + \frac{0.09}{{R_e}^{0.3}}$$

where R_e is given Reynolds number. The kinematic viscosity of water = 0.01 stoke

Solution:

Dia. of pipe, d = 200mm = 0.20 m

Length of pipe, L = 5m

Velocity V= 3 m/s

Kinematic viscosity, ν = 0.01 stoke = 0.01 cm^2/s

= $0.01*10^{-4}$ m^2/s

Reynolds number,

$$R_e = \frac{V \times d}{\nu} = \frac{3 \times 0.20}{0.01 \times 10^{-4}} = 6 \times 10^5$$

Value of

$$f = .002 + \frac{0.09}{R_e^{0.3}}$$

$$= 0.02 + \frac{0.09}{(6 \times 10^5)^{.3}} = 0.02 + \frac{0.9}{54.13}$$

$$= 0.02 + 0.00166$$

$$= 0.02166$$

Head lost due to friction,

$$h_f = \frac{4.f.L.V^2}{d \times 2g} = \frac{4 \times 0.02166 \times 5 \times 3^2}{0.20 \times 2 \times 9.81}$$

$$= 0.993 m\ of\ water\ .Answer$$

8)An oil of sp.gr.0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200 mm at the rate of 60 liters/s. Find the head lost due to friction for a 500 m length of pipe. Find the power required to maintain this flow.

Solution:

Given: Sp.gr. of oil, $s_{oil} = 0.9$

Viscosity

$$\mu = 0.06\, poise = \frac{0.06}{10}\, Ns/m^2$$

Dia. of pipe, d = 200mm = 0.20 m

Discharge,

$$Q = 60\frac{litres}{s} = 0.06\,m^3/s$$

Length of pipe,

$$L = 500m$$

Density,

$$\rho = 0.9 \times 1000 = 900kg/m^3$$

Reynolds number,

$$R_e = \frac{\rho V d}{\mu} = 900 \times \frac{V \times 0.2}{\frac{0.006}{10}}$$

Where

$$V = \frac{Q}{Area} = \frac{0.06}{\frac{\pi}{4}d^2} = \frac{0.06 \times 4}{\pi \times 0.2^2}$$

$$= 1.909m/s \cong 1.91m/s$$

$$R_e = 900 \times \frac{1.91 \times 0.2 \times 10}{0.06} = 57300$$

As $\mathbf{R_e}$ lies between 4000 and 10^5

The value of **co-efficient of friction, f** is given by

$$R_e = \frac{0.079}{R_e^{\ 0.25}} = \frac{0.079}{(57300)^{0.25}} = 0.0051$$

Head lost due to friction,

$$h_f = \frac{4.f.L.V^2}{d \times 2g} = \frac{4 \times 0.0051 \times 500 \times 1.91^2}{0.20 \times 2 \times 9.81}$$

$$= 9.48m\ of\ water\ .Answer$$

Power required =

$$\frac{\rho g.Q.h_f}{1000} = \frac{900 \times 9.81 \times 0.06 \times 9.48}{1000} = 5.02 \text{ kW}$$

Minor energy (head) losses:

The minor losses of energy are caused in the velocity of flowing fluid (either in magnitude or direction).In case of long pipes these losses are usually quite small as compared with the loss of energy due to friction and hence there are termed minor losses which may even be neglected without serious error. However in small pipes these losses may sometime overweigh the friction loss. Some of the losses of energy which may be caused due to the change of velocity are indicated below,

Loss of head due to sudden enlargement:

$$h_e = (v_1^2 - v_2^2)/2g$$

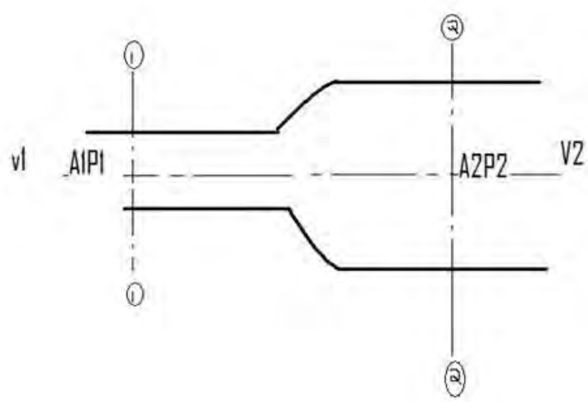


Fig.1: sudden enlargement

Loss of head due to sudden contraction:

when C_c given then use $h_c = v_2^2/2g[v_c/v_2-1]^2$

otherwise use $h_c = 0.5v_2^2/2g$

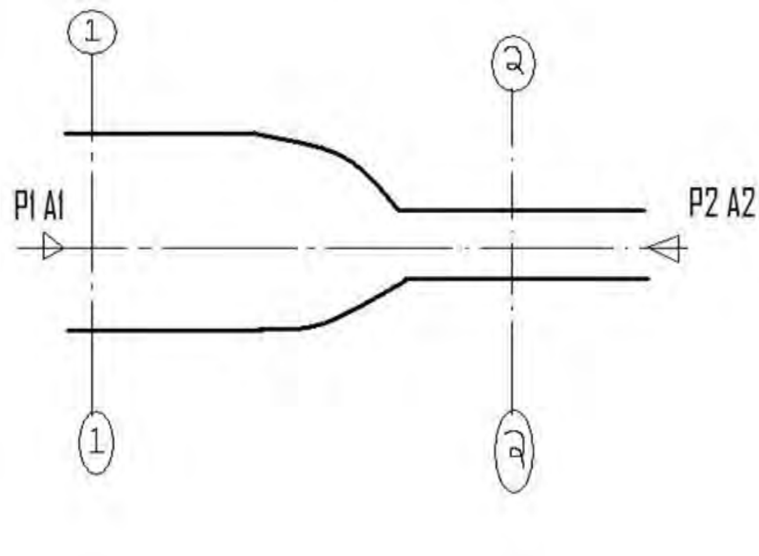


Fig.2: sudden contraction

Loss of energy at the entrance of a pipe:

$$h_i = 0.5v^2/2g$$

Loss of head at the exit of a pipe:

$$h_o = v^2/2g$$

Loss of energy due to gradual contraction or enlargement:

$$h_l = k(v_1 - v_2)^2/2g$$

Loss of energy in bends:

$$h_b = kv^2/2g$$

Loss of head in various pipe fittings:

$=kv^2/2g$ where V= velocity of flow

k= co efficient

Problems on head loss due to minor losses:

1)Find the loss of head when a pipe of diameter 200mm is suddenly enlarged to a diameter of 400mm. The rate of flow of water through the pipe is 250 liters/s.

Solution:

Given:

Dia. of smaller pipe,

$$D_1 = 200mm = 0.20m$$

Area

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.20)^2$$
$$= 0.03141m^2$$

Dia. of large pipe,

$$D_2 = 400mm = 0.4m$$

Area

$$A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564m^2$$

Discharge,

$$Q = 250\frac{litres}{s} = 0.25\, m^3/s$$

Velocity,

$$V_1 = \frac{Q}{A_1} = \frac{0.25}{0.3141} = 7.96m/s$$

Velocity,

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99m/s$$

Loss of head due to enlargement is given by equation

$$h_c = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g}$$
$$= 1.816\, m\, of\, water. answer$$

2) At a sudden enlargement of a water main from 240mm to 480mm diameter, the hydraulic gradient rises by 10mm. Estimate the rate of flow.

Solution:

Given

Dia. of smaller pipe,

$$D_1 = 240mm = 0.24m$$

Area

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.24)^2$$

Dia. of large pipe,

$$D_2 = 480mm = 0.48m$$

Area,

$$A_2 = \frac{\pi}{4} \times (0.48)^2$$

Rise of **hydraulic gradient** ,i.e.

$$\left(z_2 + \frac{p_2}{\rho g}\right) - \left(\frac{p_1}{\rho g} + z_1\right) = 10mm$$

$$= \frac{10}{1000} = \frac{1}{100}\ m$$

Let the rate of flow = Q

Applying **Bernoulli's equation** to both section,

i.e., smaller pipe section, and large pipe section.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Head loss due to enlargement} \qquad \text{-------(1)}$$

But head loss due to enlargement

$h_c = \frac{(V_1 - V_2)^2}{2g}$ ....(2)

From continuity equation, we have

$A_1 V_1 = A_2 V_2$

$$V_1 = \frac{A_2 V_2}{A_1} = \frac{\frac{\pi}{4} D_2^2 \times V_2}{\frac{\pi}{4} D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V^2 \quad = \left(\frac{0.48}{0.24}\right)^2 \times V_2 = 2V_2^2 = 4V_2$$

substituting this value in (ii), we get

$$h_c = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g}$$
$$= \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now substituting the value of h_c and V_1 in equation (1)

$$\frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{9V_2^2}{2g}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left(z_2 + \frac{p_2}{\rho g}\right) - \left(\frac{p_1}{\rho g} + z_1\right)$$

But **hydraulic gradient** rise

$$= \left(z_2 + \frac{p_2}{\rho g}\right) - \left(\frac{p_1}{\rho g} + z_1\right) = \frac{1}{100}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \frac{1}{100} \text{ or } \frac{6V_2^2}{2g} = \frac{1}{100}$$

$V_2 = \sqrt{\frac{2\times9.81}{6\times100}} = 0.1808$ = 0.181 m/s

Discharge, $Q = A_2 \times V_2$

$$= \frac{\pi}{4} D_2^2 \times V_1 = \frac{\pi}{4}(0.48)^2 \times .181$$

$= 0.03275\ m^3/s = 32.75$ liters/sec

3) The rate of flow of water through A horizontal pipe is 0.25 m^3/sec The pipe of diameter 200mm is suddenly enlarged to a diameter of pressure intensity in the smaller pipe is 11.772 N/cm^2 . **Determine:**

- Loss of head due to sudden enlargement
- Pressure intensity in the larger pipe,
- Power lost due to enlargement.

Solution

Given:

Discharge, $Q = 0.25 m^3/s$

Dia. of smaller pipe, $D_1 = 200mm = 0.20m$

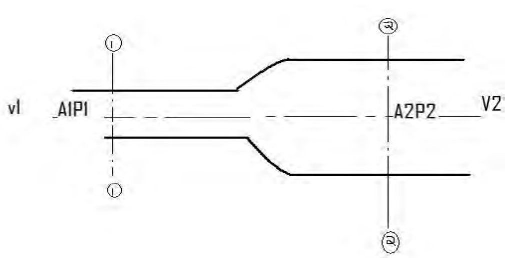
Area $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.03141$

Dia. of large pipe, $D_2 = 400mm = 0.4m$

Area $A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12566 m^2$

Pressure in smaller pipe,

$$p_1 = \frac{11.772N}{cm^2} = 11.772\times10^4 N/m^2$$



Now Velocity. $V_1 = \frac{Q}{A_1} = \frac{0.25}{0.3141} = 7.96 m/s$

Velocity. $V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99 m/s$

(i)Loss of head due to sudden enlargement

$$h_c = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96-1.99)^2}{2g}$$
$$= 1.816\ m\ of\ water.answer$$

(ii)Let the pressure intensity in large pipe=p_2

Then applying **Bernoulli's equation before and after the sudden enlargement,**

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_c$$

[h_c = h_e sudden enlargement]

(Given horizontal pipe)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

$$\frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_c$$

$$= \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{7.96^2}{2 \times 9.81} - \frac{1.99^2}{2 \times 9.81} - 1.816$$

$$= 12.0 + 3.229 - 0.2018 - 1.8160$$

$$= 15.229 - 2.0178$$

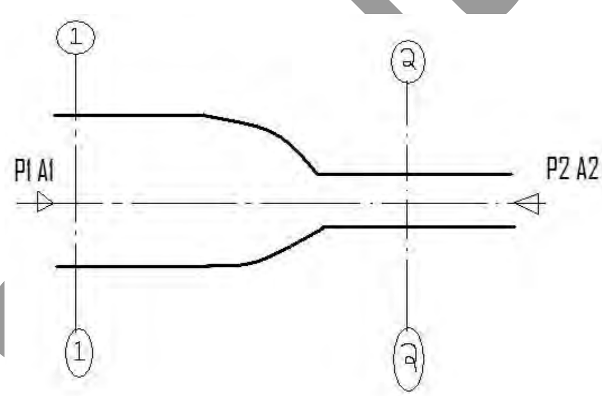
$$= 13.21\ m\ of\ water$$

$$p_2 = 13.21 \times \rho g$$
$$= 13.21 \times 1000 \times 9.81 N/m^2$$
$$= 13.21 \times 1000 \times 9.81 \times 10^{-4} N/cm^2$$
$$= 12.96\ N/cm^2 .$$

Power lost due to sudden enlargement

$$p = \frac{\rho g . Q . h_c}{1000} = \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000}$$
$$= \mathbf{4.453}\ \textbf{KW} \quad \textbf{Answer}$$

4)A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm^2 and 11.772 N/cm^2 respectively. Find the loss of head due to contraction if C_c=0.62.Aiso determine the rate of flow of water.



Solution:

Given:

Diameter of large pipe, D_1=500 mm =0.5m

$$\text{Area } A_1 = \frac{\pi \times d^2}{4} = \frac{\pi \times 0.5^2}{4} = 0.1963\ m^2$$

Diameter of smaller pipe, D_2=250 mm=0.25 m

Therefore

$$\text{Area } A_2 = \frac{\pi \times d^2}{4} = \frac{\pi \times 0.25^2}{4} = 0.04908\ m^2$$

Pressure in larger pipe,p_1=13.734N/cm^2=13.734 10^{4}N/m^2

Pressure in larger pipe, p_2=11.772N/cm^2=11.772 10^{4}N/m^2

C_c=0.62

Head loss due to contraction,

$$\frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1.0\right)^2 = \frac{V_2^2}{2g}\left(\frac{1}{0.62} - 1.0\right)^2 = 0.375\frac{V_2^2}{2g}$$

From **continuity equation**, we have $A_1V_1=A_2V_2$

$$V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V^2 = \left(\frac{0.25}{0.50}\right)^2 V_2 = \frac{V_2}{4}$$

Appling Bernoulli's equation before and after contraction

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_{2+h_c}$$

But $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

$h_c=0.375\frac{V_2^2}{2g}$ and

$V_1 = \frac{V_2}{4}$

Substituting these values in the above equation,
we get

$$\frac{13.734\times10^4}{9.81\times1000} + \frac{(\frac{V_2}{4})^2}{2g} = \frac{11.772\times10^4}{9.81\times1000} + \frac{V_2^2}{2g} + 0.375\ \frac{V_2^2}{2g}$$

$$14 + \frac{V_2^2}{16*2g} = 12.0 + 1.3125\frac{V_2^2}{2g}$$

or $$14 - 12 = 1.375\ \frac{V_2^2}{2g} - \frac{1}{16}\frac{V_2^2}{2g} = 1.375\ \frac{V_2^2}{2g}$$

or $$2.0 = 1.3125\times\frac{V_2^2}{2g}\ or\ V_2 = \sqrt{\frac{2.0\times2\times9.81}{1.3125}} = 5.467\frac{m}{s}$$

1. Loss of head due to contraction,

$$h_c = 0.375\ \frac{V_2^2}{2g}$$

$$= \frac{0.375\times(5.467)^2}{2\times9.81} = 0.571\text{ m}$$

2. Rate of flow of water,

Q $= A_2V_2 = 0.04908\times5.476 = 268.3$ lit/s.

5)If in the previous problem, the rate of flow of water is 300 liters/s, other data reaming the same, find the value of co-efficient of contraction,c_c.

Solution:

Given,

D_1=0.5 m, D2=0.25 m

p_1=13.734 10^4N/m^2

p_2=11.772 10^4N/m^2

Q=300 lit/s=0.3 m^3/sec

Also, from the previous problem,

$V_1 = \frac{V_2}{4}$, where $V_1 = \frac{Q}{A_1} = \frac{0.30}{\frac{\pi}{4}(0.5)^2}$ =1.528 m/s

$V_2 = 4 \times V_1$ = 4 x 1.528 = 6.112m/s

From **Bernoulli's equation**, we have

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$ [as $Z_1 = Z_2$]

$$\frac{13.734 \times 10^4}{9.81 \times 1000} + \frac{(1.528)^2}{2 \times 9.81} = \frac{11.772 \times 10^4}{9.81 \times 1000} + \frac{(6.112)^2}{2 \times 9.81} + h_c$$

$$h_c = 14.119 - 13.904 = 0.215$$

But $h_c = \frac{V_2^2}{2g}(\frac{1}{C_c} - 1)^2$

Hence equating the two values of h_c, we get

$\frac{V_2^2}{2g}(\frac{1}{C_c} - 1)^2$ = 0.215

V_2=6.112

Therefore $\frac{6.112^2}{2 \times 9.81}(\frac{1}{C_c} - 1)^2$=0.215

$[1/c_c - 1]^2$ = 0.215 x 2.0 x 9.81/6.112^2 = 0.1129

C_c = 1.0/1.336 = 0.748

6)150mm diameter pipe reduces in diameter abruptly to 100mm diameter. If the pipe carries Water at 30 liters per second, calculate the pressure loss across the contraction. Take the co-efficient of contraction as 0.6.

Solution

Given:

Dia of large pipe. D_1=150 mm = 0.15 m

Area of large pipe. $A_1 = \pi(0.15)^2/4 = 0.01767\ m^2$

Dia.of smaller pipe D_2=100 mm=1.00 m

Area of smaller pipe, $= A_1 \frac{\Pi}{4}(.10)^2 = 0.007854\ m^3$

Discharge Q=30 liters/s=0.3 m^3/s

Co-efficient of contraction, $C_c = 0.6$

From continuity equation, we have

$$A_1V_1 = A_2V_2 = Q$$

$$V_1 = \frac{Q}{A_1} = \frac{0.03}{0.01767} = 1.697\frac{m}{s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.03}{0.007854} = 3.82\frac{m}{s}$$

Applying Bernoulli's equation before and after the contraction,

$$(p_1/\rho g) + (v_1^2/2g)+z_1 = (p_2/\rho g) + (v_2^2/2g)z_2 + h_c \text{ ----(1)}$$

But $z_1=z_2$

And the h_c head loss due to contraction is given by

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c}-1\right)^2 = \frac{3.82^2}{2\times 9.81}\left(\frac{1}{0.6}-1\right)^2 = 0.33$$

Substituting these values in eqn.1, we get

$$(p_1/\rho g)+\frac{1.697^2}{2\times 9.81} = (p_2/\rho g)+\frac{3.82^2}{2\times 9.81}+0.33$$

$(p_1/\rho g) - (p_2/\rho g) = 0.7438+.33-0.1467=0.9271$ m of water

$p_1 - p_2 = \rho g \times 0.9271 = 1000 \times 9.81 \times 0.9271\ N/m^2$

Pressure loss across contraction

$p_1 - p_2 = 0.909*10^4$ N/m2 $= 0.909\ N/cm^2$

7) In Fig. (3) show, when a sudden contraction is introduced in a horizontal pipe line from 50 cm to 25cm, the pressure changes from 10,500kg/m^2 (103005 N/m^2) to 6900 kg/m^2(67689 N/m^2). Calculate the rate of flow. Assume co-efficient of contraction of jet to be 0.65.

Following this If there is a sudden enlargement from 25 cm to 50 cm and if the pressure at the 25 cm section is 6900 kg/m^2(67689 N/m^2) what is the pressure at the 50cm enlarged section?

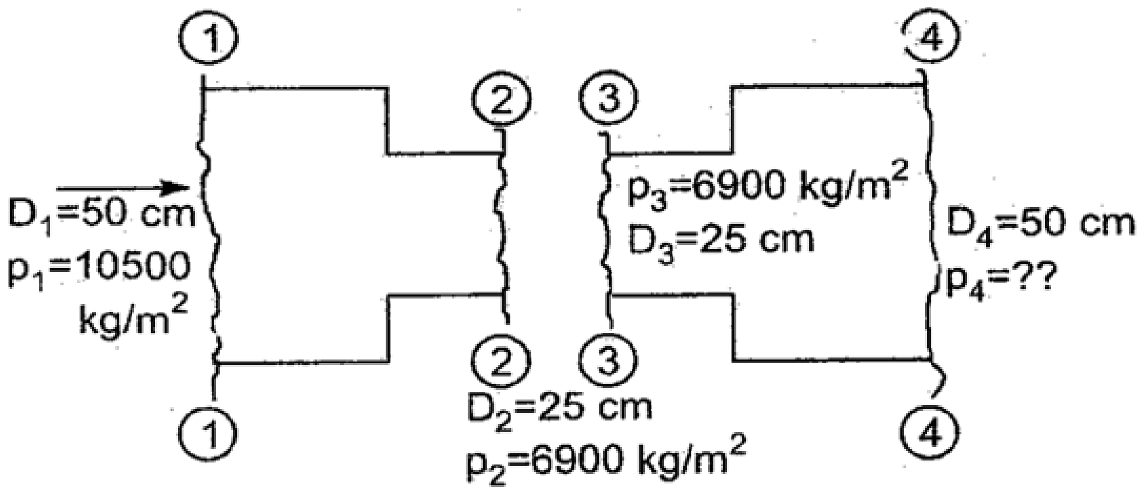


Fig 3

Solution:

Dia of large pipe d_1 = 50 cm = 0.5m

Area A_1 = 0.1963 m^2

Dia of smaller pipe , D2 = 25cm =0.25m

Area A_2 = 0.04908 m^2

Pressure in large pipe,p_1 = 10500 kg/m^2 = 103005 N/m^2

Pressure in smaller pipe, p_2 = 6900 kg/m^2 = 67689 N/m^2

Co-efficient of contraction, C_c = 0.65

Head lost due to contraction is given by equation

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 = \frac{V_2^2}{2g}\left(\frac{1}{0.65} - 1\right)^2 = 0.2899\frac{V_2^2}{2g}$$

From **continuity equation**, we have

$$A_1V_1 = A_2V_2$$

$$\text{or } V_1 = \frac{A_2V_2}{A_1} \quad \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2}$$

$$\left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{.5}{.25}\right)^2 \times V_2 = \frac{V_2}{4}$$

Applying Bernoulli's equation to sections 1-1 and 2-2,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_c$$

(as pipe is horizontal) $z_1 = z_2$

$$\text{But } \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

Substituting the values of p_1, p_2, h_c and V_1 we get

$$\frac{103005}{1000\times 9.81} \times \frac{\left(\frac{V_2}{4}\right)^2}{2g} = \frac{67689}{1000\times 9.81} + \frac{V_2^2}{2g} + .2899\frac{V_2^2}{2g}$$

$$V_2 = \sqrt{\frac{3.6 \times 2 \times 9.81}{1.2274}} = 7.586\, m/s$$

(i) Rate of flow of water ,

Q = $A_2 V_2 = 0.04908 \times 7.586\ m^3/s$ or 372.3 lit/s

(ii) Appling Bernoulli's equation to section 3-3 and 4-4

$$\frac{p_3}{\rho g} + \frac{V_3^2}{2g} + z_3 =$$

$$\frac{p_4}{\rho g} + \frac{V_4^2}{2g} + z_4 + head\ loss\ due\ to\ sudden\ enlargement (h_c)$$

But $p_3 = 6900 \frac{kg}{m^2}, or\ 67689 \frac{N}{m^2}$ $Z_3 = Z_4$

$V_3 = V_2 = 7.586$ m/s $V_4 = V_1 = \frac{V_2}{4} = \frac{7.586}{4} = 1.896$ m/sec

And

head loss due to sudden enlargement is given by

$$h_e = \frac{(V_3 - V_4)^2}{2g} \quad \frac{(7.586 - 1.8965)^2}{2 \times 9.81} = 1.65\ m$$

Substituting these values in Bernoulli's equation, we get

$$\frac{67689}{1000 \times 9.81} + \frac{7.586^2}{2 \times 9.81} = \frac{P_4}{1000 \times 9.81} + \frac{1.8965^2}{2 \times 9.81} + 1.65$$

$$P_4 = 8 \times 1000 \times 9.81 = 78480\ N/m^2$$

8) Determine the rate of flow of water through a pipe diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the Height of water in the tank is 4 m above the centre of the pipe. Consider all minor losses and take f=0.009 in the formula,

$$\frac{4.f.L.V^2}{d \times 2g}.$$

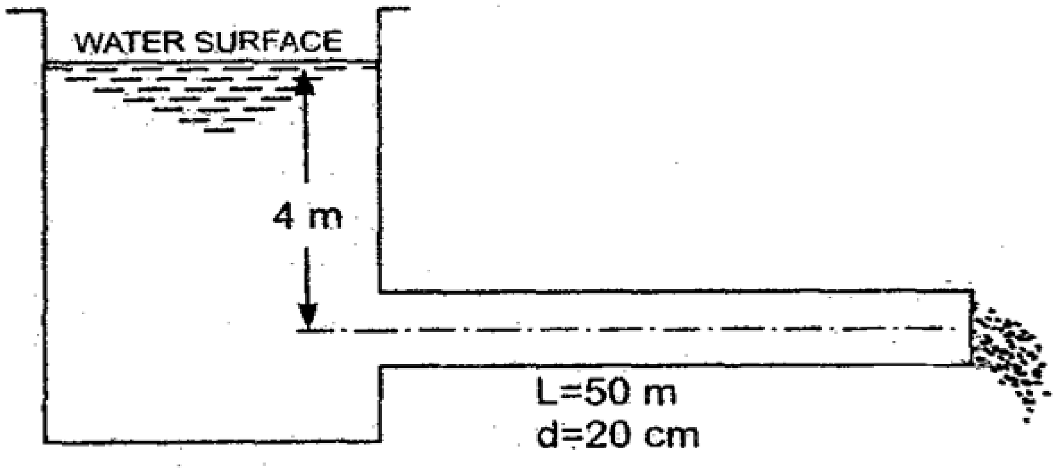


Fig 4

Solution:

Dia of pipe d = 20cm = 0.20m

Length of pipe L= 50 m

Height of water H=4 m

Co-efficient of friction, f=0.009

Let the velocity of water in the pipe =V m/s

Applying Bernoulli's equation at the top of water surface in the tank and at the outlet of pipe, we have [taking point 1 on the top point 2 at the outlet of pipe]

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + all\ losses$$

Considering datum line passing through the centre of pipe

$$0 + 0 + 4.0 = 0 + \frac{V_2^2}{2g} + h_i + h_f$$

or $$4.0 = \frac{V_2^2}{2g} + h_i + h_f$$

But the velocity in pipe =V,
therefore V= V_2

Therefore $$4.0 = \frac{V^2}{2g} + h_i + h_f$$

From the equation for loss of head at the entrance of pipe we have,

$h_i = 0.5\frac{V^2}{2g}$ and $h_f = \frac{4.f.L.V^2}{d \times 2g}$

Substituting these values, we have

$$4.0 = \frac{V^2}{g} + 0.5\frac{V^2}{2g} + \frac{4.f.L.V^2}{d \times 2g}$$

$$\frac{V^2}{2g}\left[1.0 + 0.5 + \frac{4 \times 0.009 \times 50}{0.2}\right] = \frac{V^2}{2g}[1.0 + 0.5 + 9.0]$$

$$\therefore \quad 4 = 10.5 \times \frac{V^2}{2g}$$

$$V = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}} = 2.734\ m/sec$$

Rate of flow, $Q = A \times V = \frac{\pi}{4} \times (0.2)^2 \times 2.734$

$$= 0.08589\ m^3/s$$

= 85.89 liters/s

9) Determine the difference in the elevations between the water surfaces in the two tanks which are connected by a horizontal pipe of diameter 300mm and length 400m. The rate of flow of water through the pipe is 300 liters/s. Consider all losses and take the value of f = 0.008.

Solution:

Dia. of pipe d = 300mm=0.30m

Length L = 400m, Discharge Q = 300 liters/s = 0.3 m^3/s

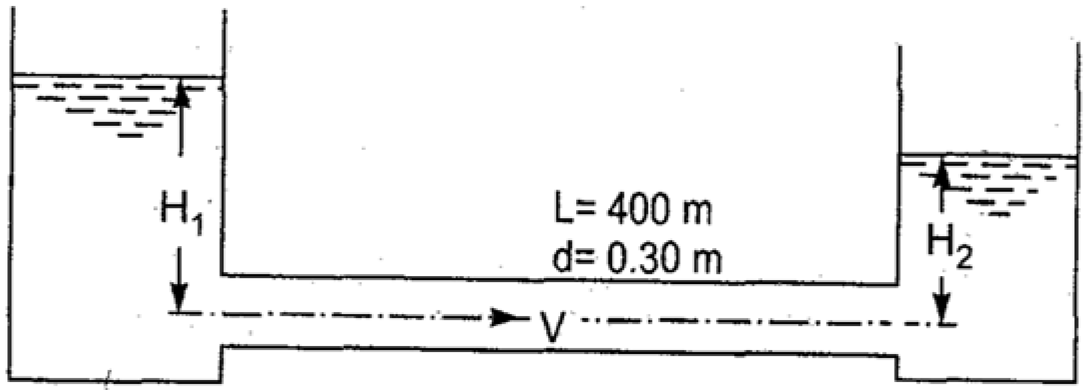


Fig. 5

Co-efficient of friction, f=0.008

Velocity, V = $\frac{Q}{Area} = \frac{0.3}{\frac{\pi}{4}\times(0.3)^2} = 4.224\frac{m}{s}$

Let the two tanks are connected by a pipe as shown in figure 3.

