Module-1

LOAD FLOW STUDIES

3.1 REVIEW OF NUMERICAL SOLUTION OF EQUATIONS

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods with respect to all forms of equations, as under:

1. Solution Linear equations:
* **Direct methods**:
- Cramer‟ s (Determinant) Method,
- Gauss Elimination Method (only for smaller systems), - LU Factorization (more preferred method), etc.

*** Iterative methods**:
- Gauss Method
- Gauss-Siedel Method (for diagonally dominant systems)

3. Solution of Nonlinear equations:
Iterative methods only:
- Gauss-Siedel Method (for smaller systems)
- Newton-Raphson Method (if corrections for variables are small)

4. Solution of differential equations:
Iterative methods only:
- Euler and Modified Euler method,
- RK IV-order method,
- Milne‟ s predictor-corrector method, etc.

It is to be observed that the nonlinear and differential equations can be solved only by the iterative methods. The iterative methods are characterized by the various performance features as under:
_ Selection of initial solution/ estimates
_ Determination of fresh/ new estimates during each iteration
_ Selection of number of iterations as per tolerance limit
_ Time per iteration and total time of solution as per the solution method selected _ Convergence and divergence criteria of the iterative solution _ Choice of the Acceleration factor of convergence, etc.

A comparison of the above solution methods is as under:
In general, the direct methods yield exact or accurate solutions. However, they are suited for only the smaller systems, since otherwise, in large systems, the possible round-off errors make the solution process inaccurate. The iterative methods are more useful when the diagonal elements of the coefficient matrix are large in comparison with the off diagonal elements. The round-off errors in these methods are corrected at the successive steps of the iterative process.The Newton-Raphson method is very much useful for solution of non –linear equations, if all the values of the corrections for the unknowns are very small in magnitude and the initial values of unknowns are selected to be reasonably closer to the exact solution.

3.2 LOAD FLOW STUDIES

Introduction: Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system. Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load flow studies play a vital role in power system studies. Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:
_ The Kirchhoff" s relations holding good,
_ Capability limits of reactive power sources,
_ Tap-setting range of tap-changing transformers,
_ Specified power interchange between interconnected systems,
_ Selection of initial values, acceleration factor, convergence limit, etc.

3.3 Classification of buses for LFA: Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Table 1. Classification of buses for LFA

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\|V\|$, δ	P_G, Q_G	$\|V\|$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\|V\|$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\|V\|$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G, Q_G, $\|V\|$	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus: The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 00 , as per flat-start procedure of iterative solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of YBUS based LFA:

The majority of load flow programs employ methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load flow analysis. It is a complex, square and symmetric matrix and hence only n(n+1)/2 elements of YBUS need to be stored for a n-bus system. Further, in the YBUS matrix, Yij = 0, if an incident element is not present in the system connecting the buses „i‟ and „j‟ . since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix, YBUS of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

$$\text{Percentage sparsity of a given matrix of } n^{th} \text{ order:} \quad = \quad \frac{\text{Total no. of zero valued elements of } Y_{BUS}}{\text{Total no. of entries of } Y_{BUS}}$$

$$S \quad = \quad (Z / n^2) \times 100 \ \% \qquad (1)$$

The percentage sparsity of Y_{BUS}, in practice, could be as high as 80-90%, especially for very large, practical power systems. This sparsity feature of YBUS is extensively used in reducing the load flow calculations and in minimizing the memory required to store the coefficient matrices. This is due to the fact that only the non-zero elements YBUS can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While YBUS is thus highly sparse, it‟ s inverse, ZBUS, the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

3.4 THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to other via the transmission lines. At any bus *i*, the complex power Si (injected), shown in figure 1, is defined as

$S_i = S_{Gi} - S_{Di}$ **(2)**

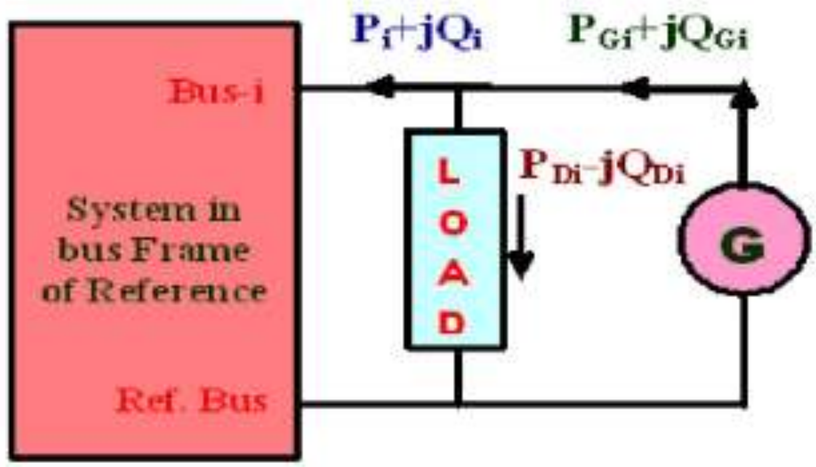


Fig.1 power flows at a bus-i

where Si = net complex power injected into bus *i*, SGi = complex power injected by the generator at bus *i*, and SDi = complex power drawn by the load at bus *i*. According to conservation of complex power, at any bus i, the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence,

Si = _Sij " i = 1, 2, ………..n
(3)

where Sij is the sum over all lines connected to the bus and *n* is the number of buses in the system (excluding the ground). The bus current injected at the bus-i is defined as

Ii = IGi – IDi " i = 1, 2, ………..n **(4)** where
IGi is the current injected by the generator at the bus and IDi is the current drawn by the load (demand) at that bus. In the bus frame of reference

IBUS = YBUS VBUS
(5)

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_n \end{bmatrix}$$ is the vector of currents injected at the buses,

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_n \end{bmatrix}$$ is the vector of complex bus voltages.

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad \forall\ i = 1, 2, \ldots\ldots n \qquad \textbf{(6)}$$

The complex power S_i is given by

$$S_i = V_i\ I_i^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}\ V_j \right)^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}^* V_j^* \right) \qquad \textbf{(7)}$$

Let $\qquad V_i \ \underline{\Delta}\ |V_i| \angle \delta_i = |V_i| (\cos \delta_i + j \sin \delta_i)$

$$\delta_{ij} = \delta_i - \delta_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^{n} |V_i| \, |V_j| \, \left(\cos\delta_{ij} + j\sin\delta_{ij}\right)\left(G_{ij} - jB_{ij}\right) \qquad \textbf{(8)}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^{n} |V_i| \, |V_j| \left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}\right) \qquad \textbf{(9)}$$

$$Q_i = \sum_{j=1}^{n} |V_i| \, |V_j| \left(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}\right) \qquad \textbf{(10)}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form

as $\qquad Y_{ij} = |Y_{ij}| \angle\theta_{ij}$ **(11)**

Again, we get from (7),

$$S_i = |V_i| \angle\delta_i \sum_{j=1}^{n} |Y_{ij}| \angle -\theta_{ij} \, |V_j| \angle -\delta_j \qquad \textbf{(12)}$$

The real part of (12) gives P_i.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \cos\left(-\theta_{ij} + \delta_i - \delta_j\right)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \cos - (\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ i = 1, 2, \ldots\ldots\ldots, n, \qquad \textbf{(13)}$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}| \, |V_j| \sin - (\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$Q_i = -\sum_{i=1}^{n} |V_i| |V_j| \, |Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(14)}$$

Equations (9)-(10) and (13)-(14) are the „power flow equations" or the „load flow equations" in two alternative forms, corresponding to the n-bus system, where each bus-*i* is characterized by four variables, Pi, Qi, |Vi|, and di. Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for any 2n unknowns, if the other 2n variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

3.4 DATA FOR LOAD FLOW

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

System data: It includes: number of buses-*n*, number of PV buses, number of loads, number of transmission lines, number of transformers, number of shunt elements, the slack bus number, voltage magnitude of slack bus (angle is generally taken as 0o), tolerance limit, base MVA, and maximum permissible number of iterations.

Generator bus data: For every PV bus *i*, the data required includes the bus number, active power generation PGi, the specified voltage magnitude V_i^{sp} , , minimum reactive power limit Qi,min, and maximum reactive power limit Qi,max.

Load data: For all loads the data required includes the the bus number, active power demand PDi, and the reactive power demand QDi.

Transmission line data: For every transmission line connected between buses *i* and *k* the data includes the starting bus number *i*, ending bus number *k*,.*r*esistance of the line, reactance of the line and the half line charging admittance.

Transformer data:
For every transformer connected between buses *i* and *k* the data to be given includes: the starting bus number *i*, ending bus number *k*, resistance of the transformer, reactance of the transformer, and the off nominal turns-ratio *a*.

Shunt element data: The data needed for the shunt element includes the bus number where element is connected, and the shunt admittance (Gsh + j Bsh).

GAUSS – SEIDEL (GS) METHOD

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \qquad \textbf{(15)}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \qquad \textbf{(16)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \qquad \forall\, i = 2, 3, \ldots\ldots\ldots n \qquad \textbf{(17)}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

3.5 Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses

(except slack bus) are taken to be $1.0\angle 0^0$. This is normally refered as the ***flat start*** solution.

4. Update the voltages. In any $(k+1)st$ iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} V_j^{(k)}\right] \forall\ i=2,3,\ldots n \qquad (18)$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current *(k+1)st* iteration. Hence these values are used. For buses *(i+1)…..n,* values from previous, *kth* iteration are used.

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall\ i = 2,3,\ldots n \qquad (19)$$

Where,e is the tolerance value. Generally it is customary to use a value of 0.0001 pu. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad (20)$$

7. Compute all line flows.
8. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (18). From (15) we have

$$Q_i = -\,\mathrm{Im}\left\{V_i^* \sum_{j=1}^{n} Y_{ij}\, V_j\right\}$$

Where *Im* stands for the imaginary part. At any *(k+1)*st iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\,\mathrm{Im}\left\{(V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij}\, V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij}\, V_j^{(k)}\right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)
2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$
3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}|\,\angle\delta_i^{(k+1)} \qquad \textbf{(22)}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:
In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e $(k+1)$ i Q computed using (21) is either less than Qi, min or greater than Qi,max, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the *(k+1)st* iteration and the voltage is calculated with the value of Qi set as follows:

If $Q_i < Q_{i,min}$	If $Q_i > Q_{i,max}$
Then $Q_i = Q_{i,min}$.	Then $Q_i = Q_{i,max}$.

(23)

If in the subsequent iteration, if Qi falls within the limits, then the bus can be switched back to PV status.

Acceleration of convergence
It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \qquad \textbf{(24)}$$

where α is a real number. When $\alpha = 1$, the value of $(k+1)$ is the computed value. If $1 < \alpha < 2$ then the value computed is extrapolated. Generally _ is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

3.6 Examples on GS load flow analysis:

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if $V_1 = 1\angle 0^0$ pu.*

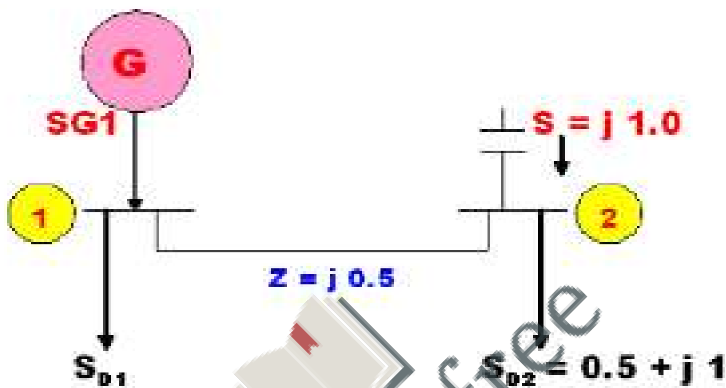


Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j\,1.0) = -\,0.5 \text{ pu.}$$

$$V_1 = 1\angle 0^0$$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^0 = 1 + j\,0.0 = 1\angle 0^0$ pu.

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 1.0 - j0.25 = 1.030776 \ \angle -14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.94118 - j\,0.23529 \ = 0.970145 \ \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.9375 - j\,0.249999 \ = 0.970261 \ \angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933612 - j\,0.248963 = 0.966237 \ \angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - \left(j2 \times 1\angle 0^0\right)\right]$$

$$= 0.933335 - j\,0.25 \ = 0.966237 \ \angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10-6 pu, the iterations can be stopped. To compute line flow

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0 - 0.966237\angle -14.995^0}{j0.5}$$

$$= 0.517472 \angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472 \angle 14.931^0$$

$$= 0.5 + j\,0.133329 \text{ pu}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^0 - 1\angle 0^0}{j0.5}$$

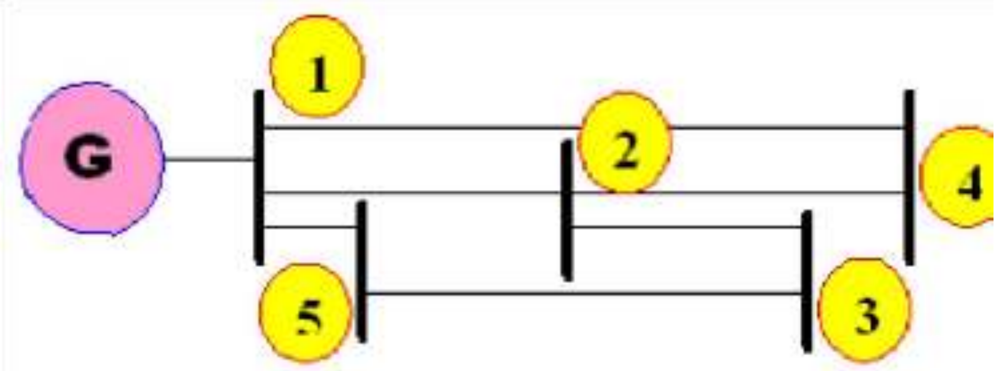
$$= 0.517472 \angle -194.93^0$$

$$S_{21} = V_2 I_{21}^* = -0.5 + j\,0.0 \text{ pu}$$

The total loss in the line is given by S12 + S21 = j 0.133329 pu Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2:

For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



Power System of Example 2

<u>Line data of example 2</u>

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

<u>Bus data of example 2</u>

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\lvert V_{SP} \rvert$ (pu)	δ
1	-	-	-	-	1.02	0°
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

<u>Solution</u>: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

  $P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$

$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$

Similarly $P_4 + jQ_4 = -0.4 - j0.1$, $\quad P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

$Y_{bus} =$

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to 1+j0.0 pu. The slack bus voltage is taken to be $V_1^0 = 1.02 + j0.0$ in all iterations.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{0*}} - Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294) \times 1.02\angle 0^\circ\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times 1.04\angle 0^\circ\} - \{(-0.58823 + j2.35294) \times 1.0\angle 0^\circ\}\right]$$

$$= 0.98140\angle -3.0665^\circ \;=\; 0.97990 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3||V_1|(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3||V_2|(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$+ |V_3|^2 (G_{33}\sin\delta_{33} - B_{33}\cos\delta_{33}) + |V_3||V_4|(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})$$

$$+ |V_3||V_5|(G_{35}\sin\delta_{35} - B_{35}\cos\delta_{35})$$

We note that $\delta_1 = 0^\circ$; $\delta_2 = -3.0665^\circ$; $\quad \delta_3 = 0^\circ$; $\quad \delta_4 = 0^\circ$ $\quad$ and $\quad \delta_5 = 0^\circ$

$\therefore \delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^\circ$ $(\delta_{ik} = \delta_i - \delta_k)$; $\quad \delta_{32} = 3.0665^\circ$

$Q_3 = 1.04\,[1.02\,(0.0+j0.0) + 0.9814\,\{-1.17647 \times \sin(3.0665^\circ) - 4.70588$ $\times\cos(3.0665^\circ)\} + 1.04\{-9.41176 \times \cos(0^\circ)\} + 1.0\,\{0.0 + j0.0\} + 1.0\{-4.70588 \times \cos(0^\circ)\}]$

$$= 1.04\,[-4.6735 + 9.78823 - 4.70588] \;=\; 0.425204 \text{ pu.}$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{0*}} - Y_{31}V_1^0 - Y_{32}V_2^1 - Y_{34}V_4^0 - Y_{35}V_5^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \{(-1.7647 + j4.70588)\times(0.98140\angle -3.0665^o)\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times(1\angle 0^o)\}\right]$$

$$= 1.05569\angle 3.077^o = 1.0541 + j0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is computed as: $V_3^1 = 1.04\angle 3.077^o$ pu

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1 - Y_{45}V_5^0\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \{(-0.39215 + j1.56862)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-0.58823 + j2.35294)\times(0.98140\angle -3.0665^o)\}\right]$$

$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715\angle -7.303^o \text{ pu} = 0.94796 - j0.12149$$

$$V_5^1 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51}V_1^o - Y_{52}V_2^1 - Y_{53}V_3^1 - Y_{54}V_4^1\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \{(-1.17647 + j4.70588)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 3.077^o\}\right]$$

$$= 0.994618\angle -1.56^o = 0.994249 - j0.027$$

Thus at end of 1st iteration, we have,

$V_1 = 1.02\angle 0^0$ pu $\qquad V_2 = 0.98140\angle -3.066^0$ pu

$V_3 = 1.04\angle 3.077^0$ pu $\qquad V_4 = 0.955715\angle -7.303^0$ pu

and $\qquad V_5 = 0.994618\angle -1.56^0$ pu

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

(i) All buses except bus 1 are PQ Buses

(ii) Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

(iii) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and 0.25_Q2_1.0 pu.

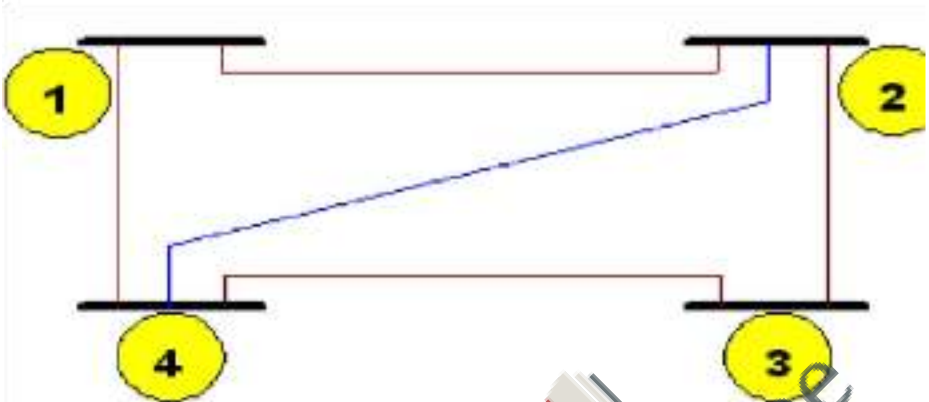


Fig. System for Example 3

Table: Line data of example 3

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04\angle 0^0$
2	0.5	– 0.2	–
3	– 1.0	0.5	–
4	– 0.3	– 0.1	–

Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

Y_{BUS} =

3.0 – j9.0	–2.0 + j6.0	– 1.0 + j3.0	0
–2.0 + j6.0	3.666 – j11.0	– 0.666 + j2.0	– 1.0 + j3.0
–1.0 + j3.0	–0.666 + j2.0	3.666 – j11.0	–2.0 + j6.0
0	–1.0 + j3.0	– 2.0 + j6.0	3.0 – j9.0

Case(i): All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0\angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}V_1^o - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5 + j0.2}{1.0 - j0.0} - \{(-2.0 + j6.0)\times(1.04\angle 0^{o})\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^{o})\} - \{(-1.0 + j3.0)\times(1.0\angle 0^{o})\}\right]$$

$$= 1.02014\angle 2.605^{o}$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^{o} - Y_{32}V_2^{1} - Y_{34}V_4^{0}\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^{o})\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.02014\angle 2.605^{o})\} - \{(-2.0 + j6.0)\times(1.0\angle 0^{0})\}\right]$$

$$= 1.03108\angle -4.831^{o}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^{o} - Y_{42}V_2^{1} - Y_{43}V_3^{1}\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.02014\angle 2.605^{o})\}\right.$$

$$\left. - \{(-2.0 + j6.0)\times(1.03108\angle -4.831^{o})\}\right]$$

$$= 1.02467\angle -0.51^{o}$$

Hence

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.02014\ \angle 2.605^0$ pu

$V_3^1 = 1.03108\ \angle -4.831^0$ pu $\qquad V_4^1 = 1.02467\ \angle -0.51^0$ pu

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2|\Big[|V_1|(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}) + |V_2|(G_{22}\sin\delta_{22} - B_{22}\cos\delta_{22})$$
$$+ |V_3|(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}) + |V_4|(G_{24}\sin\delta_{24} - B_{24}\cos\delta_{24})\Big]$$

= 1.04 [1.04 {–6.0} + 1.04 {11.0}+1.0{– 2.0} + 1.0 {–3.0}= 0.208 pu.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{0.5 - j0.208}{1.04\angle 0^0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\}\right.$$
$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^0)\}\right]$$

= 1.051288 + j0.033883

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04\angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0\angle 0^0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\}\right.$$
$$\left. - \{(-0.666 + j2.0)\times(1.04\angle 1.846^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$

$= 1.035587\angle -4.951^o$ pu.

