KINEMATICS OF MACHINES

GAJANANA PUBLICATIONS

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MODULE – 1

INTRODUCTION :

The study of a mechanism involves its analysis as well as synthesis.

Analysis is the study of motions and forces concerning different parts of an existing mechanism. Whereas **Synthesis** involves the design of its different parts.

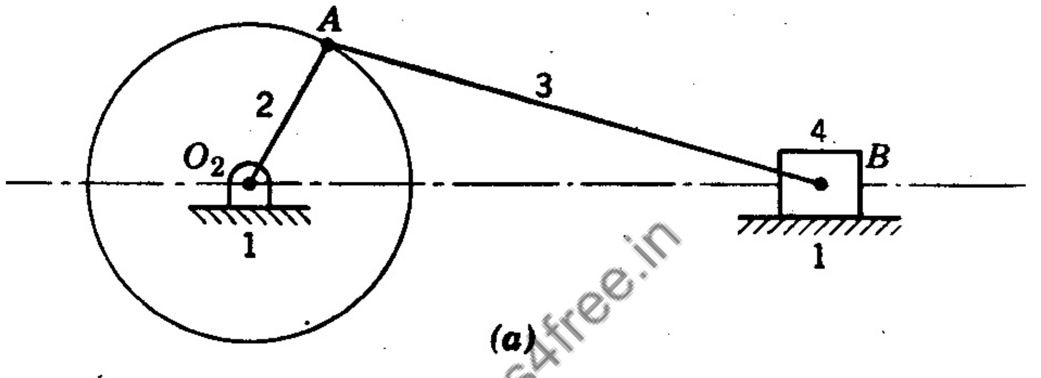
Mechanics: It is that branch of scientific analysis which deals with motion, time and force.

Kinematics is the study of motion, without considering the forces which produce that motion. Kinematics of machines deals with the study of the relative motion of machine parts. It involves the study of position, displacement, velocity and acceleration of machine parts.

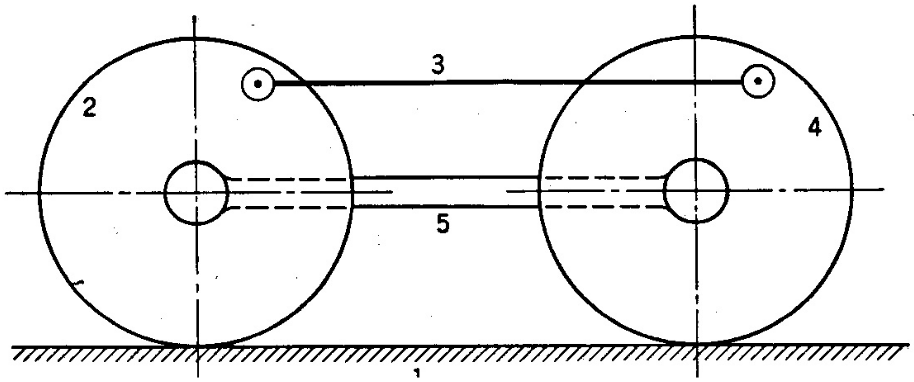
Dynamics of machines involves the study of forces acting on the machine parts and the motions resulting from these forces.

Plane motion: A body has plane motion, if all its points move in planes which are parallel to some reference plane. A body with plane motion will have only three degrees of freedom. i.e., linear along two axes parallel to the reference plane and rotational/angular about the axis perpendicular to the reference plane. (eg. linear along X and Z and rotational about Y.)The reference plane is called plane of motion. Plane motion can be of three types. 1) Translation 2) rotation and 3) combination of translation and rotation.

Translation: A body has translation if it moves so that all straight lines in the body move to parallel positions. Rectilinear translation is a motion wherein all points of the body move in straight lie paths.
Eg. The slider in slider crank mechanism has rectilinear translation. (link 4 in fig.1.1)



Translation, in which points in a body move along curved paths, is called curvilinear translation. The tie rod connecting the wheels of a steam locomotive has curvilinear translation. (link 3 in fig.1.2)



Rotation: In rotation, all points in a body remain at fixed distances from a line which is perpendicular to the plane of rotation. This line is the axis of rotation and points in the body describe circular paths about it. (Eg. link 2 in Fig.1.1 and links 2 & 4 in Fig.1.2)

Translation and rotation: It is the combination of both translation and rotation which is exhibited by many machine parts. (Eg. link 3 in Fig.1.1)

Link or element: It is the name given to any body which has motion relative to another. All materials have some elasticity. A rigid link is one, whose deformations are so small that they can be neglected in determining the motion parameters of the link.
A link or element need not to be rigid body, but it must be a resistant body. A body is said to be a resistant body if it is capable of transmitting the required forces with negligible deformation. Thus a link should have the following two characteristics:

1. It should have relative motion.
2. It must be resistant body.

TYPES OF LINKS:

In order to transmit motion, driver and the follower may be connected by the following three types of links:

(1) Rigid link: A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

(2) Flexible link: A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. Eg: belts, ropes, chains and wires are flexible links and transmit tensile forces only.

(3) Fluid link: A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only. Eg: hydraulic presses, hydraulic jacks and fluid brakes.

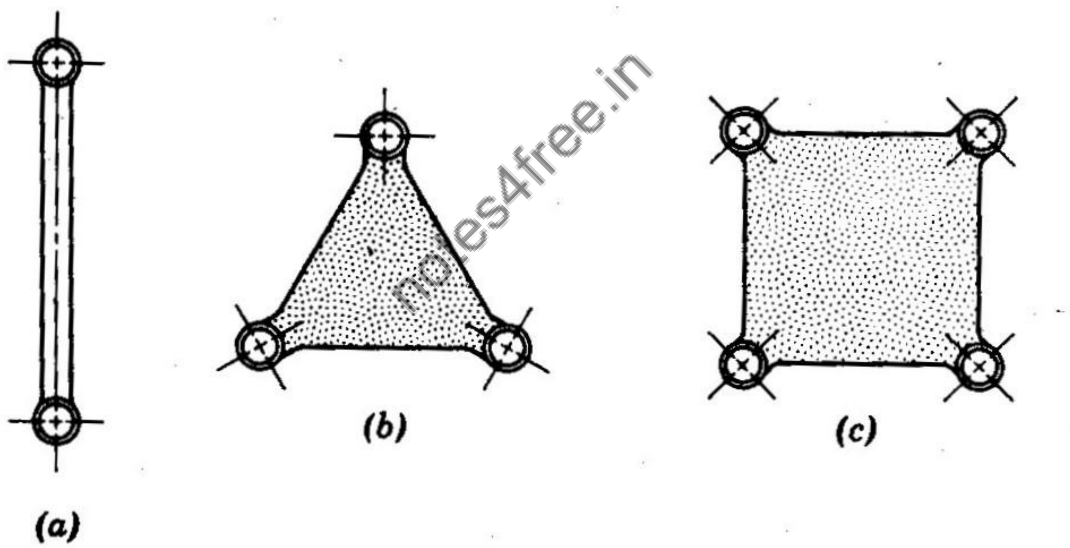


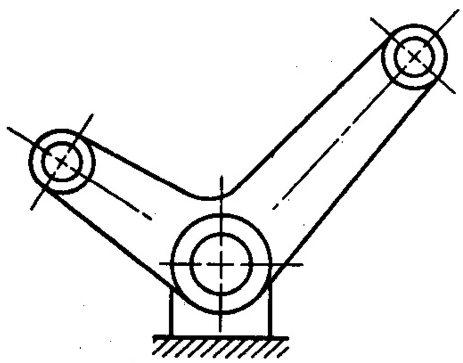
Fig.1.3

Binary link: Link which is connected to other links at two points. (Fig.1.3 a)

Ternary link: Link which is connected to other links at three points. (Fig.1.3 b)

Quaternary link: Link which is connected to other links at four points. (Fig1.3 c)

Pairing elements: the geometrical forms by which two members of a mechanism are joined together, so that the relative motion between these two is consistent are known as pairing elements and the pair so formed is called kinematic pair. Each individual link of a mechanism forms a pairing element.



(a)

Fig.1.4 Kinematic pair

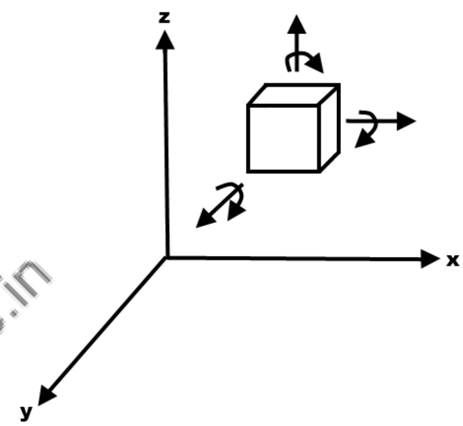


Fig.1.5

Degrees of freedom (DOF): It is the number of independent coordinates required to describe the position of a body in space. A free body in space (fig 1.5) can have six degrees of freedom. I.e., linear positions along x, y and z axes and rotational/angular positions with respect to x, y and z axes.

In a kinematic pair, depending on the constraints imposed on the motion, the links may loose some of the six degrees of freedom.

Each part of a machine, which moves relative to some other part, is known as a **kinematic link** (or simply link) or **element**. A link may consist of several parts, which are rigidly fastened together, so that they do not move relative to one another.

Mechanism:

A mechanism is a combination of rigid or resistant bodies so formed and connected that they move upon each other with definite relative motion. such as the crank-connecting rod mechanism of the I.C. engines, steering mechanisms of automobiles....... etc.

Machine:

machine is a device which receives energy and transforms it into some useful work. A machine consists of a number of parts or bodies with successfully constrained motion which is used to transmit or transform motion to do some useful work.

A machine is a mechanism or collection of mechanisms which transmit force from the source of power to the resistance to be overcome.

Structure:

It is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action.

Eg: railway bridge, a roof truss, machine frames etc.

Difference between a Machine and Structure:

MACHINE	STRUCTURE
1. The parts of a machine moves relative to one another.	1. The members of a structure do not move relative to one another.
2. A machine transforms the available energy into some useful work.	2. In a structure no energy is transformed into useful work.
3. The link of a machine may transmit both power and motion. Eg: Lathe, shaper, steam engine etc.	3. The members of a structure transmit forces only. Eg: Railway bridges, roof trusses, machine frame.

Difference between a Machine and Mechanism

MACHINE	MECHANISM
1. Machine modifies mechanical work.	1. Mechanism transmits and modifies motion.
2. A machine is a practical development of any mechanism.	2. A mechanism is a part of a machine.
3. A machine may have number of mechanisms for transmitting mechanical work or power. Eg: Lathe, Shaper, Steam engine etc.	3. A mechanism is the skeleton outline of the machine to produce motion between various links. Eg: Clock work, type-writer, an indicator to draw P.V diagrams of an engine etc.

Rigid Body: is that body whose changes in shape are negligible compared with its overall dimensions or with the changes in position of the body as a whole, such as rigid link, rigid disc.....etc.

Kinematic pair:

When two elements or links are connected together in such a way that their relative motion is constrained, form a kinematic pair. Therefore, in order to compel a body to move in a definite path, it must be paired with another. If the constraint is not complete (not definite path) the pair is termed as incomplete or unsuccessful.

Kinematic Pair

The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (*i.e.* in a definite direction), the pair is known as ***kinematic pair***.

Types of kinematic pairs:

(i) Based on nature of contact between elements:

(a) Lower pair. If the joint by which two members are connected has surface contact, the pair is known as lower pair. Eg. pin joints, shaft rotating in bush, slider in slider crank mechanism.

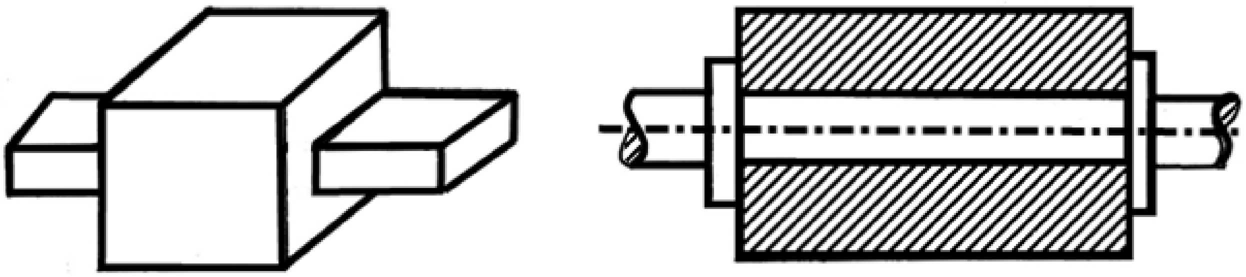


Fig.1.6 Lower pairs

(b) Higher pair. If the contact between the pairing elements takes place at a point or along a line, such as in a ball bearing or between two gear teeth in contact, it is known as a higher pair.

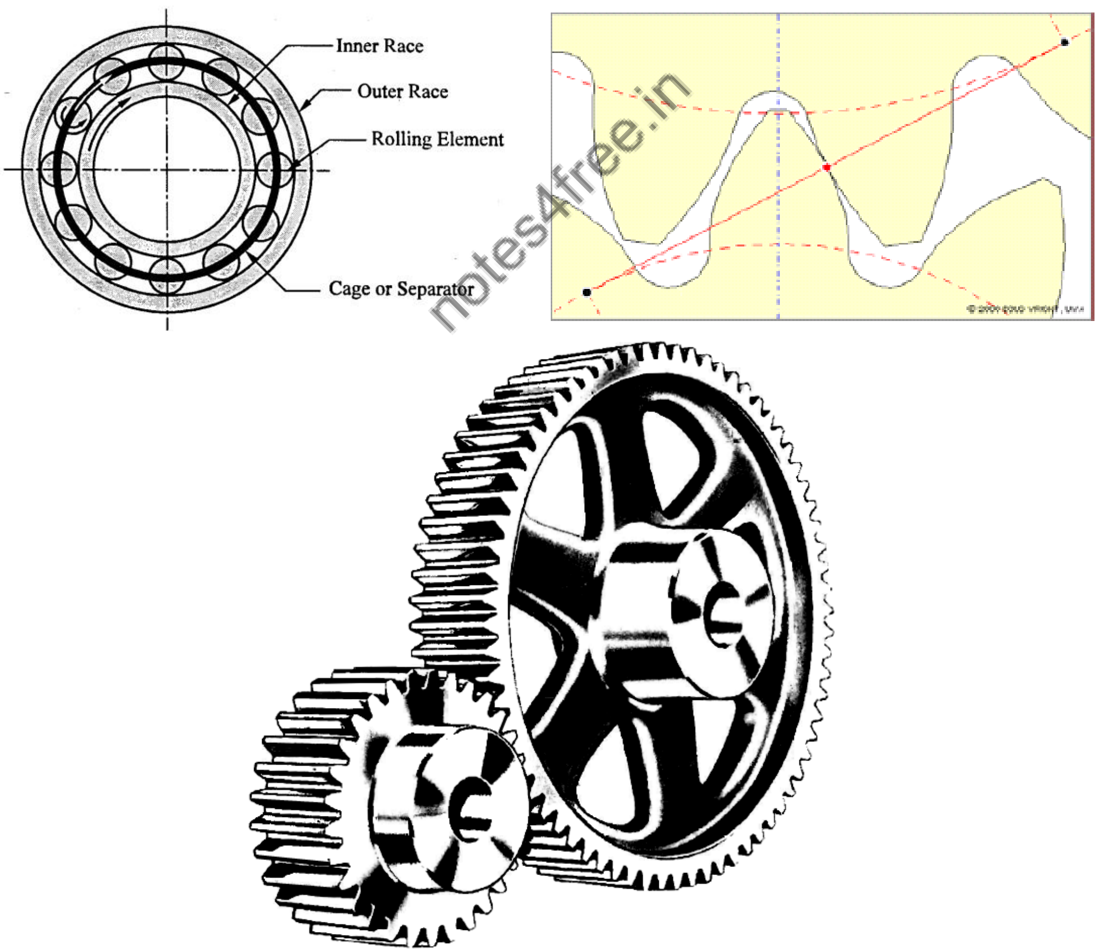


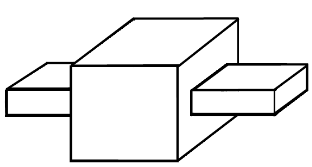
Fig.1.7 Higher pairs

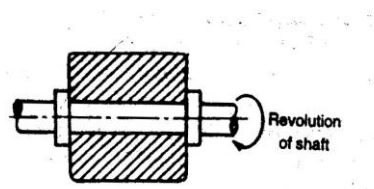
(ii) Based on relative motion between pairing elements:

(a) Sliding pair : Sliding pair is constituted by two elements so connected that one is constrained to have a sliding motion relative to the other. DOF = 1
Eg: rectangular rod in a rectangular hole, the piston and cylinder of an engine, the cross-head and guides of a steam engine, the ram and its guides in a shaper, the tailstock on the lathe bed etc.

(b) Turning pair (revolute pair). When connections of the two elements are such that only a constrained motion of rotation of one element with respect to the other is possible, the pair constitutes a turning pair. DOF = 1
Eg: circular shaft revolving inside a bearing, a shaft with a collar at both ends revolving in a circular hole, cycle wheels revolving over their axles etc.

(c) Cylindrical pair. If the relative motion between the pairing elements is the combination of turning and sliding, then it is called as cylindrical pair. DOF = 2





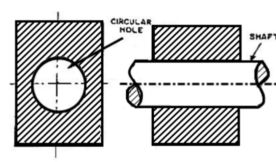


Fig.1.8 Sliding pair Fig.1.9 Turning pair Fig.1.10 Cylindrical pair

(d) Rolling pair. When the pairing elements have rolling contact, the pair formed is called rolling pair. Eg. Bearings, Belt and pulley. DOF = 1
Eg: rolling wheel on a flat surface, ball and roller bearings etc.

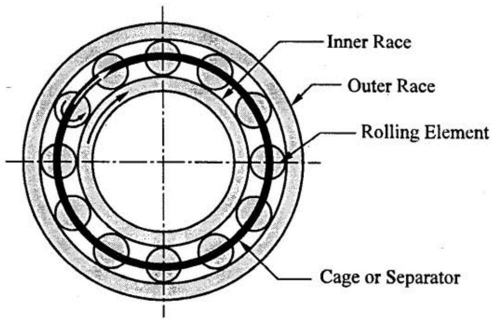


Fig.1.11 (a) Ball bearing

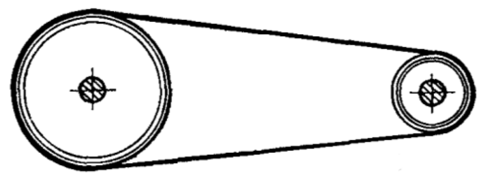


Fig.1.11(b) Belt and pulley

(e) Spherical pair. A spherical pair will have surface contact and three degrees of freedom. Eg. Ball and socket joint. DOF = 3
Eg: ball and socket joint, a mirror attachment of vehicles etc.

Picture of ball and socket joint.

(f) Helical pair or screw pair. When the nature of contact between the elements of a pair is such that one

element can turn about the other by screw threads, it is known as screw pair. Eg. Nut and bolt. DOF = 1
Eg: lead screw and the nut of a lathe, a bolt with a nut, a screw with the nut of a jack.

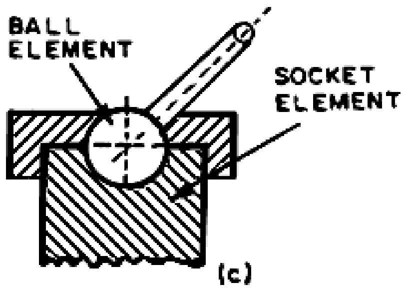


Fig.1.12 Ball and socket joint

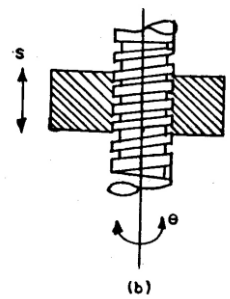


Fig.1.13 Screw pair

(iii) Based on the nature of mechanical constraint.

(a) Closed pair. Elements of pairs held together mechanically due to their geometry constitute a closed pair. They are also called form-closed or self-closed pair. All the lower pairs and some of the higher pairs are closed pairs. A cam and follower pair(higher pair) and screw pair (lower pair) belong to the closed pair.

(b) Unclosed or force closed pair. Elements of pairs held together by the action of external forces constitute unclosed or force closed pair .Eg. Cam and follower.

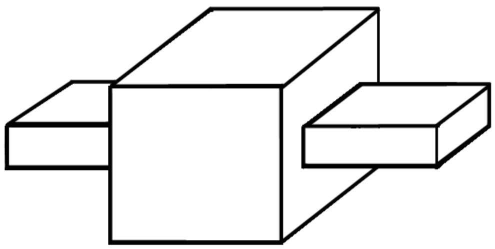


Fig.1.14 Closed pair

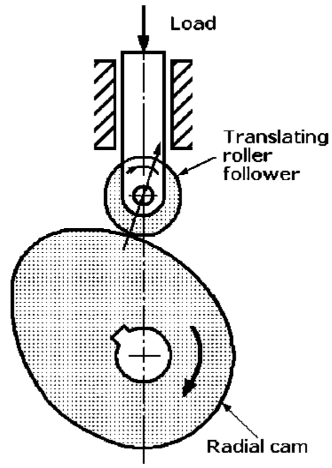


Fig. 1.15 Force closed pair (cam & follower)

Constrained motion: In a kinematic pair, if one element has got only one definite motion relative to the other, then the motion is called constrained motion.

(a) Completely constrained motion. If the constrained motion is achieved by the pairing elements themselves, then it is called completely constrained motion. When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is called completely constrained motion.

Eg: piston and cylinder (in a steam engine) form a pair and motion of the piston is limited to a definite direction (i.e., it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank.

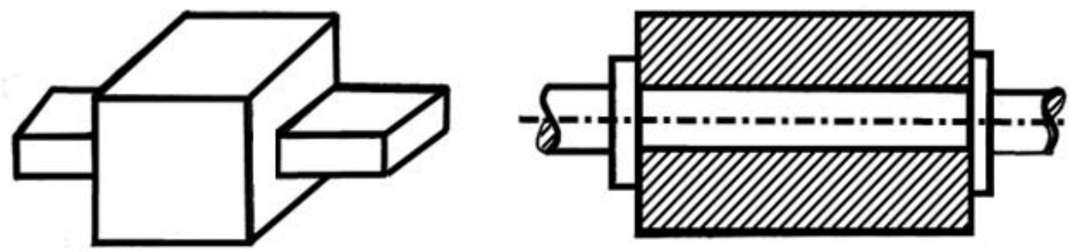


Fig.1.16 Completely constrained motion

(b) Incompletely constrained motion. When relative motion between pairing elements takes place in more than one direction, it is called incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair.

Eg. Shaft in a circular hole. (it may either rotate or slide in a hole. These both motions have no relationship with the other).

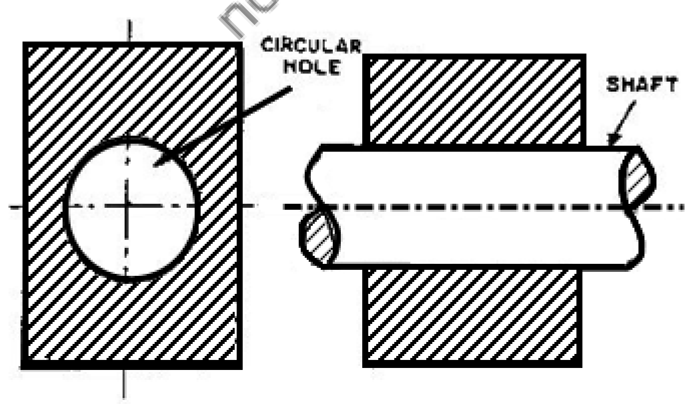


Fig.1.17 Incompletely constrained motion

(c) Successfully constrained motion. When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is called successfully constrained motion. If constrained motion is not achieved by the pairing elements themselves, but by some other means, then, it is called successfully constrained motion. Eg. Foot step bearing, where shaft is constrained from moving upwards, by its self weight.

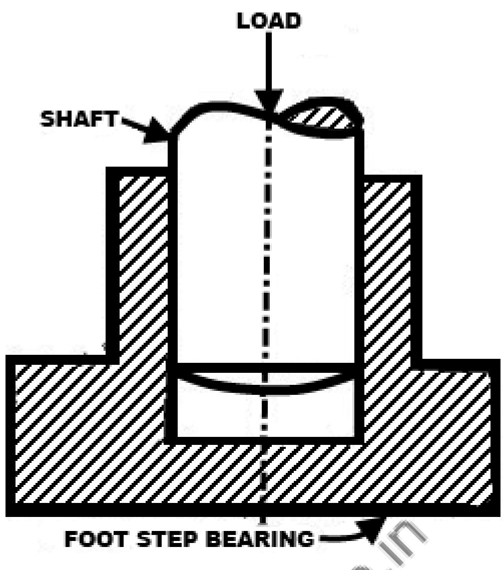


Fig.1.18 Foot strep bearing

Kinematic chain: A kinematic chain is a group of links either joined together or arranged in a manner that permits them to move relative to one another. If the links are connected in such a way that no motion is possible, it results in a locked chain or structure.

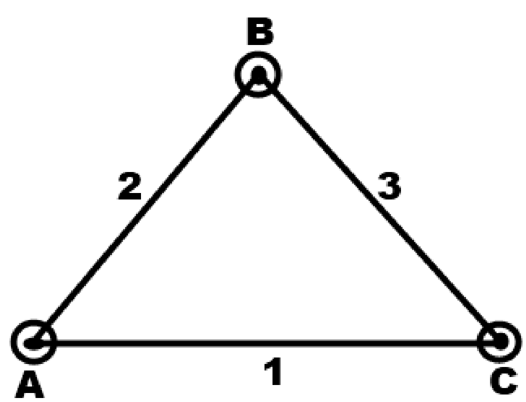


Fig.1.19 Locked chain or structure

Mechanism: A mechanism is a constrained kinematic chain. This means that the motion of any one link in the kinematic chain will give a definite and predictable motion relative to each of the others. Usually one of the links of the kinematic chain is fixed in a mechanism.

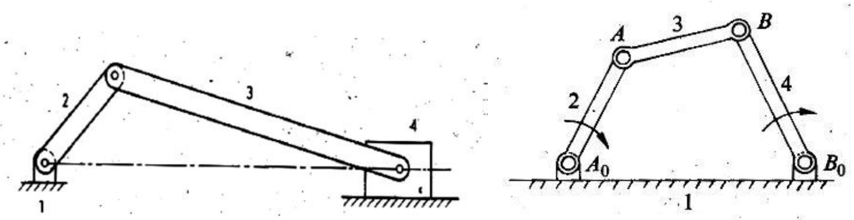


Fig.1.20 Slider crank and four bar mechanisms.

If, for a particular position of a link of the chain, the positions of each of the other links of the chain can not be predicted, then it is called as unconstrained kinematic chain and it is not mechanism.

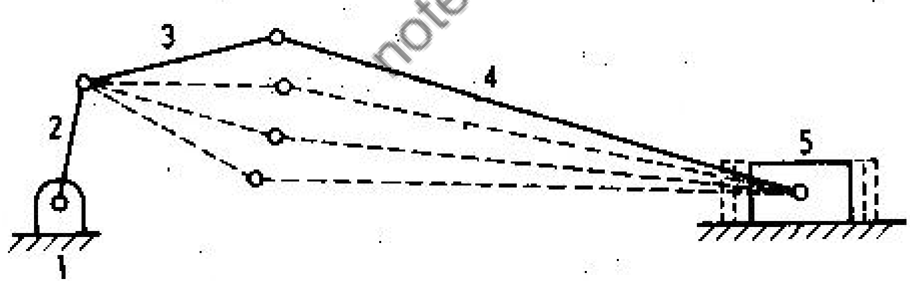


Fig.1.21 Unconstrained kinematic chain

Machine: A machine is a mechanism or collection of mechanisms, which transmit force from the source of power to the resistance to be overcome. Though all machines are mechanisms, all mechanisms are not machines. Many instruments are mechanisms but are not machines, because they do no useful work nor do they transform energy. Eg. Mechanical clock, drafter.

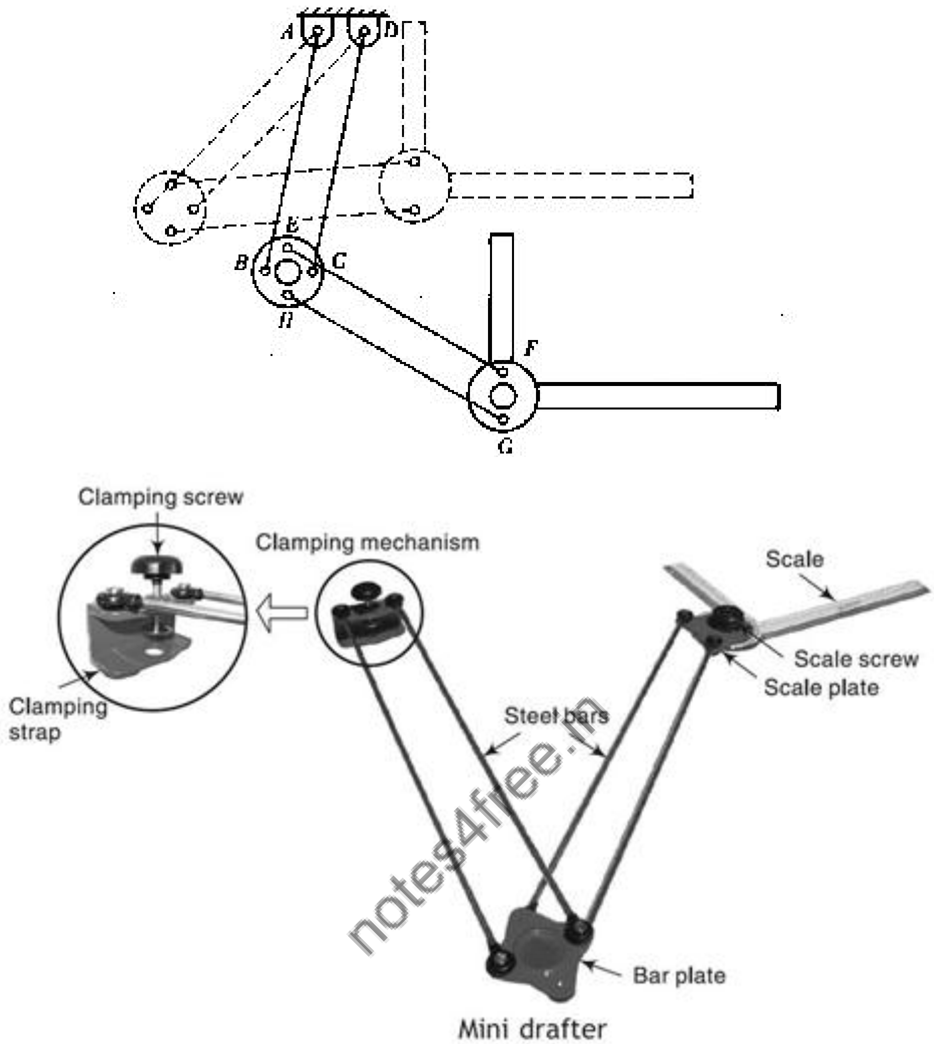


Fig.1.21 Drafter

Planar mechanisms: When all the links of a mechanism have plane motion, it is called as a planar mechanism. All the links in a planar mechanism move in planes parallel to the reference plane.

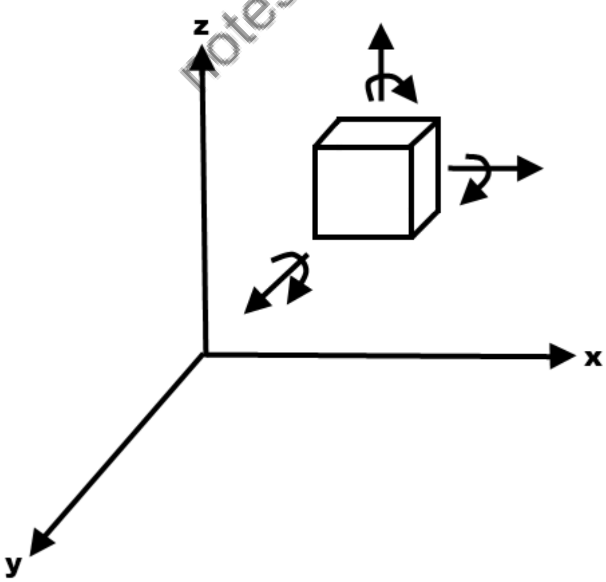
Degrees of freedom/mobility of a mechanism: It is the number of inputs (number of independent coordinates) required to describe the configuration or

position of all the links of the mechanism, with respect to the fixed link at any given instant.
An unconstrained rigid body moving in space can describe following independent motions:

1. Translational motions along any three mutually perpendicular axes x, y and z.
2. Rotational motions about these axes

Thus a rigid body possesses six degrees of freedom. The connection of a link with another imposes certain constraints on their relative motion. The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational. A pair can have degrees of freedom = 6 – number of restraints.

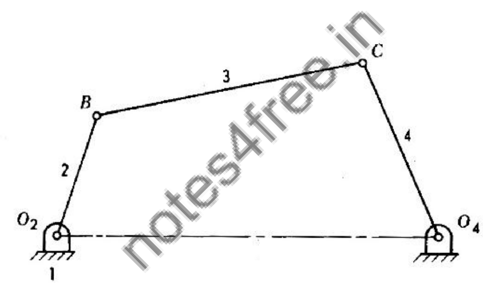


Grubler's equation: Number of degrees of freedom of a mechanism is given by
F = 3(n-1)-2l-h.

Where,
F = Degrees of freedom
n = Number of links = n2 + n3 +......+nj,
 where, n2 = number of binary links,
 n3 = number of ternary links...etc.
l = Number of lower pairs, which is obtained by counting the number of joints. If more than two links are joined together at any point, then, one additional lower pair is to be considered for every additional link.
h = Number of higher pairs

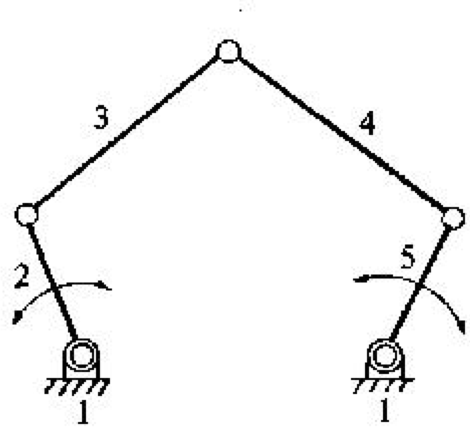
Examples of determination of degrees of freedom of planar mechanisms:

(i)



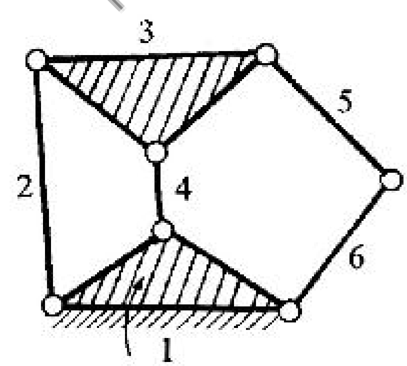
F = 3(n-1)-2l-h
Here, n2 = 4, n = 4, l = 4 and h = 0.
F = 3(4-1)-2(4) = 1
I.e., one input to any one link will result in definite motion of all the links.

(ii)



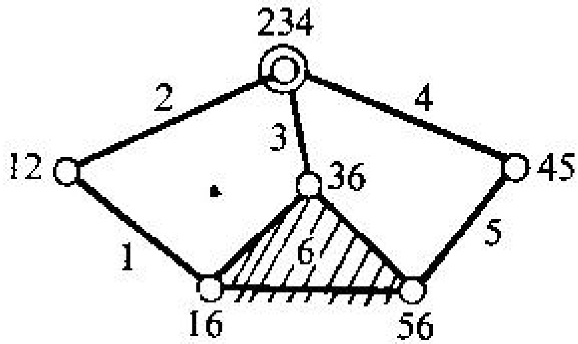
F = 3(n-1)-2l-h
Here, n2 = 5, n = 5, l = 5 and h = 0.
F = 3(5-1)-2(5) = 2
I.e., two inputs to any two links are required to yield definite motions in all the links.

(iii)



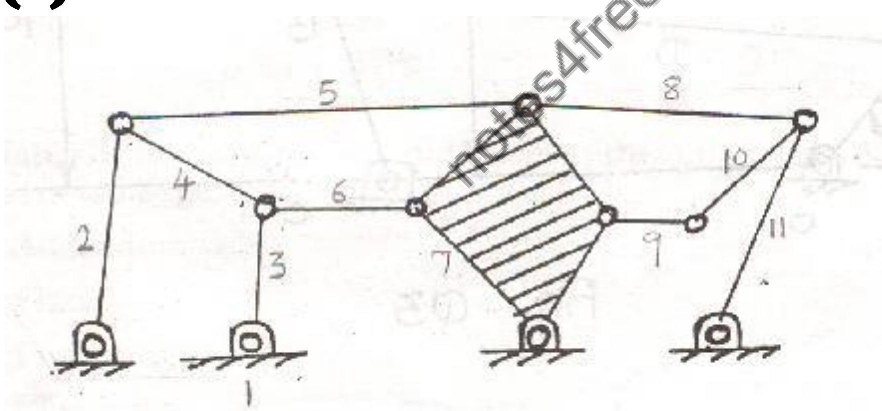
F = 3(n-1)-2l-h
Here, n2 = 4, n3 =2, n = 6, l = 7 and h = 0.
F = 3(6-1)-2(7) = 1
I.e., one input to any one link will result in definite motion of all the links.

(iv)



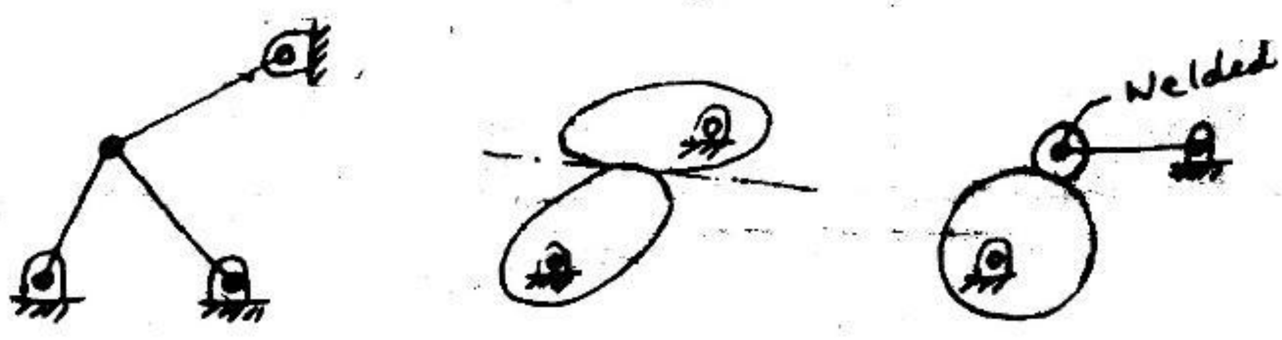
F = 3(n-1)-2l-h
Here, n2 = 5, n3 =1, n = 6, l = 7 (at the intersection of 2, 3 and 4, two lower pairs are to be considered) and h = 0.
F = 3(6-1)-2(7) = 1

(v)



F = 3(n-1)-2l-h
Here, n = 11, l = 15 (two lower pairs at the intersection of 3, 4, 6; 2, 4, 5; 5, 7, 8; 8, 10, 11) and h = 0.
F = 3(11-1)-2(15) = 0

(vi) Determine the mobility of the following mechanisms.



(a)

F = 3(n-1)-2l-h
Here, n = 4, l = 5 and h = 0.
F = 3(4-1)-2(5) = -1
I.e., it is a structure

(b)

F = 3(n-1)-2l-h
Here, n = 3, l = 2 and h = 1.
F = 3(3-1)-2(2)-1 = 1

(c)

F = 3(n-1)-2l-h
Here, n = 3, l = 2 and h = 1.
F = 3(3-1)-2(2)-1 = 1

Inversions of mechanism: A mechanism is one in which one of the links of a kinematic chain is fixed. Different mechanisms can be obtained by fixing different links of the same kinematic chain. These are called as inversions of the mechanism. By changing the fixed link, the number of mechanisms which can be obtained is equal to the number of links.

Excepting the original mechanism, all other mechanisms will be known as inversions of original mechanism. The inversion of a mechanism does not change the motion of its links relative to each other.

Four bar chain:

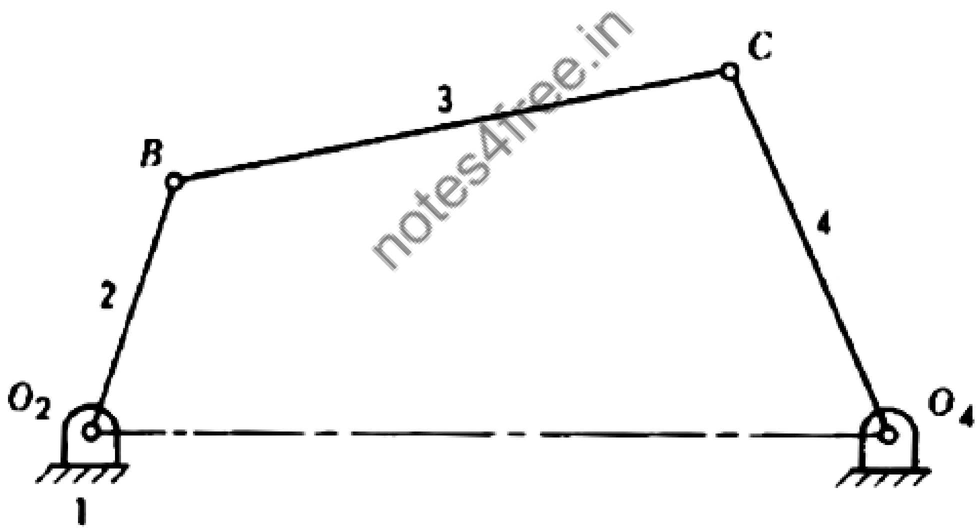


Fig 1.22 Four bar chain

One of the most useful and most common mechanisms is the four-bar linkage. In this mechanism, the link which can make complete rotation is known as crank (link 2). The link which oscillates is known as rocker or lever (link 4). And the link connecting these two is known as coupler (link 3). Link 1 is the frame.

Inversions of four bar chain:

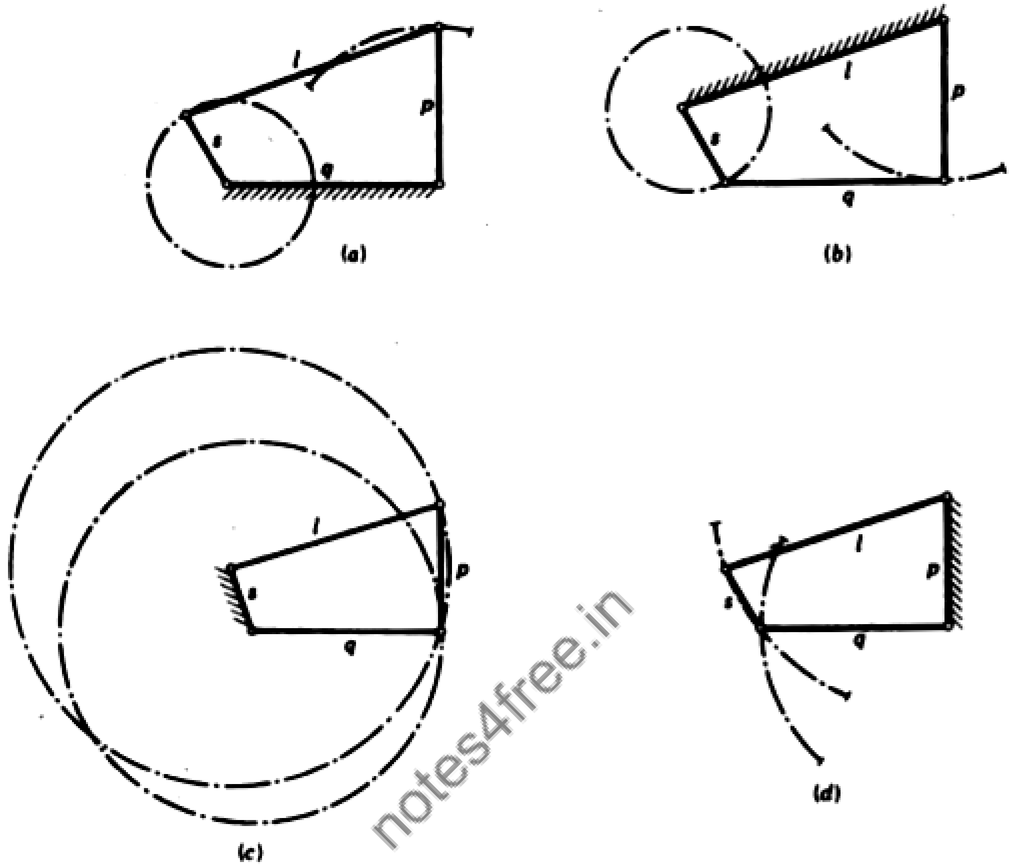


Fig.1.23 Inversions of four bar chain.

Crank-rocker mechanism: In this mechanism, either link 1 or link 3 is fixed. Link 2 (crank) rotates completely and link 4 (rocker) oscillates. It is similar to (a) or (b) of fig.1.23.

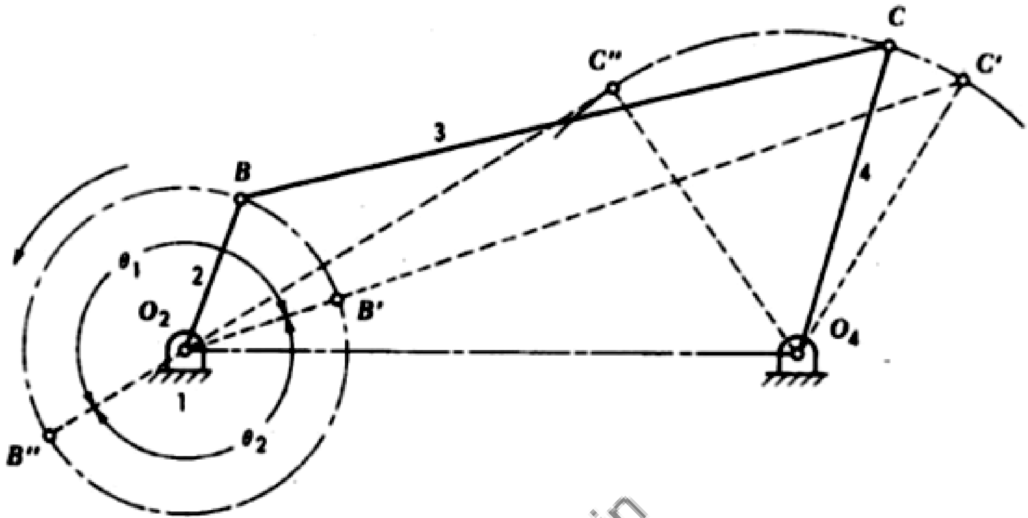


Fig.1.24

Drag link mechanism. Here link 2 is fixed and both links 1 and 4 make complete rotation but with different velocities. This is similar to 1.23(c).

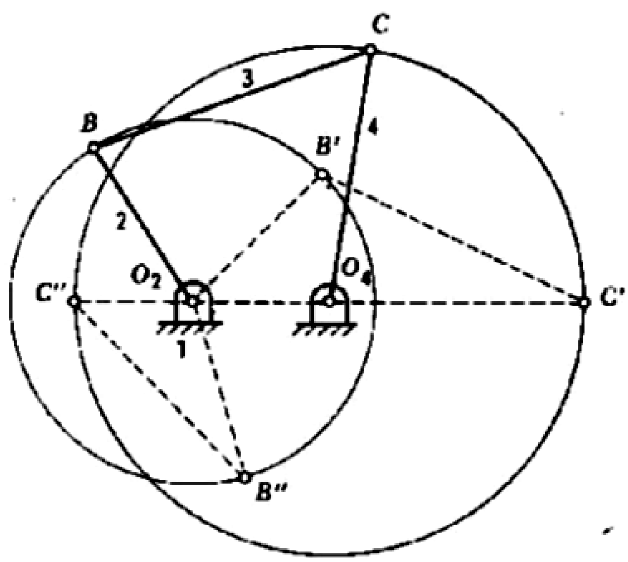


Fig.1.25

Double crank mechanism (Coupling rod of Locomotive) . This is one type of drag link mechanism, where, links 1& 3 are equal and parallel and links 2 & 4 are equal and parallel. When AB rotates about A, the crank DC rotates about D. This mechanism is used for coupling locomotive wheels. Since links AB and CD work as cranks, this mechanism is also known as double crank or crank-crank or drag-crank mechanism.

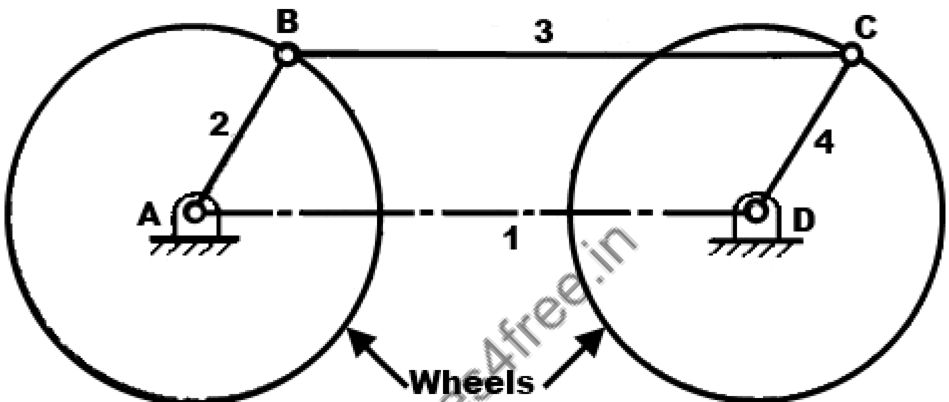


Fig.1.26

Double rocker mechanism. In this mechanism, link 4 is fixed. Link 2 makes complete rotation, whereas links 3 & 4 oscillate (Fig.1.23d)

Slider crank chain: This is a kinematic chain having four links. It has one sliding pair and three turning pairs. Link 2 has rotary motion and is called crank. Link 3 has got combined rotary and reciprocating motion and is called connecting rod. Link 4 has reciprocating motion and is called slider. Link 1 is frame (fixed). This mechanism is used to convert rotary motion to reciprocating and vice versa.

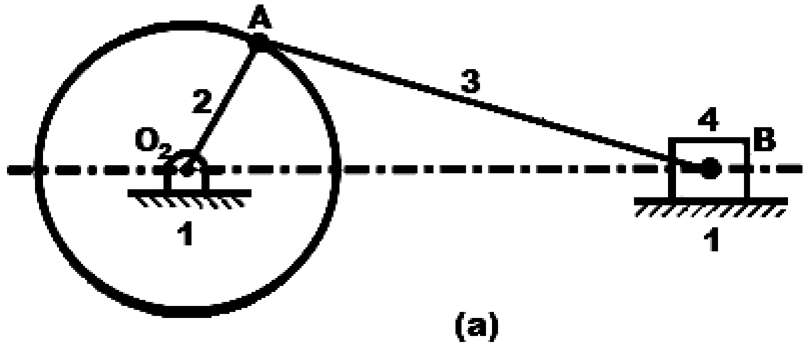
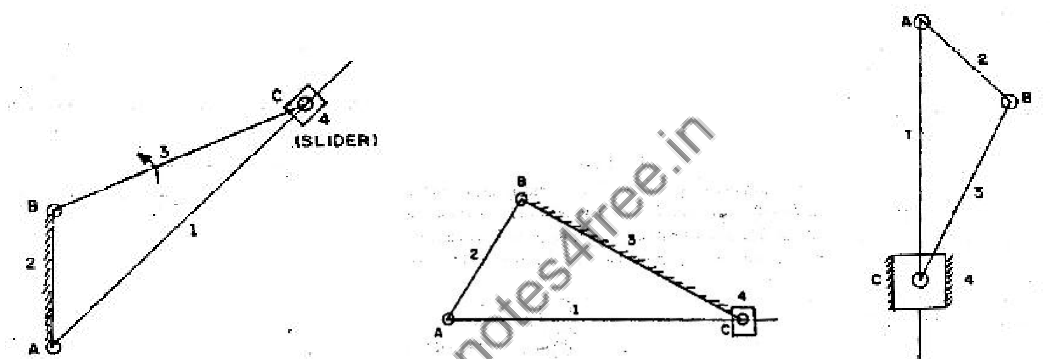


Fig1.27

Inversions of slider crank chain: Inversions of slider crank mechanism is obtained by fixing links 2, 3 and 4.



(a) crank fixed (b) connecting rod fixed (c) slider fixed

Fig.1.28

Rotary engine (Gnome Engine) – I inversion of slider crank mechanism. (crank fixed)

It is a rotary cylinder V-type internal combustion engine. Sometimes back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines are used in its place. It consists of seven cylinders in one plane and all revolves about fixed centre *A* as shown in Fig. 1.29, while the crank OA (link 2) is fixed. In this mechanism, when the connecting rod (link 4) from pistons are connected to A rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.

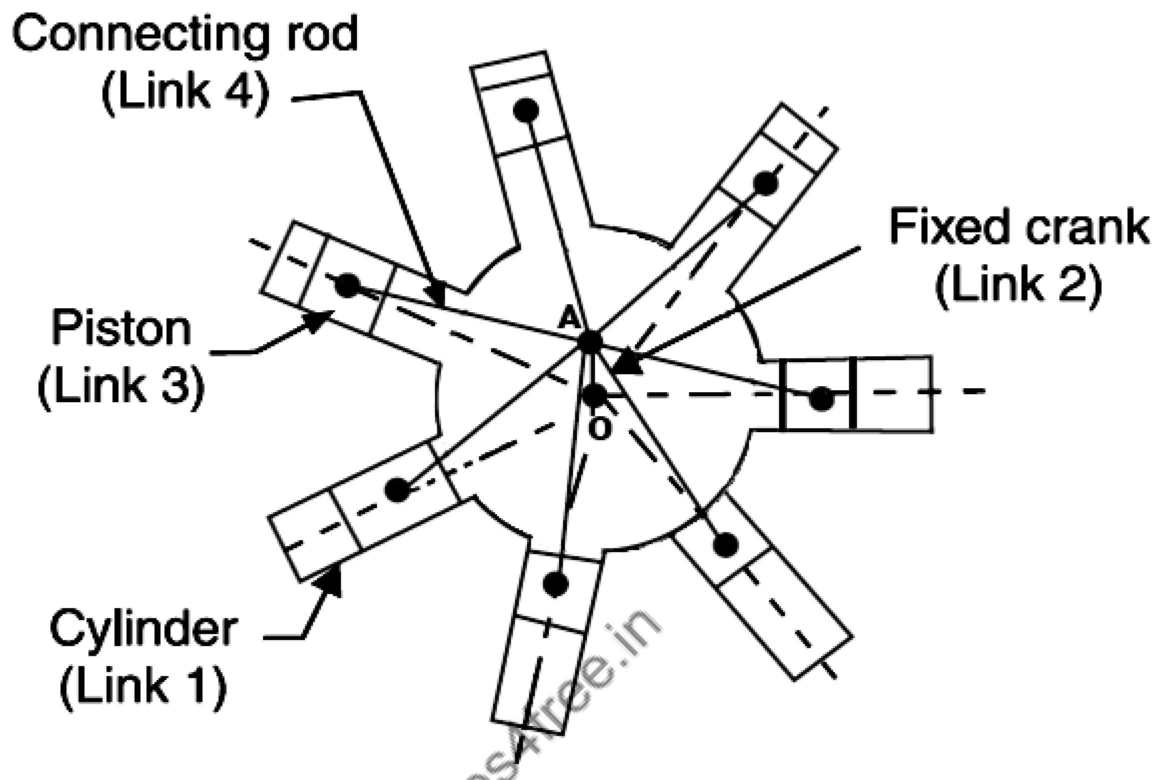


Fig. 1.29

Whitworth quick return motion mechanism–I inversion of slider crank mechanism.

This is first inversion of slider mechanism, where, crank 1 is fixed. Input is given to link 2, which moves at constant speed. Point C of the mechanism is connected to the tool post D of the machine. During cutting stroke, tool post moves from D' to D''. The corresponding positions of C are C' and C'' as shown in the fig. 1.30. For the point C to move from C' to C'', point B moves from B' to B'', in anti-clockwise direction. I.E., cutting stroke takes place when input link moves through angle $B'O_2B''$ in anticlockwise direction and return stroke takes place when input link moves through angle $B''O_2B'$ in anti-clockwise direction.

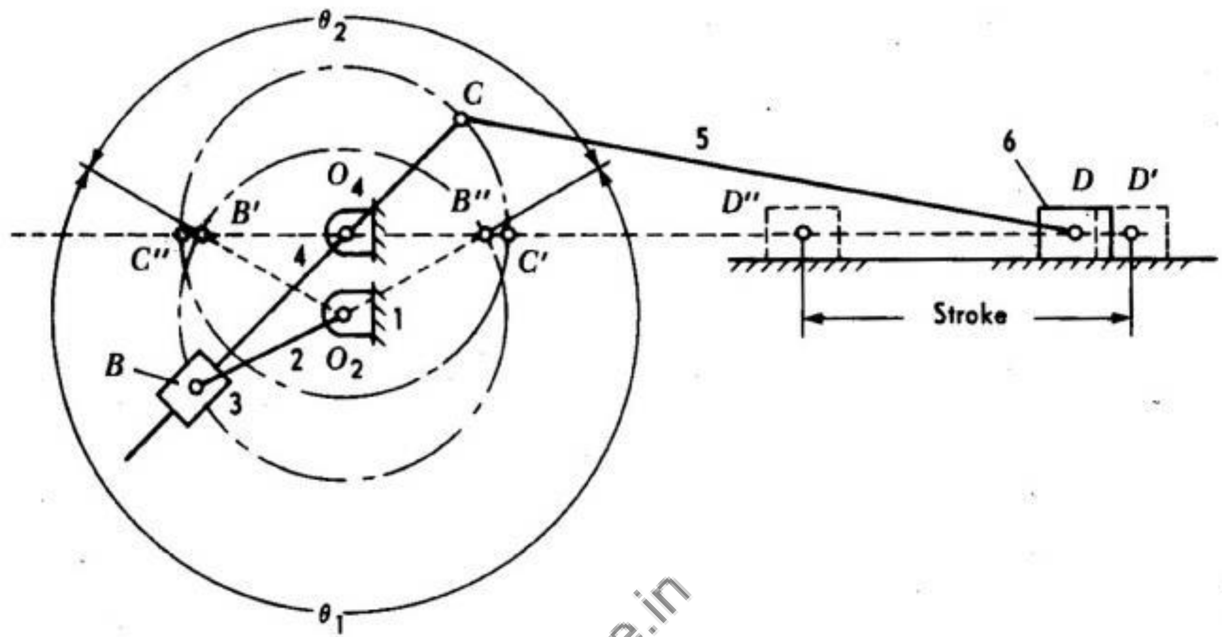


Fig.1.30

Crank and slotted lever quick return motion mechanism – II inversion of slider crank mechanism (connecting rod fixed).

This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

In this mechanism, the link *AC* (*i.e.* link 3) forming the turning pair is fixed, as shown in Fig. 1.31. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank *CB* revolves with uniform angular speed about the fixed centre *C*. A sliding block attached to the crank pin at *B* slides along the slotted bar *AP* and thus causes *AP* to oscillate about the pivoted point *A*. A short link *PR* transmits the motion from *AP* to the ram which carries the tool and reciprocates along the line of stroke R_1R_2. The line of stroke of the ram (*i.e.* R_1R_2) is perpendicular to AC produced.

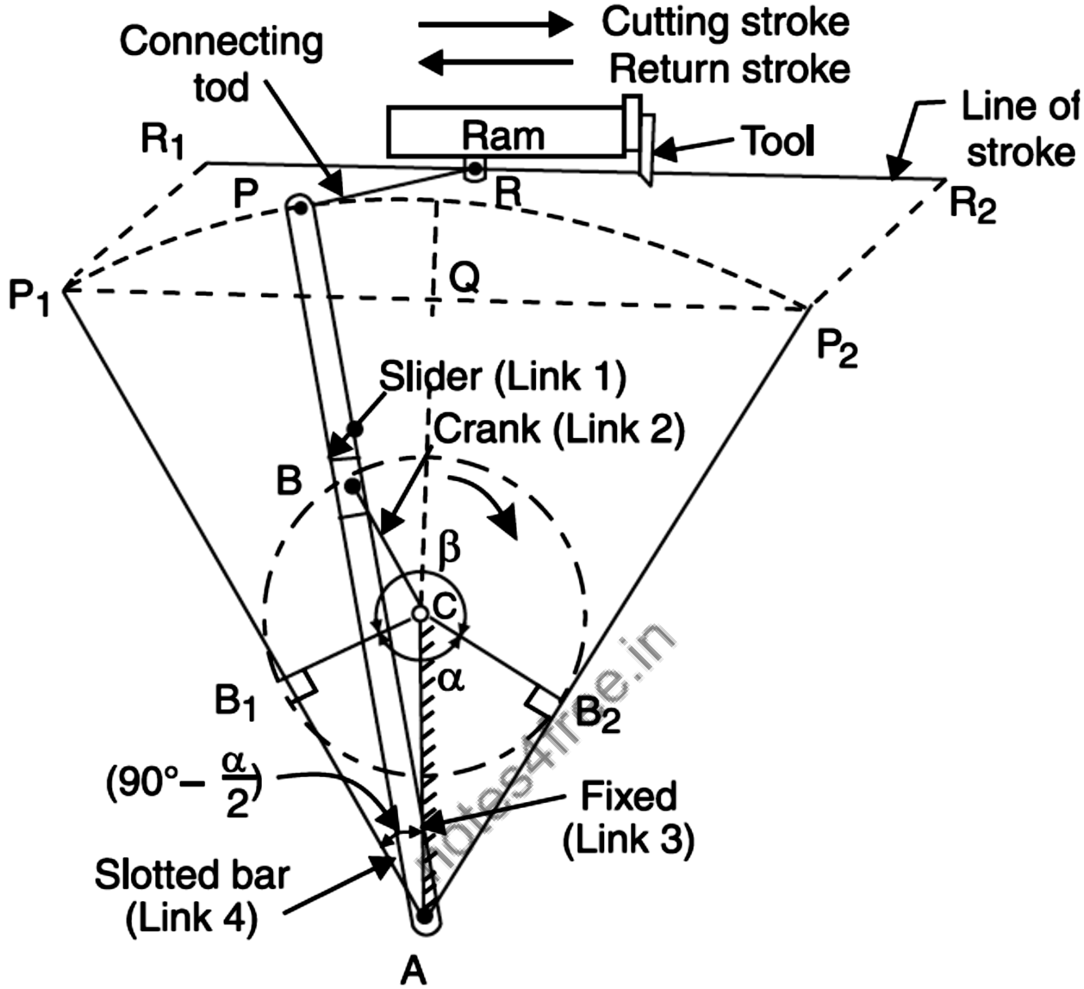


Fig.1.31

Oscillating cylinder engine–II inversion of slider crank mechanism (connecting rod fixed).