Let H_1 = height of water in 1st tank above t

H_2= height of water in 2nd tank above the centre of pipe

Then the difference in elevations between water surfaces= $H_1 - H_2$

Appling Bernoulli's equation to the free surface of water in the two tanks, we have

$H_1 = H_2 + \text{losses}$

$= H_2 + h_i + H_{f_1} + h_0$ .... (i)

Where h_i = loss of head at entrance $= 0.5\frac{V^2}{2g}$

$= \frac{0.5\times4.244^2}{2\times9.81} = .459\ m$

h_{f1} = Loss of head due to friction = $\frac{4\times f\times L\times V^2}{d\times2g}$

$= \frac{4\times.008\times400\times4.244^2}{0.3\times2\times9.81}$ = 39.16 m

10) The friction factor for turbulent flow through rough pipes can be determined by karman-prandtl $\frac{1}{\sqrt{f}} = 2\ log_{10}\left(\frac{R_0}{k}\right) + 1.74$

Where **f = friction factor**, R_0 = pipe radius, k = average roughness Two reservoirs with a surface level difference of 20 m are to be connected by 1 m diameter pipe6 km long. What will be the discharge when a cast iron pipe of roughness k = 0.3 mm is used? What will be the percentage increase in the discharge if the cast iron pipe is replaced by a steel pipe of Roughness k = 0.1 mm? Neglect all local losses.

Solution:

Difference in levels, h=20 mm

Dia of pipe, d=1.0 m

Length of pipe, L =6km=6 x 1000=6000m

Roughness of cast iron pipe, k=0.3 mm

Roughness of steel pipe, k=0.1 mm

1st case. Cast iron pipe.

First find the value of friction factor using

$$\frac{1}{\sqrt{f}} = 2\ log_{10}\left(\frac{R_0}{k}\right) + 1.74 \quad ....(i)$$

$$= 2\ log_{10}\left(\frac{500}{0.3}\right) + 1.74 = 8.1837$$

f = $\left(\frac{1}{8.1837}\right)^2 = 0.0149$

Local losses are to be neglected. This means only head loss due to friction is to be consider, head loss due to friction is

$$20 = \frac{f \times L \times V^2}{d \times 2g}$$

[Here f is the friction factor and not co-efficient of friction because Friction factor = 4 x co-efficient of friction]

$$20 = \frac{0.0149 \times 6000 \times V^2}{1.0 \times 2 \times 9.81} = 4.556\,V^2$$

$$V = \sqrt{\frac{20}{4.556}} = 2.095\ m/\ \text{sec}$$

$$\therefore \text{Discharge}, Q = V \times \text{Area} = 2.095 \times \frac{\pi}{4} \times d^2$$

$$= 2.095 \times \frac{\pi}{4} \times 1^2 = 1.645\,\frac{m^3}{s}$$

2nd case. Steel pipe. K=0.1mm, R_0 = 500 mm

Substituting these values in equation (i), we get

$$= 2\log_{10}\left(\frac{500}{0.1}\right) + 1.74 = 9.1379$$

f = $\left(\frac{1}{9.1379}\right)^2 = 0.0119$

Head loss due to friction, $20 = \frac{f \times L \times V^2}{d \times 2g}$

$$20 = \frac{0.0119 \times 6000 \times V^2}{1.0 \times 2 \times 9.81} = 3.639V^2$$

$\therefore$ V= $\sqrt{\frac{60}{3.639}} = 2.344$ m/s

$\therefore$ Discharge,

$$Q = V \times \text{Area} = 2.344 \times \frac{\pi}{4} \times 1^2$$

$$= 1.841 \text{ m}^3/\text{s}$$

Percentage increase in the discharge

$$= \frac{Q_2 - Q_1}{Q_1} \times 100 = \frac{(1.841 - 1.645)}{1.645} \times 100$$

$$= 11.91\%.$$

11).A horizontal pipeline 40m long is connected to a water tank at one end and discharge freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150mm diameter and its diameter is suddenly enlarged to 300mm. The height of water level in the tank is 8m above the centre of the pipe. Considering all losses of head which occurs determine the rate of flow, take f=0.01 for both section of the pipe.

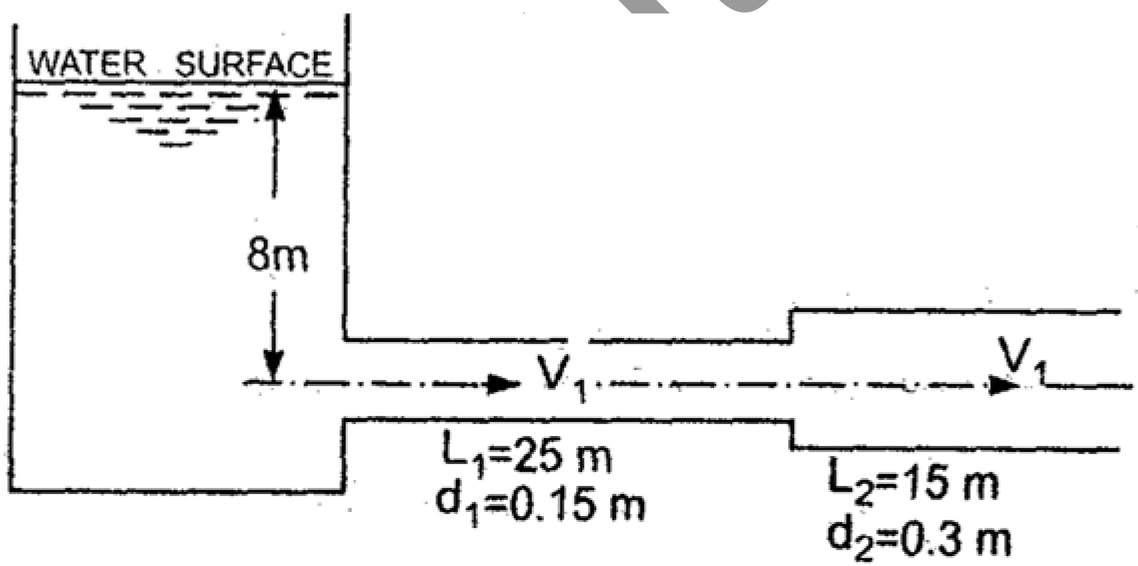


Fig. 6

Solution:

Given:

Total length of pipe, L = 40m

Length of 1st pipe L_1= 25m

Dia. Of the 1st pipe, d1 = 150mm = 0.15m

Length of 2nd pipe, L_2 = 40 – 25 = 15m

Dia. Of 2nd pipe d_2 = 300mm = 0.3 m

Height of water, H = 8m,

Co-efficient of friction, f = 0.01

Applying Bernoulli's equation to the **free surface of water on the tank** and **outlet of pipe** as shown in Fig. 4 and

Taking reference line passing through the centre of pipe.

[Taking point 1 on the top point 2 at the outlet of pipe]

$\mathbf{P_1/w+v_1^2/2g+z_1= p_2/w+v_2^2/2g+0+all\ losses}$

$0 + 0 + 8 = 0+ V_2^2/2g +0+\mathbf{h_i+h_{f1}+h_e+h_{f2}}$------(1)

Where $\mathbf{h_i}$=loss of head at entrance=$0.5v_1^2/2g$

$\mathbf{h_{f1}}$=head loss due to friction in pipe 1= $\mathbf{4f\ L_1v_1^2/2gd_1}$

$\mathbf{h_e}$=loss of head due to sudden enlargement

= $\mathbf{(v_1-v_2)^2/2g}$

$\mathbf{h_{f2}}$= head loss due to friction in pipe 2= $\mathbf{4f\ L_2v_2^2/2gd_2}$

From continuity equation, we have

$A_1V_1= A_2V_2$

$\mathbf{V_1}= A_2V_2/V_1= (D_2/D_1)^2*V_2 = (0.3/0.15)^2*V_2= \mathbf{4V_2}$

Substituting the value of V_1 in different head losses, we have

$h_i = 0.5v_1^2/2g=0.5(4v_2^2)/2g$

$= 8v_2^2/2g$

$h_{f1}=4f\ L_1v_1^2/2gd_1=(4*0.01*25*(4v_2)^2)/(0.15*2g)$

$=106.6v_2^2/2g$

$h_e= (v_1-v_2)^2/2g=(4v_2-v_2)^2/2g$

$= 9v_2^2/2g$

$h_{f2}= 4fl_2v_2^2/2gd_2=4*0.01*15*v_2^2/(0.3*2g)$

$= 2v_2^2/2g$

Substituting the value of the losses in equation (1), we get

$8=v_2^2/2g+ 8v_2^2/2g+106.6v_2^2/2g+9v_2^2/2g+2v_2^2/2g$

$=v_2^2/2g(1+8+106.6+9+2)$

$=126.6v_2^2/2g$

$V_2=\sqrt{(8*2g)}/126.6$

$=\sqrt{(8*2*9.81)}/126.6$

V_2 =1.11 m/s

Hence rate of flow '**Q'=$A_2v_2=(\pi*(0.3)^2/4)*1.11$**

$Q=0.078m^3/s$

Q= 78.67 liters/s

12) Design the diameter of a steel pipe to carry water having kinematic viscosity $\nu =10^{-6}$ m^2/sec With a mean velocity of 1 m/s. The head loss is to be limited to 5 m per 100 m length of pipe. Consider the equivalent sand roughness height of pipe k_s= 45 x 10^{-4} cm. Assume that the Darcy weisbach friction factor over the whole range of turbulent flow can be expressed as

$$f = 0.0055 [1+(20 \times 10^3 \frac{k_s}{D} + \frac{10^6}{R_e})^{\frac{1}{3}}]$$

Where D =Diameter of pipe and R_e=Reynolds number.

Solution:

Given:

Kinematic viscosity, $\nu = \frac{10^{-6} m^2}{s}$

Mean velocity, V =1 m/s.

Head loss, h_f = 5 m in a length L = 100m

Value of k_s= 45 x 10^{-4} cm=45 x 10^{-6} m

$$\text{Value of } f = 0.0055[1+(20\times10^3\frac{k_s}{D}+\frac{10^6}{R_e})^{\frac{1}{3}}]\text{.................(i)}$$

Using Darcy weisbach equation, $h_f = \frac{4\times f\times L\times V^2}{D\times 2g}$

or $f = \frac{h_f\times D\times 2g}{4\times L\times V^2} \quad \frac{5\times D\times 2\times 9.81}{4\times 100\times 1^2} = 0.2452\ D$

Now the Reynolds number is given by,

$R_e = \frac{\rho VD}{\mu} = \frac{V\times D}{\gamma}$ As ($\nu = \mu/\rho$)

$= \frac{1\times D}{10^{-6}} = 10^6 D$

Substituting the values of f, R_e, and k_s in equation (i), we get

$0.2452\ D = 0.0055\ [1+(20\times10^3\times\frac{45\times10^{-6}}{D}+\frac{10^6}{10^6 D})^{\frac{1}{3}}]$

$\frac{0.2452}{0.0055} D = [1+(\frac{0.9}{D}+\frac{1}{D})^{\frac{1}{3}}]$

or $44.58\ D = [1+(\frac{1.9}{D})^{\frac{1}{3}}]$ or $44.58\ D - 1 = (\frac{1.9}{D})^{\frac{1}{3}}$

or $\frac{1.9}{D}$

or $D(44.58\ D-1)^3 = 1.9$ .... (ii)

The equation (ii) is solved by hit and trail method.

$\therefore$ correct value of D = **0.0854 m**

13) A pipe line AB of diameter 300 mm and length 400m carries water at the rate of 50 liters/s. The flow takes place from A to B where point B is 30 m above A.

Find the pressure at A if the pressure at B is 19.62 N/cm^2 Take f = 0.008

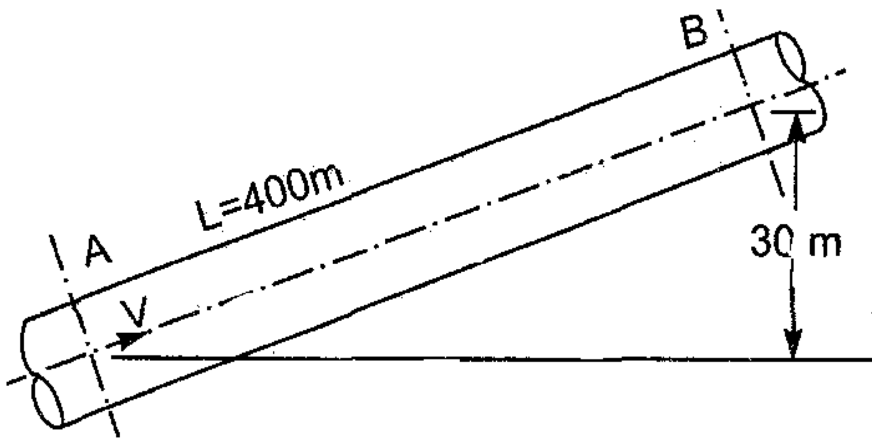


Fig 7

Solution:

Given:

Dia.of pipe, $d = 300\text{ mm} = 0.30\text{ m}$

Length of pipe, $L = 400\text{ m}$

Discharge, $Q = 50\text{ liters/s} = 0.05\text{ m}^3/\text{sec}$

$$\text{Velocity,} \quad V = \frac{Q}{Area} = \frac{0.05}{\frac{\pi}{4}d^2} = \frac{.05}{\frac{\pi}{4}\times(.3)^2} = 0.7074\, m/s$$

Pressure at B, $P_B = 19.62\text{N/cm}^2 = 19.62 \times 104\text{ N/m}^2$

$F = 0.008$

Appling Bernoulli's equation at points A and B

And taking datum line passing through A, we have

$$\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+z_A=\frac{P_B}{\rho g}+\frac{V_B^2}{2g}+z_B+h_f$$

But $V_A = V_B$ $\because Dia. is\ same]$

$z_A = 0, z_B = 30$

and $h_f = \frac{4\times f\times L\times V^2}{d\times 2g}$

$$\therefore \frac{P_A}{\rho g}+0=\frac{19.62\times 10^4}{1000\times 9.81}+30+\frac{4\times .008\times 400\times .7074^2}{0.3\times 2\times 9.81}$$

= 20+30+1.088 = 51.088

$$P_A = 51.088\times 1000\times 9.81\ N/m^2$$

$$P_A = \frac{51.088\times 1000\times 9.81}{10^4} = 50.12\ N/cm^2$$

Hydraulic gradient line (H.G.L):

It is defined as the line which gives the sum of pressure head (p/w) and datum head (z) of a flowing fluid in pipe with respect to some reference or it is line which is obtained by joining the top of all vertical ordinates, showing the pressure head(p/w) of a flowing fluid in a pipe from the centre of the pipe. The line so obtain is called the H.G.L.

Total energy loss (TEL or EGL)

It is known that the total head (which is also total energy per unit weight) with respect to any arbitrary datum, is the sum of the elevation (potential) head, pressure head and velocity head.

Total head = $P/w+Z+v^2/2g$

When the fluid flows along the pipe there is loss of head (energy) and the total energy decreases in the direction of flow. If the total energy at various parts along the axis of the pipe is plotted and joined by a line, the line so determined is called the Energy gradient (E.G.L) is also called total energy line (TEL).

Points are worth noting

- Energy gradient like always drops in direction of flow because of loss of head.
- HGL may rise or fall depending upon the pressure change.

- HGL is always below the Energy Gradient

Problems on H.G.L and T.E.L

1) Determine the rate of flow of water through a pipe diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the Height of water in the tank is 4 m above the centre of the pipe. Consider all minor losses and take f=0.009 in the formula,

$$\frac{4.f.L.V^2}{d\times 2g}.$$

Draw the hydraulic gradient line (H.G.L) and total energy line (T.E.L)

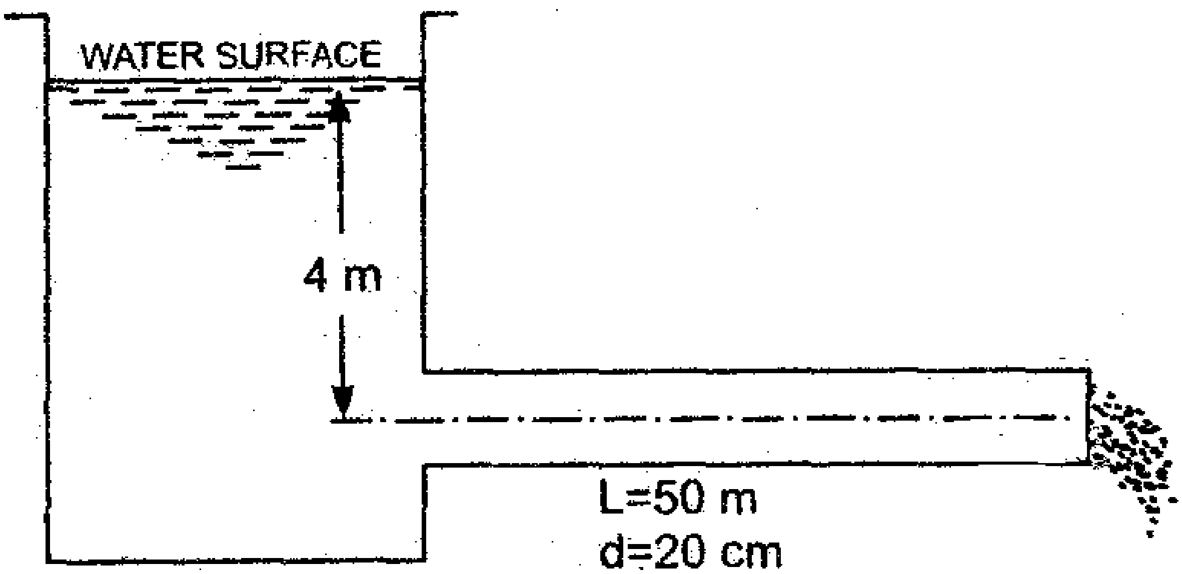


Fig 1

Solution:

Dia of pipe d = 20cm = 0.20m

Length of pipe L= 50 m

Height of water H=4 m

Co-efficient of friction, f=0.009

Let the velocity of water in the pipe =V m/s

Applying Bernoulli's equation at the top of water

Surface in the tank and at the outlet of pipe, we have

[Taking point 1 on the top point 2 at the outlet of pipe]

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+all\ losses$$

Considering datum line passing through the centre of pipe

$$0+0+4.0=0+\frac{V_2^2}{2g}+h_i+h_f$$

$$4.0=\frac{V_2^2}{2g}+h_i+h_f$$

But the velocity in pipe =V,

Therefore V= V_2

Therefore

$$4.0=\frac{V^2}{2g}+h_i+h_f$$

From the equation for loss of head at the entrance of pipe

We have,

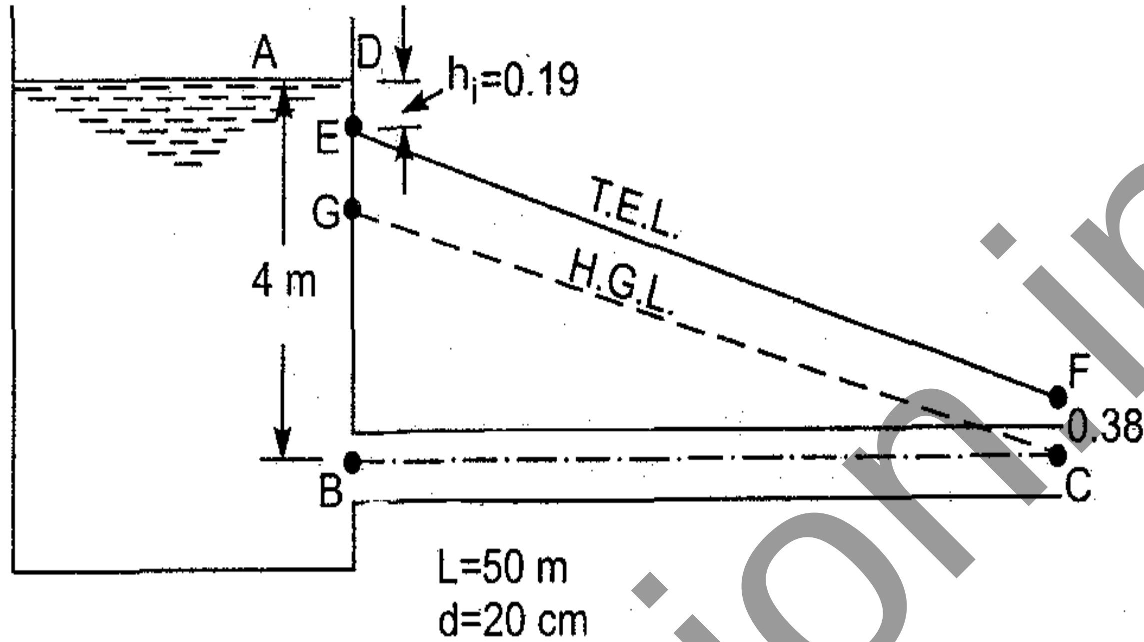
$$h_i=0.5\frac{V^2}{2g}$$

and

$$h_f=\frac{4.f.L.V^2}{d\times 2g}$$

Substituting these values, we have

$$4.0=\frac{V^2}{2g}\left[1.0+0.5+\frac{4\times0.009\times50}{0.2}\right]=\frac{V^2}{2g}[1.0+0.5+9.0]$$



Velocity, V through pipe is calculated and its value is V=2.734 m/s.

$$= 0.5\frac{V^2}{2g} = \frac{0.5\times 2.734^2}{2\times 9.81} = 0.19\ m$$

And

h_f = head loss due to friction

$$\frac{4\times f\times L\times V^2}{d\times 2g} = \frac{4\times 0.009\times 50\times (2.734)^2}{0.2\times 2\times 9.81} = 3.428\ m$$

(a) Total energy line (T. E. L.). consider three points, A, B, and C on the free surface of water in the tank, at the inlet of the pipe and at the outlet of the pipe respectively as shown in the fig. us find total energy at these points, taking the centre of pipe as the reference line.

1) Total energy at A

A $= \frac{p}{\rho g} + \frac{V^2}{2g} + z = 0 + 0 + 4.0 = 4\ m$

2) Total energy at B = Total energy at A- h_i

3) Total energy at C,

$$\frac{p_c}{\rho g}+\frac{V_c^2}{2g}+z_c=0+\frac{V^2}{2g}+0=\frac{2.734^2}{2\times 9.81}=0.38\ m.$$

Hence total energy line will be coinciding with free surface of water in the tank. At the inlet of the pipe, it will decrease by

h_i (=.19 m) from free surface and at outlet of pipe total energy is 0.38 m. Hence in the fig.

- Point D represents total energy at A
- Point E, where DE = h_i represents total energy at inlet of the pipe
- Point F, where CF = 0.38 represents total energy at outlet of the pipe

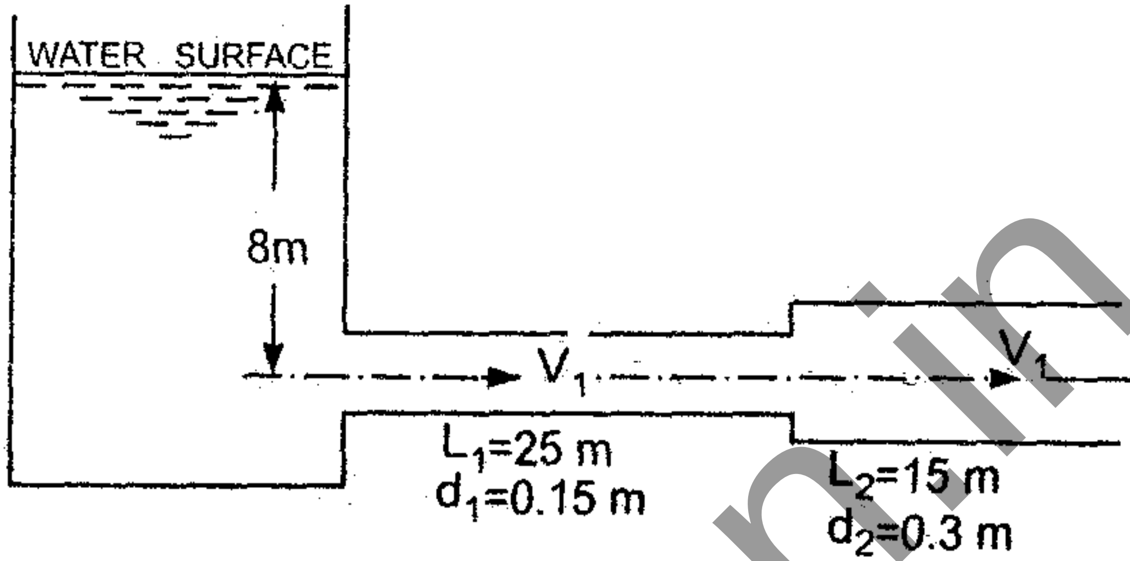
Join D to E and E to F .Then

DFE represents the total energy line.

(b) Hydraulic gradient line (H.G.L.). H.G.L. gives the sum ($\frac{p}{w}+z$) with reference to the datum- line.

Hence hydraulic gradient line is obtained by subtracting from total energy line. $V^2/2g$
At the outlet of the pipe, total energy is $V^2/2g$ By subtracting $V^2/2g$
From total energy at this point, we shall get point C. which lies on the center line of pipe.
From C, draw a line CG parallel to EF.
Then CG represents the hydraulic gradient line.

2)A horizontal pipeline 40m long is connected to a water tank at one end and discharge freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150mm diameter and its diameter is suddenly enlarged to 300mm. The height of water level in the tank is 8m above the centre of the pipe. Considering all losses of head which occurs determine the rate of flow, take f=0.01 for both section of the pipe. Draw the hydraulic gradient and total energy line.



Solution:

Given:

Total length of pipe, L = 40m

Length of 1st pipe L_1= 25m

Dia. of the 1st pipe, d1 = 150mm = 0.15m

Length of 2nd pipe, $L_2 = 40 – 25 = 15$m

Dia. of 2nd pipe d_2 = 300mm = 0.3 m

Height of water, H = 8m,

Co-efficient of friction, f = 0.01

Applying Bernoulli's equation to the free surface of water on the tank and outlet of pipe as shown in Fig. 4 and

Taking reference line passing through the centre of pipe.