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 1.846^o)\}\right.$$
$$\left. - \{(-2.0 + j6.0)\times(1.035587\angle -4.951^o)\}\right]$$

$= 0.9985\angle -0.178^o$

Hence at end of 1st iteration we have:

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.04\ \angle 1.846^0$ pu

$V_3^1 = 1.035587\ \angle -4.951^0$ pu $\qquad V_4^1 = 0.9985\angle -0.178^0$ pu

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \le Q_2 \le 1$ pu. If $0.25 < Q_2 < 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04 \angle 0^0 \text{ pu} \qquad V_2^1 = 1.05645 \angle 1.849^0 \text{ pu}$$

$$V_3^1 = 1.038546 \angle -4.933^0 \text{ pu} \qquad V_4^1 = 1.081446 \angle 4.896^0 \text{ pu}$$

Limitations of GS load flow analysis

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

3.7

NEWTON –RAPHSON METHOD

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form f(x) =0. Consider a set of *n* non-linear algebraic equations given by

$$f_i(x_1, x_2 \ldots\ldots x_n) = 0 \qquad i = 1,2\ldots\ldots n \qquad \textbf{(25)}$$

Let $x_1^0, x_2^0 \ldots\ldots\ldots\ldots x_n^0$, be the initial guess of unknown variables and $\Delta x_1^0, \Delta x_2^0 \ldots\ldots \Delta x_n^0$ be the respective corrections. Therefore,

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0 \ldots\ldots\ldots x_n^0 + \Delta x_n^0) = 0 \qquad i = 1,2\ldots\ldots n \qquad \textbf{(26)}$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^0, x_2^0 \ldots\ldots x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2}\right) \Delta x_2^0 + \ldots\ldots + \left(\frac{\partial f_i}{\partial x_n}\right) \Delta x_n^0 \right]$$

$$+ \text{ Higher order terms} = 0 \qquad \forall i = 1,2\ldots\ldots n \qquad \textbf{(27)}$$

Where, $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \ldots\ldots\ldots\ldots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2 \ldots\ldots x_n$ respectively, evaluated at $(x_1^0, x_2^0 \ldots\ldots x_n^0)$. If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ \cdot \\ \cdot \\ \cdot \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \left(\frac{\partial f_n}{\partial x_1}\right) & \left(\frac{\partial f_n}{\partial x_2}\right) & \cdot\ \cdot\ \cdot & \left(\frac{\partial f_n}{\partial x_n}\right) \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_n^0 \end{bmatrix} = 0 \qquad \textbf{(28)}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or $$F^0 = -J^0 \Delta X^0$$

Or $$\Delta X^0 = -[J^0]^{-1} F^0 \qquad \textbf{(29)}$$

And $$X^1 = X^0 + \Delta X^0 \qquad \textbf{(30)}$$

Here, the matrix [J] is called the **Jacobian** matrix. The vector of unknown variables is updated using (30). The process is continued till the difference between two successive iterations is less than the tolerance value.

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, ..., x_n) = 0$, where $x_1, x_2, ..., x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i, the angle of the complex bus voltage at bus *i*, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus *i*, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \quad \textbf{(31)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \quad \textbf{(32)}$$

Where $P_{i,sp}$ = Specified active power at bus *i*

$Q_{i,sp}$ = Specified reactive power at bus *i*

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32) are of the form F(x) = 0. Thus NR method can be applied to solve them. Equations (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \tag{33}$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respect to corresponding δ and $|V|$. The negated partial derivative of ΔP, is same as the partial derivative of P_{cal}, since P_{sp} is a constant. The various computations involved are discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \tag{34}$$

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \tag{35}$$

The powers are computed at any $(r+1)^{st}$ iteration by using the voltages available from previous iteration. The elements of the Jacobian are found using the above equations as:

Elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik}\}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k|\{G_{ik}(-\sin\delta_{ik})(-1) + B_{ik}(\cos\delta_{ik})(-1)\}$$

Elements of J_3

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J_2

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i| \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i + |V_i|^2 G$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J_4

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) = Q_i - |V_i|^2$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

Thus, the linearized form of the equation could be considered agai

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial \delta_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2$

3. 8

ALGORITHM FOR NR METHOD
IN POLAR COORDINATES

1. Formulate the Y_{BUS}
2. Assume initial voltages as follows:

$$V_i = |V_{i,sp}|\angle 0^0 \quad \text{(at all PV buses)}$$

$$V_i = 1\angle 0^0 \qquad \text{(at all PQ buses)}$$

3. At $(r+1)^{st}$ iteration, calculate $P_i^{(r+1)}$ at all the PV and PQ buses and $Q_i^{(r+1)}$ at all the PQ buses, using voltages from previous iteration, $V_i^{(r)}$. The formulae to be used are

$$P_{i,Cal} = P_i = G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$Q_{i,Cal} = Q_i = -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

4. Calculate the power mismatches (power residues)

$$\Delta P_i^{(r)} = P_{i,sp} - P_{i,cal}^{(r+1)} \quad \text{(at PV and PQ buses)}$$

$$\Delta Q_i^{(r)} = Q_{i,sp} - Q_{i,cal}^{(r+1)} \quad \text{(at PQ buses)}$$

5. Calculate the Jacobian $[J^{(r)}]$ using $V_i^{(r)}$ and its elements spread over H, N, M, L sub- matrices using the relations derived as in (36).

6. Compute

$$\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V|} \end{bmatrix} = [J^{(r)}]^{-1}\begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

7. Update the variables as follows:

$$\delta_i^{(r+1)} = \delta_i^{(r)} + \Delta\delta_i^{(r)} \quad \text{(at all buses)}$$

$$|V_i|^{(r+1)} = |V_i|^{(r)} + \Delta|V_i|^{(r)}$$

8. Go to step 3 and iterate till the power mismatches are within acceptable tolerance.

DECOUPLED LOAD FLOW

In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are,

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\frac{\partial Q_i}{\partial |V_j|}$ is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix} \qquad \textbf{(37)}$$

From (37) it is obvious that the voltage angle corrections $\Delta\delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. This equation can be solved through two alternate strategies as under:

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update δ and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta\delta^{(r)} = \left[H^{(r)}\right]^{-1} \Delta P^{(r)}$

(ii) Up date δ using $\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r)}$.

(iii) Use $\delta^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} = \left[L^{(r)}\right]^{-1} \Delta Q^{(r)}$

(v)Update, $|V^{(r+1)}| = |V^{(r)}| + |\Delta V^{(r)}|$

(vi) Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for $\Delta\delta$ and using updated values of δ to calculate $\Delta|V|$. Hence, the second strategy results in faster convergence, compared to the first strategy.

FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)

- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$H_{ik} = L_{ik} = -|V_i||V_k| B_{ik} \qquad (i \neq k)$

$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$

The matrix (37) reduces to

$$[\Delta P] = [|V_i||V_j|B'_{ij}][\Delta\delta]$$

$$[\Delta Q] = [|V_i||V_j|B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(38)}$$

Where B'_{ij} and B''_{ij} are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = [B'_{ij}][\Delta\delta]$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(39)}$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B'_{ij}, omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the Y_{bus}.

With these assumptions we obtain a loss-less network. In the FDLF method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations.

REPRESENTATION OF TAP CHANGING TRANSFORMERS

Consider a tap changing transformer represented by its admittance connected in series with an ideal autotransformer as shown (a= turns ratio of transformer)

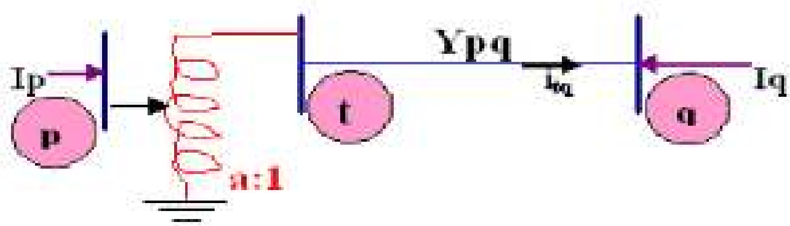


Fig. 2. Equivalent circuit of a tap setting transformer

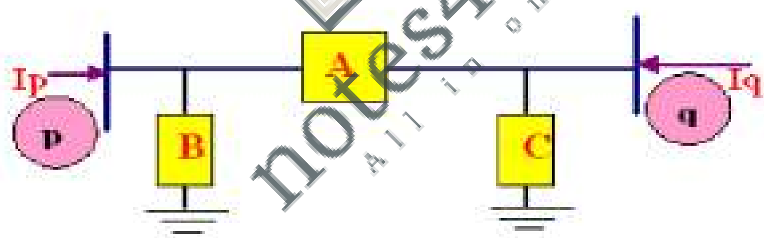


Fig. 3. π-Equivalent circuit of Fig.2 above.

By equating the bus currents in both the mutually equivalent circuits as above, it can be shown that the π-equivalent circuit parameters are given by the expressions as under:

(i) Fixed tap setting transformers (on no load)

A = Ypq/ a

B = 1/a (1/a -1) Ypq

C = (1-1/a) Ypq

(i) Tap changing under load (TCUL) transformers (on load)

A = Ypq

B = (1/a -1) (1/a + 1 – Eq/Ep) Ypq

C = (1-1/a) (Ep/Eq) Ypq

Thus, here, in the case of TCUL transformers, the shunt admittance values are observed to be a function of the bus voltages.

COMPARISON OF LOAD FLOW METHODS

The comparison of the methods should take into account the computing time required for preparation of data in proper format and data processing, programming ease, storage requirements, computation time per iteration, number of iterations, ease and time required for modifying network data when operating conditions change, etc. Since all the methods presented are in the bus frame of reference in admittance form, the data preparation is same for all the methods and the bus admittance matrix can be formed using a simple algorithm, by the rule of inspection. Due to simplicity of the equations, Gauss-Seidel method is relatively easy to program. Programming of NR method is more involved and becomes more complicated if the buses are randomly numbered. It is easier to program, if the PV buses are ordered in sequence and PQ buses are also ordered in sequence.

The storage requirements are more for the NR method, since the Jacobian elements have to be stored. The memory is further increased for NR method using rectangular coordinates. The storage requirement can be drastically reduced by using sparse matrix techniques, since both the admittance matrix and the Jacobian are sparse matrices. The time taken for a single iteration depends on the number of arithmetic and logical operations required to be performed in a full iteration. The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure below. Computation time can be reduced if the Jacobian is updated once in two or three iterations. In FDLF method, the Jacobian is constant and needs to be computed only once. In both NR and FDLF methods, the time per iteration increases directly as the number of buses.

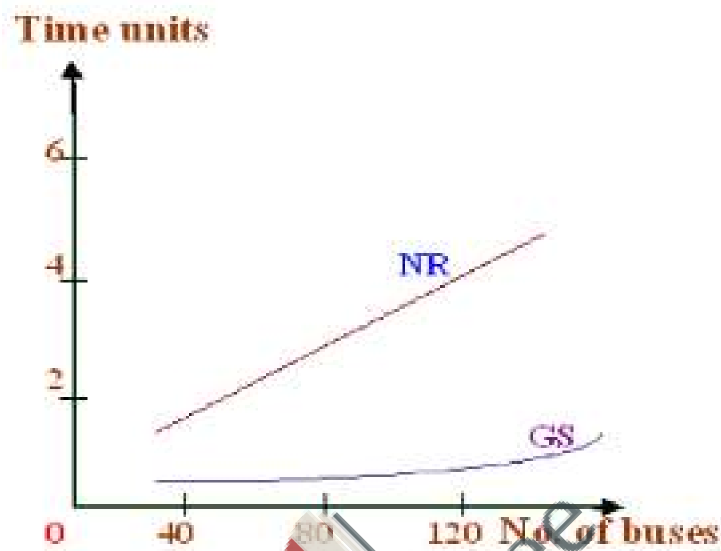


Figure 4. Time per Iteration in GS and NR methods

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. In contrast, the number of iterations is relatively constant in NR and FDLF methods. They require about 5-8 iterations for convergence in large systems. A significant increase in rate of convergence can be obtained in the GS method if an acceleration factor is used. All these variations are shown graphically in figure below. The number of iterations also depends on the required accuracy of the solution. Generally, a voltage tolerance of 0.0001 pu is used to obtain acceptable accuracy and the real power mismatch and reactive power mismatch can be taken as 0.001 pu. Due to these reasons, the NR method is faster and more reliable for large systems. The convergence of FDLF method is geometric and its speed is nearly 4-5 times that of NR method.

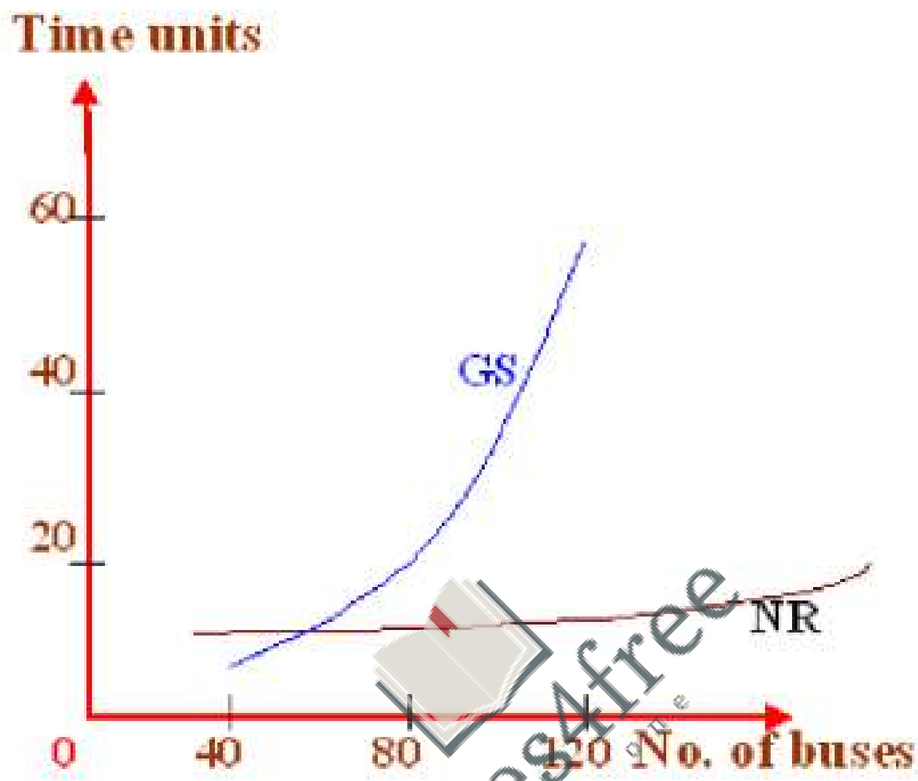


Figure 5. Total time of Iteration in GS and NR methods

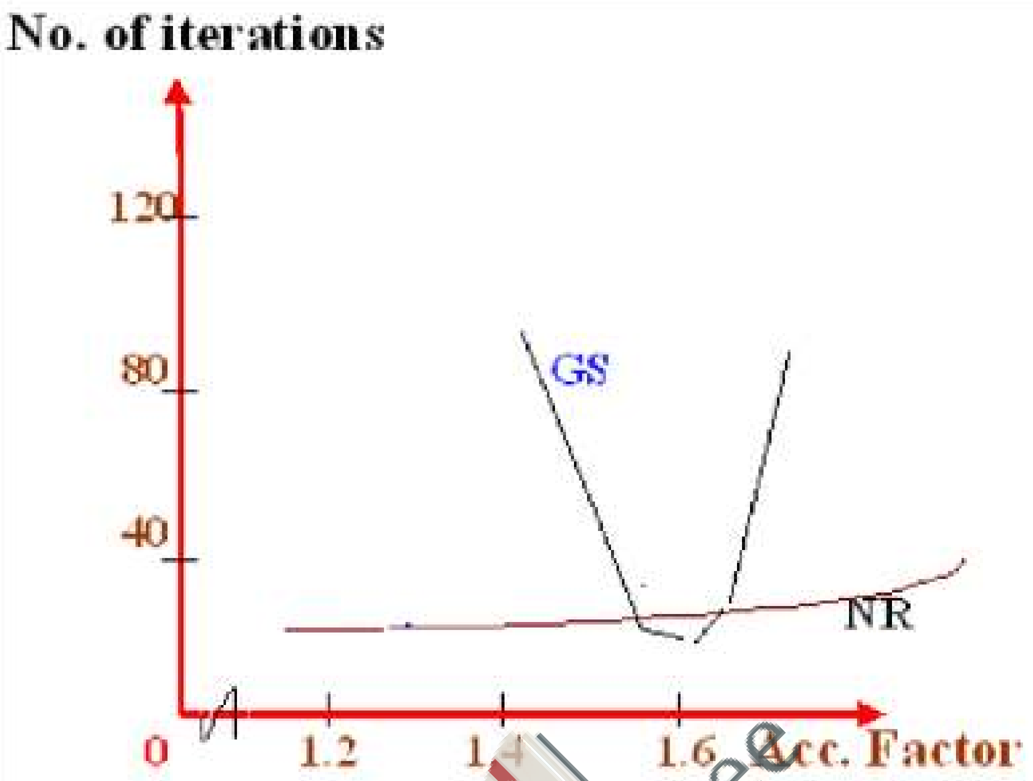


Figure 6. Influence of acceleration factor on load flow methods

FINAL WORD

In this chapter, the load flow problem, also called as the power flow problem, has been considered in detail. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines. Though, algorithms are available using the impedance form of the equations, the sparsity of the bus admittance matrix and the ease of building the bus admittance matrix, have made algorithms using the admittance form of equations more popular. The most popular methods are the Gauss-Seidel method, the Newton-Raphson method and the Fast Decoupled Load Flow method. These methods have been discussed in detail with illustrative examples. In smaller systems, the ease of programming and the memory requirements, make GS method attractive. However, the computation time increases with increase in the size of the system. Hence, in large systems NR and FDLF methods are more popular. There is a trade off between various requirements like speed, storage, reliability, computation time, convergence characteristics etc. No single method has all the desirable features. However, NR method is most popular because of its versatility, reliability and accuracy.

UNIT-3&4

1. Using generalized algorithm expressions for each case of analysis, explain the load flow studies procedure, as per the G-S method for power system having PQ and PV buses, with reactive power generations constraints.
2. Derive the expression in polar form for the typical diagonal elements of the sub matrices of the Jacobian in NR method of load flow analysis.
3. Compare NR and GS method for load flow analysis procedure in respect of the following i) Time per iteration ii) total solution time iii) acceleration factor iv)number of iterations
4. Explain briefly fast decoupled load flow solution method for solving the non linear load flow equations.
5. Draw the flow chart of NR method for load flow analysis.
6. Explain the representation of transformer with fixed tap changing during the load flow studies
7. What are the assumptions made in fast decoupled load flow method? Explain the algorithm briefly, through a flow chart.
8. Explain the NR load flow method? Explain the algorithm briefly, through a flow chart.
9. What is load flow analysis? What is the data required to conduct load flow analysis? Explain how buses are classified to carry out load flow analysis in power system. What is the significance of slack bus.
10. The following is the system data for a load flow analysis. The line data are shown and also real and reactive powers data is given. Determine the voltage at the end of first iteration using GS method. Take α=1.6

MODULE - 2

LOAD FLOW STUDIES

Newton - Raphson method :-

Introduction:

* Newton - Raphson (NR) method is a powerful method of solving non linear algebraic equations.
* It works faster due to quadratic convergence.
* For large power systems, the NR method is found to be more efficient and practical.
* The no. of iterations required to obtain a solution is independent of the system size but more functional evaluations are required at each iteration.
* Convergence can be considerably speeded up by performing the first iteration through the GS method, and using the values so obtained for solving the NR iterations.
* Large requirement of computer memory.

Consider a set of 'n' non-linear algebraic equations:

$f_i(x_1, x_2, \ldots x_n) = 0 \quad ; i = 1, 2, 3 \ldots n$

Assume initial values of unknowns as $x_1^0, x_2^0, \ldots x_n^0$.

Let $\Delta x_1^0, \Delta x_2^0, \ldots \Delta x_n^0$ be the corrections, which on being added to the initial guess, give the actual solution.

Therefore, $f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \ldots\ldots x_n^0 + \Delta x_n^0) = 0$;

$i = 1, 2, \ldots n$

Taylor series expansion of the above equation gives,

$$f_i(x_1^0, x_2^0, \ldots x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2}\right) \Delta x_2^0 + \ldots + \left(\frac{\partial f_i}{\partial x_n}\right)^0 \Delta x_n^0\right] + \text{higher order terms} = 0$$

where,

$\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \ldots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the derivatives of f_i with respect to $x_1, x_2, \ldots x_n$ evaluated at $(x_1^0, x_2^0, \ldots x_n^0)$

Neglecting higher order terms, above equation can be written in matrix form as:

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ \vdots \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \vdots & \vdots & & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} \simeq 0$$

In vector matrix form,

$$f^0 + J^0 \Delta x^0 \simeq 0$$

where, $J^0 \longrightarrow$ Jacobian matrix; $J \longrightarrow \frac{\partial f}{\partial x}$

Newton - Raphson (NR) method for load flow solution :

The complex power injected by the source into the ith bus of a power system is,

$$S_i = P_i + jQ_i = V_i I_i^* \quad ; \; i = 1,2,\ldots\ldots n$$

$$= V_i \left[\sum_{j=1}^{n} Y_{ij} V_j \right]^*$$

We have, $V_i = |V_i| \angle \delta_i$ and $V_j = |V_j| \angle \delta_j$; $Y_{ij} = |Y_{ij}| \angle \theta_{ij}$

$$\therefore \; P_i + jQ_i = |V_i| \angle \delta_i \left[\sum_{j=1}^{n} |Y_{ij}| \angle \theta_{ij} \; |V_j| \angle \delta_j \right]^*$$

$$= |V_i| \angle \delta_i \left[\sum_{j=1}^{n} |Y_{ij}| \angle -\theta_{ij} \; |V_j| \angle -\delta_j \right]$$

$$P_i + jQ_i = \sum_{j=1}^{n} |V_i| |Y_{ij}| |V_j| \angle \delta_i - \theta_{ij} - \delta_j$$

where,

$$P_i = \sum_{j=1}^{n} |V_i| |Y_{ij}| |V_j| \cos \angle \delta_i - \theta_{ij} - \delta_j \longrightarrow (1)$$

$$Q_i = \sum_{j=1}^{n} |V_i| |Y_{ij}| |V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j \longrightarrow (2)$$

Equations (1) and (2) represents a set of non-linear equations interms of the independent variables δ and $|V|$. (phase angle in radians and voltage magnitude in pu)

There are $2n$ power flow equations.