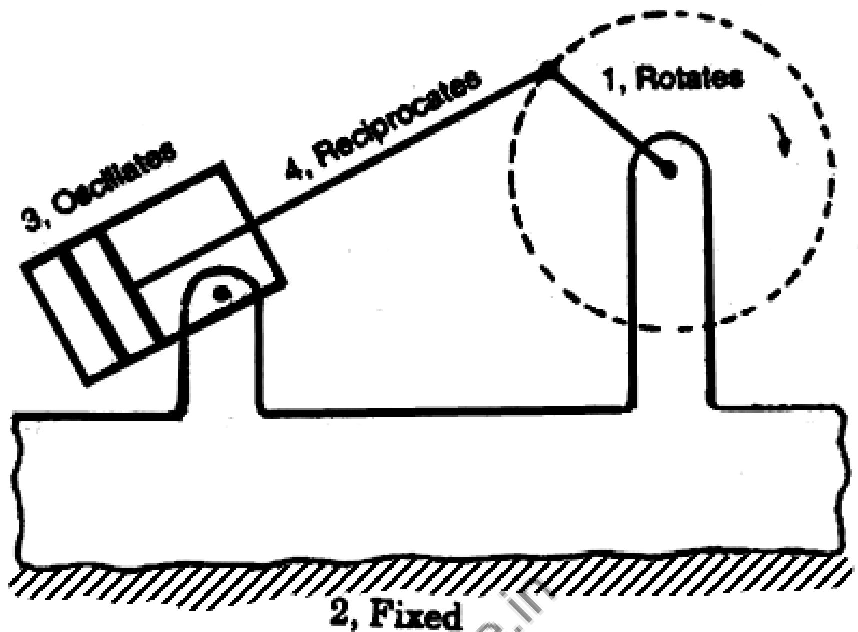


Fig.1.32

Pendulum pump or bull engine–III inversion of slider crank mechanism (slider fixed).

In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (*i.e.* sliding pair), as shown in Fig. 1.33. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at *A* and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig. 1.33.

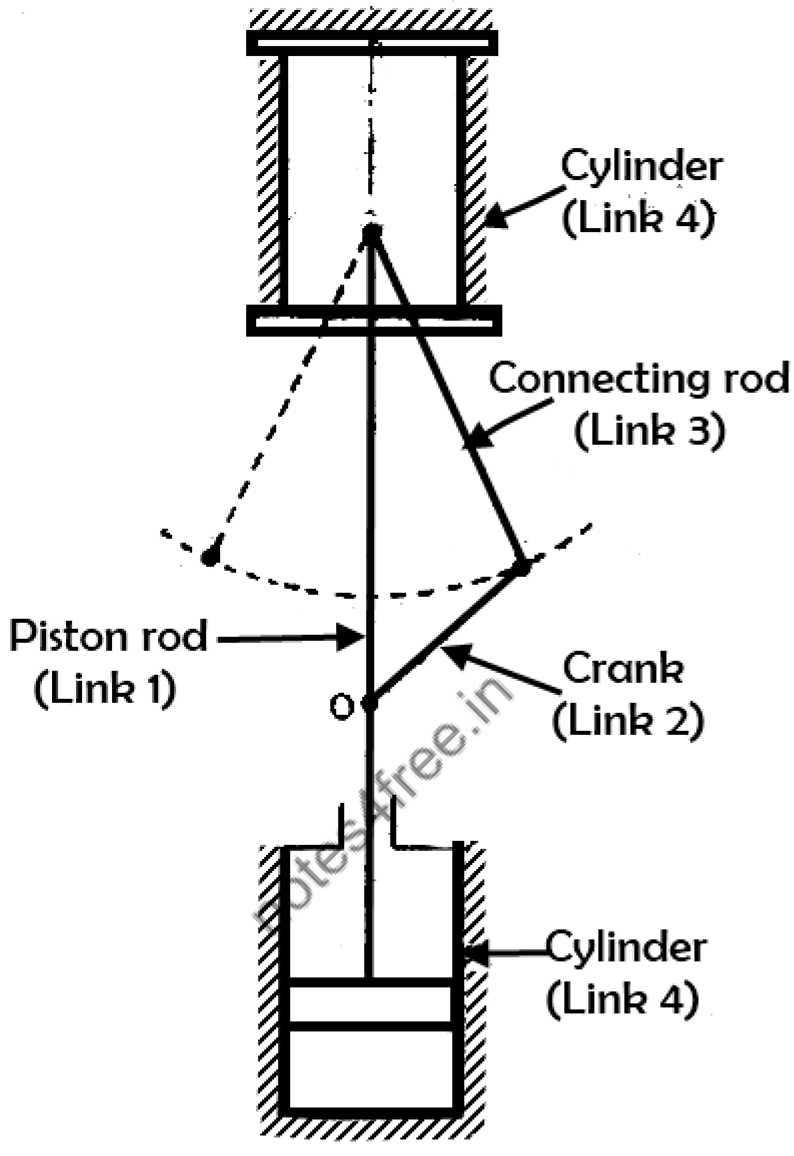


Fig.1.33

Double slider crank chain: It is a kinematic chain consisting of two turning pairs and two sliding pairs.

Scotch –Yoke mechanism.

This mechanism is used for converting rotary motion into a reciprocating motion.

Turning pairs – 1&2, 2&3; Sliding pairs – 3&4, 4&1.

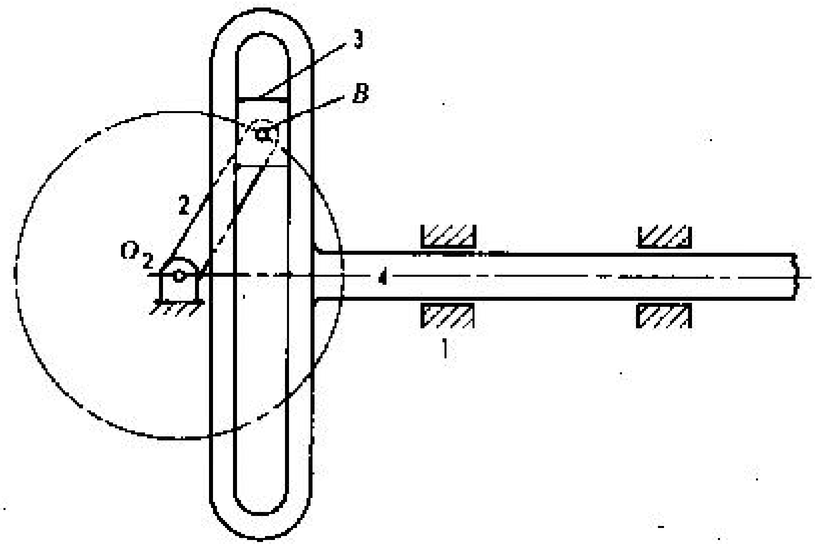
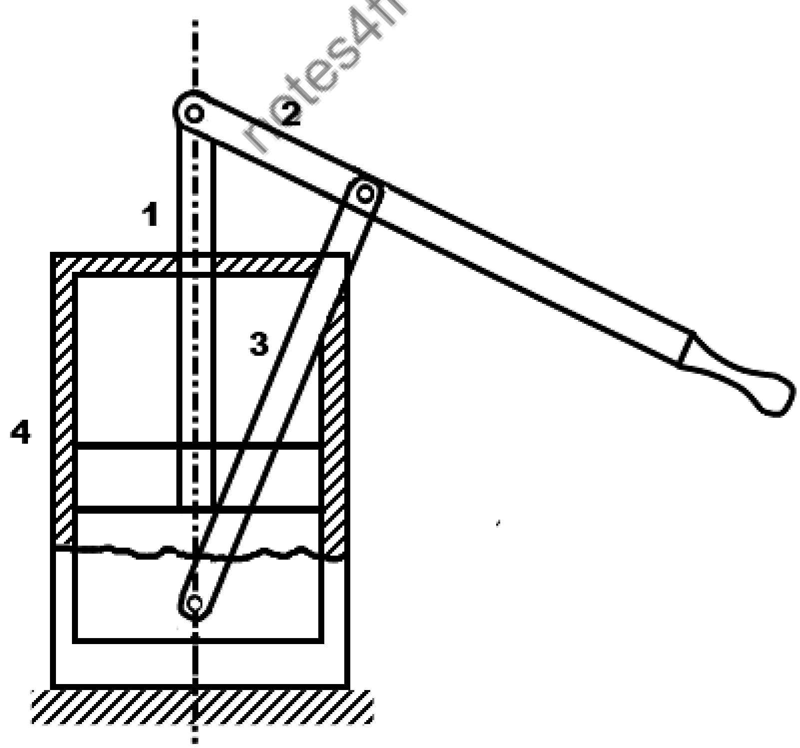


Fig.1.34

Hand pump:



Hand pump is an application of fourth inversion. The slotted link shape is given to the slider and vice versa.

Since the slider i.e., link 4 is fixed, it is possible for the link 1 to reciprocate along a vertical straight line. At the same time, link 2 will rotate and link 3 will oscillate.

Inversions of double slider crank mechanism:

Elliptical trammel. This is a device which is used for generating an elliptical profile.

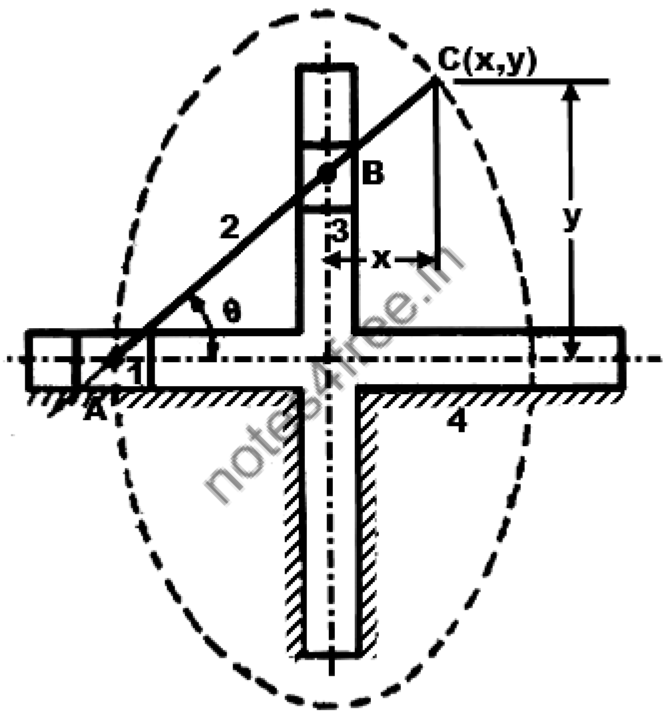


Fig.1.35

In fig. 1.35, if AC = p and BC = q, then, x = q.cosθ and y = p.sinθ.

Rearranging , $\left(\frac{x}{q}\right)^2 + \left(\frac{y}{p}\right)^2 = cos^2\theta + sin^2\theta$ = 1 This is the equation of an ellipse. The path traced by point C is an ellipse, with major axis and minor axis equal to 2p and 2q respectively.

Oldham coupling. This is an inversion of double slider crank mechanism, An oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed.

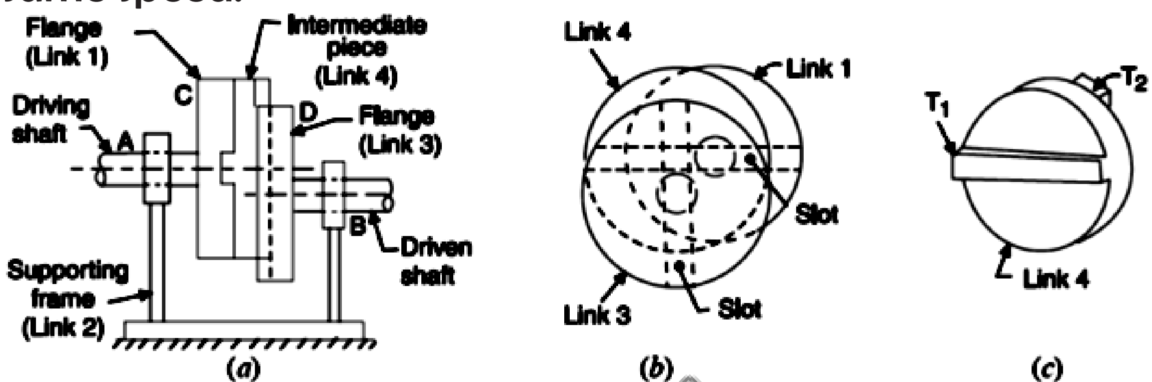


Fig.1.36

Straight line motion mechanisms

Straight line motion mechanisms are mechanisms, having a point that moves along a straight line, or nearly along a straight line, without being guided by a plane surface.

Condition for exact straight line motion:

If point B (fig.1.37) moves on the circumference of a circle with center O and radius OA, then, point C, which is an extension of AB traces a straight line perpendicular to AO, provided product of AB and AC is constant.

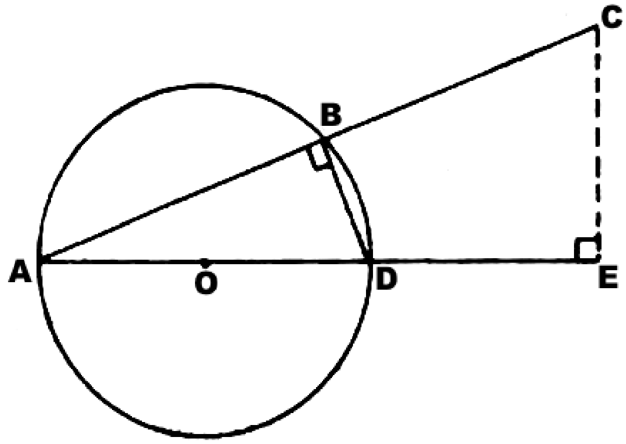


Fig.1.37

Locus of pt.C will be a straight line, ⊥ to AE if, $AB \times AC$ is constant

Proof:

$$\Delta AEC \equiv \Delta ABD$$

$$\therefore \frac{AD}{AC} = \frac{AB}{AE}$$

$$\therefore AE = \frac{AB \times AC}{AD}$$

$$but\ AD = constant.$$

$$\therefore AE = constant., if\ AB \times AC = constant.$$

Peaucellier exact straight line motion mechanism:

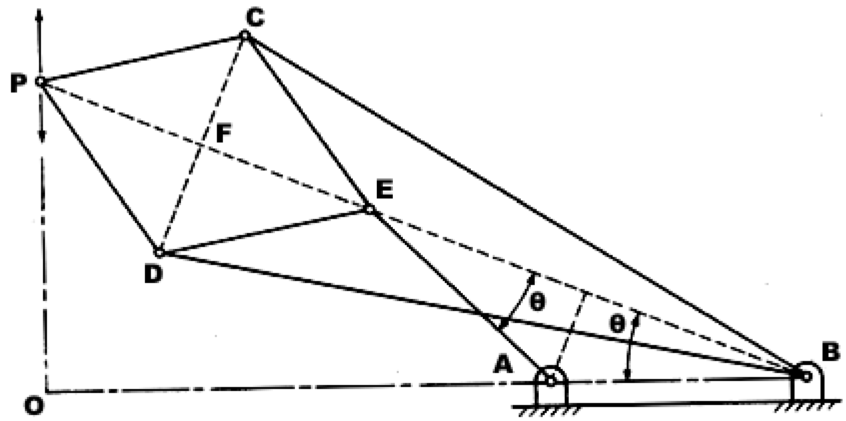


Fig.1.38

Here, AE is the input link and point E moves along a circular path of radius AE = AB. Also, EC = ED = PC = PD and BC = BD. Point P of the mechanism moves along exact straight line, perpendicular to BA extended.

To prove B, E and P lie on same straight line:

Triangles BCD, ECD and PCD are all isosceles triangles having common base CD and apex points being B, E and P. Therefore points B, E and P always lie on the perpendicular bisector of CD. Hence these three points always lie on the same straight line.

To prove product of BE and BP is constant.

In triangles BFC and PFC,

$BC^2 = FB^2 + FC^2$ and $PC^2 = PF^2 + FC^2$

$\therefore BC^2 - PC^2 = FB^2 - PF^2 = (FB + PF)(FB - PF) = BP \times BE$

But since BC and PC are constants, product of BP and BE is constant, which is the condition for exact straight line motion. Thus point P always moves along a straight line perpendicular to BA as shown in the fig.1.38.

Approximate straight line motion mechanism: A few four bar mechanisms with certain modifications provide approximate straight line motions.

Robert's mechanism

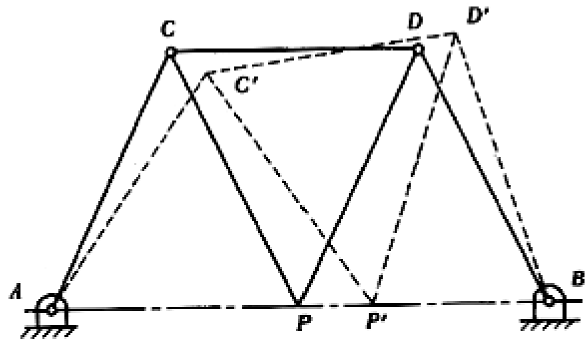


Fig.1.39

This is a four bar mechanism, where, PCD is a single integral link. Also, dimensions AC, BD, CP and PD are all equal. Point P of the mechanism moves very nearly along line AB.

Intermittent motion mechanisms

An intermittent-motion mechanism is a linkage which converts continuous motion into intermittent motion. These mechanisms are commonly used for indexing in machine tools.

Geneva wheel mechanism

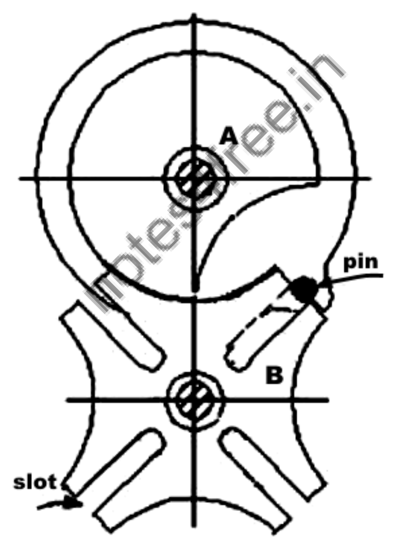


Fig.1.40

In the mechanism shown (Fig.1.40), link A is driver and it contains a pin which engages with the slots in the driven link B. The slots are positioned in such a manner, that the pin enters and leaves them tangentially avoiding impact loading during transmission of motion. In the mechanism shown, the driven member makes one-fourth of a revolution for each revolution of the driver. The locking plate, which is mounted on the driver, prevents the driven member from rotating except during the indexing period.

Ratchet and pawl mechanism:

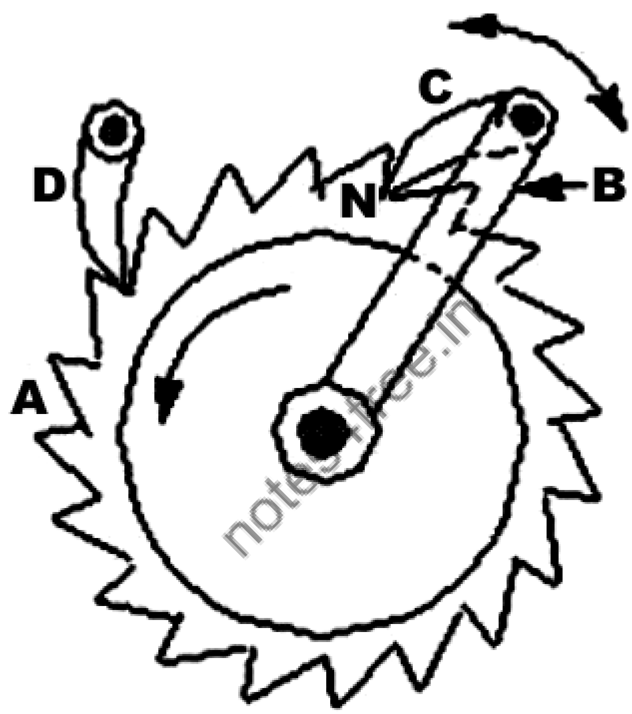


Fig.1.41

Ratchets are used to transform motion of rotation or translation into intermittent rotation or translation. In the fig.1.41, A is the ratchet wheel and C is the pawl. As lever B is made to oscillate, the ratchet wheel will rotate anticlockwise with an intermittent motion. A holding pawl D is provided to prevent the reverse motion of ratchet wheel.

Other mechanisms:

Toggle mechanism:

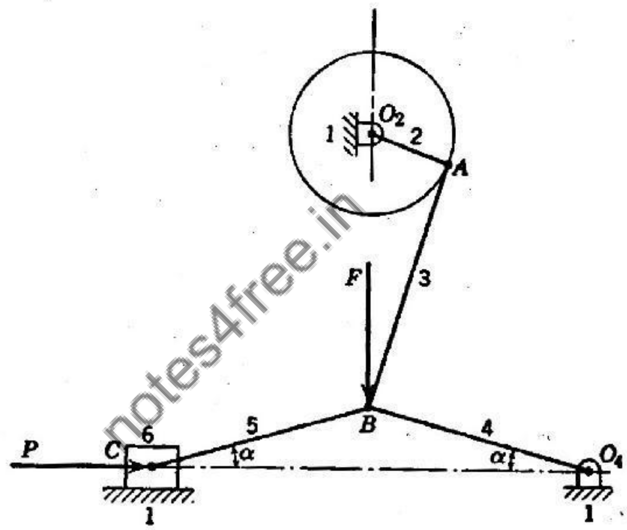


Fig.1.42

Toggle mechanisms are used, where large resistances are to be overcome through short distances. Here, effort applied will be small but acts over large distance. In the mechanism shown in fig.1.42, 2 is the input link, to which, power is supplied and 6 is the output link, which has to overcome external resistance. Links 4 and 5 are of equal length.

Considering the equilibrium condition of slider 6,

$$\tan \alpha = \frac{F/2}{P}$$

$$\therefore \quad F = 2\,P \tan \alpha$$

For small angles of α, F (effort) is much smaller than P(resistance).

This mechanism is used in rock crushers, presses, riveting machines etc.

Pantograph

Pantographs are used for reducing or enlarging drawings and maps. They are also used for guiding cutting tools or torches to fabricate complicated shapes.

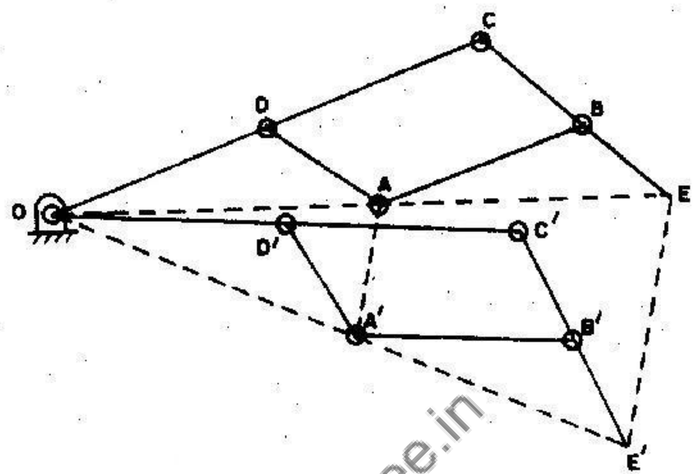


Fig.1.43

In the mechanism shown in fig.1.43 path traced by point A will be magnified by point E to scale, as discussed below.

In the mechanism shown, AB = CD; AD =BC and OAE lie on a straight line.

When point A moves to A' , E moves to E' and

$OA'E'$ also lies on a straight line.

From the fig.1.43,

$\Delta ODA \equiv \Delta OCE$ and $\Delta OD'A' \equiv \Delta OC'E'$.

$$\therefore \frac{OD}{OC} = \frac{OA}{OE} = \frac{DA}{CE} \ and \ \frac{OD'}{OC'} = \frac{OA'}{OE'} = \frac{D'A'}{C'E'}$$

But, $\frac{OD}{OC} = \frac{OD'}{OC'}$; $\therefore \frac{OA}{OE} = \frac{OA'}{OE'}$; $\therefore \Delta OAA' \equiv \Delta OEE'$

$\therefore EE' \parallel AA'$

and $\frac{EE'}{AA'} = \frac{OE}{OA} = \frac{OC}{OD}$

$$\therefore EE' = AA'\left(\frac{OC}{OD}\right)$$

Where $\left(\frac{OC}{OD}\right)$ *is the magnification factor.*

STEERING MECHANISMS

Hooke's joint (Universal joints)

Hooke's joints is used to connect two nonparallel but intersecting shafts. In its basic shape, it has two U –shaped yokes 'a' and 'b' and a center block or cross-shaped piece, C. (fig.1.44(a))

The universal joint can transmit power between two shafts intersecting at around 30° angles (α). However, the angular velocity ratio is not uniform during the cycle of operation. The amount of fluctuation depends on the angle (α) between the two shafts. For uniform transmission of motion, a pair of universal joints should be used (fig.1.44(b)). Intermediate shaft 3 connects input shaft 1 and output shaft 2 with two universal joints. The angle α between 1 and 2 is equal to angle α between 2 and 3. When shaft 1 has uniform rotation, shaft 3 varies in speed; however, this variation is compensated by the universal joint between shafts 2 and 3. One of the important applications of universal joint is in automobiles, where it is used to transmit power from engine to the wheel axle.

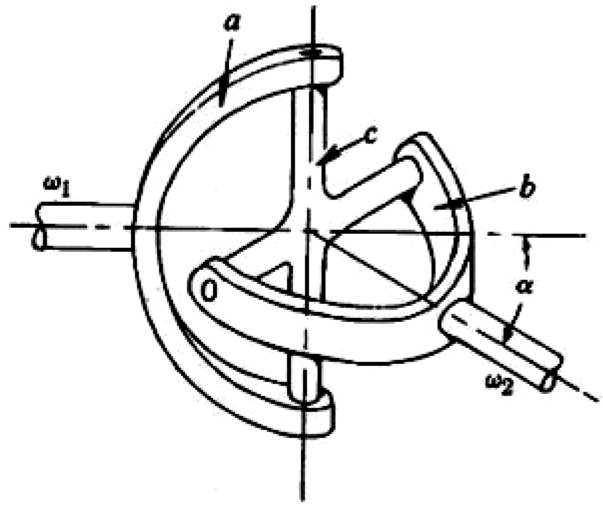


Fig.1.44(a)

Fig.1.44(b): Hooke's joint

Applications of Hooke's joint:

1. Transmission of power from the engine gear box to the rear axle of the automobile.
2. Transmission of drives to different spindle in multiple drilling machine.
3. Transmission of torque to the rolls in the rolling mills.
4. Knee joint in a milling machine.

Steering gear mechanism

The steering mechanism is used in automobiles for changing the directions of the wheel axles with reference to the chassis, so as to move the automobile in the desired path. Usually, the two back wheels will have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of front wheels.

In automobiles, the front wheels are placed over the front axles (stub axles), which are pivoted at the points A & B as shown in the fig.1.45. When the vehicle takes a turn, the front wheels, along with the stub axles turn about the pivoted points. The back axle and the back wheels remain straight.

Always there should be absolute rolling contact between the wheels and the road surface. Any sliding motion will cause wear of tyres. When a vehicle is taking turn, absolute rolling motion of the wheels on the road surface is possible, only if all the wheels describe concentric circles. Therefore, the two front wheels must turn about the same instantaneous centre I which lies on the axis of the back wheel.

Condition for perfect steering

The steering gear mechanism in an automobile is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path. Usually, the two back wheels have a common axle, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels.

There are two types of steering mechanisms:

1. Ackerman steering gear mechanism.
2. Davis steering gear mechanism.

The Ackerman steering gear mechanism is much simpler than Davis steering gear and hence quite popular. The difference between the Ackerman and Davis steering gear mechanism are:-

1. The whole mechanism of the Ackerman steering gear is mounted at the back of the front wheels, where as in Davis steering gear, it is in the front of the wheel.
2. The Ackerman steering gear consists of turning pair, whereas Davis steering gear consists of sliding member

The condition for perfect steering is that all the four wheels must turn about the same instantaneous centre. While negotiating a curve, the inner wheel makes a larger turning angle θ than the angle ϕ subtended by the axis of the outer wheel.

In the fig.1.45, a = wheel track, L = wheel base, w =_distance between the pivots of front axles.

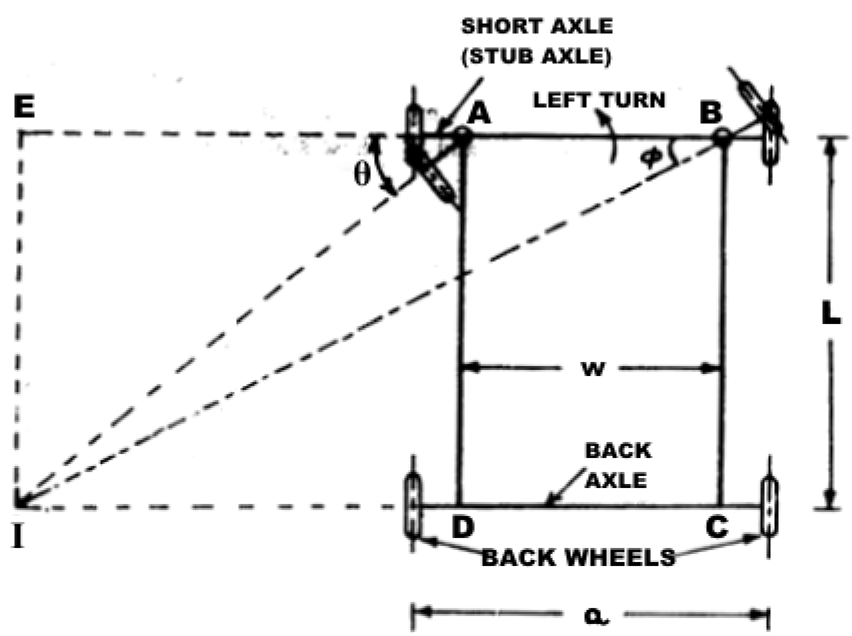


Fig.1.45: Condition for perfect steering

From ΔIAE, cot θ = $\frac{AE}{EI}$ = $\frac{AE}{L}$ and

From ΔBEI, cot φ = $\frac{EB}{EI} = \frac{(EA+AB)}{EI} = \frac{(EA+w)}{L} = \frac{EA}{L} + \frac{w}{L} = cot\theta + \frac{w}{L}$

$\therefore cot\varphi - cot\theta = \frac{w}{L}$. This is the fundamental equation for correct steering. If this condition is satisfied, there will be no skidding of the wheels when the vehicle takes a turn.

Ackermann steering gear mechanism:

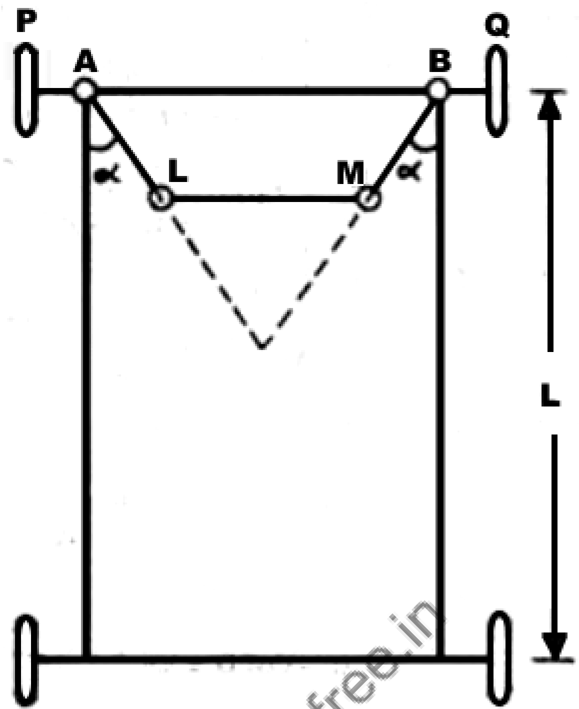


Fig.1.49: Ackermann steering gear mechanism

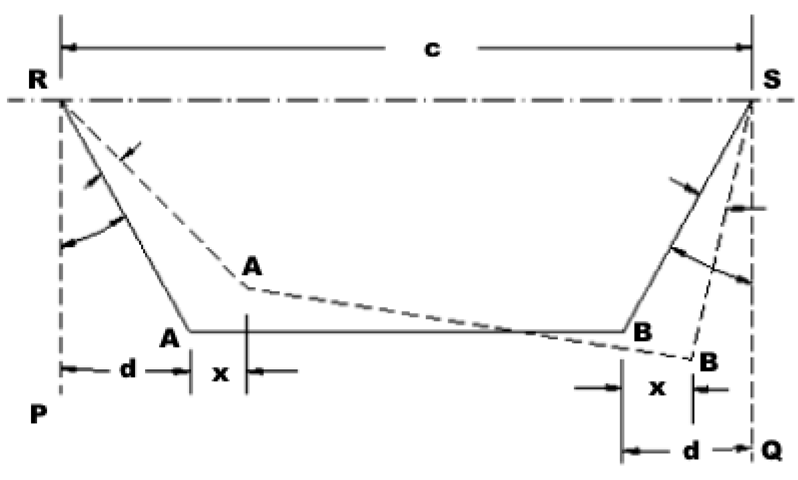


fig.1.50: Ackermann steering gear mechanism

Ackerman steering mechanism, RSAB is a four bar chain as shown in fig.1.50. Links RA and SB which are equal in length are integral with the stub axles. These links are connected with each other through track rod AB. When the vehicle is in straight ahead position, links RA and SB make equal angles α with the center line of the vehicle. The dotted lines in fig.1.50 indicate the position of the mechanism when the vehicle is turning left.

Let AB=1, RA=SB=r; $P\hat{R}A = Q\hat{S}B = \alpha$ and in the turned position, $A\hat{R}A' = \theta$ & $B\hat{S}B' = \emptyset$. IE, the stub axles of inner and outer wheels turn by θ and ϕ angles respectively.

Neglecting the obliquity of the track rod in the turned position, the movements of A and B in the horizontal direction may be taken to be same (x).

Then, $\sin(\alpha+\theta) = \frac{d+x}{r}$ and $\sin(\alpha-\varphi) = \frac{d-x}{r}$

Adding, $\sin(\alpha+\theta)+\sin(\alpha-\varphi) = \frac{2d}{r} = 2\sin\alpha$ [1]

Angle α can be determined using the above equation. The values of θ and ϕ to be taken in this equation are those found for correct steering using the equation

$\cot\Phi - \cot\theta = \frac{w}{L}$. [2]

This mechanism gives correct steering in only three positions. One, when θ = 0 and other two each corresponding to the turn to right or left (at a fixed turning angle, as determined by equation [1]).

The correct values of ϕ, [ϕ_c] corresponding to different values of θ, for correct steering can be determined using equation [2]. For the given dimensions of the mechanism, actual values of ϕ, [ϕ_a] can be obtained for different values of θ. The difference between ϕ_c and ϕ_a will be very small for small angles of θ, but the difference will be substantial, for larger values of θ. Such a difference will reduce the life of tyres because of greater wear on account of slipping.

But for larger values of θ, the automobile must take a sharp turn; hence is will be moving at a slow speed. At low speeds, wear of the tyres is less. Therefore, the greater difference between ϕ_c and ϕ_a larger values of θ ill not matter.

As this mechanism employs only turning pairs, friction and wear in the mechanism will be less. Hence its maintenance will be easier and is commonly employed in automobiles.

IMPORTANT QUESTIONS:

1. Define the following: (VTU Feb 2006, July 2008, Jan 2009, July 2009, Jan 2010, June 2010, Dec 2011, Dec 2012, July 2013, Jan 2014, July 2014, July 2015, Jan 2016, July 2016)
   a. Link.
   b. Kinematic chain.
   c. Mechanism.
   d. Structure.
   e. Inversion.
   f. Degree of Freedom.
   g. Kinematic link.
   h. Kinematic pair.
   i. Kinematic mechanism.
   j. Machine.
   k. Higher pair.
   l. Lower pair.
   m. Mobility of mechanism.
   n. Self closed pair.
   o. Forced closed pair.
2. What do you mean by rigid link? Explain two types of links with examples. (VTU Jan 2015)
3. Distinguish between:
   a. Mechanism and Machine.
   b. Structure and Kinematic chain.
   c. Open pair and Closed pair.
   d. Complete constraint and successful constraint.

4. Describe with neat figures two inversions of double slider-crank chain. (VTU July 2016)
5. With neat sketch, explain crank and slotted lever quick return mechanism. (VTU July 2016)
6. Explain the pantograph mechanism, with a neat sketch. State its applications. (VTU July 2016)
7. Draw a line diagram and explain peaucellier's straight line mechanism. (VTU July 2016)
8. Explain with a neat sketch, the double slider crank chain mechanism and its inversions (any two inversions with application). (VTU Jan 2016)
9. Sketch peaucellier's mechanism, and prove that it can trace a straight line. (VTU Jan 2016)
10. With a neat sketch, explain the condition for Ackermann's mechanism. (VTU Jan 2016)
11. Define inversion of a kinematic chain. With the help of a neat sketch explain inversions of single slider crank chain. (VTU July 2016)
12. With the help of a neat sketch, explain the working principle of crank and slotted lever mechanism. (VTU July 2015)
13. List various straight line generating mechanisms. With the help of a neat sketch along with proof, explain how a peaucellier mechanism generates a straight line. (VTU July 2015)
14. Explain the following inversions with neat sketch:
    i) Double rocker mechanism.

15. Crank and slotted lever type quick return motion mechanism. (VTU Jan 2015)
16. Define degrees of freedom and state the relation for the same for planar mechanisms having only turning and sliding pairs. (VTU Jan 2015)
17. Sketch peaucellier's mechanism and prove that it can trace a straight line. (VTU Jan 2015)
18. Explain pawl and ratchet wheel mechanism with neat sketch. (VTU Jan 2015)
19. With neat sketch, explain the conditions for correct steering for Ackermann-mechanism. (VTU Jan 2015)
20. Sketch and explain the working of an elliptical trammel. Prove that it traces an ellipse. (VTU July 2014)
21. Explain with a neat sketch. Crank and slotted lever quick return motion mechanism. (VTU July 2014)
22. Explain with a neat sketch. Pantograph mechanism. State its applications. (VTU July 2014)
23. Explain with a neat sketch, Geneva mechanism. (VTU July 2014)
24. Describe with neat sketch two inversion of double slider – crank chain mechanism. (VTU Jan 2014)
25. Derive an expression for necessary condition of correct steering and explain Ackermann steering gear with neat sketch. (VTU Jan 2014)
26. Explain with the help of neat sketches, the following mechanisms: (VTU Jan 2008, Jan 2009, July 2009, Jan 2010, June 2010, Dec 2010, July 2011, June 2012, Dec 2012, July 2013)

i) Single slider crank mechanism.
ii) Intermittent motion mechanism.
iii) Two inversions of double slider-crank chain.
iv) parallel crank mechanism.
v) Quick return mechanism.
vi) Gnome engine mechanism.
vii) Four bar mechanism.
viii) Elliptical trammel.
ix) Peaucellier's straight line mechanism.
x) Crank and slotted lever quick return motion mechanism.
xi) Scotch yoke mechanism.
xii) Pantograph.
xiii) Hooke's joint
xiv) Toggle mechanism.
xv) Roberts mechanism.
xvi) Whitworth quick return motion mechanism.
xvii) Ratchet and pawl mechanism.
xviii) Geneva wheel mechanism.

27. With the help of neat sketch, explain the working principle of Crank and slotted lever mechanism. (VTU July 2013)

28. Derive the condition for the correct steering mechanism. For a four wheeled vehicle. (VTU July 2013)

Module 2

Velocity and Acceleration by Graphical and Instantaneous Centre Method

In kinematic analysis of mechanisms, acceleration analysis is usually performed following a velocity analysis; i.e., the positions and orientations, and the velocities of all the links in a mechanism are assumed known. In this chapter we concentrate on one graphical method for acceleration analysis of planar mechanisms.

We start this chapter with some exercises to ensure that the fundamentals of acceleration analysis are well understood. You may review these fundamentals in Chapter 2 of these notes.

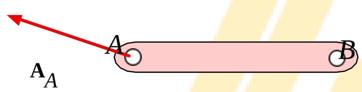
Exercises

In these exercises take direct measurements from the figures for link lengths and the magnitudes of velocity and acceleration vectors. If it is stated that the angular velocity and acceleration are known, assume $\omega = 1$ rad/sec CCW and $\alpha = 1$ rad/sec^2 CW unless it is stated otherwise. Write the position, velocity, and acceleration vector equations, and then graphically determine the unknown acceleration(s).

P.1

Known: $\mathbf{A}_A$, α and ω

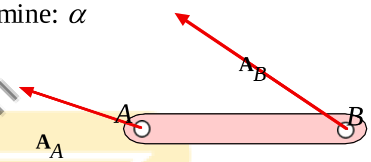
Determine: $\mathbf{A}_B$



P.2

Known: $\mathbf{A}_A$ and $\mathbf{A}_B$

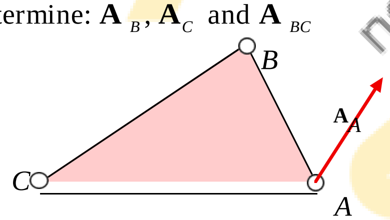
Determine: α



P.3

Known: $\mathbf{A}_A$, α, and ω

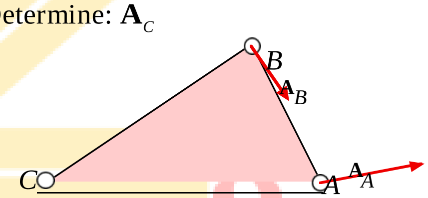
Determine: $\mathbf{A}_B$, $\mathbf{A}_C$ and $\mathbf{A}_{BC}$



P.4

Known: $\mathbf{A}_A$ and $\mathbf{A}_B$

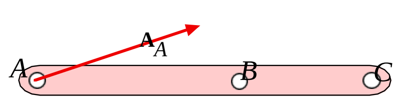
Determine: $\mathbf{A}_C$



P.5

Known: $\mathbf{A}_A$, α, and ω

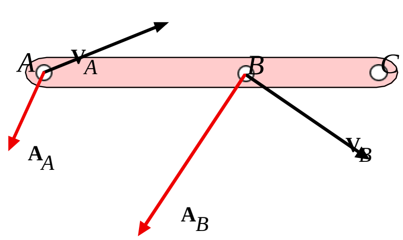
Determine: $\mathbf{A}_B$, $\mathbf{A}_C$ and $\mathbf{A}_{BC}$



P.6

Known: $\mathbf{V}_A$, $\mathbf{V}_B$, $\mathbf{A}_A$ and $\mathbf{A}_B$

Determine: $\mathbf{A}_C$



P.7

Known: ω, $\mathbf{V}^s_{BA}$ $\mathbf{A}_A$, α, $\mathbf{A}^s_{BA}$,

$V^s_{BA} = 1$ unit/sec positive, and

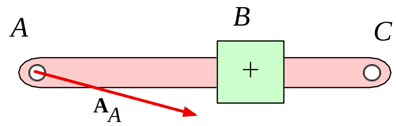
P.8

Known: ω, $\mathbf{V}^s_{BA}$ $\mathbf{A}_A$, α, $\mathbf{A}^s_{BA}$,

$V^s_{BA} = 1$ unit/sec positive, and

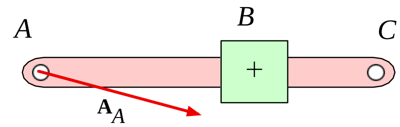
A^s_{BA} = 1 unit/sec^2 positive

Determine: $\mathbf{A}_B$ and $\mathbf{A}_C$



A^s_{BA} = 1 unit/sec^2 negative

Determine: $\mathbf{A}_B$ and $\mathbf{A}_C$

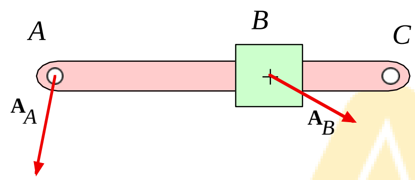


P.9

Known: ω, $\mathbf{V}^s_{BA}$, $\mathbf{A}_A$, $\mathbf{A}_B$,

V^s_{BA} = 1 unit/sec negative

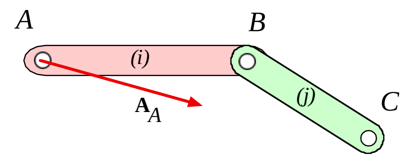
Determine: α and $\mathbf{A}_C$



P.10

Known: ω_i = 1 rad/sec CCW, ω_j = 1 rad/sec CW, α_i = 1 rad/sec^2 CCW, α_j = 1 rad/sec^2 CW, and $\mathbf{A}_A$

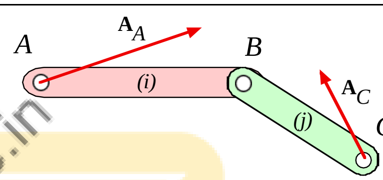
Determine: $\mathbf{A}_B$ and $\mathbf{A}_C$



P.11

Known: ω_i, ω_j, $\mathbf{A}_A$, and $\mathbf{A}_C$

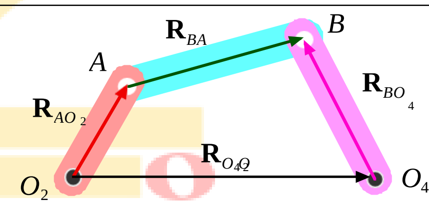
Determine: $\mathbf{A}_B$, α_i and α_j



Polygon Method

Four-bar Mechanism

For a known four-bar mechanism, in a given configuration and known velocities, and a given angular acceleration of the crank, α_2 (say CCW), construct the acceleration polygon. Determine α_3 and α_4.

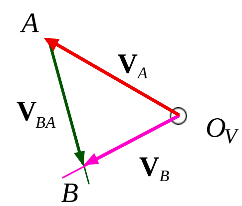


The position and velocity vector loop equations are:

$$\mathbf{R}_{AO_2} + \mathbf{R}_{BA} - \mathbf{R}_{BO_4} - \mathbf{R}_{O_4O_2} = \mathbf{0}$$

$$\mathbf{V}_A + \mathbf{V}_{BA} - \mathbf{V}_B = \mathbf{0}$$

It is assumed that for this given configuration a velocity analysis has already been performed (e.g., velocity polygon) and all of the unknown velocities have been determined.



The acceleration equation is obtained from the time derivative of the velocity equation as $\mathbf{A}_A + \mathbf{A}_{BA} = \mathbf{A}_B$. Since $\mathbf{R}_{AO_2}$, $\mathbf{R}_{BA}$, and $\mathbf{R}_{BO_2}$ are moving vectors with constant lengths, their acceleration vectors have normal and tangential components:

$$\mathbf{A}^n_A + \mathbf{A}^t_A + \mathbf{A}^n_{BA} + \mathbf{A}^t_{BA} - \mathbf{A}^n_B - \mathbf{A}^t_B = \mathbf{0}$$

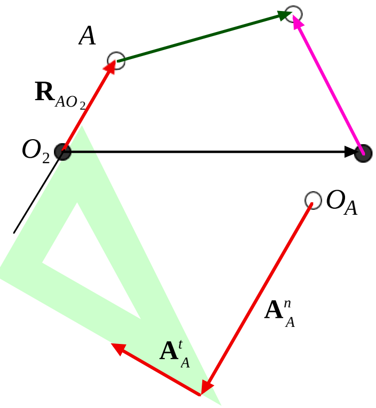
Or,

$$-\omega_2^2\mathbf{R}_{AO_2} + \alpha_2\dot{\mathbf{R}}_{AO_2} - \omega_3^2\mathbf{R}_{BA} + \alpha_3\dot{\mathbf{R}}_{BA} - (-\omega_4^2\mathbf{R}_{BO_4}) - \alpha_4\dot{\mathbf{R}}_{BO_4} = \mathbf{0}$$

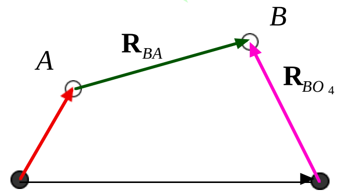
We note that since ω_2, ω_3, ω_4, and α_2 are known, $\mathbf{A}^n_A$, $\mathbf{A}^t_A$, $\mathbf{A}^n_{BA}$, and $\mathbf{A}^n_B$ can completely be constructed. The remaining components, $\mathbf{A}_{BA}^t$ and $\mathbf{A}_B^t$, have known axes but unknown magnitudes. We rearrange the terms such that these unknown terms appear as the **last component** in the equation:

$$-\omega_2^2\mathbf{R}_{AO_2}+\alpha_2\dot{\mathbf{R}}_{AO_2}-\omega_3^2\mathbf{R}_{BA}-(-\omega_4^2\mathbf{R}_{BO_4})+\alpha_3\dot{\mathbf{R}}_{BA}-\alpha_4\dot{\mathbf{R}}_{BO_4}=\mathbf{0}$$

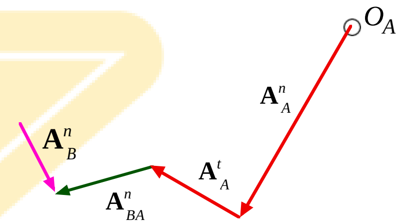
Acceleration polygon

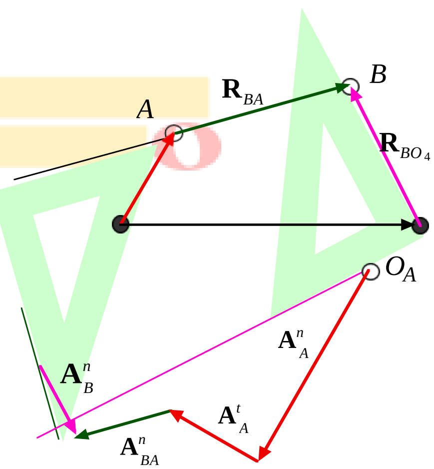


1. Select a point in a convenient position as the reference for zero acceleration. Name this point O_A (origin of accelerations).
2. Compute the magnitude of $\mathbf{A}_A^n$ as $R_{AO_2}\omega_2^2$. From O_V construct vector $\mathbf{A}_A^n$ in the opposite direction of $\mathbf{R}_{AO_2}$.
3. Compute the magnitude of $\mathbf{A}_A^t$ as $R_{AO_2}\alpha_2$. The direction of $\mathbf{A}_A^t$ is determined by rotating $\mathbf{R}_{AO_2}$ 90° in the direction of α_2. Add this vector to $\mathbf{A}_A^n$. Note that the sum of $\mathbf{A}_A^n$ and $\mathbf{A}_A^t$ is $\mathbf{A}_A$.

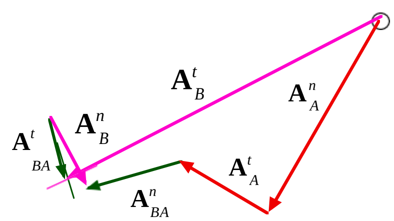


3. Compute the magnitude of $\mathbf{A}_{BA}^n$ as $R_{BA}\omega_3^2$. Add this vector in the opposite direction of $\mathbf{R}_{BA}$ to the other two vectors.
4. Compute the magnitude of $\mathbf{A}_B^n$ as $R_B\omega_4^2$. Note that $\mathbf{A}_B^n$ is in the opposite direction of $\mathbf{R}_{BO_4}$. Since $\mathbf{A}_B^n$ itself appears with a negative sign in the acceleration equation, it should be added to the other vectors in the diagram as shown; i.e., head-to-tail.





5. Since $\mathbf{A}_{BA}^t$ must be perpendicular to $\mathbf{R}_{BA}$, draw a line perpendicular to $\mathbf{R}_{BA}$ in anticipation of adding $\mathbf{A}_{BA}^t$ to the diagram.
6. Since $\mathbf{A}_B^t$ must be perpendicular to $\mathbf{R}_{BO_4}$, draw a line perpendicular to $\mathbf{R}_{BO_4}$ closing (completing) the polygon.
7. Construct vectors $\mathbf{A}_{BA}^t$ and $\mathbf{A}_B^t$ on the polygon.
8. Determine the magnitude of $\mathbf{A}_{BA}^t$ from the polygon. Compute α_3 as $\alpha_3 = A_{BA}^t / R_{BA}$ (in this diagram it is CW).



9. Determine the magnitude of $\mathbf{A}_B^t$ from the polygon. Compute α_4 as $\alpha_4 = A_B^t / R_{BO_4}$ (in this diagram it is CCW).

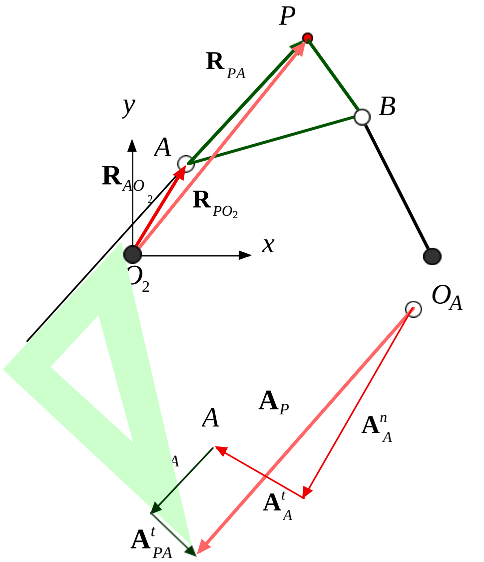
Secondary equation(s)

We can use the polygon method to determine the acceleration of a coupler point, such as *P*. It is assumed that all the angular velocities and accelerations have already been determined.

For the position vector $\mathbf{R}_{PO_2} = \mathbf{R}_{AO_2} + \mathbf{R}_{PA}$, the acceleration expression becomes

$$\mathbf{A}_P = \mathbf{A}_A + \mathbf{A}_{PA} = \mathbf{A}^n_A + \mathbf{A}^t_A + \mathbf{A}^n_{PA} + \mathbf{A}^t_{PA}$$
$$= -\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \dot{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{PA} + \alpha_3 \dot{\mathbf{R}}_{PA}$$

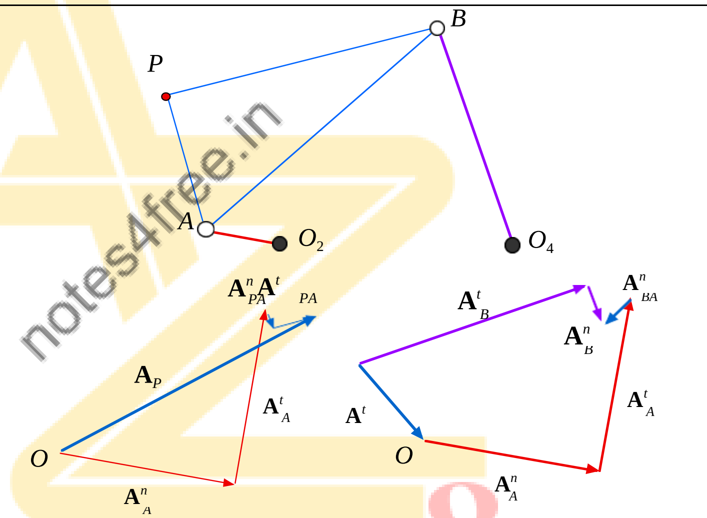
All four vectors can be constructed graphically. The vector sum is the acceleration of *P*.



Example FB-AP-1

This is a continuation of Example FB-VP-1. Assume an angular acceleration of $\alpha_2 = 1$ rad/sec CW for the crank.

Acceleration polygons are constructed and the following accelerations are obtained from the polygons: $\alpha_3 = 0.14$ CW, $\alpha_4 = 0.46$ CW, $A_P = 1.7$ in the direction shown on the polygon.



Slider-crank (inversion 1)

For a known slider-crank mechanism (inversion 1) in a given configuration and for known velocities, the acceleration of the crank, α_2, is given. Construct the acceleration polygons, then determine α_3 and the acceleration of the slider block. Assume α_2 is given to be CCW.

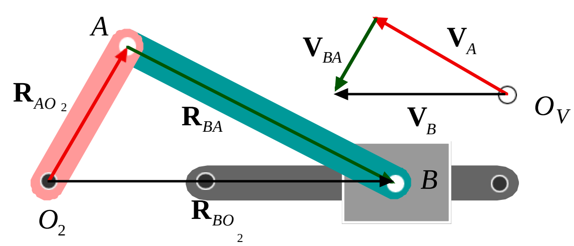
The position and velocity vector loop equations are:

$$\mathbf{R}_{AO_2} + \mathbf{R}_{BA} - \mathbf{R}_{BO_2} = \mathbf{0}$$
$$\mathbf{V}_A + \mathbf{V}_{BA} - \mathbf{V}_B = \mathbf{0}$$

Assume that all the velocities have already been obtained.

The acceleration equation is obtained from the time derivative of the velocity equation:

$$\mathbf{A}_A + \mathbf{A}_{BA} - \mathbf{A}_B = \mathbf{0}$$



Or, in terms of the components of the acceleration vectors, we have

$$\mathbf{A}_A^n + \mathbf{A}_A^t + \mathbf{A}_{BA}^n + \mathbf{A}_{BA}^t - \mathbf{A}_B^s = \mathbf{0}$$

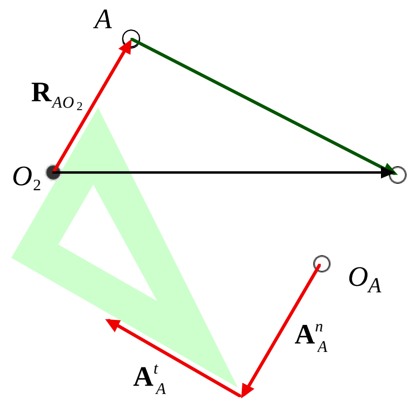
Or,

$$-\omega_2^2 \mathbf{R}_{AO_2} + \alpha_2 \dot{\mathbf{R}}_{AO_2} - \omega_3^2 \mathbf{R}_{BA} + \alpha_3 \dot{\mathbf{R}}_{BA} - \mathbf{A}_B^s = \mathbf{0}$$

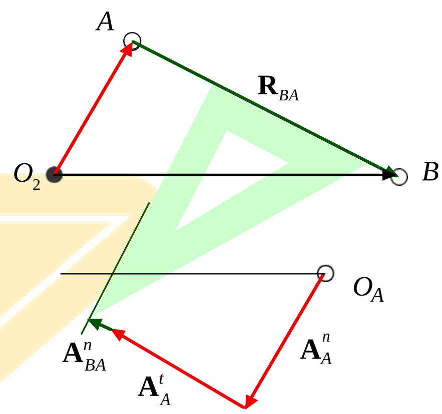
The first three components are completely and the last two components are partially known.

<u>Acceleration polygon</u>

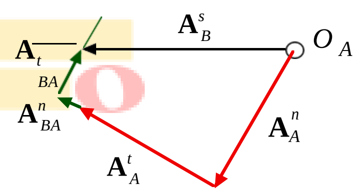
1. Select a point in a convenient position as the reference for zero acceleration, O_A.
2. Compute $A_A^n = R_{AO_2}\,\omega_2^2$. From O_V construct $\mathbf{A}_A^n$ in the opposite direction of $\mathbf{R}_{AO_2}$.
3. Compute $A_A^t = R_{AO_2}\,\alpha_2$. The direction of $\mathbf{A}_A^t$ is determined by rotating $\mathbf{R}_{AO_2}$ 90° in the direction of α_2. Add this vector to the diagram.

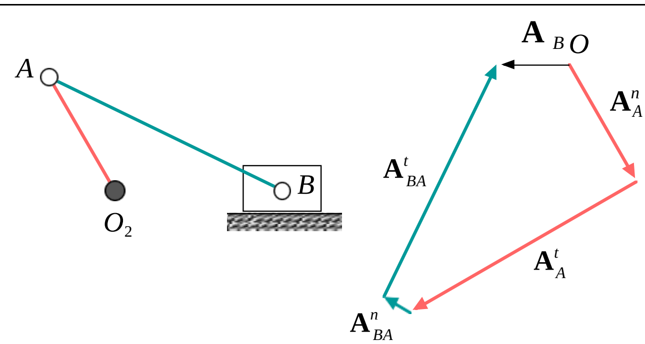


4. Compute $A_{BA}^n = R_{BA}\,\omega_3^2$. Construct $\mathbf{A}_{BA}^n$ in the opposite direction of $\mathbf{R}_{BA}$.
5. $\mathbf{A}_{BA}^t$ must be perpendicular to $\mathbf{R}_{BA}$. Draw a line perpendicular to $\mathbf{R}_{BA}$ in anticipation of adding $\mathbf{A}_{BA}^t$ to $\mathbf{A}_{BA}^n$.
6. From O_A draw a line parallel to the sliding axis. $\mathbf{A}_B$ must reside on this line.