[Taking point 1 on the top point 2 at the outlet of pipe]

$P_1/w+v_1^2/2g+z_1= p_2/w+v_2^2/2g+0+\text{all losses}$

$0 + 0 \quad + 8 = \quad 0+ V_2^2 /2g +0+h_i+h_{f1}+h_e+h_{f2}$------ (1)

Where h_i=loss of head at entrance=$0.5v_1^2/2g$

h_{f1}=head loss due to friction in pipe 1= $4f\ L_1v_1^2/2gd_1$

h_e=loss of head due to sudden enlargement

$= (v_1-v_2)^2/2g$

h_{f2}= head loss due to friction in pipe 2= $4f\ L_2v_2^2/2gd_2$

From continuity equation, we have

$A_1V_1= A_2V_2$

$V_1= A_2V_2/V_1= (D_2/D_1)^2*V_2 = (0.3/0.15)^2*V_2= 4V_2$

Substituting the value of V_1 in different head losses, we have

$h_i = 0.5v_1^2/2g=0.5(4v_2^2)/2g$

$= 8v_2^2/2g$

$h_{f1}=4f\ L_1v_1^2/2gd_1=(4*0.01*25*(4v_2)^2)/(0.15*2g)$

$=106.6v_2^2/2g$

$h_e= (v_1-v_2)^2/2g=(4v_2-v_2)^2/2g$

$= 9v_2^2/2g$

$h_{f2}= 4fL_2v_2^2/2gd_2=4*0.01*15*v_2^2/(0.3*2g)$

$= 2v_2^2/2g$

Substituting the value of the losses in equation (1), we get

$8=v_2^2/2g+ 8v_2^2/2g+106.6v_2^2/2g+9v_2^2/2g+2v_2^2/2g$

$=v_2^2/2g\ (1+8+106.6+9+2)$

$=126.6v_2^2/2g$

$V_2=\sqrt{(8*2g)/126.6}$

$=\sqrt{(8*2*9.81)/126.6}$

$V_2 =1.11$ m/s

Hence rate of flow 'Q'$=A_2v_2= (\pi*(0.3)^2/4)*1.11$

$Q=0.078m^3/s$

Q= 78.67 liters/s

$V_1 = 4V_2 = 4 \times 1.113 = 4.452$ m/s

The various head losses are

$$h_i = 0.5 \times \frac{V_1^2}{2g} = \frac{0.5\times4.452^2}{2\times9.81} = 0.50\ m$$

$$h_{f_1} = \frac{4f\times L_1\times V_1^2}{d_1\times 2g} = \frac{4\times.01\times25\times4.452^2}{.15\times2\times9.81} = 6.73\ m$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(4.452 - 1.11)^2}{2\times9.81}$$

$$= 0.568\ m$$

$$h_{f_2} = \frac{4\times f\times L_2\times V_2^2}{d_2\times 2g}$$

$$= \frac{4\times.01\times15\times1.113^2}{.3\times2\times9.81} = 0.126\ m$$

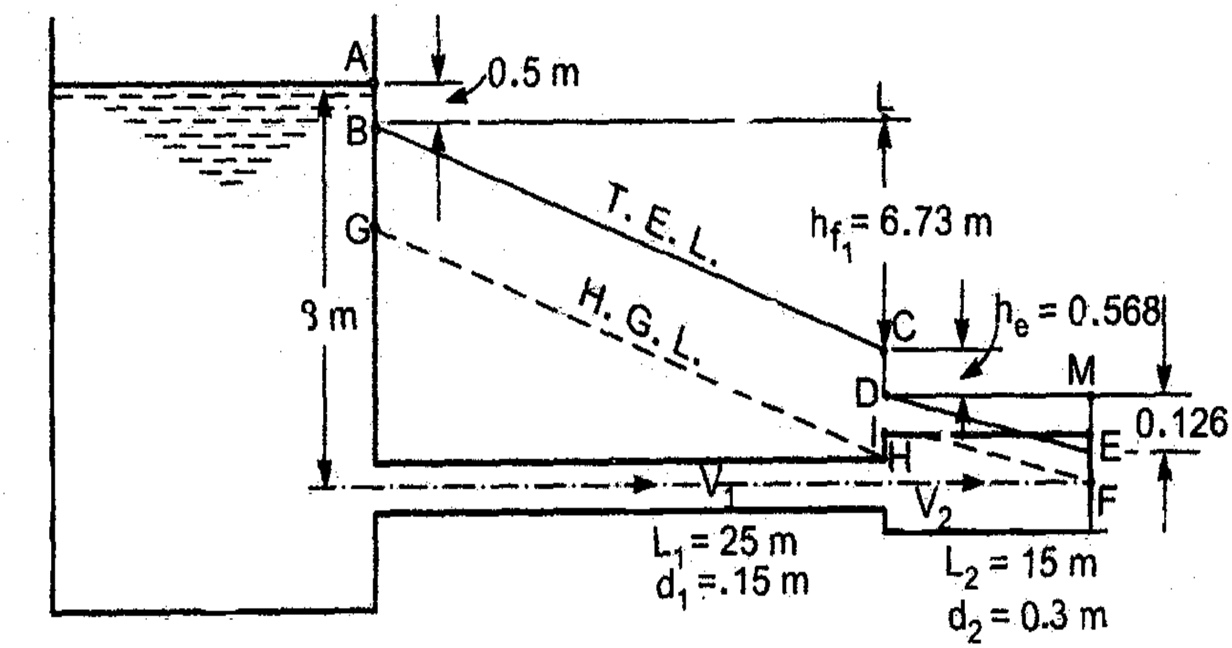
$$ho = \frac{V_2^2}{2g} = \frac{1.113^2}{2\times9.81} = 0.063\ m$$

$$\frac{V_1^2}{2g} = \frac{4.452^2}{2\times9.81} = 1.0\ m.$$

Total Energy Line

- Point A lies on free surface of water.
- Take AB = h_i = 0.5 m

- From B, draw a horizontal line. Take BL equal to the
- Length of Pipe i.e., L_1. From L draw a vertical line downward.
- Cut the line LC = h_{f1} = 6.73m
- Join the point B to C. take a line CD vertically downward equal to h_e = 0.568 m
- From D, draw DM horizontal and from point F which is lying on the centre of the pipe, draw a vertical line in the upward direction, meeting at M. From M, take a distance ME = h_{f2} = 0.126. Join DE. Then line ABCDE represents the total energy line (TEL).
  Hydraulic gradient line (H.G.L.):
- From B, take BG = $V_1^2 / 2g$ = 1.0m
- Draw the line GH parallel to the line BC
- From F, draw a line Fl parallel to the ED.
- Join the point H and I.
- Then the line GHIF represents the hydraulic gradient line (HGL).

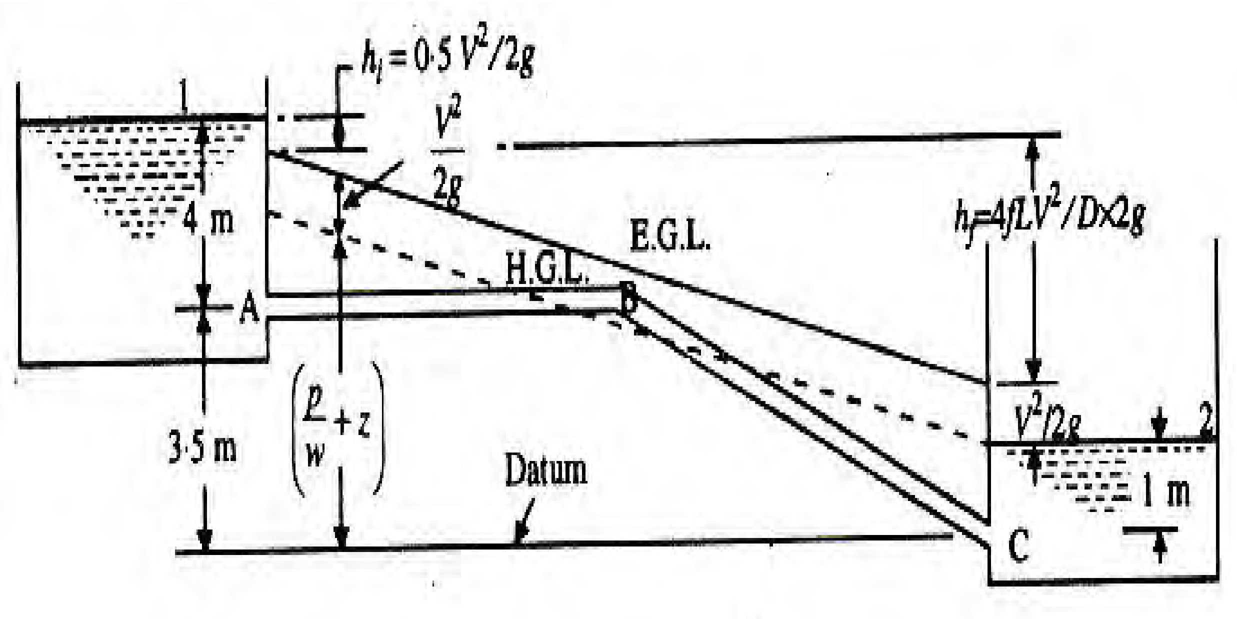


3) A pipe ABC connecting two reservoirs is 80 mm in diameter. From A to B the pipe is horizontal as shown in fig. 12.7 and from B to C it falls by 3.5 meters. The lengths AB and BC are 25m and 15 m respectively. If the water surface in the reservoirs at A is 4 m above the centerline of the pipe and at C 1 m above the centre line of the pipe, *calculate:*

(1)The rate of flow and

(2)The pressure head in the pipe at B

Neglect the loss at the bend but consider all other losses. Also draw the energy and hydraulic gradient lines. Take Darcy friction factor F = 0.024 and $E_{ntrance}$ = 0.5



Solution:

Diameter of the pipe, D = 80 mm = 0.8 m

Area, A = $\pi/4 * 0.08^2 = 0.00026\ m^2$

Friction factor (=4f), = 0.024

$K_{entrance}$ = 0.5

(1) The rate of flow Q:

Applying Bernoulli's equation between the water surfaces 1 and 2 in the two reservoirs (considering horizontal plane through C as datum), we get

$p_1/w + v_1^2/(2g) + z_1 = p_2/w + v_2^2/(2g) + z_2$ + loss at entrance + h_f (loss due to friction) + $v^2/(2g)$

$0+0+(4+3.5-1)=0+0+0+0.5v^2/(2g)+\{4fLv^2/(D*2g)\}+v^2/(2g)$

(Where v = velocity of flow in the pipe)

or $6.5 = 0.5v^2/(2g)+ \{0.024*(25+15)*v^2\}/(0.08*2g) + v^2/(2g)$

$= v^2/(2g)(0.5+12+1) + 13.5\ v^2/(2g)$

$v^2 = 6.5*2*9.81/13.5= 9.446$

$v = 3.073$ m/s

Therefore, flow rate= A* V = 0.005026*3.073 = 0.01544 m^3/s

(2)Pressure head in the pipe at B,

Applying Bernoulli's equation at 1 and B, we get

$p_1/w+v_1^2/(2g) +z_1 = p_B/w+v_B^2/(2g)+z_B+0.5v^2/(2g)+h_f$

$0+0+4 = p_B/w+v^2/(2g)+z+0.5v^2/(2g)+4fLv^2/(D*2g)$

$4 = p_B/w+(v^2/2g)+(0.5v^2/2g)+\{0.024*25*v^2/(0.08*2g)\}$

$(v_B = v = 3.073m/s)$

$4 = p_B/w+(v^2/2g)+(0.5v^2/2g)+(7.5v^2/2g)$

$= p_B/w+(9v^2/2g)$

$p_B/w = 4 - (9v^2/2g)$

$= 4 - (9*3.073^2/2*9.81)$

$p_B/w = - 0.33$ m of water (below atmosphere)

Energy gradient and hydraulic gradient lines (E.G.L. and H.G.L):

For plotting E.G.L and H.G.L., we require the velocity head, (same throughout)

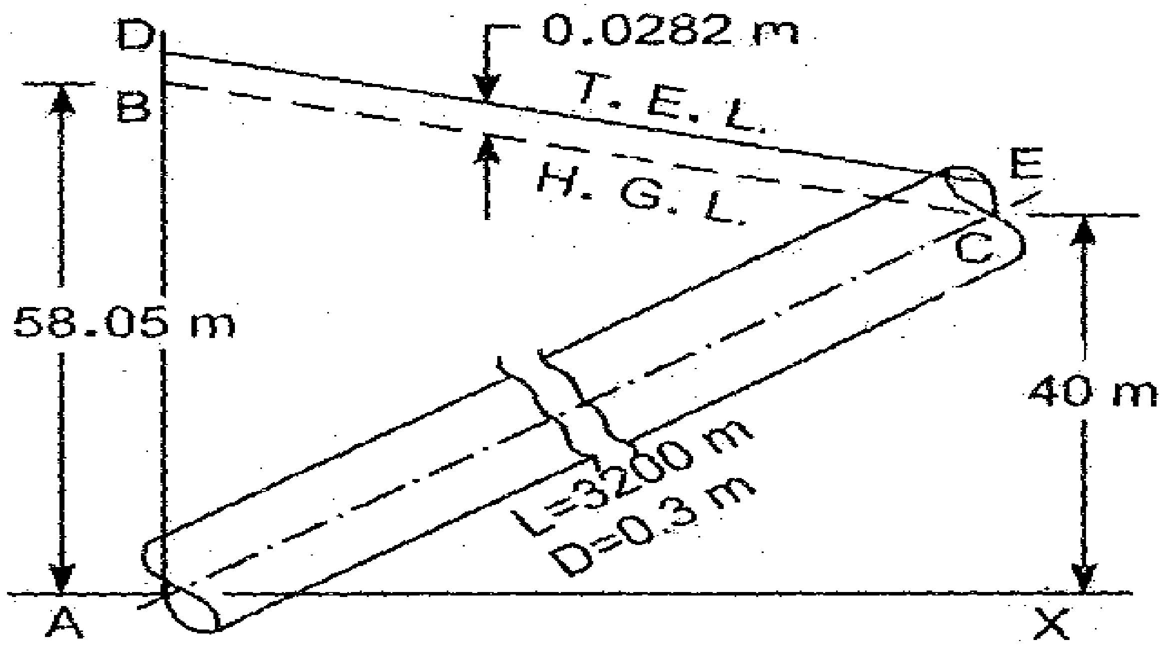
$v^2/2g = 3.073^2/(2*9.81) = 0.481$ m

Total energy at B with respect to horizontal datum through C

$= 3.5+p_B/w+v^2/2g = 3.5- 0.33+ 3.073^2/(2*9.81)$

$= 3.65$ m

4) A pipe line, 300 mm in diameter and 3200 m long is used to pump up 50kg per second of oil whose density is 950 kg/m^3 and whose kinematic viscosity is 2.1 strokes. The centre of the pipe line at the upper end is 40m above than that at the lower end. The discharge at the upper end is atmospheric. Find the pressure at the lower end and draw the hydraulic gradient and the total energy line.



Solution

Given:

Dia. of pipe. $D = 300mm = 0.3m$

Length of pipe, $L = 3200m$

Mass, $M = 50kg/s = \rho . Q$

Density, $\rho = 950kg/m^3$

Discharge, $Q = 50/\rho = 50/950 = 0.0523\ m^3/s$

Kinematic viscosity, $\nu = 2.1$ stokes $= 2.1\ cm^2/s$

$= 2.1*10^{-4}\ m^2/s$

Height of upper end = 40m

Pressure at upper end = atmospheric = 0

Reynolds number, $R_e = V*d/\nu$

Where V=Discharge/area= $0.0526/3.14*0.3^2=0.744$ m/s

$R_e = 0.744*0.30/2.1*10^{-4}=1062.8$

Coefficient of friction, f =16/R_e=16/1062.8 = 0.015

Head lost due to friction, $h_f = 4*f*L*V^2/d*2g$

$h_f = 4*0.015*3200*0.744^2/0.3*2*9.81 = 18.05$ m of oil

Applying the Bernoulli's equation at the lower and upper end of the pipe and taking datum line passing through the lower end, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

$p_2 = 0$, $h_f = 18.05$m

But $Z_1= 0$, $Z_2= 40$m, $V_1=V_2$ as diameter is same

Substituting these values, we have

$\frac{P_1}{\rho g}$=40+18.05=58.05 m of oil

$p_1 = 58.05*\rho g = 58.05*950*9.81$ [ρ for oil =950]

= 540997 N/m^2 =540997/10^{-4} N/cm^2

= 54.099 N/cm^2. Ans

H.G.L and T.E.L

$V_2/2g = 0.744^2 / 2 \times 9.81 = 0.0282$ m

H.G.L and T.E.L

$V_2/2g = 0.744^2 / 2 \times 9.81 = 0.0282$ m

$P_1/ \rho g$ = 58.05 m of oil

$P_2/ \rho g = 0$

Draw a horizontal line AX as shown in fig.8 . From A, draw a centre line of the pipe in such a way that point C is a distance of 40m above the horizontal line. Draw a vertical line AB through A such that AB = 58.05m. Join B with C. Then BC is the hydraulic gradient line .

Draw a line DE parallel to BC at a height of 0.0282 m above the hydraulic gradient line. Then DE is the total energy line.

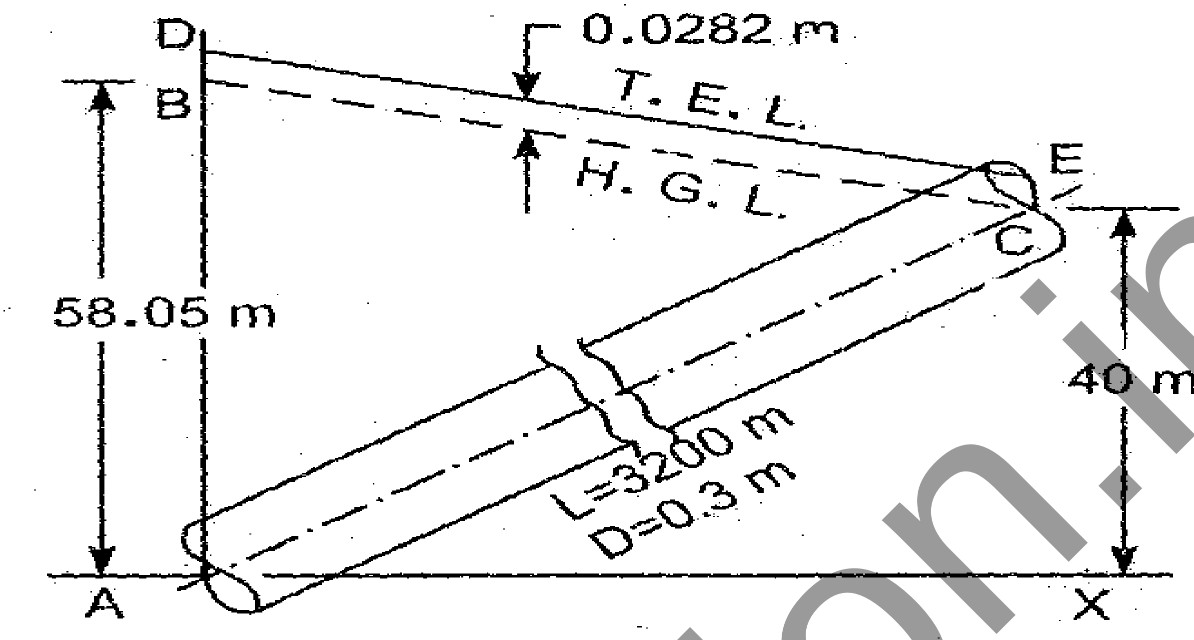


Fig 8

5)Two reservoirs A and C having a difference of level of 15.5 m are connected by a pipe line ABC the elevation of point B being 4.0 m below the level of water in reservoir A. The length AB of the pipe line is 250 m, the pipe being made of mild steel having a friction co-efficient f_1 , while the length BC is 450 m the pipe having made of cast iron having a friction co-efficient f_2. Both the lengths AB and BC have a diameter of 200mm. A partially Closed valve is located BC at a distance of 150m from reservoir C. If the flow through the pipeline is 3m3/min, the pressure head at B is 0.5m and the head loss at the valve is 5.0m.

Find the friction co- efficient f_1 and f_2; Draw the hydraulic grade line of the pipe line and indicate on the diagram head loss values at significant points. Take into account head loss at entrance and exit points of the pipeline.

Solution:

Difference of water level between two reservoirs = 15.5m

Diameter of the pipeline ABC, D=200mm=0.2m

Length AB, L_{AB}=250m

Length BC, L_{BC}=450m

Discharge through the pipe,Q=3m^3/min=0.05m^3/sec

Pressure head at B,

h_B= (p_B/w) = 0.5m

Head loss the valve =5.0m

Friction co-efficients f_1 and f_2:

Velocity in the pipes ABC,

V=Q/A=1.59 m/sec

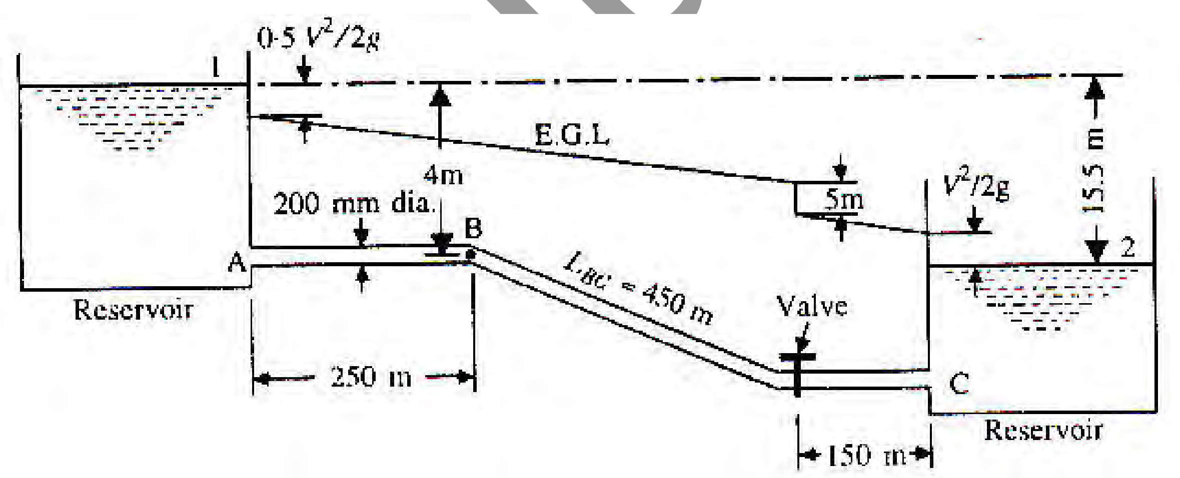
Applying Bernoulli's equation at 1 and at B we get

$P_1/w+V_1^2/2g+z_1 = p_B/w+V_B^2/2g+z_2(0.5\ V_B^2/2g+(h_f)_{AB}$

f_1=0.0051

Applying Bernoulli's equation between 1 and 2 and considering all losses in the pipeline ABC in the exit loss, we have

$p_1/w+V_1^2/2g+z_1=p_2/w+V_2^2/2g+z_2+0.5V^2/2g+(4f_1L_{AB}V^2/D*2)+ (4f_2L_{BC}V^2/D*2g)+5.0+V^2/2g.$



15.5=0.0644+3.28+1159.6f_2+5.0+0.1288

f_2=0.0066

H.G.L (hydraulic gradient line):

Above figure shows the E.G.L(energy gradient line), H.G.L will be $V^2/2g$ below the E.G.L

6) The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 300m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively, is 12 m. Determine the rate of flow of water if the co-efficient of friction are 0.005, 0.0052 and 0.0048 respectively, considering:

(i) Minor losses

(ii) Neglecting minor losses

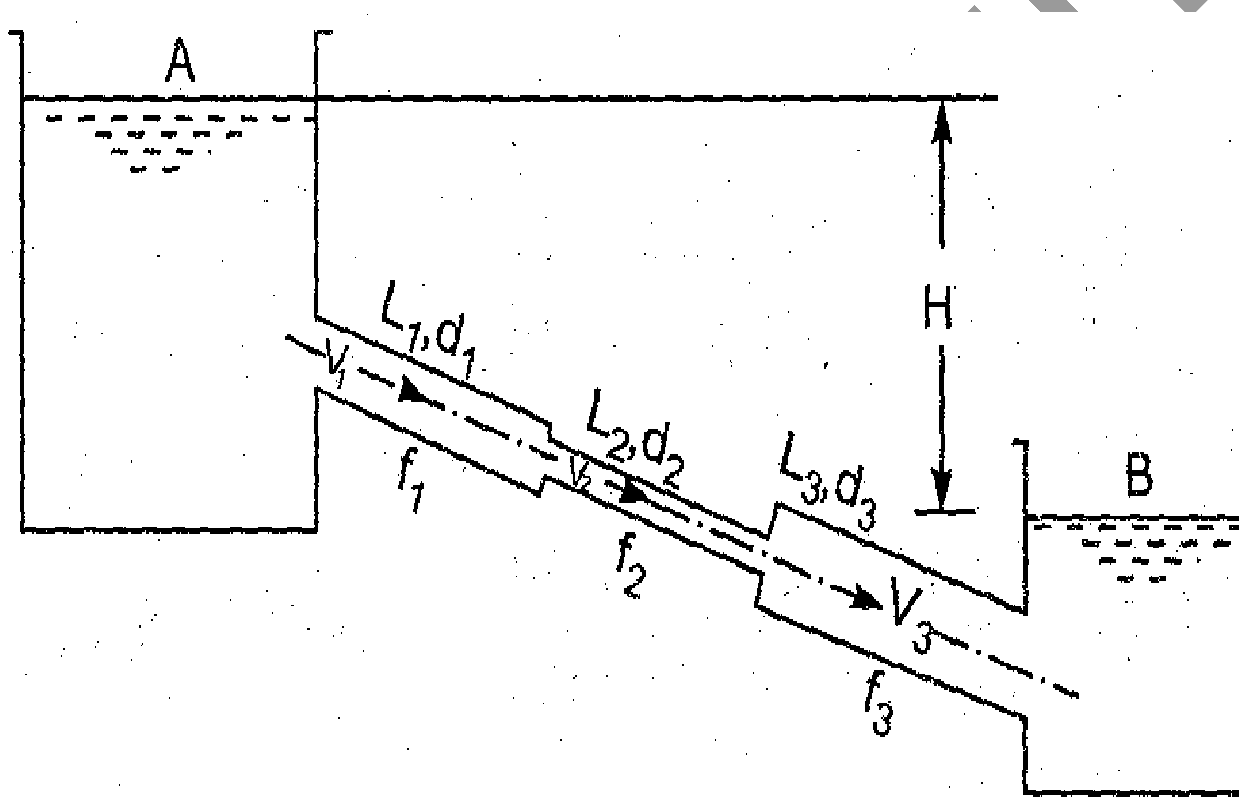


Fig 9

Solution

Given :

Difference of water level, $H = 12$ m

Length of pipe 1, $L_1 = 300$ m and

dia. of pipe 1, $d_1 = 300$ mm $= 0.3$ m

Length of pipe 2, $L_2 = 170$ m and

dia. of pipe 2 $d_2 = 200$ mm $= 0.2$ m

Length of pipe 3, $L_3 = 210$ m and

dia. of pipe 3, $d_3 = 400$ mm $= 0.4$ m

Also,

$f_1 = 0.005$, $f_2 = 0.0052$ and $f_3 = 0.0048$

(i) Considering Minor Losses. Let V_1, V_2 and V_3 are the

Velocities in the first second and third pipe respectively

From continuity equation, we have

$A_1V_1 = A_2V_2 = A_3V_3$

$V_2 = A_1V_1/A_2 = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2} V_1 = d_1^2/d_2^2 (V_1)$

$= (0.3/0.2)^2 * V_1 = 2.25V_1$

$V_3 = A_1V_1/A_3 = d_1^2/d_3^2(v_1)$

$= (90.3/0.4)^2\ V_1 = 0.5625\ V_1$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes

$$H = \left[\frac{0.5V_1^2}{2g} + \frac{4\ f_1\ \ L_1\ V_1^2}{d_1\ 2g} + \frac{0.5\ V_2^2}{2g} + \frac{4\ f_2\ \ L_2\ V_2^2}{d_2 * 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4 * f_3 * L_3 * V_3^2}{d_3 * 2g} + \frac{V_3^2}{2g} \right]$$

Substituting V_2 and V_3,

$$12.0 = \frac{0.5V_1^2}{2g} + \frac{4 * 0.005 * 300 * V_1^2}{0.3 * 2g} + \frac{0.5*(2.25\ V_1^2)^2}{2g} + 4 * 0.0052 * 170$$

or

$$12.0 = \frac{V_1^2}{2g}[0.5 + 20.0 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316]$$

$$12 = \frac{V_1^2}{2g}[118.887]$$

$$V_1 = \sqrt{\frac{12*2*9.81}{118.887}} = 1.407 \text{ m/s}$$

Therefore,

Rate of flow, $Q = \text{Area} * \text{velocity} = A_1 * V_1$

$= \pi/4 (d_1)^2 * V_1 = \pi/4 (0.3)^2 * 1.407 = 0.09945 \text{ m}^3/\text{s}$

Q = 99.45 liters/s.

(ii) Neglecting Minor Losses. Using equation we have

$h = 4 \times f_1 \times L_1 \times v_1^2 / d_1 \times 2 \times 9.81 + 4 \times f_2 \times L_2 \times v_2^2 / d_2 \times 2 \times 9.81$

$+ 4 \times f_3 \times L_2 \times v_2^2 / d_3 \times 2 \times 9.81$

$$12.0 = \frac{V_1^2}{2g}\left(\frac{4*0.005*300}{0.3} + \frac{4*0.0052*170*2.25^2}{0.2} + \frac{4*0.0048*210*0.5625^2}{0.4}\right)$$

$$= \frac{V_1^2}{2g}[20.0+89.505+3.189] = \frac{V_1^2}{2g} * 112.694$$

$$V_1 = \sqrt{\frac{2*9.81*12.0}{112.694}} = 1.445 \text{ m/s}$$

Discharge, $Q = V_1 * A_1$

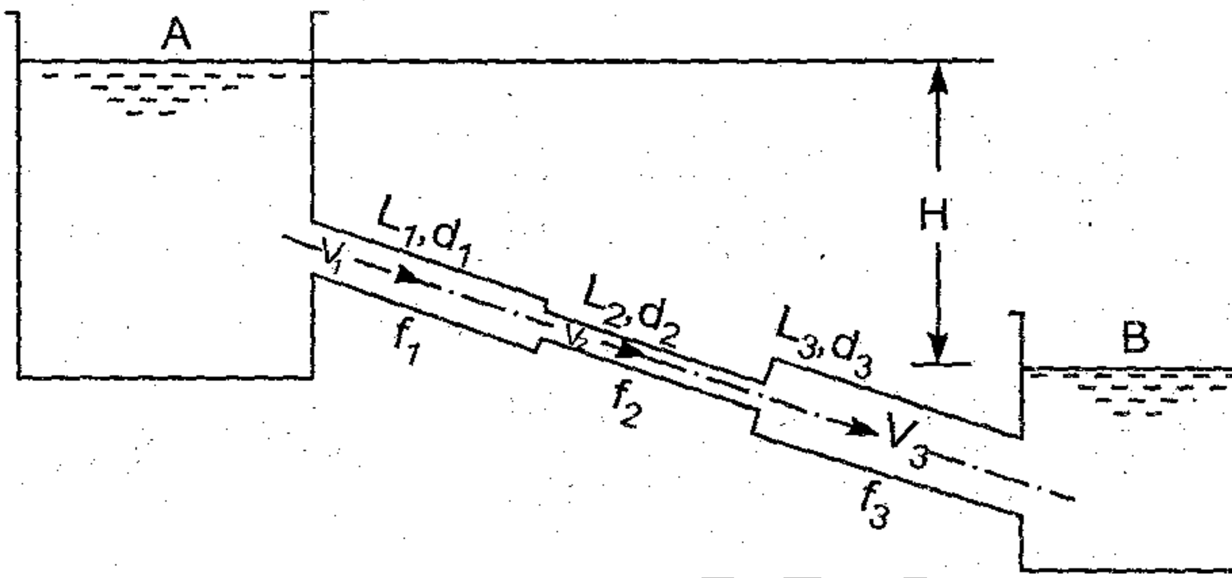
$= 1.445 * \pi / 4 (0.3^2)$

$= 0.1021 \text{ m}^3/\text{s}$

Q = 102.1 litres /s

7) Three pipes of 400 mm, 200 mm and 300 mm diameters have lengths of 400m, 200m and 300 m respectively. They are connected in series to make a compound pipe. The ends of this compound pipe are connected with two tanks whose difference of water levels is 16 m.