Equations (1) and (2) can be expanded in Taylor series and can be written in matrix form as:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \hline \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix} = \left[\begin{array}{cccc|cccc} \frac{\partial P_2}{\partial \delta_2} & \cdots & \cdots & \frac{\partial P_2}{\partial \delta_n} & \frac{\partial P_2}{\partial |V_2|} & \cdots & \cdots & \frac{\partial P_2}{\partial |V_n|} \\ \frac{\partial P_3}{\partial \delta_2} & \cdots & \cdots & \frac{\partial P_3}{\partial \delta_n} & \frac{\partial P_3}{\partial |V_2|} & \cdots & \cdots & \frac{\partial P_3}{\partial |V_n|} \\ \vdots & & & \vdots & \vdots & & & \vdots \\ \frac{\partial P_n}{\partial \delta_2} & \cdots & \cdots & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial |V_2|} & \cdots & \cdots & \frac{\partial P_n}{\partial |V_n|} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \cdots & \cdots & \frac{\partial Q_2}{\partial \delta_n} & \frac{\partial Q_2}{\partial |V_2|} & \cdots & \cdots & \frac{\partial Q_2}{\partial |V_n|} \\ \frac{\partial Q_3}{\partial \delta_2} & \cdots & \cdots & \frac{\partial Q_3}{\partial \delta_n} & \frac{\partial Q_3}{\partial |V_2|} & \cdots & \cdots & \frac{\partial Q_3}{\partial |V_n|} \\ \vdots & & & \vdots & \vdots & & & \vdots \\ \frac{\partial Q_n}{\partial \delta_2} & \cdots & \cdots & \frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial |V_n|} & \cdots & \cdots & \frac{\partial Q_n}{\partial |V_n|} \end{array}\right] \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \vdots \\ \Delta \delta_n \\ \hline \Delta |V_2| \\ \Delta |V_3| \\ \vdots \\ \Delta |V_n| \end{bmatrix}$$

Bus 1 $\longrightarrow$ slack bus

In general,

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$J_1, J_2, J_3, J_4 \longrightarrow$ elements of the Jacobian matrix.

$J_1 \longrightarrow$ matrix of order $(n-1) \times (n-1)$

$J_2 \longrightarrow$ matrix of order $(n-1) \times (n-1-m)$

$J_3 \longrightarrow$ matrix of order $(n-1-m) \times (n-1)$

$J_4 \longrightarrow$ matrix of order $(n-1-m) \times (n-1-m)$

where, $n \longrightarrow$ Total no. of buses ; $m \longrightarrow$ no. of voltage controlled bus.

$J_1 = \frac{\partial P}{\partial \delta}$; $P_i = |V_i|^2 |Y_{ii}| \cos \underline{/-\theta_{ii}} + \sum_{\substack{j=1 \\ \neq i}}^{n} |V_i||Y_{ij}||V_j| \cos \underline{/\delta_i - \theta_{ij} - \delta_j}$

Diagonal elements of J_1:

$$\frac{\partial P_i}{\partial \delta_i} = -\sum_{\substack{j=1 \\ \neq i}}^{n} |V_i||Y_{ij}||V_j| \sin \underline{/\delta_i - \theta_{ij} - \delta_j}$$

Off-diagonal elements of J_1:

$$\frac{\partial P_i}{\partial \delta_j} = |V_i||Y_{ij}||V_j| \sin \underline{/\delta_i - \theta_{ij} - \delta_j}$$

$(j \neq i)$

$J_2 = \frac{\partial P}{\partial |V|}$

Diagonal elements of J_2:

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \cos \underline{/-\theta_{ii}} + \sum_{\substack{j=1 \\ \neq i}}^{n} |Y_{ij}||V_j| \cos \underline{/\delta_i - \theta_{ij} - \delta_j}$$

Off-diagonal elements of J_2:

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}| \cos \underline{/\delta_i - \theta_{ij} - \delta_j}$$

$(j \neq i)$

$J_3 = \frac{\partial Q}{\partial \delta}$; $Q_i = |V_i|^2 |Y_{ii}| \sin \underline{/-\theta_{ii}} + \sum_{\substack{j=1 \\ \neq i}}^{n} |V_i||Y_{ij}||V_j| \sin \underline{/\delta_i - \theta_{ij} - \delta_j}$

Diagonal elements of J_3:

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ \neq i}}^{n} |V_i||Y_{ij}||V_j| \cos \underline{/\delta_i - \theta_{ij} - \delta_j}$$

Off-diagonal elements of J_3:

$$\frac{\partial Q_i}{\partial |\delta_j|} = -|V_i||Y_{ij}||V_j| \cos \angle \delta_i - \theta_{ij} - \delta_j$$

$(j \neq i)$

$$J_4 = \frac{\partial Q}{\partial |V|}$$

Diagonal elements of J_4:

$$\frac{\partial Q_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \sin \angle -\theta_{ii} + \sum_{\substack{j=1 \\ \neq i}}^{n} |Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

Off-diagonal elements of J_4:

$$\frac{\partial Q_i}{\partial |V_j|} = |V_i||Y_{ij}| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

$(j \neq i)$

The terms ΔP_i and ΔQ_i are the difference between the scheduled and calculated values and are known as the power residuals given by,

$$\Delta P_i = P_{i\,(scheduled)} - P_{i\,(calculated)}$$

$$\Delta Q_i = Q_{i\,(scheduled)} - Q_{i\,(calculated)}$$

The new estimates of the bus voltages are:

$$V_{i\,(new)} = V_{i\,(old)} + \Delta |V|$$

$$\delta_{i\,(new)} = \delta_{i\,(old)} + \Delta \delta$$

Gauss Seidel Method :-

Advantages :

(1) The technique is very simple.
(2) Computer memory requirement is less.
(3) Computation time per iteration is less.
(4) Ease in programming and most efficient use of core memory.
(5) It requires less number of arithmetic operations to complete one iteration.

Disadvantages :

(1) Number of iterations more for convergence and hence rate of convergence is slow.
(2) Convergence is seriously affected by the selection of slack bus and the selection of a particular bus as slack bus may result in poor convergence.
(3) Number of iterations increases directly with the increase in the number of buses.
(4) Convergence is affected by series capacitors.
(5) This method is limited to small sized systems.

Newton Raphson method:

Advantages:

(1) Superior convergence because of quadratic convergence.

(2) Usually employed for large sized systems.

(3) NR method has an 1:8 iteration ratio compared to GS method.

(4) More accurate

(5) Smaller number of iterations.

(6) The number of iterations are almost independent of system size.

(7) The method is insensitive to factors like slack bus selection, regulating transformer and presence of series capacitors.

(8) The number of iterations remain constant irrespective of the number of buses.

(9) It is faster and more reliable.

(10) It works for any size and any kind of problem.

Disadvantages:

(1) The solution technique is difficult.

(2) The calculations involved in each iteration are more and hence computation time per iteration is large.

(3) The computer memory requirement is large.

(4) NR method may fail in some ill conditioned problems.

(5) Its programming logic is more complex.

(6) As the elements of Jacobian matrix are to be computed in each iteration, the time taken for each iteration is considerably longer.

Comparision of Gauss-Seidel and Newton Raphson method:-

	Gauss-Seidel method	Newton Raphson method
(i)	The variables are expressed in Rectangular co-ordinates	The variables are expressed in Polar co-ordinates.
(ii)	The number of mathematical operations per iteration is less in G/S method.	The number of mathematical operations per iteration will be more in NR method.
(iii)	GS method has linear convergence characteristics, hence slow in convergence.	NR method has quadratic convergence characteristics, hence faster in convergence.
(iv)	No. of iterations increases with the no. of buses.	No. of iterations remains constant and it does not depend on the size of the system.

(v) Convergence is affected by choice of slack bus and the presence of series capacitance.	NR method is less sensitive to the choice of slack bus and series capacitance.
(vi) G-S method requires large number of iterations for same level of accuracy.	NR method needs only 3-5 iterations to reach an acceptable solution for a large system.
(vii) Computer memory requirement is less.	Computer memory requirement is large.

Decoupled Load Flow methods:

* Real power is more dependent on the changes in bus voltage angles (δ) at various buses $\left[\because \frac{\partial P}{\partial |V|} \simeq 0\right]$
* Reactive power is more dependent on the changes in voltage magnitudes ($|V|$) at various buses $\left[\frac{\partial Q}{\partial \delta} \simeq 0\right]$
* The above property gave the motivation in developing the Decoupled Load Flow (DLF) methods.
* The resulting decoupled linear matrix becomes,

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix}$$

where, $[\Delta P] = [H][\Delta \delta]$; $[\Delta Q] = [L]\left[\frac{\Delta |V|}{|V|}\right]$

Advantages of DLF :-

(i) Reduced memory requirement in storing Jacobian matrix

(ii) Computation time per iteration is less than NR method.

Fast Decoupled Load Flow (FDLF)

The Jacobian of the decoupled Newton load flow can be made constant in value, based on physically justifiable assumptions. Hence, the matrix needs to be calculated only once per solution. FDLF is developed by Stott B in 1974.

Assumptions:

(1) The assumptions which are valid in normal power system operation are:

$\cos \delta_{ij} \simeq 1$ and $\sin \delta_{ij} \simeq 0$

$G_{ij} \sin \delta_{ij} << B_{ij}$ and

$Q_i << B_{ii} |V_i|^2$

With these assumptions, the entries of J_1 and J_4 submatrices becomes considerably simplified.

(2) $|V_i|^2 \simeq |V_i|$ since $|V_i| \simeq 1.00$ pu.

(3) Real power is more dependant on changes in voltage angles and

Reactive power is more dependant on changes in voltage magnitude.

$\therefore J_2 \rightarrow \frac{\partial P}{\partial |V|} \simeq 0$ and $J_3 \rightarrow \frac{\partial Q}{\partial \delta} \simeq 0$

Further simplification of the FDLF algorithm is achieved by,

(4) Omitting the elements of [B′] that predominantly affect reactive power flows i.e, shunt reactances and transformer off-nominal in-phase taps.

(5) Omitting from [B″] the angle shifting effect of phase shifter (that which predominantly affects the real power flow).

(6) Ignoring the series resistance in calculating the elements of [B′], which then becomes the DC approximation of the power flow matrix.

Decoupled matrix is given by,

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

where,

$$J_1 \Rightarrow \frac{\partial P_i}{\partial \delta_i} = - \sum_{\substack{j=1 \\ \neq i}}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

$$= -|V_i|^2 |Y_{ii}| \sin \angle -\theta_{ii} + |V_i|^2 |Y_{ii}| \sin \angle -\theta_{ii}$$

$$- \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

$$= -\sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j + |V_i|^2 |Y_{ii}| \sin \angle -\theta_{ii}$$

$$J_1 = -Q_i + |V_i|^2 |Y_{ii}| \sin \angle -\theta_{ii}$$

By assumption (1), $Q_i << B_{ii}|V_i|^2 \Rightarrow Q_i \to 0$

$\therefore J_1 \Rightarrow |V_i|^2 |Y_{ii}| \sin \angle -\theta_{ii}$

$\Rightarrow |V_i| |Y_{ii}| \sin \angle -\theta_{ii} \Rightarrow -|V_i||Y_{ii}| \sin \angle \theta_{ii}$

Also, $\theta_{ii} = 90° \Rightarrow \sin \angle \theta_{ii} = 1$ since series resistance is neglected.

$J_1 \Rightarrow -|V_i||Y_{ii}| \Rightarrow -|V_i| B_{ii}$ (diagonal elements)

Off-diagonal elements of J_1:

$$\frac{\partial P_i}{\partial \delta_j} = |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

As $\delta_i \simeq \delta_j \simeq 0$; $|V_i| \simeq 1.0$ pu ;

$$\frac{\partial P_i}{\partial \delta_j} = |Y_{ij}||V_j| \sin \angle -\theta_{ij}$$

$$= -|V_j||Y_{ij}| = -|V_j| B_{ij}$$

$J_4 \to \frac{\partial Q}{\partial |V|}$

Diagonal elements of J_4:

$$\frac{\partial Q_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \sin \angle -\theta_{ii} + \sum_{\substack{j=1 \\ \neq i}}^{n} |Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

$$|V_i| \frac{\partial Q_i}{\partial |V_i|} = 2|V_i|^2 |Y_{ii}| \sin \angle -\theta_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

$$= |V_i|^2 |Y_{ii}| \sin \underline{\angle -\theta_{ii}} + \sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \sin \underline{\angle \delta_i - \theta_{ij} - \delta_j}$$

$$= |V_i|^2 |Y_{ii}| \sin \underline{\angle -\theta_{ii}} + Q_i$$

By assumption (1), $Q_i \longrightarrow 0$

$$\therefore |V_i| \frac{\partial Q_i}{\partial |V_i|} = |V_i|^2 |Y_{ii}| \sin \underline{\angle -\theta_{ii}}$$

Also, $|V_i|^2 \simeq |V_i|$

$$|V_i| \frac{\partial Q_i}{\partial |V_i|} = |V_i||Y_{ii}| \sin \underline{\angle -\theta_{ii}}$$

$$= -|V_i||Y_{ii}| \sin \underline{\angle \theta_{ii}}$$

$$= -|V_i||Y_{ii}| \qquad \left(\sin \underline{\angle \theta_{ii}} = \sin 90^\circ = 1\right)$$

$$= -|V_i| B_{ii}$$

<u>Off-diagonal elements of J_4</u>:

$$\frac{\partial Q_i}{\partial |V_j|} = |V_i||Y_{ij}| \sin \underline{\angle \delta_i - \theta_{ij} - \delta_j}$$

$$|V_j| \frac{\partial Q_i}{\partial |V_j|} = |V_i||Y_{ij}||V_j| \sin \underline{\angle \delta_i - \theta_{ij}} - \delta_j$$

We have $\delta_i \simeq 0$; $\delta_j \simeq 0$; $\sin \underline{\angle -\theta_{ij}} = -\sin \underline{\angle \theta_{ij}} = -1$

$$\therefore |V_j| \frac{\partial Q_i}{\partial |V_j|} = -|V_j||Y_{ij}| \sin \underline{\angle \theta_{ij}}$$

$$= -|V_j| B_{ij}$$

$\therefore [\Delta P] = -[B'][\Delta \delta]$; $B' \longrightarrow$ susceptance of the $(n-1)\times(n-1)$ elements of Y_{BUS} ;

$[\Delta Q] = -[B''][\Delta |V|]$; $(n-1) \longrightarrow$ no. of PV buses + PQ buses.

$[B''] \longrightarrow$ susceptance of the $(m-1)\times(m-1)$ elements of Y_{BUS}.
$(m-1) \longrightarrow$ no. of PQ buses.

Algorithm for NR method / FDLF method :-

Step 1 : Find Y_{BUS} matrix by Inspection method and convert into polar form.

Step 2 : Assume a flat voltage profile $(1+j0)$ for all the buses whose voltage is not specified. The voltage of the slack bus is specified and it is not modified in any iteration. Assume $\epsilon \rightarrow$ convergence criterion.

Step 3 : Set iteration count $k=0$ and assumed voltage profile of the buses are denoted as $V_1^0, V_2^0, \ldots V_n^0$ except the slack bus.

Step 4 : Set Bus count $P=1$

Step 5 : check for slack bus, if it is slack bus then go to step 19, otherwise go to next step.

Step 6 : Calculate the real and reactive power of bus 'p' using

$$P_{i(cal)} = \sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \cos \angle \delta_i - \theta_{ij} - \delta_j \quad \text{for all } (n-1) \text{ buses.}$$

$$Q_{i(cal)} = \sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j \quad \text{for all } (n-1) \text{ buses.}$$

Step 7 : Calculate the change in real power and reactive power.

S

$$\Delta P_i = P_{i(spec)} - P_{i(cal)} \quad ; \text{for all } (n-1) \text{ buses}$$

$$\Delta Q_i = Q_{i(spec)} - Q_{i(cal)} \quad ; \text{for all } (n-1-m) \text{ buses or load buses.}$$

Step 8: check for generator buses. If it is generator bus go to next step 9, or if load bus go to step 11.

Step 9: check Q_i with specified values calculated in step 6.
If Q_i limits is violated go to step 11, otherwise go to next step 10.

Step 10: Calculate $\frac{\partial P_i}{\partial \delta_i}$, $\frac{\partial P_i}{\partial |V_i|}$ for all generator buses.

Step 11: If reactive power (Q_i) limit is violated then treat this bus as the load bus. Now the specified reactive power for this bus will correspond to the limit violated.

ie, if $Q_i < Q_{i\,min}$, then $Q_{i(spec)} = Q_{i\,min}$

if $Q_i > Q_{i\,max}$, then $Q_{i(spec)} = Q_{i\,max}$

Step 12: Calculate change in reactive power.

$$\Delta Q_i = Q_{i(spec)} - Q_{i\,(cal)}$$

Step 13: Repeat steps 5 to 12 until all residues ΔP_i, ΔQ_i are calculated. For this, increment bus count by 1, and go to step 5 until bus count is 'n'.

Step 14: Determine the largest of the absolute values of the residue. Let this largest change be ΔE.

Step 15: Compare ΔE and ϵ. If $\Delta E < \epsilon$ then go to step 20; If $\Delta E > \epsilon$ go to next step 16.

Step 16 : Determine the elements of J matrix using the equations.

Step 17 : Calculate the increments in real and reactive part of voltages (ie, $\Delta|V|$ and $\Delta\delta$) by solving the following matrix equation.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$$

Step 18 : Calculate the new bus voltages as shown below:

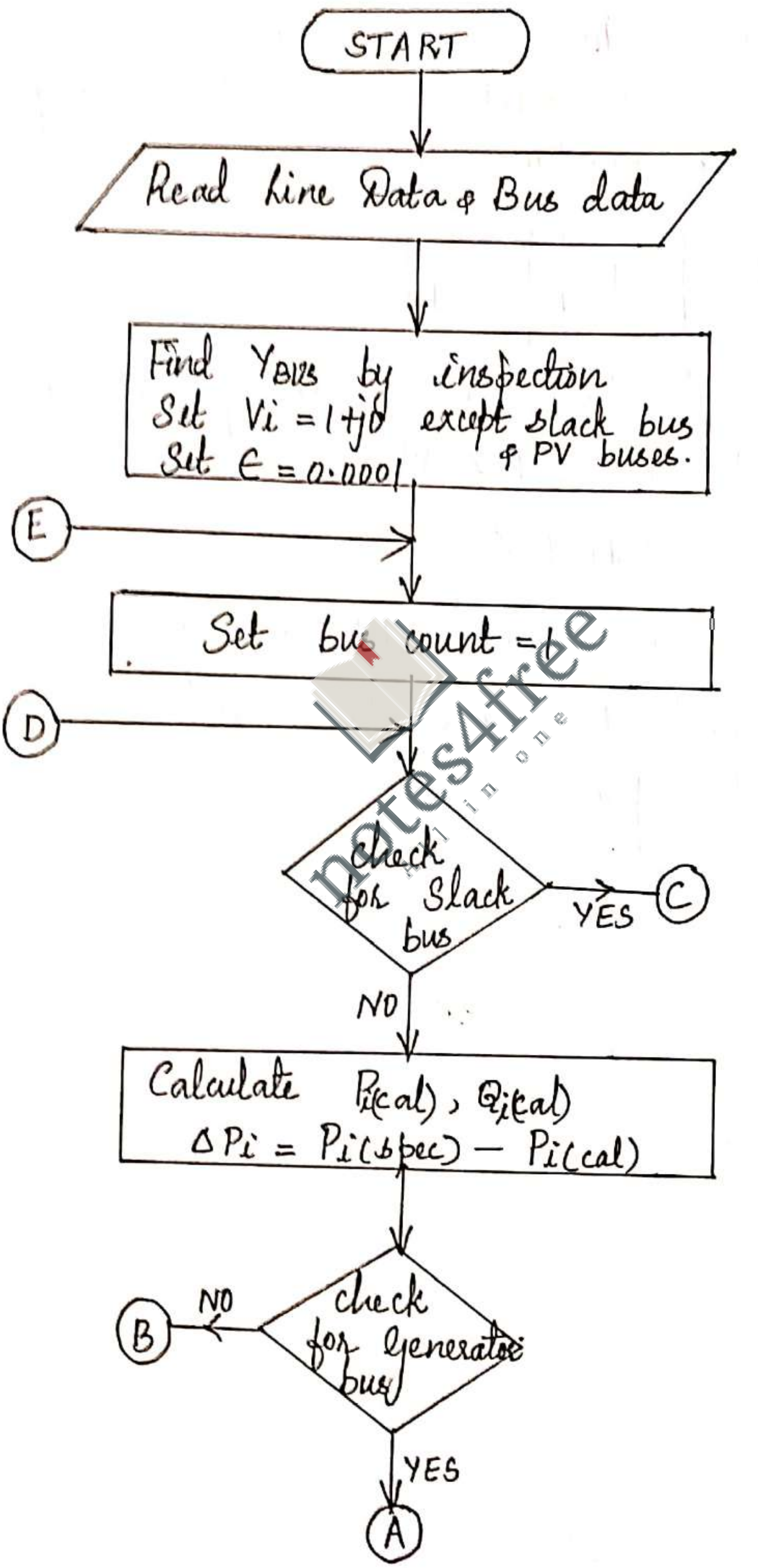
$$|V_{new}| = |V_{old}| + \Delta|V|$$

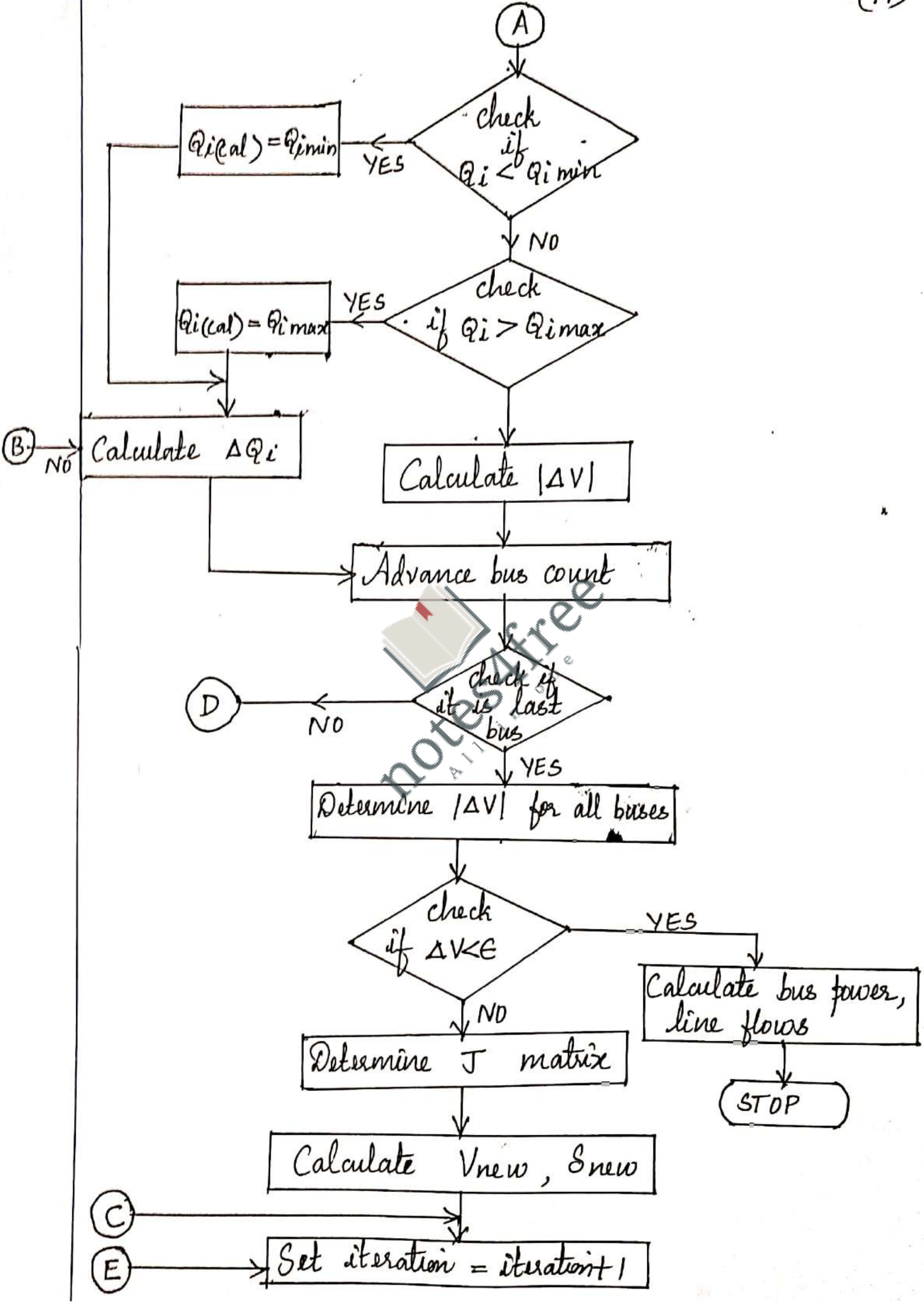
$$\delta_{new} = \delta_{old} + \Delta\delta$$

Step 19 : Iteration count = K+1 and go to step 4.

Step 20 : Calculate the line flows, line losses and also slack bus power.

Flow chart for NR method / FDLF method :-




A
check if Qi < Qi min
YES
Qi(cal) = Qimin
NO
check if Qi > Qimax
YES
Qi(cal) = Qimax
B
NO
Calculate ΔQi
Calculate |ΔV|
Advance bus count
check if it is last bus
NO
D
YES
Determine |ΔV| for all buses
check if ΔV<E
YES
Calculate bus power, line flows
STOP
NO
Determine J matrix
Calculate Vnew, δnew
C
E
Set iteration = iteration+1

Problems on NR method :-

1) Obtain the load flow solution by NR method for the data given below :-

Line data :-

SB	EB	R	X
1	2	0	0.1
1	3	0	0.2
2	3	0	0.2

Bus Data :-

Bus No.	P_G	P_L	Q_G	Q_L	$\lvert V \rvert$
1	—	—	—	—	1.0
2	5.3217	—	—	—	1.1
3	—	3.6392	—	0.5339	—

Soln:-

Step 1 : Form Y_{BUS} by Inspection and write it in polar form.

$$Y_{BUS} = \begin{bmatrix} \frac{1}{j0.1} + \frac{1}{j0.2} & -\frac{1}{j0.1} & -\frac{1}{j0.2} \\ -\frac{1}{j0.1} & \frac{1}{j0.1} + \frac{1}{j0.2} & -\frac{1}{j0.2} \\ -\frac{1}{j0.2} & -\frac{1}{j0.2} & \frac{1}{j0.2} + \frac{1}{j0.2} \end{bmatrix}$$

$$= \begin{bmatrix} -j15 & j10 & j5 \\ j10 & -j15 & j5 \\ j5 & j5 & -j10 \end{bmatrix} = \begin{bmatrix} 15\angle -90^\circ & 10\angle 90^\circ & 5\angle 90^\circ \\ 10\angle 90^\circ & 15\angle -90^\circ & 5\angle 90^\circ \\ 5\angle 90^\circ & 5\angle 90^\circ & 10\angle -90^\circ \end{bmatrix}$$

Step 2 : Assume flat voltage profile except slack bus & PV buses.

$V_1 = 1+j0$; $V_2 = 1.1+j0$; $V_3 = 1+j0$

Step 3 :

$P_2\,(\text{specified}) = P_{G2} - P_{L2} = 5.3217 - 0 = 5.3217$ pu

$P_3\,(\text{specified}) = P_{G3} - P_{L3} = 0 - 3.6392 = -3.6392$ pu

$Q_3\,(\text{specified}) = Q_{G3} - Q_{L3} = 0 - 0.5339 = -0.5339$ pu

Step 4: $P_{i(cal)} = \sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \cos \angle \delta_i - \theta_{ij} - \delta_j$

$\therefore P_{2(cal)} = \sum_{j=1}^{3} |V_2||Y_{2j}||V_j| \cos \angle \delta_2 - \theta_{2j} - \delta_j$

$= |V_2||Y_{21}||V_1| \cos \angle \delta_2 - \theta_{21} - \delta_1 + |V_2||Y_{22}||V_2| \cos \angle \delta_2 - \theta_{22} - \delta_2$

$+ |V_2||Y_{23}||V_3| \cos \angle \delta_2 - \theta_{23} - \delta_3$

$= 1.1 * 10 * 1 * \cos \angle 0 - 90 - 0 + 1.1 * 15 * 1.1 * \cos \angle 0 + 90 - 0$

$+ 1.1 * 5 * 1 * \cos \angle 0 - 90 - 0$

$= 0$ pu

$P_{3(cal)} = |V_3||Y_{31}||V_1| \cos \angle \delta_3 - \theta_{31} - \delta_1 + |V_3||Y_{32}||V_2| \cos \angle \delta_3 - \theta_{32} - \delta_2$

$+ |V_3||Y_{33}||V_3| \cos \angle \delta_3 - \theta_{33} - \delta_3$

$= 1 * 5 * 1 * \cos \angle 0 - 90 - 0 + 1 * 5 * 1.1 * \cos \angle 0 - 90 - 0$

$+ 1 * 10 * 1 * \cos \angle 0 + 90 - 0$

$= 0$ pu

$Q_{i(cal)} = \sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$

$Q_{3(cal)} = |V_3||Y_{31}||V_1| \sin \angle \delta_3 - \theta_{31} - \delta_1 + |V_3||Y_{32}||V_2| \sin \angle \delta_3 - \theta_{32} - \delta_2$

$+ |V_3||Y_{33}||V_3| \cos \angle \delta_3 - \theta_{33} - \delta_3$

$= 1 * 5 * 1 * \sin \angle 0 - 90 - 0 + 1 * 5 * 1.1 * \sin \angle 0 - 90 - 0$

$+ 1 * 10 * 1 * \sin \angle 0 + 90 - 0$

$= -0.5$ pu

Step 5: Calculate change in power.

$\Delta P_2 = P_{2(spec)} - P_{2(cal)} = 5.3217 - 0 = 5.3217$ pu

$\Delta P_3 = P_{3(spec)} - P_{3(cal)} = -3.6392 - 0 = -3.6392$ pu

$\Delta Q_3 = Q_{3(spec)} - Q_{3(cal)} = -0.5339 - (-0.5)$

$= -0.039$ pu

Step 6: Jacobian Matrix calculation

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_3| \end{bmatrix}$$

$$J_1 = \frac{\partial P}{\partial \delta}$$

$$\frac{\partial P_i}{\partial \delta_i} = -\sum_{\substack{j=1 \\ \neq i}}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

$$\frac{\partial P_i}{\partial \delta_j} = |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j \quad (i \neq j)$$

$$\frac{\partial P_2}{\partial \delta_2} = -\left\{ |V_2||Y_{21}||V_1| \sin \angle \delta_2 - \theta_{21} - \delta_1 + |V_2||Y_{23}||V_3| \sin \angle \delta_2 - \theta_{23} - \delta_3 \right\}$$

$$= -\left\{ 1.1 * 10 * 1 * \sin \angle 0 - 90 - 0 + 1.1 * 5 * 1 * \sin \angle 0 - 90 - 0 \right\}$$

$$= \underline{16.5}$$

$$\frac{\partial P_2}{\partial \delta_3} = |V_2||Y_{23}||V_3| \sin \angle \delta_2 - \theta_{23} - \delta_3$$

$$= 1.1 * 5 * 1 * \sin \angle 0 - 90 - 0$$

$$= \underline{-5.5}$$

$$\frac{\partial P_3}{\partial \delta_2} = \frac{\partial P_2}{\partial \delta_3} = -5.5$$

$$\frac{\partial P_3}{\partial \delta_3} = -\left\{ |V_3||Y_{31}||V_1| \sin \angle \delta_3 - \theta_{31} - \delta_1 + |V_3||Y_{32}||V_2| \sin \angle \delta_3 - \theta_{32} - \delta_2 \right\}$$

$$= -\left\{ 1 * 5 * 1 * \sin \angle 0 - 90 - 0 + 1 * 5 * 1.1 * \sin \angle 0 - 90 - 0 \right\}$$

$$= \underline{10.5}$$

$$J_2 \Rightarrow \frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}|\cos\angle{-\theta_{ii}} + \sum_{\substack{j=1\\ \neq i}}^{n} |Y_{ij}||V_j|\cos\angle{\delta_i-\theta_{ij}-\delta_j}$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos\angle{\delta_i-\theta_{ij}-\delta_j} \quad (j \neq i)$$

$$\frac{\partial P_2}{\partial |V_2|} = 2|V_2||Y_{22}|\cos\angle{-\theta_{22}} + |Y_{21}||V_1|\cos\angle{\delta_2-\theta_{21}-\delta_1} + |Y_{23}||V_3|\cos\angle{\delta_2-\theta_{23}-\delta_3}$$

$$= 2*1.1*15*\cos\angle{90} + 10*1*\cos\angle{0-90-0} + 5*1*\cos\angle{0-90-0}$$

$$= \underline{0}$$

$$\frac{\partial P_2}{\partial |V_3|} = |V_2||Y_{23}|\cos\angle{\delta_2-\theta_{23}-\delta_3}$$

$$= 1.1*5*\cos\angle{0-90-0}$$

$$= \underline{0}$$

$$J_3 \longrightarrow \frac{\partial Q}{\partial \delta}$$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1\\ \neq i}}^{n} |V_i||Y_{ij}||V_j|\cos\angle{\delta_i-\theta_{ij}-\delta_j}$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||Y_{ij}||V_j|\cos\angle{\delta_i-\theta_{ij}-\delta_j} \quad (j \neq i)$$

$$\frac{\partial Q_3}{\partial \delta_3} = |V_3||Y_{31}||V_1|\cos\angle{\delta_3-\theta_{31}-\delta_1} + |V_3||Y_{32}||V_2|\cos\angle{\delta_3-\theta_{32}-\delta_2}$$

$$= \underline{0} \qquad [\because \cos(90) = 0]$$

$$\frac{\partial Q_3}{\partial \delta_2} = -|V_3||Y_{32}||V_2|\cos\angle{\delta_3-\theta_{32}-\delta_2}$$

$$= \underline{0}$$

$J_4 \rightarrow \frac{\partial Q}{\partial |V|}$; $\frac{\partial Q_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \sin \angle -\theta_{ii} + \sum_{\substack{j=1 \\ \neq i}}^{n} |Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$

$$\frac{\partial Q_3}{\partial |V_3|} = 2|V_3||Y_{33}| \sin \angle -\theta_{33} + |Y_{31}||V_1| \sin \angle \delta_3 - \theta_{31} - \delta_1 + |Y_{32}||V_2| \sin \angle \delta_3 - \theta_{32} - \delta_2$$

$$= 2 * 1 * 10 * \sin \angle 90 + 5 * 1 * \sin \angle -90^\circ + 5 * 1.1 * \sin \angle -90^\circ$$

$$= \underline{9.5}$$

Step 7: Calculate $\Delta\delta_2$, $\Delta\delta_3$, $\Delta|V_3|$

$$\begin{bmatrix} 5.3217 \\ -3.6392 \\ -0.0339 \end{bmatrix} = \underbrace{\begin{bmatrix} 16.5 & -5.5 & 0 \\ -5.5 & 10.5 & 0 \\ 0 & 0 & 9.5 \end{bmatrix}}_{J} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix}$$

Inverse of Jacobian matrix,

$$J^{-1} = \begin{bmatrix} 0.0734 & 0.0385 & 0 \\ 0.0385 & 0.1154 & 0 \\ 0 & 0 & 0.1053 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix} = \begin{bmatrix} 0.0734 & 0.0385 & 0 \\ 0.0385 & 0.1154 & 0 \\ 0 & 0 & 0.1053 \end{bmatrix}_{3\times1} \begin{bmatrix} 5.3217 \\ -3.6392 \\ -0.0339 \end{bmatrix}_{3\times1}$$

$$= \begin{bmatrix} 0.2508 \\ -0.2152 \\ -0.0036 \end{bmatrix}$$; $\Delta\delta_2 = 0.2508$ radians

$\Delta\delta_3 = -0.2152$ radians

Converting $\Delta\delta_2$ & $\Delta\delta_3$ into degrees,

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_3| \end{bmatrix} = \begin{bmatrix} 0.2508 * \frac{180}{\pi} \\ -0.2152 * \frac{180}{\pi} \\ -0.0036 \end{bmatrix} = \begin{bmatrix} 14.37 \\ -12.33 \\ -0.0036 \end{bmatrix}$$

$\delta_2(new) = \delta_2(old) + \Delta\delta_2 = 0 + 14.37 = 14.37$

$\delta_3(new) = \delta_3(old) + \Delta\delta_3 = 0 - 12.33 = -12.33$

$|V_3|_{new} = |V_3|(new) + \Delta|V_3| = 1 - 0.0036 = \underline{0.9964}$

Ans:-

At the end of first iteration,

$V_1 = 1.0 \angle 0^\circ$

$V_2 = 1.1 \angle 14.37^\circ$

$V_3 = 0.9964 \angle -12.33^\circ$

2) Obtain the load flow analysis for the given data :-

Bus code	Impedance (pu)	Line charging admittance
1−2	0.08 + j0.24	0
1−3	0.02 + j0.06	0
2−3	0.06 + j0.18	0

Bus No.	Bus voltage	Generation		Load	
		MW	MVAr	MW	MVAr
1	1.05 + j0	—	—	—	—
2	0	0	0	50	20
3	0	0	0	60	25

Soln:-

Step 1 : Form Y_{BUS} by Inspection; Convert into polar form:-

$$Y_{BUS} = \begin{bmatrix} 6.25-j18.75 & 1.25+j3.75 & -5+j15 \\ -1.25+j3.75 & 2.9167-j8.75 & -1.667+j5 \\ -5+j15 & -1.6667+j5.0 & 6.66-j20 \end{bmatrix}$$

$$= \begin{bmatrix} 19.7642\angle -71.6 & 3.95285\angle 108.4 & 15.8114\angle 108.4 \\ 3.95286\angle 108.4 & 9.2233\angle -71.6 & 5.27046\angle 108.4 \\ 15.8114\angle 108.4 & 5.27046\angle 108.4 & 21.0819\angle -71.63 \end{bmatrix}$$

Step 2 : Assume flat voltage profile for all buses except slack bus and PV bus.

$V_1 = 1.05+j0$; $V_2 = 1+j0$; $V_3 = 1+j0$

Step 3:

$$P_{2(spec)} = P_{G2} - P_{L2} = 0-50 = \frac{-50}{100} = -0.5 \text{ pu}$$

$$Q_{2(spec)} = Q_{G2} - Q_{L2} = 0-20 = \frac{-20}{100} = -0.2 \text{ pu}$$

$$P_{3(spec)} = 0-60 = -\frac{60}{100} = -0.6 \text{ pu}$$

$$Q_{3(spec)} = 0-25 = -\frac{25}{100} = -0.25 \text{ pu}.$$

Step 4:

$$P_{i(cal)} = \sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \cos \angle \delta_i - \theta_{ij} - \delta_j$$

$$Q_{i(cal)} = \sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

$$P_{2(cal)} = |V_2||Y_{21}||V_1| \cos\angle \delta_2-\theta_{21}-\delta_1 + |V_2||Y_{22}||V_2| \cos\angle \delta_2-\theta_{22}-\delta_2$$
$$+ |V_2||Y_{23}||V_3| \cos \angle \delta_2-\theta_{23}-\delta_3$$

$$= 1*3.95286*1.05*\cos\angle -108.4 + 1*9.2233*1*\cos\angle 71.6$$
$$+ 1*5.27046*1*\cos\angle -108.4 = -0.0627 \text{ pu}$$

$$P_{3(cal)} = |V_3||Y_{31}||V_1|\cos\angle\delta_3-\theta_{31}-\delta_1 + |V_3||Y_{32}||V_2|\cos\angle\delta_3-\theta_{32}-\delta_2 + |V_3||Y_{33}||V_3|\cos\angle\delta_2-\theta_{33}-\delta_3$$

$$= 1\times 15.8114\times 1.05\times\cos\angle -108.4 + 1\times 5.27046\times 1\times\cos\angle -108.4 + 1\times 21.0819\times 1\times\cos\angle 71.63$$

$$= -0.26 \text{ pu}$$

$$Q_{2(cal)} = |V_2||Y_{21}||V_1|\sin\angle\delta_2-\theta_{21}-\delta_1 + |V_2||Y_{22}||V_2|\sin\angle\delta_2-\theta_{22}-\delta_2 + |V_2||Y_{23}||V_3|\sin\angle\delta_2-\theta_{23}-\delta_3$$

$$= 1\times 3.95286\times 1.05\times\sin\angle -108.4 + 1\times 9.2233\times 1\sin\angle 71.6 + 1\times 5.27046\times 1\times\sin\angle -108.4$$

$$= -0.1875 \text{ pu}$$

$$Q_{3(cal)} = |V_3||Y_{31}||V_1|\sin\angle\delta_3-\theta_{31}-\delta_1 + |V_3||Y_{32}||V_2|\sin\angle\delta_3-\theta_{32}-\delta_2 + |V_3||Y_{33}||V_3|\sin\angle\delta_2-\theta_{33}-\delta_3$$

$$= 1\times 15.8114\times 1.05\times\sin\angle -108.4 + 1\times 5.27046\times 1\times\sin\angle -108.4 + 1\times 21.0819\times 1\times\sin\angle 71.63$$

$$= -0.7466 \text{ pu}$$

Step 5: Calculate the change in power.

$$\Delta P_2 = P_{2(sch)} - P_{2(cal)} = -0.5 - (-0.0627) = -0.4373$$

$$\Delta P_3 = -0.6 - (-0.26) = -0.34 \text{ pu}$$

$$\Delta Q_2 = -0.2 - (-0.1875) = -0.0125 \text{ pu}$$

$$\Delta Q_3 = -0.25 - (-0.7466) = -0.4966 \text{ pu}$$

$J_2 = \frac{\partial P}{\partial |V|}$; $\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}|\cos\angle{-\theta_{ii}} + \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij}||V_j| \cos\angle{\delta_i - \theta_{ij} - \delta_j}$

$\frac{\partial P_2}{\partial |V_2|} = 2|V_2||Y_{22}|\cos\angle{-\theta_{22}} + |Y_{21}||V_1|\cos\angle{\delta_2 - \theta_{21} - \delta_1} + |Y_{23}||V_3|\cos\angle{\delta_2 - \theta_{23} - \delta_3}$

$= 2 \times 1 \times 9.2233 \times \cos\angle{71.63} + 3.95286 \times 1.05 \times \cos\angle{-108.4} + 5.27046 \times 1 \times \cos\angle{-108.4}$

$= \underline{\underline{2.8489}}$

$\frac{\partial P_3}{\partial |V_3|} = 2|V_3||Y_{33}|\cos\angle{-\theta_{33}} + |Y_{31}||V_1|\cos\angle{\delta_3 - \theta_{31} - \delta_1} + |Y_{32}||V_2|\cos\angle{\delta_3 - \theta_{32} - \delta_2}$

$= 2 \times 1 \times 21.0819 \times \cos\angle{71.63} + 15.8114 \times 1.05 \times \cos\angle{-108.4} + 5.27046 \times 1 \times \cos\angle{-108.4}$

$= \underline{6.3841}$

$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos\angle{\delta_i - \theta_{ij} - \delta_j} \quad (j \neq i)$

$\frac{\partial P_2}{\partial |V_3|} = |V_2||Y_{23}|\cos\angle{\delta_2 - \theta_{23} - \delta_3}$

$= 1 \times 5.27046 \times \cos\angle{-108.4}$

$= \underline{\underline{-1.6636}}$

$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos\angle{\delta_3 - \theta_{32} - \delta_2}$

$= 1 \times 5.27046 \times \cos\angle{-108.4}$

$= \underline{\underline{-1.6636}}$

Step 6 : Calculate the Jacobian matrix.