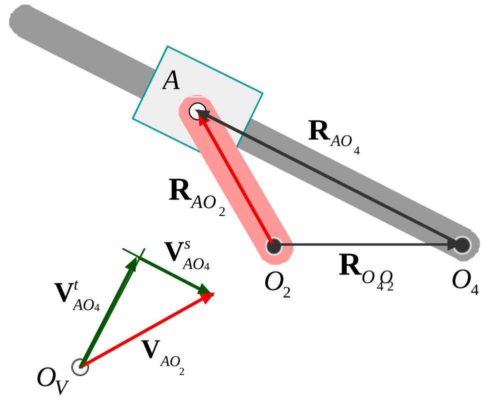


7. Construct vectors $\mathbf{A}_{BA}^t$ and $\mathbf{A}_B$
8. Determine the magnitude of $\mathbf{A}_{BA}^t$. Compute α_3 as $\alpha_3 = A_{BA}^t / R_{BA}$. Determine the direction of α_3 (in this example it is CCW).
9. Determine the magnitude of $\mathbf{A}_B$ from the polygon. The direction in this example is to the left.



Example SC_AP-1
Continue with Example SC-VP-1 from the velocity polygon section. Assume an angular acceleration of $\alpha_2 = 1$ rad/sec² CCW for the crank. Using the results from the velocity analysis, the acceleration polygon is constructed. The results are: $\alpha_3 = 1.0$ rad/sec² CCW; $A_B = 0.76$ to the left. 

Slider-crank (inversion 2)



For a known slider-crank mechanism (inversion 2), in a given configuration and for known velocities, the acceleration of the crank, α_2, is given (say CW). Construct the acceleration polygons and determine α_3.

We draw the slider-crank in the given configuration and define position vectors to form a vector loop equation:

$$\mathbf{R}_{AO_2} - \mathbf{R}_{AO_4} - \mathbf{R}_{O_4O_2} = \mathbf{0}$$

The velocity equation is:

$$\mathbf{V}^t_{AO_2} - \mathbf{V}^t_{AO_4} - \mathbf{V}^s_{AO_4} = \mathbf{0}$$

The velocity polygon for this mechanism has already been obtained; i.e., ω_4 and $\mathbf{V}^s_{AO_4}$ are assumed known.

The acceleration equation is obtained from the time derivative of the velocity equation:

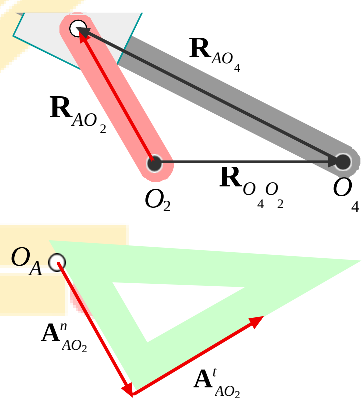
$$\mathbf{A}^n_{AO_2} + \mathbf{A}^t_{AO_2} - \mathbf{A}^n_{AO_4} - \mathbf{A}^t_{AO_4} - \mathbf{A}^s_{AO_4} - \mathbf{A}^c_{AO_4} = \mathbf{0}$$

Or,

$$-\omega_2^2\mathbf{R}_{AO_2} + \alpha_2\dot{\mathbf{R}}_{AO_2} - (-\omega_4^2\mathbf{R}_{AO_4}) - \alpha_4\dot{\mathbf{R}}_{AO_4} - \mathbf{A}^s_{AO_4} - 2\omega_4\dot{\mathbf{V}}^s_{AO_4} = \mathbf{0}$$

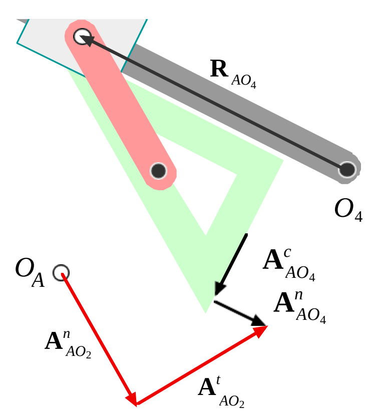
All the terms are fully known except for $\mathbf{A}^s_{AO_4}$ and $\mathbf{A}^t_{AO_4}$. Re-arranging the terms in order to have the partially known terms as the last two terms:

$$-\omega_2^2\mathbf{R}_{AO_2} + \alpha_2\dot{\mathbf{R}}_{AO_2} - (-\omega_4^2\mathbf{R}_{AO_4}) - 2\omega_4\dot{\mathbf{V}}^s_{AO_4}$$
$$- \mathbf{A}^s_{AO_4} - \alpha_4\dot{\mathbf{R}}_{AO_4} = \mathbf{0}$$



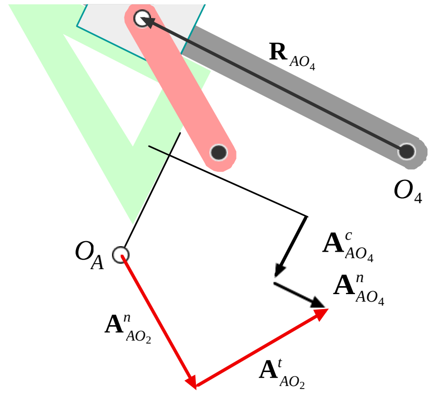
<u>Acceleration polygon</u>

1. Select the origin of accelerations, O_A, in a convenient position.
2. Compute $A^n_A = R_{AO_2}\omega_2^2$. From O_V construct vector $\mathbf{A}^n_A$ in the opposite direction of $\mathbf{R}_{AO_2}$.
3. Compute $A^t_A = R_{AO_2}\alpha_2$. Determine the direction of $\mathbf{A}^t_A$. Add this vector to $\mathbf{A}^n_A$.

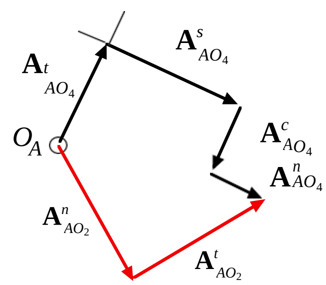


4. Compute $A^n_{AO_4} = R_{AO_4}\omega_4^2$. Construct vector $\mathbf{A}^n_{AO_4}$ in the opposite direction of $\mathbf{R}_{AO_4}$ and add it to the polygon.
5. Determine the Coriolis acceleration $\mathbf{A}^c_{AO_4}$. The magnitude of this vector is $2V^s_{AO_4}\omega_4$, and its direction is found by rotating $\mathbf{V}^s_{AO_4}$ 90° in the direction of ω_4. Add this vector to the polygon.

6. Draw an axis for $\mathbf{A}^s_{AO_4}$ parallel to $\mathbf{R}_{AO_4}$.
7. Draw another axis perpendicular to $\mathbf{R}_{AO_4}$ $\mathbf{A}^t_{AO_4}$ will be on this axis.



8. Construct vectors $\mathbf{A}^s_{AO_4}$ and $\mathbf{A}^t_{AO_4}$ to complete the polygon.
9. Determine the magnitude of $\mathbf{A}^t_{A_4O_4}$ from the polygon. Compute α_4 as $\alpha_4 = A^t_{A_4O_4} / R_{A_4O_4}$. Determine the direction of α_4 (in this example it is CW).
10. Determine the magnitude of $\mathbf{A}^s_{AO_4}$ from the polygon.



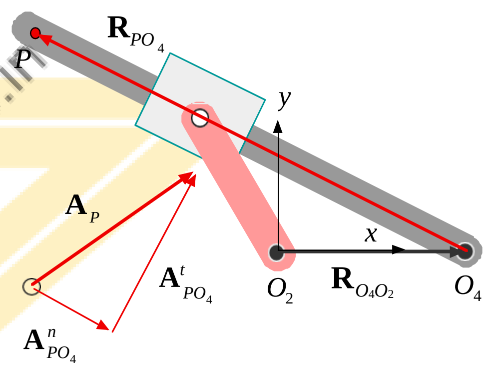
<u>Secondary point</u>

In order to determine the acceleration of point P on link 4, we express its acceleration as

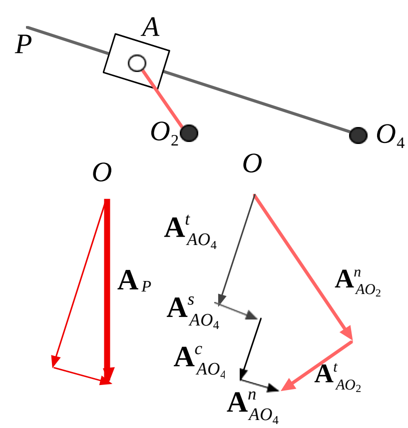
$$\mathbf{R}_P = \mathbf{R}_{O_4O_2} + \mathbf{R}_{PO_4}$$

$$\mathbf{A}_P = \mathbf{A}_{PO_4} = -\omega_4^2 \mathbf{R}_{PO_4} + \alpha_4 \dot{\mathbf{R}}_{PO_4}$$

Since the angular velocity and acceleration of link *4* are both known, the two components of the acceleration vector can be constructed.

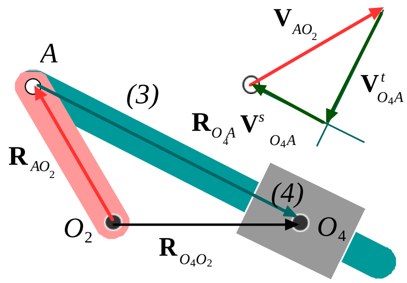


Example SC-AP-2
This is a continuation of Example SC-VP-2. Assume an angular acceleration of 0.5 rad/sec^2, CCW, for the crank. The acceleration polygon is constructed and the following accelerations are determined from the polygon: $\alpha_3 = \alpha_4 = 0.24$ rad/sec^2, CCW; $A^s_{AO_4} = 3.9$ in the direction shown. The acceleration of point P is determined from a second polygon. This acceleration has a magnitude of $A_P = 1.0$ in the direction shown.



Slider-crank (inversion 3)

For a known slider-crank mechanism (inversion 3), in a given configuration and for known angular velocity and acceleration of the crank, ω_2 and α_2 (assume both CW), construct the velocity and acceleration polygons, and then determine ω_3 and α_3.



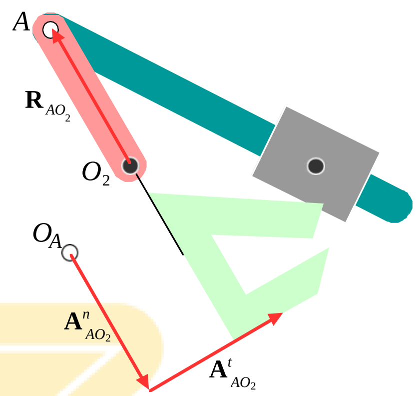
Draw the slider-crank in the given configuration and define position vectors to form a vector loop equation:

$$\mathbf{R}_{AO_2} + \mathbf{R}_{O_4A} - \mathbf{R}_{O_4O_2} = \mathbf{0}$$

The velocity equation is

$$\mathbf{V}^t_{AO_2} + \mathbf{V}^t_{O_4A} + \mathbf{V}^s_{O_4A} = \mathbf{0}$$

The velocity polygon for this mechanism has already been obtained. From this polygon, ω_3 and the velocity of the slider-block have been determined.



The acceleration equation is obtained from the time derivative of the velocity equation:

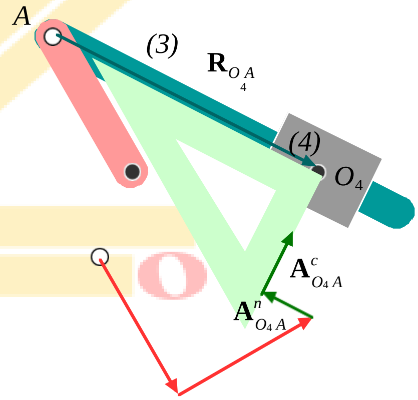
$$\mathbf{A}^n_{AO_2} + \mathbf{A}^t_{AO_2} + \mathbf{A}^n_{O_4A} + \mathbf{A}^t_{O_4A} + \mathbf{A}^s_{O_4A} + \mathbf{A}^c_{O_4A} = \mathbf{0}$$

or,

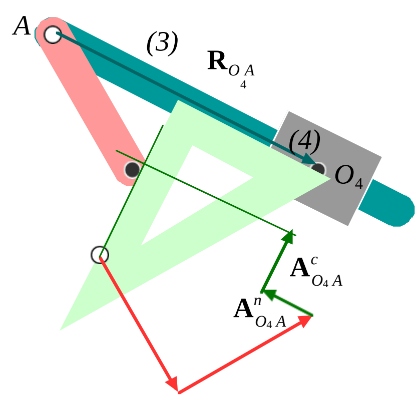
$$-\omega_2^2\mathbf{R}_{AO_2} + \alpha_2\dot{\mathbf{R}}_{AO_2} - \omega_3^2\mathbf{R}_{O_4A} + 2\omega_3\dot{\mathbf{V}}^s_{O_4A} + \mathbf{A}^s_{O_4A} + \alpha_3\dot{\mathbf{R}}_{O_4A} = \mathbf{0}$$

<u>Acceleration polygon</u>

1. Select the origin of accelerations, O_A, in a convenient position.
2. Compute $A^n_A = R_{AO_2}\,\omega_2^2$. From O_V construct vector $\mathbf{A}^n_A$ in the opposite direction of $\mathbf{R}_{AO_2}$.
3. Compute $A^t_A = R_{AO_2}\alpha_2$. Determine the direction of $\mathbf{A}^t_A$ based on the direction of α_2. Add this vector to $\mathbf{A}^n_A$.



4. Compute $A^n_{O_4A} = \omega_3^2 R_{O_4A}$. The direction of $\mathbf{A}^n_{O_4A}$ is opposite of $\mathbf{R}_{O_4A}$. Add this vector to the polygon.
5. Determine the Coriolis acceleration $\mathbf{A}^c_{O_4A}$. The magnitude of this vector is $2\omega_3 V^s_{O_4A}$, and its direction is found by rotating $\mathbf{V}^s_{O_4A}$ 90° in the direction of ω_3. Add this vector to the polygon.



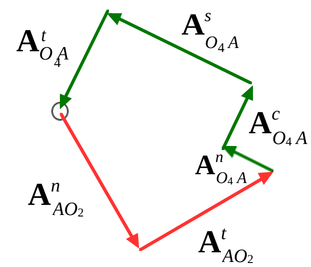
6. Draw an axis for $\mathbf{A}^s_{O_4A}$ parallel to $\mathbf{R}_{AO_4}$.
7. Draw an axis for $\mathbf{A}^t_{O_4A}$ perpendicular to $\mathbf{R}_{AO_4}$.

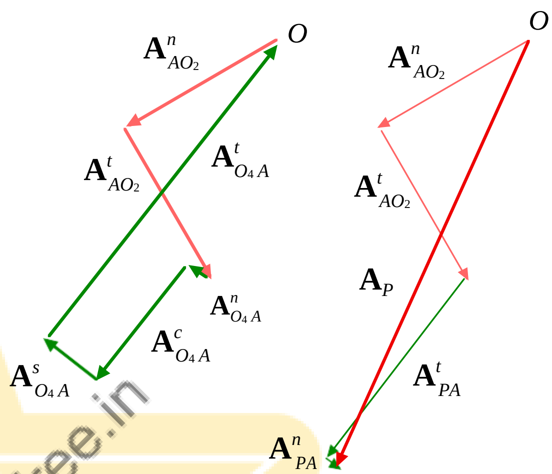
8. Construct vectors $\mathbf{A}^t_{AO_3}$ and $\mathbf{A}^s_{O_3O_4}$.

9. Determine the magnitude of $\mathbf{A}^t_{O_4A}$ from the polygon.

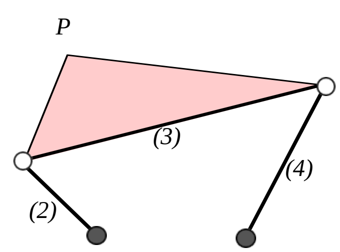
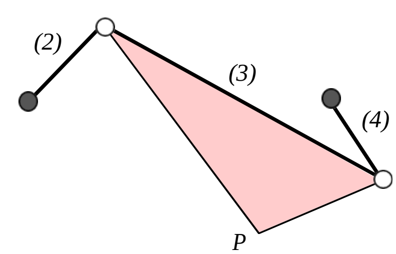
Compute α_3 as $\alpha_3 = A^t_{O_4A} / R_{O_4A}$.

10. Determine the direction of α_3. In this example it is CW since $\mathbf{R}_{O_4A}$ must rotate 90° CW to line up with $\mathbf{A}^t_{O_4A}$.

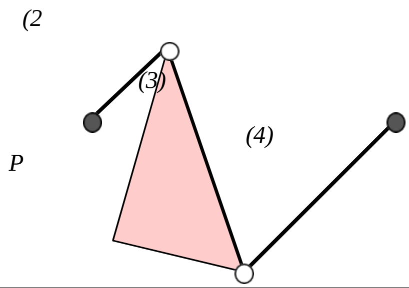


Example SC-AP-3
This is a continuation of Example SC-VP-3. Assume an angular acceleration of 1 rad/sec² CW for the crank. Acceleration polygon (right) is constructed and the following accelerations are determined from the polygon: $\alpha_3 = \alpha_4 = 2.67$ rad/sec² CW; $A^s_{O_4A} = 3.8$, and A_P = 0.4 in the direction shown. A second polygon (left) is constructed to determine the acceleration of point P as: $A_P = 2.7$ in the direction shown. 

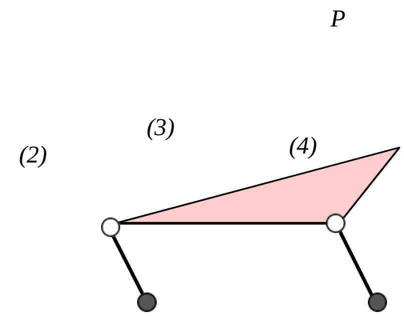
Exercises

In these exercises take direct measurements from the figures for link lengths and the magnitudes of velocity and acceleration vectors.	
Exercises P.1 – P.4 are examples of four-bar mechanism. Assume ω_2 and α_2 are given. Determine α_3, α_4, and $\mathbf{A}_P$.	
P.1 	P.2 

P.3

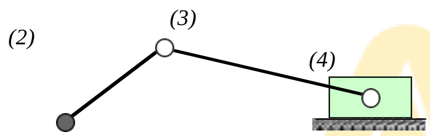


P.4

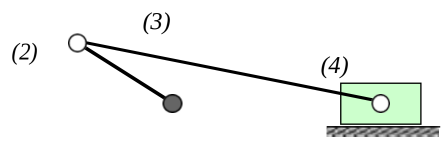


Exercises P.5 – P.8 are examples of slider-crank mechanism. Assume ω_2 and α_2 are given: For P.5 and P.6 determine α_3, α_4, and the acceleration of the slider block; For P.7 and P.8 determine α_3, α_4, and $\mathbf{A}_P$.

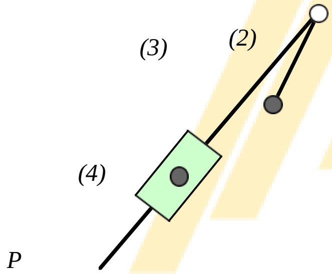
P.5



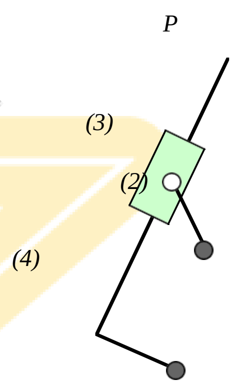
P.6



P.7

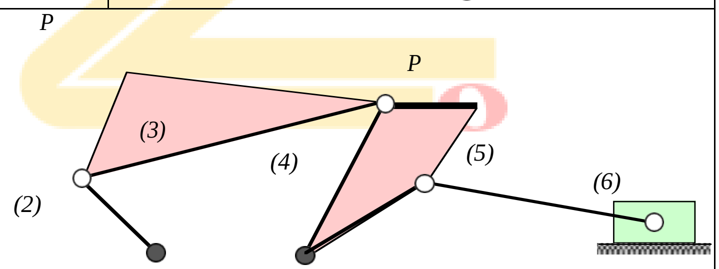


P.8



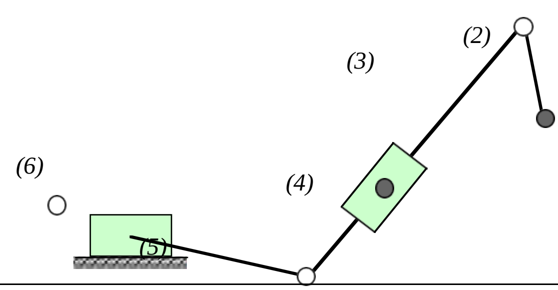
P.9

For this six-bar mechanism ω_2 and α_2 are given. Determine α_5, acceleration of *P*, and the acceleration of the slider block *6*.



P.10

For this six-bar mechanism ω_2 and α_2 are given. Determine α_5 and the acceleration of the slider block *6*.

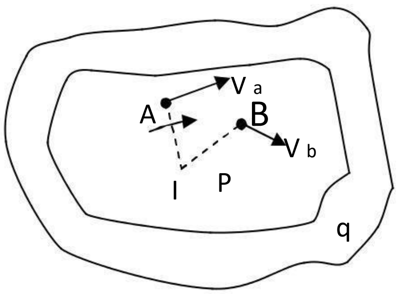


Definition, Kennedy's Theorem

II Method Instantaneous

- **Method**

To explain instantaneous centre let us consider a plane body P having a nonlinear motion relative to another body q consider two points A and B on body P having velocities as V_a and V_b respectively in the direction shown.



If a line is drawn to V_a, at A the body can be imagined to rotate about some point on the line. Thirdly, centre of rotation of the body also lies on a line to the direction of V_b at B. If the intersection of the two lines is at I, the body P will be rotating about I at that instant. The point I is known as the instantaneous centre of rotation for the body P. The position of instantaneous centre changes with the motion of the body.

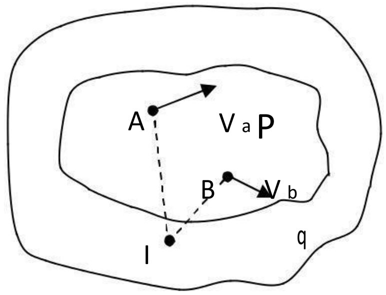


Fig. 2

In case of the □r lines drawn from A and B meet outside the body P as shown in Fig 2.

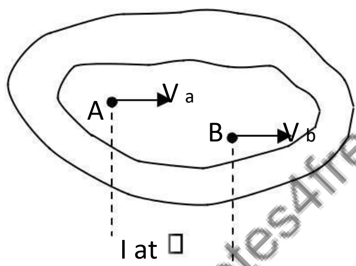


Fig. 3

If the direction of V_a and V_b are parallel to the □r at A and B met at □. This is the case when the body has linear motion.

- **Number of Instantaneous Centers**

  The number of instantaneous centers in a mechanism depends upon number of links. If N is the number of instantaneous centers and n is the number of links.

- **Types of Instantaneous Centers**

  There are three types of instantaneous centers namely fixed, permanent and neither fixed nor permanent.

Fixed instantaneous center I_{12}, I_{14}

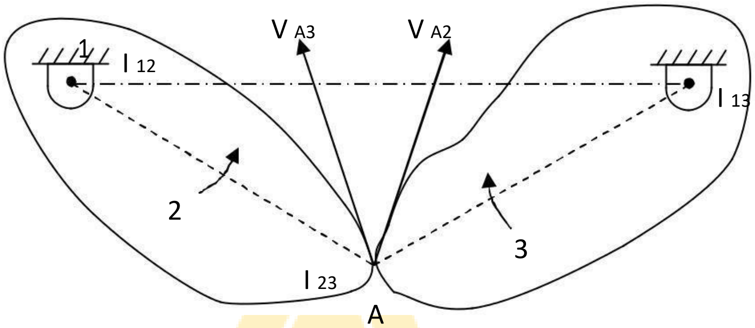
Permanent instantaneous center I_{23}, I_{34}

Neither fixed nor permanent instantaneous center I_{13}, I_{24}

- **Arnold Kennedy theorem of three centers:**

Statement: If three bodies have motion relative to each other, their instantaneous centers should lie in a straight line.

Proof:



Consider a three link mechanism with link 1 being fixed link 2 rotating about I_{12} and link 3 rotating about I_{13}. Hence, I_{12} and I_{13} are the instantaneous centers for link 2 and link 3. Let us assume that instantaneous center of link 2 and 3 be at point A i.e. I_{23}. Point A is a coincident point on link 2 and link 3.

Considering A on link 2, velocity of A with respect to I_{12} will be a vector V_{A2}

link A I_{12}. Similarly for point A on link 3, velocity of A with respect to I_{13} will be I_{13}. It seen that velocity vector of V_{A2} and V_{A3} are in different directions which is impossible. Hence, the instantaneous center of the two links cannot be at the assumed position.

It can be seen that when I_{23} lies on the line joining I_{12} and I_{13} the V_{A2} and V_{A3} will be same in magnitude and direction. Hence, for the three links to be in relative motion all the three centers should lie in a same straight line. Hence, the proof.

Determination of linear and angular velocity using instantaneous center method

Steps to locate instantaneous centers:

Step 1: Draw the configuration diagram.

Step 2: Identify the number of instantaneous centers by using the relation □n □1 n

N 2

Step 3: Identify the instantaneous centers by circle diagram.

Step 4: Locate all the instantaneous centers by making use of Kennedy's theorem.

To illustrate the procedure let us consider an example.

A slider crank mechanism has lengths of crank and connecting rod equal to 200 mm and 200 mm respectively locate all the instantaneous centers of the mechanism for the position of the crank when it has turned through 30^o from IOC. Also find velocity of slider and angular velocity of connecting rod if crank rotates at 40 rad/sec.

Step 1: Draw configuration diagram to a suitable scale.

Step 2: Determine the number of links in the mechanism and find number of instantaneous centers.

Module 4

Velocity and Acceleration Analysis of Mechanisms

SYLLUBUS:

"Displacement, velocity and acceleration - analysis in simple mechanisms - Graphical Method velocity and acceleration polygons - Kinematic analysis by Complex Algebra methods-Vector Approach, Computer applications in the Kinematic analysis of simple mechanisms-Coincident points- Coriolis Acceleration".

Content:

- Describe a mechanism.
- Define relative and absolute velocity.
- Define relative and absolute acceleration.
- Define radial and tangential velocity.
- Define radial and tangential acceleration.
- Describe a four bar chain.
- Solve the velocity and acceleration of points within a mechanism.
- Use mathematical and graphical methods.
- Construct velocity and acceleration diagrams.
- Define the Coriolis Acceleration.
- Solve problems involving sliding links.

It is assumed that the student is already familiar with the following concepts.

- Vector diagrams.
- Simple harmonic motion.
- Angular and linear motion.
- Inertia force.
- Appropriate level of mathematics.

INTRODUCTION

A mechanism is used to produce mechanical transformations in a machine. This transformation could be any of the following.

- It may convert one speed to another speed.
- It may convert one force to another force.
- It may convert one torque to another torque.
- It may convert force into torque.
- It may convert one angular motion to another angular motion.
- It may convert angular motion into linear motion.
- It may convert linear motion into angular motion.

A good example is a crank, connecting rod and piston mechanism.

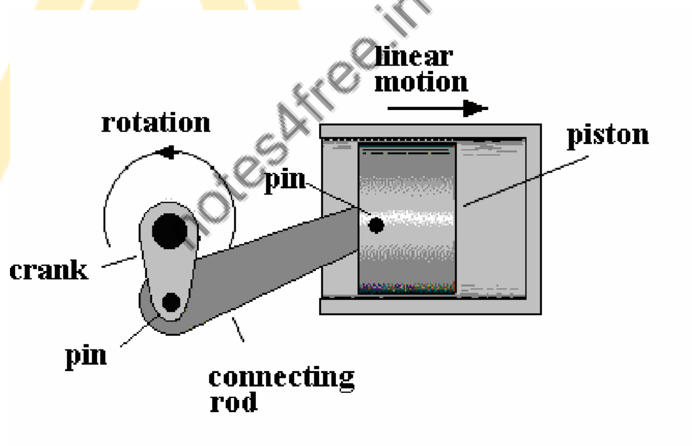


Figure 1

- If the crank is turned, angular motion is converted into linear motion of the piston and input torque is transformed into force on the piston.
- If the piston is forced to move, the linear motion is converted into rotary motion and the force into torque. The piston is a sliding joint and this is called ***PRISMATIC*** in some fields of engineering such as robotics.
- The pin joints allow rotation of one part relative to another. These are also called ***REVOLUTE*** joints in other areas of engineering.

Consider the next mechanism used in shaping machines and also known as the

Whitworth quick- return mechanism.

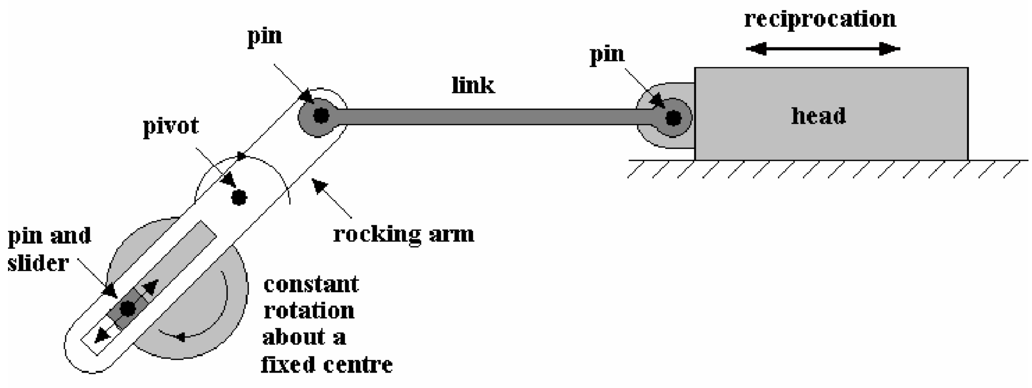


Figure 2

- The input is connected to a motor turning at constant speed. This makes the rocking arm move back and forth and the head (that carries the cutting tool) reciprocates back and forth.
- Depending on the lengths of the various parts, the motion of the head can be made to move forwards at a fairly constant cutting speed but the return stroke is quick.
- Note that the pin and slider must be able to slide in the slot or the mechanism would jam. This causes problems in the solution because of the sliding link and this is covered later under Coriolis acceleration.
- The main point is that the motion produced is anything but simple harmonic motion and at any time the various parts of the mechanism have a displacement, velocity and acceleration.
- The acceleration gives rise to inertia forces and this puts stress on the parts in addition to the stress produced by the transmission of power.
- For example the acceleration of a piston in an internal combustion engine can be enormous and the connecting rod is subjected to high stresses as a result of the inertia as well as due to the power transmission.
- You will find in these studies that the various parts are referred to as links and it can be shown that all mechanisms are made up of a series of four links.
- The basic four bar link is shown below. When the input link rotates the output link may for example swing back and forth. Note that the fourth link is the frame of the machine and it is rigid and unable to move.

- With experience you should be able to identify the four bar chains in a mechanism. All the links shown are rigid links which means they may push or pull. It is possible to have links made of chain or rope which can only pull.

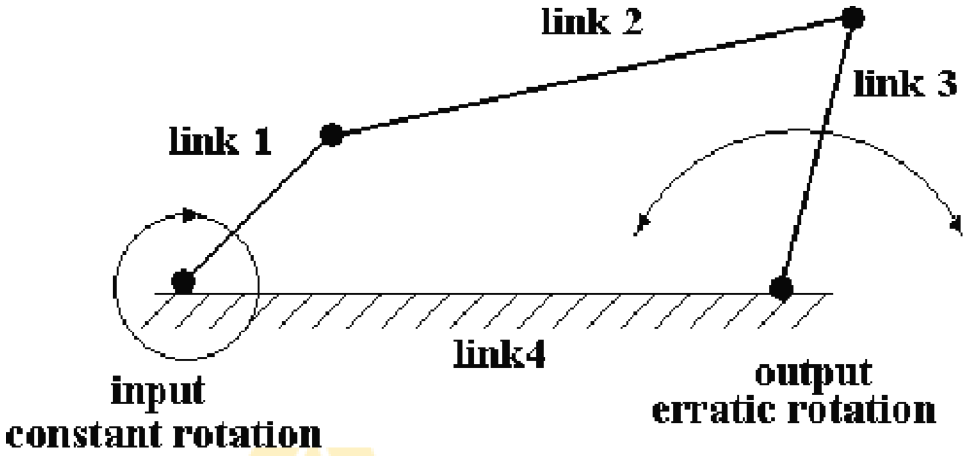
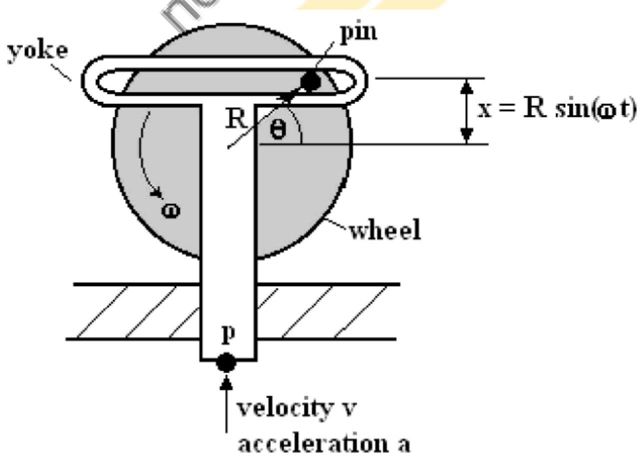


Figure 3

2. **DISPLACEMENT, VELOCITY AND ACCELERATION**

- All parts of a mechanism have displacement, velocity and acceleration. In the tutorial on free vibration, a mechanism called the Scotch Yoke was examined in order to explain sinusoidal or harmonic motion.
- The wheel turns at a constant speed and the yoke moves up and down.



3. It was shown that the displacement 'x', velocity 'v' and acceleration 'a' of point p was given as follows. Angle 0 = mt
4. Displacement x = R sin (mt).
5. Velocity v = dx/dt = mR cos(mt)
6. Acceleration a = dv/dt =m^2Rsin(mt)

- The values can be calculated for any angle or moment of time. The acceleration could then be used to calculate the inertia force needed to accelerate and decelerate the link.
- Clearly it is the maximum values that are needed. Other mechanisms can be analyzed mathematically in the same way but it is more difficult.
- The starting point is to derive the equation for displacement with respect to angle or time and then differentiate twice to get the acceleration.
- Without the aid of a computer to do this, the mathematics is normally much too difficult and a graphical method should be used as shown later.

7. VELOCITY DIAGRAMS

This section involves the construction of diagrams which needs to be done accurately and to a suitable scale. Students should use a drawing board, ruler, compass, protractor and triangles and possess the necessary drawing skills.

ABSOLUTE AND RELATIVE VELOCITY

An absolute velocity is the velocity of a point measured from a fixed point (normally the ground or anything rigidly attached to the ground and not moving). Relative velocity is the velocity of a point measured relative to another that may itself be moving.

TANGENTIAL VELOCITY

Consider a link A B pinned at A and revolving about A at angular velocity m. Point B moves in a circle relative to point A but its velocity is always tangential and hence at 90^o to the link. A convenient method of denoting this tangential velocity is (vB)A meaning the velocity of B relative to A. This method is not always suitable.

Figure 5

❖ Consider a sliding link C that can slide on link AB. The direction can only be radial relative to point A as shown.

❖ If the link AB rotates about A at the same time then link C will have radial and tangential velocities.

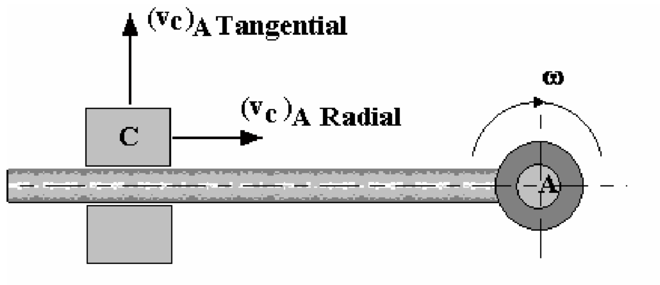


Figure 6

❖ Note that both the tangential and radial velocities are denoted the same so the tags radial and tangential are added.

❖ The sliding link has two relative velocities, the radial and the tangential. They are normal to each other and the true velocity relative to A is the vector sum of both added as shown.

❖ ***Note that lower case letters are used on the vector diagrams.*** The two vectors are denoted by c_1 and c_2. The velocity of link C relative to point A is the vector a c_2.

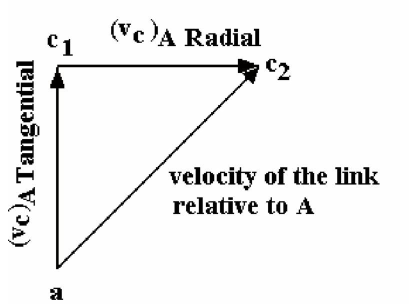


Figure 7

CRANK, CONNECTING ROD AND PISTON

Consider this mechanism again. Let's freeze the motion (snap shot) at the position shown. The diagram is called a space diagram.

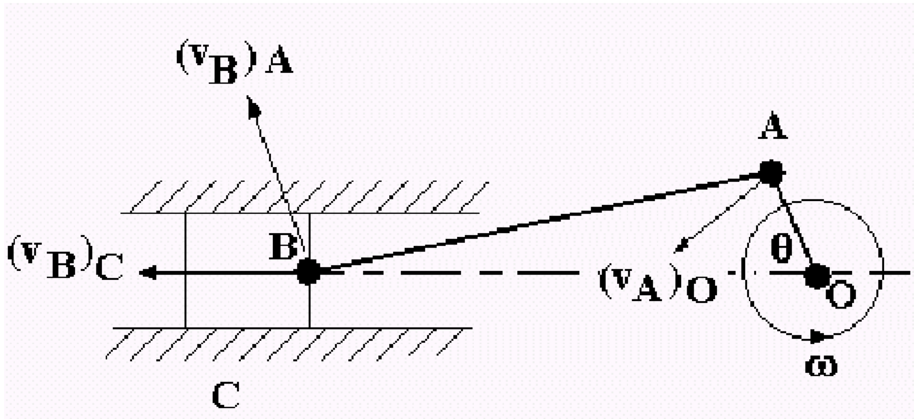
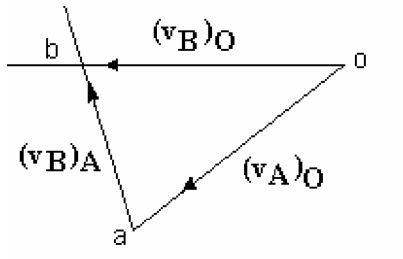


Figure 8

- Every point on every link has a velocity through space. First we label the centre of rotation, often this is the letter O. Point A can only move in a tangential direction so the velocity of A relative to O is also its absolute velocity and the vector is normal to the crank and it is designated $(v_A)_O$. (Note the rotation is anticlockwise).
- Now suppose that you are sat at point A and everything else moves relative to you. Looking towards B, it would appear the B is rotating relative to you (in reality it is you that is rotating) so it has a tangential velocity denoted $(V_B)_A$.
- The direction is not always obvious except that it is normal to the link.Consider the fixed link OC. Since both points are fixed there is no velocity between them so $(v_C)_O = 0$.
- Next consider that you at point C looking at point B. Point B is a sliding link and will move in a straight line in the direction fixed by the slider guides and this is velocity $(v_B)_C$. It follows that the velocity of B seen from O is the same as that seen from C so $(v_B)_C = (v_B)_O$.
- The absolute velocity of B is $(v_B)_C = (v_B)_O$ and this must be the vector sum of $(V_A)_O$ and $(v_B)_A$ and the three vectors must form a closed triangle as shown. The velocity of the piston must be in the direction in which it slides (conveniently horizontal here). This is a velocity diagram.



- First calculate the tangential velocity $(v_A)_O$ from v = m x radius = m x OA
- Draw the vector o - a in the correct direction (note lower case letters).
- We know that the velocity of B relative to A is to be added so the next vector ab starts at point a. At point a draw a line in the direction normal to the connecting rod but of unknown length.
- We know that the velocity of B relative and absolute to O is horizontal so the vector ob must start at a. Draw a horizontal line (in this case) through o to intersect with the other line. This is point b. The vectors ab and ob may be measured or calculated. Usually it is the velocity of the slider that is required.
- In a design problem, this velocity would be evaluated for many different positions of the crank shaft and the velocity of the piston determined for each position.
- Remember that the slider direction is not always horizontal and the direction of o - b must be the direction of sliding.

Worked Examples

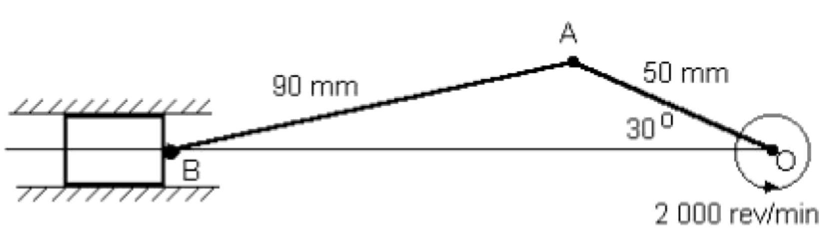
- The mechanism shown has a crank 50 mm radius which rotates at 2000 rev/min. Determine the velocity of the piston for the position shown. Also determine the angular velocity of link AB about A.
- 

Figure 10

SOLUTION

- Note the diagrams are not drawn to scale. The student should do this using a suitable scale for example 1 cm = 1 m/s.
- This is important so that the direction at 90^o to the link AB can be transferred to the velocity diagram.
- Angular speed of the crank m = 2wN/60 = 2w x 2000/60 = 209.4 rad/s $(v_A)_O$ = m x radius = 209.4 x 0.05 = 10.47 m/s.

First draw vector oa. (Diagram a)

- Next add a line in the direction ab (diagram b)
- Finally add the line in the direction of ob to find point b and measure ob to get the velocity.

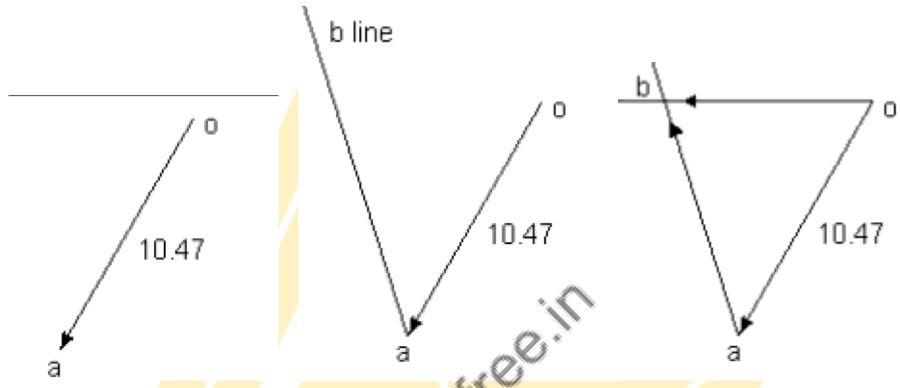


Figure 11a Figure 11b Figure 11c

- The velocity of B relative to O is 7 m/s.
- The tangential velocity of B relative to A is the vector ab and this gives 9.2 m/s.
- The angular velocity of B about A is found by dividing by the radius (length of AB).
- m for AB is then 9.2/0.09 = 102.2 rad/s. (note this is relative to A and not an absolute angular velocity)

BAR CHAIN

- The input link rotates at a constant angular velocity m_1. The relative velocity of each point relative to the other end of the link is shown.
- Each velocity vector is at right angles to the link. The output angular velocity is m_2 and this will not be constant. The points A and D are fixed so they will appear as the same point on the velocity diagram.

- The methodology is the same as before and best shown with another example.

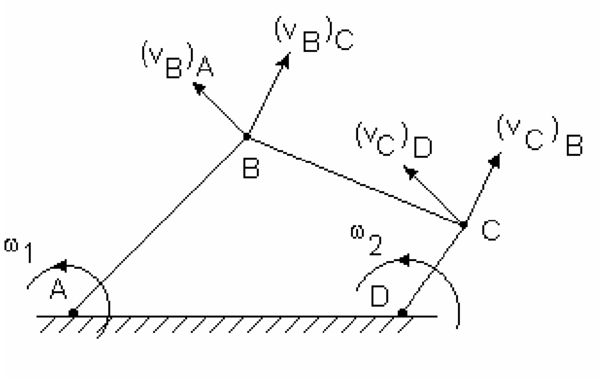
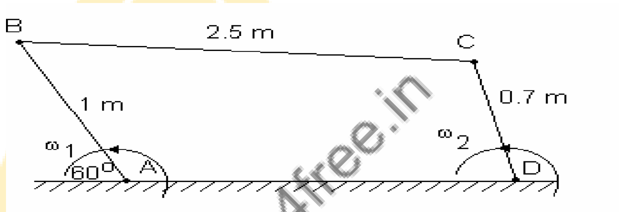


Figure 12

WORKED EXAMPLE No. 2

Find the angular velocity of the output link when the input rotates at a constant speed of 500 rev/min. The diagram is not to scale.



SOLUTION

First calculate m_1.

m_1 = 2w x 500/60 = 52.36 rad/s.

Next calculate the velocity of point B relative to A. $(V_B)_A$ = m_1 x AB = 52.36 x 1 = 52.36 m/s.

Draw this as a vector to an appropriate scale.

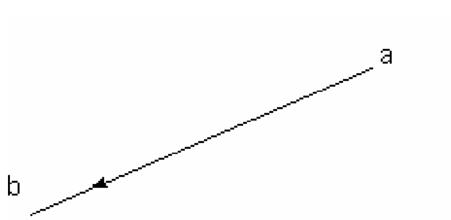


Figure 14a

Next draw the direction of velocity C relative to B at right angles to the link BC passing through point b on the velocity diagram.

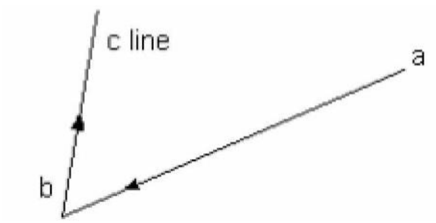


Figure 14 b

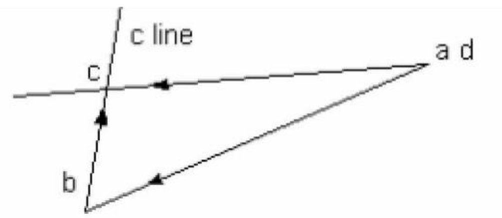


Figure 14 c

- Next draw the direction of the velocity of C relative to D at right angles to link DC passing through point a (which is the same as point d). Point c is where the two lines intersect,
- Determine velocity cd by measurement or any other method. The velocity of point C relative to D and is 43.5 m/s.
- Convert this into angular velocity by dividing the length of the link DC into it.

$$m_2 = 43.5/0.7 = 62 \text{ rad/s.}$$

4. ACCELERATION DIAGRAMS

- It is important to determine the acceleration of links because acceleration produces inertia forces in the link which stress the component parts of the mechanism.
- Accelerations may be relative or absolute in the same way as described for velocity.
- We shall consider two forms of acceleration, tangential and radial. Centripetal acceleration is an example of radial.

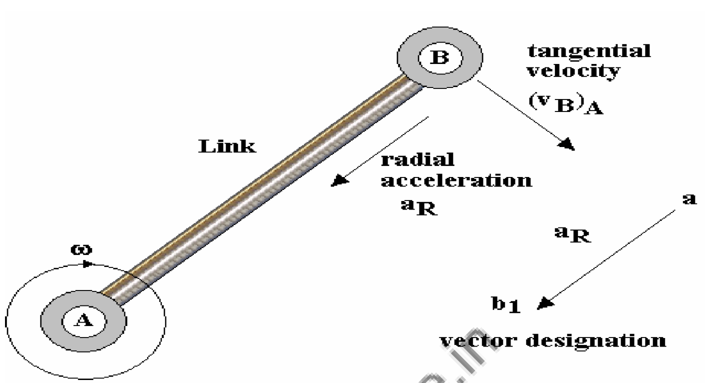
CENTRIPETAL ACCELERATION

- A point rotating about a centre at radius R has a tangential velocity v and angular velocity m and it is continually accelerating towards the centre even though it never moves any closer. This is centripetal acceleration and it is caused by the constant change in direction.
- It follows that the end of any rotating link will have a centripetal acceleration towards the opposite end.

The relevant equations are: $v = mR$ $a = m^2 R$ or $a = v^2/R$.

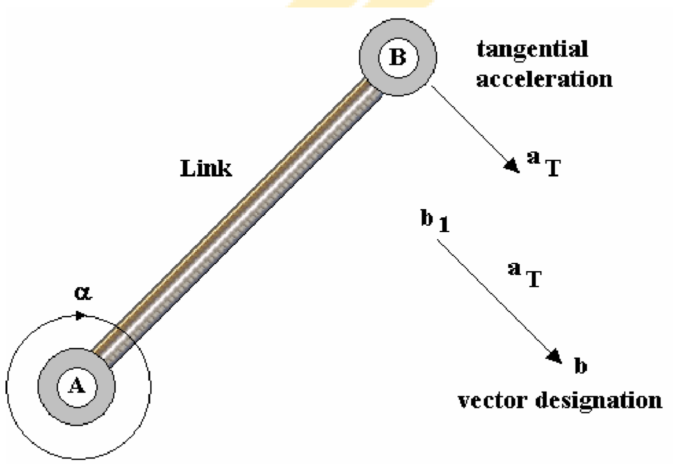
- The construction of the vector for radial acceleration causes confusion so the rules must be strictly followed. Consider the link AB. The velocity of B relative to A is tangential (vB)A.
- The centripetal acceleration of B relative to A is in a radial direction so a suitable notation might be aR. It is calculated using aR = m x AB or aR = v^2/AB.

Note the direction is towards the centre of rotation but the vector starts at a and ends at b_1. It is very important to get this the right way round otherwise the complete diagram will be wrong.



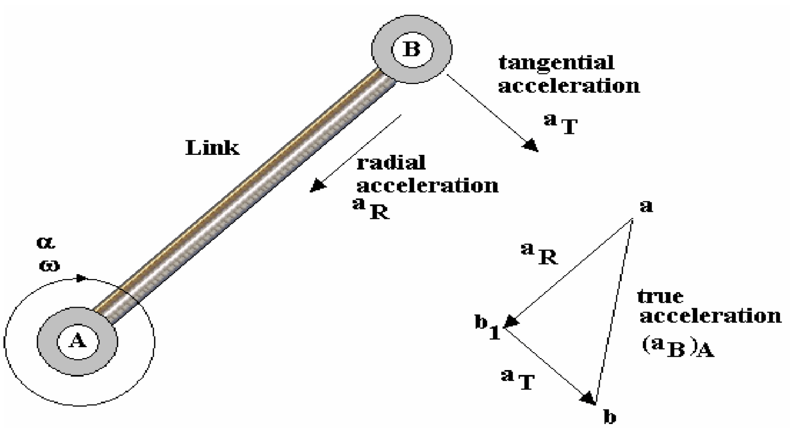
TANGENTIAL ACCELERATION

Tangential acceleration only occurs if the link has an angular acceleration a rad/s^2. Consider a link AB with an angular acceleration about A.



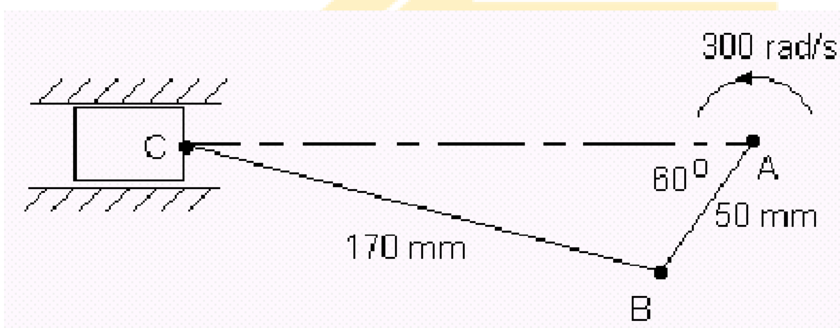
- Point B will have both radial and tangential acceleration relative to point A. The true acceleration of point B relative to A is the vector sum of them. This will require an extra point. We will use b_1 and b on the vector diagram as shown.

- ❖ Point B is accelerating around a circular path and its direction is tangential (at right angles to the link). It is designated aT and calculated using aT = a x AB.
- ❖ The vector starts at b1 and ends at b. The choice of letters and notation are arbitrary but must be logical to aid and relate to the construction of the diagram.



WORKED EXAMPLE No.3

A piston, connecting rod and crank mechanism is shown in the diagram. The crank rotates at a constant velocity of 300 rad/s. Find the acceleration of the piston and the angular acceleration of the link BC. The diagram is not drawn to



scale.

SOLUTION:

First calculate the tangential velocity of B relative to A. $(v_B)_A$ = m x radius = 300 x 0.05 = 15 m/s.

Next draw the velocity diagram and determine the velocity of C relative to B.

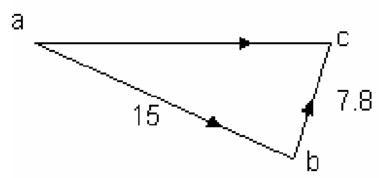


Figure 19

From the velocity diagram $(v_C)_B = 7.8$ m/s

- Next calculate all accelerations possible and construct the acceleration diagram to find the acceleration of the piston.
- The tangential acceleration of B relative to A is zero in this case since the link has no angular acceleration (a = 0).
- The centripetal acceleration of B relative to A

$$a_R = m^2 x\ AB = 300^2\ x\ 0.05 = 4500\ m/s^2.$$

The tangential acceleration of C relative to B is unknown.

The centripetal acceleration of C to B

$$a_R = v^2/BC = 7.8^2\ /0.17 = 357.9\ m/s^2.$$

The stage by stage construction of the acceleration diagram is as follows.

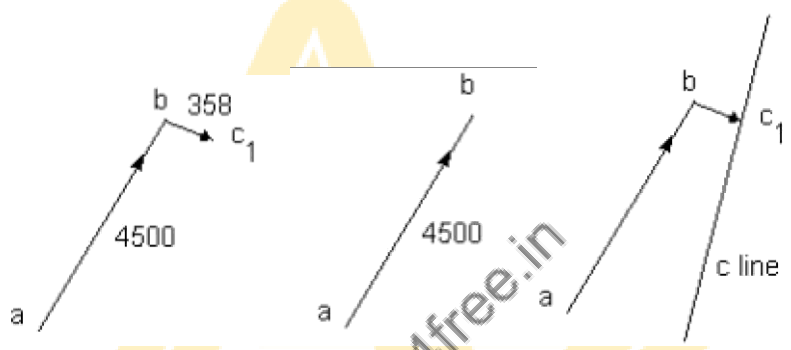


Figure 20a **Figure 20b** **Figure 20c**

- First draw the centripetal acceleration of link AB (Fig.a). There is no tangential acceleration so designate it ab. Note the direction is the same as the direction of the link towards the centre of rotation but is starts at a and ends at b.
- Next add the centripetal acceleration of link BC (Figure b). Since there are two accelerations for point C designate the point c_1. Note the direction is the same as the direction of the link towards the centre of rotation.
- Next add the tangential acceleration of point C relative to B (Figure c). Designate it c_1 c. Note the direction is at right angles to the previous vector and the length is unknown. Call the line a c line.
- Next draw the acceleration of the piston (figure d) which is constrained to be in the horizontal direction. This vector starts at a and must intersect the c line. Designate this point c.

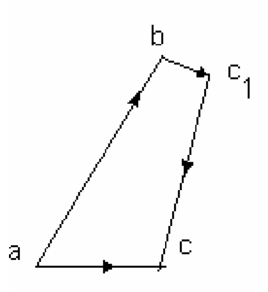


Figure 20d

- The acceleration of the piston is vector ac so $(a_C)_B = 1505\ m/s^2$. The tangential acceleration of C relative to B is $c_1 c = 4000\ m/s^2$.
- At the position shown the connecting rod has an angular velocity and acceleration about its end even though the crank moves at constant speed.
- The angular acceleration of BC is the tangential acceleration divided by the length BC.

$$a_{(BC)} = 4000 / 0.17 = 23529\ rad/s^2.$$

WORKED EXAMPLE No.4

The diagrams shows a "rocking lever" mechanism in which steady rotation of the wheel produces an oscillating motion of the lever OA. Both the wheel and the lever are mounted in fixed centers. The wheel rotates clockwise at a uniform angular velocity (ω) of 100 rad/s. For the configuration shown, determine the following.

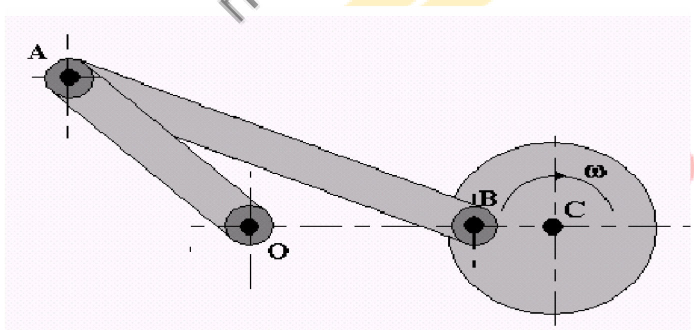


Figure 21

(i) The angular velocity of the link AB and the absolute velocity of point A.

(ii) The centrifugal accelerations of BC, AB and OA.

(iii) The magnitude and direction of the acceleration of point A. The lengths of the links are as follows.

BC = 25 mm AB = 100 mm OA = 50 mm OC = 90 mm

SOLUTION

The solution is best done graphically. First draw a line diagram of the mechanism to scale. It should look like this.

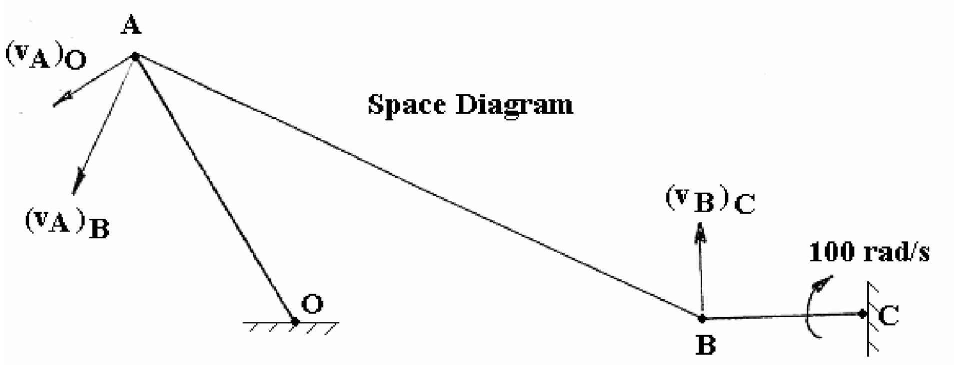


Figure 22

Next calculate the velocity of point B relative to C and construct the velocity diagram.

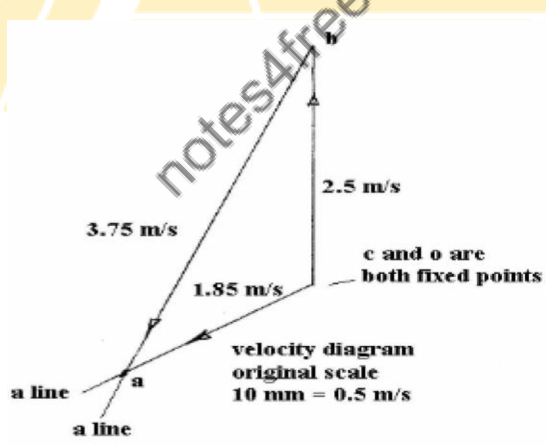


Figure 23

$(v_B)C$ = m x radius = 100 x 0.025 = 2.5

m/s Scale the following velocities from the diagram.

$(v_A)O$ = 1.85 m/s {answer (i)} (vA)B = 3.75 m/s

Angular velocity = tangential velocity/radius

For link AB, m = 3.75/0.1 = 37.5 rad/s. {answer (i)} Next calculate all the accelerations possible.

- Radial acceleration of BC = m^2 x BC = 100^2 x 0.025 = 250 m/s^2.
- Radial acceleration of AB = v^2/AB = 3.75^2/0.1 = 140.6 m/ s^2.
- Check same answer from m^2 x AB = 37.5^2 x 0.1 = 140.6 m/s^2.
- Radial Acceleration of OA is v^2/OA = 1.85^2/0.05 = 68.45 m/

s^2. Construction of the acceleration diagram gives the result shown.

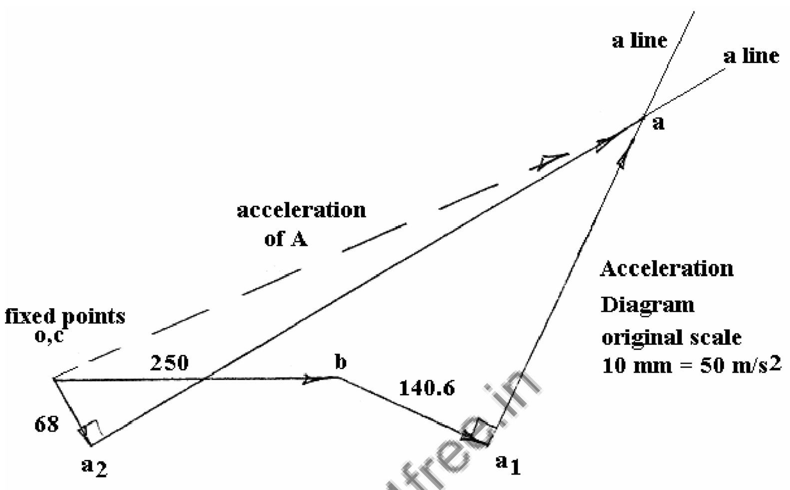


Figure 24

***The acceleration of point A is the vector o- a shown as a dotted line. Scaling this we get 560 m/s²*.**

MODULE-4
CAMS

Cams

Features

20.1. Introduction

A ***cam*** is a rotating machine element which gives reciprocating or oscillating motion to another element known as ***follower***. The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at uniform speed by a shaft, but the follower motion is pre- determined and will be according to the shape of the cam. The cam and follower is one of the simplest as well as one of the most important mechanisms found in modern machinery today. The cams are widely used for operating the inlet and exhaust valves of internal combustion engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathes etc.