If co-efficient of friction for these pipes is same and equal to 0.005, determine the discharge through the compound pipe neglecting first the minor losses and then including them.



Solution:

Given:

Difference of water levels, $H = 16$ m

Length and dia. of pipe 1, $L_1 = 400$ m and

$d_1 = 400$ mm $= 0.4$ m

Length and dia. of pipe 2, $L_1 = 200$ m and

$d_2 = 200$ mm $= 0.2$ m

Length and dia. of pipe 3, $L_1 = 300$ m and

$d_3 = 300$ mm $= 0.3$ m

Also $f_1 = f_2 = f_3 = 0.005$

Discharge through the compound pipe

(i) first neglecting minor losses

Let V_1, V_2 and V_3 are the velocities in the first, second and third pipe respectively.

From continuity equation, we have

$A_1V_1 = A_2V_2 = A_3V_3$

$$V_2 = A_1V_1/A_2 = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2}(V_1) = d_1^2 / d_2^2 (V_1)$$

$= (0.4/0.2)^2 V_1 = 4V_1$

$$V_3 = A_1V_1/A_3 = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_3^2}(V_1) = d_1^2 / d_3^2 (V_1)$$

$= (0.4/0.2)^2 V_1 = 1.77 V_1$

Using equation, we have

$h = 4 \times f_1 \times L_1 \times v_1^2 / d_1 \times 2 \times 9.81 + 4 \times f_2 \times L_2 \times v_2^2 / d_2 \times 2 \times 9.81 + 4 \times f_3 \times L_2 \times v_2^2 / d_3 \times 2 \times 9.81$

$$16 = \frac{V_1^2}{2*9.81}\left(\frac{4*0.005*400}{0.4} + \frac{4*0.005*200*200*16}{0.2} + \frac{4*0.005*300*3.157}{0.3}\right)$$

$$16 = \frac{V_1^2}{2*9.81}(20 + 320 + 63.14) = \frac{V_1^2}{2*9.81} * (403.14)$$

$$\mathbf{V_1} = \sqrt{\frac{16*2*9.81}{403.14}} = \mathbf{0.882\ m/s}$$

Therefore, Discharge, $Q = A_1 * V_1 = \pi / a *(0.4)^2 * 0.882$

$= \mathbf{0.1108\ m^3/s}$

Discharge through the compound pipe

(ii) Considering minor losses

Minor losses are:

(a) At inlet, $\mathbf{h_i}$ $(0.5*V_1^2)/2*g$

(b) Between first pipe and second pipe, due to contraction,

$h_c=(0.5V_2^2)/(2*g)=(0.5*(4V_1^2))/(2*g)$

$$h_c = \frac{0.5V_2^2}{2*g} = \frac{0.5*(4V_1^2)}{2*g}$$

$$h_c = \frac{0.5*16*V_1^2}{2*g} = 8 * \frac{V_1^2}{2*g}$$

(c) Between second pipe and third pipe, due to sudden enlargement,

$$h_e = \frac{(V_2 - V_3)^2}{2*g} = \frac{(4V_1 - 1.77V_1)^2}{2*g}$$

$$\frac{V_1^2}{2*g} = 4.973\frac{V_1^2}{2*g}$$

(d) At the outet of third pipe,

$$h_o = \frac{(1.77 * V_1^2)^2}{2*g}$$

$$= 1.77^2 * \frac{V_1^2}{2*g} = 3.1323\frac{V_1^2}{2*g}$$

The major losses are =

$h = 4 \times f_1 \times L_1 \times v_1^2 / d_1 \times 2 \times 9.81 + 4 \times f_2 \times L_2 \times v_2^2 / d_2 \times 2 \times 9.81 + 4 \times f_3 \times L_2 \times v_2^2 / d_3 \times 2 \times 9.81$

$$= \frac{V_1^2}{2g}\left[\frac{4*0.005*400}{0.4} + \frac{4*0.005*200*200*16}{0.2} + \frac{4*0.005*300*3.157}{0.3}\right]$$

$$= 403.14 * \frac{V_1^2}{2*9.81}$$

Therefore, Sum of minor losses and major losses

$$= \left[\frac{0.5V_1^2}{2*g} + 8*\frac{V_1^2}{2*g} + 4.973\frac{V_1^2}{2*g} + 3.1329\frac{V_1^2}{2*g} + 403.14\frac{V_1^2}{2*g}\right]$$

$$= 419.746 \quad \frac{V_1^2}{2*g}$$

But total loss must be equal to H

$$\text{Therefore } 419.746 * \frac{V_1^2}{2*g} = 16 \therefore V1 = \sqrt{\frac{16*2*9.81}{419.746}}$$

$$= 0.864 \text{ m/s}$$

Therefore Discharge, $Q = A_1 V_1$

$= \pi / 4 (0.4)^2 * 0.864$

$= 0.1085 \text{ m}^3/\text{s}$

Problem on equivalent pipe

8) Three pipes of lengths 800 m 500m and 400 m and of diameters 500 mm, 400 mm and 300 mm respectively are connected in series. These pipes are to be replaced by a single pipe of length 170 m. Find the diameter of the single pipe.

Solution:

Given :

Length of pipe 1, L_1 = 800 m and

Dia., of pipe 1, d_1 = 500 mm = 0.5 m

Length of pipe 2, L_2 = 500 m and

Dia., of pipe 2, d_2 = 400 mm = 0.4 m

Length of pipe 3, L_3 = 400 m and

Dia., of pipe 3, d_3 = 300 mm = 0.3 m

Length of the single pipe, L = 1700 m

Let the diameter of equivalent single pipe = d

Applying equation , $\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$

$$\frac{1700}{d^5} = \frac{800}{0.5^5} + \frac{500}{0.4^5} + \frac{400}{0.3^5} = 25600 + 48828.125 + 164609$$

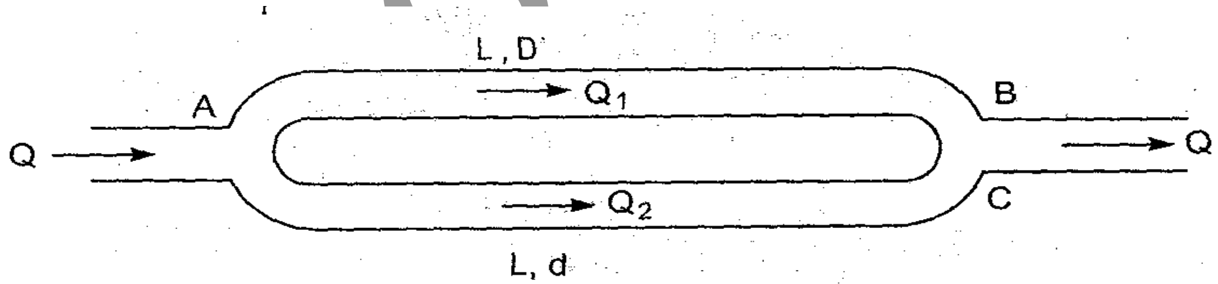
$$= 239037$$

Therefore, $d^5 = \frac{1700}{239037} = 0.007118$

Therefore $d = (0.007188)^{0.2} = 0.3718$ m

d = 371.8 mm

9).A main pipe divided into two parallel pipes which again forms one pipe as shown in fig. 3. The length and diameter for the first parallel pipe are 2000m and 1.0 m respectively, while the length and diameter of 2nd parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3.0 m^3/s . The co efficient of friction for each parallel pipe is same and equal to 0.005.



Solution:

Given:

Length of pipe 1,

Dia of pipe 1,

Length of pipe 2,

Dia of pipe 2 $d_2 = 0.8\ m$

Total flow, $Q = 3.0\ m^3/s$

$f_1 = f_2 = f = .005$

Let $Q_1 = discharge\ in\ pipe\ 1$

$Q_2 = discharge\ in\ pipe\ 2$

The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes =

We have, $Q = Q_1 + Q_2$ ......(i)

h = 4 x f_1 xL_1 x v_1^2 / d_1 x 2 x 9.81 + 4 x f_2 x L_2 x v_2^2 / d_2 x 2 x 9.81

or $\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8}$ or $V_1^2 = \frac{V_2^2}{0.8}$

Therefore $V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{.894}$

Now $Q_1 = \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4}(1)^2 \times \frac{V_2}{.894}$ [As $V_1 = \frac{V_2}{.894}$)

and $Q_1 = \frac{\pi}{4} d_2^2 \times V_2 = \frac{\pi}{4}(0.8)^2 \times V_2 = \frac{\pi}{4} \times .64 \times V_2$

Substituting the value of Q_1 and Q_2 in equation (i),we get

$$\frac{\pi}{4} \times \frac{V_2}{.894} + \frac{\pi}{4} \times .64V_2 = 3.0\ or\ 0.8785V_2 + .5026V_2 = 3.0$$

or $V_2[.8785 + .5026] = 3.0\ OR\ V = \frac{3.0}{1.3811} = 2.17\frac{m}{s}.$

Substituting this value in equation (ii)

$$V_1 = \frac{V_2}{.894} = \frac{2.17}{.894} = 2.427\ \frac{m}{s}.$$

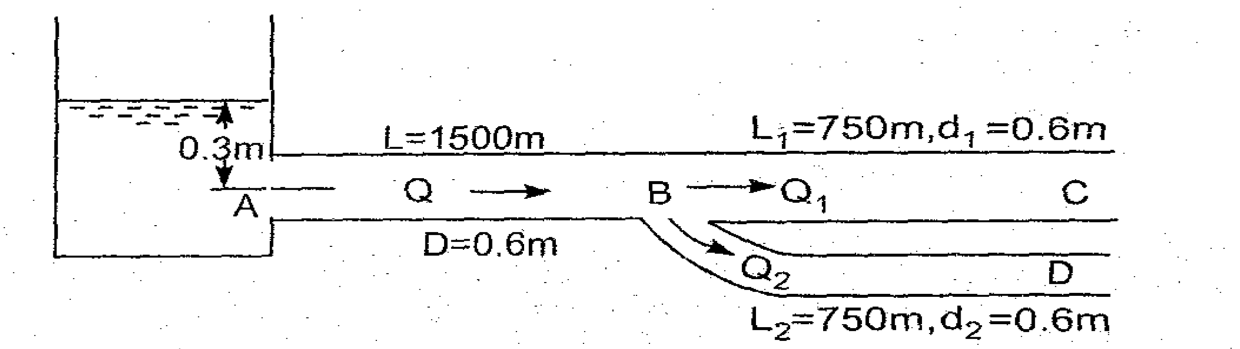
Hence $Q_1 = \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} \times 1^2 \times 2.427$

$= \mathbf{1.906}\ \frac{m^3}{s}.\ \boldsymbol{Ans.}$

$Q_2 = Q - Q_1 = 3.0 - 1.906 =$

$\mathbf{1.094}\frac{m^3}{s}.\ \boldsymbol{Ans.}$

10). A pipe line of 0.6 m diameter is 1.5 km long. To increase the discharge, another line of the same diameter is introduced parallel to the first in the second half of the length. Neglecting minor losses. Find the increase in discharge if 4f = 0.04. The head at inlet is 300mm.



1^{st} case: Discharge for a single pipe of length 1500m and dia. = 0.6m

This loss of head due to friction in single pipe is $h_f = 4flv^{*2}/2gd$

Where v* = velocity of flow for single pipe

or $0.3 = 4 \times .01 \times 1500 \times v^{*2} / 0.6 \times 2g$

$$v^* = \sqrt{\frac{0.3*0.6*2*9.81}{4*0.01*1500}} = 0.2426\, m/s$$

Discharge Q* = v* x Area = 0.2426 x π x $0.6^2/4$ = 0.0685 m^3/s

2^{nd} case: When an addition pipe of length 750m and diameter 0.6 m is connected in parallel with the last half length of the pipe

Let Q_1 = discharge in 1^{st} parallel pipe

Let Q_2 = discharge in 2^{nd} parallel pipe

Therefore $Q = Q_1 + Q_2$

Where Q = discharge in main pipe when pipes are parallel

But as the length and diameters of each parallel is same

$Q_1 = Q_2 = Q/2$

Consider the flow through pipe ABC or ABD

Head loss through ABC = Head lost through AB + head lost through BC ……….(ii)

But head lost due to friction through ABC = 0.3 m given

Head lost due to friction through AB = $4 \times f \times 750 \times v^2 / 0.6 \times 2 \times 9.81$

Where v = velocity of flow through AB

$= Q/\text{Area} = Q/ \pi \times 0.6^2/4 = 40/ \pi \times 0.36$

Head lost due to friction through AB

$= 4 \times 0.01 \times 750/0.6 \times 2 \times 9.81 \times (4Q/ \pi \times 36)^2$

$= 31.87\ Q^2$

Head lost due to friction through BC $= 4 \times f \times L_1 \times v_1^2 \ / \ d \times 2 \times g$

$= 4 \times 0.01 \times 750/0.6 \times 2 \times 9.81\ [(Q/2 \times \pi/4 \times 0.6^2\ ]$

(as v1 = Discharge / $(\pi/4(0.6)^2 = Q/2 \times \pi/4(0.6)^2$)

$= 7.969\ Q^2$

Substituting these values in equation (ii), we get

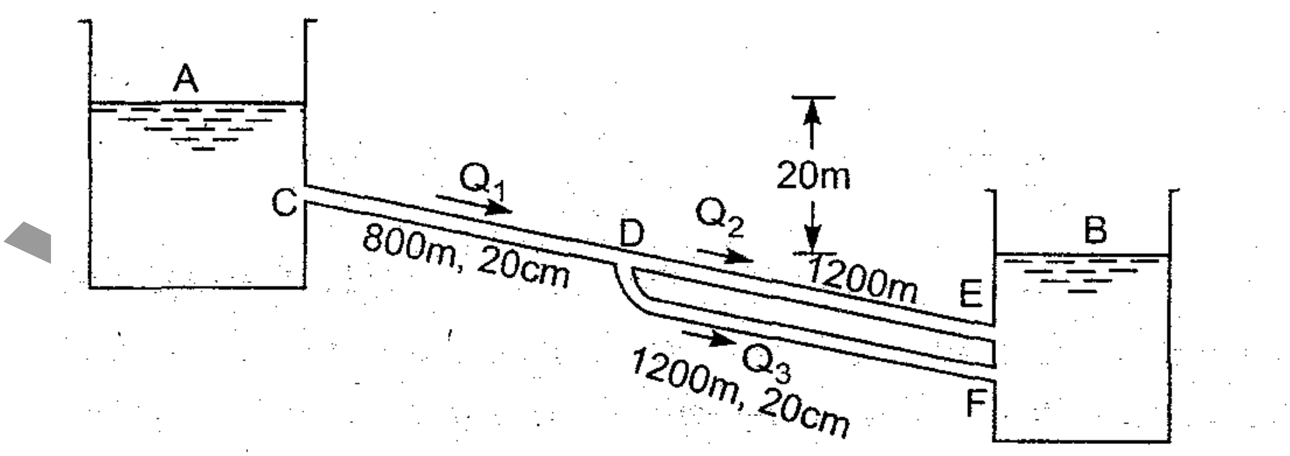
$0.3 = 31.87\ Q^2 + 7.969\ Q^2 = 39.839$

$Q = \sqrt{0.3/39.839}$

$Q = 0.0867\ m^3/s$

Increase in discharge = $Q - Q^* = 0.0867 - 0.0685$
$= 0.0182\ m^3/s$

11). A pipe of diameter 20 cm and length 2000m connects two reservoirs, having difference of water levels as 20. Determine the discharge through the pipe. If an additional pipe of diameter 20 cm and length 2000 m is attached to the last 1200m length of the existing pipe, find the increase in the discharge . Take f = 0.015 and Neglect minor losses



1st case: When a single pipe connects the reservoirs

$H = 4\ f\ L\ v^2/2gd = 4f\ LV^2/2gd\ (Q/\ \pi\ /4\ d^2\ )^2$

$[\text{ as } V = Q/\ \pi \times d^2/4\ ]$

$= 32\ f\ L\ Q^2/\ \pi^2\ g\ d^5$

$20 = 32 \times 0.015 \times 2000 \times Q^2/\pi^2 \times 9.81 \times (0.2)5$

$Q = 0.0254\ m^3/s$

2nd case:

Let Q_1 = discharge through pipe CD

Q_2 = discharge through pipe DE

Q_3 = discharge through pipe DF

Length of pipe CD, $L_1 = 800m$ and its dia, $d_1 = 0.20$ m

Length of pipe DE, $L_2 = 800m$ and its dia, $d_2 = 0.20$ m

Length of pipe DF $L_3 = 800m$ and its dia, $d_3 = 0.20$ m

Since the diameters and lengths of the pipes DE and DF are equal. Hence Q_2 will be equal to Q_3.

Also, for parallel pipes, we have

$Q_1 = Q_2 + Q_3 = Q_2 + Q_2 = 2Q_2$ [as $Q_2 = Q_3$]

Therefore ,$Q_2 = Q_1/2$

Applying Bernoulli's equation to points A and B and taking the flow through CDE, we have

$20 = 4 \times f \times L_1 \times v_1^2\ / d_1 \times 2 \times 9.81 + 4 \times f \times L_2 \times v_2^2\ / d_2 \times 2 \times 9.81$

Where $v_1 = Q_1/\ \pi \times 0.2^2/4 = 4 \times Q_1\ /\ \pi \times 0.04$

$v_2 = Q_2/\ \pi \times 0.2^2/4 = 4 \times Q_2\ /\ \pi \times 0.04 = 4 \times Q_1/2\ /\ \pi \times 0.04$

$= 2 \times Q_1/2\ /\ \pi \times 0.04$

$$= 4 \times .015 \times 800/0.2 \times 2 \times 9.81 \times (4 \times Q_1 / \pi \times 0.04)^2 + 4 \times .015 \times 1200/0.2 \times 2 \times 9.81 \times (2 \times Q_1 / \pi \times 0.04)^2$$

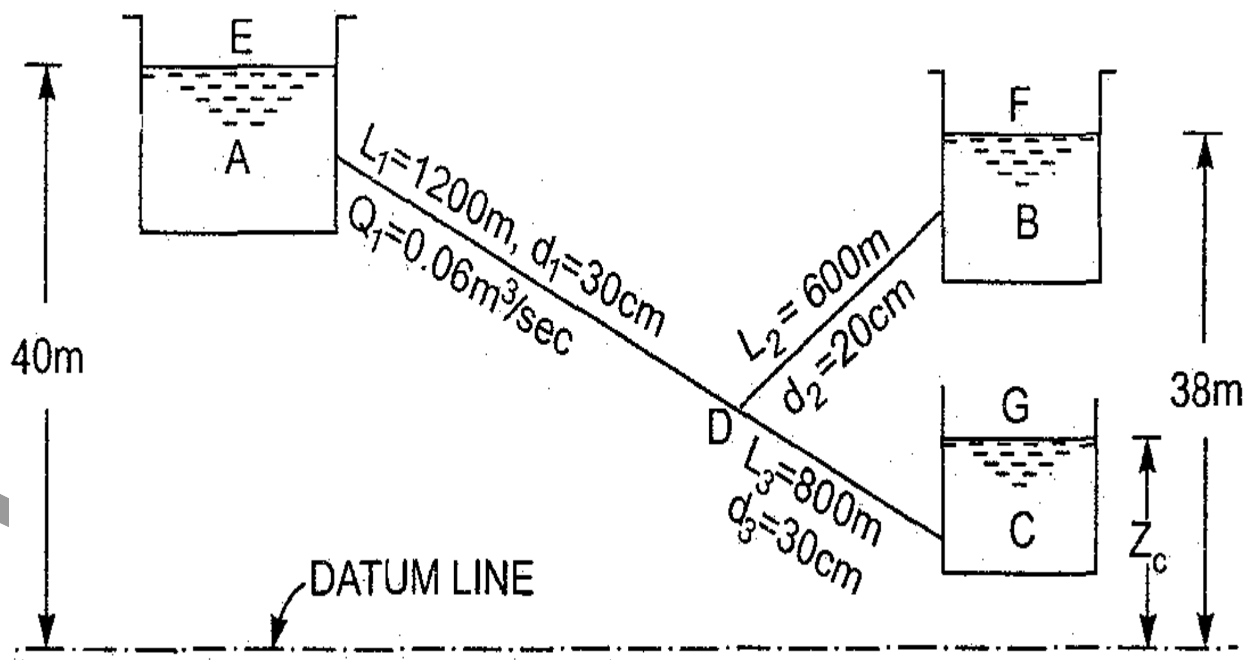
$$= 12394\ Q_1^2 + 4647\ Q_1^2 = 17041\ Q_1^2$$

$$Q_1 = \sqrt{20/17041}$$

Increase in discharge = $Q_1 - Q$ = 0.0342 – 0.0254
= 0.0088 m^3/s

Flow Through branched pipes

12) Three reservoirs A, B, and C are connected by a pipe system shown in the fig 17.find the discharge into or from the reservoirs B and C if the rate of flow from reservoirs A is 60 liters/s. **Find the height of water level in the reservoir C.** Take f=.006 for all pipes.



Solution:

Given:

Length of pipe AD, L_1=1200m

Dia. of pipe AD,

D_1 = 30 cm =0.30 m

Discharge through AD, Q_1 = 60 liters = 0.06 m^3/s

Height of water level in A from reference line, z_A = 40m

For pipe DB, length, L_2 = 600m, d_2 = 20cm=0.20m, Z_B = 38.0

For pipe DC, length, L_3 = 800m, d_3 = 30cm=0.30m

Appling Bernoulli's equations to point E and D,
Where

$$Z_A = Z_D + \frac{p_D}{\rho g} + h_{f_1}$$

where $h_{f_1} = \frac{4.f.L_1.V_1^2}{d_1 \times 2g}$

Where $V_1 = \frac{Q_1}{Area} = \frac{0.06}{\frac{\pi}{4}(0.3)^2}$ $0.848\frac{m}{sec}$.

$$h_f = \frac{4\times0.006\times1200\times.848^2}{0.3\times2\times9.81} = 3.518 \text{ m}$$

Therefore

$$Z_A = Z_D + \frac{p_D}{\rho g} + 3.518$$

$$40.0 = Z_D + \frac{p_D}{\rho g} + 3.518$$

$$\therefore \left(Z_D + \frac{p_1}{\rho g}\right) = 40.0 - 3.518 = 36.482 \text{ m}$$

Hence piezometric head at D = 36.482.

but Z_B=38m

Hence water flows from B to D.

Appling **Bernoulli's equations to point B and D**

$$Z_B = \left(Z_D + \frac{p_D}{\rho g}\right) + h_{f_2}$$

$$38 = 36.482 + h_{f_2}$$

$$\therefore h_{f_2} = 38 - 36.482 = 1.518\ m$$

But $h_{f_2} = \frac{4.f.L_2.V_2^2}{d_2 \times 2g} = \frac{4 \times 0.006 \times 600 \times V_2^2}{0.2 \times 2 \times 9.81}$

Therefore $1.518 = \frac{4 \times 0.006 \times 600 \times V_2^2}{0.2 \times 2 \times 9.81}$

$$V_2 = \sqrt{\frac{1.518 \times 0.2 \times 2 \times 9.81}{4 \times .006 \times 600}}$$

$$= 0.643 \frac{m}{s}.$$

Therefore, discharge,

$$Q_2 = V_2 \times \frac{\pi}{4}(d_2)^2 = 0.643 \times \frac{\pi}{2} \times (0.2)^2$$

$Q_2 = 0.0202\ m^3/s = 20.2$ liters/s

Appling Bernoulli's equations to point D and C,

$$Z_D + \frac{p_D}{\rho g} = Z_c + h_{f_3}$$

or $36.482 = \frac{4.f.L_3.V_3^2}{d_3 \times 2g}$

where, $V_3 = Q_3 / (\pi/4 \times d_3^2)$

But from continuity $Q_1 + Q_2 = Q_3$

$$Q_3 = Q_1 + Q_2 = 0.06 + .0202 = .0802\ m^3/s$$

$$V_3 = \frac{Q_3}{\frac{\pi}{4}(0.3)^2} = \frac{0.0802}{\frac{\pi}{4}(.09)}$$

$$= 1.134\ m/s$$

$$\text{Therefore } 36.482 = Z_C + \frac{4 \times .006 \times 800 \times 1.134^2}{0.3 \times 2 \times 9.81} = Z_C + 4.194$$

$$Z_C = 36.482 - 4.194 = \mathbf{32.288\ m. Ans.}$$

Unit 7: Intoduction to compressible flow

Introduction :

Consider a *ideal gas equation*

$$\rho = \frac{p}{RT}$$

It seen that density is depends directly on pressure and inversely on temperature. Thus density changes in the flow can in fact occur . Such flows called compressible flows

Compressible flow is defined as the flow in which the density of the fluid **does not remain constant during flow**. This means that the density changes from point to point in compressible flow. But in case of incompressible flow, the density of the fluid is assumed to be constant. In fluid flow measurements, flow passed immersed bodies, viscous flow etc,

Attributes of Compressible Flow

- **Density can no longer be regarded as constant.**
- **Bernoulli's principle** doesn't hold for compressible flow.
- **Coupling between Internal energy and Kinetic energy** can

no longer be ignored.

- **The change in density of a fluid** is accompanied by the changes in pressure and temperature and hence the thermodynamic behavior of the fluid will have to be taken into consideration

A study of compressible flow is so important because of the wide range examples that exist:

- natural gas piped from producer to consumer,
- high speed flight through air,
- discharging of compressed gas tanks,
- flow of air through compressor,

- flow of gases/steam through turbine, in machines , and many others
- Flow of gases through orifices and nozzles,
- Projectiles and airplanes flying at high altitudes with high

velocities, the density of the fluid changes during the flow.

Basic Thermodynamic Relations

(1)Equation of state- is defined as the equation which gives the relationship between the pressure, temperature ans specific volume of gas. For the perfect gas, the equation of state is

where

Vs = Specific volume or volume per unit mass = 1/ρ

p = Absolute pressure of gas in kgf/m^2 abs

T = Absolute temperature=(t+273)˚C ,absolute =Degrees Kelvin (˚K)

R = Gas constant in kgf-m/kg ˚K or (j/kg K)

- The value of **gas constant R is different for each gas**. For air having specific weight w of 1.293 (12.68) at a pressure of 760 mm of Hg (or 10,332 kgf/ or 101,300) and temperature 0˚C, the gas constant will be

$$R = \frac{pv_s}{T} = \frac{p}{\gamma T} = \frac{10{,}332}{1.293 * 273} \text{ or } \frac{101{,}300}{12.68 * 273}$$

Therefore **R = 29.27** if the value of R is given as 29.27 kgf-m/kg ˚K for air, value of **p and ρ** should be taken Kgf/m^2 and kg/m^3

- If the weight of gas is known as W, then total volume of the gas is v. Then equation (1) can be written in the form

$$pv = WRT \quad \text{………. (2)}$$

In addition to above equation of state, certain other basic relationships of thermodynamics are also required. They are briefly described below for the purpose of revision.

2 (a) Isothermal Process - In this process the temperature remains always constant. It means that when a gas is compressed sufficient time is allowed to dissipate the amount of heat generated due to compression, to the atmosphere.

Equation of state for this case is

pv_s= constant or

$p_1v_{s1} = p_2v_{s2}$ = constant….(when t = constant) …….(3)

which is also known as Boyle's Law.