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \hdashline \Delta Q_2 \\ \Delta Q_3 \end{bmatrix} = \left[\begin{array}{cc:cc} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} \\ \hdashline \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} \end{array}\right] \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \hdashline \Delta |V_2| \\ \Delta |V_3| \end{bmatrix}$$

$$J_1 = \frac{\partial P}{\partial \delta} \; ; \; \frac{\partial P_i}{\partial \delta_i} = -\sum_{\substack{j=1 \\ \neq i}}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

$$\frac{\partial P_2}{\partial \delta_2} = -\left\{ |V_2||Y_{21}||V_1| \sin \angle \delta_2 - \theta_{21} - \delta_1 + |V_2||Y_{23}||V_3| \sin \angle \delta_2 - \theta_{23} - \delta_3 \right\}$$

$$= -\left\{ 1 \times 3.95286 \times 1.05 \times \sin \angle -108.4 + 1 \times 5.27046 \times 1 \times \sin \angle -108.4 \right\}$$

$$= \underline{\underline{8.939}}$$

$$\frac{\partial P_3}{\partial \delta_3} = -\left\{ |V_3||Y_{31}||V_1| \sin \angle \delta_3 - \theta_{31} - \delta_1 + |V_3||Y_{32}||V_2| \sin \angle \delta_3 - \theta_{32} - \delta_2 \right\}$$

$$= -\left\{ 1 \times 15.8114 \times 1.05 \times \sin \angle -108.4 + 1 \times 5.27046 \times 1 \times \sin \angle -108.4 \right\}$$

$$= \underline{\underline{20.7542}}$$

$$\frac{\partial P_i}{\partial \delta_j} = |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j \qquad (j \neq i)$$

$$\frac{\partial P_2}{\partial \delta_3} = |V_2||Y_{23}||V_3| \sin \angle \delta_2 - \theta_{23} - \delta_3$$

$$= 1 \times 5.27046 \times 1 \times \sin \angle -108.4$$

$$= \underline{\underline{-5}}$$

$$\frac{\partial P_3}{\partial \delta_2} = |V_3||Y_{32}||V_2| \sin \angle \delta_3 - \theta_{32} - \delta_2$$

$$= 1 \times 5.27046 \times 1 \times \sin \angle -108.4$$

$$= \underline{\underline{-5}}$$

$J_3 = \frac{\partial Q}{\partial \delta}$; $\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ \neq i}}^{n} |V_i||Y_{ij}||V_j| \cos \angle \delta_i - \theta_{ij} - \delta_j$

$\frac{\partial Q_2}{\partial \delta_2} = |V_2||Y_{21}||V_1| \cos \angle \delta_2 - \theta_{21} - \delta_1 + |V_2||Y_{23}||V_3| \cos \angle \delta_2 - \theta_{23} - \delta_3$

$= 1 * 3.95286 * 1.05 * \cos \angle -108.4 + 1 * 5.27046 * 1 * \cos \angle -108.4$

$= -2.9737$

$\frac{\partial Q_3}{\partial \delta_3} = |V_3||Y_{31}||V_1| \cos \angle \delta_3 - \theta_{31} - \delta_1 + |V_3||Y_{32}||V_2| \cos \angle \delta_3 - \theta_{32} - \delta_2$

$= 1 * 15.8114 * 1.05 * \cos \angle -108.4 + 1 * 5.27046 * 1 * \cos \angle -108.4$

$= -6.904$

$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||Y_{ij}||V_j| \cos \angle \delta_i - \theta_{ij} - \delta_j \quad (j \neq i)$

$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||Y_{23}||V_3| \cos \angle \delta_2 - \theta_{23} - \delta_3$

$= -1 * 5.27046 * 1 * \cos \angle -108.4 = 1.6636$

$\frac{\partial Q_3}{\partial \delta_2} = -|V_3||Y_{32}||V_2| \cos \angle \delta_3 - \theta_{32} - \delta_2$

$= -1 * 5.27046 * 1 * \cos \angle -108.4 = 1.6636$

$J_4 = \frac{\partial Q}{\partial |V|}$;

$\frac{\partial Q_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \sin \angle -\theta_{ii} + \sum_{\substack{j=1 \\ \neq i}}^{n} |Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$

$\frac{\partial Q_2}{\partial |V_2|} = 2|V_2||Y_{22}| \sin \angle -\theta_{22} + |Y_{21}||V_1| \sin \angle \delta_2 - \theta_{21} - \delta_1$
$+ |Y_{23}||V_3| \sin \angle \delta_2 - \theta_{23} - \delta_3$

$= 2 * 1 * 9.2233 * \sin \angle 71.6 + 3.95286 * 1.05 * \sin \angle -108.4$
$+ 5.27046 * 1 * \sin \angle -108.4$

$= 8.5642$

$$\frac{\partial Q_3}{\partial |V_3|} = 2|V_3||Y_{33}|\sin\angle -\theta_{33} + |Y_{31}||V_1|\sin\angle \delta_3 - \theta_{31} - \delta_1 + |Y_{32}||V_2|\sin\angle \delta_3 - \theta_{32} - \delta_2$$

$$= 2*1*21.0819*\sin\angle 71.63 + 15.8114*1.05*\sin\angle -108.4 + 5.27046*1*\sin\angle -108.4$$

$$= \underline{\underline{19.2609}}$$

$$\frac{\partial Q_i}{\partial |V_j|} = |V_i||Y_{ij}|\sin\angle \delta_i - \theta_{ij} - \delta_j \quad (j \neq i)$$

$$\frac{\partial Q_2}{\partial |V_3|} = \frac{\partial Q_3}{\partial |V_2|} = |V_2||Y_{23}|\sin\angle \delta_2 - \theta_{23} - \delta_3$$

$$= 1*5.27046*\sin\angle -108.4)$$

$$= \underline{\underline{-5}}$$

Step 7: Calculate $\Delta\delta_2, \Delta\delta_3, \Delta|V_2|, \Delta|V_3|$

$$\begin{bmatrix} -0.4373 \\ -0.34 \\ -0.0125 \\ -0.4966 \end{bmatrix} = \begin{bmatrix} 8.939 & -5 & 2.8489 & -1.6636 \\ -5 & 20.7542 & -1.6636 & 6.3841 \\ -2.9737 & 1.6636 & 8.5642 & -5 \\ 1.6636 & -6.904 & -5 & 19.2609 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \\ \Delta|V_3| \end{bmatrix}$$

$$J^{-1} = \begin{bmatrix} 0.1164 & 0.0281 & -0.0387 & -0.0093 \\ 0.0281 & 0.0502 & -0.0093 & -0.0166 \\ 0.0412 & 0.0107 & 0.1239 & 0.0322 \\ 0.0107 & 0.0183 & 0.0322 & 0.0551 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \\ \Delta|V_3| \end{bmatrix} = \begin{bmatrix} 0.1164 & 0.0281 & -0.0387 & -0.0093 \\ 0.0281 & 0.0502 & -0.0093 & -0.0166 \\ 0.0412 & 0.0107 & 0.1239 & 0.0322 \\ 0.0107 & 0.0183 & 0.0322 & 0.0551 \end{bmatrix}_{4\times4} \begin{bmatrix} -0.4373 \\ -0.3400 \\ -0.0125 \\ -0.4966 \end{bmatrix}_{4\times1}$$

$$= \begin{bmatrix} -0.0554 \\ -0.0210 \\ -0.0392 \\ -0.0387 \end{bmatrix}$$

Step 8: Calculate the new values of $\Delta\delta_2, \Delta\delta_2, \Delta|V_2|, \Delta|V_3|$

$\delta_{2(new)} = \delta_{2(old)} + \Delta\delta_2 = 0 - 0.0554 = -0.0554$ radians $= -3.176^\circ$

$\delta_{3(new)} = 0 + (-0.0210) = -1.204^\circ$

$|V_2|_{new} = 1 - 0.0392 = 0.9608$ pu

$|V_3|_{new} = 1 - 0.0387 = 0.9613$ pu.

At the end of 1st iteration,

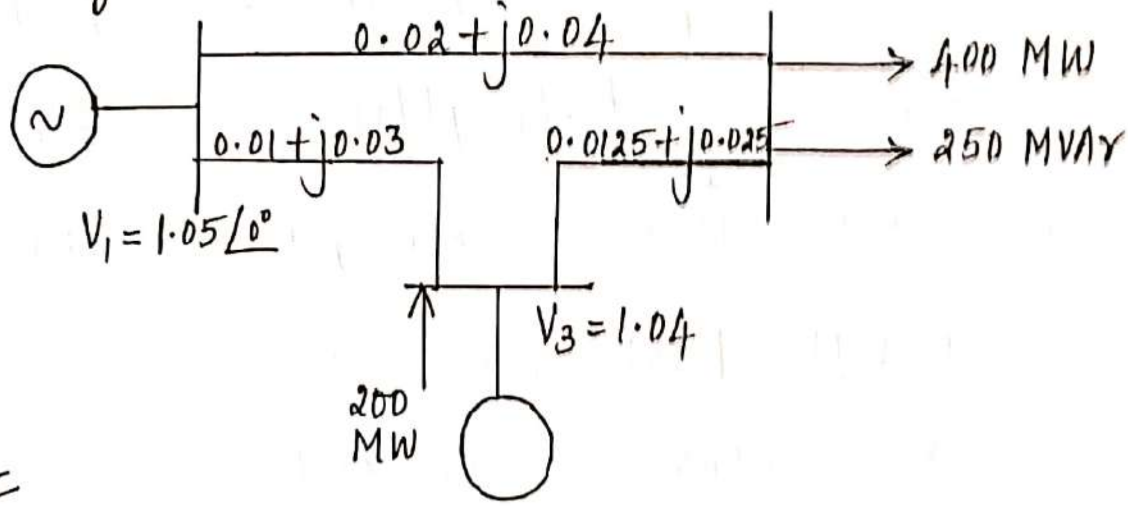
$V_1 = 1.05\angle 0^\circ = 1.05 + j0$

$V_2 = 0.9608\angle -3.176^\circ = 0.9593 - j0.0532$

$V_3 = 0.9613\angle -1.204^\circ = 0.9611 - j0.0202$

Problems on FDLF method:

1) Obtain the values of voltage magnitudes and angles at various buses using FDLF method for the following power system.



Soln:-

Step 1: Form Y_{BUS} by Inspection; Convert it in polar form.

$$Y_{BUS} = \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

$$= \begin{bmatrix} 53.85\angle 68.19 & 22.36\angle 116.57 & 31.62\angle 108.43 \\ 22.36\angle 116.57 & 58.14\angle -63.43 & 35.77\angle 116.56 \\ 31.62\angle 108.43 & 35.77\angle 116.56 & 67.23\angle -67.24 \end{bmatrix}$$

Step 2: Assume flat voltage profile except slack and PV bus.

$$V_1 = 1.05+j0 \;;\; V_2 = 1+j0 \;;\; V_3 = 1.04+j0$$

Step 3:

$$P_{2(spec)} = P_{G2} - P_{L2} = 0 - 400\text{ MW} = \frac{-400}{100} = -4\text{ pu}$$

$$Q_2(spec) = Q_{G2} - Q_{L2} = -\frac{250}{100} = -2.5 \text{ pu}$$

$$P_3(spec) = P_{G3} - P_{L3} = 200 - 0 = \frac{200}{100} = 2.0 \text{ pu}$$

Step 4:

$$P_i(cal) = \sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \cos \angle \delta_i - \theta_{ij} - \delta_j$$

$$Q_i(cal) = \sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

$$P_2(cal) = |V_2||Y_{21}||V_1| \cos \angle \delta_2 - \theta_{21} - \delta_1 + |V_2||Y_{22}||V_2| \cos \angle \delta_2 - \theta_{22} - \delta_2$$

$$+ |V_2||Y_{23}||V_3| \cos \angle \delta_2 - \theta_{23} - \delta_3$$

$$= 1 * 22.36 * 1.05 * \cos(-116.57) +$$
$$1 * 58.14 * 1 * \cos(63.43) +$$
$$1 * 35.78 * 1.04 * \cos(-116.56)$$

$$= -10.5 + 26.006 - 16.64 = -1.14 \text{ pu}$$

$$Q_2(cal) = |V_2||Y_{21}||V_1| \sin \angle \delta_2 - \theta_{21} - \delta_1 + |V_2||Y_{22}||V_2| \sin \angle \delta_2 - \theta_{22} - \delta_2$$

$$+ |V_2||Y_{23}||V_3| \sin \angle \delta_2 - \theta_{23} - \delta_3$$

$$= 1 * 22.36 * 1.05 * \sin(-116.57) +$$
$$1 * 58.14 * 1 * \sin(63.43) +$$
$$1 * 35.78 * 1.04 * \sin(-116.56)$$

$$= -2.28 \text{ pu}$$

$$P_3(cal) = |V_3||Y_{31}||V_1| \cos \angle \delta_3 - \theta_{31} - \delta_1 + |V_3||Y_{32}||V_2| \cos \angle \delta_3 - \theta_{32} - \delta_2$$

$$+ |V_3||Y_{33}||V_3| \cos \angle \delta_3 - \theta_{33} - \delta_3$$

$= 1.04 * 31.62 * 1.05 * \cos(-108.43) +$

$1.04 * 35.77 * 1 * \cos(-116.56) +$

$1.04 * 67.23 * 1.04 * \cos(+67.24)$

$= -10.916 - 16.634 + 28.132 = \underline{0.5817}$

Step 5: Calculate the change in power.

$\Delta P_2 = P_{2(spec)} - P_{2(cal)} = -4 - (-1.14) = -2.86$ pu.

$\Delta P_3 = P_{3(spec)} - P_{3(cal)} = 2 - 0.5817 = 1.4183$ pu

$\Delta Q_2 = Q_{2(spec)} - Q_{2(cal)} = -2.5 - (-2.28) = \underline{-0.22}$ pu

Step 6:

$[\Delta P] = -[B'][\Delta \delta]$

$[\Delta Q] = -[B''][\Delta V]$

$[\Delta \delta] = -[B']^{-1}[\Delta P]$

$[\Delta V] = -[B'']^{-1}[\Delta Q]$

$$B' = \begin{matrix} & 2 & 3 \\ 2 & -52 & 32 \\ 3 & 32 & -62 \end{matrix} \quad ; \quad B'' = 2\left[-52\right] \text{ (column 2)}$$

$$[B']^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014543 & -0.023636 \end{bmatrix} \quad ; \quad [B'']^{-1} = [-0.019]$$

Step 7:

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} = - \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014546 & -0.023636 \end{bmatrix} \begin{bmatrix} -2.86 \\ 1.4384 \end{bmatrix}$$

$$= \begin{bmatrix} -0.060483 \\ -0.0076 \end{bmatrix} \text{ radians}$$

Step 8:

$$[\Delta V_2] = - [-0.019][-0.22] = -0.00418 \text{ pu}$$

Step 9:

$$\delta_{2(new)} = \delta_{2(old)} + \Delta\delta_2 = 0 + (-0.060483)$$
$$= -0.060483 \text{ radians}$$
$$= -3.467^\circ$$

$$\delta_{3(new)} = \delta_{3(old)} + \Delta\delta_3 = 0 + (-0.0076) = -0.0076 \text{ radians}$$
$$= 0.4357^\circ$$

$$V_2(new) = V_2(old) + \Delta|V_3| = 1.0 + (-0.00418)$$
$$= 0.99582 \text{ pu}$$

At the end of 1st iteration,

$$V_1 = 1.05 \angle 0^\circ$$

$$V_2 = 0.99582 \angle -3.467^\circ$$

$$V_3 = 1.04 \angle 0.4357^\circ$$

2) Obtain the load flow analysis for the given data by FDLF method:-

Bus code	Impedance (pu)	Line charging admittance
1–2	0.08+j0.24	0
1–3	0.02+j0.06	0
2–3	0.06+j0.18	0

Bus No.	Bus Voltage	Generation		Load	
		MW	MVAr	MW	MVAr
1	1.05+j0	–	–	–	–
2	–	0	0	50	20
3	–	0	0	60	25

Soln:

Step 1: Form Y_{BUS} by Inspection.

$$Y_{BUS} = \begin{bmatrix} 6.25-j18.75 & 1.25+j3.75 & -5+j15 \\ -1.25+j3.75 & 2.9167-j8.75 & -1.667+j5 \\ -5+j15 & -1.6667+j5.0 & 6.66-j20 \end{bmatrix}$$

$$= \begin{bmatrix} 19.7642\angle -71.6 & 3.95285\angle 108.4 & 15.8114\angle 108.4 \\ 3.95286\angle 108.4 & 9.2233\angle -71.6 & 5.27046\angle 108.4 \\ 15.8114\angle 108.4 & 5.27046\angle 108.4 & 21.0819\angle -71.63 \end{bmatrix}$$

Step 2: Assume flat voltage profile for all buses except slack & PV bus.

$V_1 = 1.05+j0$; $V_2 = 1+j0$; $V_3 = 1+j0$

Step 3:

$P_2(spec) = -0.5$ pu ; $P_3(spec) = -0.6$ pu

$Q_3(spec) = -0.25$ pu ; $Q_2(spec) = -0.2$ pu.

Step 4:

$$P_i(cal) = \sum_{\substack{j=1 \\ \neq i}}^{n} |V_i||Y_{ij}||V_j| \cos \angle \delta_i - \theta_{ij} - \delta_j$$

$$Q_i(cal) = \sum_{j=1}^{n} |V_i||Y_{ij}||V_j| \sin \angle \delta_i - \theta_{ij} - \delta_j$$

(For calculations refer problem (2) of NR method – same)

$P_2(cal) = -0.0627$ pu

$P_3(cal) = -0.26$ pu

$Q_2(cal) = -0.1875$ pu

$Q_3(cal) = -0.7466$ pu

Step 5:

$\Delta P_2 = -0.4373$; $\Delta P_3 = -0.34$ pu

$\Delta Q_2 = -0.0125$; $\Delta Q_3 = -0.4966$ pu.

Step 6:

$$[\Delta P] = -[B'][\Delta\delta]$$

$$[\Delta Q] = -[B''][\Delta V]$$

$$\therefore [\Delta\delta] = -[B']^{-1}[\Delta P]$$

$$[\Delta V] = -[B'']^{-1}[\Delta Q]$$

$$B' = \begin{bmatrix} -8.75 & 5 \\ 5 & -20 \end{bmatrix} ; \quad B'' = \begin{bmatrix} -8.75 & 5 \\ 5 & -20 \end{bmatrix}$$

$$[B']^{-1} = \begin{bmatrix} -0.1333 & -0.0333 \\ -0.0333 & -0.0583 \end{bmatrix}$$

Step 7:

$$\therefore \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} = -\begin{bmatrix} -0.1333 & -0.0333 \\ -0.0333 & -0.0583 \end{bmatrix}\begin{bmatrix} -0.4373 \\ -0.34 \end{bmatrix} = \begin{bmatrix} -0.0696 \\ -0.03438 \end{bmatrix}$$

Step 8:

$$\begin{bmatrix} \Delta|V_2| \\ \Delta|V_3| \end{bmatrix} = -\begin{bmatrix} -0.1333 & -0.0333 \\ -0.0333 & -0.0583 \end{bmatrix}\begin{bmatrix} -0.0125 \\ -0.4966 \end{bmatrix} = \begin{bmatrix} -0.0182 \\ -0.0294 \end{bmatrix}$$

Step 9:

$$\delta_{2(new)} = \delta_{2(old)} + \Delta\delta_2 = 0 + (-0.0696) = -0.0696 \text{ radians}$$

$$\delta_{3(new)} = \delta_{3(old)} + \Delta\delta_3 = 0 + (-0.03438) = -0.03438 \text{ radians}$$

$$|V_{2(new)}| = |V_{2(old)}| + \Delta|V_2| = 1 + (-0.0182) = 0.9818 \text{ pu}$$

$$|V_{3(new)}| = |V_{3(old)}| + \Delta|V_3| = 1 + (-0.0294) = 0.9706 \text{ pu}$$

At the end of 1st iteration,

$$V_1 = 1.05 \angle 0^\circ$$

$$V_2 = 0.9818 \angle -3.99^\circ$$

$$V_3 = 0.9706 \angle -1.97^\circ$$

Voltage control methods:

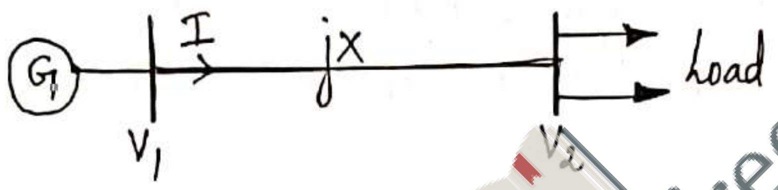
→ Voltage at various buses in power system should remain constant or vary within prescribed limits.