20.2. Classific ation of Followers

The followers may be classified as discussed below :

1. *According to the surface in contact*. The followers, according to the surface in contact, are as follows :

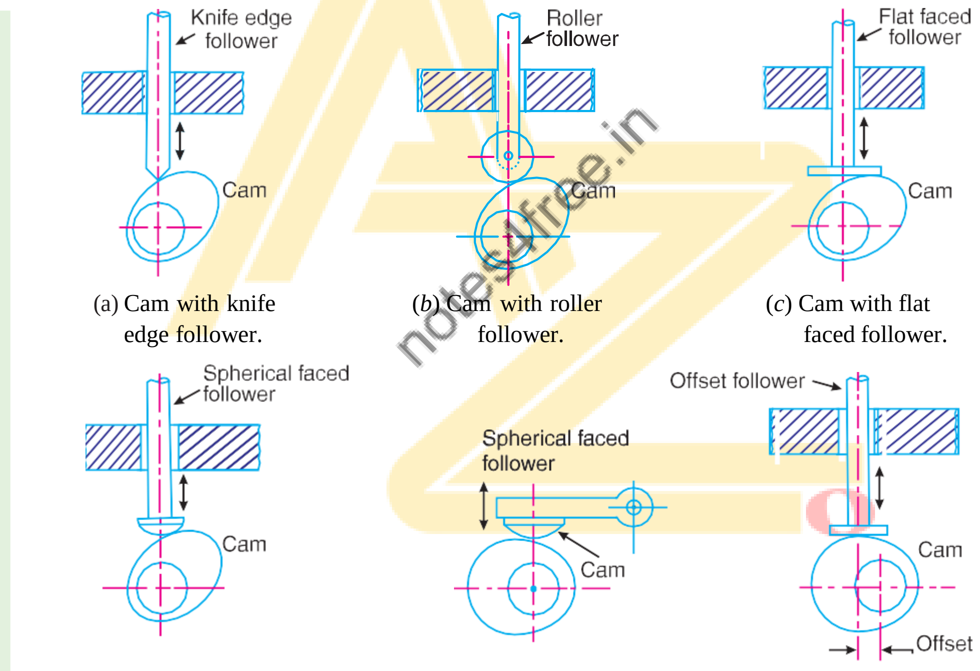
(a) ***Knife edge follower*.** When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower, as shown in Fig. 20.1 (*a*). The sliding motion takes place between the contacting surfaces (*i.e.* the knife edge and the cam surface). It is seldom used in practice because the small area of contacting surface results in excessive wear. In knife edge followers, a considerable side thrust exists between the follower and the guide.

(b) ***Roller follower*****.** When the contacting end of the follower is a roller, it is called a roller follower, as shown in Fig. 20.1 (*b*). Since the rolling motion takes place between the contacting surfaces (*i.e.* the roller and the cam), therefore the rate of wear is greatly reduced. In roller followers also the side thrust exists between the follower and the guide. The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircraft engines.

(c) ***Flat faced or mushroom follower*****.** When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower, as shown in Fig. 20.1 (*c*). It may be noted that the side thrust between the follower and the guide is much reduced in case of flat faced followers. The only side thrust is due to friction between the contact surfaces of the follower and the cam. The relative motion between these surfaces is largely of sliding nature but wear may be reduced by off-setting the axis of the follower, as shown in Fig. 20.1 (*f*) so that when the cam rotates, the follower also rotates about its own axis. The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobile engines.

Note : When the flat faced follower is circular, it is then called a mushroom follower.

(d) ***Spherical faced follower*****.** When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in Fig. 20.1 (*d*). It may be noted that when a flat-faced follower is used in automobile engines, high surface stresses are produced. In order to minimise these stresses, the flat end of the follower is machined to a spherical shape.



(a) Cam with knife edge follower. (*b*) Cam with roller follower. (*c*) Cam with flat faced follower.

(*d*) Cam with spherical faced follower. (*e*) Cam with spherical faced follower. (*f*) Cam with offset follower.

Fig. 20.1. Classification of followers.

2. ***According to the motion of the follower*****.** The followers, according to its motion, are of the following two types

(a) ***Reciprocating or translating follower*.** When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. The followers as shown in Fig. 20.1 (*a*) to (*d*) are all reciprocating or translating followers.

(b) ***Oscillating or rotating follower*.** When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower. The follower, as shown in Fig 20.1 (*e*), is an oscillating or rotating follower.

3. ***According to the path of motion of the follower*.** The followers, according to its path of motion, are of the following two types:

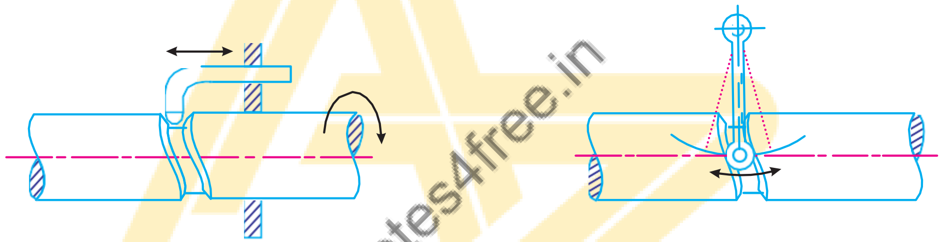
(a) ***Radial follower*.** When the motion of the follower is along an axis passing through the centre of the cam, it is known as radial follower. The followers, as shown in Fig. 20.1 (*a*) to (*e*), are all radial followers.

(b) ***Off-set follower*.** When the motion of the follower is along an axis away from the axis of the cam centre, it is called off-set follower. The follower, as shown in Fig. 20.1 (*f*), is an off-set follower.

Note : In all cases, the follower must be constrained to follow the cam. This may be done by springs, gravity or hydraulic means. In some types of cams, the follower may ride in a groove.

20.3. Classific ation of Cams

Though the cams may be classified in many ways, yet the following two types are important from the subject point of view :



(*a*) Cylindrical cam with reciprocating follower. (*b*) Cylindrical cam with oscillating follower.

Fig. 20.2. Cylindrical cam.

1. ***Radial or disc cam*.** In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis. The cams as shown in Fig. 20.1 are all radial cams.

2. ***Cylindrical cam*.** In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower is shown in Fig. 20.2 (*a*) and (*b*) respectively.

Note : In actual practice, radial cams are widely used. Therefore our discussion will be only confined to radial cams.

In IC engines, cams are widely used to operate valves.

20.4. Terms Used in Radial Cams

Fig. 20.3 shows a radial cam with reciprocating roller follower. The following terms are important in order to draw the cam profile.

1. *Base circle.* It is the smallest circle that can be drawn to the cam profile.

2. *Trace point.* It is a reference point on the follower and is used to generate the *pitch curve*. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centre of the roller represents the trace point.

3. *Pressure angle.* It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

4. *Pitch point.* It is a point on the pitch curve having the maximum pressure angle.

5. *Pitch circle.* It is a circle drawn from the centre of the cam through the pitch points.

6. *Pitch curve.* It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.

7. *Prime circle.* It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

8. *Lift or stroke.* It is the maximum travel of the follower from its lowest position to the topmost position.

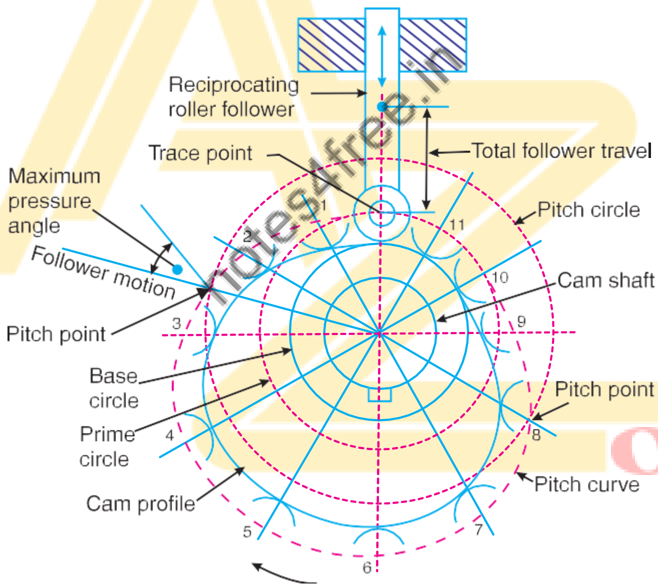


Fig. 20.3. Terms used in radial cams.

20.5. Motion of the Follower

The follower, during its travel, may have one of the following motions.

1. Uniform velocity, **2.** Simple harmonic motion, **3.** Uniform acceleration and retardation, and **4.** Cycloidal motion.

We shall now discuss the displacement, velocity and acceleration diagrams for the cam when the follower moves with the above mentioned motions.

20.6. Displa cement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velo city

The displacement, velocity and acceleration diagrams when a knife-edged follower moves with uniform velocity are shown in Fig. 20.4 (*a*), (*b*) and (*c*) respectively. The abscissa (base) represents the time (*i.e.* the number of seconds required for the cam to complete one revolution) or it may represent the angular displacement of the cam in degrees. The ordinate represents the displacement, or velocity or acceleration of the follower.

Since the follower moves with uniform velocity during its rise and return stroke, therefore the slope of the displacement curves must be constant. In other words, AB_1 and C_1D must be straight lines. A little consideration will show that the follower remains at rest during part of the cam rotation. The periods during which the follower remains at rest are known as ***dwell periods,*** as shown by lines B_1C_1 and *DE* in Fig. 20.4 (*a*). From Fig. 20.4 (*c*), we see that the acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This is due to the fact that the follower is required to start from rest and has to gain a velocity within no time. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite. These conditions are however, impracticable.

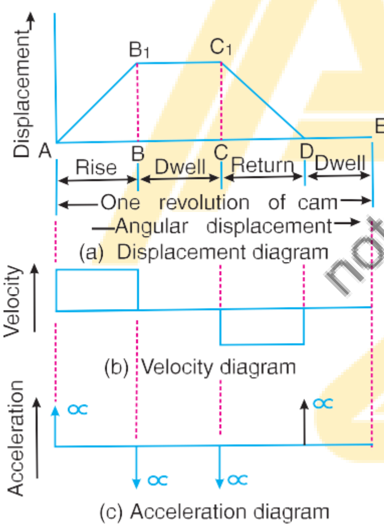


Fig. 20.4. Displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

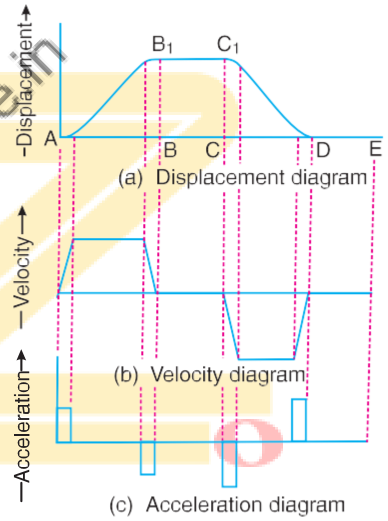


Fig. 20.5. Modified displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

In order to have the acceleration and retardation within the finite limits, it is necessary to modify the conditions which govern the motion of the follower. This may be done by rounding off the sharp corners of the displacement diagram at the beginning and at the end of each stroke, as shown in Fig. 20.5 (*a*). By doing so, the velocity of the follower increases gradually to its maximum value at the beginning of each stroke and decreases gradually to zero at the end of each stroke as shown in Fig. 20.5 (*b*). The modified

Camshaft of an IC engine.

displacement, velocity and acceleration diagrams are shown in Fig. 20.5. The round corners of the displacement diagram are usually parabolic curves because the parabolic motion results in a very low acceleration of the follower for a given stroke and cam speed.

20.7. Displa cement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion

The displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion are shown in Fig. 20.6 (*a*), (*b*) and (*c*) respectively. The displacement diagram is drawn as follows :

1. Draw a semi-circle on the follower stroke as diameter.
2. Divide the semi-circle into any number of even equal parts (say eight).
3. Divide the angular displacements of the cam during out stroke and return stroke into the same number of equal parts.
4. The displacement diagram is obtained by projecting the points as shown in Fig. 20.6 (*a*).

The velocity and acceleration diagrams are shown in Fig. 20.6 (*b*) and (*c*) respectively. Since the follower moves with a simple harmonic motion, therefore velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve. We see from Fig. 20.6 (*b*) that the velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke. On the other hand, the acceleration of the follower is maximum at the beginning and at the ends of the stroke and diminishes to zero at mid-stroke.

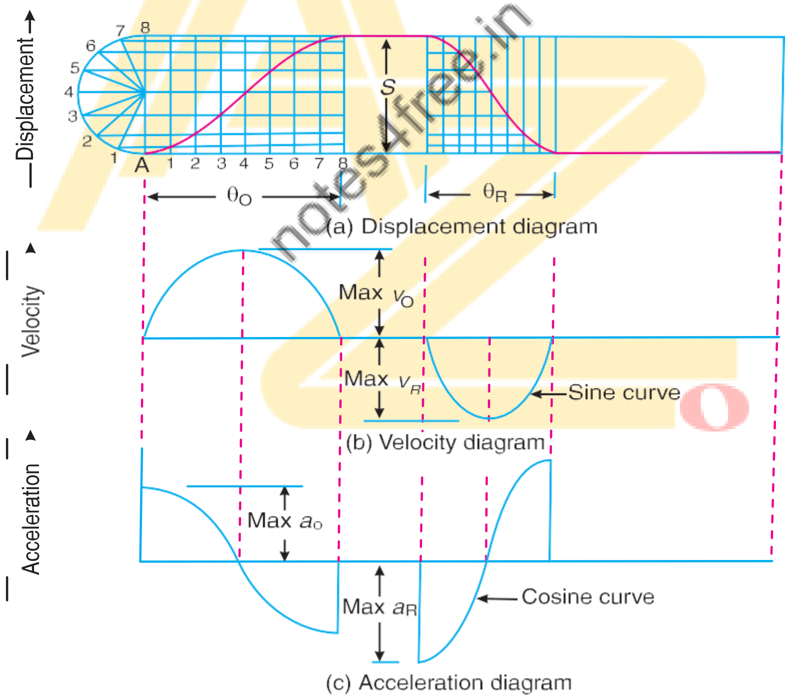


Fig. 20.6. Displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion.

Let S = Stroke of the follower,

θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians, and

ω = Angular velocity of the cam in rad/s.

∴ Time required for the out stroke of the follower in seconds,

$$t_O = \theta_O / \omega$$

Consider a point *P* moving at a uniform speed ω_P radians per sec round the circumference of a circle with the stroke *S* as diameter, as shown in Fig. 20.7. The point *P′* (which is the projection of a point *P* on the diameter) executes a simple harmonic motion as the point *P* rotates. The motion of the follower is similar to that of point *P′*.

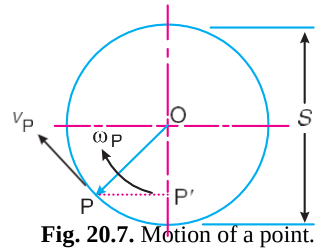


Fig. 20.7. Motion of a point.

∴ Peripheral speed of the point *P′*,

$$v_P = \frac{\pi S}{2} \times \frac{1}{t_O} = \frac{\pi S}{2} \times \frac{\omega}{\theta_O}$$

and maximum velocity of the follower on the outstroke,

$$v_O = v_P = \frac{\pi S}{2} \times \frac{\omega}{\theta_O} = \frac{\pi \omega . S}{2 \theta_O}$$

We know that the centripetal acceleration of the point *P*,

$$a_P = \frac{(v_P)^2}{OP} = \left(\frac{\pi \omega . S}{2\,\theta_O}\right)^2 \times \frac{2}{S} = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

∴ Maximum acceleration of the follower on the outstroke,

$$a_O = a_P = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi\, \omega . S}{2\,\theta_R}$$

and maximum acceleration of the follower on the return stroke,

$$a_R = \frac{\pi^2 \omega^2 . S}{2\,(\theta_R)^2}$$

20.8. Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation

The displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation are shown in Fig. 20.8 (*a*), (*b*) and (*c*) respectively. We see that the displacement diagram consists of a parabolic curve and may be drawn as discussed below :

1. Divide the angular displacement of the cam during outstroke (θ_O) into any even number of equal parts (say eight) and draw vertical lines through these points as shown in Fig. 20.8 (*a*).
2. Divide the stroke of the follower (*S*) into the same number of equal even parts.
3. Join *Aa* to intersect the vertical line through point 1 at *B*. Similarly, obtain the other points *C*, *D* etc. as shown in Fig. 20.8 (*a*). Now join these points to obtain the parabolic curve for the out stroke of the follower.
4. In the similar way as discussed above, the displacement diagram for the follower during return stroke may be drawn.

Since the acceleration and retardation are uniform, therefore the velocity varies directly with the time. The velocity diagram is shown in Fig. 20.8 (*b*).

Let S = Stroke of the follower,

θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, and

ω = Angular velocity of the cam.

We know that time required for the follower during outstroke,

$$t_O = \theta_O / \omega$$

and time required for the follower during return stroke,

$$t_R = \theta_R / \omega$$

Mean velocity of the follower during outstroke

$$= S/t_O$$

and mean velocity of the follower during return stroke

$$= S/t_R$$

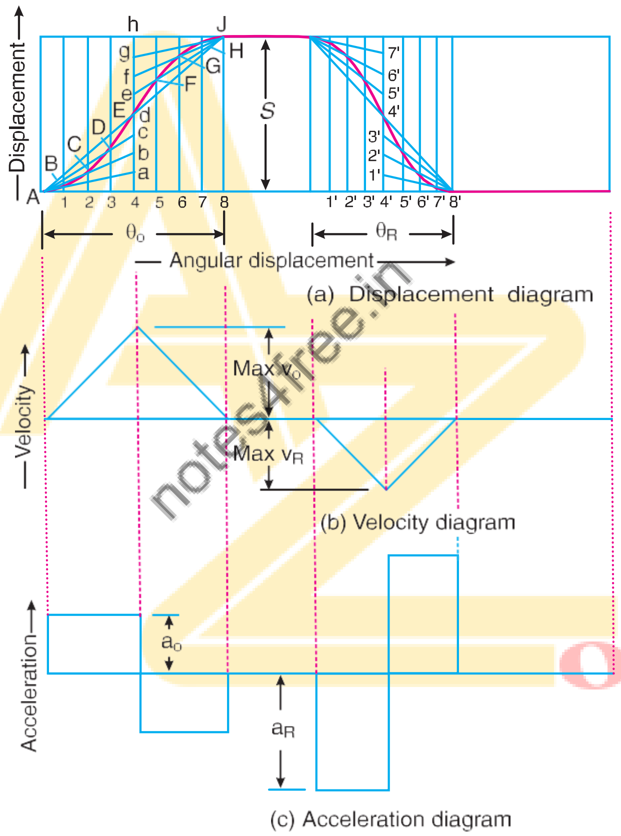


Fig. 20.8. Displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation.

Since the maximum velocity of follower is equal to twice the mean velocity, therefore maximum velocity of the follower during outstroke,

$$v_O = \frac{2S}{t_O} = \frac{2\omega.S}{\theta_O}$$

Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\ \omega.S}{\theta_R}$$

We see from the acceleration diagram, as shown in Fig. 20.8 (*c*), that during first half of the outstroke there is uniform acceleration and during the second half of the out stroke there is uniform retardation. Thus, the maximum velocity of the follower is reached after the time $t_O / 2$ (during out stroke) and $t_R / 2$ (during return stroke).

∴ Maximum acceleration of the follower during outstroke,

$$a_O = \frac{v_O}{t_O/2} = \frac{2 \times 2\,\omega.S}{t_O \theta_O} = \frac{4\,\omega^2.S}{(\theta_O)^2} \qquad \ldots (\because \quad t_O = \theta_O / \omega)$$

Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\,\omega^2.S}{(\theta_R)^2}$$

20.9. Displa cement, Velocity and Acceleration Diagrams when the Follower Moves with Cycloidal Motion

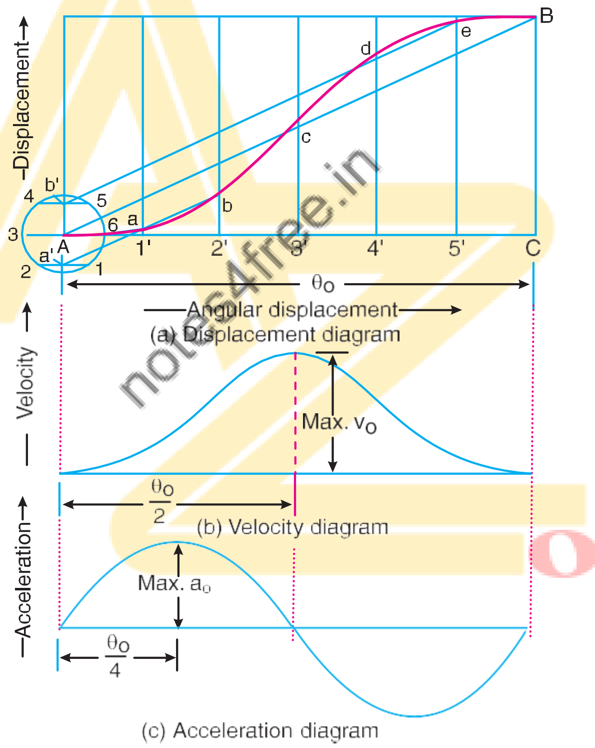


Fig. 20.9. Displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion.

The displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion are shown in Fig. 20.9 (*a*), (*b*) and (*c*) respectively. We know that cycloid is a curve traced by a point on a circle when the circle rolls without slipping on a straight line.

In case of cams, this straight line is a stroke of the follower which is translating and the circumference of the rolling circle is equal to the stroke (*S*) of the follower. Therefore the radius of the rolling circle is $S / 2\pi$. The displacement diagram is drawn as discussed below :

1. Draw a circle of radius $S / 2\pi$ with A as centre.
2. Divide the circle into any number of equal even parts (say six). Project these points horizontally on the vertical centre line of the circle.

   These points are shown by a' and b' in Fig. 20.9 (*a*).
3. Divide the angular displacement of the cam during outstroke into the same number of equal even parts as the circle is divided. Draw vertical lines through these points.
4. Join *AB* which intersects the vertical line through $3'$ at *c*.

   From a' draw a line parallel to *AB* intersecting the vertical lines through $1'$ and $2'$ at *a* and *b* respectively.
5. Similarly, from b' draw a line parallel to *AB* intersecting the vertical lines through $4'$ and $5'$ at *d* and *e* respectively.
6. Join the points *A a b c d e B* by a smooth curve. This is the required cycloidal curve for the follower during outstroke.

Cams are used in Jet and aircraft engines. The above picture shows an aircraft engine.

Let θ = Angle through which the cam rotates in time t seconds, and

ω = Angular velocity of the cam.

We know that displacement of the follower after time t seconds,

$$x = S\left[\frac{\theta}{\theta_O} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\theta_O}\right)\right] \qquad \ldots (i)$$

$\therefore$ Velocity of the follower after time t seconds,

$$\frac{dx}{dt} = S\left[\frac{1}{\theta_O}\times\frac{d\theta}{dt} - \frac{2\pi}{2\pi\theta_O}\cos\left(\frac{2\pi\theta}{\theta_O}\right)\frac{d\theta}{dt}\right]$$

. . . [Differentiating equation (*i*)]

$$= \frac{S}{\theta_O}\times\frac{d\theta}{dt}\left[1-\cos\left(\frac{2\pi\theta}{\theta_O}\right)\right] = \frac{\omega.S}{\theta_O}\left[1-\cos\left(\frac{2\pi\theta}{\theta_O}\right)\right] \qquad \ldots (ii)$$

The velocity is maximum, when

$$\cos\left(\frac{2\pi\theta}{\theta_O}\right) = -1 \quad \text{or} \quad \frac{2\pi\theta}{\theta_O} = \pi \quad \text{or} \quad \theta = \theta_O/2$$

Substituting $\theta = \theta_O/2$ in equation ***(ii)***, we have maximum velocity of the follower during outstroke,

$$v_O = \frac{\omega.S}{\theta_O}(1+1) = \frac{2\,\omega.S}{\theta_O}$$

Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\,\omega.S}{\theta_R}$$

Now, acceleration of the follower after time *t* sec,

$$\frac{d^2x}{dt^2} = \frac{\omega.S}{\theta_O}\left[\frac{2\pi}{\theta_O}\sin\left(\frac{2\pi\theta}{\theta_O}\right)\frac{d\theta}{dt}\right] \qquad \ldots \text{[Differentiating equation } (\textbf{\textit{ii}})]$$

$$= \frac{2\pi\omega^2 S}{(\theta_O)^2}\sin\left(\frac{2\pi\theta}{\theta_O}\right) \qquad \ldots \left(\because \frac{d\theta}{dt} = \omega\right) \qquad \ldots (\textbf{\textit{iii}})$$

The acceleration is maximum, when

$$\sin\left(\frac{2\pi\theta}{\theta_O}\right) = 1 \quad \text{or} \quad \frac{2\pi\theta}{\theta_O} = \frac{\pi}{2} \quad \text{or} \quad \theta = \theta_O/4$$

Substituting $\theta = \theta_O/4$ in equation (*iii*), we have maximum acceleration of the follower during outstroke,

$$a_O = \frac{2\pi\omega^2.S}{(\theta_O)^2}$$

Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{2\pi\omega^2.S}{(\theta_R)^2}$$

The velocity and acceleration diagrams are shown in Fig. 20.9 (*b*) and (*c*) respectively.

20.10. Construction of Cam Profile for a Radial Cam

In order to draw the cam profile for a radial cam, first of all the displacement diagram for the given motion of the follower is drawn. Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam is drawn.

In constructing the cam profile, the principle of kinematic inversion is used, *i.e.* the cam is imagined to be stationary and the follower is allowed to rotate in the ***opposite direction*** to the ***cam rotation.***

The construction of cam profiles for different types of follower with different types of motions are discussed in the following examples.

Example 20.1. *A cam is to give the following motion to a knife-edged follower :*

1. *Outstroke during 60° of cam rotation ;* ***2.*** *Dwell for the next 30° of cam rotation ;* ***3.*** *Return stroke during next 60° of cam rotation, and* ***4.*** *Dwell for the remaining 210° of cam rotation.*

The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes. Draw the profile of the cam when ***(a)*** *the axis of the follower passes through the axis of the cam shaft, and* ***(b)*** *the axis of the follower is offset by 20 mm from the axis of the cam shaft.*

Construction

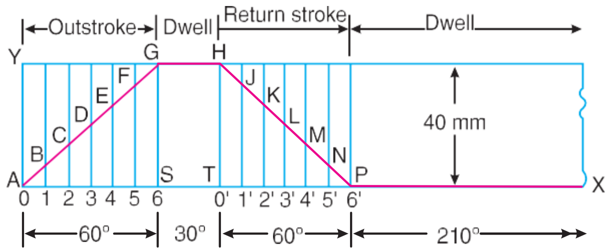


Fig. 20.10

First of all, the displacement diagram, as shown in Fig. 20.10, is drawn as discussed in the following steps :

1. Draw a horizontal line *AX* = 360° to some suitable scale. On this line, mark *AS* = 60° to represent outstroke of the follower, *ST* = 30° to represent dwell, *TP* = 60° to represent return stroke and *PX* = 210° to represent dwell.
2. Draw vertical line *AY* equal to the stroke of the follower (*i.e.* 40 mm) and complete the rectangle as shown in Fig. 20.10.
3. Divide the angular displacement during outstroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
4. Since the follower moves with uniform velocity during outstroke and return stroke, therefore the displacement diagram consists of straight lines. Join *AG* and *HP*.
5. The complete displacement diagram is shown by *AGHPX* in Fig. 20.10.

(a) Profile of the cam when the axis of follower passes through the axis of cam shaft

The profile of the cam when the axis of the follower passes through the axis of the cam shaft, as shown in Fig. 20.11, is drawn as discussed in the following steps :

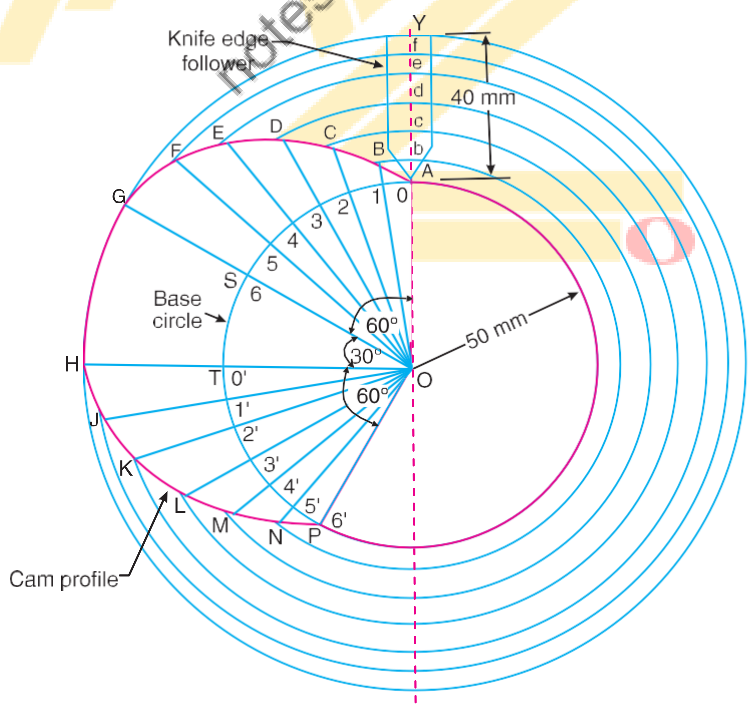


Fig. 20.11

1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with *O* as centre.
2. Since the axis of the follower passes through the axis of the cam shaft, therefore mark trace point *A*, as shown in Fig. 20.11.
3. From *OA*, mark angle *AOS* = 60° to represent outstroke, angle *SOT* = 30° to represent dwell and angle *TOP* = 60° to represent return stroke.
4. Divide the angular displacements during outstroke and return stroke (*i.e.* angle *AOS* and angle *TOP*) into the same number of equal even parts as in displacement diagram.
5. Join the points 1, 2, 3 ...etc. and 0′ ,1′ , 2′ , 3′ , ... etc. with centre *O* and produce beyond the base circle as shown in Fig. 20.11.
6. Now set off 1*B*, 2*C*, 3*D* ... etc. and 0′ *H*,1′ *J* ... etc. from the displacement diagram.
7. Join the points *A*, *B*, *C*,... *M*, *N*, *P* with a smooth curve. The curve *AGHPA* is the complete profile of the cam.

Notes : The points *B*, *C*, *D*......*L*, *M*, *N* may also be obtained as follows :

1. Mark *AY* = 40 mm on the axis of the follower, and set of *Ab*, *Ac*, *Ad*....etc. equal to the distances 1*B*, 2*C*, 3*D*... etc. as in displacement diagram.

2. From the centre of the cam *O*, draw arcs with radii *Ob*, *Oc*, *Od* etc. The arcs intersect the produced lines *O*1, *O*2... etc. at *B*, *C*, *D*..... *L*, *M*, *N*.

(b) Profile of the cam when the axis of the follower is offset by 20 mm from the axis of the cam shaft

The profile of the cam when the axis of the follower is offset from the axis of the cam shaft, as shown in Fig. 20.12, is drawn as discussed in the following steps :

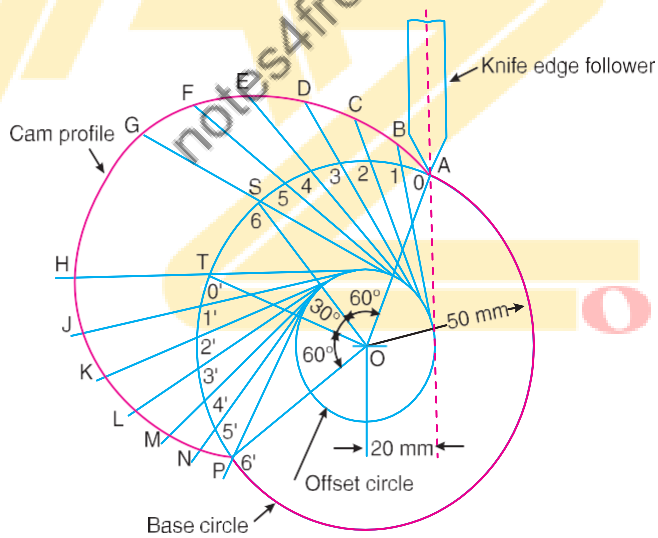


Fig. 20.12

1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with *O* as centre.
2. Draw the axis of the follower at a distance of 20 mm from the axis of the cam, which intersects the base circle at *A*.
3. Join *AO* and draw an offset circle of radius 20 mm with centre *O*.
4. From *OA*, mark angle *AOS* = 60° to represent outstroke, angle *SOT* = 30° to represent dwell and angle *TOP* = 60° to represent return stroke.

5. Divide the angular displacement during outstroke and return stroke (*i.e.* angle *AOS* and angle *TOP*) into the same number of equal even parts as in displacement diagram.

6. Now from the points 1, 2, 3 ... etc. and 0′,1′, 2′, 3′ ... etc. on the base circle, draw tangents to the offset circle and produce these tangents beyond the base circle as shown in Fig. 20.12.

7. Now set off 1*B*, 2*C*, 3*D* ... etc. and 0′ *H*,1′ *J* ... etc. from the displacement diagram.

8. Join the points *A*, *B*, *C* ...*M*, *N*, *P* with a smooth curve. The curve *AGHPA* is the complete profile of the cam.

Example 20.2. *A cam is to be designed for a knife edge follower with the following data :*

1. Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion.

2. Dwell for the next 30°.

3. During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion.

4. Dwell during the remaining 180°.

Draw the profile of the cam when

(a) the line of stroke of the follower passes through the axis of the cam shaft, and

(b) the line of stroke is offset 20 mm from the axis of the cam shaft.

The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

Solution. Given : S = 40 mm = 0.04 m; θ_O = 90° = π /2 rad = 1.571 rad ; θ_R = 60° = π /3 rad = 1.047 rad ; N = 240 r.p.m.

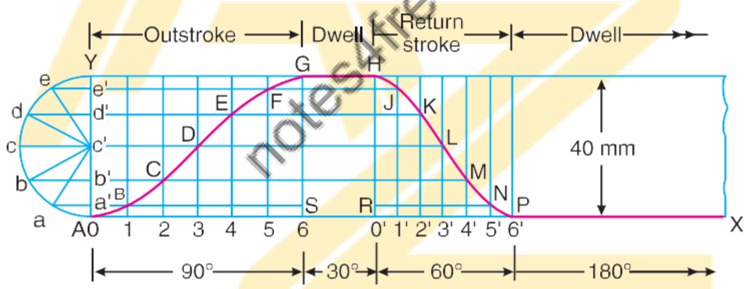


Fig. 20.13

First of all, the displacement diagram, as shown in Fig 20.13, is drawn as discussed in the following steps :

1. Draw horizontal line *AX* = 360° to some suitable scale. On this line, mark *AS* = 90° to represent out stroke ; *SR* = 30° to represent dwell ; *RP* = 60° to represent return stroke and *PX* = 180° to represent dwell.

2. Draw vertical line *AY* = 40 mm to represent the cam lift or stroke of the follower and complete the rectangle as shown in Fig. 20.13.

3. Divide the angular displacement during out stroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.

4. Since the follower moves with simple harmonic motion, therefore draw a semicircle with *AY* as diameter and divide into six equal parts.

5. From points *a*, *b*, *c* ... etc. draw horizontal lines intersecting the vertical lines drawn through 1, 2, 3 ... etc. and 0′ , 1′ , 2′ ...etc. at *B*, *C*, *D* ... *M*, *N*, *P*.

6. Join the points *A*, *B*, *C* ... etc. with a smooth curve as shown in Fig. 20.13. This is the required displacement diagram.

(a) Profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft

The profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft, as shown in Fig. 20.14, is drawn in the similar way as is discussed in Example 20.1.

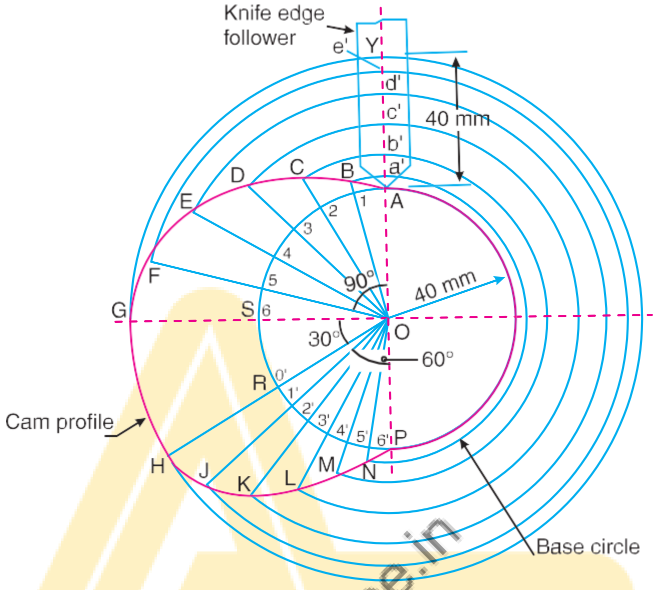


Fig. 20.14

(b) Profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft

The profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft, as shown in Fig. 20.15, is drawn in the similar way as discussed in Example 20.1.

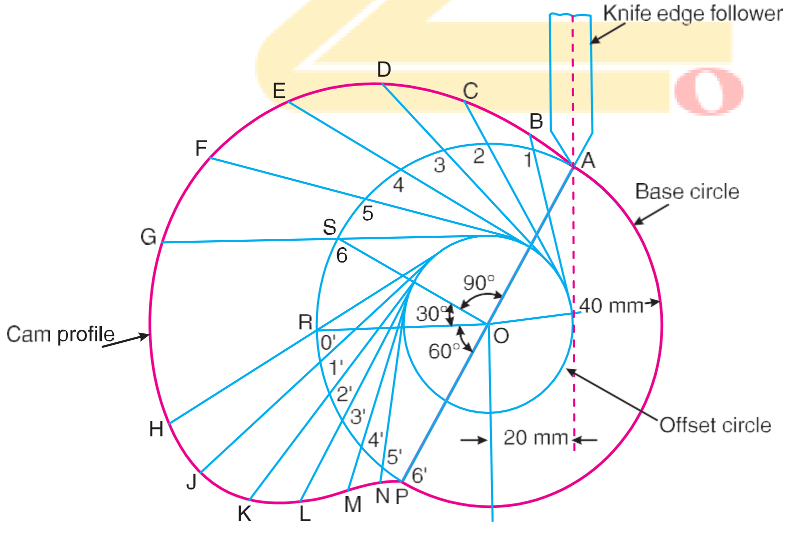


Fig. 20.15

Maximum velocity of the follower during its ascent and descent

We know that angular velocity of the cam,

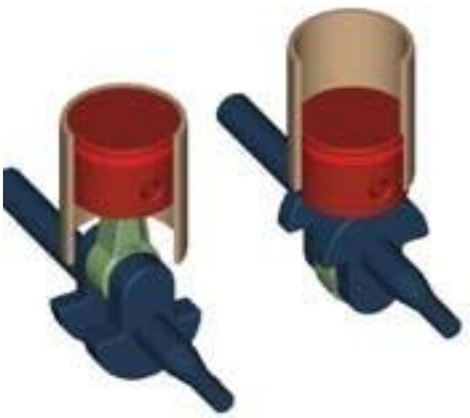
$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.14 \text{ rad/s}$$

We also know that the maximum velocity of the follower during its ascent,

$$v_O = \frac{\pi \omega . S}{2\theta_O} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/s } \textbf{Ans.}$$

and maximum velocity of the follower during its descent,

$$v_R = \frac{\pi \omega . S}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/s } \textbf{Ans.}$$



Role of cams in piston movement.

Maximum acceleration of the follower during its ascent and descent

We know that the maximum acceleration of the follower during its ascent,

$$a_O = \frac{\pi^2 \omega^2 . S}{2 (\theta_O)^2} = \frac{\pi^2 (25.14)^2 \, 0.04}{2 (1.571)^2} = 50.6 \text{ m/s}^2 \textbf{ Ans.}$$

and maximum acceleration of the follower during its descent,

$$a_R = \frac{\pi^2 \omega^2 . S}{2 (\theta_R)^2} = \frac{\pi^2 (25.14)^2 \, 0.04}{2 (1.047)^2} = 113.8 \text{ m/s}^2 \textbf{ Ans.}$$

Example 20.3. *A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a roller follower, at the end of a valve rod, motion described below :*

1. *To raise the valve through 50 mm during 120° rotation of the cam ;*

2. *To keep the valve fully raised through next 30°;*

3. *To lower the valve during next 60°; and*

4. *To keep the valve closed during rest of the revolution i.e. 150° ;*

The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm.

Draw the profile of the cam when ***(a)*** *the line of stroke of the valve rod passes through the axis of the cam shaft, and* ***(b)*** *the line of the stroke is offset 15 mm from the axis of the cam shaft.*

The displacement of the valve, while being raised and lowered, is to take place with simple harmonic motion. Determine the maximum acceleration of the valve rod when the cam shaft rotates at 100 r.p.m.

Draw the displacement, the velocity and the acceleration diagrams for one complete revolution of the cam.

Solution. Given : S = 50 mm = 0.05 m ; θ_O = 120° = 2 π /3 rad = 2.1 rad ; θ_R = 60° = π /3 rad = 1.047 rad ; N = 100 r.p.m.

Since the valve is being raised and lowered with simple harmonic motion, therefore the displacement diagram, as shown in Fig. 20.16 (*a*), is drawn in the similar manner as discussed in the previous example.

(a) ***Profile of the cam when the line of stroke of the valve rod passes through the axis of the cam shaft***

The profile of the cam, as shown in Fig. 20.17, is drawn as discussed in the following steps :

1. Draw a base circle with centre *O* and radius equal to the minimum radius of the cam (*i.e.* 25 mm).

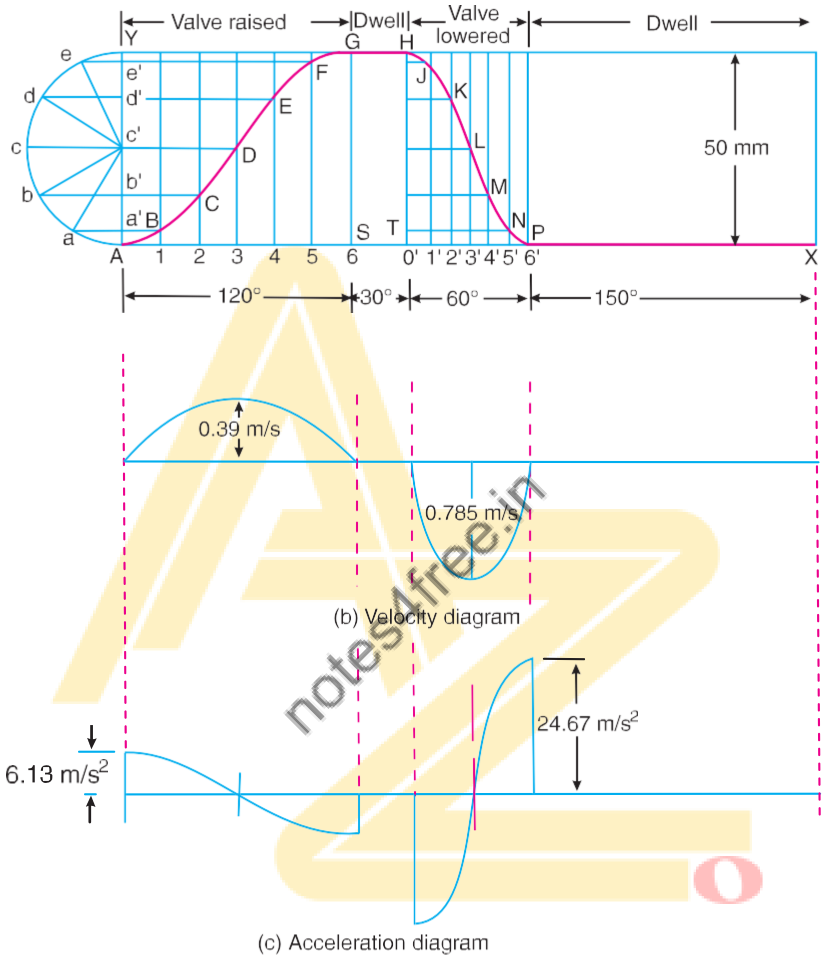


Fig. 20.16

2. Draw a prime circle with centre *O* and radius,

$$OA = \text{Min. radius of cam} + \frac{1}{2}\ \text{Dia. of roller} = 25 + \frac{1}{2} \times 20 = 35 \text{ mm}$$

3. Draw angle *AOS* = 120° to represent raising or out stroke of the valve, angle *SOT* = 30° to represent dwell and angle *TOP* = 60° to represent lowering or return stroke of the valve.
4. Divide the angular displacements of the cam during raising and lowering of the valve (*i.e.* angle *AOS* and *TOP*) into the same number of equal even parts as in displacement diagram.
5. Join the points 1, 2, 3, etc. with the centre *O* and produce the lines beyond prime circle as shown in Fig. 20.17.
6. Set off 1*B*, 2*C*, 3*D* etc. equal to the displacements from displacement diagram.
7. Join the points *A*, *B*, *C* ... *N*, *P*, *A*. The curve drawn through these points is known as ***pitch curve.***

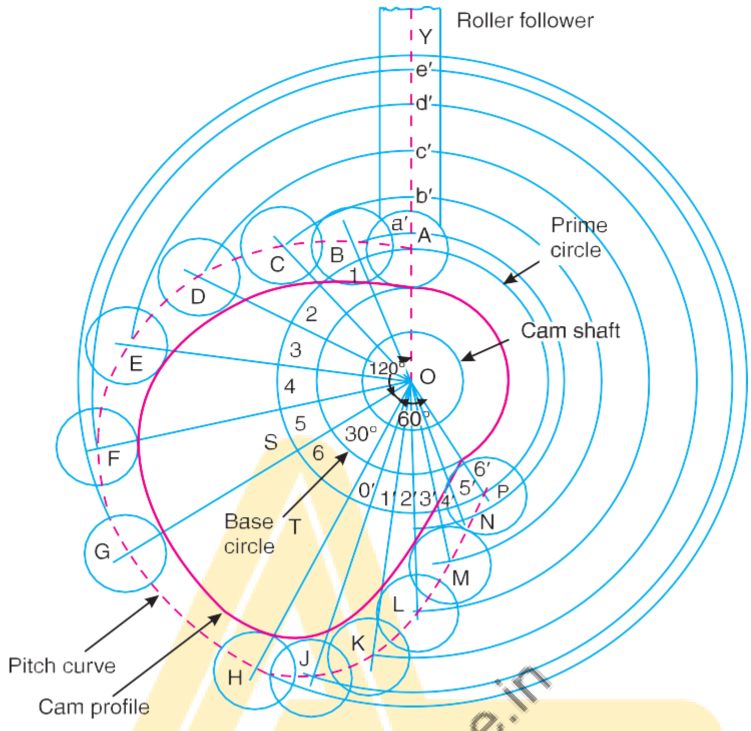


Fig. 20.17

8. From the points *A*, *B*, *C* ... *N*, *P*, draw circles of radius equal to the radius of the roller.
9. Join the bottoms of the circles with a smooth curve as shown in Fig. 20.17. This is the required profile of the cam.

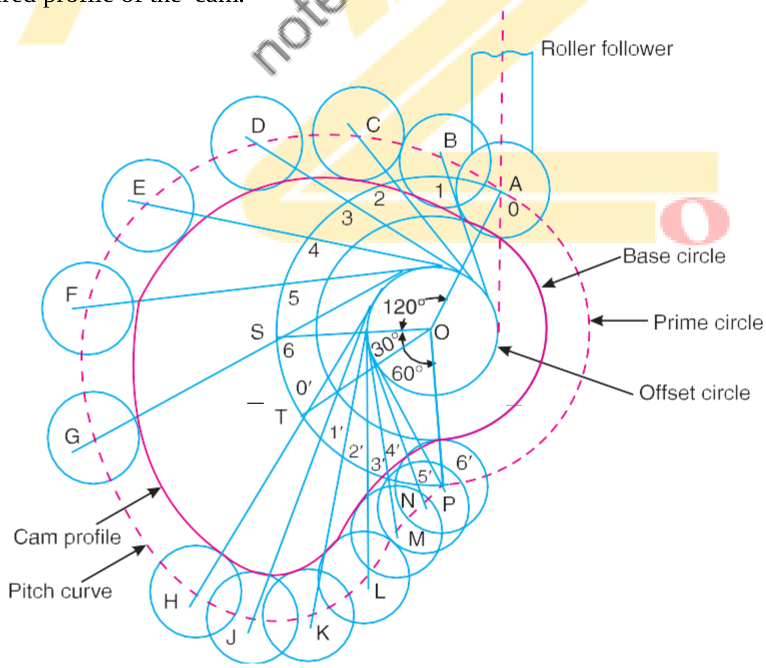


Fig. 20.18

(b) Profile of the cam when the line of stroke is offset 15 mm from the axis of the cam shaft

The profile of the cam when the line of stroke is offset from the axis of the cam shaft, as shown in Fig. 20.18, may be drawn as discussed in the following steps :

1. Draw a base circle with centre *O* and radius equal to 25 mm.
2. Draw a prime circle with centre *O* and radius *OA* = 35 mm.
3. Draw an off-set circle with centre *O* and radius equal to 15 mm.
4. Join *OA*. From *OA* draw the angular displacements of cam *i.e.* draw angle *AOS* = 120°, angle *SOT* = 30° and angle *TOP* = 60°.
5. Divide the angular displacements of the cam during raising and lowering of the valve into the same number of equal even parts (*i.e.* six parts) as in displacement diagram.
6. From points 1, 2, 3 .... etc. and 0′ ,1′ , 3′ , ...etc. on the prime circle, draw tangents to the offset circle.
7. Set off 1*B*, 2*C*, 3*D*... etc. equal to displacements as measured from displacement diagram.
8. By joining the points *A*, *B*, *C* ... *M*, *N*, *P*, with a smooth curve, we get a ***pitch curve.***
9. Now *A*, *B*, *C*...etc. as centre, draw circles with radius equal to the radius of roller.
10. Join the bottoms of the circles with a smooth curve as shown in Fig. 20.18. This is the required profile of the cam.

Maximum acceleration of the valve rod

We know that angular velocity of the cam shaft,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} = 10.47 \text{ rad/s}$$

We also know that maximum velocity of the valve rod to raise valve,

$$v_O = \frac{\pi\omega.S}{2\theta_O} = \frac{\pi \times 10.47 \times 0.05}{2 \times 2.1} = 0.39 \text{ m/s}$$

and maximum velocity of the valve rod to lower the valve,

$$v_R = \frac{\pi\omega.S}{2\theta_R} = \frac{\pi \times 10.47 \times 0.05}{2 \times 1.047} = 0.785 \text{ m/s}$$

The velocity diagram for one complete revolution of the cam is shown in Fig. 20.16 (*b*).

We know that the maximum acceleration of the valve rod to raise the valve,

$$a_O = \frac{\pi^2\omega^2.S}{2(\theta_O)^2} = \frac{\pi^2(10.47)^2 0.05}{2\,(2.1)^2} = 6.13 \text{ m/s}^2 \textbf{ Ans.}$$

and maximum acceleration of the valve rod to lower the valve,

$$a_R = \frac{\pi^2\omega^2.S}{2\,(\theta_R)^2} = \frac{\pi^2(10.47)^2 0.05}{2(1.047)^2} = 24.67 \text{ m/s}^2 \textbf{ Ans.}$$

The acceleration diagram for one complete revolution of the cam is shown in Fig. 20.16 (*c*).

Example 20.4. *A cam drives a flat reciprocating follower in the following manner :*

During first 120° rotation of the cam, follower moves outwards through a distance of 20 mm with simple harmonic motion. The follower dwells during next 30° of cam rotation. During next 120° of cam rotation, the follower moves inwards with simple harmonic motion. The follower dwells for the next 90° of cam rotation.

The minimum radius of the cam is 25 mm. Draw the profile of the cam.

Construction

Since the follower moves outwards and inwards with simple harmonic motion, therefore the displacement diagram, as shown in Fig. 20.19, is drawn in the similar manner as discussed earlier.

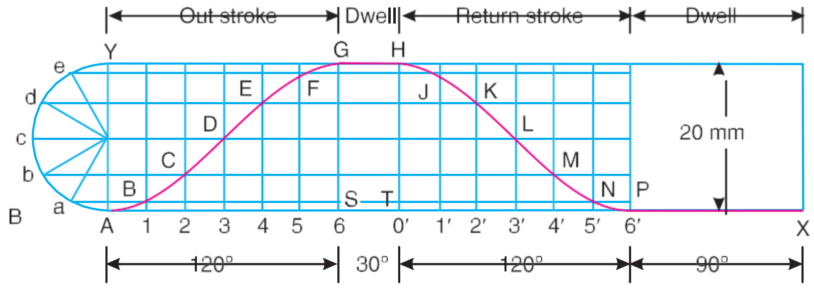


Fig. 20.19

Now the profile of the cam driving a flat reciprocating follower, as shown in Fig. 20.20, is drawn as discussed in the following steps :

1. Draw a base circle with centre *O* and radius *OA* equal to the minimum radius of the cam (*i.e.* 25 mm).
2. Draw angle *AOS* = 120° to represent the outward stroke, angle *SOT* = 30° to represent dwell and angle *TOP* = 120° to represent inward stroke.
3. Divide the angular displacement during outward stroke and inward stroke (*i.e.* angles *AOS* and *TOP*) into the same number of equal even parts as in the displacement diagram.

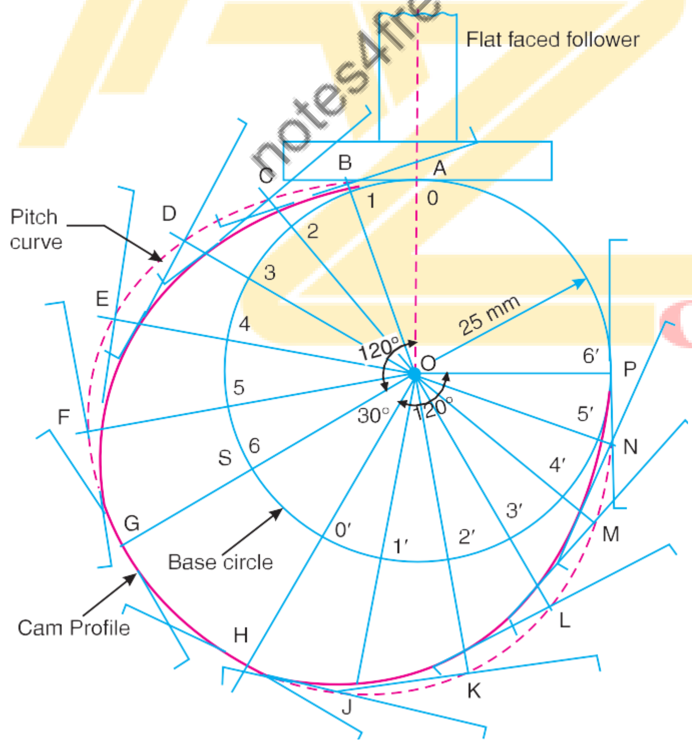


Fig. 20.20

4. Join the points 1, 2, 3 . . . etc. with centre *O* and produce beyond the base circle.
5. From points 1, 2, 3 . . . etc., set off 1*B*, 2*C*, 3*D* . . . etc. equal to the distances measured from the displacement diagram.
6. Now at points *B*, *C*, *D* . . . *M*, *N*, *P*, draw the position of the flat-faced follower. The axis of the follower at all these positions passes through the cam centre.
7. The curve drawn tangentially to the flat side of the follower is the required profile of the cam, as shown in Fig. 20.20.

Example 20.5. *Draw a cam profile to drive an oscillating roller follower to the specifications given below :*

(a) Follower to move outwards through an angular displacement of 20° during the first 120° rotation of the cam ;

(b) Follower to return to its initial position during next 120° rotation of the cam ;

(c) Follower to dwell during the next 120° of cam rotation.

The distance between pivot centre and roller centre = 120 mm ; distance between pivot centre and cam axis = 130 mm ; minimum radius of cam = 40 mm ; radius of roller = 10 mm ; inward and outward strokes take place with simple harmonic motion.

Construction

We know that the angular displacement of the roller follower

$$= 20° = 20 \times \pi / 180 = \pi / 9 \text{ rad}$$

Since the distance between the pivot centre and the roller centre (*i.e.* the radius $A_1 A$) is 120 mm, therefore length of the arc AA_2, as shown in Fig. 20.21, along which the displacement of the roller actually takes place

$$= 120 \times \pi / 9 = 41.88 \text{ mm}$$

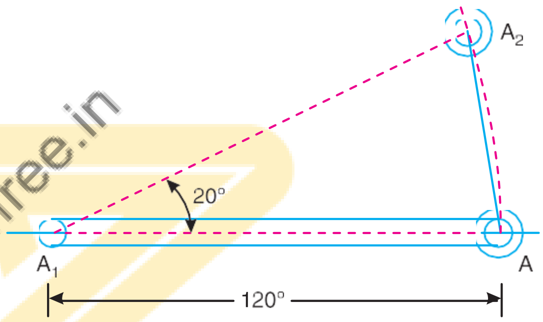


Fig. 20.21

. . . (∵ Length of arc = Radius of arc × Angle subtended by the arc at the centre in radians)

Since the angle is very small, therefore length of chord AA_2 is taken equal to the length of arc AA_2. Thus in order to draw the displacement diagram, we shall take lift of the follower equal to length of chord AA_2 *i.e.* 41.88 mm.

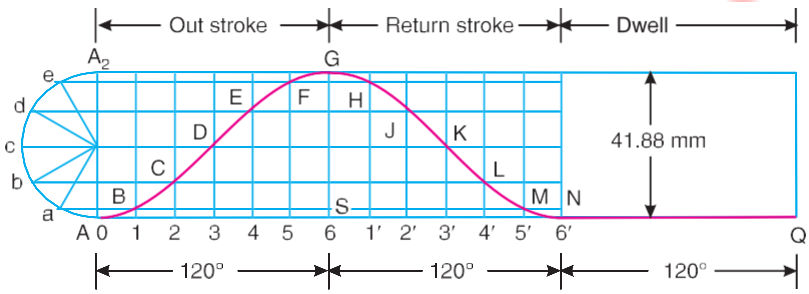


Fig. 20.22

The outward and inward strokes take place with simple harmonic motion, therefore the displacement diagram, as shown in Fig. 20.22, is drawn in the similar way as discussed in Example 20.4.

The profile of the cam to drive an oscillating roller follower, as shown in Fig. 20.23, is drawn as discussed in the following steps :

1. First of all, draw a base circle with centre O and radius equal to the minimum radius of the cam (*i.e.* 40 mm)
2. Draw a prime circle with centre O and radius OA

   = Min. radius of cam + radius of roller = 40 + 10 = 50 mm
3. Now locate the pivot centre A_1 such that OA_1 = 130 mm and AA_1 = 120 mm. Draw a pivot circle with centre O and radius OA_1 = 130 mm.

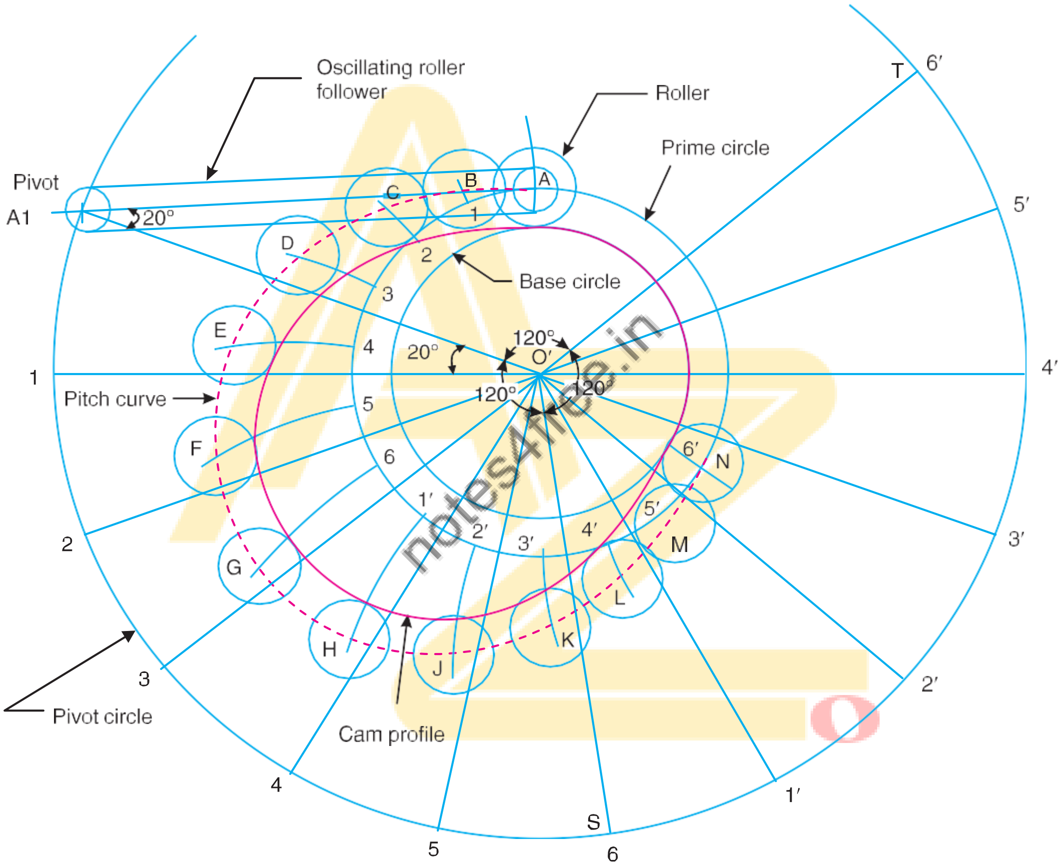


Fig. 20.23

4. Join OA_1. Draw angle A_1OS = 120° to represent the outward stroke of the follower, angle SOT = 120° to represent the inward stroke of the follower and angle TOA_1 = 120° to represent the dwell.
5. Divide angles A_1OS and SOT into the same number of equal even parts as in the displacement diagram and mark points 1, 2, 3 . . . 4′ , 5′ , 6′ on the pivot circle.
6. Now with points 1, 2, 3 . . . 4′ , 5′ , 6′ (on the pivot circle) as centre and radius equal to A_1A (*i.e.* 120 mm) draw circular arcs to intersect the prime circle at points 1, 2, 3 . . . 4′ , 5′ , 6′ .