(b) Adiabatic process- In this process no heat is added or taken away from the flow system by its surroundings, that is, in this process, the expansion or compression of gas is done without allowing sufficient time. A good insulated system is an example from which neither heat can go out nor enter in. The relationship between pressure and specific volume is given by the following relationship:

$$(\Upsilon = k) = \text{Bulk modulus} = 1.4 \text{ for air}$$

Pv_s^k = constant

or $p_1v_{s1}^k = p_2v_s^k$ = constant ……4

$$\frac{p_1}{p_2} = \left(\frac{v_{s2}}{v_{s1}}\right)^k \quad \text{…. (4.a)}$$

The **equation of state** gives the following relationship for this process

$$\frac{p_1}{p_2} = \left(\frac{V_{s2}}{V_{s1}}\right)^k \quad \text{………….. (5)}$$

This is also known as Charles' Law.

Substituting the value of $\frac{p_1}{p_2}$ from Equation (4.a) in (5), the following relationships are obtained : $\frac{T_1}{T_2} = \left(\frac{V_{s2}}{V_{s1}}\right)^{k-1}$ … (6)

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} \quad \text{…..(7)}$$

(c) specific Heats- In above equations, k is given by

$$\mathbf{k = c_p/c_v}$$

Where cp =specific heat at constant pressure and

c_v = specific heat at constant volume

(a)c_p is defined as that amount of heat which is required to change the temperature of a unit weight of fluid throug 1˚C, when **pressure remains constant.**

$$c_p = \left(\frac{1}{k-1}\right)^R \quad .....(8)$$

(b) c_v is defined as the amount of heat required to change the temperature of a unit weight of fluid through 1˚C when **volume remains constant**

$$c_v = \left(\frac{1}{k-1}\right)^R \quad ......(9)$$

Physical properties of Various Gases

Gas	Chemical formula	c_p	c_v	K	R
Air	---	0.242	0.171	1.4	29.57
Carbon dioxide	CO_2	0.203	0.156	1.3	19.50
Carbon monoxide	CO	0.244	0.174	1.4	30.64
Hydrogen	H_2	3.410	2.420	1.41	425.00
oxygen	O_2	0.217	0.155	1.39	26.80
Nitrogen	N_2	0.244	0.173	1.4	30.60
Water vapour	H_2O	0.451	0.339	1.33	47.55

Entropy is defined as a property of a gas which **measures the availability of heat energy for conversion into work.** It is denoted by **S**. It cannot be measured by instruments but like moment of inertia it is quite real and very useful in thermodynamic calculations. **Like heat it has no**

definite value, but it is measured above an arbitrary chosen datum. Its absolute value is not important. The change of entropy during a given process is a quantity which is of interest.

If $\mathbf{d_\theta}$ is the amount of heat absorbed or transferred by a unit weight of gas in a small time interval and **T** is the absolute temperature of gas at that instant, then the change of entropy during the process is

$$d_s = \frac{d\theta}{T} \quad \text{and} \quad s_2 - s_1 = \int_{T_1}^{T_2} \frac{d\theta}{T} \qquad \ldots\ldots\ldots(10)$$

Basic Equations of Compressible Fluid Flow:

These are the same as continuity equation and momentum equation derived from three basic principles. The only change from incompressible fluid cases is that **thermodynamics of mass, energy and momentum.**

Equation of Continuity:

In deriving the equation $a_1V_1 = a_2V_2 = Q$ = constant, it was assumed that flowing fluid is incompressible i.e. $\rho_1=\rho_2$, hence the volumetric rate of i.e. flow volumetric discharge passing through any section

This is based on law of conservation of mass which states that matter cannot be created nor be destroyed. Or in other words, the matter or mass is constant. For 1-D steady flow, the mass per second = ρAV

Where ρ = mass density, A = area of cross section, V= velocity

As mass or mass per second is constant according to law of conservation of mass, Hence

ρAV = constant ----(a)

Differentiating eqn. (a), $d(\rho AV) = 0$ **or** $\rho\mathbf{d(AV)} + AV\mathbf{d\,\rho} = 0$

or $\rho[AdV + VdA] + AV\,d\,\rho = 0$ **or** $\rho AdV + AVd\,\rho = 0$

Dividing by ρAV, we get $\mathbf{dV/V + dA/A + d\,\rho/\,\rho = 0}$ -→ **continuity equation in differential form**

Bernoulli's equation

Total energy = pressure energy (p/ρ)+ potential or elevation energy (z) +kinetic energy ($V^2/2g$)

In case of compressible flow , with the change of density ρ, the pressure p also changes for compressible fluids.The Bernoulli's equation will be different for **isothermal process** and for **adiabatic process**

(a) Bernoulli's equation for Isothermal process:

For isothermal process, the relation between pressure(p) and density(ρ) is given by equation

$p/\rho = \text{constant}$

Finally, $p/\rho g \log_e p + V^2/2g + Z = \text{constant}$

Bernoulli's equation for compressible flow under going Isothermal process.

For the two points 1 and 2, this equation is written as

$p_1/\rho_1 g \log_e p_1 + V_1^2/2g + Z_1 = p_2/\rho_2 g \log_e p_2 + V_2^2/2g + Z_2$

(b) Bernoulli's equation for Adiabatic (or Isentropic) Process:

For adiabatic process, the relation between pressure(p) and density(ρ) is given by equation

$p/\rho^k = \text{constant}$

Finally, $$\left[\frac{k}{k-1}\right]\frac{p_1}{\rho_1 g} + V_1^2 + Z_1 = \left[\frac{k}{k-1}\right]\frac{p_2}{\rho_2 g} + V_2^2 + Z_2$$

Bernoulli's equation for compressible flow under going Isothermal process

For the two points 1 and 2, this equation is written as

$$\left[\frac{k}{k-1}\right]\frac{p_1}{\rho_1 g} + V_1^2 + Z_1 = \left[\frac{k}{k-1}\right]\frac{p_2}{\rho_2 g} + V_2^2 + Z_2$$

Momentum Equations:

The momentum per second of a flowing fluid (momentum flux) Is equal to the product of mass per second and the velocity of the flow.

Mathematically, the momentum per second of a flowing fluid (compressible or incompressible) is = ρAV x **V, where** ρAV = mass per second

The term **ρAV** is constant at every section of flow due to continuity equation. This means the momentum per second at any section is equal to the **product of a constant quantity** and the **velocity**. This also implies that momentum per second is independent of compressible effect. Hence the momentum equation for incompressible and compressible fluid is the same. The momentum

Equation for compressible fluid for any direction may be expressed as,

Net force in the direction of S

= rate of change of momentum in the direction of S

= Mass per second [change of velocity]

= $\rho AV [V_2 - V_1]$

where v2 = Final velocity in the direction of S

V1 = Initial velocity in the direction of S

Propagation of Disturbances in Fluid and Sonic Velocity of Flow:

Both solid and fluid as transmitting media consist of molecules with a difference that in case of a solid the molecules are close together and in a fluid the molecules are relatively apart. Whenever a minor disturbance takes place, it is transmitted through a solid body instantaneously, but in case of fluid its molecules change in position before the disturbance is transmitted or propagated.

Thus the propagation of disturbance depends upon the elastic properties of a fluid. **This propagation of disturbance is similar to the propagation of sound through a media.** The speed of propagation of sound in a media is known as acoustic or sonic velocity which is due to pressure difference. The **sonic velocity** is considered as an important factor in compressible flow.

Derivation of sonic velocity:

Consider one-dimensional flow through a long straight cylinder made of rigid material and having uniform thickness (refer Fig 2). Let a frictionless piston work in the cylinder with **velocity v**. It compresses the fluid and produces disturbance propagating along its length in the form of a pressure wave travelling with the **velocity of sound C.**

Let A = cross sectional area of the pipe

V = velocity of the piston

p = pressure of the fluid in pipe before movement of the

piston

ρ = Density of fluid before the movement of the piston

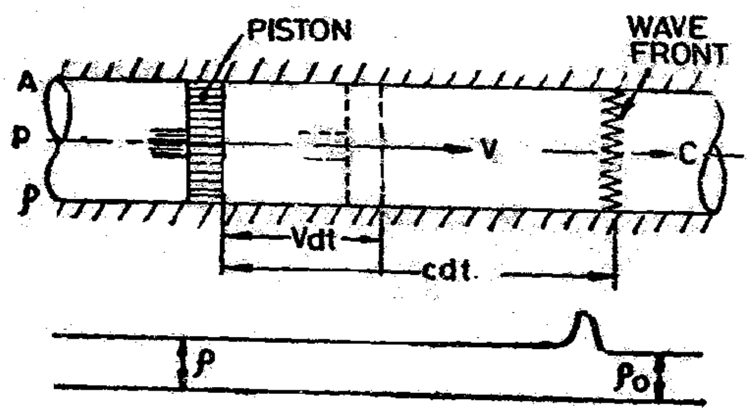


Fig 2. One dimensional pressure wave propagation

Let C = Velocity pressure wave or sound wave travelling in fluid

vdt = distance travelled by the piston in time dt

dp = Increase in pressure

Dρ =Increase in fluid density between piston and wave front

dt = A small interval of time with which piston is moved

Neglect friction and heat transfer, if any.

By applying momentum equation

$(p + dp)dA - pdA = dA\rho c\{c - (c - dv)\}$

$dp = \rho c dv$ **……(a)**

Now, ρVA = constant ….Equation of continuity

(neglecting terms of higher order)

i.e $\rho v A = (\rho + d\rho)(c - dv).A$

or

$c d\rho = \rho dV$ (neglecting terms of higher order)

Therefore $$dv = \frac{c.\,d\rho}{\rho}$$

Substituting the value of **dv** in eqn. (a)

$$dp = \rho c.\,c\frac{d\rho}{\rho} = c^2 d\rho$$

Or $$C = \sqrt{\frac{dp}{d\rho}} \qquad \text{........(b)}$$

Equation (b) gives the velocity of sound wave which is the square root of the ratio of change of pressure to the change of density of a fluid disturbance

Velocity of sound in terms of bulk modulus.

Bulk modulus k is defined as K = (Increase in pressure/(Decrease in volume/Original volume)

$= dp/-(dv_s/v_s)$ …(c)

Where dv_s = Decrease in volume,

v_s= Original volume

Negative sign is taken as with the increase of pressure, volume decreases

Now we know mass of the fluid is constant. Hence ρ * volume = constant (since mass = ρ * volume) $\rho * v_s$= constant

Differentiating the above equation (ρ and v_s are variables)

$\rho dv_s + v_s d\rho = 0$ **or** $\rho dv_s = -v_s d\rho$ **or** $dv_s/v_s = dp/\rho$

substituting the value $(-dv_s/v_s)$ in equation (c), we get $K=dp/(d\rho/\rho) = \rho(dp/d\rho)$ **or** $(dp/d\rho) = (K/\rho)$

the velocity of sound wave is given by

$$C = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} \qquad \text{.....(d)}$$

Equation (d) gives the velocity of sound wave in terms of bulk modulus and density. This equation is applicable for liquids and gases.

Velocity of sound for isothermal process

For isothermal process , p/ρ = constant

$p\,\rho^{-1}$ = constant

Differentiating the above equation, we get

$P^{(-1)}\,\rho^{-2}\,d\rho + \rho^{-1}\,dp = 0$

Dividing by ρ^{-1} , we get $-p\,\rho^{-1}\,d\rho + dp$ **or** $-p\,d\rho + dp = 0$

$dp = p/\rho\; d\rho$ **or** $dp/d\rho = p/\rho = RT$ (as $p/\rho = RT$…Eqn. of state)

Substituting the value of **dp/dρ** in equation $C = \sqrt{\dfrac{dp}{d\rho}}$

We get $C = \sqrt{\dfrac{K}{\rho}} = \sqrt{RT}$

Velocity of sound for adiabatic (isentropic process)

p/ρ^{k} = constant

$p\,\rho^{-k}$ = constant

Differentiating the above equation, we get

$P^{(-k)}\,\rho^{-k-1}\,d\rho + \rho^{-k}\,dp = 0$

Dividing by ρ^{-k} , we get $-p^{k}\,\rho^{-1}\,d\rho + dp$ or $dp = p^{k}/\rho\; d\rho$

$dp/d\rho = p/\rho\; k = RTk$ (as $p/\rho = RT$…Eqn. of state)

Substituting the value of **dp/dρ** in equation $C = \sqrt{\dfrac{dp}{d\rho}}$

We get $c = \sqrt{kRT}$ **…….(e)**

Note 1. For the propagation of the minor disturbances through air, the process is assumed to be adiabatic. The velocity of disturbances (pressure wave) through air is very high and hence there is no time for any appreciable heat transfer.

2. isothermal process is considered for calculation of the velocity of the sound waves (or pressure waves) only when it is given in the numerical problem that process is isothermal. If no process is mentioned, it is assumed to be adiabatic.

Mach number:

Mach number is defined as the square root of the inertia force of a flowing fluid to the elastic force.Then,

Mach number = $$M = \sqrt{\frac{Inertiaforce}{Elasticforce}} = \sqrt{\frac{\rho AV^2}{KA}}$$

$$= \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C} \qquad \because \sqrt{\frac{K}{\rho}} = C$$

Thus mach number = M [From Equation eqn. (d)]

Mach number, M = Velocity of fluid or body moving in fluid / Velocity of sound in the fluid

$$\mathbf{M = \frac{V}{c}}$$

If the Mach number of fluid flow is less than one(M<1), it is flowing with a velocity which is less than velocity of sound.Such a flow is called subsonic flow. For M>1, the flow is known as Supersonic flow. For M=1, the flow is sonic flow.

Problems on Mach number

1)Find the **sonic velocity** of the following fluid :

(i)Crude oil of specific gravity 0.8 and bulk modulus 153036 N/cm^2

(ii)Mercury having a bulk modulus of 2648700 N/cm^2

Solution:

Given:

(i) Crude oil: Specific gravity =0.8

Therefore density of oil, ρ =0.8 * 1000=800 kg/m^3

Bulk modulus, K = 153036 N/cm^2 = 153036 $*10^4$ N/m^2

Using the equation for sonic velocity, as

$C = \sqrt{(k/\rho)} = \sqrt{((153036 *10^4)/800)}$

= 1383.09 = **1383 m/s**

(ii) Mercury: Bulk modulus, K= 2648700 N/cm^2 = 2648700 * 10^4 N/m^2

Specific gravity = 13.

Density of mercury, ρ =13.6 * 1000 =13600 kg/m^3

The sonic velocity, C is : given by C = √(k/ρ)

= √((2648700 * 10^4)/13600)

= **1395.55 m/s**

2) Find the **sonic velocity** for the following fluids:

(i) Crude oil of specific gravity 0.8 and bulk modulus 1.5 GN/m^2

(ii) Mercury having a bulk modulus of 27 GN/m^2

Solution:

Crude oil: Specific gravity =0.8

Therefore density of oil, ρ =0.8 * 1000 =800 kg/m^3

Bulk modulus, K = 1.5 GN/m^2

Mercury: Bulk modulus, K= 27 GN/m^2

Density of mercury, ρ =13.6 * 1000 =13600 kg/m^3

Sonic velocity, C_{oil} , C_{Hg} :

Sonic velocity is given by the relation :

C = √(k/ρ)

C_{oil} = √((1.5 *10^9)/800) = 1369.3 m/s

C_{Hg} = √((27 *10^9)/13600) = 1409 m/s

3) Find the **speed of the sound wave** in air at sea-level where the pressure and temperature are 10.1043 N/cm^2 (absolute) and 15°C respectively. Take R =287 J/kg K and k=1.4.

Solution: Given :

Pressure, p =10.1043 N/cm^2

= 10.1043 * 10^4 N/m^2

Temperature, t = 15^{o}C

Therefore T = 273+15 = 288 K

R = 287 J/kg K, k=1.4.

For **adiabatic process**, the velocity of sound is given by

$C=\sqrt{(kRT)} = \sqrt{(1.4 * 287 * 288)}$

= **340.17 m/s**

4) Calculate the **Mach number** at a point on a jet propelled aircraft, which is flying at 1100 km/hour at sea level where air temperature is 20^{o}C. Take k = 1.4 and R = 287 J/kg K.

Solution: Given :

Speed of aircraft,V = 1100 km/hour

= (1100 * 1000)/(60 * 60) = 305.55 m/s

Temperature, t = 20^{o}C

Therefore T = 273+20 = 293 K

k = 1.4, R = 287 J/kg K.

The **velocity of sound** is given by the equation

$C = \sqrt{(kRT)}$

$= \sqrt{(1.4 * 287 * 293)}$

= **343.11 m/s**

Mach number is given as

M = (V/C) = (305.55/343.11) = **0.89**

5) An aero plane is flying at an height of 14 km where the temperature is -50ºC. The speed of the is corresponding to M = 2.0. Assuming k = 1.4 and R = 287 k/kg K, find the speed of the plane

Solution:

Height of plane, Z = 15 km(extra data)

Temperature, t = -50oC

Therefore, T = -50+273 = 223oC

Mach number, M = 2.0, k = 1.4, R=287 j/kg K

Using equation, we get the velocity of sound as C = √(kRT)

$$= \sqrt{1.4 \times 287 \times 223} = 299.33 m/s$$

We have Mach number M = V/C

2.0= V/299.33

V = 2.0 x 299.33 = 598.66 m/s

= 598.66 x 60x60/1000 = 2155.17 km/hr

6)Find the Mach number of rocket travelling in standard air with a Speed of 1600 km/hr.

Solution:

Same in MKS and SI units

$$v = 1600 \frac{km}{hr} = \frac{1600 \times 1000}{60 \times 60} = 444.4 m/sec$$

Standard air has the following values at sea level;

$$p = 1.0332 \frac{kgf}{cm^2 \left(or\ 101.3 \frac{kN}{m^2}\right)}$$

$$t = 15°C ; \gamma = 1.226 \frac{kgf}{m^3} (or 12\ N/m^3)$$

$$\rho = 0.125 \frac{msl}{m^3} \left(or\ 1.226 \frac{kgm}{m^3}\right)$$

$$R = 29.27 \frac{m}{°K}$$

Nature of propagation of pressure Waves or disturbances) in a compressible Fluid

Whenever any disturbance is produced in a compressible fluid, the disturbance is propagated in all direction with a Velocity of sound. The nature of propagation of the disturbance depends upon the Mach number. Let a projectile travel in a straight line with a steady velocity V. It will produce disturbance propagating in all directions.

At time t = 0 sec, the body is at point A

At time t = 1 sec, the body is at point 1, then distance $s_1 = V\Delta t$

At time t = 2 sec, the body is at point 2, then $s_2 = 2V\Delta t$

At time t = 3 sec, the body is at point 3, then $s_3 = 3V\Delta t$ etc.

With point 3 as center, draw a circle with radius CΔ t, and with point 2 as center, draw another circle with radius 2cΔt. similarly with point A as center draw circle with radius to 4c Δt. From this it will be seen that point B remains within the sphere of radius 4c

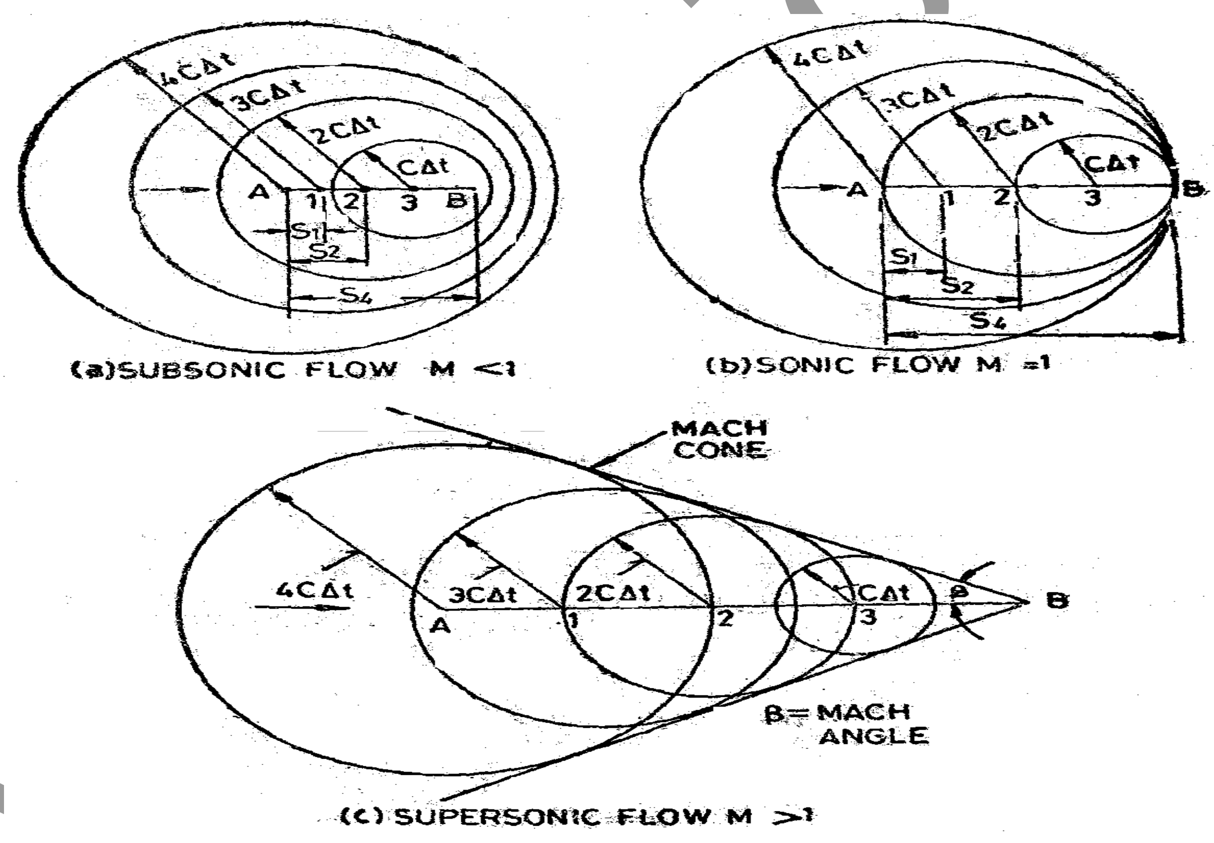


Figure 3. Nature of disturbances in compressible flow

(a) M<1, when V is less than the velocity of sound C,which means the projectile lags behind the pressure wave.

(b) M=1 Since V=C, then circles drawn join to points as shown in Fig 1.2(b). The circle drawn with center A will pass through B.

(c) M>1, i.e V>C, then the sphere of propagation of disturbance is smaller and the velocity of projectile is higher.

Drawing the circles as earlier, if tangents are drawn to the circles, the spherical pressure waves form a cone with its vertex at B. It is known as Mach cone. Half cone angle is known as mach angle and denoted by β and $\beta = \sin^{-1}\frac{c}{v} = \sin^{-1}\frac{1}{Ma}$

In such a case the disturbance takes place inside the coneand outside it there is no disturbance which is then called Silence zone. It is seen that when an aero plane is moving with supersonic speed, the noise of the plane is heard only after the plane has already passed over us.

When M >1, the effect of the disturbance is felt only in the region Inside the mach cone. This region is called the zone of action

Stagnation Properties:

When the fluid flowing past an immersed body, and at a point on the body if the resultant velocity becomes zero, the value of pressure, temperature and density at that point are called **Stagnation point.** The values of pressure, temperature and density are called **stagnation pressure, stagnation temperature and stagnation density** respectively. They are denoted as **p_s, ρs and Ts** respectively.

Expression for stagnation pressure (p_s)

Consider a compressible fluid flowing past an immersed body under frictionless adiabatic conditions.

Consider points 1 and 2 on a stream line.

Let p_1 = pressure of compressible fluid at point 1

V_1 = Velocity of fluid at 1 and

ρ_1= Density of fluid at 1

p_1, v_2 ρ_2 = corresponding values of pressure, velocity and density at point 2

By applying Bernoulli's equation for adiabatic flow by equation at 1 and 2, we get

$$\left[\frac{k}{k-1}\right]\frac{p_1}{\rho_1 g}+V_1^2+Z_1=\left[\frac{k}{k-1}\right]\frac{p_2}{\rho_2 g}+V_2^2+Z_2$$

Finally

$$p_s = p_1\left[1+\frac{k-1}{2}M_1^2\right]^{\left(\frac{k}{k-1}\right)}$$

Where P_s =stagnation pressure

Expression for stagnation Density (ρ_s)

$p_s / \rho_s = RT_s$

Expression for stagnation (T_s)

Equation fo state is given by $p_s/\rho_s = RT$

Finally

$$T_s = T_1\left[1+\frac{k-1}{2}M_1^2\right]$$

1) A projectile is travelling in air having pressure and temperature as 88.3 kN/m^2 and -2^{o}C. If the Mach angle is 40^o, find the velocity of the projectile. Take k =1.4 and R = 287 J/kg K.

Solution:

Pressure, p= 88.3 kN/m^2

Temperature of air, t= -2ºC

T = -2+273 = 271K

Mach angle, α = 40^o

k =1.4, R = 287 J/kg K.

L et **Velocity of the projectile = V**

Sonic velocity, C = √(kRT)

= √(1.4*287 *271)

= 330 m/s

Now, sinα= C/V

or $\sin 40^o = 330/V$

or $V = 330/\sin 40^o$

V = 513.4 m/s

2) A projectile travels in air of pressure 10.1043 N/cm^2 at 10^oC at a speed of 1500 km/hour. Find the **Mach number** and the **Mach angle**. Take k = 1.4 and R = 287 J/kg K.

Solution:

Given :

Pressure, $p = 10.1043\ N/cm^2$

$= 10.1043 * 10^4\ N/m^2$

Temperature, $t = 10^oC$

$T = 10+273 = 283\ K$

Speed of projectile, $V=1500$ km/hour

$= (1500 * 1000)/(60 * 60)$ m/s

$= 416.67$ m/s

$k = 1.4$, $R = 287$ J/kg K.

For **adiabatic process** velocity of sound C is given by

$C = \sqrt{(kRT)}$

$= \sqrt{(1.4 * 287 * 283)}$

$= 337.20$ m/s

Therefore Mach number, $M = (V/C)$

$= (416.67/337.20)$

= **1.235.**

Therefore Mach angle is obtained as

$\sin\alpha = (C/V)$

= (1/M)

= (1/1.235)

= 0.8097

Therefore Mach angle, $\alpha = \sin^{-1}(0.8097)$

$= \mathbf{54.06^o}$

3) Find the velocity of bullet fired in standard air if the Mach angle is 30^o. Take R = 287.14 J/kg k and k = 1.4 for air. Assume temperature as 15^oc.

Solution:

Given :

Mach angle $\alpha = 30^o$

R = 287.14 J/kg k

k = 1.4

Temperature, $t = 15^oC$

Therefore T = 15 + 273 = 288 k

Velocity of sound is given as

$C = \sqrt{(kRT)}$

$= \sqrt{(1.4 * 287.14 * 288)}$

= 340.25 m/s

Using the relation, $\sin \alpha = C/V$

$\sin 30^o = 340.25/V$

V = 340.25/sin 30

= **680.5 m/s**

4)An air plane is flying at an altitude of 15 km where the temperature is -50°C. The speed of the plane corresponds to Mach number of 1.6. Assuming k= 1.4 and R = 287 J/kg K for air. Find the **speed of the plane and Mach angle α.**

Solution:

Given :

Height of plane, H = 15 km = 15 * 1000 = 15000 m

Temperature, t = -50°C

therefore T = -50+273 = 223 K

Mach number, M = 1.6, k=1.4 and R = 287 J/kg K

Find : (i) speed of plane (V)

(ii) Mach angle, α

Velocity of sound wave is given as

$C = \sqrt{(kRT)} = \sqrt{(1.4 * 287 * 223)} = 229.33$ m/s

(ii) Mach angle, α

Using the relation for Mach angle, we get

$\sin \alpha = C/V = 1/(V/C) = 1/M = 1/1.6 = 0.625$

$\alpha = \sin^{-1} 0.625 =$ **38.68°**

(i) Speed of plane, V

We know, M=V/C

1.6 = V/299.33

V = 1.6 * 299.33 = 478.928 m/s

= (478.98 * 3600)/1000 = **1724.14 m/s**

5)Find the Mach number when an aeroplane is flying at 1100 km/hour through still air having a pressure of 7 N/cm^2 and temperature -5°C. Wind velocity may be taken as zero. Take R = 287.14 J/kg K. **calculate the pressure, temperature and density of air at stagnation point** on the nose of the plane. Take k=1.4.