→ Voltage variation in a bus or node is directly related to reactive power Q.

→ If Q injected at bus is less than Q drawn from it, then voltage at bus decreases and vice versa.

Proof:

Consider a two-bus system.



Complex power, $S = P + jQ = V_1 I^*$

$$P - jQ = V_1^* I$$

$$\therefore \quad I = \frac{P - jQ}{V_1^*} = \frac{P - jQ}{V_1} \qquad \left[\because V_1^* \simeq V_1 \text{ as } \delta = 0\right]$$

Also, $V_2 = V_1 - jIX$

$$= V_1 - j\left(\frac{P - jQ}{V_1}\right)X$$

$$= V_1 - j\frac{PX}{V_1} - \frac{QX}{V_1}$$

$$V_2 = \left(V_1 - \frac{Q}{V_1}X\right) - j\frac{P}{V_1}X$$

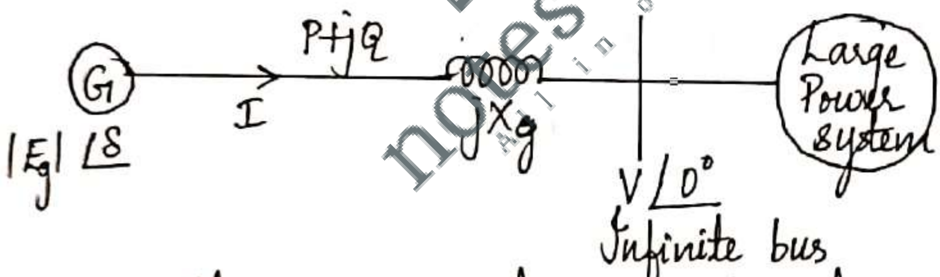
From the above equation, voltage drop and voltage at bus is directly proportional to reactive power Q.

Methods:

(1) By adjusting the excitation of generators.
(2) Using shunt capacitors
(3) Using series capacitors.
(4) Using synchronous condenser
(5) Using Tap changing transformers
(6) Using regulating and Booster transformers.

(1) Voltage control by adjusting the excitation of generators.

* Over excited generator supply reactive power to the system
* Under excited generator ~~supply~~ absorb reactive power from the system.



Let us consider a generator connected to an infinite bus bar.

→ Infinite bus bar is one whose voltage remain constant and cannot be altered by any changes in Generator excitation.

→ So any changes in excitation will change only E_g and not the terminal voltage of generator [since terminal point is connected to infinite bus]

→ Generator at the bus is modelled by a synchronous reactance X_g.

Let $|E_g|\angle\delta \longrightarrow$ Generated emf.

$$I = \frac{|E_g|\angle\delta - |V|\angle 0^\circ}{jX_g}$$

$$I^* = \frac{|E_g|\angle -\delta - |V|}{-jX_g}$$

Complex power, $S = P+jQ = VI^*$

$$P+jQ = |V|\angle 0 \left[\frac{|E_g|\angle -\delta - |V|}{-jX_g}\right]$$

$$= \frac{|V||E_g|\angle -\delta}{-jX_g} - \frac{|V|^2}{-jX_g}$$

$$P+jQ = \frac{|V||E_g|\angle 90^\circ-\delta}{X_g} - \frac{|V|^2\angle 90^\circ}{X_g}$$

where, $P = \frac{|V||E_g|}{X_g}\cos(90^\circ-\delta)$

$$Q = \frac{|V||E_g|}{X_g}\sin(90-\delta) - \frac{|V|^2}{X_g}$$

Hence, power angle δ can be varied by varying the generator excitation.

At a bus with generation, voltage can be conveniently controlled by adjusting generator excitation.

(2) Voltage control by shunt capacitors :

→ Bus voltage can be controlled by providing shunt capacitors at buses where reactive power demand is high.

→ The capacitor banks may be permanently connected to the system or can be varied by switching ON OR OFF the parallel connected capacitors, depending on the load demand.

→ The switching can be manual or automatic.

→ When the capacitor banks are connected parallel to a lagging load, they supply part or full reactive power.

→ Thus capacitors reduce line current and also reduce the voltage drop in the line which also results in improving power factor.

$$I_C = \frac{E_{TH}}{Z_{TH} - jX_C}$$

$$V = E_{TH} + I_C (R_{TH} + jX_{TH})$$

→ Shunt capacitors are connected either directly to a bus bar or through a tertiary winding of the main transformer.

Drawback :

→ Capacitor output voltage is large even with less reactive power requirement.

(3) **Voltage control by Series capacitor :**

When the transmission line has a high value of X/R ratio, the inductive reactance of the line can be reduced by introducing series capacitor.

Drawback :-

* Series capacitor may get damaged due to the high voltage produced under fault (short circuit) conditions.

(4) **Voltage control by synchronous capacitors :**

* Synchronous capacitors are over excited synchronous motors running on no-load.

* $$P_g = \frac{|E_g||V|}{X_g}\cos(90-\delta) = \frac{|E_g||V|}{X_g}\sin\delta$$

* $$Q_g = \frac{|V||E_g|}{X_g}\sin(90-\delta) - \frac{|V|^2}{X_g} = \frac{|V|}{X_g}\left[|E_g|\cos\delta - |V|\right]$$

* If $|E_g|\cos\delta > |V|$, then reactive power $Q_g > 0$ and the synchronous machine produces reactive power. i.e, It acts as a shunt capacitor. This is called over excitation.

* If $|E_g|\cos\delta < |V|$, then $Q_g < 0$ and the synchronous machine absorbs reactive power. i.e, It acts as an inductor. This is called under excited condition.

* The synchronous motor is operated on no-load for reactive power control. Therefore $\delta \simeq 0$ and so $|E_g| \cos \delta \simeq |E_g|$.

* Hence for reactive power control the $|E_g|$ alone can be varied by varying the excitation of the synchronous machine.

Advantages:

* It is flexible for use at all loads (both leading and lagging)

Disadvantages:

* It increases the fault level of the system.
* It can fall out of step which may result in a large sudden change in voltage.

(5) **Voltage control by Tap changing transformer:**

→ In tap changing transformer, the system voltage can be varied upto ± 10% by changing the tap setting in the secondary winding.

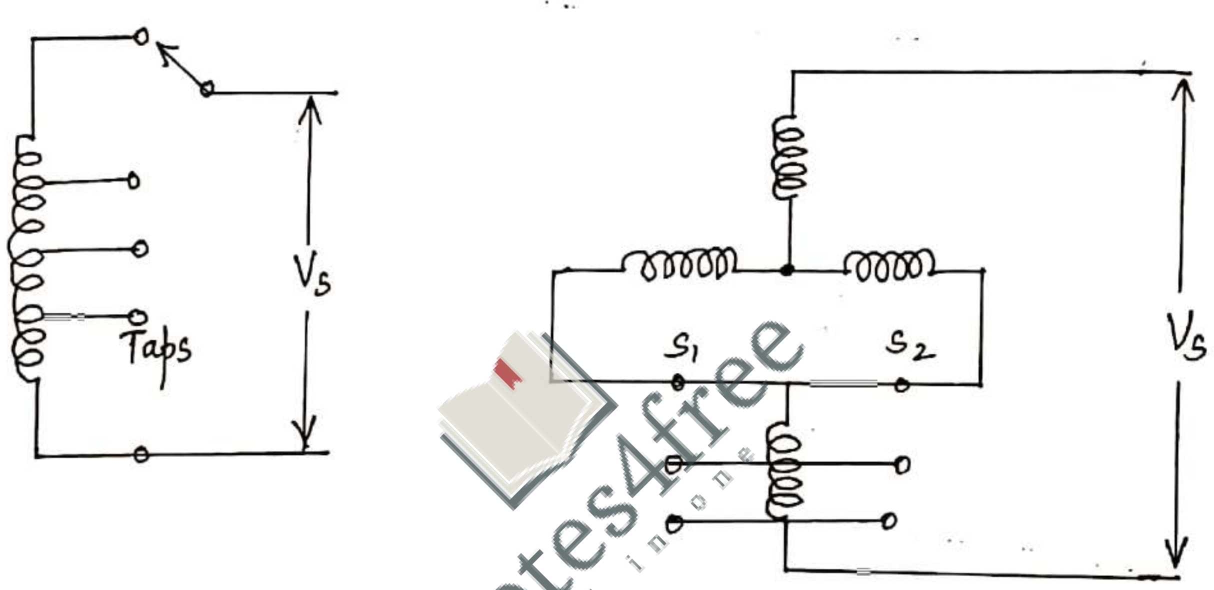
→ The two types are:

(i) On load tap changing (or) Load Tap changing (or) Tap changing under load [LTC or TCUL]

In on load tap changing transformer, special circuits are provided to change the tap settings without interrupting the load currents.

(ii) Off load tap changing transformer :

The load has to be disconnected when the tap setting is changed.



(6) Voltage control by regulating and Booster transformer:

→ The type of transformer designed for small adjustments of voltage rather than for changing the voltage levels is called a regulating transformer.

→ For small outputs and voltages, the regulating transformer is an auto transformer with necessary tappings.

→ For large outputs, the regulating transformer is a two winding transformer with tappings in secondary.

→ The Booster transformer is a transformer used for boosting or bucking the voltages in a transmission line by connecting one of the winding in series with the line. Usually the other winding of the booster transformer is excited by regulating transformer

Module-3

ECONOMIC OPEARATION OF POWER SYSTEM

5.1 INTRODUCTION

One of the earliest applications of on-line centralized control was to provide a central facility, to operate economically, several generating plants supplying the loads of the system. Modern integrated systems have different types of generating plants, such as coal fired thermal plants, hydel plants, nuclear plants, oil and natural gas units etc. The capital investment, operation and maintenance costs are different for different types of plants. The operation economics can again be subdivided into two parts.

i) Problem of *economic dispatch*, which deals with determining the power output of each plant to meet the specified load, such that the overall fuel cost is minimized.

ii) Problem of *optimal power flow*, which deals with minimum – loss delivery, where in the power flow, is optimized to minimize losses in the system. In this chapter we consider the problem of economic dispatch.

During operation of the plant, a generator may be in one of the following states: i) Base supply without regulation: the output is a constant.
ii) Base supply with regulation: output power is regulated based on system load. iii) Automatic non-economic regulation: output level changes around a base setting as area control error changes. iv) Automatic economic regulation: output level is adjusted, with the area load and area control error, while tracking an economic setting.

Regardless of the units operating state, it has a contribution to the economic operation, even though its output is changed for different reasons. The factors influencing the cost of generation are the generator efficiency, fuel cost and transmission losses. The most efficient generator may not give minimum cost, since it may be located in a place where fuel cost is high. Further, if the plant is located far from the load centers, transmission losses may be high and running the plant may become uneconomical. The economic dispatch problem basically determines the generation of different plants to minimize total operating cost.

Modern generating plants like nuclear plants, geo-thermal plants etc, may require capital investment of millions of rupees. The economic dispatch is however determined in terms of fuel cost per unit power generated and does not include capital investment, maintenance, depreciation, start-up and shut down costs etc.

5.2 PERFORMANCE CURVES

INPUT-OUTPUT CURVE

This is the fundamental curve for a thermal plant and is a plot of the input in British thermal units (Btu) per hour versus the power output of the plant in MW as shown in Fig1.

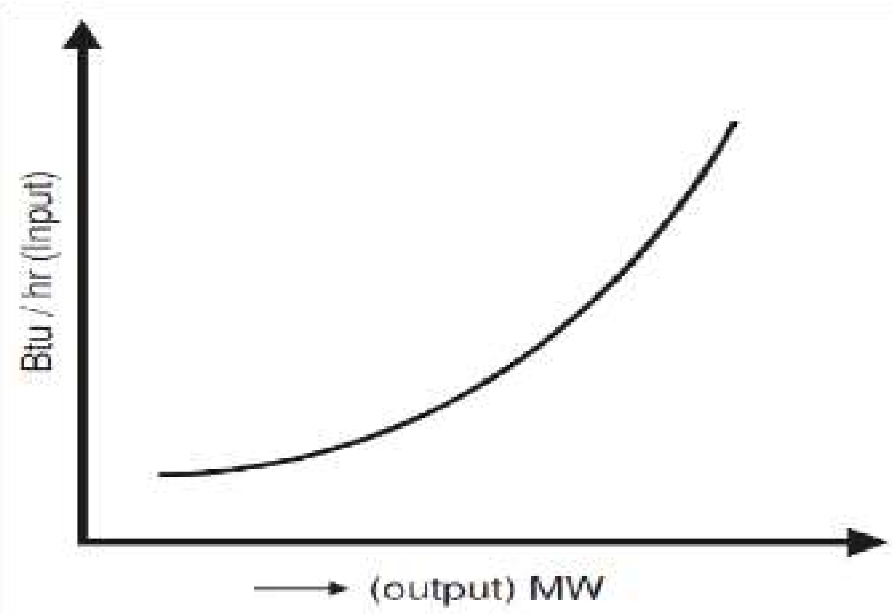


Fig 1: Input – output curve

HEAT RATE CURVE

The heat rate is the ratio of fuel input in Btu to energy output in KWh. It is the slope of the input – output curve at any point. The reciprocal of heat – rate is called fuel –efficiency. The heat rate curve is a plot of heat rate versus output in MW. A typical plot is shown in Fig .2

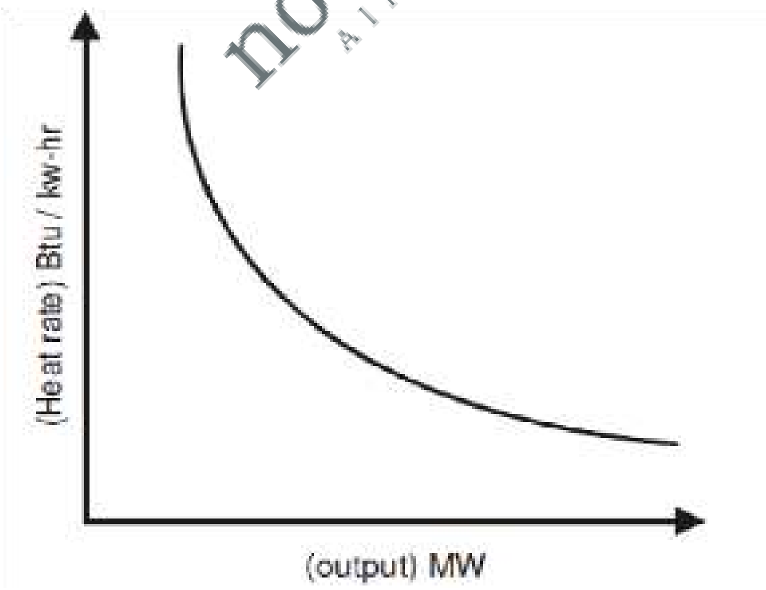


Fig .2 Heat rate curve.

INCREMENTAL FUEL RATE CURVE

The incremental fuel rate is equal to a small change in input divided by the corresponding change in output.

$$\text{Incremental fuel rate} = \frac{\Delta Input}{\Delta Output}$$

The unit is again Btu / KWh. A plot of incremental fuel rate versus the output is shown in Fig 3

Fig 3: Incremental fuel rate curve

<u>**Incremental cost curve**</u>

The incremental cost is the product of incremental fuel rate and fuel cost (Rs / Btu or $ / Btu). The curve in shown in Fig. 4. The unit of the incremental fuel cost is Rs / MWh or $ /MWh.

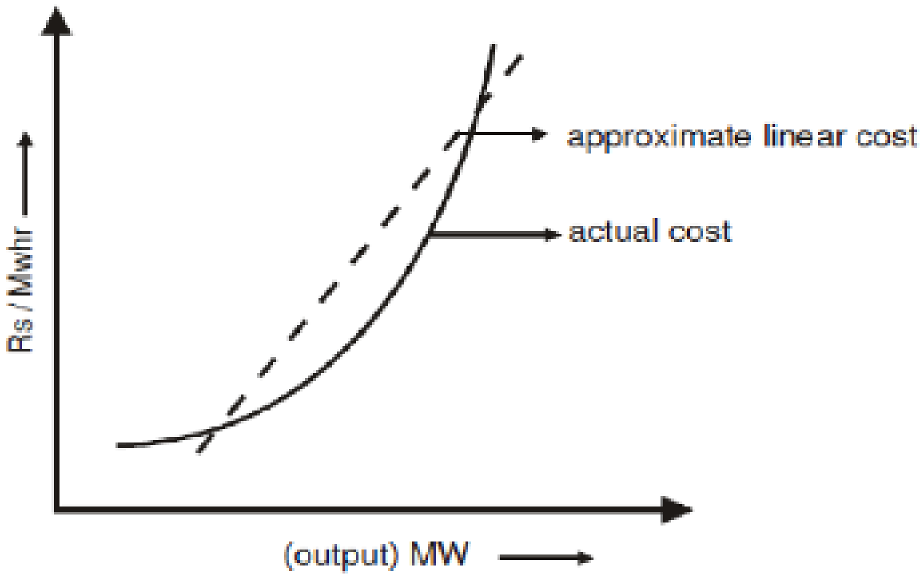


Fig 4: Incremental cost curve

In general, the fuel cost Fi for a plant, is approximated as a quadratic function of the generated output PGi.

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \text{ Rs / h}$$

The incremental fuel cost is given by

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \quad \text{Rs / MWh}$$

The incremental fuel cost is a measure of how costly it will be produce an increment of power. The incremental production cost, is made up of incremental fuel cost plus the incremental cost of labour, water, maintenance etc. which can be taken to be some percentage of the incremental fuel cost, instead of resorting to a rigorous mathematical model. The cost curve can be approximated by a linear curve. While there is negligible operating cost for a hydel plant, there is a limitation on the power output possible. In any plant, all units normally operate between PGmin, the minimum loading limit, below which it is technically infeasible to operate a unit and PGmax, which is the maximum output limit.

5.3 ECONOMIC GENERATION SCHEDULING NEGLECTING LOSSES AND GENERATOR LIMITS

The simplest case of economic dispatch is the case when transmission losses are neglected. The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand PD. Consider a system with **n**g number of generating plants supplying the total demand PD. If Fi is the cost of plant ***i*** in Rs/h, the mathematical formulation of the problem of economic scheduling can be stated as follows:

Minimize $\quad F_T = \sum_{i=1}^{n_g} F_i$

Such that $\quad \sum_{i=1}^{n_g} P_{Gi} = P_D$

where F_T = total cost.
P_{Gi} = generation of plant i.
P_D = total demand.

This is a constrained optimization problem, which can be solved by Lagrange‟ s method.

LAGRANGE METHOD FOR SOLUTION OF ECONOMIC SCHEDULE

The problem is restated below:

Minimize $\quad F_T = \sum_{i=1}^{n_g} F_i$

Such that $\quad P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$

The augmented cost function is given by

$$£ = F_T + \lambda\left(P_D - \sum_{i=1}^{n_g} P_{Gi} \right)$$

The minimum is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = 0 \text{ and } \frac{\partial £}{\partial \lambda} = 0$$

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0$$

$$\frac{\partial £}{\partial \lambda} = P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

The second equation is simply the original constraint of the problem. The cost of a plant Fi depends only on its own output PGi, hence

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Using the above,

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} = \lambda \ ; \qquad i = 1 \ldots\ldots n_g$$

We can write

$$b_i + 2c_i P_{Gi} = \lambda \qquad i = 1 \ldots\ldots n_g$$

The above equation is called the co-ordination equation. Simply stated, for economic generation scheduling to meet a particular load demand, when transmission losses are neglected and generation limits are not imposed, all plants must operate at equal incremental production costs, subject to the constraint that the total generation be equal to the demand. From we have

$$P_{Gi} = \frac{\lambda - b_i}{2c_i}$$

We know in a loss less system

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

Substituting (8.16) we get

$$\sum_{i=1}^{n_g} \frac{\lambda - b_i}{2c_i} = P_D$$

An analytical solution of λ is obtained from (8.17) as

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{b_i}{2c_i}}{\sum_{i=1}^{n_g} \frac{1}{2c_i}}$$

It can be seen that l is dependent on the demand and the coefficients of the cost function.

Example 1.