7. Set off the distances 1*B*, 2*C*, 3*D*... 4′*L*, 5′*M* along the arcs drawn equal to the distances as measured from the displacement diagram.
8. The curve passing through the points *A*, *B*, *C*... *L*, *M*, *N* is known as pitch curve.
9. Now draw circles with *A*, *B*, *C*, *D*... *L*, *M*, *N* as centre and radius equal to the radius of roller.
10. Join the bottoms of the circles with a smooth curve as shown in Fig. 20.23. This is the required profile of the cam.

Example 20.6. *A cam, with a minimum radius of 50 mm, rotating clockwise at a uniform speed, is required to give a knife edge follower the motion as described below :*

1. *To move outwards through 40 mm during 100° rotation of the cam ;* ***2.*** *To dwell for next 80° ;* ***3.*** *To return to its starting position during next 90°, and* ***4.*** *To dwell for the rest period of a revolution i.e. 90°.*

Draw the profile of the cam

(i) when the line of stroke of the follower passes through the centre of the cam shaft, and

(ii) when the line of stroke of the follower is off-set by 15 mm.

The displacement of the follower is to take place with uniform acceleration and uniform retardation. Determine the maximum velocity and acceleration of the follower when the cam shaft rotates at 900 r.p.m.

Draw the displacement, velocity and acceleration diagrams for one complete revolution of the cam.

Solution. Given : S = 40 mm = 0.04 m; $\theta_O = 100° = 100 \times \pi/180 = 1.745$ rad ; $\theta_R = 90° = \pi/2 = 1.571$ rad ; N = 900 r.p.m.

First of all, the displacement diagram, as shown in Fig. 20.24 (*a*), is drawn as discussed in the following steps :

1. Draw a horizontal line *ASTPQ* such that *AS* represents the angular displacement of the cam during outward stroke (*i.e.* 100°) to some suitable scale. The line *ST* represents the dwell period of 80° after outward stroke. The line *TP* represents the angular displacement of the cam during return stroke (*i.e.* 90°) and the line *PQ* represents the dwell period of 90° after return stroke.
2. Divide *AS* and *TP* into any number of equal even parts (say six).
3. Draw vertical lines through points 0, 1, 2, 3 etc. and equal to the lift of the valve *i.e.* 40 mm.
4. Divide the vertical lines 3-*f* and 3′- *f* ′ into six equal parts as shown by points *a*, *b*, *c* . . . and *a*′ , *b*′ , *c*′ . . . in Fig. 20.24 (*a*).
5. Since the follower moves with equal uniform acceleration and uniform retardation, therefore the displacement diagram of the outward and return stroke consists of a double parabola.
6. Join *Aa*, *Ab* and *Ac* intersecting the vertical lines through 1, 2 and 3 at *B*, *C* and *D* respectively.
7. Join the points *B*, *C* and *D* with a smooth curve. This is the required parabola for the half outstroke of the valve. Similarly the other curves may be drawn as shown in Fig. 20.24.
8. The curve *A B C* . . . *N P Q* is the required displacement diagram.

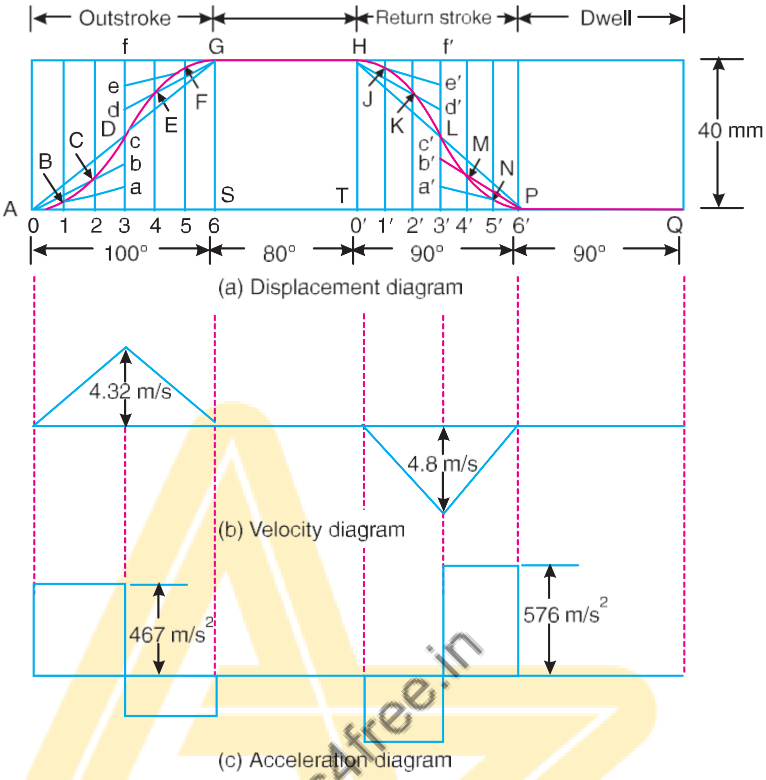


Fig. 20.24

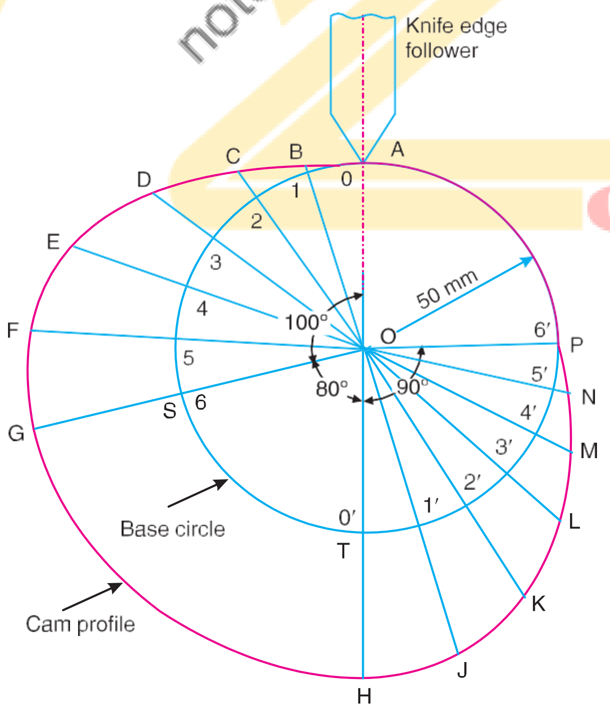


Fig. 20.25

(i) Profile of the cam when the line of stroke of the follower passes through the centre of the cam shaft

The profile of the cam when the line of stroke of the follower passes through the centre of cam shaft, as shown in Fig. 20.25, may be drawn as discussed in the following steps :

1. Draw a base circle with centre *O* and radius 50 mm (equal to minimum radius of the cam).
2. Divide the base circle such that angle *AOS* = 100° ; angle *SOT* = 80° and angle *TOP* = 90°.
3. Divide angles *AOS* and *TOP* into the same number of equal even parts as in displacement diagram (*i.e.* six parts).
4. Join the points 1, 2, 3 . . . and $1', 2', 3'$, . . . with centre *O* and produce these lines beyond the base circle.
5. From points 1, 2, 3 . . . and $1', 2', 3'$, . . . mark the displacements 1*B*, 2*C*, 3*D* . . . etc. as measured from the displacement diagram.
6. Join the points *A*, *B*, *C* . . . *M*, *N*, *P* with a smooth curve as shown in Fig. 20.25. This is the required profile of the cam.

(ii) Profile of the cam when the line of stroke of the follower is offset by 15 mm

The profile of the cam when the line of stroke of the follower is offset may be drawn as discussed in Example 20.2. The profile of cam is shown in Fig. 20.26.

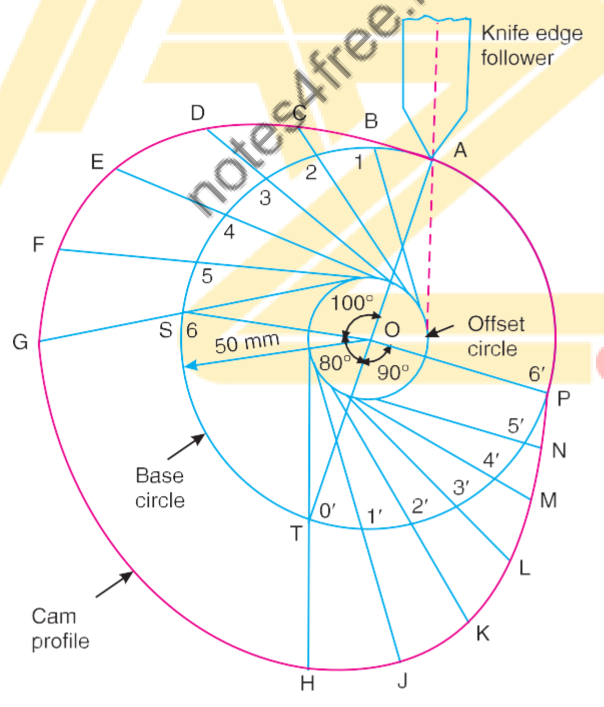


Fig. 20.26

Maximum velocity of the follower during out stroke and return stroke

We know that angular velocity of the cam shaft,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 900}{60} = 94.26 \text{ rad/s}$$

We also know that the maximum velocity of the follower during out stroke,

$$v_O = \frac{2\omega.S}{\theta_O} = \frac{2 \times 94.26 \times 0.04}{1.745} = 4.32 \text{ m/s } \textbf{Ans.}$$

A type of roller follower.

and maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega.S}{\theta_R} = \frac{2 \times 94.26 \times 0.04}{1.571} = 4.8 \text{ m/s } \textbf{Ans.}$$

The velocity diagram is shown in Fig. 20.24 (*b*).

Maximum acceleration of the follower during out stroke and return stroke

We know that the maximum acceleration of the follower during out stroke,

$$a_O = \frac{4\omega^2 .S}{(\theta_O)^2} = \frac{4\,(94.26)^2\, 0.04}{(1.745)^2} = 467 \text{ m/s}^2 \textbf{ Ans.}$$

and maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\omega^2 .S}{(\theta_R)^2} = \frac{4\,(94.26)^2\, 0.04}{(1.571)^2} = 576 \text{ m/s}^2 \textbf{ Ans.}$$

The acceleration diagram is shown in Fig. 20.24 (*c*).

Example 20.7. *Design a cam for operating the exhaust valve of an oil engine. It is required to give equal uniform acceleration and retardation during opening and closing of the valve each of which corresponds to 60° of cam rotation. The valve must remain in the fully open position for 20° of cam rotation.*

The lift of the valve is 37.5 mm and the least radius of the cam is 40 mm. The follower is provided with a roller of radius 20 mm and its line of stroke passes through the axis of the cam.

Construction

First of all, the displacement diagram, as shown in Fig. 20.27, is drawn as discussed in the following steps :

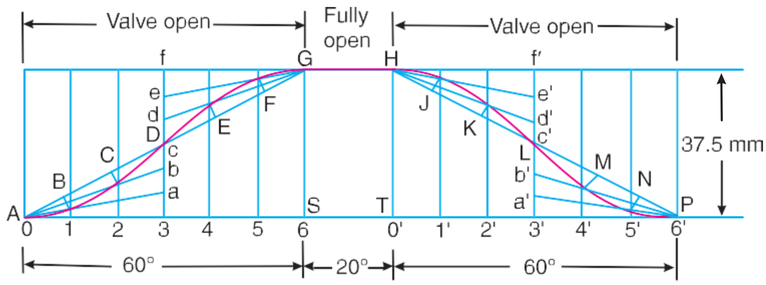


Fig. 20.27

1. Draw a horizontal line *ASTP* such that *AS* represents the angular displacement of the cam during opening (*i.e.* out stroke) of the valve (equal to 60°), to some suitable scale. The line *ST* represents the dwell period of 20° *i.e.* the period during which the valve remains

fully open and *TP* represents the angular displacement during closing (*i.e.* return stroke) of the valve which is equal to 60°.

2. Divide *AS* and *TP* into any number of equal even parts (say six).
3. Draw vertical lines through points 0, 1, 2, 3 etc. and equal to lift of the valve *i.e.* 37.5 mm.
4. Divide the vertical lines 3*f* and 3′ *f* ′ into six equal parts as shown by the points *a*, *b*, *c* . .. and *a*′, *b*′, *c*′ . . . in Fig. 20.27.
5. Since the valve moves with equal uniform acceleration and retardation, therefore the displacement diagram for opening and closing of a valve consists of double parabola.
6. Complete the displacement diagram as shown in Fig. 20.27.

Now the profile of the cam, with a roller follower when its line of stroke passes through the axis of cam, as shown in Fig. 20.28, is drawn in the similar way as discussed in Example 20.3.

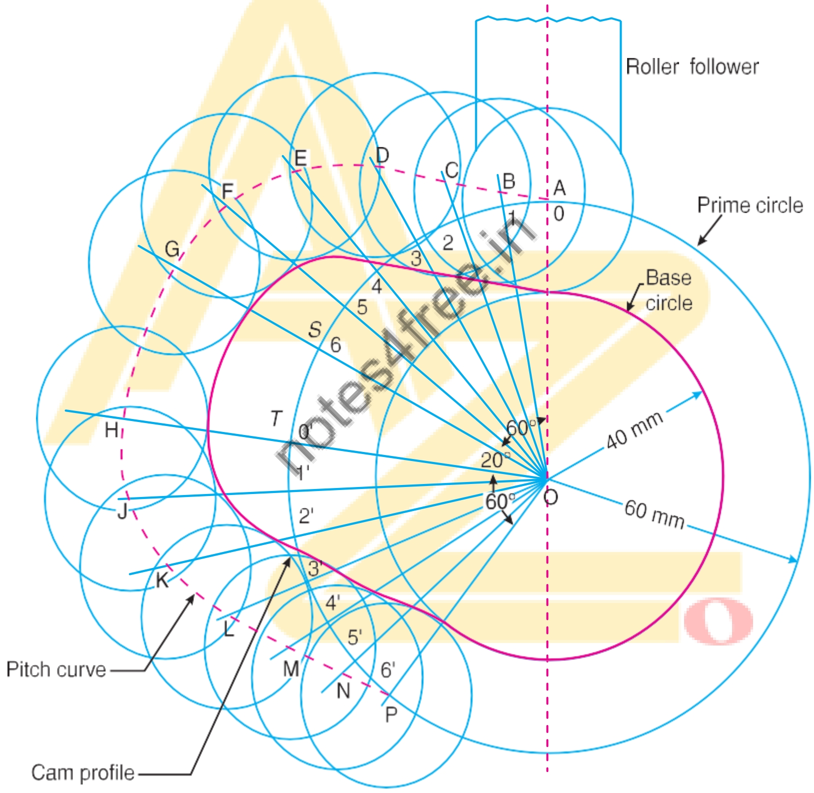


Fig. 20.28

Example 20.8. *A cam rotating clockwise at a uniform speed of 1000 r.p.m. is required to give a roller follower the motion defined below :*

1. *Follower to move outwards through 50 mm during 120° of cam rotation,*

2. *Follower to dwell for next 60° of cam rotation,*

3. *Follower to return to its starting position during next 90° of cam rotation,*

4. *Follower to dwell for the rest of the cam rotation.*

The minimum radius of the cam is 50 mm and the diameter of roller is 10 mm. The line of stroke of the follower is off-set by 20 mm from the axis of the cam shaft. If the displacement of the *follower takes place with uniform and equal acceleration and retardation on both the outward and return strokes, draw profile of the cam and find the maximum velocity and acceleration during out stroke and return stroke.*

Solution. Given : N = 1000 r.p.m. ; S = 50 mm = 0.05 m ; θ_O = 120° = 2 π /3 rad = 2.1 rad ; θ_R = 90° = π /2 rad = 1.571 rad

Since the displacement of the follower takes place with uniform and equal acceleration and retardation on both outward and return strokes, therefore the displacement diagram, as shown in Fig. 20.29, is drawn in the similar manner as discussed in the previous example. But in this case, the angular displacement and stroke of the follower is divided into eight equal parts.

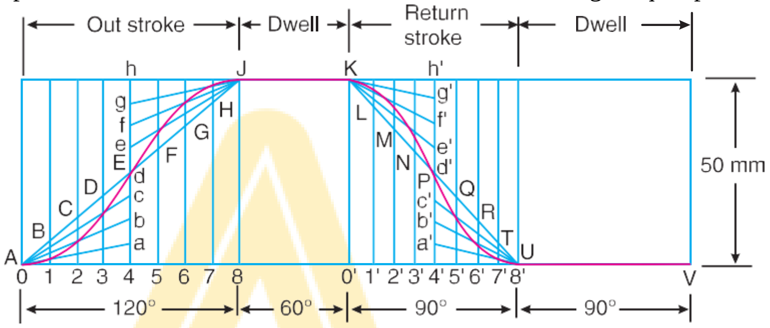
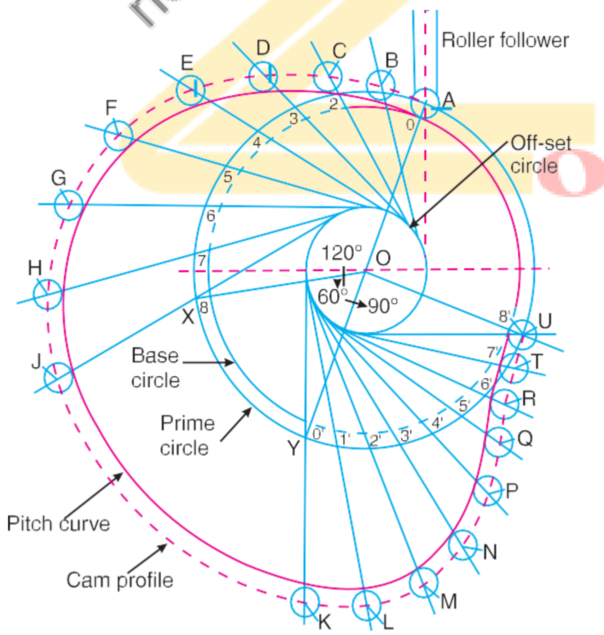


Fig. 20.29

Now, the profile of the cam, as shown in Fig. 20.30, is drawn as discussed in the following steps :

1. Draw a base circle with centre O and radius equal to the minimum radius of the cam (*i.e.* 50 mm).



2. Draw a prime circle with centre *O* and radius

$$OA = \text{Minimum radius of the cam + radius of roller} = 50 + 5 = 55 \text{ mm}$$

3. Draw an off-set circle with centre *O* and radius equal to 20 mm.
4. Divide the angular displacements of the cam during out stroke and return stroke into eight equal parts as shown by points 0, 1, 2 . . . and 0′ ,1′ , 2′ . . . etc. on the prime circle in Fig. 20.30.
5. From these points draw tangents to the off-set circle.
6. Set off 1*B*, 2*C*, 3*D* . . . etc. equal to the displacements as measured from the displacement diagram.
7. By joining the points *A*, *B*, *C* . . . *T*, *U*, *A* with a smooth curve, we get a pitch curve.
8. Now from points *A*, *B*, *C* . . . *T*, *U*, draw circles with radius equal to the radius of the roller.
9. Join the bottoms of these circles with a smooth curve to obtain the profile of the cam as shown in Fig. 20.30.

Maximum velocity of the follower during out stroke and return stroke

We know that angular velocity of the cam,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1000}{60} = 104.7 \text{ rad/s.}$$

We also know that the maximum velocity of the follower during outstroke,

$$v_O = \frac{2\omega . S}{\theta_O} = \frac{2 \times 104.7 \times 0.05}{2.1} = 5 \text{ m/s} \textbf{ Ans.}$$

and maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega . S}{\theta_R} = \frac{2 \times 104.7 \times 0.05}{1.571} = 6.66 \text{ m/s} \textbf{ Ans.}$$

Maximum acceleration of the follower during out stroke and return stroke

A rocker using a cam.

We know that the maximum acceleration of the follower during out stroke,

$$a_O = \frac{4\omega^2 . S}{(\theta_O)^2} = \frac{4(104.7)^2 0.05}{(2.1)^2} = 497.2 \text{ m/s}^2 \textbf{ Ans.}$$

and maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\omega^2 . S}{(\theta_R)^2} = \frac{4(104.7)^2 0.05}{(1.571)^2} = 888 \text{ m/s}^2 \textbf{ Ans.}$$

Example 20.9. *Construct the profile of a cam to suit the following specifications :*

Cam shaft diameter = 40 mm ; Least radius of cam = 25 mm ; Diameter of roller = 25 mm; Angle of lift = 120° ; Angle of fall = 150° ; Lift of the follower = 40 mm ; Number of pauses are two of equal interval between motions.

During the lift, the motion is S.H.M. During the fall the motion is uniform acceleration and deceleration. The speed of the cam shaft is uniform. The line of stroke of the follower is off-set 12.5 mm from the centre of the cam.

Construction

First of all the displacement diagram, as shown in Fig. 20.31, is drawn as discussed in the following steps :

1. Since the follower moves with simple harmonic motion during lift (*i.e.* for 120° of cam rotation), therefore draw the displacement curve *ADG* in the similar manner as discussed in Example 20.2.
2. Since the follower moves with uniform acceleration and deceleration during fall (*i.e.* for 150° of cam rotation), therefore draw the displacement curve *HLP* consisting of double parabola as discussed in Example 20.6.

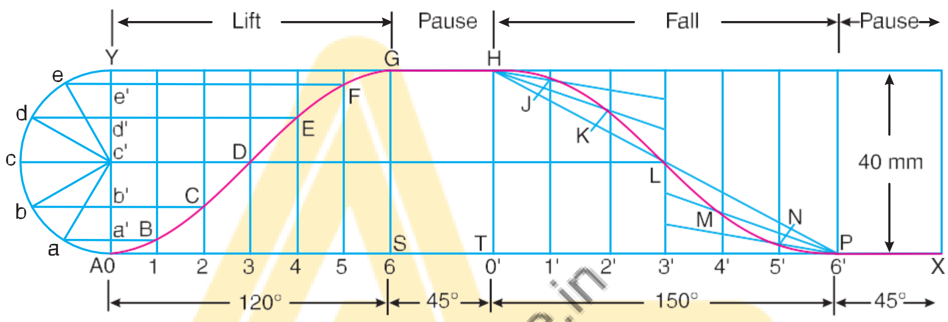


Fig. 20.31

Now the profile of the cam, when the line of stroke of the follower is off-set 12.5 mm from the centre of the cam, as shown in Fig. 20.32, is drawn as discussed in the following steps :

1. Draw a base circle with centre *O* and radius equal to the least radius of cam (*i.e.* 25 mm).

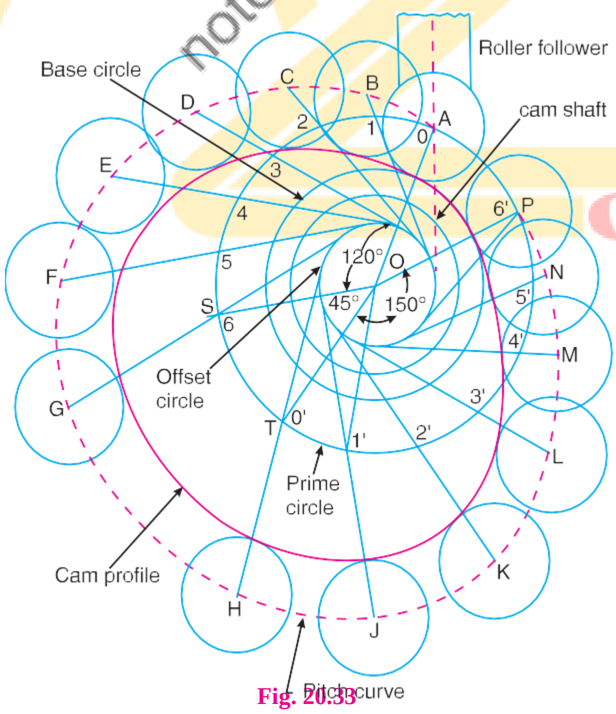


Fig. 20.33

2. Draw a prime circle with centre O and radius,

$$OA = \text{Least radius of cam + radius of roller} = 25 + 25/2 = 37.5 \text{ mm}$$

3. Draw a circle with centre O and radius equal to 20 mm to represent the cam shaft.
4. Draw an offset circle with centre O and radius equal to 12.5 mm.
5. Join OA. From OA draw angular displacements of the cam, *i.e.* draw angle AOS = 120° to represent lift of the follower, angle SOT = 45° to represent pause, angle TOP = 150° to represent fall of the follower and angle POA = 45° to represent pause.

   Note. Since the number of pauses are two of equal interval between motions (*i.e.* between lift and fall of the follower), therefore angular displacement of each pause

$$= \frac{360° - (120° + 150°)}{2} = 45°$$

6. Divide the angular displacements during lift and fall (*i.e.* angle AOS and TOP) into the same number of equal even parts (*i.e.* six parts) as in the displacement diagram.
7. From points 1, 2, 3 . . . etc. and 0′, 1′, 2′, 3′ . . . etc. on the prime circle, draw tangents to the off-set circle.
8. Set off 1B, 2C, 3D . . . etc. equal to the displacements as measured from the displacement diagram.
9. By joining the points A, B, C . . . M, N, P with a smooth curve, we get a pitch curve.
10. Now with A, B, C . . . etc. as centre, draw circles with radius equal to the radius of roller.
11. Join the bottoms of the circles with a smooth curve as shown in Fig. 20.32. This is the required profile of the cam.

Example 20.10. *It is required to set out the profile of a cam to give the following motion to the reciprocating follower with a flat mushroom contact face :*

(i) Follower to have a stroke of 20 mm during 120° of cam rotation ;

(ii) Follower to dwell for 30° of cam rotation ;

(iii) Follower to return to its initial position during 120° of cam rotation ; and

(iv) Follower to dwell for remaining 90° of cam rotation.

The minimum radius of the cam is 25 mm. The out stroke of the follower is performed with simple harmonic motion and the return stroke with equal uniform acceleration and retardation.

Construction

Since the out stroke of the follower is performed with simple harmonic motion and the return stroke with uniform acceleration and retardation, therefore the displacement diagram, as shown in Fig. 20.33, is drawn in the similar manner as discussed in the previous example.

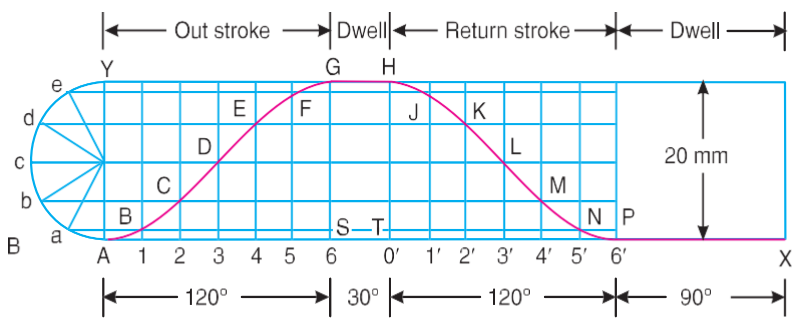


Fig. 20.33

The profile of the cam with a flat mushroom contact face reciprocating follower, as shown in Fig. 20.34, is drawn in the similar way as discussed in Example 20.4.

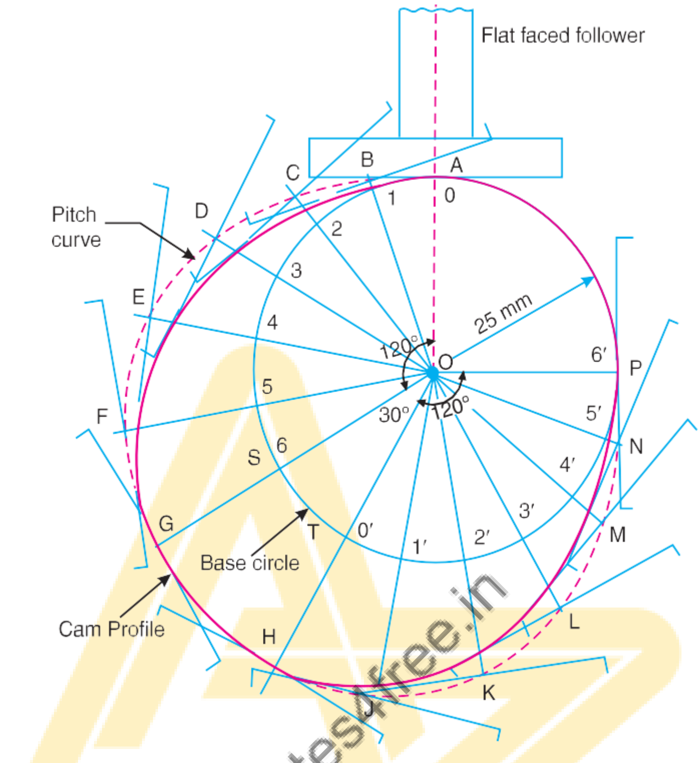


Fig. 20.34

Example 20.11. *It is required to set out the profile of a cam with oscillating follower for the following motion :*

(a) *Follower to move outward through an angular displacement of 20° during 90° of cam rotation ;* ***(b)*** *Follower to dwell for 45° of cam rotation ;* ***(c)*** *Follower to return to its original position of zero displacement in 75° of cam rotation ; and* ***(d)*** *Follower to dwell for the remaining period of the revolution of the cam.*

The distance between the pivot centre and the follower roller centre is 70 mm and the roller diameter is 20 mm. The minimum radius of the cam corresponds to the starting position of the follower as given in ***(a).*** *The location of the pivot point is 70 mm to the left and 60 mm above the axis of rotation of the cam. The motion of the follower is to take place with S.H.M. during out stroke and with uniform acceleration and retardation during return stroke.*

Construction

We know that the angular displacement of the roller follower,

$$= 20° = 20 \times \pi / 180 = \pi / 9 \text{ rad}$$

Since the distance between the pivot centre and the roller centre (*i.e.* radius A_1A) is 70 mm, therefore length of arc AA_2, as shown in Fig. 20.35, along which the displacement of the roller actually takes place

$$= 70 \times \pi / 9 = 24.5 \text{ mm}$$

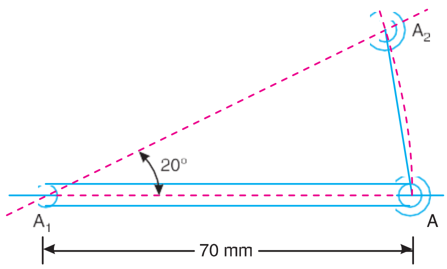


Fig. 20.35

Fig. 20.37

Since the angle is very small, therefore length of chord AA_2 is taken equal to the length of arc AA_2. Thus in order to draw the displacement diagram, we shall take lift of the follower equal to the length of chord AA_2 *i.e.* 24.5 mm.

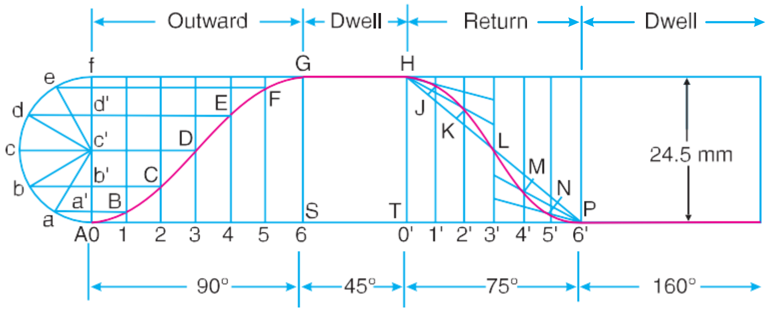


Fig. 20.36

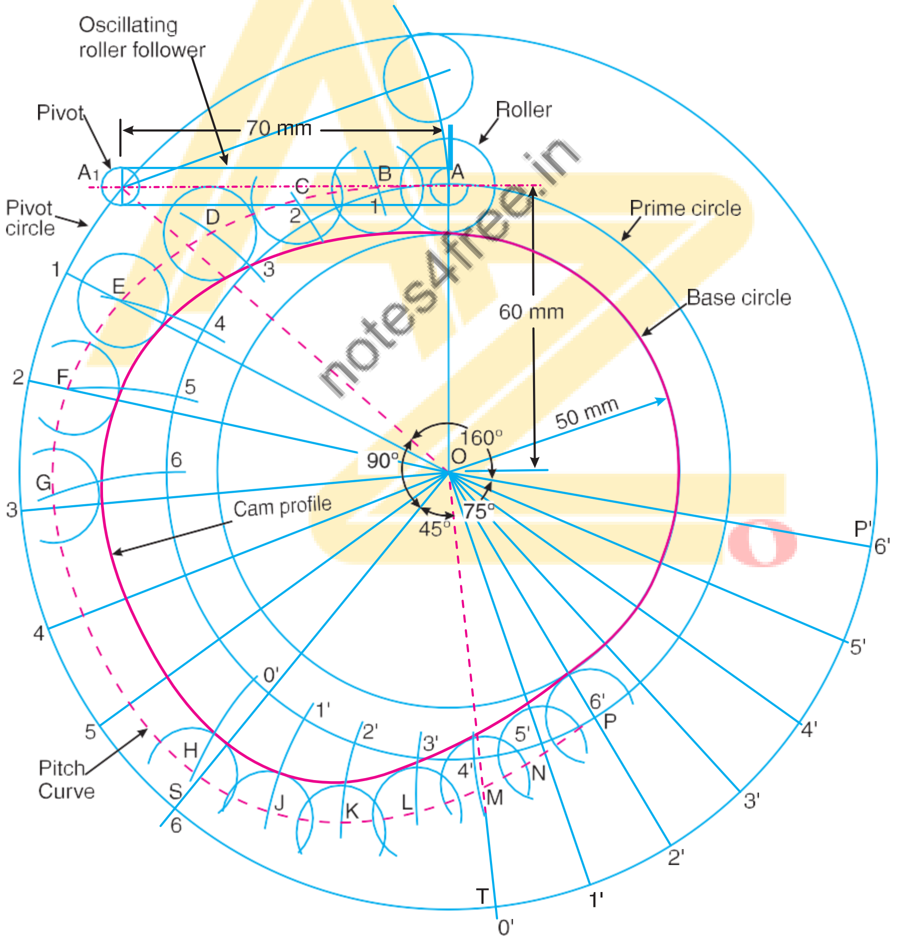


Fig. 20.37

The follower moves with simple harmonic motion during out stroke and with uniform acceleration and retardation during return stroke. Therefore, the displacement diagram, as shown in Fig. 20.36, is drawn in the similar way as discussed in the previous example.

The profile of the cam, as shown in Fig. 20.37, is drawn as discussed in the following steps :

1. First of all, locate the pivot point A_1 which is 70 mm to the left and 60 mm above the axis of the cam.
2. Since the distance between the pivot centre A_1 and the follower roller centre A is 70 mm and the roller diameter is 20 mm, therefore draw a circle with centre A and radius equal to the radius of roller *i.e.* 10 mm.
3. We find that the minimum radius of the cam

$$= 60 - 10 = 50 \text{ mm}$$

$\therefore$ Radius of the prime circle,

$$OA = \text{Min. radius of cam} + \text{Radius of roller} = 50 + 10 = 60 \text{ mm}$$

4. Now complete the profile of the cam in the similar way as discussed in Example 20.5.

Example 20.12. *Draw the profile of the cam when the roller follower moves with cycloidal motion during out stroke and return stroke, as given below :*

1. Out stroke with maximum displacement of 31.4 mm during 180° of cam rotation,

2. Return stroke for the next 150° of cam rotation,

3. Dwell for the remaining 30° of cam rotation.

The minimum radius of the cam is 15 mm and the roller diameter of the follower is 10 mm. The axis of the roller follower is offset by 10 mm towards right from the axis of cam shaft.

Construction

First of all, the displacement diagram, as shown in Fig. 20.38, is drawn as discussed in the following steps :

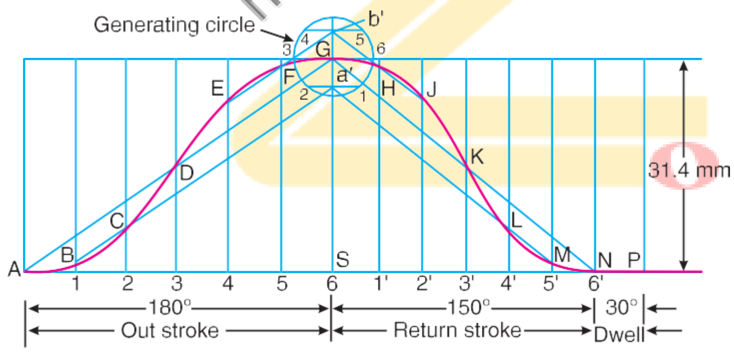


Fig. 20.38

1. Draw horizontal line *ASP* such that *AS* = 180° to represent the out stroke, *SN* = 150° to represent the return stroke and *NP* = 30° to represent the dwell period.
2. Divide *AS* and *SN* into any number of even equal parts (say six).
3. From the points 1, 2, 3 . . . etc. draw vertical lines and set-off equal to the stroke of the follower.
4. From a point *G* draw a generating circle of radius,

Fig. 20.37

$$r = \frac{\text{Stroke}}{2\pi} = \frac{31.4}{2\pi} = 5 \text{ mm}$$

5. Divide the generating circle into six equal parts and from these points draw horizontal lines to meet the vertical diameter at a' , G and b' .

6. Join AG and GN. From point a' , draw lines parallel to AG and GN to intersect the vertical lines drawn through 1, 2, 4′ and 5′ at B, C, L and M respectively. Similarly draw parallel lines from b' intersecting the vertical lines through 4, 5, 1′ and 2′ at E, F, H and J respectively.

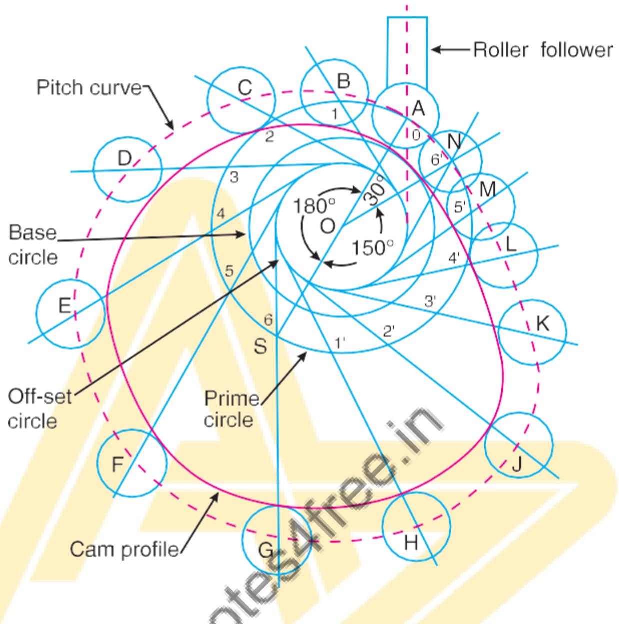


Fig. 20.39

7. Join the points A, B, C . . . L, M, N with a smooth curve.

8. The curve A B C . . . L M N is the required displacement diagram.

Now the profile of the cam, as shown in Fig. 20.39, may be drawn in the similar way as discussed in Example 20.9.

20.11. Cams with Specified Contours

In the previous articles, we have discussed about the design of the profile of a cam when the follower moves with the specified motion. But, the shape of the cam profile thus obtained may be difficult and costly to manufacture. In actual practice, the cams with specified contours (cam profiles consisting of circular arcs and straight lines are preferred) are assumed and then motion of the follower is determined.

20.12. Tangent Cam with Reciprocating Roller Follower

When the flanks of the cam are straight and tangential to the base circle and nose circle, then the cam is known as a ***tangent cam,*** as shown in Fig. 20.40. These cams are usually symmetrical about the centre line of the cam shaft. Such type of cams are used for operating the inlet and exhaust valves of internal combustion engines. We shall now derive the expressions for displacement, velocity and acceleration of the follower for the following two cases :

1. When the roller has contact with the straight flanks ; and

2. When the roller has contact with the nose.

Let r_1 = Radius of the base circle or minimum radius of the cam,

r_2 = Radius of the roller,

r_3 = Radius of nose,

α = Semi-angle of action of cam or angle of ascent,

θ = Angle turned by the cam from the beginning of the roller displacement,

ϕ = Angle turned by the cam for contact of roller with the straight flank, and

ω = Angular velocity of the cam.

1. *When the roller has contact with straight flanks.* A roller having contact with straight flanks is shown in Fig. 20.40. The point *O* is the centre of cam shaft and the point *K* is the centre of nose. *EG* and *PQ* are straight flanks of the cam. When the roller is in lowest position, (*i.e.* when the roller has contact with the straight flank at *E*), the centre of roller lies at *B* on the pitch curve. Let the cam has turned through an angle* θ (less than ϕ) for the roller to have contact at any point (say *F*) between the straight flanks *EG*. The centre of roller at this stage lies at *C*. Therefore displacement (or lift or stroke) of the roller from its lowest position is given by

$$x = OC - OB = \frac{OB}{\cos\theta} - OB = OB\left(\frac{1-\cos\theta}{\cos\theta}\right)$$

$$= (r_1 + r_2)\left(\frac{1-\cos\theta}{\cos\theta}\right) \quad \ldots (\because OB = OE + EB = r_1 + r_2) \quad \ldots \textbf{(i)}$$

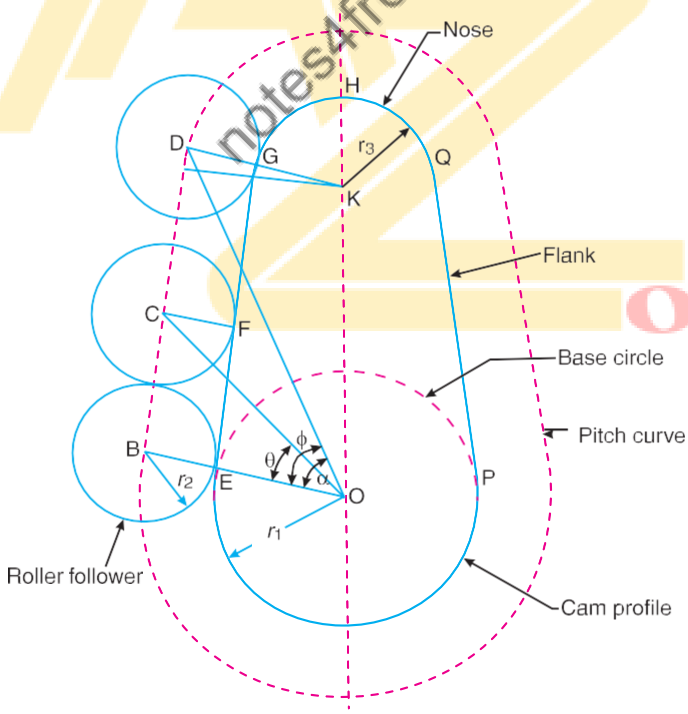


Fig. 20.40. Tangent cam with reciprocating roller follower having contact with straight flanks.

* Since the cam is assumed to be stationary, the angle θ is turned by the roller.

Differentiating equation (*i*) with respect to *t*, we have velocity of the follower,

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = (r_1 + r_2)\left(\frac{\sin\theta}{\cos^2\theta}\right)\frac{d\theta}{dt}$$

$$= \omega(r_1 + r_2)\left(\frac{\sin\theta}{\cos^2\theta}\right) \qquad \ldots (\because d\theta/dt = \omega) \quad \ldots \textbf{(ii)}$$

From equation ***(ii)***, we see that when θ increases, sin θ increases and cos θ decreases. In other words, $\sin\theta/\cos^2\theta$ increases. Thus the velocity is maximum where θ is maximum. This happens when θ = φ *i.e.* when the roller just leaves contact with the straight flank at *G* or when the straight flank merges into a circular nose.

∴ Maximum velocity of the follower,

$$v_{max} = \omega(r_1 + r_2)\left(\frac{\sin\phi}{\cos^2\phi}\right)$$

A car in the assembly line.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Now differentiating equation ***(ii)*** with respect to *t*, we have acceleration of the follower,

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt}$$

$$= \omega(r_1 + r_2)\left(\frac{\cos^2\theta.\cos\theta - \sin\theta \times 2\cos\theta \times -\sin\theta}{\cos^4\theta}\right)\frac{d\theta}{dt}$$

$$= \omega^2 (r_1 + r_2)\left(\frac{\cos^2\theta + 2\sin^2\theta}{\cos^3\theta}\right) \qquad \ldots \left(\because \frac{d\theta}{dt} = \omega\right)$$

$$= \omega^2 (r_1 + r_2)\left[\frac{\cos^2\theta + 2(1 - \cos^2\theta)}{\cos^3\theta}\right]$$

$$= \omega^2 (r_1 + r_2)\left(\frac{2 - \cos^2\theta}{\cos^3\theta}\right) \qquad \ldots \textbf{(iii)}$$

A little consideration will show that the acceleration is minimum when $\frac{2 - \cos^2\theta}{\cos^3\theta}$ is minimum. This is only possible when $(2 - \cos^2\theta)$ is minimum and $\cos^3\theta$ is maximum. This happens when $\theta = 0°$, *i.e.* when the roller is at the beginning of its lift along the straight flank (or when the roller has contact with the straight flank at *E*).

∴ Minimum acceleration of the follower,

$$a_{min} = \omega^2 (r_1 + r_2)$$

The acceleration is maximum when $\theta = \phi$, *i.e.* when the roller just leaves contact with the straight flank at *G* or when the straight flank merges into a circular nose.

∴ Maximum acceleration of the follower,

$$a_{max} = \omega^2 (r_1 + r_2) \left(\frac{2 - \cos^2 \phi}{\cos^3 \phi} \right)$$

2. ***When the roller has contact with the nose.*** A roller having contact with the circular nose at *G* is shown in Fig 20.41. The centre of roller lies at *D* on the pitch curve. The displacement is usually measured from the top position of the roller, *i.e.* when the roller has contact at the apex of the nose (point *H*) and the centre of roller lies at *J* on the pitch curve.

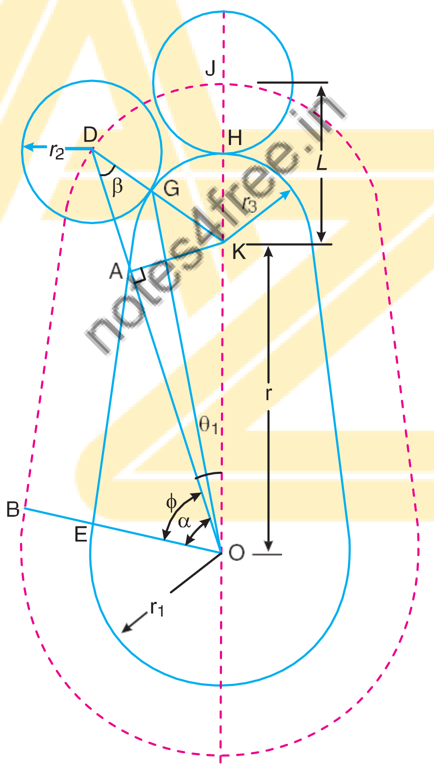


Fig. 20.41. Tangent cam with reciprocating roller follower having contact with the nose.

Let θ_1 = Angle turned by the cam measured from the position when the roller is at the top of the nose.

The displacement of the roller is given by

$$x = OJ - OD = OJ - (OA + AD) = (OK + KJ) - (OA + AD)$$

Substituting $OK = r$ and $KJ = KH + HJ = r_3 + r_2 = L$, we have

$$x = (r + L) - (OK \times \cos\theta_1 + DK \cos\beta)$$

$$= (r + L) - (r\cos\theta_1 + L\cos\beta) \qquad \ldots (\because DK = KJ = r_3 + r_2 = L)$$

$$= L + r - r\cos\theta_1 - L\cos\beta \qquad \ldots \textbf{(i)}$$

Now from right angled triangles *OAK* and *DAK*,

$$AK = DK\sin\beta = OK\sin\theta_1$$

or $$L\sin\beta = r\sin\theta_1$$

Squaring both sides,

$$L^2\sin^2\beta = r^2\sin^2\theta_1 \text{ or } L^2(1-\cos^2\beta) = r^2\sin^2\theta_1$$

$$L^2 - L^2\cos^2\beta = r^2\sin^2\theta_1 \text{ or } L^2\cos^2\beta = L^2 - r^2\sin^2\theta_1$$

$$\therefore \quad L\cos\beta = (L^2 - r^2\sin^2\theta_1)^{\frac{1}{2}}$$

Substituting the value of $L\cos\beta$ in equation **(*i*)**, we get

$$x = L + r - r\cos\theta_1 - (L^2 - r^2\sin^2\theta_1)^{\frac{1}{2}} \qquad \ldots \textbf{(ii)}$$

Differentiating equation **(*ii*)** with respect to *t*, we have velocity of the follower,

$$v = \frac{dx}{dt} = \frac{dx}{d\theta_1} \times \frac{d\theta_1}{dt}$$

$$= -r \times -\sin\theta_1 \times \frac{d\theta_1}{dt} - \frac{1}{2}(L^2 - r^2\sin^2\theta_1)^{-\frac{1}{2}}(-r^2 \times 2\sin\theta_1\cos\theta_1)\frac{d\theta_1}{dt}$$

$$= r\sin\theta_1 \times \frac{d\theta_1}{dt} + \frac{1}{2}(L^2 - r^2\sin^2\theta_1)^{-\frac{1}{2}} r^2 \times \sin 2\theta_1 \times \frac{d\theta_1}{dt}$$

$$= \omega . r\left[\sin\theta_1 + \frac{r\sin 2\theta_1}{2(L^2 - r^2\sin^2\theta_1)^{\frac{1}{2}}}\right] \qquad \ldots \left(\text{Substituting } \frac{d\theta_1}{dt} = \omega\right) \quad \ldots \textbf{(iii)}$$

Now differentiating equation **(*iii*)** with respect to *t*, we have acceleration of the follower,

$$a = \frac{dv}{dt} = \frac{dv}{d\theta_1} \times \frac{d\theta_1}{dt}$$

$$= \omega . r\left[\cos\theta_1 + \frac{(L^2 - r^2\sin^2\theta_1)^{\frac{1}{2}}(r \times 2\cos 2\theta_1) + r\sin 2\theta_1 \times \frac{1}{2}(L^2 - r^2\sin^2\theta_1)^{-\frac{1}{2}}(r^2 \times 2\sin\theta_1\cos\theta_1)}{2(L^2 - r^2\sin^2\theta_1)}\right]\frac{d\theta_1}{dt}$$

Substituting $\frac{d\theta_1}{dt} = \omega$ and multiplying the numerator and denominator of second term by $(L^2 - r^2 \sin^2 \theta_1)^{\frac{1}{2}}$, we have

$$a = \omega^2 r \left[\cos \theta_1 + \frac{(L^2 - r^2 \sin^2 \theta_1)(2\, r \cos 2\theta_1) + \frac{1}{2} \times r^3 \sin^2 2\theta_1}{2(L^2 - r^2 \sin^2 \theta_1)^{3/2}} \right]$$

$$= \omega^2 r \left[\cos \theta_1 + \frac{L^2 \times 2r \cos 2\theta_1 - 2r^3 \sin^2 \theta_1 . \cos 2\theta_1 + \frac{1}{2} \times r^3 (2 \sin \theta_1 \cos \theta_1)^2}{2(L^2 - r^2 \sin^2 \theta_1)^{3/2}} \right]$$

$$= \omega^2 . r \left[\cos \theta_1 + \frac{2L^2 . r \cos 2\theta_1 - 2r^3 . \sin^2 \theta_1 (1 - 2 \sin^2 \theta_1) + 2r^3 \sin^2 \theta_1 (1 - \sin^2 \theta_1)}{2(L^2 - r^2 \sin^2 \theta_1)^{3/2}} \right]$$

$$= \omega^2 . r \left[\cos \theta_1 + \frac{L^2 . r \cos 2\theta_1 + r^3 \sin^4 \theta_1}{(L^2 - r^2 \sin^2 \theta_1)^{3/2}} \right]$$

Notes : 1. Since θ_1 is measured from the top position of the roller, therefore for the roller to have contact at the apex of the nose (*i.e.* at point *H*), then $\theta_1 = 0$, and for the roller to have contact where straight flank merges into a nose (*i.e.* at point *G*), then $\theta_1 = \alpha - \phi$.

2. The velocity is zero at *H* and maximum at *G*.

3. The acceleration is minimum at *H* and maximum at *G*.

4. From Fig 20.41, we see that the distances *OK* and *KD* remains constant for all positions of the roller when it moves along the circular nose. In other words, a tangent cam operating a roller follower and having contact with the nose is equivalent to a slider crank mechanism (*i.e. ODK*) in which the roller is assumed equivalent to the slider *D*, crank *OK* and connecting rod *DK*. Therefore the velocity and acceleration of the roller follower may be obtained graphically as discussed in Chapters 7 and 8.

Example 20.13. *In a symmetrical tangent cam operating a roller follower, the least radius of the cam is 30 mm and roller radius is 17.5 mm. The angle of ascent is 75° and the total lift is 17.5 mm. The speed of the cam shaft is 600 r.p.m. Calculate :* **1.** *the principal dimensions of the cam ;* **2.** *the accelerations of the follower at the beginning of the lift, where straight flank merges into the circular nose and at the apex of the circular nose. Assume that there is no dwell between ascent and descent.*

Solution. Given : $r_1 = 30$ mm ; $r_2 = 17.5$ mm ; $\alpha = 75°$; Total lift = 17.5 mm ; $N = 600$ r.p.m. or $\omega = 2\,\pi \times 600/60 = 62.84$ rad/s

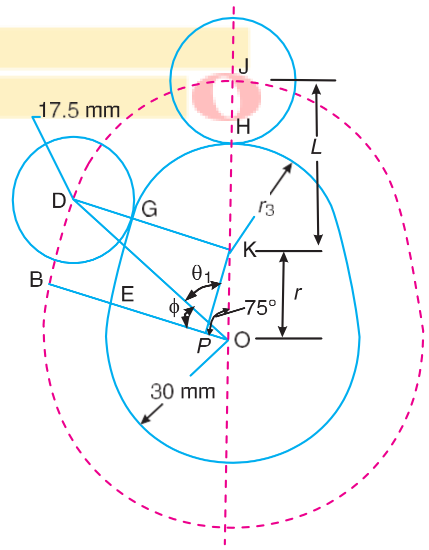


Fig. 20.42

1. *Principal dimensions of the cam*

Let $r = OK$ = Distance between cam centre and nose centre,

r_3 = Nose radius, and

ϕ = Angle of contact of cam with straight flanks.

From the geometry of Fig. 20.42,

$$r + r_3 = r_1 + \text{Total lift}$$

$$= 30 + 17.5 = 47.5 \text{ mm}$$

$$\therefore \quad r = 47.5 - r_3 \quad \ldots (i)$$

Also, $OE = OP + PE$ or $r_1 = OP + r_3$

$$\therefore \quad OP = r_1 - r_3 = 30 - r_3 \quad \ldots (ii)$$

Now from right angled triangle *OKP*,

$$OP = OK \times \cos\alpha \quad \ldots (\because \cos\alpha = OP / OK)$$

or

$$30 - r_3 = (47.5 - r_3)\cos 75° = (47.5 - r_3)0.2588 = 12.3 - 0.2588\, r_3 \quad \ldots (\because OK = r)$$

$$\therefore \quad r_3 = 23.88 \text{ mm } \textbf{Ans.}$$

and

$$r = OK = 47.5 - r_3 = 47.5 - 23.88 = 23.62 \text{ mm } \textbf{Ans.}$$

Again, from right angled triangle *ODB*,

$$\tan\phi = \frac{DB}{OB} = \frac{KP}{OB} = \frac{OK \sin\alpha}{r_1 + r_2} = \frac{23.62 \sin 75°}{30 + 17.5} = 0.4803$$

$$\therefore \quad \phi = 25.6° \textbf{ Ans.}$$

2. *Acceleration of the follower at the beginning of the lift*

We know that acceleration of the follower at the beginning of the lift, *i.e.* when the roller has contact at *E* on the straight flank,

$$a_{min} = \omega^2 (r_1 + r_2) = (62.84)^2 (30 + 17.5)^2 = 187\ 600 \text{ mm/s}^2$$

$$= 187.6 \text{ m/s}^2 \textbf{ Ans.}$$

Acceleration of the follower where straight flank merges into a circular nose

We know that acceleration of the follower where straight flank merges into a circular nose *i.e.* when the roller just leaves contact at *G*,

$$a_{max} = \omega^2 (r_1 + r_2)\left[\frac{2 - \cos^2\phi}{\cos^3\phi}\right] = (62.84)^2 (30 + 17.5)\left(\frac{2 - \cos^2 25.6°}{\cos^3 25.6°}\right)$$

$$= 187\ 600\left(\frac{2 - 0.813}{0.733}\right) = 303\ 800 \text{ mm/s}^2 = \textbf{303.8 m/s}^2 \textbf{ Ans.}$$

Acceleration of the follower at the apex of the circular nose

We know that acceleration of the follower for contact with the circular nose,

$$a = \omega^2 . r \left[\cos\theta_1 + \frac{L^2 . r\cos 2\theta_1 + r^3 \sin^4\theta_1}{(L^2 - r^2 \sin^2\theta_1)^{3/2}}\right]$$

Since θ_1 is measured from the top position of the follower, therefore for the follower to have contact at the apex of the circular nose (*i.e.* at point *H*), $\theta_1 = 0$.

∴ Acceleration of the follower at the apex of the circular nose,

$$a = \omega^2 . r\left(1 + \frac{L^2 . r}{L^3}\right) = \omega^2 . r\left(1 + \frac{r}{L}\right) = \omega^2 . r\left(1 + \frac{r}{r_2 + r_3}\right)$$

$$= (62.84)^2\ 23.62\left(1 + \frac{23.62}{17.5 + 23.88}\right) = 146\ 530 \text{ mm/s}^2 \qquad \ldots (\because L = r_2 + r_3)$$

$= 146.53$ m/s^2 **Ans.**

Example 20.14. *A cam has straight working faces which are tangential to a base circle of diameter 90 mm. The follower is a roller of diameter 40 mm and the centre of roller moves along a straight line passing through the centre line of the cam shaft. The angle between the tangential faces of the cam is 90° and the faces are joined by a nose circle of 10 mm radius. The speed of rotation of the cam is 120 revolutions per min.*

Find the acceleration of the roller centre **1.** *when during the lift, the roller is just about to leave the straight flank ; and* **2.** *when the roller is at the outer end of its lift.*

Solution. Given : $d_1 = 90$ mm or $r_1 = 45$ mm ; $d_2 = 40$ mm or $r_2 = 20$ mm ; $2\,\alpha = 90°$ or $\alpha = 45°$; $r_3 = 10$ mm ; $N = 120$ r.p.m. or $\omega = 2\,\pi \times 120/60 = 12.57$ rad/s

The tangent cam operating a roller follower is shown in Fig. 20.43.

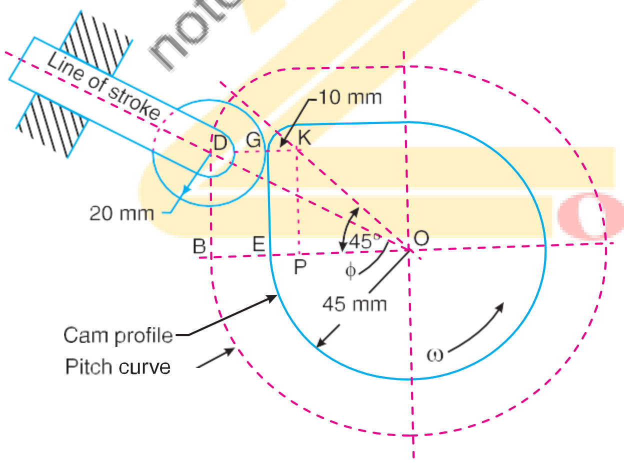


Fig. 20.43

First of all, let us find the *angle turned by the cam (ϕ) when the roller is just about to leave the straight flank at *G*. The centre of roller at this position lies at *D*.

* Since the cam is assumed to be stationary, ϕ is the angle turned by the roller when it is just about to leave the straight flank at *G*.


In aircraft engines roller followers are widely used.