Solution:

Given :

Speed of aeroplane, V=1100 km/hour

$= (1100 * 1000)/(60 *60)$

$= 305.55$ m/s

Pressure of air, $p_1 = 7\ N/cm^2 = 7*10^4\ N/m^2$

Temperature, $t_1 = -5^oC$

Therefore $T_1 = -5 + 273 = 268$ K

$R = 287.14$ J/kg K

$K=1.4$

Using relation $C = \sqrt{(kRT)}$ for velocity of sound for adiabatic process, we have

$C_1 = \sqrt{(1.4 *287.14 *268)} = 328.2$ m/s

Therefore Mach number, $M_1 = (V_1/C_1)$

$= (305.55/328.20) = 0.9309 =$ **0.931**

Stagnation pressure, p_s. using equation for stagnation pressure,

$p_s = p_1[1+((k-1)/2)M_1^2]^{k/(k-1)}$

$= 7.0 *10^4[1+((1.4-1)/2)(0.931)^2]^{1.4/(1.4-1)}$

$= 7.0 *10^4[1+0.1733]^{1.4/0.4}$

$= 7.0 *10^4[1.1733]^{3.5} = 12.24 * 10^4\ N/m^2$

$=$ **12.24 N/cm^2**

Stagnation temperature, T_s. using the equation for stagnation temperature,

$T_s = T_1[1+((k-1)/2)\ M_1^2]$

$= 268\ [1+((1.4-1)/2)\ (0.931)^2]$

$= 268\ [1.1733] = 314.44$ k

Therefore

$t_s = T_s - 273 = 314.44 - 273 =$ **41.44 °C**

Stagnation density , ρ_s. Using equation of state for stagnation density, $p_s/\rho_s = RT_s$

$\rho_s = p_s/(RT_s)$

In the above equation given, if R is taken as 287.14 J/kg K, then pressure should be taken in N/m^2 so that the value of ρ is in kg/m^3. Hence $p_s = 12.24 * 10^4\ N/m^2$ and Ts = 314.44 k.

$\rho_s = 12.24 * 10^4/(287.14 * 314.44)$ **= 1.355 kg/m³**

6) Calculate the stagnation pressure, temperature, density on the stagnation point on the nose of a plane, which is flying at 800 km/hour through still air having a pressure 8.0 N/cm^2 (abs.) and temperature -10 ^{0}C.Take R=287 J/Kg K and k = 1.4.

Solution:

Given:

Speed of plane, V = 800 km/hour

= (800*1000)/(60*60) = 222.22 m/s

Pressure of air, $p_1 = 8.0\ N/cm^2 = 8.0*10^4\ N/cm^2$

Temperature, $t_1 = -10\ ^0C$

$T_1 = -10 + 273 = 263^0$ C

R= 287 J/Kg^0 K

k = 1.4

For adiabatic flow, the velocity of sound is given by

$C = \sqrt{(KRT)}$

$= \sqrt{(1.4*287*263)}$

=325.07 m/s.

Mach number, M = V/C

= 222.22/325.07 =0.683

This **Mach number is the local Mach number and hence equal to M_1.**

Therefore M_1 = 0.683

Using equation for **stagnation pressure**,

$p_s = p_1 [1+ ((k-1)/2) M_1^2] (k/(k-1))$

$= 8.0*104[1+((1.4-1.0)/2.0)*(0.683)^2](1.4/(1.4-1.0))$

=8.0*104[1.0933]3.5 = 10.93*104 N/m2

= 10.93 N/cm2

Using equation for **stagnation temperature**

$T_s = T_1 [1+ ((k-1)/2) M_1^2]$ =263 $[1+ ((1.4-1.0)/2.0)*(0.683)^2]$

=263 [1.0933] =287.5 K

Therefore $t_s = T_S - 273$ =287.5 – 723

=**14.5 ^{0}C**

Using equation of state, $p/\rho = RT$

For stagnation point, $p_s /\rho_s=RT_S$

Therefore $\rho_s= p_s / RT_S$

As R =287 J/Kg K, the value of p_s should be taken in N/m^2 so that the value of ρ_s is obtained in Kg/m^3.

$p_s =10.93*10^4 N/m^2$

Therefore **Stagnation density**,

$\rho_s = (10.93*10^4) / (287*287.5)$

= **1.324 Kg/m^3**

<u>Area-velocity Relationship for Compressible Flow</u>:

The are velocity relation ship for incompressible fluid is given by the continuity equation as A * V = constant

From the above equation, it is clear that with the increase of area, velocity decreases. But in case of compressible fluid, the continuity equation is given by, ρAV = constant ……(i)

Differentiating equation(i), we get

$\rho d(AV) + Avd\rho = 0$ or $\rho[AdV + VdA] + Avd\rho = 0$

or $\rho AdV + \rho VdA + Avd\rho = 0$

Dividing by ρAV, we get $dV/V + dA/A + d\rho/\rho = 0$ …..(ii)

The **Euler's equation for compressible fluid** is given by equation, as $dp/\rho + VdV + gdZ = 0$

Neglecting the Z term, the above equation is written as

$$dp/\rho + VdV = 0$$

This equation can also be written as

$$dp/\rho \times d\rho/d\rho + VdV = 0$$

(Dividing and multiplying by $d\rho$)

or $dp/d\rho \times d\rho/\rho + VdV = 0$

But **$dp/d\rho = C^2$**

Hence above equation becomes as $C^2\ d\rho/\rho + VdV = 0$

or $C^2\ d\rho/\rho = -VdV$

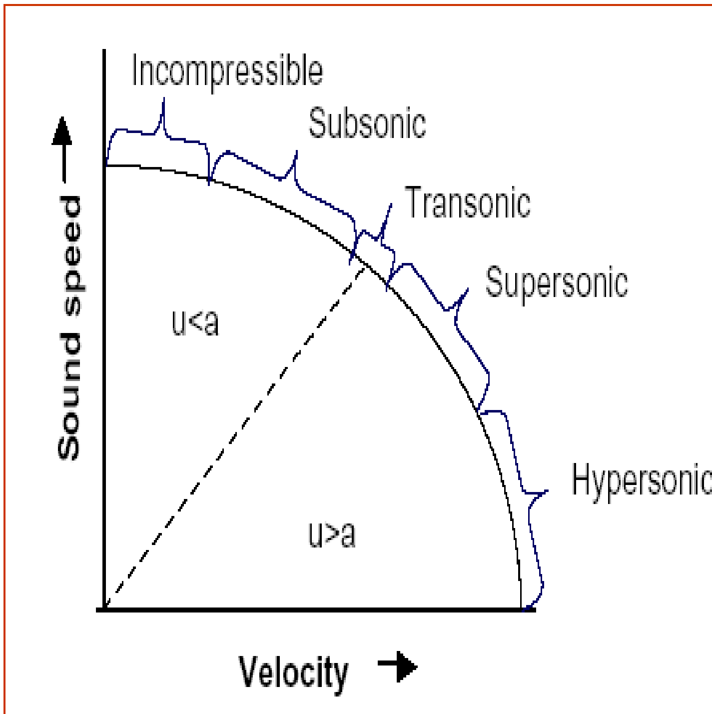
or $d\rho/\rho = VdV/C^2$

Substituting the value of **$d\rho/\rho$** in equation (ii), we get

$dV/V + dA/A - VdV/C^2 = 0$

or $dA/A = VdV/C^2 - dV/V = dV/V[V^2/C^2 - 1]$

$$\mathbf{dA/A = dV/V[M^2-1]}$$



Ma < 1 : Subsonic

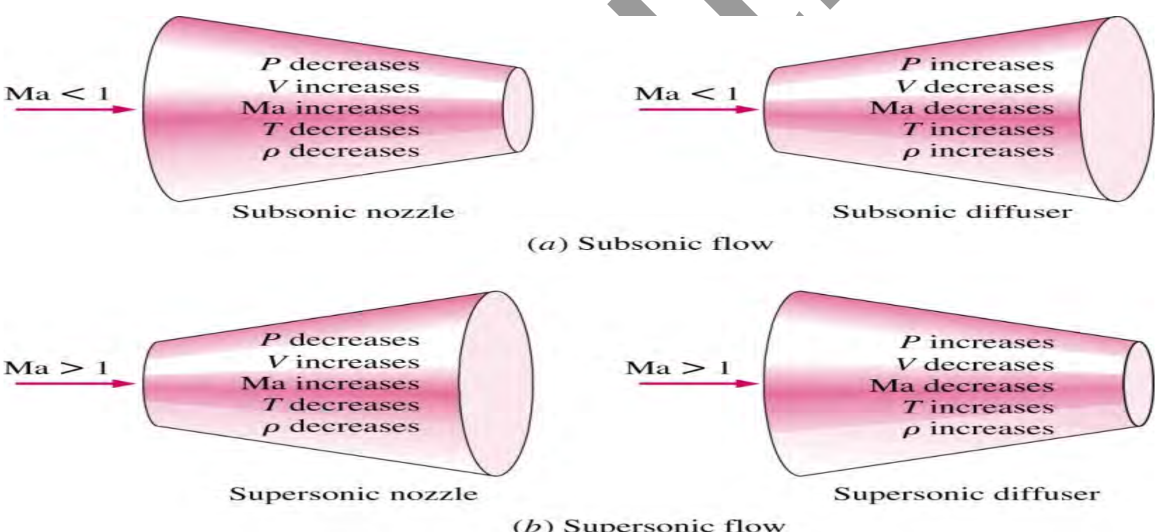
Ma = 1 : Sonic

Ma > 1 : Supersonic

Ma >> 1 : Hypersonic

Ma ≈ 1 : Transonic

Comparison of flow properties in subsonic and supersonic nozzles and diffusers



Pitot-static tube in a compressible flow

The pitot - static tube, when used for determining the velocity at any point in a compressible fluid, gives only the difference between the stagnation head and static head. From this difference, the velocity of the incompressible fluid at that point is obtained from the relation

$V = \sqrt{2gh}$, where h = difference in two heads.

But when the pitot - static tube is used for finding velocity at any point in a compressible fluid, the actual pressure difference shown by the gauges of the pitot – static should be multiplied by a factor, for obtaining correct velocity at that point. The value of the factor depends upon the mach number of the flow. Let us find an expression for the correction factor for sub-sonic flow.

At a point in pitot-static tube, the pressure becomes stagnation pressure, denoted by p_s. The expression for stagnation pressure, p_s is given by equation, as

$$\mathbf{p_s = p_1 [1+ ((k-1)/2) M_1^2]^{(k/(k-1))}} \quad \text{.......(i)}$$

Where, p_1 = pressure of fluid far away from stagnation point,

M_1 = Mach number at point 1, far away from

stagnation point,

For M<1,the term $(k-1)/2 * M_1^2$ will be the less than 1 and hence the right-hand side of the equation (i) can be expressed by binomial theorem as

$p_s = [1 +((k-1)/2)M_1^2 * k/(k-1)*((k/(k-1)*((k/(k-1)-1))/2)$

$* (((k-1)/2) M_1^2)+((k/k-1)(k/(k-1)-1)(k/(k-1)-2)((k-1/2)M_1^2)^3+....]$

$=p_1 [1+ (k/2) M_1^2+ (k/8) M_1^4+ (k (2-k)/48) M_1^6+...]$

$=p_1+p_1 [(k/2) M_1^2+ (k/8) M_1^4+ (k (2-k)/48) M_1^6+...]$

$p_s-p_1=p_1*(k/2) M_1^2[1+(M_1^2/4)+((2-k)/24)M_1^4+....]$ ..(ii)

But $M_1^2=V_1^2$ where $C_1^2=kp_1/\rho_1$

$=(V_1^2/ (kp_1/\rho_1))= V_1^2 \rho_1/kp_1$

Substituting the value of M_1^2 in equation, we get

$p_s-p_1=p_1*(k/2) * (V_1^2 \rho_1/kp_1)[1+(M_1^2/4)+((2-k)/24)M_1^4+....]$

$= V_1^2\rho_1/2[1+(M_1^2/4)+((2-k)/24)M_1^4+...]$

The term $[1+(M_1^2/4)+((2-k)/24)M_1^4+...]$ is known as compressible correction factor. And $(V_1^2 \rho_1/2)$ is the **reading of the pitot-static tube**. Thus the readings of the pitot-tube must be multiplied by a correction factor given below for correct value of velocity measured by pitot-tube.

Compressibility Correction Factor,

$C.C.F=[1+(M_1^2/4)+((2-k)/24)M_1^4+...]$

1) Calculate the **numerical factor** by which the actual pressure difference shown by the gauge of a pitot-static tube must be multiplied to allow for compressibility when the value of the Mach number is 0.9. Take k=1.4

Solution :

Mach number, $M_1 = 0.9$

$K = 1.4$

Using equation for, **Compressibility Correction Factor** is

C.C.F $=[\ 1+(M_1^2/4)+((2-k)/24)M_1^4+\ldots]$ $=1+((0.9^2)/4)+((21.4)/24)(0.9)^4+\ldots$

$=1.0+0.2025+0.0164+\ldots =$ **1.2189.**

Numerical factor by which the actual pressure difference is to be multiplied = 1.2189

=============@@@@@@@@@@@@@===================

FLUID MECHANICS

LAMINAR FLOW AND VISCOUS FLOWS

Viscosity plays an important role.
Low velocity flows.
Laminar flow - where each fluid layer glides over the adjacent layer.
Shear stress (τ) = $\mu(du/dy)$.
Ex., Flow of viscous fluid through circular pipe, two parallel plates, bearings etc.,

NO SLIP CONDITION

In ideal fluids, when fluid passes over a boundary, it slips over the boundary and velocity distribution is uniform over the boundary.
In real fluids, due to viscosity, there is no relative motion between the boundary and fluid. The fluid at the boundary has the same velocity as the boundary – This is known as “No Slip Condition”.
In a real fluid flow with stationary boundary, the velocity is zero at the boundary and increases as we go away from the boundary. This change in velocity gives rise to a velocity gradient and hence the viscous shear resistance opposing the motion. Due to this resistance to motion, power is required to maintain flow of real fluids. Hence, in many fluid flow problems, effect of viscosity cannot be neglected near the boundaries.

LAMINAR AND TURBULENT FLOWS

Depending upon the relative magnitudes of the viscous forces and inertia forces, flow can exist in two types- Laminar Flow and Turbulent Flow

REYNOLDS EXPERIMENTS

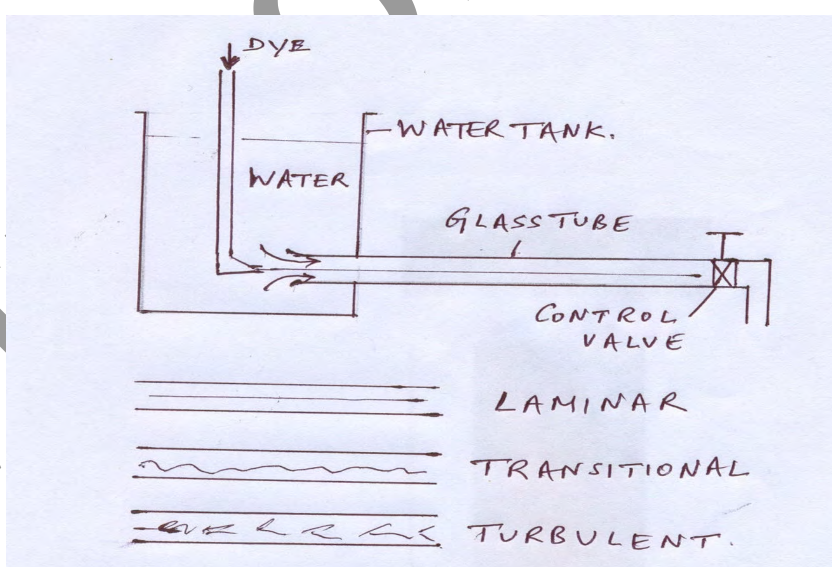


Fig. Reynolds Experiments

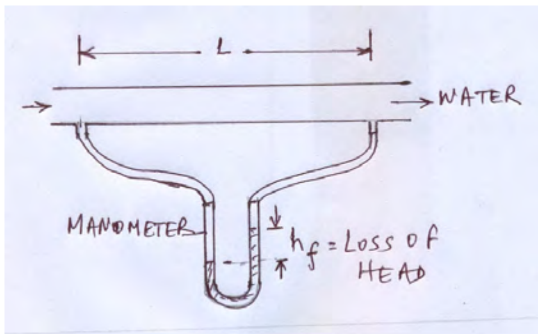
The nature of dye filament was observed at different velocities:

1. At low velocities, the dye remained in the form of straight stable filament parallel to the axis of the tube: (a) – The flow is laminar.
2. At higher velocities, dye filament showed irregularities and wavy nature: (b)- The flow is transitional.
3. With further increase in velocity, the filaments become more and more irregular, and finally dye is diffused over the complete cross section: (c) The flow is turbulent.

At low velocities, flow takes place in number of sheets or laminae. This flow is called Laminar Flow. At high velocities, the flow is disturbed and inter-mixing of particles takes place. The flow is called Turbulent Flow.

TYPES OF FLOW AND LOSS OF HEAD

Loss of head, h_f is measured in a pipe of length (L) for various values of velocity (v) in the pipe and (h_f/L) vs (v) is plotted in a log – log plot



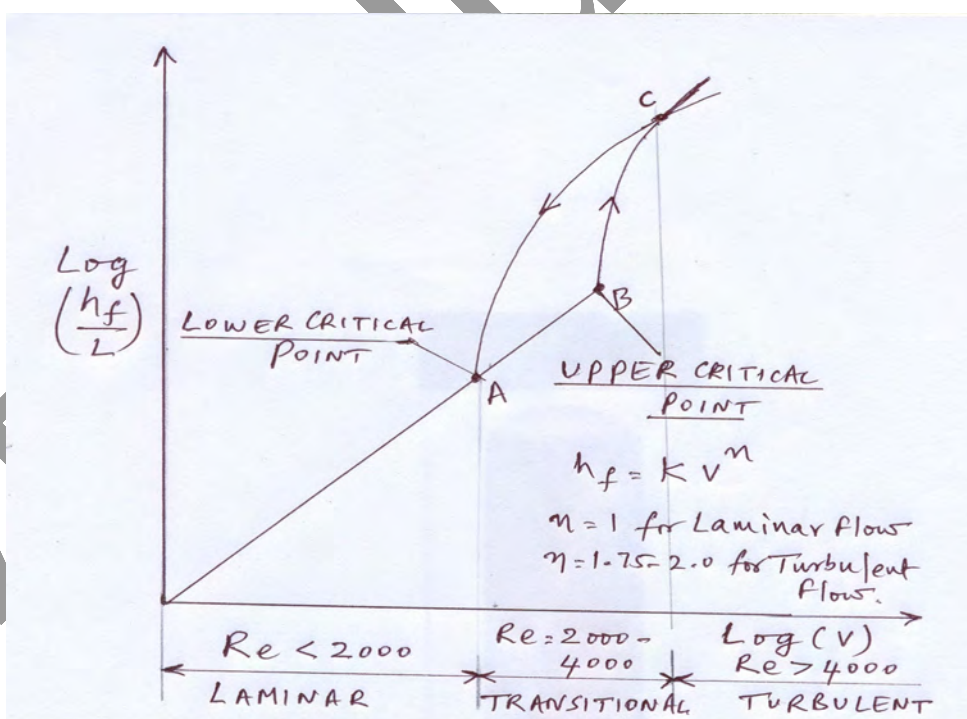


Fig. Types of Flow and Loss of Head

Loss of head (h_f) is measured in a pipe of length (L) for various values of flow velocity (v) in the pipe and (h_f/L) Vs (v) are plotted on a log – log scale. At low velocities, up to point (B), the curve is a straight line. (h_f) is proportional to (v) up to point (B). We see transition up to point (C). After (C) again, the curve obtained has a slope varying from 1.75 to 2.

Up to (B), it is one type of flow called the laminar flow in which (h_f) is proportional to (v). Beyond (C), it is another type of flow in which (h_f) is proportional to (V^n) where n=1.75 to 2. This is called turbulent flow. However, if the velocity is reduced from a high value, line BC is not retraced. Instead, the points lie along line CA.

Point (B) is called as higher (or upper) critical point and the corresponding velocity is called as upper critical velocity. Point (A) is called as lower critical point and the corresponding velocity is called as lower critical velocity. Reynolds Number, which is the ratio of inertia force to viscous force is the criterion which decides whether the flow is laminar or turbulent.

Re= (ρVL/μ)= (VL/ν). For pipes, L=d, the diameter of the pipe which is a characteristic dimension.

The Upper Critical Reynolds Number corresponding to point (B) is not definite. Its value depends upon how carefully the initial disturbance affecting the flow is prevented. Normally, Upper Critical Reynolds Number for pipe flow is about 4000. (Note: with proper precaution, values as high as 50,000 can be achieved.

The Lower Critical Reynolds Number corresponding to point (A) is definite. For a straight pipe, its value is about 2000. This Reynolds Number is the true Critical Reynolds Number, which is the dividing line between the laminar and turbulent flows. The Reynolds Number below which the flow is definitely laminar is called the Critical Reynolds Number (For pipe flow, Re (critical) around 2000.

LAMINAR FLOW

Definition: The flow in which the particles of fluid behave in orderly manner with out intermixing with each other and the flow takes place in number of sheets, layers or laminae, each sliding over the other is called as laminar flow.

Characteristics of Laminar Flow:

1. Particle of fluid behave in disciplined manner. No inter-mixing of particle.Flow takes place in layers which glide over one another.
2. Velocity of flow at a point is nearly constant in magnitude and direction.
3. Viscous force plays an important role in fluid flow (as compared to other forces).
4. Shear stress is obtained by the Newton's Law of Viscosity.
5. Any disturbance caused is quickly damped by viscous forces
6. Due to No-slip condition, velocity across the section is not uniform. Velocity gradient and hence, the shear stress gradient is established at right angles to the direction of flow.
7. Loss of head is proportional to the velocity of flow.
8. Velocity distribution is parabolic in nature (pipe flow)

Practical examples of laminar flow: Flow of oil in lubricating mechanisms, capillary tubes, blood flow in vein etc.,

SHEAR AND PRESSURE GRADIENT IN LAMINAR FLOW

Because of No-Slip Condition at the wall, different layers move over each other with different velocities in the flow near the wall. The relative motion between the layers gives rise to shear stress. Shear stress varies from layer to layer and it is maximum at the wall.

Shear stress (τ) = μ(du/dy)

Shear stress gradient exists across the flow. Also, along the flow, pressure will vary to maintain the flow and pressure gradient exists along the flow.

RELATION BETWEEN SHEAR AND PRESSURE GRADIENTS IN LAMINAR FLOW

Consider the free body of the fluid element with sides (dx, dy, dz) as shown in the Fig. For ex, in the flow inside a pipe.

(τ) = Shear stress; p=pressure

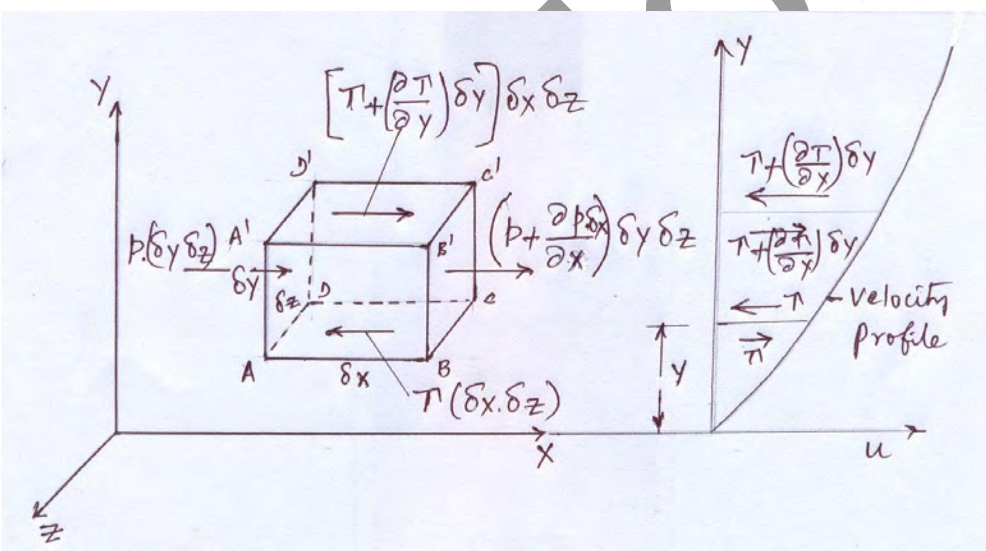


Fig. Shear and Pressure Gradients in Laminar Flow

Various forces acting on the element are shown in Fig. For steady uniform flow, there is no acceleration and the sum of forces acting in the direction of motion must be equal to zero. (Forces in +(x) direction are taken as positive)

Pressure forces + Shear forces =0

$[p.\,dydz - \{p + (\partial p/\partial x)dx\}dydz] + [\{\tau+(\partial\tau/\partial y)dy\}\,dxdz - \tau dxdz] = 0$

Simplifying,

$-(\partial p/\partial x)dxdydz + (\partial\tau/\partial y)dxdydz = 0$; Dividing by dxdydz, the volume of the parallelepiped,

$$(\partial p/\partial x) = (\partial\tau/\partial y)$$

For a two- dimensional steady uniform laminar flow, the pressure gradient in the direction of flow is equal to the shear stress gradient in the normal direction. Since p=p(x) and $\tau = \tau$ (y) only, and according to the Newton's law of viscosity, $(\tau) = \mu(du/dy)$; we get $(\partial p/\partial x) = \mu(\partial^2 u/\partial y^2)$

Problems on steady uniform laminar flows can be analyzed by integrating this equation.

LAMINAR FLOW THROUGH A CIRCULAR PIPE

Consider a steady laminar flow of fluid through a horizontal circular pipe as shown. Consider a concentric cylindrical element having radius (r) and length (dx)
Note: Shear stress resists motion.

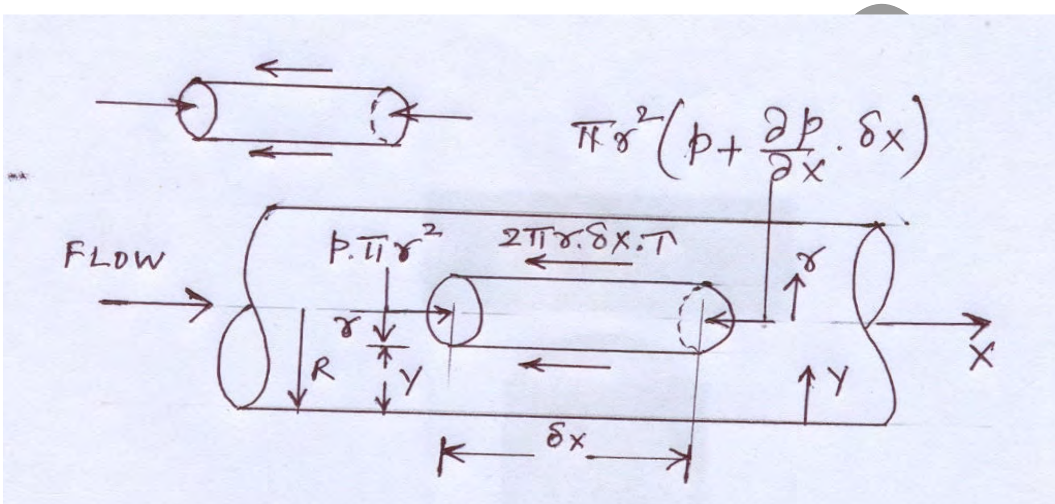


Fig. Laminar Flow through Circular Pipe

Since the flow is steady and uniform, there is no acceleration and the sum of all forces acting on the element in x-direction must be zero.

$p.\pi r^2 - (p+(\partial p/\partial x)dx)\pi r^2 - \tau 2\pi r dx = 0$

Or $-(\partial p/\partial x)dx\, \pi r^2 - \tau 2\pi r dx = 0$

Or $\tau = -(\partial p/\partial x)(r/2)$; Gives the variation of shear stress with respect to radius.

At the center, r=0, and shear stress is zero. At the pipe wall, r=R, the shear stress is maximum. $\tau_o = -(\partial p/\partial x)(R/2)$

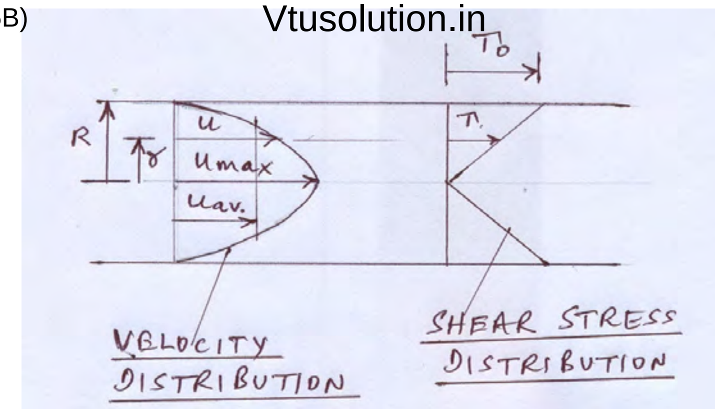


Fig. Velocity and Shear Stress Distributions

VELOCITY DISTRIBUTIONS

According to the Newton's Law of Viscosity,
$(\tau) = \mu(du/dy)$; But $y = (R - r)$ and $\partial y = -\partial r$
Therefore, $(\tau) = -\mu(\partial u/\partial r) = -(\partial p/\partial x)(r/2)$
Therefore, $(\partial u/\partial r) = (1/2\mu)(\partial p/\partial x)r$
Integrating w.r.t. (r) we get,
$u = (1/2\mu)(\partial p/\partial x)(r^2/2) + C$; where C=Constant
$(\partial p/\partial x)$ is independent of (r)
Now, let us find constant C
At r = R, that is at the wall, u=0.
$0 = (1/4\mu)(\partial p/\partial x)R^2 + C$
Or $C = -(1/4\mu)(\partial p/\partial x) R^2$
$u = (1/4\mu)(\partial p/\partial x)(r^2 - R^2)$ or
$u = (1/4\mu)(-\partial p/\partial x)(R^2 - r^2)$

Since μ, $(\partial p/\partial x)$ and R are constant, u varies with square of r.
Thus, for steady laminar flow through a circular pipe, the velocity variation across the section is parabolic in nature.
At r=R, u=0
At r=0,i.e.,at center of the pipe, $u = u_{max}$
$u_{max} = (R^2/4\mu)(-\partial p/\partial x)$
$u = (1/4\mu)(-\partial p/\partial x)(R^2 - r^2)$
$= (R^2/4\mu)(-\partial p/\partial x)[1 - (r/R)^2]$
But $(R^2/4\mu)(-\partial p/\partial x) = u_{max}$ Therefore, $u = u_{max}[1 - (r/R)^2]$
Gives the velocity distribution in the pipe, which is parabolic.