The fuel costs of two units are given by

$F_1 = 1.5 + 20\ P_{G1} + 0.1\ P_{G1}^2$ Rs/h

$F_2 = 1.9 + 30\ P_{G2} + 0.1\ P_{G2}^2$ Rs/h

P_{G1}, P_{G2} are in MW. Find the optimal schedule neglecting losses, when the demand is 200 MW.

Solution:

$$\frac{dF_1}{dP_{G1}} = 20 + 0.2P_{G1} \quad \text{Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 30 + 0.2P_{G2} \quad \text{Rs / MWh}$$

$$P_D = P_{G1} + P_{G2} = 200\ \text{MW}$$

For economic schedule

$$\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}} = \lambda$$

$$20 + 0.2\ P_{G1} = 30 + 0.2\ (200 - P_{G1})$$

Solving we get, $P_{G1} = 125$ MW

$P_{G2} = 75$ MW

$$\lambda = 20 + 0.2\ (125) = 45\ \text{Rs / MWh}$$

Example 2

The fuel cost in \$ / h for two 800 MW plants is given by

$F_1 = 400 + 6.0\ P_{G1} + 0.004\ P_{G1}^2$

$F_2 = 500 + b_2 P_{G2} + c_2 P_{G2}^2$

where P_{G1}, P_{G2} are in MW

(a) The incremental cost of power λ is \$8 / MWh when total demand is 550MW. Determine optimal generation schedule neglecting losses.

(b) The incremental cost of power is \$10/MWh when total demand is 1300 MW. Determine optimal schedule neglecting losses.

(c) From (a) and (b) find the coefficients b_2 and c_2.

Solution:

a) $$P_{G1} = \frac{\lambda - b_1}{2c_1} = \frac{8.0 - 6.0}{2 \times 0.004} = 250\ \text{MW}$$

$$P_{G2} = P_D - P_{G1} = 550 - 250 = 300\ \text{MW}$$

b) $P_{G1} = \frac{\lambda - b_1}{2C_1} = \frac{10-6}{2 \times 0.004} = 500 \text{ MW}$

$$P_{G2} = P_D - P_{G1} = 1300 - 500 = 800 \text{ MW}$$

c) $P_{G2} = \frac{\lambda - b_2}{2c_2}$

From (a) $300 = \frac{8.0 - b_2}{2c_2}$

From (b) $800 = \frac{10.0 - b_2}{2c_2}$

Solving we get $b_2 = 6.8$
$c_2 = 0.002$

5.4 ECONOMIC SCHEDULE INCLUDING LIMITS ON GENERATOR(NEGLECTING LOSSES)

The power output of any generator has a maximum value dependent on the rating of the generator. It also has a minimum limit set by stable boiler operation. The economic dispatch problem now is to schedule generation to minimize cost, subject to the equality constraint.

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

and the inequality constraint

$$P_{Gi\,(min)} \leq P_{Gi} \leq P_{Gi\,(max)} \,;\, i = 1, \ldots\ldots\ldots n_g$$

The procedure followed is same as before i.e. the plants are operated with equal incremental fuel costs, till their limits are not violated. As soon as a plant reaches the limit (maximum or minimum) its output is fixed at that point and is maintained a constant. The other plants are operated at equal incremental costs.

Example 3

Incremental fuel costs in $ / MWh for two units are given below:

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 2.0 \ \$ / \text{MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.012P_{G2} + 1.6 \ \$ / \text{MWh}$$

The limits on the plants are P_{min} = 20 MW, P_{max} = 125 MW. Obtain the optimal schedule if the load varies from 50 – 250 MW.

Solution:

The incremental fuel costs of the two plants are evaluated at their lower limits and upper limits of generation.

At $P_{G(min)}$ = 20 MW.

$$\lambda_{1(min)} = \frac{dF_1}{dP_{G1}} = 0.01 \times 20 + 2.0 = 2.2 \ \$ / \text{MWh}$$

$$\lambda_{2(min)} = \frac{dF_2}{dP_{G2}} = 0.012 \times 20 + 1.6 = 1.84 \ \$ / \text{MWh}$$

At $P_{G(Max)}$ =125 Mw

$$\lambda_{1(max)} = 0.01 \times 125 + 2.0 = 3.25 \ \$ / \text{MWh}$$

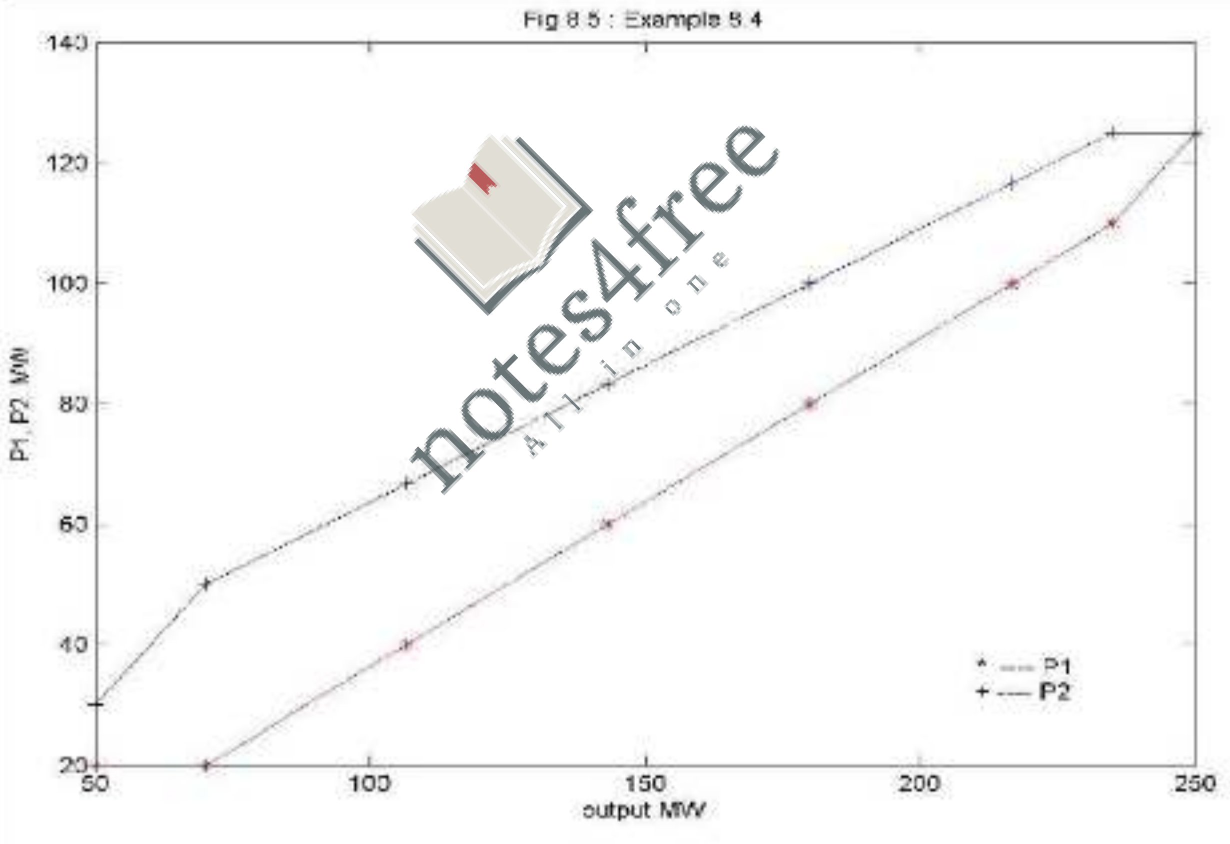
$$\lambda_{2(max)} = 0.012 \times 125 + 1.6 = 3.1 \ \$ / \text{MWh}$$

Now at light loads unit 1 has a higher incremental cost and hence will operate at its lower limit of 20 MW. Initially, additional load is taken up by unit 2, till such time its incremental fuel cost becomes equal to 2.2$ / MWh at PG2 = 50 MW. Beyond this, the two units are operated with equal incremental fuel costs. The contribution of each unit to meet the demand is obtained by assuming different values of l; When l = 3.1 $ / MWh, unit 2 operates at its upper limit. Further loads are taken up by unit 1. The computations are show in Table

Table Plant output and output of the two units

$\frac{dF_1}{dP_{G1}}$ \$/MWh	$\frac{dF_2}{dP_{G2}}$ \$/MWh	Plant λ \$/MWh	P_{G1} MW	P_{G2} MW	Plant Output MW
2.2	1.96	1.96	20^+	30	50
2.2	2.2	2.2	20^+	50	70
2.4	2.4	2.4	40	66.7	106.7
2.6	2.6	2.6	60	83.3	143.3
2.8	2.8	2.8	80	100	180
3.0	3.0	3.0	100	116.7	216.7
3.1	3.1	3.1	110	125^*	235

For a particular value of l, PG1 and PG2 are calculated using (8.16). Fig 8.5 Shows plot of each unit output versus the total plant output.



For any particular load, the schedule for each unit for economic dispatch can be obtained.

Example 4.

In example 3, what is the saving in fuel cost for the economic schedule compared to the case where the load is shared equally. The load is 180 MW.

Solution:

From Table it is seen that for a load of 180 MW, the economic schedule is PG1 = 80 MW and PG2 = 100 MW. When load is shared equally PG1 = PG2 = 90 MW. Hence, the generation of unit 1 increases from 80 MW to 90 MW and that of unit 2 decreases from 100 MW to 90 MW, when the load is shared equally. There is an increase in cost of unit 1 since PG1 increases and decrease in cost of unit 2 since PG2 decreases.

$$\text{Increase in cost of unit 1} = \int_{80}^{90} \left(\frac{dF_1}{dP_{G1}} \right) dP_{G1}$$

$$= \int_{80}^{90} (0.01 P_{G1} + 2.0) dP_{G1} = 28.5 \,\$/\text{h}$$

$$\text{Decrease in cost of unit 2} = \int_{100}^{90} \left(\frac{dF_2}{dP_{G2}} \right) dP_{G2}$$

$$= \int_{100}^{90} (0.012 P_{G2} + 1.6) dP_{G2} = -27.4 \,\$/\text{h}$$

Total increase in cost if load is shared equally = 28.5 – 27.4 = 1.1 $ / h

Hence the saving in fuel cost is 1.1 $ / h if coordinated economic schedule is used.

5.5 ECONOMIC DISPATCH INCLUDING TRANSMISSION LOSSES

When transmission distances are large, the transmission losses are a significant part of the generation and have to be considered in the generation schedule for economic operation. The mathematical formulation is now stated as

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such That $$\sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$$

where P_L is the total loss.

The Lagrange function is now written as

$$£ = F_T - \lambda\left(\sum_{i=1}^{n_g} P_{Gi} - P_D - P_L\right) = 0$$

The minimum point is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda\left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right) = 0;\ i = 1\ldots\ldots n_g$$

$$\frac{\partial £}{\partial \lambda} = \sum_{i=1}^{n_g} P_{Gi} - P_D + P_L = 0 \quad \text{(Same as the constraint)}$$

Since $\frac{\partial F_T}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$, (8.27) can be written as

$$\frac{dF_i}{dP_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \lambda$$

$$\lambda = \frac{dF_i}{dP_{Gi}}\left(\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}}\right)$$

The term $\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}}$ is called the penalty factor of plant i, L_i. The coordination equations including losses are given by

$$\lambda = \frac{dF_i}{dP_{Gi}} L_i;\ i = 1\ldots\ldots n_g$$

The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same, when losses are considered. A rigorous general expression for the loss PL is given by

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn} + \Sigma_n P_{Gn} B_{no} + B_{oo}$$

where Bmn, Bno , Boo called loss – coefficients , depend on the load composition. The assumption here is that the load varies linearly between maximum and minimum values. A simpler expression is

$$P_L = \Sigma_m \Sigma_n P_{Gm} B_{mn} P_{Gn}$$

The expression assumes that all load currents vary together as a constant complex fraction of the total load current. Experiences with large systems has shown that the loss of accuracy is not significant if this approximation is used. An average set of loss coefficients may be used over the complete daily cycle in the coordination of incremental production costs and incremental transmission losses. In general, Bmn = Bnm and can be expanded for a two plant system as

$$P_L = B_{11} P_{G1} + 2 B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2$$

Example 5

A generator is supplying a load. An incremental change in load of 4 MW requires generation to be increased by 6 MW. The incremental cost at the plant bus is Rs 30 /MWh. What is the incremental cost at the receiving end?

Solution:

$$\frac{dF_1}{dP_{G1}} = 30$$

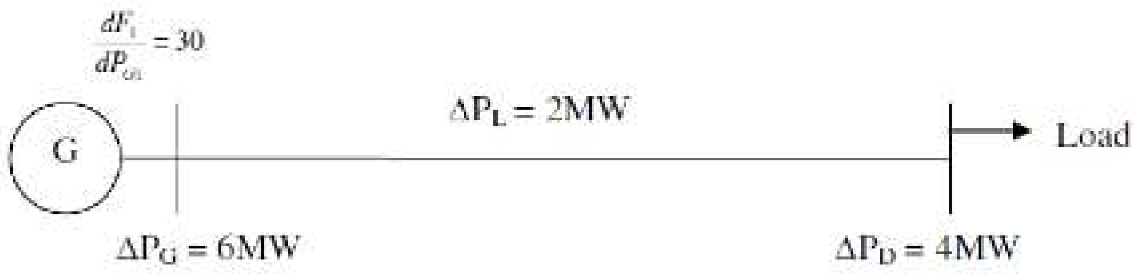


Fig ; One line diagram of example 5

$\Delta P_L = \Delta P_G - \Delta P_D = 2$ MW

λ at receiving end is given by

$$\lambda = \frac{dF_1}{dP_{G1}} \times \frac{\Delta P_G}{\Delta P_D} = 30 \times \frac{6}{4} = 45 \text{ Rs / MWh}$$

$$\text{or } \lambda = \frac{dF_1}{dP_{G1}} \times \frac{1}{1 - \frac{\Delta P_L}{\Delta P_G}} = 30 \times \frac{1}{1 - \frac{2}{6}} = 45 \text{ Rs / MWh}$$

Example 6

In a system with two plants, the incremental fuel costs are given by

$$\frac{dF_1}{dP_{G1}} = 0.01P_{G1} + 20 \text{ Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.015P_{G2} + 22.5 \text{ Rs / MWh}$$

The system is running under optimal schedule with $P_{G1} = P_{G2} = 100$ MW.

If $\frac{\partial P_L}{\partial P_{G2}} = 0.2$, find the plant penalty factors and $\frac{\partial P_L}{\partial P_{G1}}$.

Solution:

For economic schedule,

$$\frac{dF_i}{dP_{Gi}} L_i = \lambda; \qquad L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}};$$

For plant 2, $P_{G2} = 100$ MW

$$\therefore (0.015 \times 100 + 22.5) \frac{1}{1 - 0.2} = \lambda$$

Solving, $\lambda = 30$ Rs / MWh

$$L_2 = \frac{1}{1 - 0.2} = 1.25$$

$$\frac{dF_1}{dP_{G1}} L_1 = \lambda \Rightarrow (0.01 \times 100 + 20)\, L_1 = 30$$

$L_1 = 1.428$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}}$$

$$1.428 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}} \text{ ; Solving } \frac{\partial P_L}{\partial P_{G1}} = 0.3$$

Example 7

A two bus system is shown in Fig. 8.8 If 100 MW is transmitted from plant 1 to the load, a loss of 10 MW is incurred. System incremental cost is Rs 30 / MWh. Find P_{G1}, P_{G2} and power received by load if

$$\frac{dF_1}{dP_{G1}} = 0.02P_{G1} + 16.0 \text{ Rs / MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.04P_{G2} + 20.0 \text{ Rs / MWh}$$

5.6 DERIVATION OF TRANSMISSION LOSS FORMULA

An accurate method of obtaining general loss coefficients has been presented by Kron. The method is elaborate and a simpler approach is possible by making the following assumptions:

(i) All load currents have same phase angle with respect to a common reference (ii) The ratio X / R is the same for all the network branches.

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig a

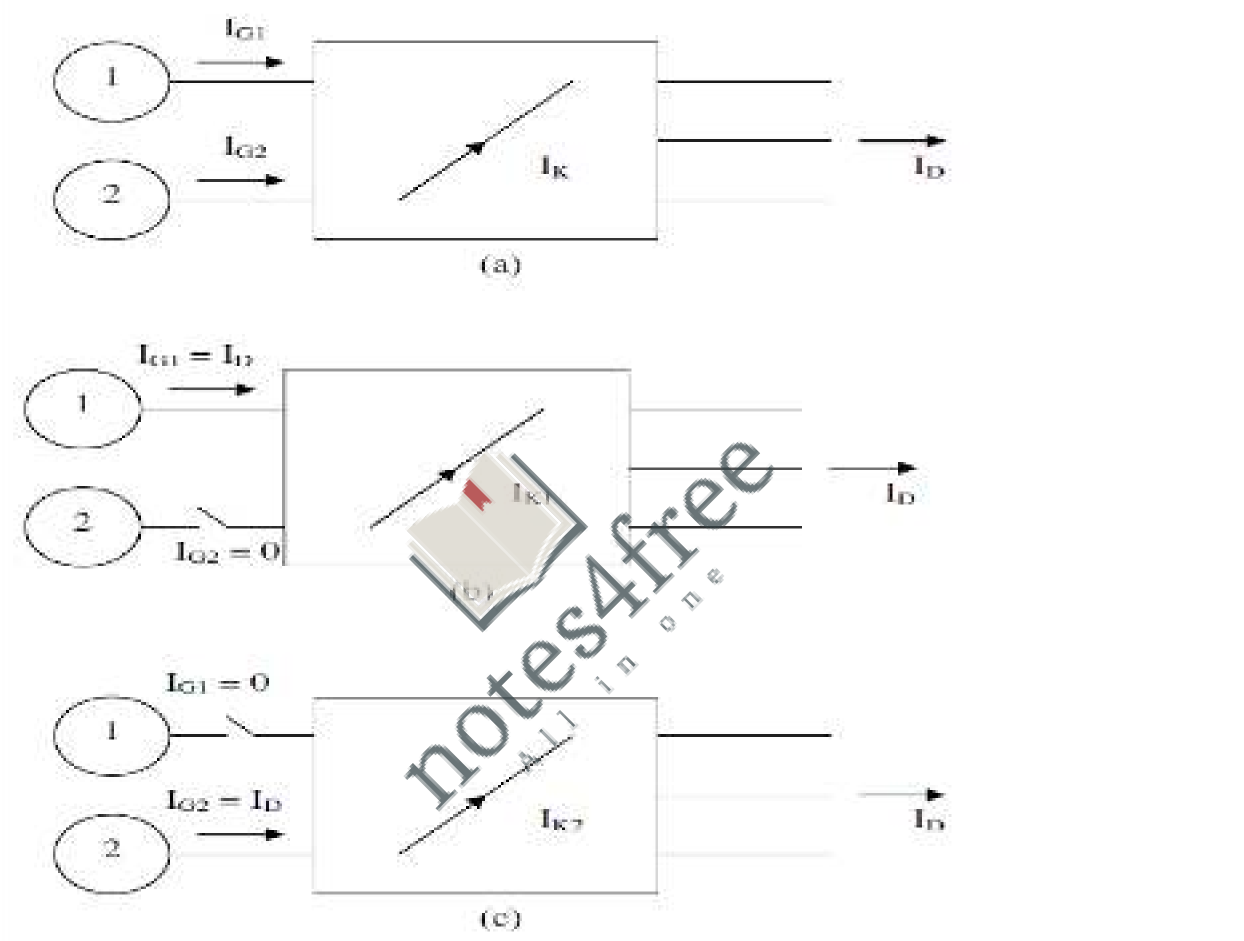


Fig Two plants connected to a number of loads through a transmission network

Let's assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be I_{K1}. We define

$$N_{K1} = \frac{I_{K1}}{I_D}$$

It is to be noted that $I_{G1} = I_D$ in this case. Similarly with only plant 2 supplying the load current I_D, as shown in Fig 8.9c, we define

$$N_{K2} = \frac{I_{K2}}{I_D}$$

N_{K1} and N_{K2} are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of I_D. When both generators are supplying the load, then by principle of superposition

$$I_K = N_{K1} I_{G1} + N_{K2} I_{G2}$$

where I_{G1}, I_{G2} are the currents supplied by plants 1 and 2 respectively, to meet the demand I_D. Because of the assumptions made, I_{K1} and I_D have same phase angle, as do I_{K2} and I_D. Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = I_{G1} \angle\sigma_1 \text{ and } I_{G2} = |I_{G2}|\angle\sigma_2 .$$

where σ_1 and σ_2 are phase angles of I_{G1} and I_{G2} with respect to a common reference. We can write

$$|I_K|^2 = \left(N_{K1}|I_{G1}|\cos\sigma_1 + N_{K2}|I_{G2}|\cos\sigma_2\right)^2 + \left(N_{K1}|I_{G1}|\sin\sigma_1 + N_{K2}|I_{G2}|\sin\sigma_2\right)^2$$

$$= \begin{array}{l} N_{K1}^2 I_{G1}^2\left[\cos^2\sigma_1 + \sin^2\sigma_1\right] + N_{K2}^2|I_{G2}|^2\left[\cos^2\sigma_2 + \sin^2\sigma_2\right] \\ + 2\left[N_{K1}|I_{G1}|\cos\sigma_1 N_{K2}|I_{G2}|\cos\sigma_2 + N_{K1}|I_{G1}|\sin\sigma_1 N_{K2}|I_{G2}|\sin\sigma_2\right] \end{array}$$

$$= N_{K1}^2|I_{G1}|^2 + N_{K2}^2|I_{G2}|^2 + 2N_{K1}N_{K2}|I_{G1}||I_{G2}|\cos(\sigma_1 - \sigma_2)$$

Now $|I_{G1}| = \dfrac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1}$ and $|I_{G2}| = \dfrac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2}$

where P_{G1}, P_{G2} are three phase real power outputs of plant 1 and plant 2; V_1, V_2 are the line to line bus voltages of the plants and ϕ_1, ϕ_2 are the power factor angles.