From the geometry of the figure,

$$BD = PK = OP = OE - PE$$
$$= OE - KG$$
$$= r_1 - r_3 = 45 - 10 = 35 \text{ mm}$$

Now from triangle *OBD*,

$$\tan\phi = \frac{BD}{OB} = \frac{BD}{OE + EB}$$
$$= \frac{BD}{r_1 + r_2} = \frac{35}{45 + 20} = 0.5385$$

$$\therefore \qquad \phi = 28.3°$$

1. Acceleration of the roller centre when roller is just about to leave the straight flank

We know that acceleration of the roller centre when the roller is just about to leave the straight flank,

$$a = \omega^2 (r_1 + r_2)\left(\frac{2 - \cos^2\phi}{\cos^3\phi}\right) = (12.57)^2 (45 + 20)\left(\frac{2 - \cos^2 28.3°}{\cos^3 28.3°}\right)$$
$$= 18\,500 \text{ mm/s}^2 = 18.5 \text{ m/s}^2 \textbf{ Ans.}$$

2. Acceleration of the roller centre when the roller is at the outer end of the lift

First of all, let us find the values of *OK* and *KD*. From the geometry of the figure,

$$OK = r = \sqrt{(OP)^2 + (PK)^2} = \sqrt{2} \times OP \qquad \ldots (\because OP = PK)$$
$$= \sqrt{2}(OE - EP) = \sqrt{2}(45 - 10) = 49.5 \text{ mm}$$
$$KD = L = KG + GD = r_3 + r_2 = 10 + 20 = 30 \text{ mm}$$

We know that acceleration of the roller centre when the roller is at the outer end of the lift, *i.e.* when the roller has contact at the top of the nose,

$$a = \omega^2 . r\left[\cos\theta_1 + \frac{L^2 . r\cos 2\theta_1 + r^3\sin^4\theta_1}{(L^2 - r^2\sin^2\theta_1)^{3/2}}\right] = \omega^2 . r\left(1 + \frac{r}{L}\right)$$
$$\ldots (\because \text{At the outer end of the lift, } \theta_1 = 0)$$
$$= (12.57)^2\, 49.5\left(1 + \frac{49.5}{30}\right) = 20\,730 \text{ mm/s}^2 = 20.73 \text{ m/s}^2 \textbf{ Ans.}$$

20.13. Circular Arc Cam with Flat-faced Follower

When the flanks of the cam connecting the base circle and nose are of convex circular arcs, then the cam is known as ***circular arc cam.*** A symmetrical circular arc cam operating a flat-faced follower is shown in Fig. 20.44, in which *O* and *Q* are the centres of cam and nose respectively. *EF* and *GH* are two circular flanks whose centres lie at *P* and *P′* respectively. The centres * *P* and *P′*

* The centres *P* and *P′* may also be obtained by drawing arcs with centres *O* and *Q* and radii equal to *OP* and *PQ* respectively. The circular flanks *EF* and *GH* are now drawn with centres *P* and *P′* and radius equal to *PE*.

lie on lines *EO* and *GO* produced.

Let r_1 = Minimum radius of the cam or radius of the base circle = *OE*,

r_2 = Radius of nose,

R = Radius of circular flank = *PE*,

2α = Total angle of action of cam = angle *EOG*,

α = Semi-angle of action of cam or angle of ascent = angle *EOK*, and

ϕ = Angle of action of cam on the circular flank.

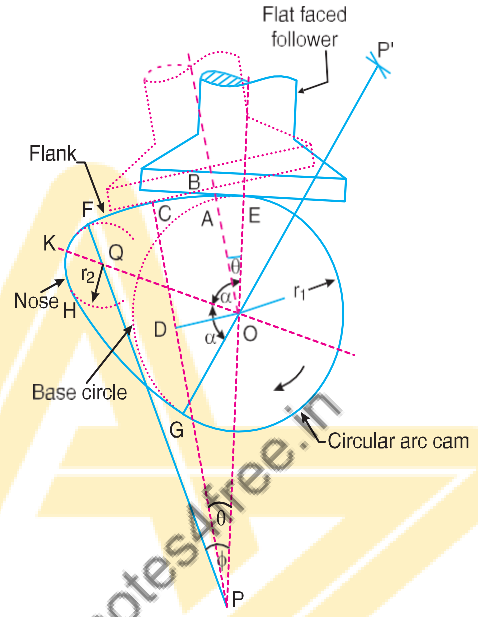


Fig. 20.44. Circular arc cam with flat face of the follower having contact with the circular flank.

We shall consider the following two cases :

1. When the flat face of the follower has contact on the circular flank, and

2. When the flat face of the follower has contact on the nose.

In deriving the expressions for displacement, velocity and acceleration of the follower for the above two cases, it is assumed that the cam is fixed and the follower rotates in the opposite sense to that of the cam. In Fig. 20.44, the cam is rotating in the clockwise direction and the follower rotates in the counter-clockwise direction.

1. ***When the flat face of the follower has contact on the circular flank.*** First of all, let us consider that the flat face of the follower has contact at *E* (*i.e.* at the junction of the circular flank and base circle). When the cam turns through an angle θ (less than ϕ) relative to the follower, the contact of the flat face of the follower will shift from *E* to *C* on the circular flank, such that flat face of the follower is perpendicular to *PC*. Since *OB* is perpendicular to *BC*, therefore *OB* is parallel to *PC*. From *O*, draw *OD* perpendicular to *PC*.

From the geometry of the figure, the displacement or lift of the follower (*x*) at any instant for contact on the circular flank, is given by

$$x = BA = BO - AO = CD - EO \qquad \ldots (i)$$

We know that

$$CD = PC - PD = PE - OP \cos\theta$$

$$= OP + OE - OP \cos\theta = OE + OP\,(1 - \cos\theta)$$

Substituting the value of CD in equation **(*i*),**

$$x = OE + OP(1-\cos\theta) - EO = OP(1-\cos\theta)$$

$$= (PE - OE)(1-\cos\theta) = (R - r_1)(1-\cos\theta) \qquad \ldots (ii)$$

Differentiating equation **(*ii*)** with respect to t, we have velocity of the follower,

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = \frac{dx}{d\theta} \times \omega \qquad \ldots \left(\text{substituting } \frac{d\theta}{dt} = \omega\right)$$

$$= (R - r_1)\sin\theta \times \omega = \omega(R - r_1)\sin\theta \qquad \ldots (iii)$$

From the above expression, we see that at the beginning of the ascent (*i.e.* when $\theta = 0$), the velocity is zero (because sin 0 = 0) and it increases as θ increases. The velocity will be maximum when $\theta = \phi$, *i.e.* when the contact of the follower just shifts from circular flank to circular nose. Therefore maximum velocity of the follower,

$$v_{max} = \omega(R - r_1)\sin\phi$$

Now differentiating equation **(*iii*)** with respect to t, we have acceleration of the follower,

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} = \frac{dv}{d\theta} \times \omega$$

$$= \omega(R - r_1)\cos\theta \times \omega = \omega^2 (R - r_1)\cos\theta \qquad \ldots (iv)$$

From the above expression, we see that at the beginning of the ascent (*i.e.* when $\theta = 0$), the acceleration is maximum (because cos 0 = 1) and it decreases as θ increases. The acceleration will be minimum when $\theta = \phi$.

$\therefore$ Maximum acceleration of the follower,

$$a_{max} = \omega^2 (R - r_1)$$

and minimum acceleration of the follower,

$$a_{min} = \omega^2 (R - r_1)\cos\phi$$

2. ***When the flat face of the follower has contact on the nose.*** The flat face of the follower having contact on the nose at C is shown in Fig. 20.45. The centre of curvature of the nose lies at Q. In this case, the displacement or lift of the follower at any instant when the cam has turned through an angle θ (greater than ϕ) is given by

$$x = AB = OB - OA = CD - OA \qquad \ldots (\because OB = CD) \ldots (i)$$

But $\quad CD = CQ + QD = CQ + OQ\cos(\alpha - \theta)$

Substituting the value of CD in equation **(*i*),** we have

$$x = CQ + OQ\cos(\alpha - \theta) - OA \qquad \ldots (ii)$$

The displacement or lift of the follower when the contact is at the apex K of the nose *i.e.* when $\alpha - \theta = 0$ is

$$^* x = CQ + OQ - OA = r_2 + OQ - r_1$$

* From the geometry of Fig. 20.45, we also find that lift of the follower when the contact is at the apex K of the nose is

$$x = JK = OQ + QK - OJ = OQ + r_2 - r_1$$

Differentiating equation ***(ii)*** with respect to *t*, we have velocity of the follower,

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = \frac{dx}{d\theta} \times \omega$$

$$= OQ \sin(\alpha - \theta)\omega = \omega \times OQ \sin(\alpha - \theta) \qquad \ldots \textbf{(iii)}$$

. . .($\because$ *CQ*, *OQ*, *OA* and α are constant)

From the above expression, we see that the velocity is zero when $\alpha - \theta = 0$ or $\alpha = \theta$ *i.e.* when the follower is at the apex *K* of the nose. The velocity will be maximum when $(\alpha - \theta)$ is maximum. This happens when the follower changes contact from circular flank to circular nose at point *F*, *i.e.* when $(\alpha - \theta) = \phi$.

Now differentiating equation ***(iii)*** with respect to *t*, we have acceleration of the follower,

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} = \frac{dv}{d\theta} \times \omega$$

$$= -\omega \times OQ \cos(\alpha - \theta)\omega = -\omega^2 \times OQ \cos(\alpha - \theta) \qquad \ldots \textbf{(iv)}$$

The negative sign in the above expression shows that there is a retardation when the follower is in contact with the nose of the cam.

From the above expression, we see that retardation is maximum when $\alpha - \theta = 0$ or $\theta = \alpha$, *i.e.* when the follower is at the apex *K* of the nose.

$\therefore$ Maximum retardation $= \omega^2 \times OQ$

The retardation is minimum when $\alpha - \theta$ is maximum. This happens when the follower changes contact from circular flank to circular nose at point *F i.e.* when $\theta = \phi$.

$\therefore$ Minimum retardation $= \omega^2 \times OQ \cos(\alpha - \phi)$

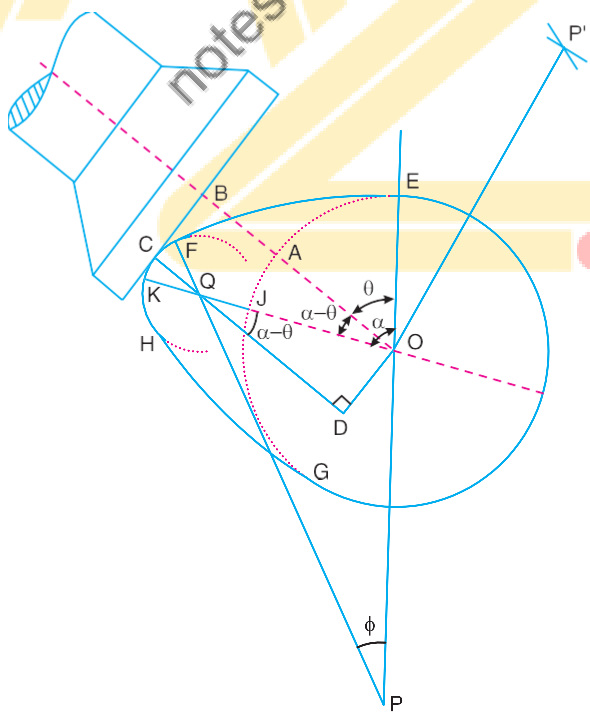


Fig. 20.45. Circular arc cam with flat face of the follower having contact on the nose.

Example 20.15. *A symmetrical circular cam operating a flat-faced follower has the following particulars :*

*Minimum radius of the cam = 30 mm ; Total lift = 20 mm ; Angle of lift = 75° ; Nose radius = 5 mm ; Speed = 600 r.p.m. Find : **1.** the principal dimensions of the cam, and **2.** the acceleration of the follower at the beginning of the lift, at the end of contact with the circular flank , at the beginning of contact with nose and at the apex of the nose.*

Solution. Given : $r_1 = OE = 30$ mm ; $x = JK = 20$ mm ; $\alpha = 75°$; $r_2 = QF = QK = 5$ mm ; $N = 600$ r.p.m. or $\omega = 2\pi \times 600 / 60 = 62.84$ rad/s

1. ***Principal dimensions of the cam***

A symmetrical circular cam operating a flat faced follower is shown in Fig. 20.46.

Let OQ = Distance between cam centre and nose centre,

$R = PE$ = Radius of circular flank, and

ϕ = Angle of contact on the circular flank.

We know that lift of the follower (x),

$$20 = OQ + r_2 - r_1 = OQ + 5 - 30 = OQ - 25$$

$\therefore$ $OQ = 20 + 25 = 45$ mm **Ans.**

We know that $PQ = PF - FQ = PE - FQ = OP + OE - FQ$

$$= OP + 30 - 5 = (OP + 25) \text{ mm}$$

Now from a triangle OPQ,

$$(PQ)^2 = (OP)^2 + (OQ)^2 - 2 \times OP \times OQ \cos\beta$$

$$(OP + 25)^2 = (OP)^2 + 45^2 - 2 \times OP \times 45 \cos(180° - 75°)$$

$$(OP)^2 + 50\,OP + 625 = (OP)^2 + 2025 + 23.3\,OP$$

$$50\,OP - 23.3\,OP = 2025 - 625$$

or $26.7\,OP = 1400$

and $OP = 1400/26.7 = 52.4$ mm

$\therefore$ Radius of circular flanks,

$R = PE = OP + OE = 52.4 + 30$

$= 82.4$ mm **Ans.**

and $PQ = OP + 25 = 52.4 + 25$

$= 77.4$ mm **Ans.**

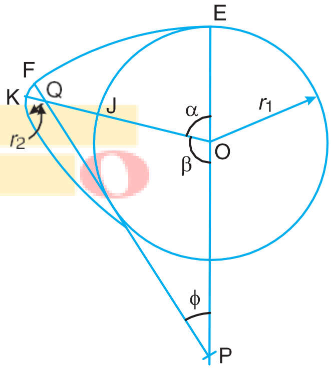


Fig. 20.46

In order to find angle ϕ , consider a triangle OPQ. We know that

$$\frac{OQ}{\sin\phi} = \frac{PQ}{\sin\beta}$$

or $$\sin\phi = \frac{OQ \times \sin\beta}{PQ} = \frac{45 \times \sin(180° - 75°)}{77.4} = 0.5616$$

$\therefore$ $\phi = 34.2°$ **Ans.**

2. ***Acceleration of the follower***

We know that acceleration of the follower at the beginning of the lift,

$$a = \omega^2 (R - r_1)\cos\theta = \omega^2 (R - r_1) \quad \ldots (\because \text{At the beginning of lift, } \theta = 0°)$$

$$= (62.84)^2 (82.4 - 30) = 206\,930 \text{ mm/s} = 206.93 \text{ m/s}^2 \textbf{ Ans.}$$

Acceleration of the follower at the end of contact with the circular flank,

$$a = \omega^2 (R - r_1) \cos\theta = \omega^2 (R - r_1) \cos\phi$$

$$\ldots (\because \text{At the end of contact with the circular flank, } \theta = \phi)$$

$$= -(62.84)^2 (82.4 - 30) \cos 34.2° = 171\,130 \text{ mm/s}^2 = 171.13 \text{ m/s}^2 \textbf{ Ans.}$$

Acceleration of the follower at the beginning of contact with nose,

$$a = -\omega^2 \times OQ \cos(\alpha - \theta) = -\omega^2 \times OQ \cos(\alpha - \phi)$$

$$\ldots (\because \text{At the beginning of contact with nose, } \theta = \phi)$$

$$= -(62.84)^2\, 45 \cos(75° - 34.2°) = -134\,520 \text{ mm/s}^2 = -134.52 \text{ m/s}^2$$

$$= 134.52 \text{ m/s}^2 \textbf{ (Retardation) Ans.}$$

and acceleration of the follower at the apex of nose,

$$a = -\omega^2 \times OQ \cos(\alpha - \theta) = -\omega^2 \times OQ \ldots (\because \text{At the apex of nose, } \alpha - \theta = 0)$$

$$= -(62.84)^2\, 45 = -177\,700 \text{ mm/s}^2 = -177.7 \text{ m/s}^2$$

$$= 177.7 \text{ m/s}^2 \textbf{ (Retardation) Ans.}$$

Example 20.16. *A symmetrical cam with convex flanks operates a flat-footed follower. The lift is 8 mm, base circle radius 25 mm and the nose radius 12 mm. The total angle of the cam action is 120°.*

***1.** Find the radius of convex flanks, **2.** Draw the profile of the cam, and **3.** Determine the maximum velocity and the maximum acceleration when the cam shaft rotates at 500 r.p.m.*

Solution. Given : $x = JK = 8$ mm ; $r_1 = OE = OJ = 25$ mm ; $r_2 = QF = QK = 12$ mm ; $2\alpha = \angle EOG = 120°$ or $\alpha = \angle EOK = 60°$; $N = 500$ r.p.m. or $\omega = 2\pi \times 500/60 = 52.37$ rad/s

1. Radius of convex flanks

Let R = Radius of convex flanks = $PE = P'G$

A symmetrical cam with convex flanks operating a flat footed follower is shown in Fig. 20.47. From the geometry of the figure,

$$OQ = OJ + JK - QK = r_1 + x - r_2$$

$$= 25 + 8 - 12 = 21 \text{ mm}$$

$$PQ = PF - QF = PE - QF = (R - 12) \text{ mm}$$

and $$OP = PE - OE = (R - 25) \text{ mm}$$

Now consider the triangle *OPQ*. We know that

$$(PQ)^2 = (OP)^2 + (OQ)^2 - 2\, OP \times OQ \times \cos\beta$$

$$(R - 12)^2 = (R - 25)^2 + (21)^2 - 2\,(R - 25)21\cos(180° - 60°)$$

$$R^2 - 24\,R + 144 = R^2 - 50\,R + 625 + 441 + 21R - 525$$

$$-24\,R + 144 = -29\,R + 541 \text{ or } 5\,R = 397$$

$\therefore$ $R = 397/5 = 79.4$ mm **Ans.**

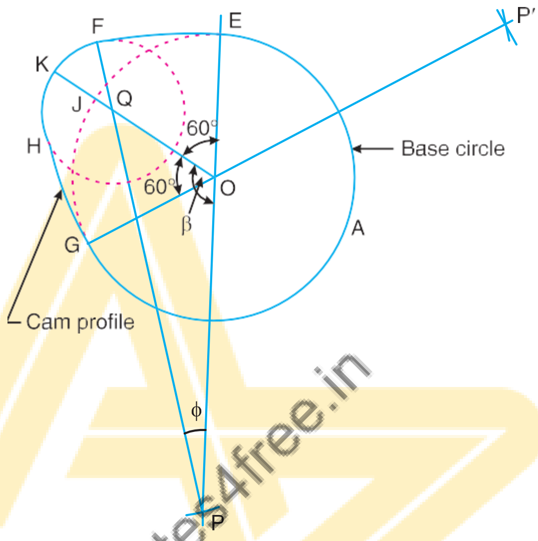


Fig. 20.47

2. Profile of the cam

The profile of the cam, as shown in Fig. 20.47, is drawn as discussed in the following steps :

(a) First of all, draw a base circle with centre *O* and radius $OE = r_1 = 25$ mm.

(b) Draw angle *EOK* = 60° and angle *KOG* = 60° such that the total angle of cam action is 120°.

(c) On line *OK* mark *OQ* = 21 mm (as calculated above). Now *Q* as centre, draw a circle of radius equal to the nose radius $r_2 = QK = QF = 12$ mm. This circle cuts the line *OK* at *J*. Now *JK* represents the lift of the follower (*i.e.* 8 mm).

(d) Produce *EO* and *GO* as shown in Fig. 20.47. Now with *Q* as centre and radius equal to $PQ = R - r_2 = 79.4 - 12 = 67.4$ mm, draw arcs intersecting the lines *EO* and *GO* produced at *P* and *P′* respectively. The centre *P′* may also be obtained by drawing arcs with centres *O* and *Q* and radii *OP* and *PQ* respectively.

(e) Now with *P* and *P′* as centres and radius equal to *R* = 79.4 mm, draw arcs *EF* and *GH* which represent the convex flanks. *EFKHGAE* is the profile of the cam.

3. Maximum velocity and maximum acceleration

First of all, let us find the angle ϕ . From triangle *OPQ*,

$$\frac{OQ}{\sin\phi} = \frac{PQ}{\sin\beta}$$

or $$\sin\phi = \frac{OQ}{PQ} \times \sin\beta = \frac{21}{79.4 - 12} \times \sin(180° - 60°) = 0.2698$$

$$\ldots (\because PQ = R - 12)$$

$\therefore$ $\phi = 15.65°$

We know that maximum velocity,

$$v_{max} = \omega (R - r_1) \sin\phi = 52.37 (79.4 - 25) \sin 15.65° = 770 \text{ mm/s}$$

$$= 0.77 \text{ m/s } \textbf{Ans.}$$

and maximum acceleration,

$$a_{max} = \omega^2 (R - r_1) = (52.37)^2 (79.4 - 25) = 149\,200 \text{ mm/s}^2 = 149.2 \text{ m/s}^2 \textbf{ Ans.}$$

Example 20.17. *The following particulars relate to a symmetrical circular cam operating a flat faced follower :*

Least radius = 16 mm, nose radius = 3.2 mm, distance between cam shaft centre and nose centre = 25 mm, angle of action of cam = 150°, and cam shaft speed = 600 r.p.m.

Assuming that there is no dwell between ascent or descent, determine the lift of the valve, the flank radius and the acceleration and retardation of the follower at a point where circular nose merges into circular flank.

Solution. Given : $r_1 = OE = OJ = 16$ mm ; $r_2 = QK = QF = 3.2$ mm ; $OQ = 25$ mm ; $2\alpha = 150°$ or $\alpha = 75°$; $N = 600$ r.p.m. or $\omega = 2\pi \times 600/60 = 62.84$ rad/s

Lift of the valve

A symmetrical circular cam operating a flat faced follower is shown in Fig. 20.48.

We know that lift of the valve,

$$x = JK = OK - OJ$$

$$= OQ + QK - OJ = OQ + r_2 - r_1$$

$$= 25 + 3.2 - 16 = 12.2 \text{ mm } \textbf{Ans.}$$

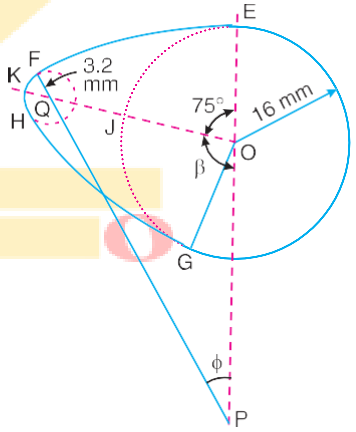


Fig. 20.48

Flank radius

Let $R = PE$ = Flank radius.

First of all, let us find out the values of OP and PQ. From the geometry of Fig. 20.48,

$$OP = PE - OE = R - 16$$

and $$PQ = PF - FQ = R - 3.2$$

Now consider the triangle OPQ. We know that

$$(PQ)^2 = (OP)^2 + (OQ)^2 - 2\, OP \times OQ \times \cos\beta$$

Substituting the values of OP and PQ in the above expression,

$$(R - 3.2)^2 = (R - 16)^2 + (25)^2 - 2 (R - 16) \times 25 \cos(180° - 75°)$$

$$R^2 - 6.4\,R + 10.24 = R^2 - 32\,R + 256 + 625 - (50\,R - 800)(-0.2588)$$

$$-6.4\,R + 10.24 = -19.06\,R + 673.96 \quad \text{or } 12.66\,R = 663.72$$

$\therefore$ $R = 52.43$ mm **Ans.**

Acceleration and retardation of the follower at a point where circular nose merges into circular flank

From Fig. 20.48 we see that at a point *F*, the circular nose merges into a circular flank. Let ϕ be the angle of action of cam at point *F*. From triangle *OPQ*,

$$\frac{OQ}{\sin\phi} = \frac{PQ}{\sin\beta}$$

or
$$\sin\phi = \frac{OQ}{PQ} \times \sin(180° - 75°) = \frac{OQ}{PF - FQ} \times \sin 105°$$

$$= \frac{25}{52.43 - 3.2} \times 0.966 = 0.4907$$

$\therefore$ $\phi = 29.4°$

We know that acceleration of the follower,

$$a = \omega^2 \times OP \times \cos\theta = \omega^2 (R - r_1)\cos\phi \qquad \ldots (\because \theta = \phi)$$

$$= (62.84)^2 (52.43 - 16) \cos 29.4° = 125\,330 \text{ mm/s}^2$$

$$= 125.33 \text{ m/s}^2 \textbf{ Ans.}$$

We also know that retardation of the follower,

$$a = \omega^2 \times OQ \cos(\alpha - \theta) = \omega^2 \times OQ \cos(\alpha - \phi) \qquad \ldots (\because \theta = \phi)$$

$$= (62.84)^2\, 25 \cos(75° - 29.4°) = 69\,110 \text{ mm/s}^2$$

$$= 69.11 \text{ m/s}^2 \textbf{ Ans.}$$

Example 20.18. *A flat ended valve tappet is operated by a symmetrical cam with circular arc for flank and nose. The straight line path of the tappet passes through the cam axis. Total angle of action = 150°. Lift = 6 mm. Base circle diameter = 30 mm. Period of acceleration is half the period of retardation during the lift. The cam rotates at 1250 r.p.m. Find :* **1.** *flank and nose radii ;* **2.** *maximum acceleration and retardation during the lift.*

Solution. Given : $2\,\alpha = 150°$ or $\alpha = 75°$; $x = JK = 6$ mm ; $d_1 = 30$ mm or $r_1 = OE = OJ = 15$ mm ; $N = 1250$ r.p.m. or $\omega = 2\,\pi \times 1250/60 = 131$ rad/s

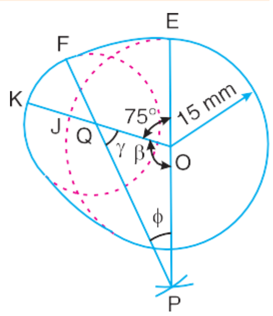


Fig. 20.49

1. ***Flank and nose radii***

The circular arc cam operating a flat ended valve tappet is shown in Fig. 20.49.

Let $R = PE$ = Flank radius, and

$r_2 = QF = QK$ = Nose radius.

First of all, let us find the values of OP, OQ and PQ. The acceleration takes place while the follower is on the flank and retardation while the follower is on nose. Since the period of acceleration is half the period of retardation during the lift, therefore

$$\phi = \frac{1}{2}\gamma \qquad \ldots (i)$$

We know that $\beta = 180° - \alpha = 180° - 75° = 105°$

$$\therefore \quad \phi + \gamma = 75° = 180° - \beta = 180° - 105° = 75° \qquad \ldots (ii)$$

From equations **(*i*)** and **(*ii*)**,

$$\phi = 25°, \quad \text{and} \quad \gamma = 50°$$

Now from the geometry of Fig. 20.49,

$$OQ = OJ + JK - QK = r_1 + x - r_2 = 15 + 6 - r_2 = 21 - r_2 \qquad \ldots (iii)$$

and

$$PQ = PF - FQ = PE - FQ = (OP + OE) - FQ = OP + 15 - r_2 \qquad \ldots (iv)$$

Now from triangle OPQ,

$$\frac{OP}{\sin\gamma} = \frac{OQ}{\sin\phi} = \frac{PQ}{\sin\beta}$$

or

$$\frac{OP}{\sin 50°} = \frac{21 - r_2}{\sin 25°} = \frac{OP + 15 - r_2}{\sin 105°}$$

$$\therefore \quad OP = \frac{21 - r_2}{\sin 25°} \times \sin 50° = \frac{21 - r_2}{0.4226} \times 0.766 = 38 - 1.8 r_2 \qquad \ldots (v)$$

Also

$$OP = \frac{OP + 15 - r_2}{\sin 105°} \times \sin 50° = \frac{OP + 15 - r_2}{0.966} \times 0.766$$

$$= 0.793 \times OP + 11.9 - 0.793\, r_2$$

$$\therefore \quad 0.207\, OP = 11.9 - 0.793\, r_2 \quad \text{or} \quad OP = 57.5 - 3.83\, r_2 \qquad \ldots (vi)$$

From equations **(*v*)** and **(*vi*)**,

$$38 - 1.8\, r_2 = 57.5 - 3.83\, r_2 \quad \text{or} \quad 2.03\, r_2 = 19.5$$

$$\therefore \quad r_2 = 9.6 \text{ mm} \quad \textbf{Ans.}$$

We know that $OP = 38 - 1.8 r_2 = 38 - 1.8 \times 9.6 = 20.7$ mm ... [From equation (*v*)]

$$\therefore \quad R = PE = OP + OE = 20.7 + 15 = 35.7 \text{ mm} \quad \textbf{Ans.}$$

2. *Maximum acceleration and retardation during the lift*

We know that maximum acceleration

$$= \omega^2 (R - r_1) = \omega^2 \times OP = (131)^2\, 20.7 = 355\,230 \text{ mm/s}^2$$

$$= 355.23 \text{ m/s}^2 \quad \textbf{Ans.}$$

and maximum retardation, $= \omega^2 \times OQ = \omega^2 (21 - r_2)$ ... [From equation **(*iii*)**]

$$= (131)^2 (21 - 9.6) = 195\,640 \text{ mm/s}^2 = 195.64 \text{ m/s}^2 \quad \textbf{Ans.}$$

Example 20.19. *A cam consists of a circular disc of diameter 75 mm with its centre displaced 25 mm from the camshaft axis. The follower has a flat surface (horizontal) in contact with the cam and the line of action of the follower is vertical and passes through the shaft axis as shown in Fig. 20.50. The mass of the follower is 2.3 kg and is pressed downwards by a spring which has a stiffness of 3.5 N/mm. In the lowest position the spring force is 45 N.*

1. Derive an expression for the acceleration of the follower in terms of the angle of rotation from the beginning of the lift.

2. As the cam shaft speed is gradually increased, a value is reached at which the follower begins to lift from the cam surface. Determine the camshaft speed for this condition.

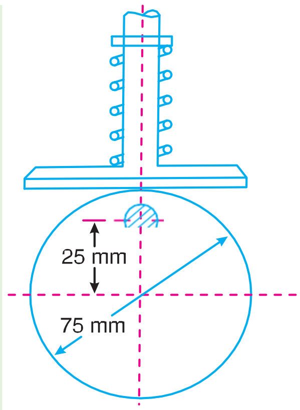


Fig. 20.50

Solution. Given : d = 75 mm or $r = OA$ = 37.5 mm ; OQ = 25 mm ; m = 2.3 kg ; s = 3.5 N/mm ; S = 45 N

1. *Expression for the acceleration of the follower*

The cam in its lowest position is shown by full lines in Fig. 20.51 and by dotted lines when it has rotated through an angle θ .

From the geometry of the figure, the displacement of the follower,

$$x = AB = OS = OQ - QS$$

$$= OQ - PQ \cos\theta$$

$$= OQ - OQ\cos\theta \qquad \ldots (\because PQ = OQ)$$

$$= OQ(1-\cos\theta) = 25(1-\cos\theta) \qquad \ldots (i)$$

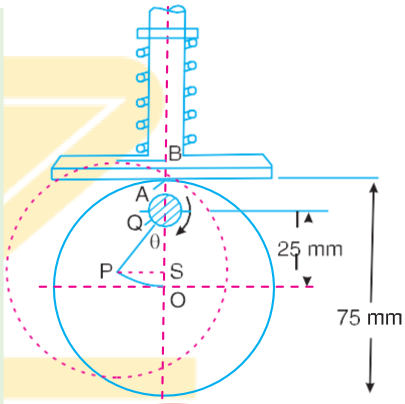


Fig. 20.51

Differentiating equation (*i*) with respect to *t*, we get velocity of the follower,

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = \frac{dx}{d\theta} \times \omega$$

$$\ldots \text{(Substituting } d\theta/dt = \omega \text{)}$$

$$= 25\sin\theta \times \omega = 25\,\omega\sin\theta \qquad \ldots (ii)$$

Now differentiating equation (*ii*) with respect to *t*, we get acceleration of the follower,

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} = 25\,\omega\cos\theta \times \omega$$

$$= 25\,\omega^2 \cos\theta \text{ mm/s}^2 = 0.025\omega^2 \cos\theta \text{ m/s}^2$$ **Ans.**

2. *Cam shaft speed*

Let N = Cam shaft speed in r.p.m.

We know that accelerating force

$$= m.a = 2.3 \times 0.025\,\omega^2 \cos\theta = 0.0575\omega^2 \cos\theta \text{ N}$$

Now for any value of θ, the algebraic sum of the spring force, weight of the follower and the accelerating force is equal to the vertical reaction between the cam and follower. When this reaction is zero, then the follower will just begin to leave the cam.

$\therefore \quad S + s.x + m.g + m.a = 0$

$$45 + 3.5 \times 25(1 - \cos\theta) + 2.3 \times 9.81 + 0.0575\omega^2 \cos\theta = 0$$

$$45 + 87.5 - 87.5 \cos\theta + 22.56 + 0.0575\,\omega^2 \cos\theta = 0$$

$$155.06 - 87.5\cos\theta + 0.0575\,\omega^2 \cos\theta = 0$$

$$2697 - 1522\cos\theta + \omega^2 \cos\theta = 0 \qquad \text{. . . (Dividing by 0.0575)}$$

$$\omega^2 \cos\theta = 1522\cos\theta - 2697 \quad \text{or} \quad \omega^2 = 1522 - 2697 \sec\theta$$

Since $\sec\theta \geq +1$ or , ≤ -1, therefore the minimum value of ω^2 occurs when θ = 180° therefore

$$\omega^2 = 1522 - (-2697) = 4219 \qquad \text{. . . [Substituting } \sec\theta = -1 \text{]}$$

$\therefore \quad \omega = 65$ rad/s

and maximum allowable cam shaft speed,

$$N = \frac{\omega \times 60}{2\pi} = \frac{65 \times 60}{2\pi} = 621 \text{ r.p.m. } \textbf{Ans.}$$

EXERCISES

1. A disc cam is to give uniform motion to a knife edge follower during out stroke of 50 mm during the first half of the cam revolution. The follower again returns to its original position with uniform motion during the next half of the revolution. The minimum radius of the cam is 50 mm and the diameter of the cam shaft is 35 mm. Draw the profile of the cam when 1. the axis of follower passes through the axis of cam shaft, and 2. the axis of follower is offset by 20 mm from the axis of the cam shaft.

2. A cam operating a knife-edged follower has the following data :

   (a) Follower moves outwards through 40 mm during 60° of cam rotation.

   (b) Follower dwells for the next 45°.

   (c) Follower returns to its original position during next 90°.

   (d) Follower dwells for the rest of the rotation.

   The displacement of the follower is to take place with simple harmonic motion during both the outward and return strokes. The least radius of the cam is 50 mm. Draw the profile of the cam when 1. the axis of the follower passes through the cam axis, and 2. the axis of the follower is offset 20 mm towards right from the cam axis. If the cam rotates at 300 r.p.m., determine maximum velocity and acceleration of the follower during the outward stroke and the return stroke.

   [Ans. 1.88 m/s, 1.26 m/s ; 177.7 m/s^2, 79 m/s^2]

3. A disc cam rotating in a clockwise direction is used to move a reciprocating roller with simple harmonic motion in a radial path, as given below :

   (i) Outstroke with maximum displacement of 25 mm during 120° of cam rotation,

   (ii) Dwell for 60° of cam rotation,

   (iii) Return stroke with maximum displacement of 25 mm during 90° of cam rotation, and

   (iv) Dwell during remaining 90° of cam rotation.

   The line of reciprocation of follower passes through the camshaft axis. The maximum radius of cam is 20 mm. If the cam rotates at a uniform speed of 300 r.p.m. find the maximum velocity and acceleration during outstroke and return stroke. The roller diameter is 8 mm.

Draw the profile of the cam when the line of reciprocation of the follower is offset by 20 mm towards right from the cam shaft axis. **[Ans. 0.59 m/s, 0.786 m/s ; 27.8 m/s^2, 49.4 m/s^2]**

4. Design a cam to raise a valve with simple harmonic motion through 50 mm in 1/3 of a revolution, keep if fully raised through 1/12 revolution and to lower it with harmonic motion in 1/6 revolution. The valve remains closed during the rest of the revolution. The diameter of the roller is 20 mm and the minimum radius of the cam is 25 mm. The diameter of the camshaft is 25 mm. The axis of the valve rod passes through the axis of the camshaft. If the camshaft rotates at uniform speed of 100 r.p.m. ; find the maximum velocity and acceleration of a valve during raising and lowering.

[Ans. 0.39 m/s, 0.78 m/s ; 6.17 m/s^2, 24.67 m/s^2]

5. A cam rotating clockwise with a uniform speed is to give the roller follower of 20 mm diameter with the following motion :

(a) Follower to move outwards through a distance of 30 mm during 120° of cam rotation ;

(b) Follower to dwell for 60° of cam rotation ;

(c) Follower to return to its initial position during 90° of cam rotation ; and

(d) Follower to dwell for the remaining 90° of cam rotation.

The minimum radius of the cam is 45 mm and the line of stroke of the follower is offset 15 mm from the axis of the cam and the displacement of the follower is to take place with simple harmonic motion on both the outward and return strokes. Draw the cam profile.

6. A cam rotating clockwise at a uniform speed of 100 r.p.m. is required to give motion to knife-edge follower as below :

(a) Follower to move outwards through 25 mm during 120° of cam rotation,

(b) Follower to dwell for the next 60° of cam rotation,

(c) Follower to return to its starting position during next 90° of cam rotation, and

(d) Follower to dwell for the rest of the cam rotation.

The minimum radius of the cam is 50 mm and the line of stroke of the follower passes through the axis of the cam shaft. If the displacement of the follower takes place with uniform and equal acceleration and retardation on both the outward and return strokes, find the maximum velocity and acceleration during outstroke and return stroke. **[Ans. 0.25 m/s, 0.33 m/s ; 2.5 m/s^2 , 4.44 m/s^2]**

7. A cam with 30 mm as minimum diameter is rotating clockwise at a uniform speed of 1200 r.p.m. and has to give the following motion to a roller follower 10 mm in diameter:

(a) Follower to complete outward stroke of 25 mm during 120° of cam rotation with equal uniform acceleration and retardation ;

(b) Follower to dwell for 60° of cam rotation ;

(c) Follower to return to its initial position during 90° of cam rotation with equal uniform acceleration and retardation ;

(d) Follower to dwell for the remaining 90° of cam rotation.

Draw the cam profile if the axis of the roller follower passes through the axis of the cam.

Determine the maximum velocity of the follower during the outstroke and return stroke and also the uniform acceleration of the follower on the out stroke and the return stoke.

[Ans. 3 m/s , 4 m/s ; 360.2 m/s^2, 640.34 m/s^2]

8. A cam rotating clockwise at a uniform speed of 200 r.p.m. is required to move an offset roller follower with a uniform and equal acceleration and retardation on both the outward and return strokes. The angle of ascent, the angle of dwell (between ascent and descent) and the angle of descent is 120°, 60° and 90° respectively. The follower dwells for the rest of cam rotation. The least radius of the cam is 50 mm, the lift of the follower is 25 mm and the diameter of the roller is 10 mm. The line of stroke of the follower is offset by 20 mm from the axis of the cam. Draw the cam profile and find the maximum velocity and acceleration of the follower during the outstroke.

9. A flat faced reciprocating follower has the following motion :

(i) The follower moves out for 80° of cam rotation with uniform acceleration and retardation, the acceleration being twice the retardation.

(ii) The follower dwells for the next 80° of cam rotation.

(iii) It moves in for the next 120° of cam rotation with uniform acceleration and retardation, the retardation being twice the acceleration.

(iv) The follower dwells for the remaining period.

The base circle diameter of the cam is 60 mm and the stroke of the follower is 20 mm. The line of movement of the follower passes through the cam centre.

Draw the displacement diagram and the profile of the cam very neatly showing all constructional details.

10. From the following data, draw the profile of a cam in which the follower moves with simple harmonic motion during ascent while it moves with uniformly accelerated motion during descent :

Least radius of cam = 50 mm ; Angle of ascent = 48° ; Angle of dwell between ascent and descent = 42° ; Angle of descent = 60° ; Lift of follower = 40 mm ; Diameter of roller = 30 mm ; Distance between the line of action of follower and the axis of cam = 20 mm.

If the cam rotates at 360 r.p.m. anticlockwise, find the maximum velocity and acceleration of the follower during descent. **[Ans. 2.88 m/s ; 207.4 m/s^2]**

11. Draw the profile of a cam with oscillating roller follower for the following motion :

(a) Follower to move outwards through an angular displacement of 20° during 120° of cam rotation.

(b) Follower to dwell for 50° of cam rotation.

(c) Follower to return to its initial position in 90° of cam rotation with uniform acceleration and retardation.

(d) Follower to dwell for the remaining period of cam rotation.

The distance between the pivot centre and the roller centre is 130 mm and the distance between the pivot centre and cam axis is 150 mm. The minimum radius of the cam is 80 mm and the diameter of the roller is 50 mm.

12. Draw the profile of the cam when the roller follower moves with cycloidal motion as given below :

(a) Outstroke with maximum displacement of 44 mm during 180° of cam rotation.

(b) Return stroke for the next 150° of cam rotation.

(c) Dwell for the remaining 30° of cam rotation.

The minimum radius of the cam is 20 mm and the diameter of the roller is 10 mm. The axis of the roller follower passes through the cam shaft axis.

13. A symmetrical tangent cam operating a roller follower has the following particulars :

Radius of base circle of cam = 40 mm, roller radius = 20 mm, angle of ascent = 75°, total lift = 20 mm, speed of cam shaft = 300 r.p.m.

Determine : 1. the principal dimensions of the cam, 2. the equation for the displacement curve, when the follower is in contact with the straight flank, and 3. the acceleration of the follower when it is in contact with the straight flank where it merges into the circular nose.

[Ans. r_3 = 33 mm ; θ = 23.5° ; 89.4 m/s^2]

14. A cam profile consists of two circular arcs of radii 24 mm and 12 mm, joined by straight lines, giving the follower a lift of 12 mm. The follower is a roller of 24 mm radius and its line of action is a straight line passing through the cam shaft axis. When the cam shaft has a uniform speed of 500 rev/min, find the maximum velocity and acceleration of the follower while in contact with the straight flank of the cam. **[Ans. 1.2 m/s ; 198 m/s^2]**

15. The following particulars relate to a symmetrical tangent cam operating a roller follower :-

Least radius = 30 mm, nose radius = 24 mm, roller radius = 17.5 mm, distance between cam shaft and nose centre = 23.5 mm, angle of action of cam = 150°, cam shaft speed = 600 r.p.m.

Assuming that there is no dwell between ascent and descent, determine the lift of the valve and the acceleration of the follower at a point where straight flank merges into the circular nose.

[Ans. 17.5 mm ; 304.5 m/s^2]

16. Following is the data for a circular arc cam working with a flat faced reciprocating follower : Minimum radius of the cam = 30 mm ; Total angle of cam action = 120° ; Radius of the circular arc = 80 mm ; Nose radius = 10 mm.

1. Find the distance of the centre of nose circle from the cam axis ; 2. Draw the profile of the cam to full scale; 3. Find the angle through which the cam turns when the point of contact moves from the junction of minimum radius arc and circular arc to the junction of nose radius arc and circular arc ; and 4. Find the velocity and acceleration of the follower when the cam has turned through an angle of θ = 20°. The angle θ is measured from the point where the follower just starts moving away from the cam. The angular velocity of the cam is 10 rad/s.

[Ans. 30 mm ; 22°; 68.4 mm/s ; 1880 mm/s^2]

17. The suction valve of a four stroke petrol engine is operated by a circular arc cam with a flat faced follower. The lift of the follower is 10 mm ; base circle diameter of the cam is 40 mm and the nose radius is 2.5 mm. The crank angle when suction valve opens is 4° after top dead centre and when the suction valve closes, the crank angle is 50° after bottom dead centre. If the cam shaft rotates at 600 r.p.m., determine: 1. maximum velocity of the valve, and 2. maximum acceleration and retardation of the valve.

[Ans. 1.22 m/s ; 383 m/s^2, 108.6 m/s^2]

[**Hint.** Total angle turned by the crankshaft when valve is open

= 180° – 4° + 50° = 226°

Since the engine is a four stroke cycle, therefore speed of cam shaft is half of the speed of the crank shaft.

∴ Total angle turned by the cam shaft during opening of valve, 2 α = 226/2 = 113° or

α = 56.5°].

18. The following particulars relate to a symmetrical circular cam operating a flat-faced follower : Least radius = 25 mm ; nose radius = 8 mm, lift of the valve = 10 mm, angle of action of cam = 120°, cam shaft speed = 1000 r.p.m.

Determine the flank radius and the maximum velocity, acceleration and retardation of the follower. If the mass of the follower and valve with which it is in contact is 4 kg, find the minimum force to be exerted by the spring to overcome inertia of the valve parts.

[Ans. 88 mm ; 1.93 m/s, 690.6 m/s^2, 296 m/s^2 ; 1184 N]

DO YOU KNOW ?

1. Write short notes on cams and followers.

2. Explain with sketches the different types of cams and followers.

3. Why a roller follower is preferred to that of a knife-edged follower ?

4. Define the following terms as applied to cam with a neat sketch :-

(a) Base circle, (*b*) Pitch circle, (*c*) Pressure angle, and (*d*) Stroke of the follower.

5. What are the different types of motion with which a follower can move ?

6. Draw the displacement, velocity and acceleration diagrams for a follower when it moves with simple harmonic motion. Derive the expression for velocity and acceleration during outstroke and return stroke of the follower.

7. Draw the displacement, velocity and acceleration diagrams for a follower when it moves with uniform acceleration and retardation. Derive the expression for velocity and acceleration during outstroke and return stroke of the follower.

8. Derive expressions for displacement, velocity and acceleration for a tangent cam operating on a radial-translating roller follower :

(i) when the contact is on straight flank, and
(ii) when the contact is on circular nose.

9. Derive the expressions for displacement, velocity and acceleration for a circular arc cam operating a flat-faced follower
(i) when the contact is on the circular flank, and
(ii) when the contact is on circular nose.

OBJECTIVE TYPE QUESTIONS

1. The size of a cam depends upon
(a) base circle (*b*) pitch circle (*c*) prime circle (*d*) pitch curve

2. The angle between the direction of the follower motion and a normal to the pitch curve is called
(a) pitch angle (*b*) prime angle
(*c*) base angle (*d*) pressure angle

3. A circle drawn with centre as the cam centre and radius equal to the distance between the cam centre and the point on the pitch curve at which the pressure angle is maximum, is called
(a) base circle (*b*) pitch circle
(*c*) prime circle (*d*) none of these

4. The cam follower generally used in automobile engines is
(a) knife edge follower (*b*) flat faced follower
(*c*) spherical faced follower (*d*) roller follower

5. The cam follower extensively used in air-craft engines is
(a) knife edge follower (*b*) flat faced follower
(*c*) spherical faced follower (*d*) roller follower

6. In a radial cam, the follower moves
(a) in a direction perpendicular to the cam axis
(b) in a direction parallel to the cam axis
(c) in any direction irrespective of the cam axis
(d) along the cam axis

7. A radial follower is one
(a) that reciprocates in the guides (*b*) that oscillates
(c) in which the follower translates along an axis passing through the cam centre of rotation.
(d) none of the above

8. Offet is provided to a cam follower mechanism to
(a) minimise the side thrust (*b*) accelerate
(*c*) avoid jerk (*d*) none of these

9. For low and moderate speed engines, the cam follower should move with
(a) uniform velocity (*b*) simple harmonic motion
(*c*) uniform acceleration and retardation (*d*) cycloidal motion

10. For high speed engines, the cam follower should move with
(a) uniform velocity (*b*) simple harmonic motion
(*c*) uniform acceleration and retardation (*d*) cycloidal motion

11. Which of the following displacement diagrams should be chosen for better dynamic performance of a cam-follower mechanism ?
(a) simple hormonic motion (*b*) parabolic motion
(*c*) cycloidal motion (*d*) none of these

12. For a given lift of the follower of a cam follower mechanism, a smaller base circle diameter is desired.

(a) because it will give a steeper cam and higher pressure angle.

(b) because it will give a profile with lower pressure angle

(c) because it will avoid jumping

(d) none of the above.

13. The linear velocity of the reciprocating roller follower when it has contact with the straight flanks of the tangent cam, is given by

(*a*) $\omega(r_1 - r_2)\sin\theta$ (*b*) $\omega(r_1 - r_2)\cos\theta$

(*c*) $\omega(r_1 + r_2)\sin\theta\sec^2\theta$ (*d*) $\omega(r_1 + r_2)\cos\theta\operatorname{cosec}^2\theta$

where ω = Angular velocity of the cam shaft,

r_1 = Minimum radius of the cam,

r_2 = Radius of the roller, and

θ = Angle turned by the cam from the beginning of the displacement for contact of roller with the straight flanks.

14. The displacement of a flat faced follower when it has contact with the flank of a circular arc cam, is given by

(*a*) $R(1 - \cos\theta)$ (*b*) $R(1 - \sin\theta)$

(*c*) $(R - r_1)(1 - \cos\theta)$ (*d*) $(R - r_1)(1 - \sin\theta)$

where R = Radius of the flank,

r_1 = Minimum radius of the cam, and

θ = Angle turned by the cam for contact with the circular flank.

15. The retardation of a flat faced follower when it has contact at the apex of the nose of a circular arc cam, is given by

(a) $\omega^2 \times OQ$ (*b*) $\omega^2 \times OQ\sin\theta$

(*c*) $\omega^2 \times OQ\cos\theta$ (*d*) $\omega^2 \times OQ\tan\theta$

where *OQ* = Distance between the centre of circular flank and centre of nose.

ANSWERS

1. (*a*)	**2.** (*d*)	**3.** (*b*)	**4.** (*c*)	**5.** (*d*)
6. (*a*)	**7.** (*a*)	**8.** (*a*)	**9.** (*b*)	**10.** (*d*)
11. (*c*)	**12.** (*d*)	**13.** (*c*)	**14.** (*c*)	**15.** (*a*)

GO To FIRST

MODULE 5

Gear Trains

Features

Gear Trains

13.1. Introduction

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train*** or ***train of toothed wheels.*** The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

13.2. Types of Gear Trains

Following are the different types of gear trains, depending upon the arrangement of wheels :

1. Simple gear train, **2.** Compound gear train, **3.** Reverted gear train, and **4.** Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

13.3. Simple Gear Train

When there is only one gear on each shaft, as shown in Fig. 13.1, it is known as ***simple gear train.*** The gears are represented by their pitch circles.

When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. 13.1 (*a*). Since the gear 1 drives the gear 2, therefore gear 1 is called the ***driver*** and the gear 2 is called the ***driven*** or ***follower***. It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

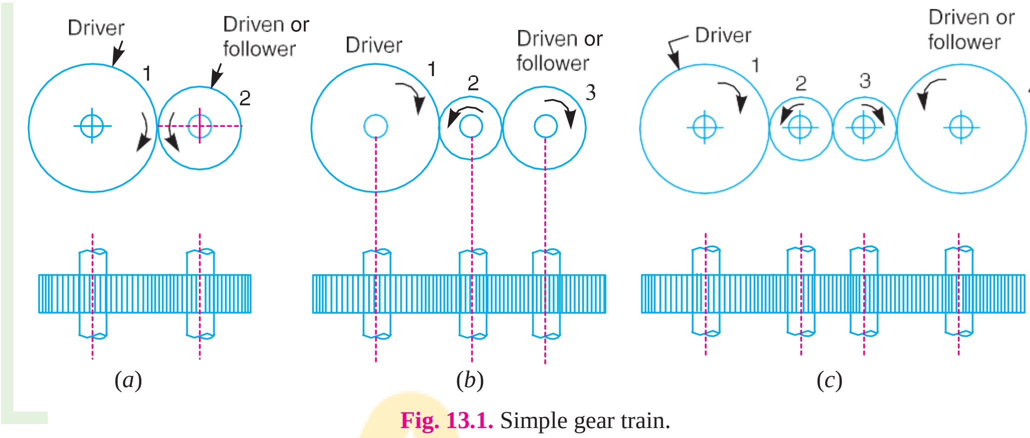


Fig. 13.1. Simple gear train.

Let N_1 = Speed of gear 1(or driver) in r.p.m.,

N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,

T_1 = Number of teeth on gear 1, and

T_2 = Number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

From above, we see that the train value is the reciprocal of speed ratio.

Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods :

1. By providing the large sized gear, or **2.** By providing one or more intermediate gears.

A little consideration will show that the former method (*i.e.* providing large sized gears) is very inconvenient and uneconomical method ; whereas the latter method (*i.e.* providing one or more intermediate gear) is very convenient and economical.

It may be noted that when the number of intermediate gears are ***odd,*** the motion of both the gears (*i.e.* driver and driven or follower) is ***like*** as shown in Fig. 13.1 (*b*).

But if the number of intermediate gears are ***even,*** the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig. 13.1 (*c*).

Now consider a simple train of gears with one intermediate gear as shown in Fig. 13.1 (*b*).

Let N_1 = Speed of driver in r.p.m.,

N_2 = Speed of intermediate gear in r.p.m.,

N_3 = Speed of driven or follower in r.p.m.,
T_1 = Number of teeth on driver,
T_2 = Number of teeth on intermediate gear, and
T_3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \ldots(i)$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \qquad \ldots(ii)$$

The speed ratio of the gear train as shown in Fig. 13.1 (*b*) is obtained by multiplying the equations **(*i*)** and **(*ii*)**.

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \quad \text{or} \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

i.e.
$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

and
$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

Similarly, it can be proved that the above equation holds good even if there are any number of intermediate gears. From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called ***idle gears,*** as they do not effect the speed ratio or train value of the system. The idle gears are used for the following two purposes :

1. To connect gears where a large centre distance is required, and

2. To obtain the desired direction of motion of the driven gear (*i.e.* clockwise or anticlockwise).

Gear trains inside a mechanical watch

13.4. Compound Gear Train

When there are more than one gear on a shaft, as shown in Fig. 13.2, it is called a ***compound train of gear.***

We have seen in Art. 13.3 that the idle gears, in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.13.2.

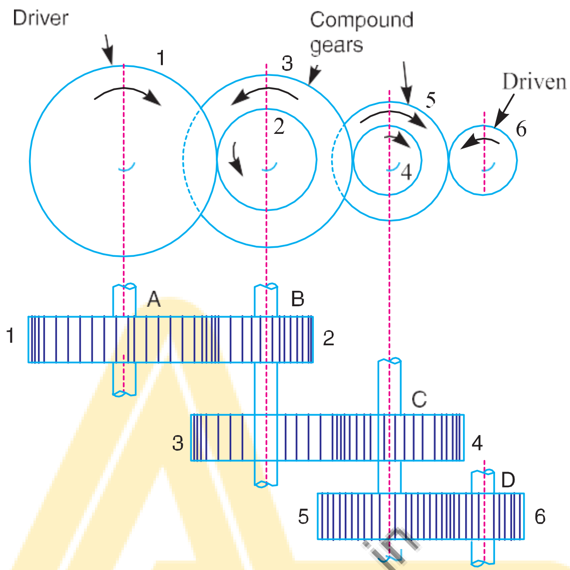


Fig. 13.2. Compound gear train.

In a compound train of gears, as shown in Fig. 13.2, the gear 1 is the driving gear mounted on shaft *A* , gears 2 and 3 are compound gears which are mounted on shaft *B*. The gears 4 and 5 are also compound gears which are mounted on shaft *C* and the gear 6 is the driven gear mounted on shaft *D*.

Let N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and

$T_2, T_3 ..., T_6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad ...(i)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad ...(ii)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad ...(iii)$$

The speed ratio of compound gear train is obtained by multiplying the equations **(*i*), (*ii*)** and **(*iii*)**,

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^{*}\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

* Since gears 2 and 3 are mounted on one shaft *B*, therefore $N_2 = N_3$. Similarly gears 4 and 5 are mounted on shaft *C*, therefore $N_4 = N_5$.

i.e. $\text{Speed ratio} = \dfrac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

and $\text{Train value} = \dfrac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$

$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

Note: The gears which mesh must have the same circular pitch or module. Thus gears 1 and 2 must have the same module as they mesh together. Similarly gears 3 and 4, and gears 5 and 6 must have the same module.

Example 13.1. *The gearing of a machine tool is shown in Fig. 13.3. The motor shaft is connected to gear A and rotates at 975 r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F ? The number of teeth on each gear are as given below :*

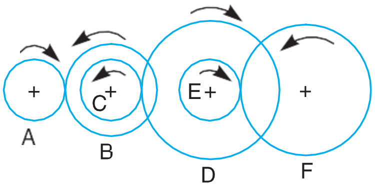


Fig. 13.3

Gear	*A*	*B*	*C*	*D*	*E*	*F*
No. of teeth	*20*	*50*	*25*	*75*	*26*	*65*

Solution. Given : $N_A = 975$ r.p.m. ; $T_A = 20$; $T_B = 50$; $T_C = 25$; $T_D = 75$; $T_E = 26$; $T_F = 65$

From Fig. 13.3, we see that gears *A* , *C* and *E* are drivers while the gears *B*, *D* and *F* are driven or followers. Let the gear *A* rotates in clockwise direction. Since the gears *B* and *C* are mounted on the same shaft, therefore it is a compound gear and the direction or rotation of both these gears is same (*i.e.* anticlockwise). Similarly, the gears *D* and *E* are mounted on the same shaft, therefore it is also a compound gear and the direction of rotation of both these gears is same (*i.e.* clockwise). The gear *F* will rotate in anticlockwise direction.

Battery Car: Even though it is run by batteries, the power transmission, gears, clutches, brakes, etc. remain mechanical in nature.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Let N_F = Speed of gear *F*, *i.e.* last driven or follower.

We know that

$$\frac{\text{Speed of the first driver}}{\text{Speed of the last driven}} = \frac{\text{Product of no. of teeth on drivens}}{\text{Product of no. of teeth on drivers}}$$

or $$\frac{N_A}{N_F} = \frac{T_B \times T_D \times T_F}{T_A \times T_C \times T_E} = \frac{50 \times 75 \times 65}{20 \times 25 \times 26} = 18.75$$

$\therefore$ $$N_F = \frac{N_A}{18.75} = \frac{975}{18.75} = 52 \text{ r. p. m.}$$ **Ans.**

13.5. Design of Spur Gears

Sometimes, the spur gears (*i.e.* driver and driven) are to be designed for the given velocity ratio and distance between the centres of their shafts.

Let x = Distance between the centres of two shafts,

N_1 = Speed of the driver,

T_1 = Number of teeth on the driver,

d_1 = Pitch circle diameter of the driver,

N_2, T_2 and d_2 = Corresponding values for the driven or follower, and

p_c = Circular pitch.

We know that the distance between the centres of two shafts,

$$x = \frac{d_1 + d_2}{2} \quad \text{...}(i)$$

and speed ratio or velocity ratio,

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1} \quad \text{...}(ii)$$

From the above equations, we can conveniently find out the values of d_1 and d_2 (or T_1 and T_2) and the circular pitch (p_c). The values of T_1 and T_2, as obtained above, may or may not be whole numbers. But in a gear since the number of its teeth is always a whole number, therefore a slight alterations must be made in the values of x, d_1 and d_2, so that the number of teeth in the two gears may be a complete number.

Example 13.2. *Two parallel shafts, about 600 mm apart are to be connected by spur gears. One shaft is to run at 360 r.p.m. and the other at 120 r.p.m. Design the gears, if the circular pitch is to be 25 mm.*

Solution. Given : x = 600 mm ; N_1 = 360 r.p.m. ; N_2 = 120 r.p.m. ; p_c = 25 mm

Let d_1 = Pitch circle diameter of the first gear, and

d_2 = Pitch circle diameter of the second gear.

We know that speed ratio,

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{360}{120} = 3 \quad \text{or} \quad d_2 = 3d_1 \quad \text{...}(i)$$

and centre distance between the shafts (x),

$$600 = \frac{1}{2}(d_1 + d_2) \quad \text{or} \quad d_1 + d_2 = 1200 \quad \text{...}(ii)$$

From equations ***(i)*** and ***(ii)***, we find that

$$d_1 = 300 \text{ mm, and } d_2 = 900 \text{ mm}$$

$\therefore$ Number of teeth on the first gear,

$$T_1 = \frac{\pi d_2}{p_c} = \frac{\pi \times 300}{25} = 37.7$$

and number of teeth on the second gear,

$$T_2 = \frac{\pi\, d_2}{p_c} = \frac{\pi \times 900}{25} = 113.1$$

Since the number of teeth on both the gears are to be in complete numbers, therefore let us make the number of teeth on the first gear as 38. Therefore for a speed ratio of 3, the number of teeth on the second gear should be 38 × 3 = 114.

Now the exact pitch circle diameter of the first gear,

$$d_1' = \frac{T_1 \times p_c}{\pi} = \frac{38 \times 25}{\pi} = 302.36 \text{ mm}$$

and the exact pitch circle diameter of the second gear,

$$d_2' = \frac{T_2 \times p_c}{\pi} = \frac{114 \times 25}{\pi} = 907.1 \text{ mm}$$

∴ Exact distance between the two shafts,

$$x' = \frac{d_1' + d_2'}{2} = \frac{302.36 + 907.1}{2} = 604.73 \text{ mm}$$

Hence the number of teeth on the first and second gear must be 38 and 114 and their pitch circle diameters must be 302.36 mm and 907.1 mm respectively. The exact distance between the two shafts must be 604.73 mm. **Ans.**

13.6. Reverted Gear Train

When the axes of the first gear (*i.e.* first driver) and the last gear (*i.e.* last driven or follower) are co-axial, then the gear train is known as ***reverted gear train*** as shown in Fig. 13.4.

We see that gear 1 (*i.e.* first driver) drives the gear 2 (*i.e.* first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (*i.e.* the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is ***like***.

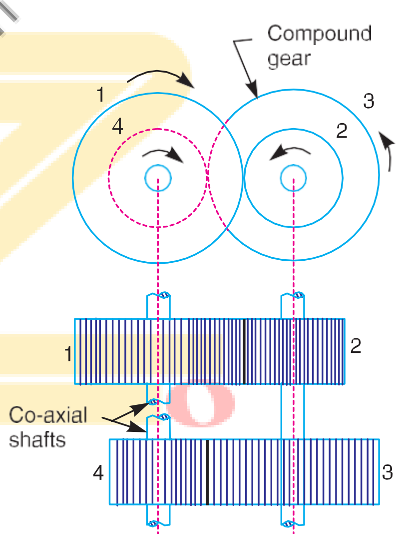


Fig. 13.4. Reverted gear train.

Let T_1 = Number of teeth on gear 1,

r_1 = Pitch circle radius of gear 1, and

N_1 = Speed of gear 1 in r.p.m.

Similarly,

T_2, T_3, T_4 = Number of teeth on respective gears,

r_2, r_3, r_4 = Pitch circle radii of respective gears, and

N_2, N_3, N_4 = Speed of respective gears in r.p.m.

Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4 \quad ...(i)$$

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

$$\therefore \quad {}^*T_1 + T_2 = T_3 + T_4 \quad ...(ii)$$

and

$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

or

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3} \quad ...(iii)$$

From equations **(*i*), (*ii*)** and **(*iii*),** we can determine the number of teeth on each gear for the given centre distance, speed ratio and module only when the number of teeth on one gear is chosen arbitrarily.