DISCHARGE AND AVERAGE VELOCITY

Discharge (q) across a section is found by integrating the discharge (dq) passing through an annular ring of width (dr) situated at a distance (r) from the center. Discharge through the annular ring=area of the ring X velocity at radius (r)

$dq=2\pi r\, dr\, [(1/4\mu)(-\partial p/\partial x)(R^2-r^2)]$

Total discharge,

$q = \int_0^R (R^2-r^2)\, r\, dr\, [(1/4\mu)(-\partial p/\partial x)\, 2\pi]$

$=(\pi/2\mu)(-\partial p/\partial x)\,[(R^2 r^2/2)-(r^4/4)]_0^R$

$=(\pi/2\mu)(-\partial p/\partial x)\,[(R^4/2)-(R^4/4)]$

$q= (\pi R^4/8\mu)(-\partial p/\partial x)$

Average velocity= u_{av}

$=(q/\text{area of pipe})=(q/\pi R^2)$

$u_{av}= (\pi R^4/8\mu)(-\partial p/\partial x)(1/\pi R^2)$

$= (R^2/8\mu)(-\partial p/\partial x)$; But $(R^2/4\mu)(-\partial p/\partial x)= u_{max}$. Therefore, $u_{av} =[u_{max}/2]$

Thus, in case of steady laminar flow through a circular pipe, average velocity is half of max. Velocity.

The radius at which the local velocity is equal to the average velocity is given by:

$u= u_{max}\,[1-(r/R)^2]= u_{av}= (u_{max}/2)$

$[r^2/R^2]= 0.5$ or $r=0.707R$

Thus the average velocity occurs at a radial distance of 0.707R from the center of the pipe.

Pressure drop over a given pipe length: We know that,

$u_{av} = (R^2/8\mu)(-\partial p/\partial x)$

$(-\partial p/\partial x)= [(8\mu u_{av})/R^2]$

$-dp=[(8\mu u_{av})/R^2]\, dx$ Integrating from 1 to 2

$\int -dp = \int [(8\mu u_{av})/R^2]\, dx$ At 1, $p=p_1$, $x=x_1$; At 2, $p=p_2$, $x=x_2$

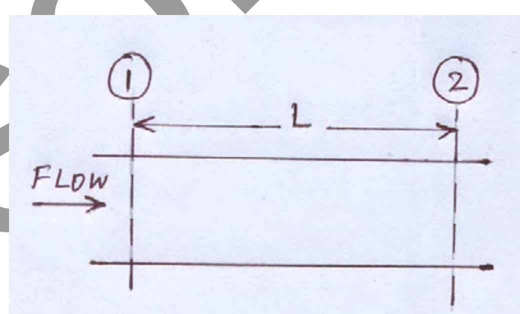


Fig. Pressure Drop over a Length of Pipe.

$(p_1 - p_2) = [(8\mu u_{av})/R^2]\,(x_2 - x_1)$

$L= (x_2 - x_1)$

$(p_1 - p_2) = [(8\mu u_{av})/R^2]\,(L)$ --------Eq.(1)

Since D=2R, $(p_1 - p_2) = (32\mu u_{av} L/D^2)$

Further, $u_{av} = [4q/ \pi D^2]$; Substituting for u_{av} , We get

$(p_1 – p_2) = [128\mu qL/(\pi D^4)]$ -----Eq.(2)

Equations.(1) or (2) are called Hagen-Poiseuille's equation for steady uniform laminar flow through a circular pipe.

To obtain H-P equation, we can also use

$q= (\pi R^4 /8\mu) (-\partial p/\partial x)$

$-dp = (8\mu q/ \pi R^4)dx = (128\mu q/ \pi D^4)dx$

Integrating between (1) and (2)

$(p_1 – p_2) = [128\mu qL/\pi D^4]$

But $(4q/ \pi D^2) = u_{av}$

Therefore,

$(p_1 – p_2) = [32\mu u_{av}L/D^2]$ - -----Eq.(3)

Another version of Hagen-Poiseuille's Equation

LAMINAR FLOW THROUGH A CIRCULAR PIPE

For a two- dimensional steady uniform laminar flow, the pressure gradient in the direction of flow is equal to the shear

LOSS OF HEAD

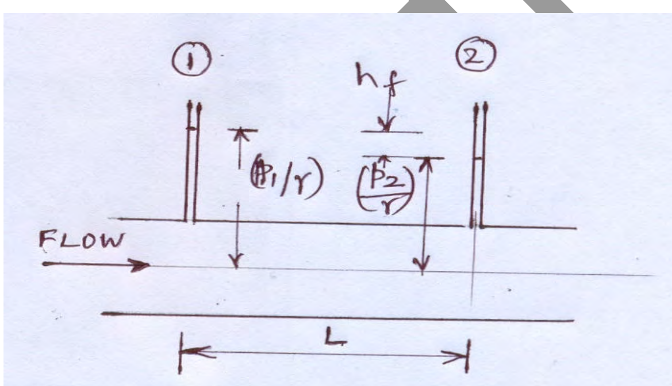


Fig. Loss of Head

Specific weight $(\gamma) = \rho g$

$(p_1 – p_2) = [32\mu u_{av}L/D^2]$

Loss of Head, $h_f = [(32\mu u_{av}L)/(\gamma D^2)]$

$= [(128\mu qL)/(\gamma\pi D^4)]$

FRICTION FACTOR (f)

The loss of head due to friction in pipe is given by Darcy- Weisbach equation
$h_f = [(4fLu_{av}^2)/(2gD)] = [(32\mu u_{av}L)/(\rho gD^2)]$
Simplifying, we get
$f = 16(\mu/\rho Du_{av}) = [16 / R_e]$; Therefore, the value of the friction factor for a steady laminar flow through a circular pipe, $f = [16 / R_e]$

POWER REQUIRED TO MAINTAIN THE FLOW

Power = Rate of doing work = (Force × Distance) /Time = Force × Velocity
Force = $(-\partial p/\partial x)AL$ where A= Area of Pipe and
L = Length of pipe.
Power (P)= $(-\partial p/\partial x)AUL$; where AU = q, the discharge, U= Average Flow Velocity.
$(-\partial p/\partial x) = (p_1 - p_2)/L$; Therefore,
Power (P) = $(p_1 - p_2)q$; But $(p_1 - p_2) = \rho gh_f$
Therefore, Power (P) = ρgqh_f

For laminar flow through an inclined pipe,

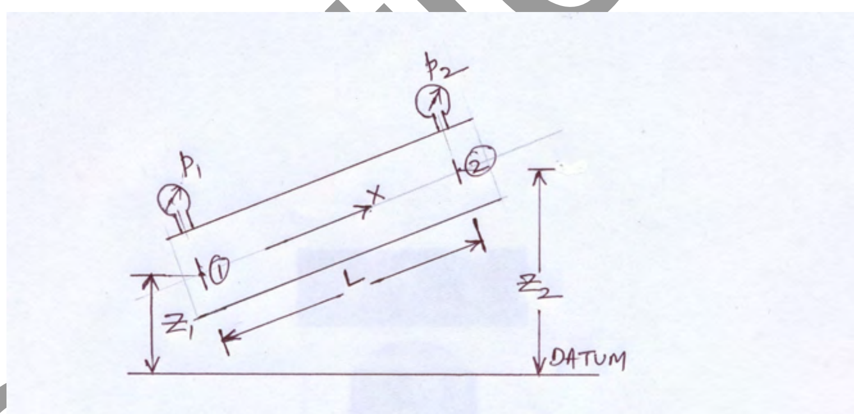


Fig. Laminar Flow through an Inclined Pipe,

We have $u=(1/4\mu)(-\partial p/\partial x)(R^2-r^2)$
Or $u=(1/4\mu)\,\rho g\,(-\partial h/\partial x)(R^2-r^2)$
Power (P) = $\rho gq\,(h_1 - h_2)$ where h_1 and h_2 are peizo-metric heads = $[(p/\rho g) + z]$

Problem-1

Calculate the loss of head in a pipe having a diameter of 15cm and a length of 2km. It carries oil of specific gravity 0.85 and viscosity of 6 Stokes at the rate of 30.48 lps (Assume laminar flow).

$(p_1 - p_2) = [128\mu qL/\pi D^4]$; $(p_1 - p_2) = \rho gqh_f$

$h_f = (p_1 - p_2)/\rho g = [128\mu qL/\rho g\pi D^4]$

$(\mu/\rho) = \nu = 6$ Stokes $= 6\times10^{-4}\ m^2/s$; (Stoke = $1cm^2$/sec.)

Substituting,

$h_f = (128\times6\times10^{-4}\times0.03048\times2000)/(\pi\times9.81\times0.15^4)$

Loss of head (h_f) = 300 m

Problem - 2

Calculate the power required to maintain a laminar flow of an oil of viscosity 10P through a pipe of 100mm diameter at the rate of 10 lps if the length of the pipe is 1 km. (assume laminar flow) (1 Ns/m^2 = 10 Poise)

$\Delta P = [128\mu qL/\pi D^4]$

$= (128\times1\times10\times10^{-3}\times10^3)/(\pi\times 0.1^4)$

$= 4.075\times10^6\ N/m^2$

Power $= \Delta P\times q = 4.075\times10^6\times10\times10^{-3} = 40.75kW$

Problem-3.

Oil of viscosity 8P and specific gravity 1.2 flows through a horizontal pipe 80mm in diameter. If the pressure drop in 100m length of the pipe is 1500 kN/m^2, determine,

1. Rate of flow of oil in lpm.
2. The maximum velocity
3. The total frictional drag over 100m length of pipe
4. The power required to maintain flow.
5. The velocity gradient at the pipe wall.
6. The velocity and shear stress at 10mm from the wall.

We have $(-\partial p/\partial x)$

$= 1500\times1000/100 = 15{,}000\ N/m^2/m$

Average velocity $= u_{av} = (R^2/8\mu)(-\partial p/\partial x) = (0.04^2/8\times0.8)(15{,}000) = 3.75m/s$.

Discharge (q) $= (\pi D^2/4)u_{av} = 0.01885\ m^3/s$

= 18.85 lpm = 1131 lpm. (D=0.08m)

Max. Velocity (u_{max}) $= 2\ u_{av} = 7.0$ m/s (at the center line)

Wall shear stress $(\tau_o) = -(\partial p/\partial x)(R/2) = 300N/m^2$

Total frictional drag for 100m-pipe length (F_D)

$= \tau_o\ \pi DL = 7540N = 7.54kN$ (L=100m)

Power required to maintain flow (P) $= F_D\times u_{av} = 28.275$ kW ;

Also, $P = q\Delta P = 0.01885\times1500 = 28.275$ kW.

Velocity gradient at pipe wall

$\tau_o = \mu(\partial u/\partial y)_{y=0}$

$(\partial u/\partial y)_{y=0} = (\tau_o/\mu) = 300/0.8 = 3.75/s$

($10P = 1\ N\text{-}m/s^2$)

Velocity and shear stress at 10mm from the wall (y=10mm, r=30mm)

At r=30mm, shear stress (τ) = τ_0 (30/40) = 225N/m^2

OR (τ) = ($-\partial p/\partial x$)(r/2) = 225N/m^2

Local velocity (at r=30mm)

u= u_{max} [1-$(r/R)^2$] = 3.28m/s.

Problem-4

Oil is transported from a tanker to the shore at the rate of 0.6m^3/s using a pipe of 32cm diameter for a distance of 20km. If the oil has the viscosity of 0.1Nm/s^2 and density of 900 kg/m^3, calculate the power necessary to maintain flow.

$(p_1 - p_2) = [128\mu qL/\pi D^4]$

Power (P) = $(p_1 - p_2)$ q

LAMINAR FLOW BETWEEN PARALLEL PLATES – BOTH PLATES FIXED

Plates are at distance (B) apart. Consider a fluid element as shown with sides (dx, dy, dz). The flow is a steady and uniform. There is no acceleration.

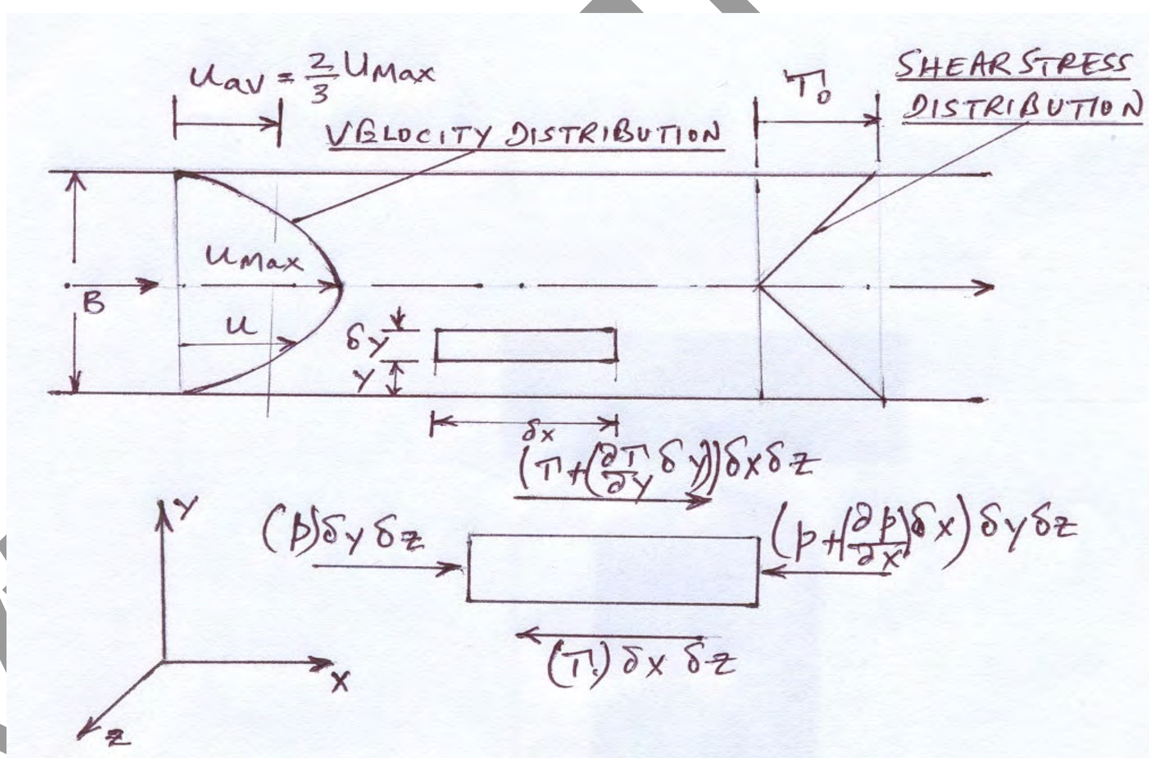


Fig. Laminar Flow through Parallel Plates

Sum of all forces in the direction of motion is zero.

Pressure forces + Shear forces =0

$[p.\,dydz - \{p + (\partial p/\partial x)dx\}dydz] + [\{\tau + (\partial \tau/\partial y)dy\}\,dxdz - \tau\,dxdz] = 0$;

Simplifying,

$-(\partial p/\partial x)dxdydz + (\partial \tau/\partial y)dxdydz = 0$; Dividing by dxdydz, The volume of the parallelepiped,

$$(\partial p/\partial x) = (\partial \tau/\partial y)$$

According to the Newton's law of viscosity,

$(\tau) = \mu(du/dy)$; Therefore, $(\partial p/\partial x) = \mu(\partial^2 u/\partial y^2)$

$(\partial^2 u/\partial y^2) = 1/\mu\ (\partial p/\partial x)$; Since $(\partial p/\partial x)$ is independent of (y), integrating the above equation, we get

$(\partial u/\partial y) = 1/\mu\ (\partial p/\partial x)y + C_1$; Integrating again,

$$u = 1/2\mu\ (\partial p/\partial x)y^2 + C_1 y + C_2$$

C_1 and C_2 are constants of integration.

At y=0, u=0; Therefore, $C_2 = 0$; At y=B (at the upper plate); u=0.

$0 = 1/2\mu\ (\partial p/\partial x)B^2 + C_1 B$ OR $C_1 = 1/2\mu\ (-\partial p/\partial x)B$; Substituting,

$u = 1/2\mu\ (-\partial p/\partial x)\ [By - y^2]$

This equation shows that the velocity distribution for steady laminar flow between fixed parallel plates is parabolic.

$(\partial p/\partial x)$ – Pressure decreases in the direction of flow and $[-(\partial p/\partial x)]$ is a positive quantity.

Max. Velocity occurs mid-way between the plates and can be obtained using y=(B/2).

$U_{max} = B^2/8\mu\ (-\partial p/\partial x)$

DISCHARGE AND AVERAGE VELOCITY

Consider an elemental strip of height (dy) situated at a distance (y) from the bottom plate as shown. Consider unit width normal to the plane of paper.

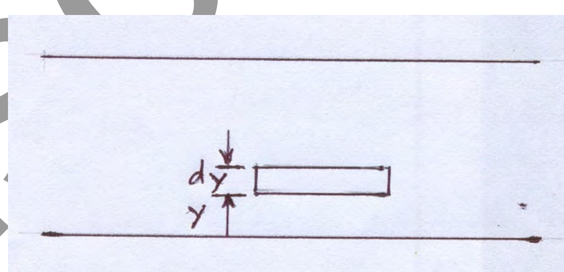


Fig. Discharge and Average Velocity

Velocity of fluid passing through the strip,

$u = 1/2\mu\ (-\partial p/\partial x)\ [By - y^2]$

Discharge through the strip per unit width =

dq = Area ´ Velocity

$= dy \times 1 \times 1/2\mu\ (-\partial p/\partial x)\ [By - y^2]$

Discharge (q) per unit width of plate

$q = {}_0\int^B [1/2\mu\ (-\partial p/\partial x)\ [By - y^2]dy$

Integrating, $q = 1/2\mu\ (-\partial p/\partial x)[B^3/2 - B^3/3]$

OR $q = (B^3/12\mu)\ (-\partial p/\partial x)$

Average velocity = u_{av} = (q/Area)

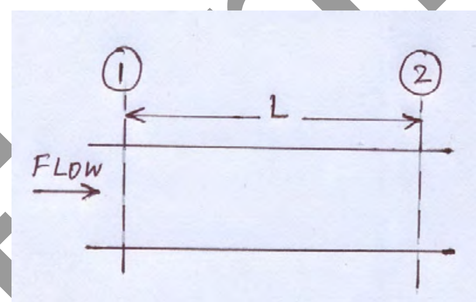
Area = $B \times 1$; $u_{av} = (B^2/12\mu)\ (-\partial p/\partial x)$

Since $U_{max} = B^2/8\mu\ (-\partial p/\partial x)$; $u_{av} = (2/3)\ U_{max}$

In the case of steady laminar flow between two fixed parallel plates, the average velocity is equal to (2/3) maximum velocity

PRESSURE DROP OVER A GIVEN LENGTH OF PLATES

In the case of steady laminar flow between two fixed parallel plates, the average velocity is
equal to



$u_{av} = (B^2/12\mu)\ (-\partial p/\partial x)$

$(-\partial p) = [12\mu\ u_{av}/B^2]\ \partial x$ OR

Integrating between sections (1) and (2)

${}_1\int^2(-\partial p) = {}_1\int^2 [12\mu\ u_{av}/B^2]\ \partial x$

At (1), $x=x_1$, $p=p_1$; At(2), $x=x_2$, $p=p_2$

$(p_1 - p_2) = [12\mu\ u_{av}/B^2]\ (x_2 - x_1)$ OR

$(p_1 - p_2) = [12\mu\ u_{av}/B^2]\ L$ where $L = (x_2 - x_1)$

LOSS OF HEAD AND SHEAR STRESS

Loss of Head: $h_f = [(p_1 - p_2)/\rho\ g] = [12\mu\ u_{av}/\rho g B^2]\ L$

Shear stress: According to the Newton's law, $(\tau) = \mu(du/dy)$

$(\tau) = \mu(d/dy)[1/2\mu\ (-\partial p/\partial x)\ \{By - y^2\}]$

OR $(\tau) = \mu[1/2\mu\ (-\partial p/\partial x)\ \{B - 2y\}]$

OR $(\tau) = (-\partial p/\partial x)\ (B/2 - y)$

Shows the variation of shear stress with distance (y) - $(\tau) = 0$ at $y = B/2$, mid-way between the plates.

Shear stress is maximum at the plates –

$(\tau) = (\tau_0)$ at $y = 0$ or B.

$(\tau_0) = (-\partial p/\partial x)\ (B/2)$

Laminar Flow through Inclined Plates:

Replace $(\partial p/\partial x)$ by $\rho g(\partial h/\partial x)$ where

$h = z + (p/\rho g)$

$u = 1/2\mu\ [-\partial\ (p + \rho g z)/\ \partial x)\ \{By - y^2\}]$

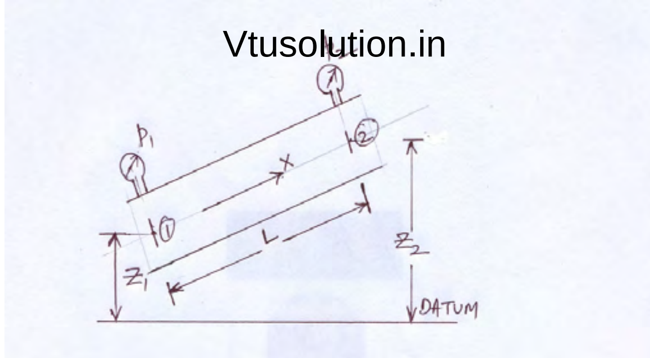


Fig. Laminar Flow through Inclined Plates

Problem-1

Oil of specific gravity 0.92 and dynamic viscosity 1.05 poise flows between two fixed parallel plates 12mm apart. If the mean velocity is 1.4m/s, calculate (a) maximum velocity (b) velocity and shear stress at a distance of 2mm from one of the plates and (c) loss of head over a distance of 25m.

For laminar flow between parallel plates, we have u_{av} = 1.4 m/s.

(a) $u_{max} = (3/2)\ u_{av} = 2.1$ m/s

(b) Velocity at 2mm from one of the plates:

$u = 1/2\mu\ (-\partial p/\partial x)\ [By - y^2]$; To get $(\partial p/\partial x)$

$u_{av} = (B^2/12\mu)\ (-\partial p/\partial x)$; $(-\partial p/\partial x) = (12\mu u_{av}/B^2)$

$= (12\times 1.05\times 10^{-1}\times 1.4)/(12\times 10^{-3})^2$

= 12,250 $N/m^2/m$; Substituting we get

u [at y=2mm] = 1.167m/s

(c) Loss of head in 25m length:

$h_f = [12\mu\ u_{av}/\rho g B^2]\ L =$

$[(12\times 0.105\times 1.4)/(0.92\times 1000\times 9.81)(1.2\times 10^{-2})^2]\ 25$

= 33.933m

Problem 2

Two parallel plates kept at 100mm apart have laminar flow of oil between them. The maximum velocity of flow is 1.5m/s. Calculate (a) Discharge per meter width (b) Shear stress at the plates (c) Pressure difference between two points 20m apart (d) Velocity gradient at the plates (e) Velocity at 20mm from the plate. Take viscosity of oil as 2.45 pa-s.

(a) Given u_{max} = 1.5m/s.; $u_{av} = 2/3\ u_{max}$ = 1m/s;

Discharge per unit width (q) = $(B\times 1\times u_{av})$ = 0.1m/s/m width

(b) Shear stress at the plate:

$(\tau_0) = (-\partial p/\partial x)\ (B/2)$;   Pa-s = $N\text{-}s/m^2$

$u_{av} = (B^2/12\mu)\ (-\partial p/\partial x)$

$(-\partial p/\partial x) = (12\mu u_{av})/B^2 = 2490\ N/m^2/m$

(τ_0) = 147 N/m^2

(c) Pressure difference between two points 20m apart:

$\Delta p = [12\mu\, u_{av}/B^2]\, L = 58{,}800\ N/m^2$

(d) Velocity gradient at the plates: $(\tau_0) = \mu(du/dy)_{y=0}$

$(du/dy)_{y=0} = (\tau_0)/\mu = 147/2.45 = 60/s$

(e) Velocity at 20mm from plates: u(at y= 20mm)

$= 1/2\mu\, (-\partial p/\partial x)\, [By - y^2] = 0.96\ m/s$

FLUID MECHANICS

FLOW PAST IMMERSED BODIES

Whenever a body is placed in a stream, forces are exerted on the body. Similarly, if the body is moving in a stationary fluid, force is exerted on the body.
Therefore, when there is a relative motion between the body and the fluid, force is exerted on the body.
Example: Wind forces on buildings, bridges etc., Force experienced by automobiles, aircraft, propeller etc.,

FORCE EXERTED BY FLOWING FLUID ON A STATIONARY BODY

Consider a stationary body placed in a stream of real fluid.
Let U = Free stream velocity.
Fluid will exert a Force F_R on the body.
The force is inclined at an angle to the direction of velocity.
The Force F_R can be resolved into TWO components – One in the direction of flow (F_D) and the other perpendicular to it (F_L).
$F_R = F_L + F_D$
Drag: The component of the total force (F_R) in the direction of motion is called as Drag (F_D). Drag is the force exerted by the fluid on the body in the direction of motion. Drag resists motion of the body or fluid.
Example: Wind resistance to a moving car, water resistance to torpedoes etc.,
Power is required to overcome drag and hence drag has to be reduced to a possible minimum.

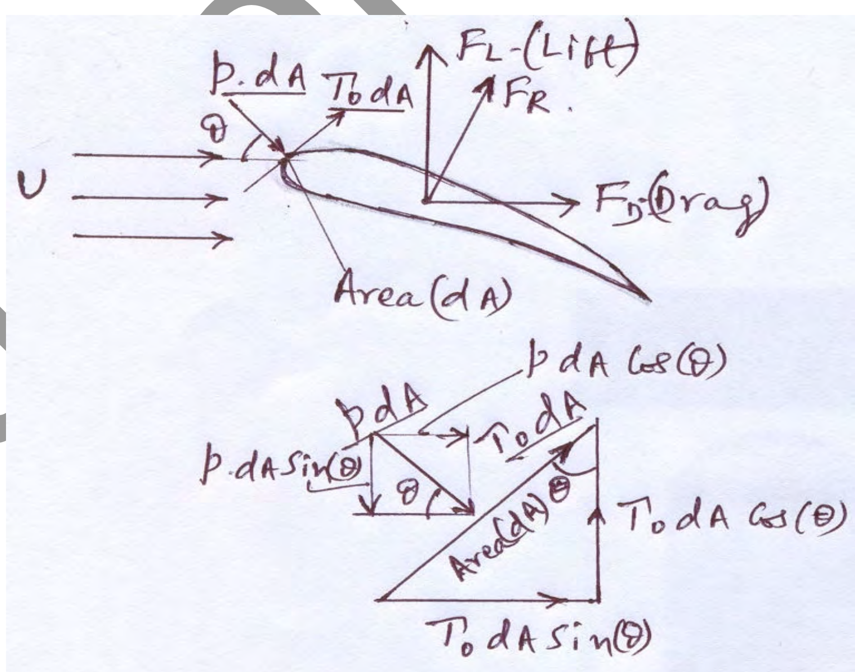


Fig. Forces Exerted by Fluid on Immersed Bodies

Lift: The component of the total force in the direction perpendicular to the direction of motion. Lift is the force exerted by the fluid normal to the direction of motion.
Lift is zero for symmetrical flow.
Lift = Weight (in the case of an airplane in cruise)

EXPRESSIONS FOR DRAG AND LIFT

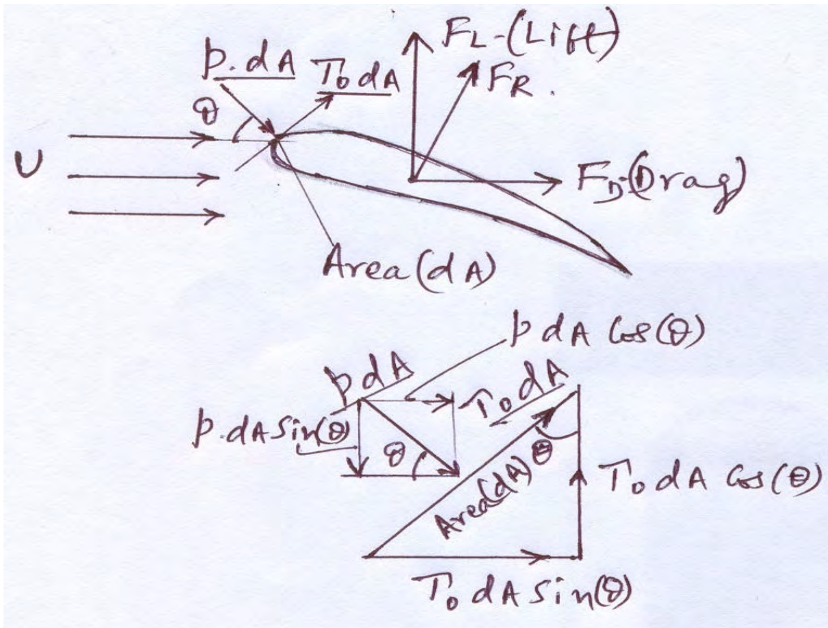


Fig. Expressions for Lift and Drag

Consider an elemental area (dA) on the surface of the body.