The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

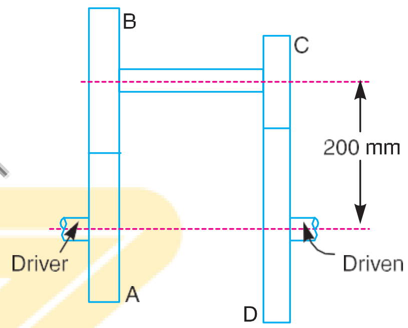


Fig. 13.5

Example 13.3. *The speed ratio of the reverted gear train, as shown in Fig. 13.5, is to be 12. The module pitch of gears A and B is 3.125 mm and of gears C and D is 2.5 mm. Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.*

Solution. Given : Speed ratio, $N_A/N_D = 12$; $m_A = m_B = 3.125$ mm ; $m_C = m_D = 2.5$ mm

Let N_A = Speed of gear *A*,

T_A = Number of teeth on gear *A* ,

r_A = Pitch circle radius of gear *A* ,

N_B, N_C, N_D = Speed of respective gears,

T_B, T_C, T_D = Number of teeth on respective gears, and

r_B, r_C, r_D = Pitch circle radii of respective gears.

* We know that circular pitch,

$$p_c = \frac{2\pi r}{T} = \pi m \quad \text{or} \quad r = \frac{m.T}{2}, \text{ where } m \text{ is the module.}$$

$$\therefore \quad r_1 = \frac{m.T_1}{2} \;;\; r_2 = \frac{m.T_2}{2} \;;\; r_3 = \frac{m.T_3}{2} \;;\; r_4 = \frac{m.T_4}{2}$$

Now from equation **(*i*),**

$$\frac{m.T_1}{2} + \frac{m.T_2}{2} = \frac{m.T_3}{2} + \frac{m.T_4}{2}$$

$$T_1 + T_2 = T_3 + T_4$$

Since the speed ratio between the gears *A* and *B* and between the gears *C* and *D* are to be same, therefore

$$^*\frac{N_A}{N_B} = \frac{N_C}{N_D} = \sqrt{12} = 3.464$$

Also the speed ratio of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\frac{T_B}{T_A} = \frac{T_D}{T_C} = 3.464 \quad \text{...}(i)$$

We know that the distance between the shafts

$$x = r_A + r_B = r_C + r_D = 200 \text{ mm}$$

or $$\frac{m_A.T_A}{2} + \frac{m_B.T_B}{2} = \frac{m_C.T_C}{2} + \frac{m_D.T_D}{2} = 200 \quad \text{...}\left(r = \frac{m.T}{2}\right)$$

$$3.125\,(T_A + T_B) = 2.5\,(T_C + T_D) = 400 \quad \text{...}(\because m_A = m_B, \text{ and } m_C = m_D)$$

$\therefore$ $$T_A + T_B = 400 / 3.125 = 128 \quad \text{...}(ii)$$

and $$T_C + T_D = 400 / 2.5 = 160 \quad \text{...}(iii)$$

From equation ***(i)***, $T_B = 3.464\,T_A$. Substituting this value of T_B in equation ***(ii)***,

$T_A + 3.464\,T_A = 128$ or $T_A = 128 / 4.464 = 28.67$ say 28 **Ans.**

and $T_B = 128 - 28 = 100$ **Ans.**

Again from equation ***(i)***, $T_D = 3.464\,T_C$. Substituting this value of T_D in equation ***(iii)***,

$T_C + 3.464\,T_C = 160$ or $T_C = 160 / 4.464 = 35.84$ say 36 **Ans.**

and $T_D = 160 - 36 = 124$ **Ans.**

Note : The speed ratio of the reverted gear train with the calculated values of number of teeth on each gear is

$$\frac{N_A}{N_D} = \frac{T_B \times T_D}{T_A \times T_C} = \frac{100 \times 124}{28 \times 36} = 12.3$$

13.7. Epicyclic Gear Train

We have already discussed that in an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. 13.6, where a gear *A* and the arm *C* have a common axis at O_1 about which they can rotate. The gear *B* meshes with gear *A* and has its axis on the arm at O_2, about which the gear *B* can rotate. If the

---

* We know that speed ratio $= \frac{\text{Speed of first driver}}{\text{Speed of last driven}} = \frac{N_A}{N_D} = 12$

Also $$\frac{N_A}{N_D} = \frac{N_A}{N_B} \times \frac{N_C}{N_D} \quad \text{...}(N_B = N_C, \text{ being on the same shaft})$$

For $\frac{N_A}{N_B}$ and $\frac{N_C}{N_D}$ to be same, each speed ratio should be $\sqrt{12}$ so that

$$\frac{N_A}{N_D} = \frac{N_A}{N_B} \times \frac{N_C}{N_D} = \sqrt{12} \times \sqrt{12} = 12$$

arm is fixed, the gear train is simple and gear A can drive gear B or ***vice- versa***, but if gear A is fixed and the arm is rotated about the axis of gear A (*i.e.* O_1), then the gear B is forced to rotate ***upon*** and ***around*** gear A. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains*** (***epi.*** means upon and ***cyclic*** means around). The epicyclic gear trains may be ***simple*** or ***compound***.

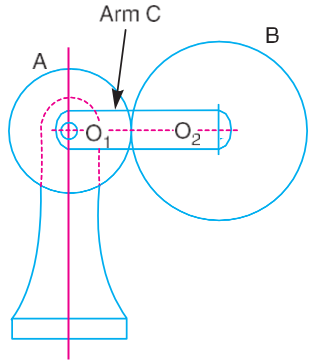


Fig. 13.6. Epicyclic gear train.

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

13.8. Velocity Ratioz of Epicyclic Gear Train

The following two methods may be used for finding out the velocity ratio of an epicyclic gear train.

1. Tabular method, and **2.** Algebraic method.

These methods are discussed, in detail, as follows :

1. Tabular method. Consider an epicyclic gear train as shown in Fig. 13.6.

Let T_A = Number of teeth on gear A , and

T_B = Number of teeth on gear B.

First of all, let us suppose that the arm is fixed. Therefore the axes of both the gears are also fixed relative to each other. When the gear A makes one revolution anticlockwise, the gear B will make *T_A / T_B revolutions, clockwise. Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear A makes + 1 revolution, then the gear B will make ($-T_A / T_B$) revolutions. This statement of relative motion is entered in the first row of the table (see Table 13.1).


Inside view of a car engine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Secondly, if the gear A makes $+ x$ revolutions, then the gear B will make $- x \times T_A / T_B$ revolutions. This statement is entered in the second row of the table. In other words, multiply the each motion (entered in the first row) by x.

Thirdly, each element of an epicyclic train is given $+ y$ revolutions and entered in the third row. Finally, the motion of each element of the gear train is added up and entered in the fourth row.

* We know that $N_B / N_A = T_A / T_B$. Since N_A = 1 revolution, therefore $N_B = T_A / T_B$.

Table 13.1. Table of motions

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

A little consideration will show that when two conditions about the motion of rotation of any two elements are known, then the unknown speed of the third element may be obtained by substituting the given data in the third column of the fourth row.

2. *Algebraic method.* In this method, the motion of each element of the epicyclic train relative to the arm is set down in the form of equations. The number of equations depends upon the number of elements in the gear train. But the two conditions are, usually, supplied in any epicyclic train *viz*. some element is fixed and the other has specified motion. These two conditions are sufficient to solve all the equations ; and hence to determine the motion of any element in the epicyclic gear train.

Let the arm *C* be fixed in an epicyclic gear train as shown in Fig. 13.6. Therefore speed of the gear *A* relative to the arm *C*

$$= N_A - N_C$$

and speed of the gear *B* relative to the arm *C*,

$$= N_B - N_C$$

Since the gears *A* and *B* are meshing directly, therefore they will revolve in ***opposite*** directions.

$$\therefore \qquad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since the arm *C* is fixed, therefore its speed, $N_C = 0$.

$$\therefore \qquad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear *A* is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

Note : The tabular method is easier and hence mostly used in solving problems on epicyclic gear train.

Example 13.4. *In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?*

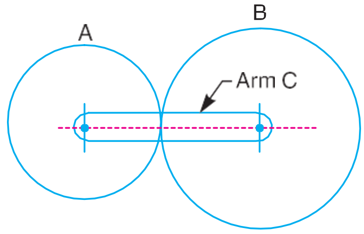


Fig. 13.7

Solution. Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m. (anticlockwise)

The gear train is shown in Fig. 13.7.

We shall solve this example, first by tabular method and then by algebraic method.

1. Tabular method

First of all prepare the table of motions as given below :

Table 13.2. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +150 \text{ r.p.m.}$$

Also the gear *A* is fixed, therefore

$$x + y = 0 \quad \text{or} \quad x = -y = -150 \text{ r.p.m.}$$

$\therefore$ Speed of gear *B*, $N_B = y - x \times \frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = +270$ r.p.m.

$= 270$ r.p.m. (anticlockwise) **Ans.**

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear *A* makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x + y = -300 \quad \text{or} \quad x = -300 - y = -300 - 150 = -450 \text{ r.p.m.}$$

$\therefore$ Speed of gear *B*,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +510 \text{ r.p.m.}$$

$= 510$ r.p.m. (anticlockwise) **Ans.**

2. Algebraic method

Let N_A = Speed of gear *A*.

N_B = Speed of gear *B*, and

N_C = Speed of arm *C*.

Assuming the arm *C* to be fixed, speed of gear *A* relative to arm *C*

$$= N_A - N_C$$

and speed of gear *B* relative to arm *C* = $N_B - N_C$

Since the gears A and B revolve in ***opposite*** directions, therefore

$$\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B} \qquad ...(i)$$

Speed of gear B when gear A is fixed

When gear A is fixed, the arm rotates at 150 r.p.m. in the anticlockwise direction, *i.e.*

$$N_A = 0, \quad \text{and} \quad N_C = +150 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{0 - 150} = -\frac{36}{45} = -0.8 \qquad ...[\text{From equation } (i)]$$

or $\quad N_B = -150 \times -0.8 + 150 = 120 + 150 = 270$ r.p.m. **Ans.**

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m. clockwise, therefore

$$N_A = -300 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{-300 - 150} = -\frac{36}{45} = -0.8$$

or $\quad N_B = -450 \times -0.8 + 150 = 360 + 150 = 510$ r.p.m. **Ans.**

Example 13.5. *In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise.*

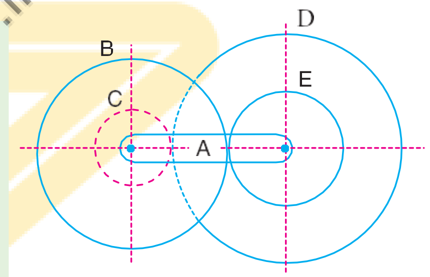


Fig. 13.8

Solution. Given : $T_B = 75$; $T_C = 30$; $T_D = 90$; $N_A = 100$ r.p.m. (clockwise)

The reverted epicyclic gear train is shown in Fig. 13.8. First of all, let us find the number of teeth on gear E (T_E). Let d_B, d_C, d_D and d_E be the pitch circle diameters of gears B, C, D and E respectively. From the geometry of the figure,

$$d_B + d_E = d_C + d_D$$

Since the number of teeth on each gear, for the same module, are proportional to their pitch circle diameters, therefore

$$T_B + T_E = T_C + T_D$$

$$\therefore \quad T_E = T_C + T_D - T_B = 30 + 90 - 75 = 45$$

The table of motions is drawn as follows :

A gear-cutting machine is used to cut gears.
Note : This picture is given as additional information and is not a direct example of the current chapter.

Table 13.3. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear *D- E* rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear *D- E* rotated through + *x* revolutions	0	+ *x*	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$

Since the gear *B* is fixed, therefore from the fourth row of the table,

$$y - x \times \frac{T_E}{T_B} = 0 \quad \text{or} \quad y - x \times \frac{45}{75} = 0$$

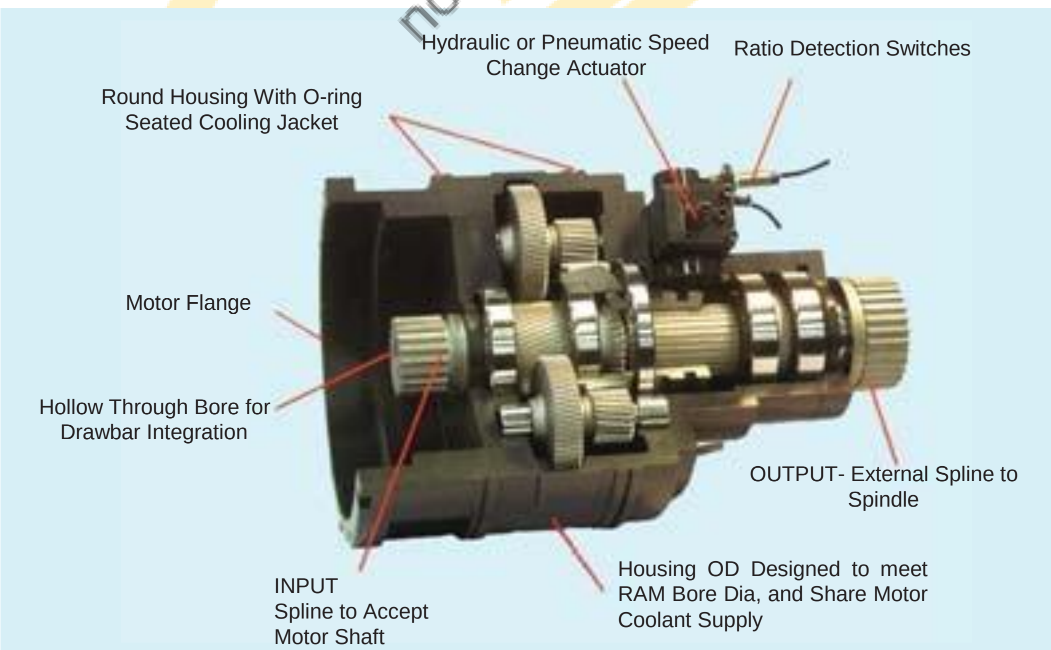
$$\therefore \qquad y - 0.6 = 0 \qquad \text{...}(i)$$

Also the arm *A* makes 100 r.p.m. clockwise, therefore

$$y = -100 \qquad \text{...}(ii)$$

Substituting *y* = – 100 in equation (*i*), we get

$$-100 - 0.6\,x = 0 \quad \text{or} \quad x = -100 / 0.6 = -166.67$$



Model of sun and planet gears.

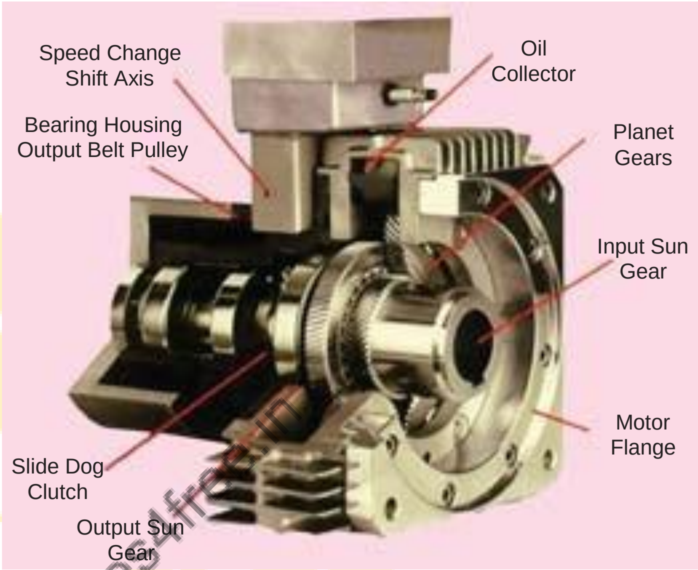
From the fourth row of the table, speed of gear *C*,

$$N_C = y - x \times \frac{T_D}{T_C} = -100 + 166.67 \times \frac{90}{30} = +400 \text{ r.p.m.}$$

$$= 400 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

13.9. Compound Epicyclic Gear Train—Sun and Planet Gear

A compound epicyclic gear train is shown in Fig. 13.9. It consists of two co-axial shafts S_1 and S_2, an annulus gear *A* which is fixed, the compound gear (or planet gear) *B*-*C*, the sun gear *D* and the arm *H*. The annulus gear has internal teeth and the compound gear is carried by the arm and revolves freely on a pin of the arm *H*. The sun gear is co-axial with the annulus gear and the arm but independent of them.



Sun and Planet gears.

The annulus gear *A* meshes with the gear *B* and the sun gear *D* meshes with the gear *C*. It may be noted that when the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as a follower.

Note : The gear at the centre is called the ***sun gear*** and the gears whose axes move are called ***planet gears***.

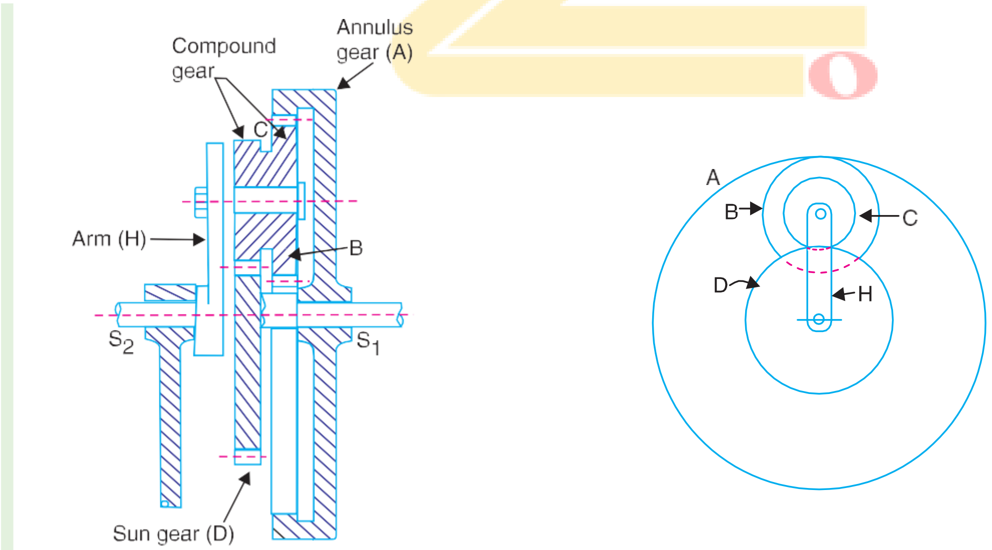


Fig. 13.9. Compound epicyclic gear train.

Let T_A , T_B , T_C , and T_D be the teeth and N_A, N_B, N_C and N_D be the speeds for the gears *A* , *B*, *C* and *D* respectively. A little consideration will show that when the arm is fixed and the sun gear *D* is turned anticlockwise, then the compound gear *B*-*C* and the annulus gear A will rotate in the clockwise direction.

The motion of rotations of the various elements are shown in the table below.

Table 13.4. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm	*Gear D*	*Compound gear B-C*	*Gear A*
1.	Arm fixed-gear *D* rotates through + 1 revolution	0	+ 1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
2.	Arm fixed-gear *D* rotates through + *x* revolutions	0	+ *x*	$-x\times\frac{T_D}{T_C}$	$-x\times\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y-x\times\frac{T_D}{T_C}$	$y-x\times\frac{T_D}{T_C}\times\frac{T_B}{T_A}$

Note : If the annulus gear *A* is rotated through one revolution anticlockwise with the arm fixed, then the compound gear rotates through T_A / T_B revolutions in the same sense and the sun gear *D* rotates through T_A / T_B × T_C / T_D revolutions in clockwise direction.

Example 13.6. *An epicyclic gear consists of three gears A, B and C as shown in Fig. 13.10. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.*

Solution. Given : T_A = 72 ; T_C = 32 ; Speed of arm *EF* = 18 r.p.m.

Considering the relative motion of rotation as shown in Table 13.5.

Table 13.5. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm EF	*Gear C*	*Gear B*	*Gear A*
1.	Arm fixed-gear *C* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B}\times\frac{T_B}{T_A}=-\frac{T_C}{T_A}$
2.	Arm fixed-gear *C* rotates through + *x* revolutions	0	+ *x*	$-x\times\frac{T_C}{T_B}$	$-x\times\frac{T_C}{T_A}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y-x\times\frac{T_C}{T_B}$	$y-x\times\frac{T_C}{T_A}$

Speed of gear C

We know that the speed of the arm is 18 r.p.m. therefore,

$$y = 18 \text{ r.p.m.}$$

and the gear A is fixed, therefore

$$y - x \times \frac{T_C}{T_A} = 0 \quad \text{or} \quad 18 - x \times \frac{32}{72} = 0$$

$$\therefore \quad x = 18 \times 72 / 32 = 40.5$$

$\therefore$ Speed of gear C $= x + y = 40.5 + 18$

$= + 58.5$ r.p.m.

$= 58.5$ r.p.m. in the direction of arm. **Ans.**

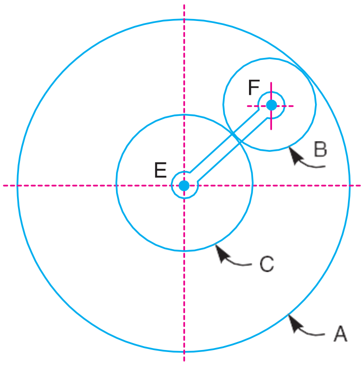


Fig. 13.10

Speed of gear B

Let d_A, d_B and d_C be the pitch circle diameters of gears A, B and C respectively. Therefore, from the geometry of Fig. 13.10,

$$d_B + \frac{d_C}{2} = \frac{d_A}{2} \quad \text{or} \quad 2\,d_B + d_C = d_A$$

Since the number of teeth are proportional to their pitch circle diameters, therefore

$$2\,T_B + T_C = T_A \quad \text{or} \quad 2\,T_B + 32 = 72 \quad \text{or} \quad T_B = 20$$

$$\therefore \text{ Speed of gear } B = y - x \times \frac{T_C}{T_B} = 18 - 40.5 \times \frac{32}{20} = -46.8 \text{ r.p.m.}$$

$= 46.8$ r.p.m. in the opposite direction of arm. **Ans.**

Example 13.7. *An epicyclic train of gears is arranged as shown in Fig.13.11. How many revolutions does the arm, to which the pinions B and C are attached, make :*

1. *when A makes one revolution clockwise and D makes half a revolution anticlockwise, and*

2. *when A makes one revolution clockwise and D is stationary ?*

The number of teeth on the gears A and D are 40 and 90 respectively.

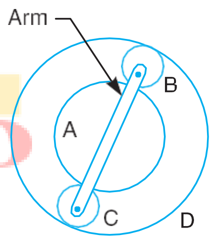


Fig. 13.11

Solution. Given : $T_A = 40$; $T_D = 90$

First of all, let us find the number of teeth on gears B and C (*i.e.* T_B and T_C). Let d_A, d_B, d_C and d_D be the pitch circle diameters of gears A, B, C and D respectively. Therefore from the geometry of the figure,

$$d_A + d_B + d_C = d_D \quad \text{or} \quad d_A + 2\,d_B = d_D \qquad \ldots(d_B = d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_A + 2\,T_B = T_D \quad \text{or} \quad 40 + 2\,T_B = 90$$

$$\therefore \quad T_B = 25, \quad \text{and} \quad T_C = 25 \qquad \ldots(T_B = T_C)$$

The table of motions is given below :

Table 13.6. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm	*Gear A*	*Compound gear B-C*	*Gear D*
1.	Arm fixed, gear *A* rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear *A* rotates through – *x* revolutions	0	– *x*	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add – *y* revolutions to all elements	– *y*	– *y*	– *y*	– *y*
4.	Total motion	– *y*	– *x* – *y*	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

1. Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise

Since the gear *A* makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad ...(i)$$

Also, the gear *D* makes half revolution anticlockwise, therefore

$$x\times\frac{T_A}{T_D}-y=\frac{1}{2} \quad \text{or} \quad x\times\frac{40}{90}-y=\frac{1}{2}$$

$\therefore$
$$40\,x-90\,y=45 \quad \text{or} \quad x-2.25\,y=1.125 \qquad ...(ii)$$

From equations *(i)* and *(ii)*, $x = 1.04$ and $y = -0.04$

$\therefore$ Speed of arm $= -y = -(-0.04) = +0.04$

= 0.04 revolution anticlockwise **Ans.**

2. Speed of arm when A makes 1 revolution clockwise and D is stationary

Since the gear *A* makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad ...(iii)$$

Also the gear *D* is stationary, therefore

$$x\times\frac{T_A}{T_D}-y=0 \quad \text{or} \quad x\times\frac{40}{90}-y=0$$

$\therefore$
$$40\,x-90\,y=0 \quad \text{or} \quad x-2.25\,y=0 \qquad ...(iv)$$

From equations *(iii)* and *(iv)*,

$$x = 0.692 \quad \text{and} \quad y = 0.308$$

$\therefore$ Speed of arm $= -y = -0.308$ = 0.308 revolution clockwise **Ans.**

Example 13.8. *In an epicyclic gear train, the internal wheels A and B and compound wheels C and D rotate independently about axis O. The wheels E and F rotate on pins fixed to the arm G. E gears with A and C and F gears with B and D. All the wheels have the same module and the number of teeth are : $T_C = 28$; $T_D = 26$; $T_E = T_F = 18$.*

***1.** Sketch the arrangement ; **2.** Find the number of teeth on A and B ; **3.** If the arm G makes 100 r.p.m. clockwise and A is fixed, find the speed of B ; and **4.** If the arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise ; find the speed of wheel B.*

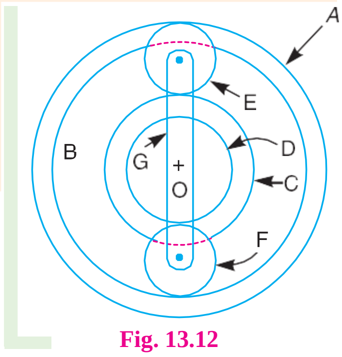


Fig. 13.12

Solution. Given : $T_C = 28$; $T_D = 26$; $T_E = T_F = 18$

1. Sketch the arrangement

The arrangement is shown in Fig. 13.12.

2. Number of teeth on wheels A and B

Let T_A = Number of teeth on wheel *A*, and

T_B = Number of teeth on wheel *B*.

If d_A, d_B, d_C, d_D, d_E and d_F are the pitch circle diameters of wheels *A* , *B*, *C*, *D*, *E* and *F* respectively, then from the geometry of Fig. 13.12,

$$d_A = d_C + 2\, d_E$$

and $$d_B = d_D + 2\, d_F$$

Since the number of teeth are proportional to their pitch circle diameters, for the same module, therefore

$$T_A = T_C + 2\, T_E = 28 + 2 \times 18 = 64 \quad \textbf{Ans.}$$

and $$T_B = T_D + 2\, T_F = 26 + 2 \times 18 = 62 \quad \textbf{Ans.}$$

3. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A is fixed

First of all, the table of motions is drawn as given below :

Table 13.7. Table of motions.

Step No.	Conditions of motion	Revolutions of elements					
		Arm G	*Wheel A*	*Wheel E*	*Compound wheel C-D*	*Wheel F*	*Wheel B*
1.	Arm fixed- wheel *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$\frac{T_A}{T_E}$	$-\frac{T_A}{T_E} \times \frac{T_E}{T_C} = -\frac{T_A}{T_C}$	$+\frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$+\frac{T_A}{T_C} \times \frac{T_D}{T_F} \times \frac{T_F}{T_B} = +\frac{T_A}{T_C} \times \frac{T_D}{T_B}$
2.	Arm fixed-wheel *A* rotates through + *x* revolutions	0	+ *x*	$+ x \times \frac{T_A}{T_E}$	$x \times \frac{T_A}{T_C}$	$+ x \times \frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y + x \times \frac{T_A}{T_E}$	$y - x \times \frac{T_A}{T_C}$	$y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$

Since the arm *G* makes 100 r.p.m. clockwise, therefore from the fourth row of the table,

$$y = -100 \qquad ...(i)$$

Also, the wheel *A* is fixed, therefore from the fourth row of the table,

$$x + y = 0 \quad \text{or} \quad x = -y = 100 \qquad ...(ii)$$

$$\therefore \text{ Speed of wheel } B = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B} = -100 + 100 \times \frac{64}{28} \times \frac{26}{62} = -100 + 95.8 \text{ r.p.m.}$$

$$= -4.2 \text{ r.p.m.} = 4.2 \text{ r.p.m. clockwise } \textbf{Ans.}$$

4. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise

Since the arm *G* makes 100 r.p.m. clockwise, therefore from the fourth row of the table

$$y = -100 \qquad ...(iii)$$

Also the wheel *A* makes 10 r.p.m. counter clockwise, therefore from the fourth row of the table,

$$x + y = 10 \quad \text{or} \quad x = 10 - y = 10 + 100 = 110 \qquad ...(iv)$$

$$\therefore \quad \text{Speed of wheel } B = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B} = -100 + 110 \times \frac{64}{28} \times \frac{26}{62} = -100 + 105.4 \text{ r.p.m.}$$

$$= +5.4 \text{ r.p.m.} = 5.4 \text{ r.p.m. counter clockwise } \textbf{Ans.}$$

Example 13.9. *In an epicyclic gear of the 'sun and planet' type shown in Fig. 13.13, the pitch circle diameter of the internally toothed ring is to be 224 mm and the module 4 mm. When the ring D is stationary, the spider A, which carries three planet wheels C of equal size, is to make one revolution in the same sense as the sunwheel B for every five revolutions of the driving spindle carrying the sunwheel B. Determine suitable numbers of teeth for all the wheels.*

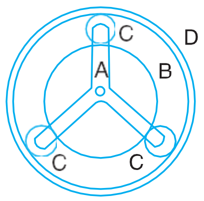


Fig. 13.13

Solution. Given : $d_D = 224$ mm ; $m = 4$ mm ; $N_A = N_B / 5$

Let T_B, T_C and T_D be the number of teeth on the sun wheel *B*, planet wheels *C* and the internally toothed ring *D*. The table of motions is given below :

Table 13.8. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Spider A	Sun wheel B	Planet wheel C	Internal gear D
1.	Spider *A* fixed, sun wheel *B* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_B}{T_C}$	$-\frac{T_B}{T_C} \times \frac{T_C}{T_D} = -\frac{T_B}{T_D}$
2.	Spider *A* fixed, sun wheel *B* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_B}{T_C}$	$-x \times \frac{T_B}{T_D}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_B}{T_C}$	$y - x \times \frac{T_B}{T_D}$

We know that when the sun wheel B makes + 5 revolutions, the spider A makes + 1 revolution. Therefore from the fourth row of the table,

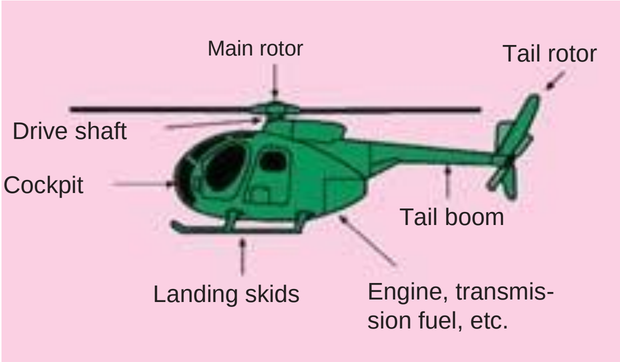
$$y = +1 \; ; \text{ and } x + y = +5$$

$$\therefore\; x = 5 - y = 5 - 1 = 4$$

Since the internally toothed ring D is stationary, therefore from the fourth row of the table,

$$y - x \times \frac{T_B}{T_D} = 0$$

or $$1 - 4 \times \frac{T_B}{T_D} = 0$$



Power transmission in a helicopter is essentially through gear trains.

Note : This picture is given as additional information and is not a direct example of the current chapter.

$$\therefore \quad \frac{T_B}{T_D} = \frac{1}{4} \quad \text{or} \quad T_D = 4\,T_B \qquad \text{...}(i)$$

We know that $T_D = d_D / m = 224 / 4 = 56$ **Ans.**

$\therefore$ $T_B = T_D / 4 = 56 / 4 = 14$ **Ans.**.................................. [From equation (*i*)]

Let d_B, d_C and d_D be the pitch circle diameters of sun wheel B, planet wheels C and internally toothed ring D respectively. Assuming the pitch of all the gears to be same, therefore from the geometry of Fig. 13.13,

$$d_B + 2\,d_C = d_D$$

Since the number of teeth are proportional to their pitch circle diameters, therefore

$$T_B + 2\,T_C = T_D \quad \text{or} \quad 14 + 2\,T_C = 56$$

$\therefore$ $T_C = 21$ **Ans.**

Example 13.10. *Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. A compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Sketch the arrangement and find the number of teeth on internal gear G assuming that all gears have the same module. If the shaft A rotates at 110 r.p.m., find the speed of shaft B.*

Solution. Given : $T_C = 50$; $T_D = 20$; $T_E = 35$; $N_A = 110$ r.p.m.

The arrangement is shown in Fig. 13.14.

Number of teeth on internal gear G

Let d_C, d_D, d_E and d_G be the pitch circle diameters of gears C, D, E and G respectively. From the geometry of the figure,

$$\frac{d_G}{2} = \frac{d_C}{2} + \frac{d_D}{2} + \frac{d_E}{2}$$

or $$d_G = d_C + d_D + d_E$$

Let T_C, T_D, T_E and T_G be the number of teeth on gears *C*, *D*, *E* and *G* respectively. Since all the gears have the same module, therefore number of teeth are proportional to their pitch circle diameters.

$\therefore \quad T_G = T_C + T_D + T_E = 50 + 20 + 35 = 105$ **Ans.**

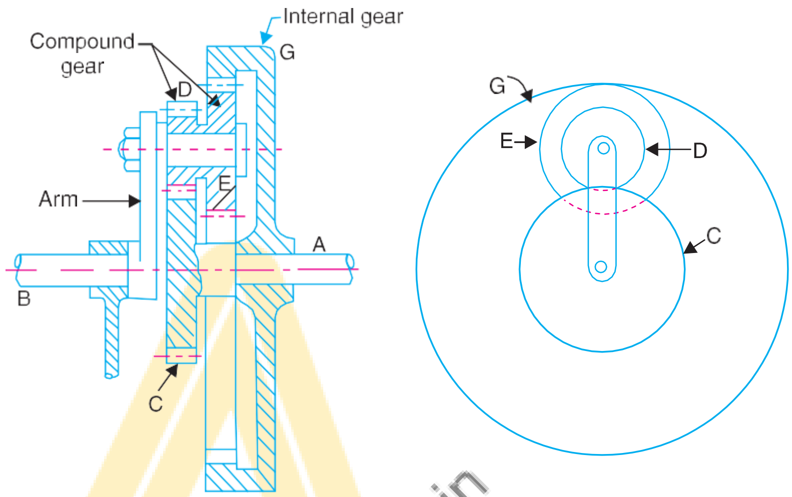


Fig. 13.14

Speed of shaft B

The table of motions is given below :

Table 13.9. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm	*Gear C (or shaft A)*	*Compound gear D-E*	*Gear G*
1.	Arm fixed - gear *C* rotates through + 1 revolution	0	+ 1	$-\frac{T_C}{T_D}$	$-\frac{T_C}{T_D} \times \frac{T_E}{T_G}$
2.	Arm fixed - gear *C* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_C}{T_D}$	$-x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_C}{T_D}$	$y - x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G}$

Since the gear *G* is fixed, therefore from the fourth row of the table,

$$y - x \times \frac{T_C}{T_D} \times \frac{T_E}{T_G} = 0 \quad \text{or} \quad y - x \times \frac{50}{20} \times \frac{35}{105} = 0$$

$$\therefore \qquad y - \frac{5}{6}x = 0 \qquad \ldots(i)$$

Since the gear C is rigidly mounted on shaft A, therefore speed of gear C and shaft A is same. We know that speed of shaft A is 110 r.p.m., therefore from the fourth row of the table,

$$x + y = 100 \quad \quad ...(ii)$$

From equations ***(i)*** and ***(ii)***, $x = 60$, and $y = 50$

$\therefore$ Speed of shaft B = Speed of arm = $+ y$ = 50 r.p.m. anticlockwise **Ans.**

Example 13.11. *Fig. 13.15 shows diagrammatically a compound epicyclic gear train. Wheels A , D and E are free to rotate independently on spindle O, while B and C are compound and rotate together on spindle P, on the end of arm OP. All the teeth on different wheels have the same module. A has 12 teeth, B has 30 teeth and C has 14 teeth cut externally. Find the number of teeth on wheels D and E which are cut internally.*

If the wheel A is driven clockwise at 1 r.p.s. while D is driven counter clockwise at 5 r.p.s., determine the magnitude and direction of the angular velocities of arm OP and wheel E.

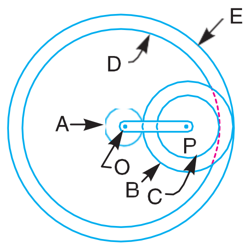


Fig. 13.15

Solution. Given : $T_A = 12$; $T_B = 30$; $T_C = 14$; $N_A = 1$ r.p.s. ; $N_D = 5$ r.p.s.

Number of teeth on wheels D and E

Let T_D and T_E be the number of teeth on wheels D and E respectively. Let d_A, d_B, d_C, d_D and d_E be the pitch circle diameters of wheels A, B, C, D and E respectively. From the geometry of the figure,

$$d_E = d_A + 2d_B \quad \text{and} \quad d_D = d_E - (d_B - d_C)$$

Since the number of teeth are proportional to their pitch circle diameters for the same module, therefore

$$T_E = T_A + 2T_B = 12 + 2 \times 30 = 72 \text{ Ans.}$$

and $$T_D = T_E - (T_B - T_C) = 72 - (30 - 14) = 56 \text{ Ans.}$$

Magnitude and direction of angular velocities of arm OP and wheel E

The table of motions is drawn as follows :

Table 13.10. Table of motions.

Step No.	Conditions of motion	*Revolutions of elements*				
		Arm	*Wheel A*	*Compound wheel B-C*	*Wheel D*	*Wheel E*
1.	Arm fixed A rotated through – 1 revolution (*i.e.* 1 revolution clockwise)	0	-1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_E} = +\frac{T_A}{T_E}$
2.	Arm fixed-wheel A rotated through – x revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$+x\times\frac{T_A}{T_E}$
3.	Add – y revolutions to all elements	$-y$	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_B}\times\frac{T_C}{T_D}-y$	$x\times\frac{T_A}{T_E}-y$

Since the wheel A makes 1 r.p.s. clockwise, therefore from the fourth row of the table,

$$-x - y = -1 \quad \text{or} \quad x + y = 1 \qquad \ldots(i)$$

Also, the wheel D makes 5 r.p.s. counter clockwise, therefore

$$x \times \frac{T_A}{T_B} \times \frac{T_C}{T_D} - y = 5 \quad \text{or} \quad x \times \frac{12}{30} \times \frac{14}{56} - y = 5$$

$$\therefore \qquad 0.1\, x - y = 5 \qquad \ldots(ii)$$

From equations ***(i)*** and ***(ii)***,

$$x = 5.45 \quad \text{and} \quad y = -4.45$$

$\therefore$ Angular velocity of arm OP

$$= -y = -(-4.45) = 4.45 \text{ r.p.s}$$

$$= 4.45 \times 2\,\pi = 27.964 \text{ rad/s (counter clockwise)} \quad \textbf{Ans.}$$

and angular velocity of wheel $E = x \times \frac{T_A}{T_E} - y = 5.45 \times \frac{12}{72} - (-4.45) = 5.36$ r.p.s.

$$= 5.36 \times 2\,\pi = 33.68 \text{ rad/s (counter clockwise)} \quad \textbf{Ans.}$$

Example 13.12. *An internal wheel B with 80 teeth is keyed to a shaft F. A fixed internal wheel C with 82 teeth is concentric with B. A compound wheel D-E gears with the two internal wheels; D has 28 teeth and gears with C while E gears with B. The compound wheels revolve freely on a pin which projects from a disc keyed to a shaft A co-axial with F. If the wheels have the same pitch and the shaft A makes 800 r.p.m., what is the speed of the shaft F ? Sketch the arrangement.*

Helicopter

Note : This picture is given as additional information and is not a direct example of the current chapter.

Solution. Given : $T_B = 80$; $T_C = 82$; $T_D = 28$; $N_A = 500$ r.p.m.

The arrangement is shown in Fig. 13.16.

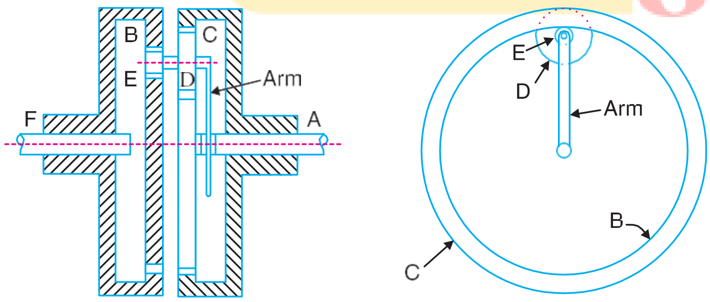


Fig. 13.16

First of all, let us find out the number of teeth on wheel E (T_E). Let d_B , d_C , d_D and d_E be the pitch circle diameter of wheels B, C, D and E respectively. From the geometry of the figure,

$$d_B = d_C - (d_D - d_E)$$

or $$d_E = d_B + d_D - d_C$$

Since the number of teeth are proportional to their pitch circle diameters for the same pitch, therefore

$$T_E = T_B + T_D - T_C = 80 + 28 - 82 = 26$$

The table of motions is given below :

Table 13.11. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm (or shaft A)	Wheel B (or shaft F)	Compound gear D-E	Wheel C
1.	Arm fixed - wheel *B* rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	$\frac{T_B}{T_E}$	$+\frac{T_B}{T_E}\times\frac{T_D}{T_C}$
2.	Arm fixed - wheel *B* rotated through + *x* revolutions	0	+ *x*	$+x\times\frac{T_B}{T_E}$	$+x\times\frac{T_B}{T_E}\times\frac{T_D}{T_C}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y + x\times\frac{T_B}{T_E}$	$y + x\times\frac{T_B}{T_E}\times\frac{T_D}{T_C}$

Since the wheel *C* is fixed, therefore from the fourth row of the table,

$$y + x\times\frac{T_B}{T_E}\times\frac{T_D}{T_C} = 0 \quad \text{or} \quad y + x\times\frac{80}{26}\times\frac{28}{82} = 0$$

$$\therefore \quad y + 1.05\,x = 0 \qquad \ldots(i)$$

Also, the shaft *A* (or the arm) makes 800 r.p.m., therefore from the fourth row of the table,

$$y = 800 \qquad \ldots(ii)$$

From equations (*i*) and (*ii*),

$$x = -762$$

$\therefore$ Speed of shaft *F* = Speed of wheel *B* = $x + y = -762 + 800 = +38$ r.p.m.

= 38 r.p.m. (anticlockwise) **Ans.**

Example 13.13. *Fig. 13.17 shows an epicyclic gear train known as Ferguson's paradox. Gear A is fixed to the frame and is, therefore, stationary. The arm B and gears C and D are free to rotate on the shaft S. Gears A, C and D have 100, 101 and 99 teeth respectively. The planet gear has 20 teeth. The pitch circle diameters of all are the same so that the planet gear P meshes with all of them. Determine the revolutions of gears C and D for one revolution of the arm B.*

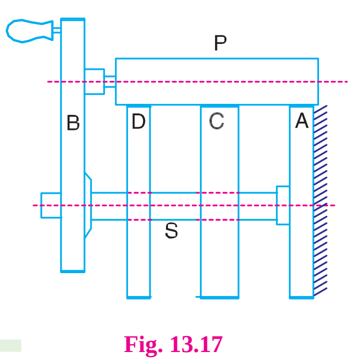


Fig. 13.17

Solution. Given : $T_A = 100$; $T_C = 101$; $T_D = 99$; $T_P = 20$

The table of motions is given below :

Table 13.12. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm B	*Gear A*	*Gear C*	*Gear D*
1.	Arm *B* fixed, gear *A* rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	$\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_C}{T_D}=+\frac{T_A}{T_D}$
2.	Arm *B* fixed, gear *A* rotated through + *x* revolutions	0	+ *x*	$+x\times\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_D}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y+x\times\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_D}$

The arm *B* makes one revolution, therefore

$$y = 1$$

Since the gear *A* is fixed, therefore from the fourth row of the table,

$$x + y = 0 \quad \text{or} \quad x = -y = -1$$

Let N_C and N_D = Revolutions of gears *C* and *D* respectively.

From the fourth row of the table, the revolutions of gear *C*,

$$N_C = y + x \times \frac{T_A}{T_C} = 1 - 1 \times \frac{100}{101} = +\frac{1}{101} \quad \textbf{Ans.}$$

and the revolutions of gear *D*,

$$N_D = y + x \times \frac{T_A}{T_D} = 1 - \frac{100}{99} = -\frac{1}{99} \quad \textbf{Ans.}$$

From above we see that for one revolution of the arm *B*, the gear *C* rotates through 1/101 revolutions in the same direction and the gear *D* rotates through 1/99 revolutions in the opposite direction.

Example 13.14. *In the gear drive as shown in Fig. 13.18, the driving shaft A rotates at 300 r.p.m. in the clockwise direction, when seen from left hand. The shaft B is the driven shaft. The casing C is held stationary. The wheels E and H are keyed to the central vertical spindle and wheel F can rotate freely on this spindle. The wheels K and L are rigidly fixed to each other and rotate together freely on a pin fitted on the underside of F. The wheel L meshes with internal teeth on the casing C. The numbers of teeth on the different wheels are indicated within brackets in Fig. 13.18.*

Find the number of teeth on wheel C and the speed and direction of rotation of shaft B.

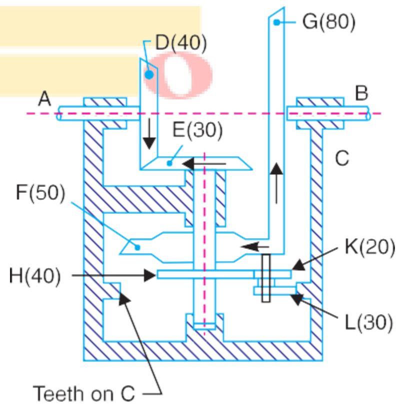


Fig. 13.18

Solution. Given : N_A = 300 r.p.m. (clockwise) ; T_D = 40 ; T_B = 30 ; T_F = 50 ; T_G = 80 ; T_H = 40 ; T_K = 20 ; T_L = 30

In the arrangement shown in Fig. 13.18, the wheels *D* and *G* are auxillary gears and do not form a part of the epicyclic gear train.

Speed of wheel E, $N_E = N_A \times \frac{T_D}{T_E} = 300 \times \frac{40}{30} = 400$ r.p.m. (clockwise)

Number of teeth on wheel C

Let T_C = Number of teeth on wheel C.

Assuming the same module for all teeth and since the pitch circle diameter is proportional to the number of teeth ; therefore from the geometry of Fig.13.18,

$$T_C = T_H + T_K + T_L = 40 + 20 + 30 = 90 \textbf{ Ans.}$$

Speed and direction of rotation of shaft B

The table of motions is given below. The wheel F acts as an arm.

Table 13.13. Table of motions.

Step No.	Conditions of motion	*Revolutions of elements*				
		Arm or wheel F	*Wheel E*	*Wheel H*	*Compound wheel K-L*	*Wheel C*
1.	Arm fixed-wheel E rotated through – 1 revolution (*i.e.* 1 revolution clockwise)	0	– 1	– 1 (E and H are on the same shaft)	$+\frac{T_H}{T_K}$	$+\frac{T_H}{T_K}\times\frac{T_L}{T_C}$
2.	Arm fixed-wheel E rotated through – x revolutions	0	$-x$	$-x$	$+x\times\frac{T_H}{T_K}$	$+x\times\frac{T_H}{T_K}\times\frac{T_L}{T_C}$
3.	Add – y revolutions to all elements	$-y$	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$-x-y$	$x\times\frac{T_H}{T_K}-y$	$x\times\frac{T_H}{T_K}\times\frac{T_L}{T_C}-y$

Since the speed of wheel E is 400 r.p.m. (clockwise), therefore from the fourth row of the table,

$$-x-y=-400 \quad \text{or} \quad x+y=400 \qquad \text{...}(i)$$

Also the wheel C is fixed, therefore

$$x\times\frac{T_H}{T_K}\times\frac{T_L}{T_C}-y=0$$

or

$$x\times\frac{40}{20}\times\frac{30}{90}-y=0$$

$$\therefore \quad \frac{2x}{3}-y=0 \qquad \text{...}(ii)$$

From equations ***(i)*** and ***(ii)***,

$$x = 240 \quad \text{and} \quad y = 160$$

$\therefore$ Speed of wheel F, $N_F = -y = -160$ r.p.m.

Since the wheel F is in mesh with wheel G, therefore speed of wheel G or speed of shaft B

$$= -N_F\times\frac{T_F}{T_G} = -\left(-160\times\frac{50}{80}\right) = 100 \text{ r.p.m.}$$

...(Wheel G will rotate in opposite direction to that of wheel F.)

= 100 r.p.m. anticlockwise *i.e.* in opposite direction of shaft A . **Ans.**

Example 13.15. *Fig. 13.19 shows a compound epicyclic gear in which the casing C contains an epicyclic train and this casing is inside the larger casing D.*

Determine the velocity ratio of the output shaft B to the input shaft A when the casing D is held stationary. The number of teeth on various wheels are as follows :

Wheel on A = 80 ; Annular wheel on B = 160 ; Annular wheel on C = 100 ; Annular wheel on D = 120 ; Small pinion on F = 20 ; Large pinion on F = 66.

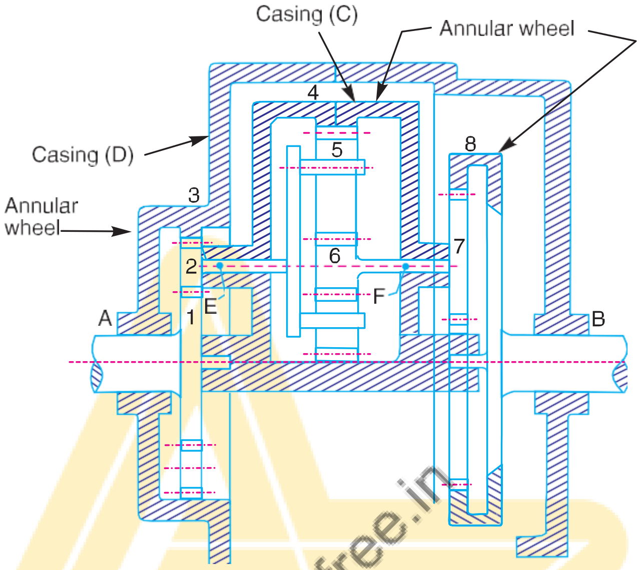


Fig. 13.19

Solution. Given : $T_1 = 80$; $T_8 = 160$; $T_4 = 100$; $T_3 = 120$; $T_6 = 20$; $T_7 = 66$

First of all, let us consider the train of wheel 1 (on *A*), wheel 2 (on *E*), annular wheel 3 (on *D*) and the arm *i.e.* casing *C*. Since the pitch circle diameters of wheels are proportional to the number of teeth, therefore from the geometry of Fig. 13.19,

$$T_1 + 2T_2 = T_3 \quad \text{or} \quad 80 + 2\,T_2 = 120$$

$$\therefore \quad T_2 = 20$$

The table of motions for the train considered is given below :

Table 13.14. Table of motions.

Step No.	Conditons of motion	Revolutions of elements			
		Arm	Wheel 1	Wheel 2	Wheel 3
1.	Arm fixed - wheel 1 rotated through + 1 revolution (anticlockwise)	0	+ 1	$-\frac{T_1}{T_2}$	$-\frac{T_1}{T_2}\times\frac{T_2}{T_3} = -\frac{T_1}{T_3}$
2.	Arm fixed - wheel 1 rotated through + *x* revolutions	0	$+x$	$-x\times\frac{T_1}{T_2}$	$-x\times\frac{T_1}{T_3}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	y	$x+y$	$y - x\times\frac{T_1}{T_2}$	$y - x\times\frac{T_1}{T_3}$

Let us assume that wheel 1 makes 1 r.p.s. anticlockwise.

$\therefore \qquad x + y = 1 \qquad \ldots(i)$

Also the wheel 3 is stationary, therefore from the fourth row of the table,

$$y - x \times \frac{T_1}{T_3} = 0 \quad \text{or} \quad y - x \times \frac{80}{120} = 0$$

$$\therefore \qquad y - \frac{2}{3}x = 0 \qquad \ldots(ii)$$

From equations *(i)* and *(ii)*, $x = 0.6$, and $y = 0.4$

$\therefore$ Speed of arm or casing $C = y = 0.4$ r.p.s.

and speed of wheel 2 or arm E $= y - x \times \dfrac{T_1}{T_2} = 0.4 - 0.6 \times \dfrac{80}{20} = -2$ r.p.s.

$= 2$ r.p.s. (clockwise)

Let us now consider the train of annular wheel 4 (on C), wheel 5 (on E), wheel 6 (on F) and arm E. We know that

$$T_6 + 2\,T_5 = T_4 \quad \text{or} \quad 20 + 2\,T_5 = 100$$

$$\therefore \qquad T_5 = 40$$

The table of motions is given below :

Table 13.15. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm E or wheel 2	Wheel 6	Wheel 5	Wheel 4
1.	Arm fixed, wheel 6 rotated through + 1 revolution	0	+ 1	$-\dfrac{T_6}{T_5}$	$-\dfrac{T_6}{T_5} \times \dfrac{T_5}{T_4} = -\dfrac{T_6}{T_4}$
2.	Arm fixed, wheel 6 rotated through $+ x_1$ revolutions	0	x_1	$-x_1 \times \dfrac{T_6}{T_5}$	$-x_1 \times \dfrac{T_6}{T_4}$
3.	Add $+ y_1$ revolutions to all elements	$+ y_1$	$+ y_1$	$+ y_1$	$+ y_1$
4.	Total motion	$+ y_1$	$x_1 + y_1$	$y_1 - x_1 \times \dfrac{T_6}{T_5}$	$y_1 - x_1 \times \dfrac{T_6}{T_4}$

We know that speed of arm E = Speed of wheel 2 in the first train

$$\therefore \qquad y_1 = -2 \qquad \ldots(iii)$$

Also speed of wheel 4 = Speed of arm or casing C in the first train

$$\therefore \qquad y_1 - x_1 \times \frac{T_6}{T_4} = 0.4 \quad \text{or} \quad -2 - x_1 \times \frac{20}{100} = 0.4 \qquad \ldots(iv)$$

or $$x_1 = (-2 - 0.4)\,\frac{100}{20} = -12$$

∴ Speed of wheel 6 (or *F*)

$$= x_1 + y_1 = -12 - 2 = -14 \text{ r.p.s.} = 14 \text{ r.p.s. (clockwise)}$$

Now consider the train of wheels 6 and 7 (both on *F*), annular wheel 8 (on *B*) and the arm *i.e.* casing *C*. The table of motions is given below :

Table 13.16. Table of motions.

Step No.	Conditions of motion	Revolutions of elements		
		Arm	Wheel 8	Wheel 7
1.	Arm fixed, wheel 8 rotated through + 1 revolution	0	+ 1	$\frac{T_8}{T_7}$
2.	Arm fixed, wheel 8 rotated through + x_2 revolutions	0	$+ x_2$	$+ x_2 \times \frac{T_8}{T_7}$
3.	Add + y_2 revolutions to all elements	$+ y_2$	$+ y_2$	$+ y_2$
4.	Total motion	y_2	$x_2 + y_2$	$y_2 + x_2 \times \frac{T_8}{T_7}$

We know that the speed of *C* in the first train is 0.4 r.p.s., therefore

$$y_2 = 0.4 \qquad ...(v)$$

Also the speed of wheel 7 is equal to the speed of *F* or wheel 6 in the second train, therefore

$$y_2 + x_2 \times \frac{T_8}{T_7} = -14 \quad \text{or} \quad 0.4 + x_2 \times \frac{160}{66} = -14 \qquad ...(vi)$$

$$\therefore \qquad x_2 = (-14 - 0.4)\frac{66}{160} = -5.94$$

∴ Speed of wheel 8 or of the shaft *B*

$$x_2 + y_2 = -5.94 + 0.4 = -5.54 \text{ r.p.s.} = 5.54 \text{ r.p.s. (clockwise)}$$

We have already assumed that the speed of wheel 1 or the shaft *A* is 1 r.p.s. anticlockwise

∴ Velocity ratio of the output shaft *B* to the input shaft *A*

$$= -5.54 \text{ **Ans.**}$$

Note : The – ve sign shows that the two shafts *A* and *B* rotate in opposite directions.

13.10. Epicyclic Gear Train with Bevel Gears

The bevel gears are used to make a more compact epicyclic system and they permit a very high speed reduction with few gears. The useful application of the epicyclic gear train with bevel gears is found in Humpage's speed reduction gear and differential gear of an automobile as discussed below :

1. *Humpage's speed reduction gear.* The Humpage's speed reduction gear was originally designed as a substitute for back gearing of a lathe, but its use is now considerably extended to all kinds of workshop machines and also in electrical machinery. In Humpage's speed reduction gear, as shown in Fig. 13.20, the driving shaft *X* and the driven shaft *Y* are co-axial. The driving shaft carries a bevel gear *A* and driven shaft carries a bevel gear *E*. The bevel gear *B* meshes with gear *A* (also known as pinion) and a fixed gear *C*. The gear *E* meshes with gear *D* which is compound with gear *B*.

This compound gear B-D is mounted on the arm or spindle F which is rigidly connected with a hollow sleeve G. The sleeve revolves freely loose on the axes of the driving and driven shafts.

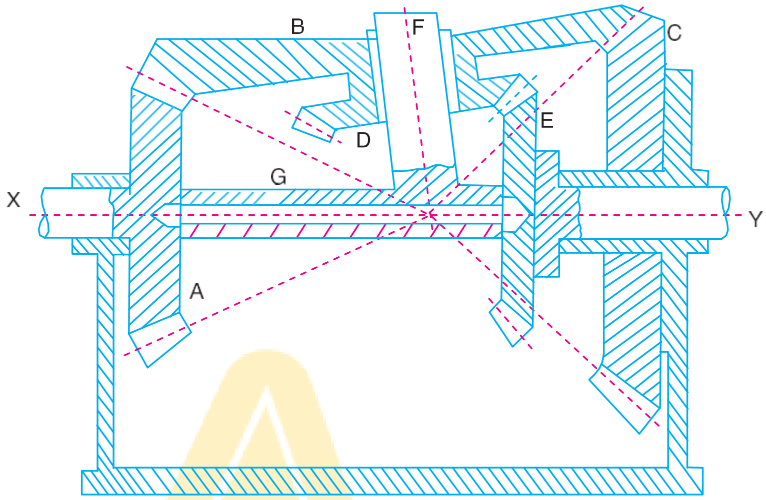


Fig. 13.20. Humpage's speed reduction gear.

2. *Differential gear of an automobile.* The differential gear used in the rear drive of an automobile is shown in Fig. 13.21. Its function is

(a) to transmit motion from the engine shaft to the rear driving wheels, and

(b) to rotate the rear wheels at different speeds while the automobile is taking a turn.

As long as the automobile is running on a straight path, the rear wheels are driven directly by the engine and speed of both the wheels is same. But when the automobile is taking a turn, the outer wheel will run faster than the * inner wheel because at that time the outer rear wheel has to cover more distance than the inner rear wheel. This is achieved by epicyclic gear train with bevel gears as shown in Fig. 13.21.

The bevel gear A (known as pinion) is keyed to the propeller shaft driven from the engine shaft through universal coupling. This gear A drives the gear B (known as crown gear) which rotates freely on the axle P. Two equal gears C and D are mounted on two separate parts P and Q of the rear axles respectively. These gears, in turn, mesh with equal pinions E and F which can rotate freely on the spindle provided on the arm attached to gear B.

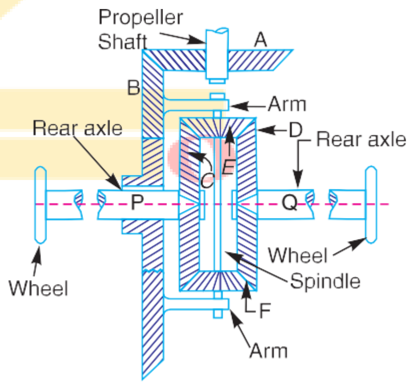


Fig. 13.21. Differential gear of an automobile.

When the automobile runs on a straight path, the gears C and D must rotate together. These gears are rotated through the spindle on the gear B. The gears E and F do not rotate on the spindle. But when the automobile is taking a turn, the inner rear wheel should have lesser speed than the outer rear wheel and due to relative speed of the inner and outer gears D and C, the gears E and F start rotating about the spindle axis and at the same time revolve about the axle axis.

Due to this epicyclic effect, the speed of the inner rear wheel decreases by a certain amount and the speed of the outer rear wheel increases, by the same amount. This may be well understood by drawing the table of motions as follows :

* This difficulty does not arise with the front wheels as they are greatly used for steering purposes and are mounted on separate axles and can run freely at different speeds.

Table 13.17. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Gear B	Gear C	Gear E	Gear D
1.	Gear *B* fixed-Gear *C* rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	$\frac{T_C}{T_E}$	$-\frac{T_C}{T_E}\times\frac{T_E}{T_D}=-1$ $(\because T_C = T_D)$
2.	Gear *B* fixed-Gear *C* rotated through + *x* revolutions	0	+ *x*	$+x\times\frac{T_C}{T_E}$	$-x$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y+x\times\frac{T_C}{T_E}$	$y-x$

From the table, we see that when the gear *B*, which derives motion from the engine shaft, rotates at *y* revolutions, then the speed of inner gear *D* (or the rear axle *Q*) is less than *y* by *x* revolutions and the speed of the outer gear *C* (or the rear axle *P*) is greater than *y* by *x* revolutions. In other words, the two parts of the rear axle and thus the two wheels rotate at two different speeds. We also see from the table that the speed of gear *B* is the mean of speeds of the gears *C* and *D*.

Example 13.16. *Two bevel gears A and B (having 40 teeth and 30 teeth) are rigidly mounted on two co-axial shafts X and Y. A bevel gear C (having 50 teeth) meshes with A and B and rotates freely on one end of an arm. At the other end of the arm is welded a sleeve and the sleeve is riding freely loose on the axes of the shafts X and Y. Sketch the arrangement.*

If the shaft X rotates at 100 r.p.m. clockwise and arm rotates at 100 r.p.m.anitclockwise, find the speed of shaft Y.

Solution. Given : $T_A = 40$; $T_B = 30$; $T_C = 50$; $N_X = N_A = 100$ r.p.m. (clockwise) ; Speed of arm = 100 r.p.m. (anticlockwise)

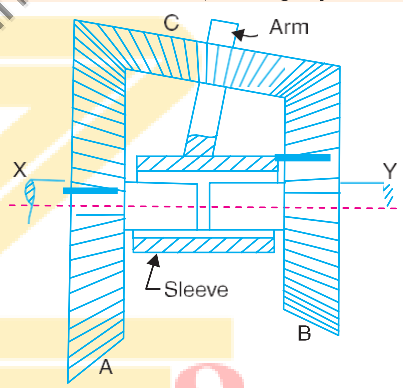


Fig. 13.22

The arangement is shown in Fig. 13.22.

The table of motions is drawn as below :

Table 13.18. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Gear C	Gear B
1.	Arm *B* fixed, gear *A* rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	* $\pm\frac{T_A}{T_C}$	$-\frac{T_A}{T_C}\times\frac{T_C}{T_B}=-\frac{T_A}{T_B}$
2.	Arm *B* fixed, gear *A* rotated through + *x* revolutions	0	+ *x*	$\pm x\times\frac{T_A}{T_C}$	$-x\times\frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y\pm x\times\frac{T_A}{T_C}$	$y-x\times\frac{T_A}{T_B}$

* The ± sign is given to the motion of the wheel *C* because it is in a different plane. So we cannot indicate the direction of its motion specifically, *i.e.* either clockwise or anticlockwise.

Since the speed of the arm is 100 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +100$$

Also, the speed of the driving shaft X or gear A is 100 r.p.m. clockwise.

$$\therefore \quad x + y = -100 \quad \text{or} \quad x = -y - 100 = -100 - 100 = -200$$

$\therefore$ Speed of the driven shaft *i.e.* shaft Y,

$$N_Y = \text{Speed of gear } B = y - x \times \frac{T_A}{T_B} = 100 - \left(-200 \times \frac{40}{30}\right)$$

$$= +366.7 \text{ r.p.m.} = 366.7 \text{ r.p.m. (anticlockwise)} \textbf{ Ans.}$$

Example 13.17. *In a gear train, as shown in Fig. 13.23, gear B is connected to the input shaft and gear F is connected to the output shaft. The arm A carrying the compound wheels D and E, turns freely on the output shaft. If the input speed is 1000 r.p.m. counter- clockwise when seen from the right, determine the speed of the output shaft under the following conditions :*

1. *When gear C is fixed, and* ***2.*** *when gear C is rotated at 10 r.p.m. counter clockwise.*

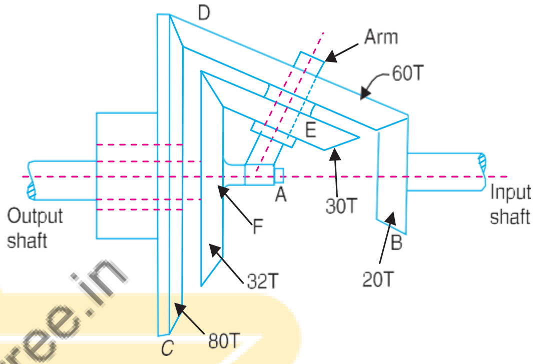


Fig. 13.23

Solution. Given : $T_B = 20$; $T_C = 80$; $T_D = 60$; $T_E = 30$; $T_F = 32$; $N_B = 1000$ r.p.m. (counter-clockwise)

The table of motions is given below :

Table 13.19. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*				
		Arm A	*Gear B (or input shaft)*	*Compound wheel D-E*	*Gear C*	*Gear F (or output shaft)*
1.	Arm fixed, gear B rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	$\frac{T_B}{T_D}$	$-\frac{T_B}{T_D} \times \frac{T_D}{T_C} = -\frac{T_B}{T_C}$	$-\frac{T_B}{T_D} \times \frac{T_E}{T_F}$
2.	Arm fixed, gear B rotated through + x revolutions	0	$+x$	$+x \times \frac{T_B}{T_D}$	$x \times \frac{T_B}{T_C}$	$-x \times \frac{T_B}{T_D} \times \frac{T_E}{T_F}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y + x \times \frac{T_B}{T_D}$	$y - x \times \frac{T_B}{T_C}$	$y - x \times \frac{T_B}{T_D} \times \frac{T_E}{T_F}$

1. Speed of the output shaft when gear C is fixed

Since the gear *C* is fixed, therefore from the fourth row of the table,

$$y - x \times \frac{T_B}{T_C} = 0 \quad \text{or} \quad y - x \times \frac{20}{80} = 0$$

$$\therefore \qquad y - 0.25\,x = 0 \qquad ...(i)$$

We know that the input speed (or the speed of gear *B*) is 1000 r.p.m. counter clockwise, therefore from the fourth row of the table,

$$x + y = +1000 \qquad ...(ii)$$

From equations **(*i*)** and **(*ii*),** $x = +800$, and $y = +200$

$$\therefore \text{ Speed of output shaft } = \text{Speed of gear } F = y - x \times \frac{T_B}{T_D} \times \frac{T_E}{T_F}$$

$$= 200 - 800 \times \frac{20}{80} \times \frac{30}{32} = 200 - 187.5 = 12.5 \text{ r.p.m.}$$

= 12.5 r.p.m. (counter clockwise) **Ans.**

2. Speed of the output shaft when gear C is rotated at 10 r.p.m. counter clockwise

Since the gear *C* is rotated at 10 r.p.m. counter clockwise, therefore from the fourth row of the table,

$$y - x \times \frac{T_B}{T_C} = +10 \quad \text{or} \quad y - x \times \frac{20}{80} = 10$$

$$\therefore \qquad y - 0.25\,x = 10 \qquad ...(iii)$$

From equations **(*ii*)** and **(*iii*),**

$$x = 792, \quad \text{and} \quad y = 208$$

$\therefore$ Speed of output shaft

$$= \text{Speed of gear } F = y - x \times \frac{T_B}{T_D} \times \frac{T_E}{T_F} = 208 - 792 \times \frac{20}{80} \times \frac{30}{32}$$

$$= 208 - 185.6 = 22.4 \text{ r.p.m.} = 22.4 \text{ r.p.m. (counter clockwise)} \textbf{ Ans.}$$

Example 13.18. *Fig. 13.24 shows a differential gear used in a motor car. The pinion A on the propeller shaft has 12 teeth and gears with the crown gear B which has 60 teeth. The shafts P and Q form the rear axles to which the road wheels are attached. If the propeller shaft rotates at 1000 r.p.m. and the road wheel attached to axle Q has a speed of 210 r.p.m. while taking a turn, find the speed of road wheel attached to axle P.*

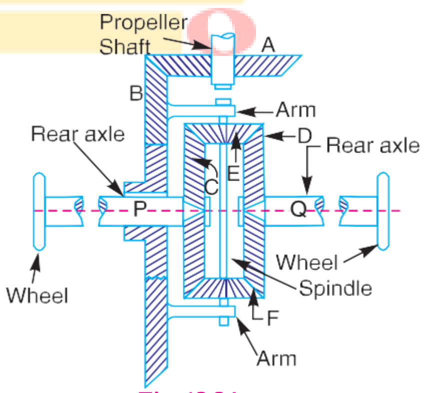


Fig. 13.24

Solution. Given : $T_A = 12$; $T_B = 60$; $N_A = 1000$ r.p.m. ; $N_Q = N_D = 210$ r.p.m.

Since the propeller shaft or the pinion *A* rotates at 1000 r.p.m., therefore speed of crown gear *B*,

$$N_B = N_A \times \frac{T_A}{T_B} = 1000 \times \frac{12}{60}$$

$$= 200 \text{ r.p.m.}$$

The table of motions is given below :

Table 13.20. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Gear B	Gear C	Gear E	Gear D
1.	Gear *B* fixed-Gear *C* rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	$\frac{T_C}{T_E}$	$-\frac{T_C}{T_E}\times\frac{T_E}{T_D}=-1$ $(\because T_C = T_D)$
2.	Gear *B* fixed-Gear *C* rotated through + *x* revolutions	0	+ *x*	$+x\times\frac{T_C}{T_E}$	– *x*
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y+x\times\frac{T_C}{T_E}$	*y* – *x*

Since the speed of gear *B* is 200 r.p.m., therefore from the fourth row of the table,

$$y = 200 \qquad \ldots(i)$$

Also, the speed of road wheel attached to axle *Q* or the speed of gear *D* is 210 r.p.m., therefore from the fourth row of the table,

$$y - x = 210 \quad \text{or} \quad x = y - 210 = 200 - 210 = -10$$

∴ Speed of road wheel attached to axle *P*

$$= \text{Speed of gear } C = x + y$$

$$= -10 + 200 = 190 \text{ r.p.m. } \textbf{Ans.}$$

13.11. Torques in Epicyclic Gear Trains

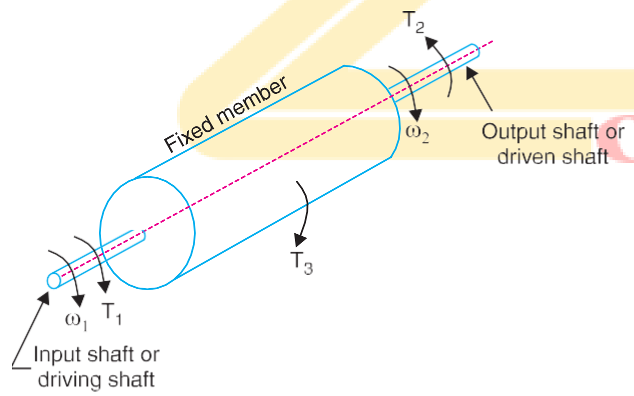


Fig. 13.25. Torques in epicyclic gear trains.

When the rotating parts of an epicyclic gear train, as shown in Fig. 13.25, have no angular acceleration, the gear train is kept in equilibrium by the three externally applied torques, *viz.*

1. Input torque on the driving member (T_1),

2. Output torque or resisting or load torque on the driven member (T_2),

3. Holding or braking or fixing torque on the fixed member (T_3).

The net torque applied to the gear train must be zero. In other words,

$$T_1 + T_2 + T_3 = 0 \qquad \text{...}(i)$$

$$\therefore \quad F_1.r_1 + F_2.r_2 + F_3.r_3 = 0 \qquad \text{...}(ii)$$

where F_1, F_2 and F_3 are the corresponding externally applied forces at radii r_1, r_2 and r_3.

Further, if ω_1, ω_2 and ω_3 are the angular speeds of the driving, driven and fixed members respectively, and the friction be neglected, then the net kinetic energy dissipated by the gear train must be zero, *i.e.*

$$T_1.\omega_1 + T_2.\omega_2 + T_3.\omega_3 = 0 \qquad \text{...}(iii)$$

But, for a fixed member, $\omega_3 = 0$

$$\therefore \quad T_1.\omega_1 + T_2.\omega_2 = 0 \qquad \text{...}(iv)$$

Notes : 1. From equations **(*i*)** and **(*iv*),** the holding or braking torque T_3 may be obtained as follows :

$$T_2 = -T_1 \times \frac{\omega_1}{\omega_2} \qquad \text{...[From equation } (iv)]$$

and

$$T_3 = -(T_1 + T_2) \qquad \text{...[From equation } (i)]$$

$$= T_1\left(\frac{\omega_1}{\omega_2} - 1\right) = T_1\left(\frac{N_1}{N_2} - 1\right)$$

2. When input shaft (or driving shaft) and output shaft (or driven shaft) rotate in the same direction, then the input and output torques will be in opposite directions. Similarly, when the input and output shafts rotate in opposite directions, then the input and output torques will be in the same direction.

Example 13.19. *Fig. 13.26 shows an epicyclic gear train. Pinion A has 15 teeth and is rigidly fixed to the motor shaft. The wheel B has 20 teeth and gears with A and also with the annular fixed wheel E. Pinion C has 15 teeth and is integral with B (B, C being a compound gear wheel). Gear C meshes with annular wheel D, which is keyed to the machine shaft. The arm rotates about the same shaft on which A is fixed and carries the compound wheel B, C. If the motor runs at 1000 r.p.m., find the speed of the machine shaft. Find the torque exerted on the machine shaft, if the motor develops a torque of 100 N-m.*

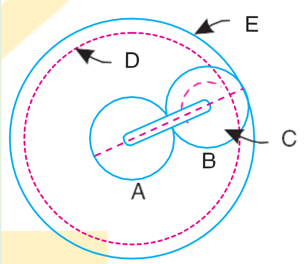


Fig. 13.26

Solution. Given : $T_A = 15$; $T_B = 20$; $T_C = 15$; $N_A = 1000$ r.p.m.; Torque developed by motor (or pinion *A*) = 100 N-m

First of all, let us find the number of teeth on wheels *D* and *E*. Let T_D and T_E be the number of teeth on wheels *D* and *E* respectively. Let d_A, d_B, d_C, d_D and d_E be the pitch circle diameters of wheels *A*, *B*, *C*, *D* and *E* respectively. From the geometry of the figure,

$$d_E = d_A + 2\,d_B \text{ and } d_D = d_E - (d_B - d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_E = T_A + 2\,T_B = 15 + 2 \times 20 = 55$$

and

$$T_D = T_E - (T_B - T_C) = 55 - (20 - 15) = 50$$

Speed of the machine shaft

The table of motions is given below :

Table 13.21. Table of motions.

Step No.	Conditions of motion	Revolutions of elements				
		Arm	*Pinion A*	*Compound wheel B-C*	*Wheel D*	*Wheel E*
1.	Arm fixed-pinion *A* rotated through + 1 revolution (anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B} \times \frac{T_C}{T_D}$	$-\frac{T_A}{T_B} \times \frac{T_B}{T_E} = -\frac{T_A}{T_E}$
2.	Arm fixed-pinion *A* rotated through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$	$-x \times \frac{T_A}{T_B} \times \frac{T_C}{T_D}$	$-x \times \frac{T_A}{T_E}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$	$y - x \times \frac{T_A}{T_B} \times \frac{T_C}{T_D}$	$y - x \times \frac{T_A}{T_E}$

We know that the speed of the motor or the speed of the pinion *A* is 1000 r.p.m. Therefore

$$x + y = 1000 \qquad \ldots(i)$$

Also, the annular wheel *E* is fixed, therefore

$$y - x \times \frac{T_A}{T_E} = 0 \quad \text{or} \quad y = x \times \frac{T_A}{T_E} = x \times \frac{15}{55} = 0.273\,x \qquad \ldots(ii)$$

From equations (*i*) and (*ii*),

$$x = 786 \quad \text{and} \quad y = 214$$

∴ Speed of machine shaft = Speed of wheel *D*,

$$N_D = y - x \times \frac{T_A}{T_B} \times \frac{T_C}{T_D} = 214 - 786 \times \frac{15}{20} \times \frac{15}{50} = +\,37.15 \text{ r.p.m.}$$

$$= 37.15 \text{ r.p.m. (anticlockwise)} \quad \textbf{Ans.}$$

Torque exerted on the machine shaft

We know that

Torque developed by motor × Angular speed of motor

= Torque exerted on machine shaft × Angular speed of machine shaft

or $\qquad 100 \times \omega_A$ = Torque exerted on machine shaft × ω_D

∴ Torque exerted on machine shaft

$$= 100 \times \frac{\omega_A}{\omega_D} = 100 \times \frac{N_A}{N_D} = 100 \times \frac{1000}{37.15} = 2692 \text{ N-m} \quad \textbf{Ans.}$$

Example 13.20. *An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star- shaped planet carrier C. The size of different toothed wheels are such that the planet carrier C rotates at 1/5th of the speed of the sunwheel S. The minimum number of teeth on any wheel is 16. The driving torque on the sun wheel is 100 N-m. Determine :* **1.** *number of teeth on different wheels of the train, and* **2.** *torque necessary to keep the internal gear stationary.*

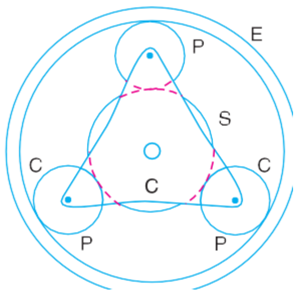


Fig. 13.27

Solution. Given : $N_C = \dfrac{N_S}{5}$

1. Number of teeth on different wheels

The arrangement of the epicyclic gear train is shown in Fig. 13.27. Let T_S and T_E be the number of teeth on the sun wheel *S* and the internal gear *E* respectively. The table of motions is given below :

Table 13.22. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Planet carrier C	*Sun wheel S*	*Planet wheel P*	*Internal gear E*
1.	Planet carrier *C* fixed, sunwheel *S* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\dfrac{T_S}{T_P}$	$-\dfrac{T_S}{T_P}\times\dfrac{T_P}{T_E}=-\dfrac{T_S}{T_E}$
2.	Planet carrier *C* fixed, sunwheel *S* rotates through + *x* revolutions	0	+ *x*	$-x\times\dfrac{T_S}{T_P}$	$-x\times\dfrac{T_S}{T_E}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y-x\times\dfrac{T_S}{T_P}$	$y-x\times\dfrac{T_S}{T_E}$

We know that when the sunwheel *S* makes 5 revolutions, the planet carrier *C* makes 1 revolution. Therefore from the fourth row of the table,

$$y = 1, \quad \text{and} \quad x + y = 5 \quad \text{or} \quad x = 5 - y = 5 - 1 = 4$$

Since the gear *E* is stationary, therefore from the fourth row of the table,

$$y - x \times \frac{T_S}{T_E} = 0 \quad \text{or} \quad 1 - 4 \times \frac{T_S}{T_E} = 0 \quad \text{or} \quad \frac{T_S}{T_E} = \frac{1}{4}$$

$\therefore \qquad T_E = 4T_S$

Since the minimum number of teeth on any wheel is 16, therefore let us take the number of teeth on sunwheel, $\qquad T_S = 16$

$\therefore \qquad T_E = 4\,T_S = 64$ **Ans.**

Let d_S, d_P and d_E be the pitch circle diameters of wheels *S*, *P* and *E* respectively. Now from the geometry of Fig. 13.27,

$$d_S + 2\,d_P = d_E$$

Assuming the module of all the gears to be same, the number of teeth are proportional to their pitch circle diameters.

$$T_S + 2\,T_P = T_E \quad \text{or} \quad 16 + 2\,T_P = 64 \quad \text{or} \quad T_P = 24 \text{ Ans.}$$

2. Torque necessary to keep the internal gear stationary

We know that

Torque on S × Angular speed of S

= Torque on C × Angular speed of C

$$100 \times \omega_S = \text{Torque on } C \times \omega_C$$

$$\therefore \quad \text{Torque on } C = 100 \times \frac{\omega_S}{\omega_C} = 100 \times \frac{N_S}{N_C} = 100 \times 5 = 500 \text{ N-m}$$

∴ Torque necessary to keep the internal gear stationary

= 500 – 100 = 400 N-m **Ans.**

Example 13.21. *In the epicyclic gear train, as shown in Fig. 13.28, the driving gear A rotating in clockwise direction has 14 teeth and the fixed annular gear C has 100 teeth. The ratio of teeth in gears E and D is 98 : 41. If 1.85 kW is supplied to the gear A rotating at 1200 r.p.m., find :* **1.** *the speed and direction of rotation of gear E, and* **2.** *the fixing torque required at C, assuming 100 per cent efficiency throughout and that all teeth have the same pitch.*

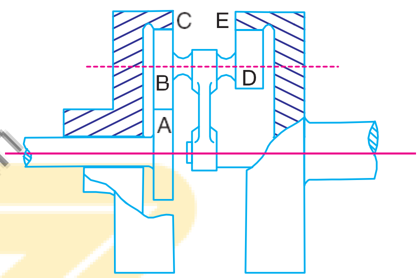


Fig. 13.28

Solution. Given : $T_A = 14$; $T_C = 100$; $T_E / T_D = 98 / 41$; $P_A = 1.85$ kW = 1850 W ; $N_A = 1200$ r.p.m.

Let d_A, d_B and d_C be the pitch circle diameters of gears A , B and C respectively. From Fig. 13.28,

$$d_A + 2\,d_B = d_C$$

Gears are extensively used in trains for power transmission.

Since teeth of all gears have the same pitch and the number of teeth are proportional to their pitch circle diameters, therefore

$$T_A + 2T_B = T_C \quad \text{or} \quad T_B = \frac{T_C - T_A}{2} = \frac{100 - 14}{2} = 43$$

The table of motions is now drawn as below :

Table 13.23. Table of motions.

Step No.	Conditions of motion	Revolutions of elements				
		Arm	*Gear A*	*Compound gear B-D*	*Gear C*	*Gear E*
1.	Arm fixed-Gear *A* rotated through – 1 revolution (*i.e.* 1 revolution clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B} \times \frac{T_B}{T_C} = +\frac{T_A}{T_C}$	$+\frac{T_A}{T_B} \times \frac{T_D}{T_E}$
2.	Arm fixed-Gear *A* rotated through – *x* revolutions	0	– *x*	$+x \times \frac{T_A}{T_B}$	$+x \times \frac{T_A}{T_C}$	$+x \times \frac{T_A}{T_B} \times \frac{T_D}{T_E}$
3.	Add – *y* revolutions to all elements	– *y*	– *y*	– *y*	– *y*	– *y*
4.	Total motion	– *y*	– *y* – *x*	$-y + x \times \frac{T_A}{T_B}$	$-y + x \times \frac{T_A}{T_C}$	$-y + x \times \frac{T_A}{T_B} \times \frac{T_D}{T_E}$

Since the annular gear *C* is fixed, therefore from the fourth row of the table,

$$-y + x \times \frac{T_A}{T_C} = 0 \quad \text{or} \quad -y + x \times \frac{14}{100} = 0$$

$$\therefore \qquad -y + 0.14\,x = 0 \qquad ...(i)$$

Also, the gear *A* is rotating at 1200 r.p.m., therefore

$$-x - y = 1200 \qquad ...(ii)$$

From equations *(i)* and *(ii)*, $x = -1052.6$, and $y = -147.4$

1. Speed and direction of rotation of gear E

From the fourth row of the table, speed of gear *E*,

$$N_E = -y + x \times \frac{T_A}{T_B} \times \frac{T_D}{T_E} = 147.4 - 1052.6 \times \frac{14}{43} \times \frac{41}{98}$$

$$= 147.4 - 143.4 = 4 \text{ r.p.m.}$$

$$= 4 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

2. Fixing torque required at C

We know that torque on $A = \frac{P_A \times 60}{2\pi N_A} = \frac{1850 \times 60}{2\pi \times 1200} = 14.7$ N-m

Since the efficiency is 100 per cent throughout, therefore the power available at *E* (P_E) will be equal to power supplied at *A* (P_A).

$\therefore$ Torque on E $= \dfrac{P_A \times 60}{2\pi \times N_E} = \dfrac{1850 \times 60}{2\pi \times 4} = 4416$ N-m

$\therefore$ Fixing torque required at C

$= 4416 - 14.7 = 4401.3$ N-m **Ans.**

Example 13.22. *An over drive for a vehicle consists of an epicyclic gear train, as shown in Fig. 13.29, with compound planets B-C. B has 15 teeth and meshes with an annulus A which has 60 teeth. C has 20 teeth and meshes with the sunwheel D which is fixed. The annulus is keyed to the propeller shaft Y which rotates at 740 rad /s. The spider which carries the pins upon which the planets revolve, is driven directly from main gear box by shaft X, this shaft being relatively free to rotate with respect to wheel D. Find the speed of shaft X, when all the teeth have the same module.*

When the engine develops 130 kW, what is the holding torque on the wheel D ? Assume 100 per cent efficiency throughout.

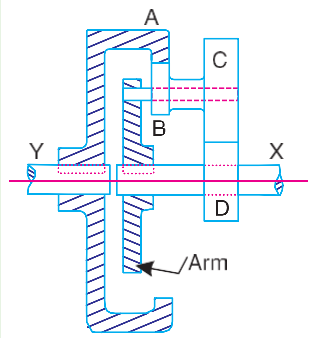


Fig. 13.29

Solution. Given : $T_B = 15$; $T_A = 60$; $T_C = 20$; $\omega_Y = \omega_A = 740$ rad /s ; $P = 130$ kW $= 130 \times 10^3$ W

First of all, let us find the number of teeth on the sunwheel D (T_D). Let d_A, d_B, d_C and d_D be the pitch circle diameters of wheels A, B, C and D respectively. From Fig. 13.29,

$$\frac{d_D}{2} + \frac{d_C}{2} + \frac{d_B}{2} = \frac{d_A}{2} \quad \text{or} \quad d_D + d_C + d_B = d_A$$

Since the module is same for all teeth and the number of teeth are proportional to their pitch circle diameters, therefore

$$T_D + T_C + T_B = T_A \quad \text{or} \quad T_D = T_A - (T_C + T_B) = 60 - (20 + 15) = 25$$

The table of motions is given below :

Table 13.24. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm (or shaft X)	Wheel D	Compound wheel C-B	Wheel A (or shaft Y)
1.	Arm fixed-wheel D rotated through + 1 revolution (anticlockwise)	0	+ 1	$-\dfrac{T_D}{T_C}$	$-\dfrac{T_D}{T_C} \times \dfrac{T_B}{T_A}$
2.	Arm fixed-wheel D rotated through + x revolutions	0	$+x$	$-x \times \dfrac{T_D}{T_C}$	$-x \times \dfrac{T_D}{T_C} \times \dfrac{T_B}{T_A}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x + y$	$y - x \times \dfrac{T_D}{T_C}$	$y - x \times \dfrac{T_D}{T_C} \times \dfrac{T_B}{T_A}$

Since the shaft Y or wheel A rotates at 740 rad/s, therefore

$$y - x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A} = 740 \quad \text{or} \quad y - x \times \frac{25}{20} \times \frac{15}{60} = 740$$

$$y - 0.3125\, x = 740 \qquad \text{...}(i)$$

Also the wheel D is fixed, therefore

$$x + y = 0 \quad \text{or} \quad y = -x \qquad ...(ii)$$

From equations **(*i*)** and **(*ii*),**

$$x = -563.8 \quad \text{and} \quad y = 563.8$$

Speed of shaft X

Since the shaft X will make the same number of revolutions as the arm, therefore

Speed of shaft X, ω_X = Speed of arm = y = 563.8 rad/s **Ans.**

Holding torque on wheel D

We know that torque on $A = P/\omega_A = 130 \times 10^3 / 740 = 175.7$ N-m

and Torque on $X = P/\omega_X = 130 \times 10^3/563.8 = 230.6$ N-m

$\therefore$ Holding torque on wheel D

$= 230.6 - 175.7 = 54.9$ N-m **Ans.**

Example 13.23. *Fig. 13.30 shows some details of a compound epicyclic gear drive where I is the driving or input shaft and O is the driven or output shaft which carries two arms A and B rigidly fixed to it. The arms carry planet wheels which mesh with annular wheels P and Q and the sunwheels X and Y. The sun wheel X is a part of Q. Wheels Y and Z are fixed to the shaft I. Z engages with a planet wheel carried on Q and this planet wheel engages the fixed annular wheel R. The numbers of teeth on the wheels are :*

P = 114, Q = 120, R = 120, X = 36, Y = 24 and Z = 30.

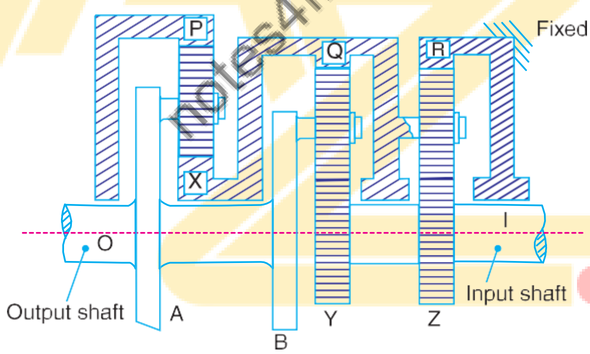


Fig. 13.30.

The driving shaft I makes 1500 r.p.m.clockwise looking from our right and the input at I is 7.5 kW.

1. *Find the speed and direction of rotation of the driven shaft O and the wheel P.*

2. *If the mechanical efficiency of the drive is 80%, find the torque tending to rotate the fixed wheel R.*

Solution. Given : $T_P = 144$; $T_Q = 120$; $T_R = 120$; $T_X = 36$; $T_Y = 24$; $T_Z = 30$; $N_I = 1500$ r.p.m. (clockwise) ; P = 7.5 kW = 7500 W ; η = 80% = 0.8

First of all, consider the train of wheels Z,R and Q (arm). The revolutions of various wheels are shown in the following table.

Table 13.25. Table of motions.

Step No.	Conditions of motion	*Revolutions of elements*		
		Q (Arm)	*Z (also I)*	*R (Fixed)*
1.	Arm fixed-wheel *Z* rotates through + 1 revolution (anticlockwise)	0	+ 1	$-\frac{T_Z}{T_R}$
2.	Arm fixed-wheel *Z* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_Z}{T_R}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_Z}{T_R}$

Since the driving shaft *I* as well as wheel *Z* rotates at 1500 r.p.m. clockwise, therefore

$$x + y = -1500 \qquad \text{...}(i)$$

Also, the wheel *R* is fixed. Therefore

$$y - x \times \frac{T_Z}{T_R} = 0 \quad \text{or} \quad y = x \times \frac{T_Z}{T_R} = x \times \frac{30}{120} = 0.25\,x \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*),

$$x = -1200, \quad \text{and} \quad y = -300$$

Now consider the train of wheels *Y* , *Q*, arm *A* , wheels *P* and *X* . The revolutions of various elements are shown in the following table.

Table 13.26. Table of motions.

Step No.	Conditions of motion	*Revolutions of elements*			
		Arm A, B and Shaft O	*Wheel Y*	*Compound wheel Q-X*	*Wheel P*
1.	Arm *A* fixed-wheel *Y* rotates through + 1 revolution (anticlockwise)	0	+ 1	$-\frac{T_Y}{T_Q}$	$+\frac{T_Y}{T_Q} \times \frac{T_X}{T_P}$
2.	Arm *A* fixed-wheel *Y* rotates through + x_1 revolutions	0	+ x_1	$-x_1 \times \frac{T_Y}{T_Q}$	$+x_1 \times \frac{T_Y}{T_Q} \times \frac{T_X}{T_P}$
3.	Add + y_1 revolutions to all elements	+ y_1	+ y_1	+ y_1	+ y_1
4.	Total motion	+ y_1	$x_1 + y_1$	$y_1 - x_1 \times \frac{T_Y}{T_Q}$	$y_1 + x_1 \times \frac{T_Y}{T_Q} \times \frac{T_X}{T_P}$

Since the speed of compound wheel *Q*-*X* is same as that of *Q*, therefore

$$y_1 - x_1 \times \frac{T_Y}{T_Q} = y = -300$$

or

$$y_1 - x_1 \times \frac{24}{120} = -300$$

$\therefore \qquad y_1 = 0.2\,x_1 - 300$ ...(*iii*)

Also Speed of wheel *Y* = Speed of wheel *Z* or shaft *I*

$\therefore \qquad x_1 + y_1 = x + y = -1500$ ...(*iv*)

$x_1 + 0.2\,x_1 - 300 = -1500$ ...[From equation (*iii*)]

$1.2\,x_1 = -1500 + 300 = -1200$

or $\qquad x_1 = -1200/1.2 = -1000$

and $\qquad y_1 = -1500 - x_1 = -1500 + 1000 = -500$

1. Speed and direction of the driven shaft O and the wheel P

Speed of the driven shaft *O*,

$$N_O = y_1 = -500 = 500 \text{ r.p.m. clockwise}$$ **Ans.**

and Speed of the wheel *P*, $N_P = y_1 + x_1 \times \dfrac{T_Y}{T_Q} \times \dfrac{T_X}{T_P} = -500 - 1000 \times \dfrac{24}{120} \times \dfrac{36}{144}$

$$= -550 = 550 \text{ r.p.m. clockwise}$$ **Ans.**

2. Torque tending to rotate the fixed wheel R

We know that the torque on shaft *I* or input torque

$$T_1 = \frac{P \times 60}{2\pi \times N_1} = \frac{7500 \times 60}{2\pi \times 1500} = 47.74 \text{ N-m}$$

and torque on shaft *O* or output torque,

$$T_2 = \frac{\eta \times P \times 60}{2\pi \times N_O} = \frac{0.8 \times 7500 \times 60}{2\pi \times 500} = 114.58 \text{ N-m}$$

Since the input and output shafts rotate in the same direction (*i.e.* clockwise), therefore input and output torques will be in opposite direction.

$\therefore$ Torque tending to rotate the fixed wheel *R*

$$= T_2 - T_1 = 114.58 - 47.74 = 66.84 \text{ N-m}$$ **Ans.**

Example 13.24. *An epicyclic bevel gear train (known as Humpage's reduction gear) is shown in Fig. 13.31. It consists of a fixed wheel C, the driving shaft X and the driven shaft Y. The compound wheel B-D can revolve on a spindle F which can turn freely about the axis X and Y.*

Show that **(*i*)** *if the ratio of tooth numbers* T_B / T_D *is greater than* T_C / T_E*, the wheel E will rotate in the same direction as wheel A, and* **(*ii*)** *if the ratio* T_B / T_D *is less than* T_C / T_E*, the direction of E is reversed.*

If the numbers of teeth on wheels A, B, C, D and E are 34, 120, 150, 38 and 50 respectively and 7.5 kW is put into the shaft X at 500 r.p.m., what is the output torque of the shaft Y, and what are the forces (tangential to the pitch cones) at the contact points between wheels D and E and between wheels B and C, if the module of all wheels is 3.5 mm ?

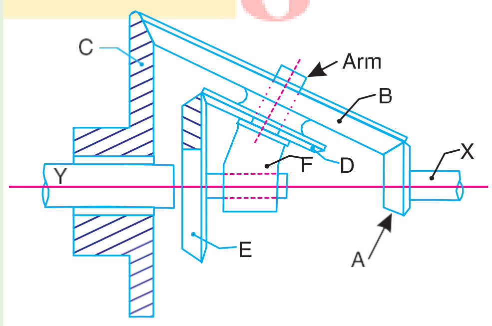


Fig. 13.31

Solution. Given : $T_A = 34$; $T_B = 120$; $T_C = 150$; $T_D = 38$; $T_E = 50$; $P_X = 7.5$ kW = 7500 W ; $N_X = 500$ r.p.m. ; $m = 3.5$ mm

The table of motions is given below :

Table 13.27. Table of motions.

Step No.	Conditions of motion	Revolutions of elements				
		Spindle F	Wheel A (or shaft X)	Compound wheel B-D	Wheel C	Wheel E (or shaft Y)
1.	Spindle fixed, wheel *A* is rotated through + 1 revolution	0	+ 1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B}\times\frac{T_B}{T_C}$ $= -\frac{T_A}{T_C}$	$-\frac{T_A}{T_B}\times\frac{T_D}{T_E}$
2.	Spindle fixed, wheel *A* is rotated through + *x* revolutions	0	+ *x*	$+x\times\frac{T_A}{T_B}$	$-x\times\frac{T_A}{T_C}$	$-x\times\frac{T_A}{T_B}\times\frac{T_D}{T_E}$
3.	Add + *y* revolutions to all elements	+ y	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y+x\times\frac{T_A}{T_B}$	$y-x\times\frac{T_A}{T_C}$	$y-x\times\frac{T_A}{T_B}\times\frac{T_D}{T_E}$

Let us assume that the driving shaft *X* rotates through 1 revolution anticlockwise, therefore the wheel *A* will also rotate through 1 revolution anticlockwise.

∴ $x + y = +1$ or $y = 1 - x$ ...(*i*)

We also know that the wheel *C* is fixed, therefore

$$y - x\times\frac{T_A}{T_C} = 0 \quad \text{or} \quad (1-x) - x\times\frac{T_A}{T_C} = 0 \qquad \text{...[From equation (i)]}$$

$$1 - x\left(1+\frac{T_A}{T_C}\right) = 0 \quad \text{or} \quad x\left(\frac{T_C+T_A}{T_C}\right) = 1$$

and
$$x = \frac{T_C}{T_C+T_A} \qquad \text{...(ii)}$$

From equation (*i*),

$$y = 1 - x = 1 - \frac{T_C}{T_C+T_A} = \frac{T_A}{T_C+T_A} \qquad \text{...(iii)}$$

We know that speed of wheel *E*,

$$N_E = y - x\times\frac{T_A}{T_B}\times\frac{T_D}{T_E} = \frac{T_A}{T_C+T_A} - \frac{T_C}{T_C+T_A}\times\frac{T_A}{T_B}\times\frac{T_D}{T_E}$$

$$= \frac{T_A}{T_C+T_A}\left(1-\frac{T_C}{T_B}\times\frac{T_D}{T_E}\right) \qquad \text{...(iv)}$$

and the speed of wheel *A*,

$$N_A = x + y = +1 \text{ revolution}$$

(*i*) If $\frac{T_B}{T_D} > \frac{T_C}{T_E}$ or $T_B\times T_E > T_C\times T_D$, then the equation (*iv*) will be positive. Therefore the wheel *E* will rotate in the same direction as wheel *A* . **Ans.**

(ii) If $\frac{T_B}{T_D} < \frac{T_C}{T_E}$ or $T_B \times T_E < T_C \times T_D$, then the equation **(*iv*)** will be negative. Therefore the wheel *E* will rotate in the opposite direction as wheel *A*. **Ans.**

Output torque of shaft Y

We know that the speed of the driving shaft *X* (or wheel *A*) or input speed is 500 r.p.m., therefore from the fourth row of the table,

$$x + y = 500 \quad \text{or} \quad y = 500 - x \qquad \text{...(v)}$$

Since the wheel *C* is fixed, therefore

$$y - x \times \frac{T_A}{T_C} = 0 \quad \text{or} \quad (500 - x) - x \times \frac{34}{150} = 0 \qquad \text{...[From equation (v)]}$$

$$\therefore \quad 500 - x - 0.227\,x = 0 \quad \text{or} \quad x = 500/1.227 = 407.5 \text{ r.p.m.}$$

and $$y = 500 - x = 500 - 407.5 = 92.5 \text{ r.p.m.}$$

Since the speed of the driven or output shaft *Y* (*i.e.* N_Y) is equal to the speed of wheel *E* (*i.e.* N_E), therefore

$$N_Y = N_E = y - x \times \frac{T_A}{T_B} \times \frac{T_D}{T_E} = 92.5 - 407.5 \times \frac{34}{120} \times \frac{38}{50}$$

$$= 92.5 - 87.75 = 4.75 \text{ r.p.m.}$$

Assuming 100 per cent efficiency of the gear train, input power P_X is equal to output power (P_Y), *i.e.*

$$P_Y = P_X = 7.5 \text{ kW} = 7500 \text{ W}$$

$\therefore$ Output torque of shaft *Y* ,

$$= \frac{P_Y \times 60}{2\pi\, N_Y} = \frac{7500 \times 60}{2\pi \times 4.75} = 15\,076 \text{ N-m} = 15.076 \text{ kN-m} \quad \textbf{Ans.}$$

Tangential force between wheels D and E

We know that the pitch circle radius of wheel *E*,

$$r_E = \frac{m \times T_E}{2} = \frac{3.5 \times 50}{2} = 87.5 \text{ mm} = 0.0875 \text{ m}$$

$\therefore$ Tangential force between wheels *D* and *E*,

$$= \frac{\text{Torque on wheel } E}{\text{Pitch circle radius of wheel } E} = \frac{15.076}{0.0875} = 172.3 \text{ kN} \quad \textbf{Ans.}$$

...($\therefore$ Torque on wheel *E* = Torque on shaft *Y*)

Tangential force between wheels B and C

We know that the input torque on shaft *X* or on wheel *A*

$$= \frac{P_X \times 60}{2\pi\, N_X} = \frac{7500 \times 60}{2\pi \times 500} = 143 \text{ N-m}$$

$\therefore$ Fixing torque on the fixed wheel *C*

$$= \text{Torque on wheel } E - \text{Torque on wheel } A$$

$$= 15\,076 - 143 = 14\,933 \text{ N-m} = 14.933 \text{ kN-m}$$

Pitch circle radius of wheel *C*,

$$r_C = \frac{m \times T_C}{2} = \frac{3.5 \times 150}{2} = 262.5 \text{ mm} = 0.2625 \text{ m}$$

Tangential force between wheels *B* and *C*

$$= \frac{\text{Fixing torque on wheel } C}{r_C} = \frac{14.933}{0.2625} = 57 \text{ kN}$$ **Ans.**

EXERCISES

1. A compound train consists of six gears. The number of teeth on the gears are as follows :

Gear :	*A*	*B*	*C*	*D*	*E*	*F*
No. of teeth :	60	40	50	25	30	24

The gears *B* and *C* are on one shaft while the gears *D* and *E* are on another shaft. The gear *A* drives gear *B*, gear *C* drives gear *D* and gear *E* drives gear *F*. If the gear *A* transmits 1.5 kW at 100 r.p.m. and the gear train has an efficiency of 80 per cent, find the torque on gear *F*. **[Ans. 30.55 N-m]**

2. Two parallel shafts are to be connected by spur gearing. The approximate distance between the shafts is 600 mm. If one shaft runs at 120 r.p.m. and the other at 360 r.p.m., find the number of teeth on each wheel, if the module is 8 mm. Also determine the exact distance apart of the shafts. **[Ans. 114, 38 ; 608 mm]**

3. In a reverted gear train, as shown in Fig. 13.32, two shafts *A* and *B* are in the same straight line and are geared together through an intermediate parallel shaft *C*. The gears connecting the shafts *A* and *C* have a module of 2 mm and those connecting the shafts *C* and *B* have a module of 4.5 mm. The speed of shaft *A* is to be about but greater than 12 times the speed of shaft *B*, and the ratio at each reduction is same. Find suitable number of teeth for gears. The number of teeth of each gear is to be a minimum but not less than 16. Also find the exact velocity ratio and the distance of shaft *C* from *A* and *B*. **[Ans. 36, 126, 16, 56 ; 12.25 ; 162 mm]**

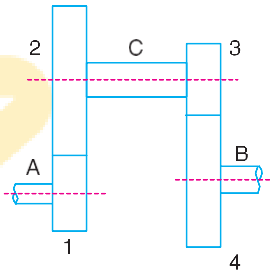


Fig. 13.32

4. In an epicyclic gear train, as shown in Fig.13.33, the number of teeth on wheels *A* , *B* and *C* are 48, 24 and 50 respectively. If the arm rotates at 400 r.p.m., clockwise, find : 1. Speed of wheel *C* when *A* is fixed, and 2. Speed of wheel *A* when *C* is fixed. **[Ans. 16 r.p.m. (clockwise) ; 16.67 (anticlockwise)]**

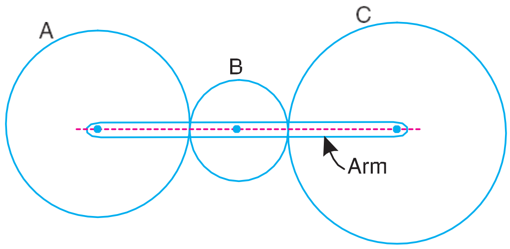


Fig. 13.33

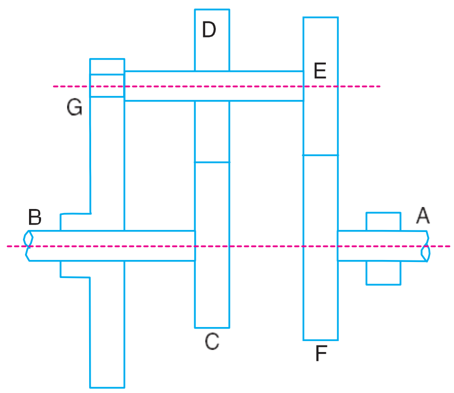


Fig. 13.34

5. In an epicyclic gear train, as shown in Fig. 13.34, the wheel *C* is keyed to the shaft *B* and wheel *F* is keyed to shaft *A*. The wheels *D* and *E* rotate together on a pin fixed to the arm *G*. The number of teeth on wheels *C*, *D*, *E* and *F* are 35, 65, 32 and 68 respectively.

If the shaft *A* rotates at 60 r.p.m. and the shaft *B* rotates at 28 r.p.m. in the opposite direction, find the speed and direction of rotation of arm *G*. **[Ans. 90 r.p.m., in the same direction as shaft *A*]**

6. An epicyclic gear train, as shown in Fig. 13.35, is composed of a fixed annular wheel *A* having 150 teeth. The wheel *A* is meshing with wheel *B* which drives wheel *D* through an idle wheel *C*, *D* being concentric with *A*. The wheels *B* and *C* are carried on an arm which revolves clockwise at 100 r.p.m. about the axis of *A* and *D*. If the wheels *B* and *D* have 25 teeth and 40 teeth respectively, find the number of teeth on *C* and the speed and sense of rotation of *C*. **[Ans. 30 ; 600 r.p.m. clockwise]**

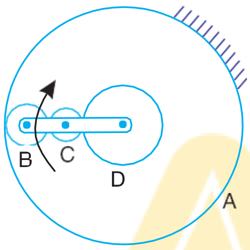


Fig. 13.35

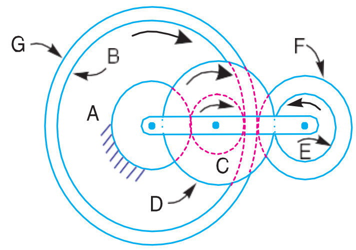


Fig. 13.36

7. Fig. 13.36, shows an epicyclic gear train with the following details :

A has 40 teeth external (fixed gear) ; *B* has 80 teeth internal ; *C* - *D* is a compound wheel having 20 and 50 teeth (external) respectively, *E*-*F* is a compound wheel having 20 and 40 teeth (external) respectively, and *G* has 90 teeth (external).

The arm runs at 100 r.p.m. in clockwise direction. Determine the speeds for gears *C*, *E*, and *B*.

[Ans. 300 r.p.m. clockwise ; 400 r.p.m. anticlockwise ; 150 r.p.m. clockwise]

8. An epicyclic gear train, as shown in Fig. 13.37, has a sun wheel *S* of 30 teeth and two planet wheels *P*-*P* of 50 teeth. The planet wheels mesh with the internal teeth of a fixed annulus *A*. The driving shaft carrying the sunwheel, transmits 4 kW at 300 r.p.m. The driven shaft is connected to an arm which carries the planet wheels. Determine the speed of the driven shaft and the torque transmitted, if the overall efficiency is 95%. **[Ans. 56.3 r.p.m. ; 644.5 N-m]**

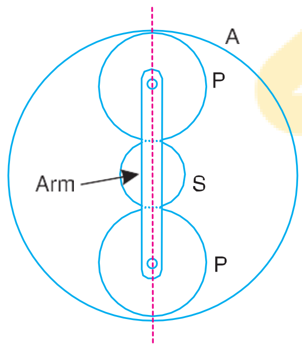


Fig. 13.37

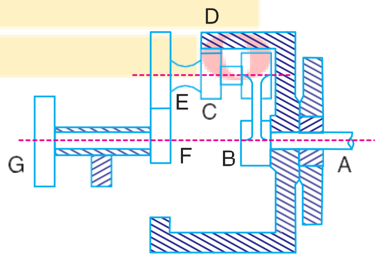


Fig. 13.38

9. An epicyclic reduction gear, as shown in Fig. 13.38, has a shaft *A* fixed to arm *B*. The arm *B* has a pin fixed to its outer end and two gears *C* and *E* which are rigidly fixed, revolve on this pin. Gear *C* meshes with annular wheel *D* and gear *E* with pinion *F*. *G* is the driver pulley and *D* is kept stationary.

The number of teeth are : *D* = 80 ; *C* = 10 ; *E* = 24 and *F* = 18.

If the pulley *G* runs at 200 r.p.m. ; find the speed of shaft *A*.

[Ans. 17.14 r.p.m. in the same direction as that of *G*]

10. A reverted epicyclic gear train for a hoist block is shown in Fig. 13.39. The arm *E* is keyed to the same shaft as the load drum and the wheel *A* is keyed to a second shaft which carries a chain wheel, the chain being operated by hand. The two shafts have common axis but can rotate independently. The wheels *B* and *C* are compound and rotate together on a pin carried at the end of arm *E*. The wheel *D* has internal teeth and is fixed to the outer casing of the block so that it does not rotate.

The wheels *A* and *B* have 16 and 36 teeth respectively with a module of 3 mm. The wheels *C* and *D* have a module of 4 mm. Find : 1. the number of teeth on wheels *C* and *D* when the speed of *A* is ten times the speed of arm *E*, both rotating in the same sense, and 2. the speed of wheel *D* when the wheel *A* is fixed and the arm *E* rotates at 450 r.p.m. anticlockwise.

[Ans. T_C = 13 ; T_D = 52 ; 500 r.p.m. anticlockwise]

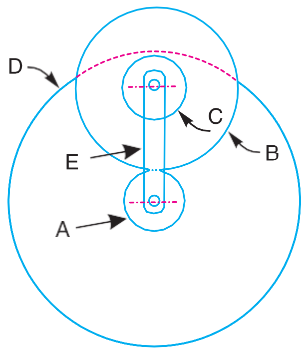


Fig. 13.39

11. A compound epicyclic gear is shown diagrammatically in Fig. 13.40. The gears *A* , *D* and *E* are free to rotate on the axis *P*. The compound gear *B* and *C* rotate together on the axis *Q* at the end of arm *F*. All the gears have equal pitch. The number of external teeth on the gears *A* , *B* and *C* are 18, 45 and 21 respectively. The gears *D* and *E* are annular gears. The gear *A* rotates at 100 r.p.m. in the anticlockwise direction and the gear *D* rotates at 450 r.p.m. clockwise. Find the speed and direction of the arm and the gear *E*. **[Ans. 400 r.p.m. clockwise ; 483.3 r.p.m. clockwise]**

12. In an epicyclic gear train of the 'sun and planet type' as shown in Fig. 13.41, the pitch circle diameter of the internally toothed ring *D* is to be 216 mm and the module 4 mm. When the ring *D* is stationary, the spider *A* , which carries three planet wheels *C* of equal size, is to make one revolution in the same sense as the sun wheel *B* for every five revolutions of the driving spindle carrying the sunwheel *B*. Determine suitable number of teeth for all the wheels and the exact diameter of pitch circle of the ring. **[Ans. T_B = 14 , T_C = 21 , T_D = 56 ; 224 mm]**

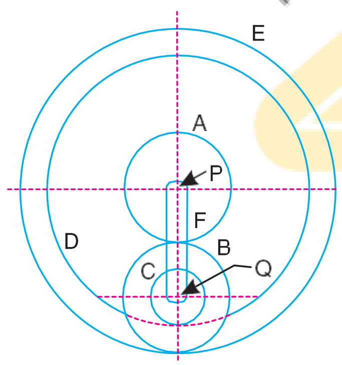


Fig. 13.40

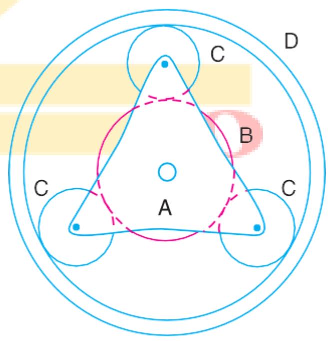


Fig. 13.41

13. An epicyclic train is shown in Fig. 13.42. Internal gear *A* is keyed to the driving shaft and has 30 teeth. Compound wheel *C* and *D* of 20 and 22 teeth respectively are free to rotate on the pin fixed to the arm *P* which is rigidly connected to the driven shaft. Internal gear *B* which has 32 teeth is fixed. If the driving shaft runs at 60 r.p.m. clockwise, determine the speed of the driven shaft. What is the direction of rotation of driven shaft with reference to driving shaft? **[Ans. 1980 r.p.m. clockwise]**

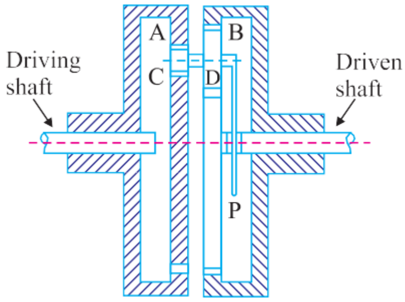


Fig. 13.42

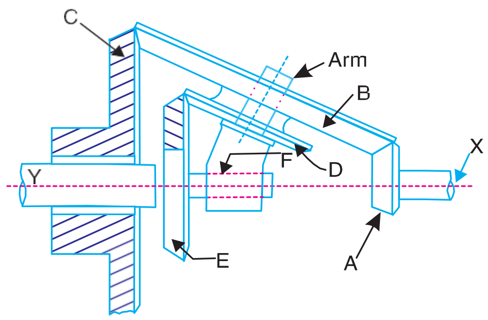


Fig. 13.43

14. A shaft *Y* is driven by a co-axial shaft *X* by means of an epicyclic gear train, as shown in Fig. 13.43. The wheel *A* is keyed to *X* and *E* to *Y* . The wheels *B* and *D* are compound and carried on an arm *F* which can turn freely on the common axes of *X* and *Y* . The wheel *C* is fixed. If the numbers of teeth on *A* , *B*, *C*, *D* and *E* are respectively 20, 64, 80, 30 and 50 and the shaft *X* makes 600 r.p.m., determine the speed in r.p.m. and sense of rotation of the shaft *Y* .

[Ans. 30 r.p.m. in the same sense as shaft *X*]

15. An epicyclic bevel gear train, as shown in Fig. 13.44, has fixed gear *B* meshing with pinion *C*. The gear *E* on the driven shaft meshes with the pinion *D*. The pinions *C* and *D* are keyed to a shaft, which revolves in bearings on the arm *A* . The arm *A* is keyed to the driving shaft. The number of teeth are : $T_B = 75$, $T_C = 20$, $T_D = 18$, and $T_E = 70$. Find the speed of the driven shaft, if 1. the driving shaft makes 1000 r.p.m., and 2. the gear *B* turns in the same sense as the driving shaft at 400 r.p.m., the driving shaft still making 1000 r.p.m.

[Ans. 421.4 r.p.m. in the same direction as driving shaft]

16. The epicyclic gear train is shown in Fig. 13.45. The wheel *D* is held stationary by the shaft *A* and the arm *B* is rotated at 200 r.p.m. The wheels *E* (20 teeth) and *F* (40 teeth) are fixed together and rotate freely on the pin carried by the arm. The wheel *G* (30 teeth) is rigidly attached to the shaft *C*. Find the speed of shaft *C* stating the direction of rotation to that of *B*.

If the gearing transmits 7.5 kW, what will be the torque required to hold the shaft *A* stationary, neglecting all friction losses?

[Ans. 466.7 r.p.m. in opposite direction of *B*; 511.5 N-m in opposite direction of *B*]

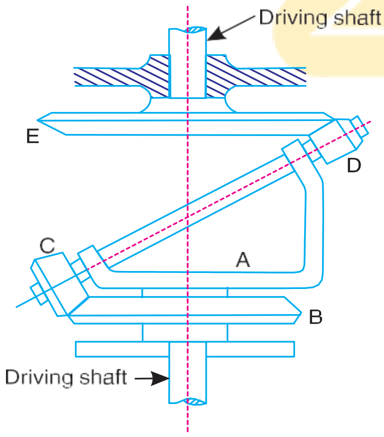


Fig. 13.44

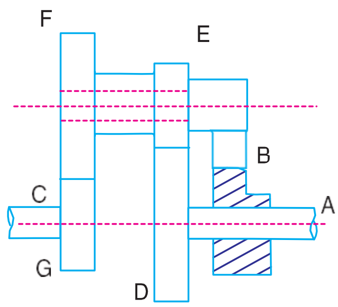


Fig. 13.45

17. An epicyclic gear train, as shown in Fig. 13.46, consists of two sunwheels *A* and *D* with 28 and 24 teeth respectively, engaged with a compound planet wheels *B* and *C* with 22 and 26 teeth. The sunwheel

D is keyed to the driven shaft and the sunwheel *A* is a fixed wheel co-axial with the driven shaft. The planet wheels are carried on an arm *E* from the driving shaft which is co-axial with the driven shaft.

Find the velocity ratio of gear train. If 0.75 kW is transmitted and input speed being 100 r.p.m., determine the torque required to hold the sunwheel *A*. **[Ans. 2.64 ; 260.6 N-m]**

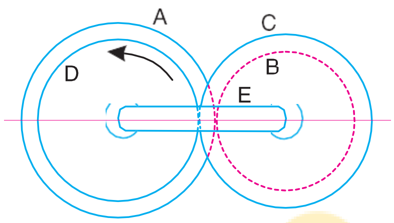


Fig. 13.46

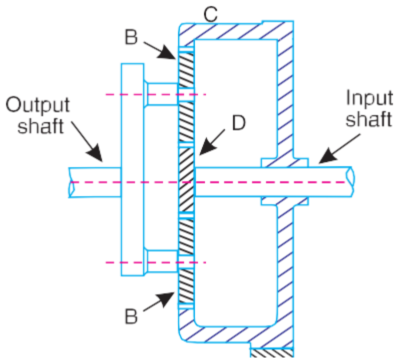


Fig. 13.47

18. In the epicyclic reduction gear, as shown in Fig. 13.47, the sunwheel *D* has 20 teeth and is keyed to the input shaft. Two planet wheels *B*, each having 50 teeth, gear with wheel *D* and are carried by an arm *A* fixed to the output shaft. The wheels *B* also mesh with an internal gear *C* which is fixed. The input shaft rotates at 2100 r.p.m. Determine the speed of the output shaft and the torque required to fix *C* when the gears are transmitting 30 kW.

[Ans. 300 r.p.m. in the same sense as the input shaft ; 818.8 N-m]

19. An epicyclic gear train for an electric motor is shown in Fig. 13.48. The wheel *S* has 15 teeth and is fixed to the motor shaft rotating at 1450 r.p.m. The planet *P* has 45 teeth, gears with fixed annulus *A* and rotates on a spindle carried by an arm which is fixed to the output shaft. The planet *P* also gears with the sun wheel *S*. Find the speed of the output shaft. If the motor is transmitting 1.5 kW, find the torque required to fix the annulus *A*. **[Ans. 181.3 r.p.m. ; 69.14 N-m]**

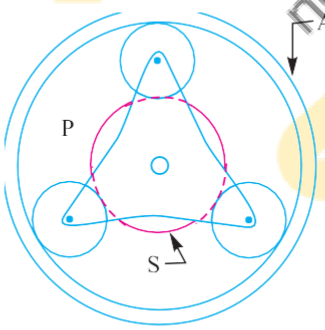


Fig. 13.48

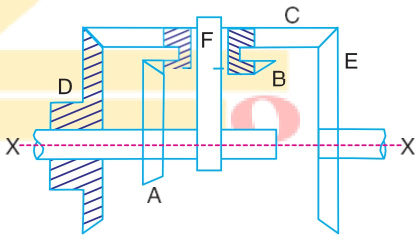


Fig. 13.49

20. An epicyclic gear consists of bevel wheels as shown in Fig. 13.49. The driving pinion *A* has 20 teeth and meshes with the wheel *B* which has 25 teeth. The wheels *B* and *C* are fixed together and turn freely on the shaft *F*. The shaft *F* can rotate freely about the main axis *X X*. The wheel *C* has 50 teeth and meshes with wheels *D* and *E*, each of which has 60 teeth. Find the speed and direction of *E* when *A* rotates at 200 r.p.m., if

1. *D* is fixed, and **2.** *D* rotates at 100 r.p.m., in the same direction as *A*.

In both the cases, find the ratio of the torques transmitted by the shafts of the wheels *A* and *E*, the friction being neglected.

[Ans. 800 r.p.m. in the opposite direction of *A* ; 300 r.p.m. in the opposite direction of *A* ; 4 ; 1.5]

DO YOU KNOW ?

1. What do you understand by 'gear train'? Discuss the various types of gear trains.
2. Explain briefly the differences between simple, compound, and epicyclic gear trains. What are the special advantages of epicyclic gear trains ?
3. Explain the procedure adopted for designing the spur wheels.
4. How the velocity ratio of epicyclic gear train is obtained by tabular method?
5. Explain with a neat sketch the 'sun and planet wheel'.
6. What are the various types of the torques in an epicyclic gear train ?

OBJECTIVE TYPE QUESTIONS

1. In a simple gear train, if the number of idle gears is odd, then the motion of driven gear will
   (a) be same as that of driving gear
   (b) be opposite as that of driving gear
   (c) depend upon the number of teeth on the driving gear
   (d) none of the above
2. The train value of a gear train is
   (a) equal to velocity ratio of a gear train (*b*) reciprocal of velocity ratio of a gear train
   (*c*) always greater than unity (*d*) always less than unity
3. When the axes of first and last gear are co-axial, then gear train is known as
   (a) simple gear train (*b*) compound gear train
   (*c*) reverted gear train (*d*) epicyclic gear train
4. In a clock mechanism, the gear train used to connect minute hand to hour hand, is
   (a) epicyclic gear train (*b*) reverted gear train
   (*c*) compound gear train (*d*) simple gear train
5. In a gear train, when the axes of the shafts, over which the gears are mounted, move relative to a fixed axis, is called
   (*a*) simple gear train (*b*) compound gear train
   (*c*) reverted gear train (*d*) epicyclic gear train
6. A differential gear in an automobile is a
   (*a*) simple gear train (*b*) epicyclic gear train
   (*c*) compound gear train (*d*) none of these
7. A differential gear in automobiles is used to
   (*a*) reduce speed (*b*) assist in changing speed
   (*c*) provide jerk-free movement of vehicle (*d*) help in turning

ANSWERS

1.	(*a*)	**2.**	(*b*)	**3.**	(*c*)	**4.**	(*b*)	**5.**	(*d*)
6.	(*b*)	**7.**	(*d*)						

GO To FIRST