1. Pressure force (PdA) acts normal to the area dA.
2. Shear force (τ_0dA) acts along the tangent to dA
3. (θ) = Angle made by force pdA with horizontal.

dF_D = Drag force on the element

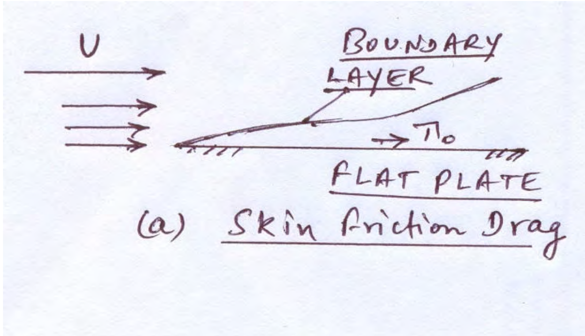
= (pdA)Cos (θ) + (τ_0dA) Sin (θ)

Therefore, Total drag on the body

= $F_D = \int dF_D = \int (pdA)\cos(\theta) + \int (\tau_0 dA)\sin(\theta)$ -----Equation (1)

Total drag (or Profile drag) = Pressure drag (or form drag) + Friction drag.

The quantity $\int$(pdA) Cos (θ) is called the pressure drag or form drag and depends upon the form or shape of the body as well as the location of the separation point.
The quantity $\int(\tau_0$dA) Sin (θ) is called as the friction drag or skin friction drag and depends upon the extent and character of the boundary layer.
The sum of the pressure drag and the friction drag is called as total drag or profile drag.



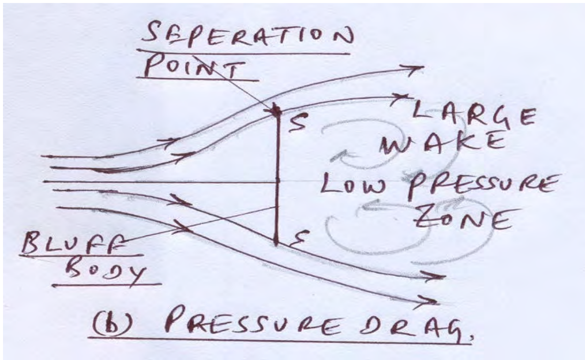


Fig. Skin Friction and Pressure Drag

In the case of a flat plate (Fig. a), $(\theta) = 90^0$. Hence, F_D is only the friction drag

If the plate is held normal to the plane (Fig. b), $(\theta) = 0^0$, Hence F_D is only the pressure drag

Lift = Force due to Pressure in the normal direction + Force due to shear in the normal direction.

$= -(pdA)\mathrm{Sin}(\theta) + (\tau_0 dA)\mathrm{Cos}(\theta)$ OR $F_L = -\int(pdA)\mathrm{Sin}(\theta) + \int(\tau_0 dA)\mathrm{Cos}(\theta)$

-----Equation (2)

Equations (1) and (2) require detailed information regarding pressure distributions and shear stress distributions to determine F_D and F_L on the body.

As a simple alternative, Drag and Lift Forces are expressed as

$F_D = C_D A\,(\rho U^2/2)$

$F_L = C_L A\,(\rho U^2/2)$

Where C_D and C_L are called Coefficient of Drag and Coefficient of Lift respectively,

ρ= Density of fluid, U = Velocity of body relative to fluid

A = Reference area or projected area of the body perpendicular to the direction of flow or it is the largest projected area in the in the case of submerged body.

$(\rho U^2/2)$ = Dynamic pressure.

GENERAL EQUATIONS FOR DRAG AND LIFT

Let force 'F' is exerted by fluid on the body.
$F = F(L, \rho, \mu, k, U, g)$ where L = Length, ρ = Density, μ = Viscosity, k = Bulk modulus of elasticity, U = Velocity and g = Acceleration due to gravity. From dimensional analysis, we get,

$F = \rho L^2 U^2 f(Re, Fr, M)$
Where Re = Reynolds Number = $(\rho UL/ m)$,
Fr = Froude Number = $(U/\sqrt{gL})$
M = Mach Number = $(U/\sqrt{k/\rho}) = (U/a)$; a = Sonic velocity
If the body is completely submerged, Fr is not important. If Mach number is relatively low (say, < 0.25), M can be neglected.
Then, $F = \rho L^2 U^2 f(Re)$ or
$F_D = C_D L^2 (\rho U^2/2) = C_D \times \text{Area} \times (\rho U^2/2)$
$F_L = C_L L^2 (\rho U^2/2) = C_L \times \text{Area} \times (\rho U^2/2)$
C_L and C_D are the coefficients of Lift and Drag respectively

TYPES OF DRAG

The type of drag experienced by the body depends upon the nature of fluid and the shape of the body:

1. Skin friction drag
2. Pressure drag
3. Profile drag
4. Wave drag
5. Induced drag

Skin Friction Drag: The part of the total drag that is due to the tangential shear stress (τ_0) acting on the surface of the body is called the skin friction drag. It is also called as friction drag or shear drag or viscous drag.

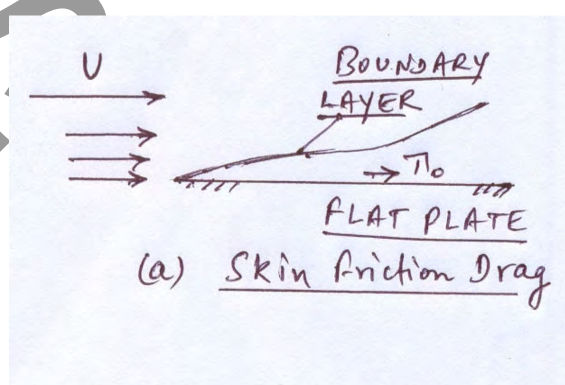


Fig. Skin Friction Drag

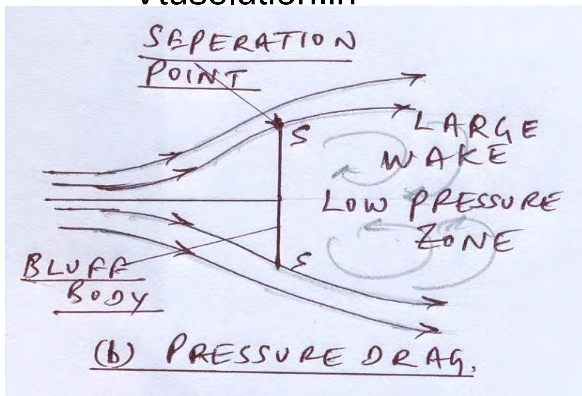


Fig. Pressure Drag

Pressure Drag: The part of the total drag that is due to pressure on the body is called as Pressure Drag. It is also called as Form Drag since it mainly depends on the shape or form of the body

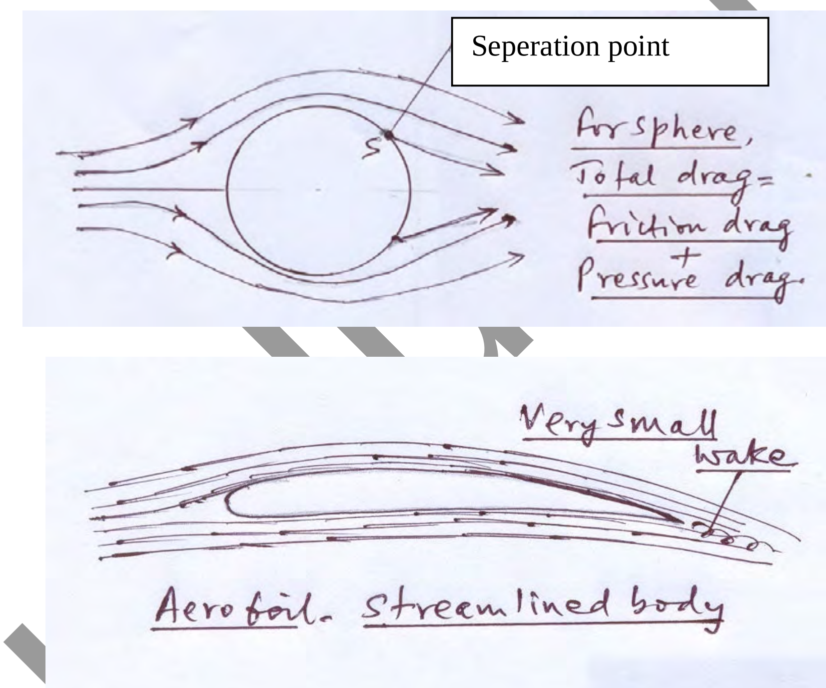


Fig. Flow over Bodies – Pressure Drag and Skin Friction Drag

For a streamlined body, pressure drag is small. Large part of drag is due to friction. Ex., Aerofoils, modern cars etc., - Streamlines match with the surface and there is very small wake behind the body.

For a bluff body, streamlines don't match with the surface. Flow separates and gives rise to large wake zone. Pressure drag is predominant compared to friction drag – Ex., Bus body.

Profile Drag or Total Drag is the sum of Pressure or Form drag and Skin Friction drag.

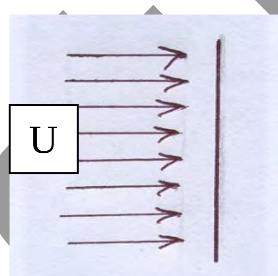
Wave Drag: When a body like ship moves through a fluid, waves are produced on the surface of the liquid. The drag caused due to these waves is called as wave drag. The wave drag is obtained by subtracting all other drags from the total drag measurements. The drag, which is caused by change in pressure due to a shock wave in supersonic flow, is also called as wave drag.

Induced Drag: When a body has a finite length (Ex., Wing of an airplane), the pattern of flow is affected due to the conditions of flow at the ends. The flow cannot be treated as two-dimensional, but has to be treated as three-dimensional flow. Due to this, body is subjected to additional drag. This drag, due to the three dimensional nature of flow and finite length of the body is called as Induced Drag.

Deformation Drag: If the body with a very small length (Ex., Sphere) moves at very low velocity through a fluid with high kinematics viscosity ($Re = (\rho UL/\mu)$ less than 0.1), the body experiences a resistance to its motion due to the wide spread deformation of fluid particles. This drag is known as Deformation Drag.

Problem –1.

A circular disc 3m in diameter is held normal to 26.4m/s wind velocity. What force is required to hold it at rest? Assume density of air = 1.2kg/m^3, and C_D = 1.1.



Force required to hold the disc = Drag = $F_D = C_D A(\rho U^2/2)$

$= 1.1\times(\pi\times3^2/4)\times(1.2\times26.4^2/2) = 3251.5$ N

Problem-2.

Calculate the power required to overcome the aerodynamic drag for the two cars both traveling at 90km/h using the following data.

Car (A) – C_D = 0.8, A (frontal) = 2m^2,

Car (B) – C_D = 0.4, A (frontal) = 1.8m^2. Take ρ = 1.164 kg/m^3.

For Car (A)

Power = Force × Velocity = $F_D \times U$

U = 90km/hr = 25m/s.

Power = $C_D\, A(\rho U^2/2) \times U$

$=0.8\times 2\times(1.164\times 25^2/2)\times 25$ = 14550W = 14.55kW

Similarly for Car (B),

Power = $0.4\times 1.8\times(1.164\times 25^2/2)\times 25$ = 6.55kW

Problem-3.

Experiments were conducted in a wind tunnel with a wind speed of 50km/h. on a flat plate of size 2m long and 1m wide. The plate is kept at such an angle that the co-efficient of lift and drag are 0.75 and 0.15 respectively. Determine (a) Lift force (b) Drag force (c) Resultant force (d) Power required to maintain flow.

Take ρ = 1.2 kg/m^3.

Given: A = 2m^2; C_L = 0.75; C_D = 0.15; ρ = 1.2 kg/m^3; U= 13.89m/s

Drag force = $F_D = C_D A(\rho U^2/2)$ = 34.72N

Lift force = $F_L = C_L A(\rho U^2/2)$ = 173.6N

Resultant force = $F_R = (F_D^2 + F_L^2)^{1/2}$ = 177.03 N

Power = $F_D \times U$ = 482.26 kW

BOUNDARY LAYER CONCEPT

Ideal fluid theory assumes that fluid is ideal, zero viscosity and constant density. Results obtained don't match with experiments.

With ideal fluid, there is no drag force. However, in practice, drag force exists.

In practice, fluids adhere to the boundary.

At wall, fluid velocity = wall velocity- this is called No Slip Condition.

The velocity of the fluid is zero at the wall and goes on increasing as we go away from the wall if the wall is stationary.

This variation in velocity near the wall gives rise to shear stresses resulting in resistance to motion of bodies.

CONCEPT OF BOUNDARY LAYER

L.Prandtl developed Boundary Layer Theory

Boundary layer theory explains the drag force experienced by the body. The fluid in the vicinity of the surface of the body may be divided into two regions – (1) Boundary layer and (2) Potential flow or Ir-rotational flow region.

BOUNDARY LAYER

Boundary layer is a very thin layer of fluid in the immediate vicinity of the wall (or boundary). When a real fluid flows past a solid boundary, there develops a thin layer very close to the boundary in which the velocity rapidly increases from zero at the boundary (due to no slip condition) to the nearly uniform velocity in the free stream. This region is called Boundary layer. In this region, the effect of viscosity is predominant due to the high values of (du/dy) and most of the energy is lost in this zone due to viscous shear.

The layer of fluid which has its velocity affected by the boundary shear is called as Boundary Layer. A thin layer of fluid in the vicinity of the boundary, whose velocity is affected due to viscous shear, is called as the Boundary layer

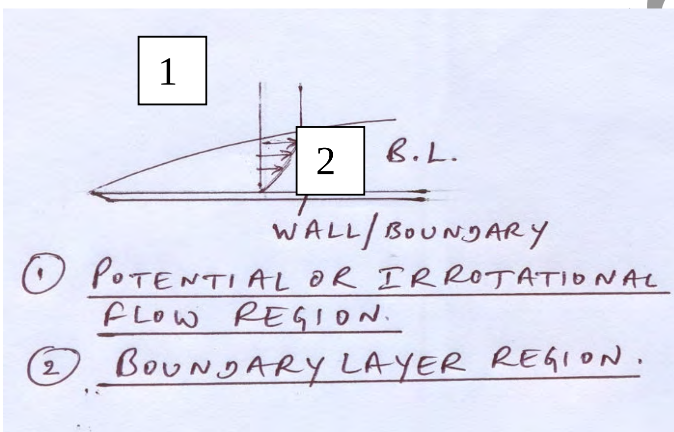


Fig. Potential and Boundary layer Flow Regions

POTENTIAL FLOW OR IRROTATIONAL FLOW REGION

The portion of the fluid outside the boundary layer where viscous effects are negligible is called potential flow or ir-rotational flow region. The flow in this region can be treated as Ideal Fluid Flow.

BOUNDARY LAYER ALONG A FLAT PLATE AND IT'S CHARECTERISTICS

Consider a steady, uniform stream of fluid moving with velocity (U) on a flat plate. Let U = Free stream velocity or Ambient velocity. At the leading edge, the thickness of the boundary layer is zero. In the down stream direction, the thickness of the boundary layer (δ) goes on increasing as shown.

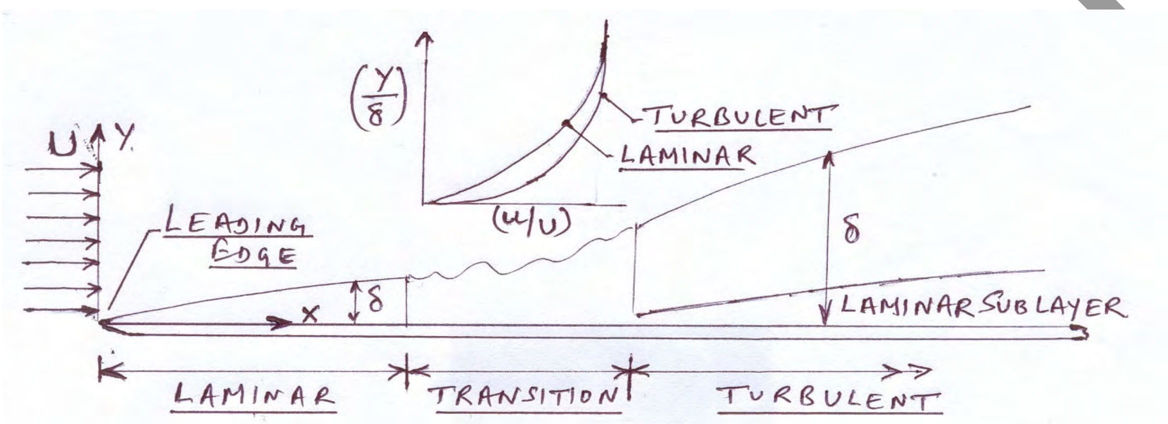


Fig. Boundary Layer Growth along a Flat Plate

Up to a certain length along the plate from the leading edge, boundary layer thickness increases and the boundary layer exhibits the characteristics of a laminar flow irrespective of whether the incoming flow is laminar or turbulent. – This is known as laminar boundary layer.
The thickness of the laminar boundary layer (δ) is given by (δ) = y (at (u/U) = 0.99) where u = local velocity.

The thickness of the laminar boundary layer is given by $(\delta) = [5x/(R_{ex})^{0.5}]$
Where R_{ex} = Reynolds number based on distance from the leading edge (x)
$R_{ex} = (Ux/\nu)$; Therefore, $(\delta) = 5(x\nu/U)^{0.5}$
In the laminar boundary layer, the Newton's law of viscosity $(\tau) = \mu\,(du/dy)$ is valid and the velocity distribution is parabolic in nature.

Beyond some distance from the leading edge, the laminar boundary layer becomes unstable and the flow in the boundary layer exhibits the characteristics between laminar and turbulent flows. This region is known as the transition region.
After this region, the thickness of the boundary layer increases rapidly and the flow in the boundary layer exhibits the characteristics of the turbulent flow
This region is known as the turbulent boundary layer. In the turbulent boundary layer, the boundary layer thickness is given by
$(\delta) = [0.377x/(R_{ex})^{0.2}]$
The velocity profile is logarithmic in the turbulent boundary layer.

The change from laminar to turbulent boundary layer depends mainly on $R_{ex} = (Ux/\nu)$. The value of critical Reynolds number varies from 3×10^5 to 6×10^5 (for a flat plate).

For all practical purposes, we can take $R_{cr} = 5\times10^5$

x= Distance from the leading edge.

If the plate is smooth, the turbulent boundary layer consists of a thin layer adjacent to the boundary in which the flow is laminar. This thin layer is known as the laminar sub-layer.

The thickness of the laminar sub-layer (δ') is given by

$(\delta') = [11.6\nu/ (\tau_0/\rho)^{0.5}] = [11.6\nu/U_*]$ where

$U_* = (\tau_0/\rho)^{0.5}$ is called the shear velocity.

The laminar sub-layer, although very thin is an important factor in deciding whether a surface is hydro-dynamically smooth or rough surface.

FACTORS AFFECTING THE GROWTH OF BOUNDARY LAYERS

1. Distance (x) from the leading edge – Boundary layer thickness varies directly with the distance (x). More the distance (x), more is the thickness of the boundary layer.
2. Free stream velocity – Boundary layer thickness varies inversely as free stream velocity.
3. Viscosity of the fluid – Boundary layer thickness varies directly as viscosity.
4. Density of the fluid – Boundary layer thickness varies inversely as density.

THICKNESSES OF THE BOUNDARY LAYER

Boundary layer thickness - It is the distance from the boundary in which the local velocity reaches 99% of the main stream velocity and is denoted by (δ).

y = (δ) when u=0.99U

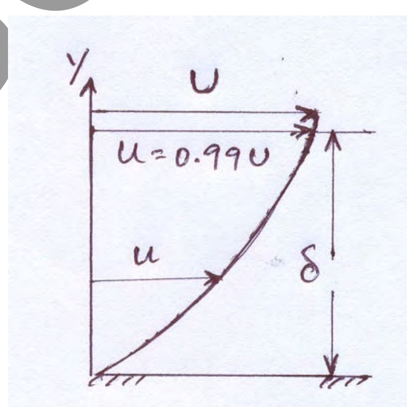


Fig. Boundary Layer Thickness

Displacement Thickness (δ*): It is defined as the distance perpendicular to the boundary by which the boundary will have to be displaced outward so that the actual discharge would be same as that of the ideal fluid past the displaced boundary. It is also defined as the distance measured perpendicular from the actual boundary such that the mass flux through this distance is equal to the deficit of mass flux due to boundary layer formation.

Deficit of mass flow (discharge) = (b.dy)(U-u)ρ

Total deficit of mass flow:

$\int_0^{\infty} \rho(b.dy)(U-u) = \rho b\delta^* U$

$\delta^* = \int_0^{\delta} (1- u/U)dy$

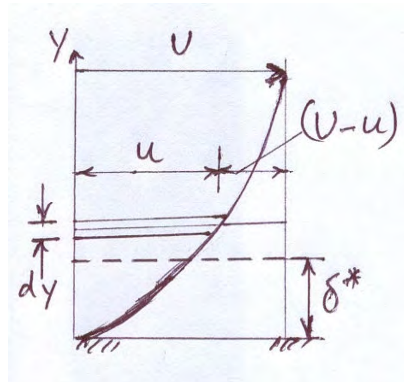


Fig. Displacement Thickness

Momentum thickness (θ): It is defined as the distance measured perpendicular from the actual boundary such that the momentum flux through this distance is equal to the deficit of the momentum flux due to the boundary layer formation.

Momentum deficit = ρ(b.dy)(U-u)u

Total momentum deficit = Moment through thickness (θ)

$\int_0^{\infty} \rho\ (b.dy)(U-u)u = \rho b\theta U^2$

$(\theta) = \int_0^{\delta} (1- u/U)(u/U)dy$

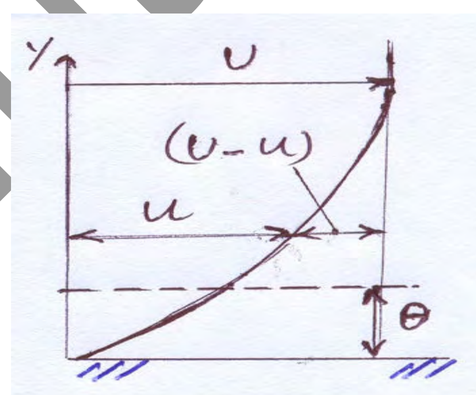


Fig. Momentum Thickness

Energy Thickness (δ)**: It is the distance perpendicular to the boundary by which the boundary has to be displaced to compensate for the reduction in the kinetic energy of the fluid caused due to the formation of the boundary layer. Energy thickness is also defined as the distance measured perpendicular from the actual boundary such that the kinetic energy flux through this distance is equal to the deficit of kinetic energy due to the boundary layer formation.

$\int_0^{\infty} 0.5\rho\ (b.dy)(U^2-u^2)u = b\ \delta^{**}U\ (\rho U^2/2)$

$(\delta^{**}) = \int_0^{\delta} \{1- (u/U)^2\}(u/U)dy$

Shape Factor (H) : It is defined as the ratio of the displacement thickness to the momentum thickness. $H = (\delta^*/\theta)$

LAMINAR BOUNDARY LAYER

Boundary layer thickness (δ)

$(\delta/x) = (5/(R_{ex})^{0.5})$ where $R_{ex} = (Ux/\nu)$

$\delta = 5(x\nu/U)^{0.5}$

Local co-efficient of drag (C_{fl}) is defined as the ratio of wall shear stress (τ_0) to the quantity $½(\rho U^2)$

$(C_{fl}) = [\tau_0 / ½(\rho U^2)]$

$= [0.664/(R_{ex})^{0.5}]$

Average Drag Co-efficient (C_{fa}) is defined as the ratio of the total drag force (F_D) to the quantity $(½(\rho AU^2)$. 'A' is a reference area.

$(C_{fa}) = [F_D/ ½(\rho AU^2)]$

$= 1.328/(R_{eL})^{0.5}$; Where $R_{eL} = (\rho UL/\mu)$

SOME VELOCITY PROFILES

For Laminar Flow:

$(u/U) = 2\eta - \eta^2$ where $\eta = (y/d)$

$= (3/2)\,\eta - ½\,\eta^3$

$= 2\eta - 2\eta^3 + \eta^4$

For Turbulent Flow:

$(u/U) = (y/d)^{1/n}$ n= 5 to 10

Problem-1

If the velocity distribution in the boundary layer over a flat plate is given by

$(u/U) = 2\eta - \eta^2$ where $\eta = (y/d)$

Find the displacement thickness, momentum thickness and the shape factor.

$(u/U) = 2\eta - \eta^2$

Displacement thickness: $\delta^* = {}_0\int^{\delta} (1- u/U)dy$

Multiply and divide by (δ) we get,

$\delta^* = \delta\ {}_0\int^1 (1- u/U)\, d(y/d)$

Let $F(\eta) = (u/U) = 2\eta - \eta^2$ Also, $\eta = (y/d)$

When y=0, η=0; y= d, η=1.

$\delta^* = \delta\ {}_0\int^1 (1- 2\eta + \eta^2)d(\eta)$; Therefore,

$\delta^* = \delta\,[\eta-\eta^2 + \eta^3/3]_0^1$

$\delta^* = (\delta/3)$

Momentum Thickness:

$(\theta) = {}_0\int^{\delta} (1- u/U)(u/U)dy$ Rearranging,

$(\theta) = \delta\, {}_0\int^{1} \{1- F(\eta)\} F(\eta)\, d(\eta)$

Substituting for $F(\eta) = 2\eta - \eta^2$;

$(\theta) = \delta\, {}_0\int^{1}\{1- 2\eta + \eta^2\} (2\eta - \eta^2)\, d(\eta)$

Integrating we get,

Momentum thickness $(\theta) = (2/15)\, \delta$

Shape Factor $(H) = (\delta^*/\theta) = (\delta/3)/(2\delta/15) = 2.5$

Problem-2

A flat plate of 1.2m wide and 1m long is held in air flow of velocity 5m/s parallel to the flow. Find the boundary layer thickness at the end of the plate and the drag on the plate.

Average co-efficient of drag is given by

$C_{fa}= (1.328/\sqrt{R_{el}})$. Assume ρ = 1.226kg/m3 and

$\mu = 0.184 \times 10^{-4}$ Pa-s

Find out the Reynolds number at the end of the plate to check whether the boundary layer is laminar.

$R_{el} = (\rho UL/\mu) = (1.226 \times 5 \times 1)/(0.184 \times 10^{-4})$

$= 3.332 \times 10^5$

Since R_{el} is less than 5×10^5 , the boundary layer on the flat plate is completely laminar.

Boundary layer thickness$(\delta) = x(5/(R_{ex})^{0.5})$ where $R_{ex} = (Ux/\nu)$

(δ) at x=L, $= (5L/\sqrt{R_{el}}) = [(5\times 1)/ (3.332 \times 10^5)^{0.5}] = 0.866 \times 10^{-2}$ m

$C_{fa}= (1.328/\sqrt{R_{el}}) = (1.328/\sqrt{(3.332 \times 10^5}\ )$

$= 0.0023$

F_D = Drag on both sides of flat plate =

$= [(C_{fa} \times \text{Area}\times(\rho U^2/2)] \times 2$

$= [0.0023 \times 1.2\times(1.226 \times 5^2 / 2)] \times 2 = 0.0846$N