The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

where the summation is taken over all branches of the network and R_K is the branch resistance. Substituting we get

$$P_L = \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K + \frac{2P_{G1}P_{G2}\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K$$
$$+ \frac{P_{G2}^2}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}^2 R_K$$

$$P_L = P_{G1}^2 B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}^2 B_{22}$$

where $$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1 \cos\phi_2} \sum_K N_{K1} N_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2 (\cos\phi_2)^2} \sum_K N_{K2}^2 R_K$$

The loss – coefficients are called the B – coefficients and have unit MW^{-1}.

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}^2}{|V_1|^2 (\cos\phi_1)^2} \sum_K N_{K1}^2 + \ldots\ldots + \frac{P_{Gn}^2}{|V_n|^2 (\cos\phi_n)^2} \sum_K N_{Kn}^2 R_K$$
$$+ 2 \sum_{\substack{p,q=1 \\ p \neq q}}^{n} \frac{P_{GP} P_{Gq} \cos(\sigma_p - \sigma_q)}{|V_p||V_q| \cos\phi_p \cos\phi_q} \sum_k N_{KP} N_{Kq} R_K$$

In a compact form

$$P_L = \sum_{p=1}^{n} \sum_{q=1}^{n} P_{Gp} B_{Pq} P_{Gq}$$

$$B_{Pq} = \frac{\cos(\sigma_p - \sigma_q)}{|V_p||V_q| \cos\phi_p \cos\phi_q} \sum_K N_{KP} N_{Kq} R_K$$

B – Coefficients can be treated as constants over the load cycle by computing them at average operating conditions, without significant loss of accuracy.

Example 8

Calculate the loss coefficients in pu and MW^{-1} on a base of 50MVA for the network of Fig below. Corresponding data is given below.

$I_a = 1.2 - j\,0.4$ pu	$Z_a = 0.02 + j\,0.08$ pu
$I_b = 0.4 - j\,0.2$ pu	$Z_b = 0.08 + j\,0.32$ pu
$I_c = 0.8 - j\,0.1$ pu	$Z_c = 0.02 + j\,0.08$ pu
$I_d = 0.8 - j\,0.2$ pu	$Z_d = 0.03 + j\,0.12$ pu
$I_e = 1.2 - j\,0.3$ pu	$Z_e = 0.03 + j\,0.12$ pu

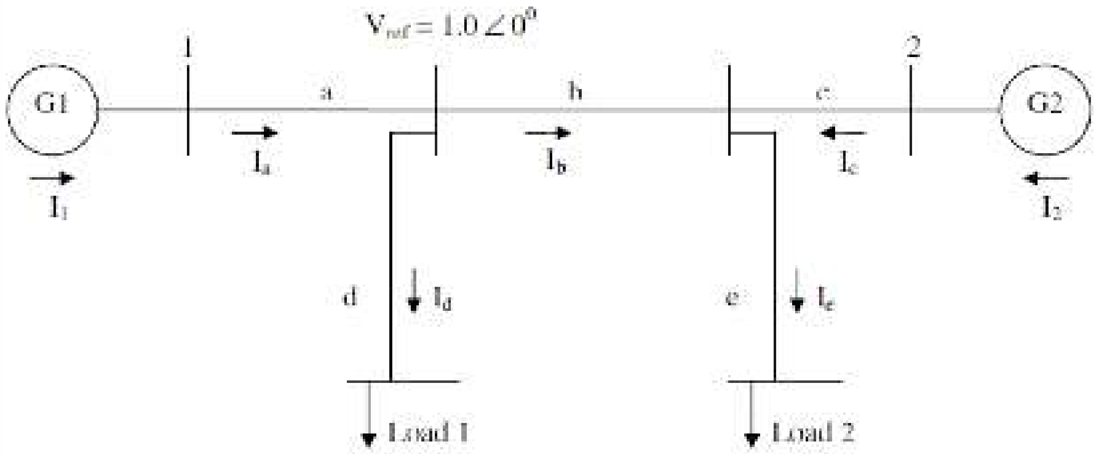


Fig : Example 8

Solution:

Total load current

$I_L = I_d + I_e = 2.0 - j\,0.5 = 2.061\ \angle -14.03^0 A$

$I_{L1} = I_d = 0.8 - j\,0.2 = 0.8246\ \angle -14.03^0\ A$

$\frac{I_{L1}}{I_L} = 0.4; \qquad \frac{I_{L2}}{I_L} = 1.0 - 0.4 = 0.6$

If generator 1, supplies the load then $I_1 = I_L$. The current distribution is shown in Fig a.

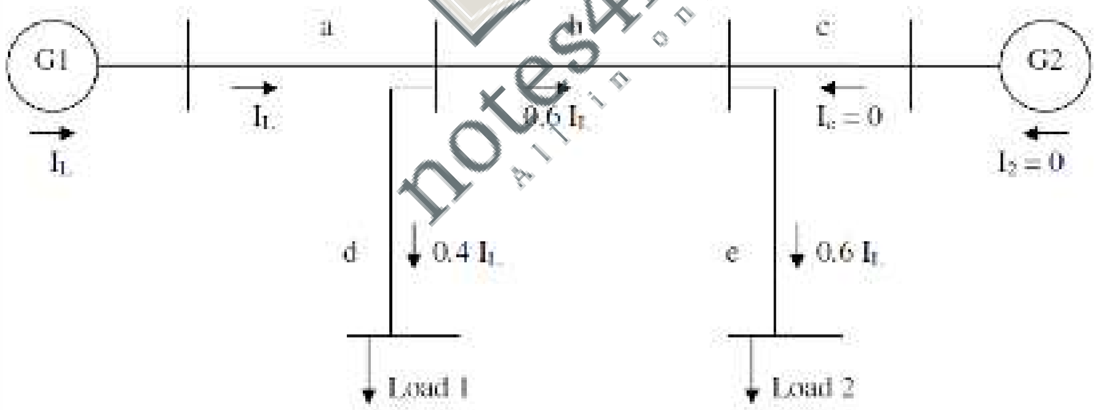


Fig a : Generator 1 supplying the total load

$N_{a1} = \frac{I_a}{I_L} = 1.0;\ N_{b1} = \frac{I_b}{I_L} = 0.6;\ N_{C1} = 0;\ N_{d1} = 0.4;\ N_{e1} = 0.6.$

Similarly the current distribution when only generator 2 supplies the load is shown in Fig b.

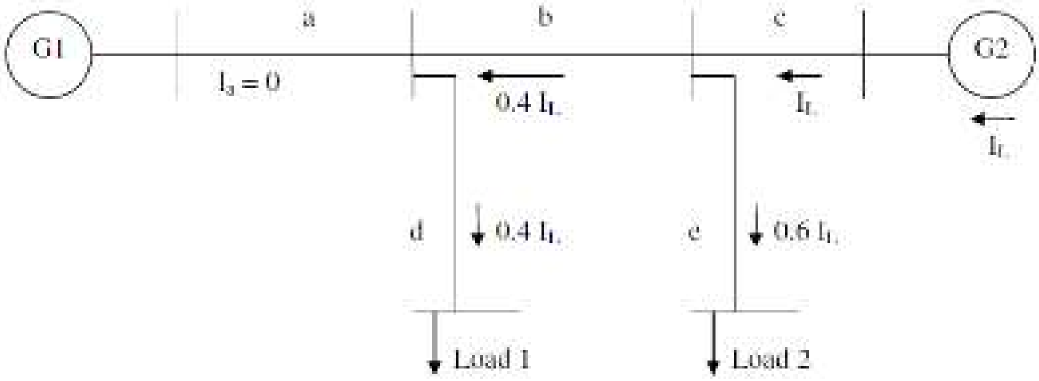


Fig b: Generator 2 supplying the total load

$N_{a2} = 0$; $N_{b2} = -0.4$; $N_{c2} = 1.0$; $N_{d2} = 0.4$; $N_{e2} = 0.6$

From Fig 8.10, $V_1 = V_{ref} + Z_a I_a$

$$= 1\angle 0^0 + (1.2 - j\,0.4)\,(0.02 + j0.08)$$

$$= 1.06\angle 4.78^0 = 1.056 + j\,0.088 \text{ pu.}$$

$$V_2 = V_{ref} - I_b Z_b + I_c Z_c$$

$$= 1.0\ \angle 0^0 - (0.4 - j\,0.2)\,(0.08 + j\,0.32) + (0.8 - j\,0.1)\,(0.02 + j\,0.08)$$

$$= 0.928 - j\,0.05 = 0.93\ \angle -3.10^0 \text{ pu.}$$

Current Phase angles

$$\sigma_1 = \text{angle of } I_1(=I_a) = \tan^{-1}\left(\frac{-0.4}{1.2}\right) = -18.43^0$$

$$\sigma_2 = \text{angle of } I_2(=I_c) = \tan^{-1}\left(\frac{-0.1}{0.8}\right) = -7.13^0$$

$$\cos(\sigma_1 - \sigma_2) = 0.98$$

Power factor angles

$$\phi_1 = 4.78^0 + 18.43 = 23.21^0; \cos\phi_1 = 0.92$$

$$\phi_2 = 7.13^0 - 3.10^0 = 4.03^0; \cos\phi_2 = 0.998$$

$$B_{11} = \frac{\sum_K N_{K1}^{\ 2} R_K}{|V_1|^2 (\cos\phi_1)^2} = \frac{1.0^2 \times 0.02 + 0.6^2 \times 0.08 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(1.06)^2 (0.920)^2}$$

$= 0.0677$ pu

$= 0.0677 \times \frac{1}{50} = 0.1354 \times 10^{-2}\ MW^{-1}$

$$B_{12} = \frac{Cos(\sigma_1 - \sigma_2)}{|V_1||V_2|(\cos\phi_1)(\cos\phi_2)} \sum_K N_{K1} N_{K2} R_K$$

$$= \frac{0.98}{(1.06)(0.93)(0.998)(0.92)}\left[-0.4\times0.6\times0.08+0.4\times0.4\times0.03+0.6\times0.6\times0.03\right]$$

$= -0.00389$ pu

$= -0.0078 \times 10^{-2}\ MW^{-1}$

$$B_{22} = \frac{\sum_{K} N_{K2}{}^{2} R_{K}}{|V_2|^2 \left(\cos\phi_2{}^2\right)}$$

$$= \frac{(-0.4)^2 0.08 + 1.0^2\times0.02 + 0.4^2\times0.03 + 0.6^2\times0.03}{(0.93)^2(0.998)^2}$$

$= 0.056\text{pu} = 0.112\times10^{-2}\,MW^{-1}$

POWER SYSTEM CONTROL

INTRODUCTION:

The preceding chapters were devoted to problems associated with the selection of a normal operating state for the power system and optimum scheduling of generation. The present chapter deals with the continuous control of active and reactive power in order to keep the system in steady state. The power system being dynamic, the demand continuously deviates from its normal value. This leads to a small change in the state of the system. The automatic control should act in a closed loop manner, to detect these changes and initiate actions to eliminate the deviations. Briefly stated, the control strategy should be designed to deliver power to an interconnected system economically and reliably, while maintaining the voltage and frequency within the permissible limits.

Changes in real power mainly affect the system frequency and changes in reactive power mainly depend on changes in voltage magnitude and are relatively less sensitive to changes in frequency. Thus, real and reactive powers can be controlled separately. The Automatic Load Frequency Control (ALFC) controls the real power and the Automatic Voltage Regulator (AVR) regulates the voltage magnitude and hence the reactive power. The two controls, along with the generator and prime mover are shown in Fig.1. Unlike the AVR, ALFC is not a single loop. A fast primary loop responds to the frequency changes and regulates the steam (water) flow via the speed governor and control valves to match the active power output with that of the load. The time period here is a few seconds. The frequency is controlled via control of the active power.

A slower secondary loop maintains fine frequency adjustment to maintain proper active power exchange with other interconnected networks via tie-lines. This loop does not respond to fast load changes but instead focuses on changes, which lead to frequency drifting over several minutes.

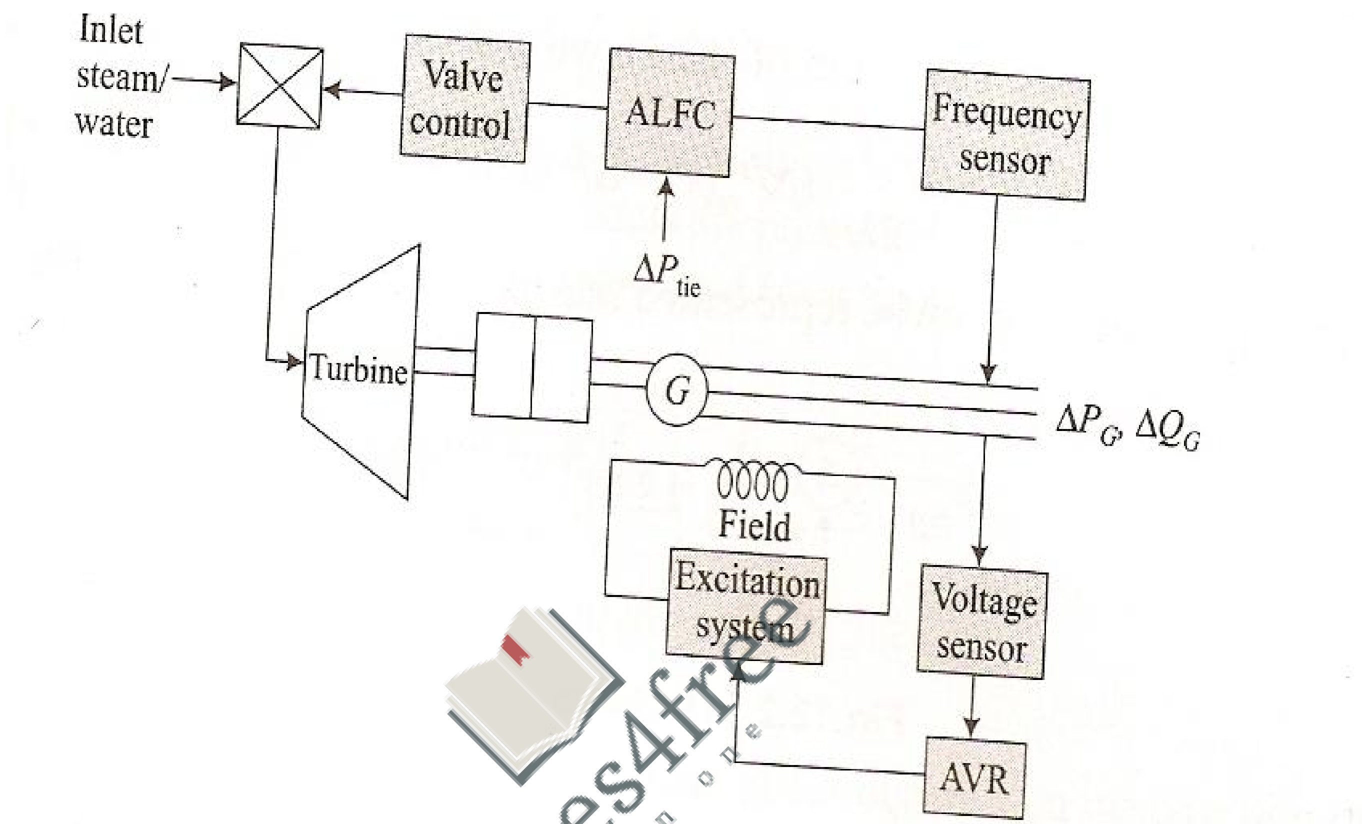


Fig.1 ALFC and AVR

Since the AVR loop is much faster than the ALFC loop, the AVR dynamics settle down before they affect the ALFC control loop. Hence, cross-coupling between the controls can be neglected. With the growth of large interconnected systems, ALFC has gained importance in recent times. This chapter presents an introduction to power system controls.

AUTOMATIC LOAD FREQUENCY CONTROL:

The functions of the ALFC are to maintain steady frequency, control tie-line power exchange and divide the load between the generators. The tie-line power deviation is given by ΔP_{tie} and the change in frequency Δf, is measured by $\Delta\delta$, the change in the rotor angle δ. The error signals Δf and ΔP_{tie} are amplified, mixed and transformed to a real power signal, which controls the valve position to generate a command signal ΔP_v. ΔP_v is sent to the prime mover to initiate change in its torque. The prime mover changes the generator output by ΔP_G , so as to bring Δf and ΔP_{tie} within acceptable limits. The next step in the analysis is to build the mathematical model for the ALFC.

Generator Model:

We can apply the swing equation to a small perturbation to obtain the linearized equation.

$$\frac{2H}{\mu_s}\frac{d^2\Delta ¢}{dt^2} = \Delta P_m - \Delta P_e$$

Expressing speed deviation in pu, can be written as

$$\frac{d\Delta\mu}{dt} = \frac{1}{2H}(\Delta P_m - \Delta P_e)$$

Taking the Laplace Transform of we get

$$\Delta\mu(s) = \frac{1}{2Hs}(\Delta P_m(s) - \Delta P_e(s))$$

Load Model:

The details of load modeling are covered in chapter 11. In general, the loads are composite. Resistive loads such as lighting and heating loads are independent of frequency. However, in case of electric motors, the power is dependent on frequency. We can arrive at a composite frequency dependent load characteristic given by

$$\Delta P_e = \Delta P_L + D\Delta\omega$$

where ΔP_L = non frequency sensitive load change

D = Load damping constant

$D\Delta\omega$ = frequency sensitive load change

The damping constant is expressed as percent change in load for one percent change in frequency. A value of D = 1.2 means a change in frequency by 1% causes the load to change by 1.2%.

Turbine Model:

The prime mover for the generator is the turbine, which is mostly a steam turbine or a hydro turbine. In the simplified model, the turbine can be represented by a first order, single time constant transfer function given by

$$G_T(s) = \frac{\Delta P_m(s)}{\Delta P_V(s)} = \frac{1}{1+\tau_T s}$$

where ΔP_V = Change in valve output

τ_T = turbine time constant

τ_T varies from 0.2 – 2 secs. The exact value of τ_T, depends on the type of turbine.

Governor Model:

If the electrical load on the generator suddenly increases, the output electrical power exceeds the input mechanical power. The difference is supplied by the kinetic energy stored in the system. The reduction in the kinetic energy causes the turbine speed and frequency to fall. The turbine governor reacts to this change in speed, and adjusts the turbine input valve/gate to change the mechanical power output to match the increased power demand and bring the frequency to its steady state value. Such a governor which brings back the frequency to its nominal value is called as *isochronous governor*. The essential elements of a conventional governor system are shown in Fig

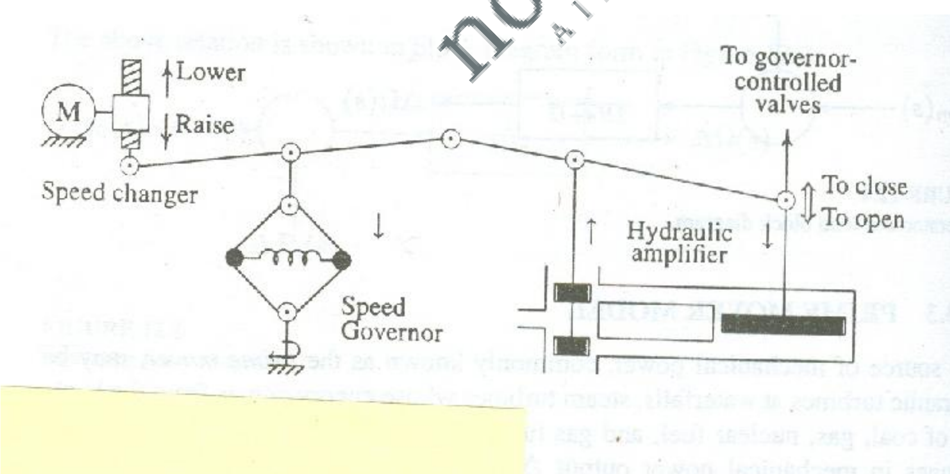


Fig Conventional governor

The major parts are

(i) **Speed Governor:** This consists of centrifugal flyballs driven directly or through gears by the turbine shaft, to provide upward and downward vertical movements proportional to the change in speed.

(ii) **Linkage mechanism:** This transforms the flyball movement to the turbine valve through a hydraulic amplifier and provides a feed back from turbine valve movement.

(iii) **Hydraulic amplifiers:** These transform the governor movements into high power forces via several stages of hydraulic amplifiers to build mechanical forces large enough to operate the steam valves or water gates.

(iv) **Speed changer:** This consists of a servomotor which is used to schedule the load at nominal frequency. By adjusting its set point, a desired load dispatch can be scheduled.

An isochronous governor works satisfactorily only when a generator is supplying an isolated load, or when only one generator is required to respond to change in load in a multi generator system. For proper power sharing between a number of generators connected to the system, the governors are designed to permit the speed to drop as the load is increased. This provides the speed – output characteristic a droop as shown in Fig. The speed regulation R is given by the slope of the speed – output characteristic.

$$R = \frac{\Delta\omega}{\Delta P}$$

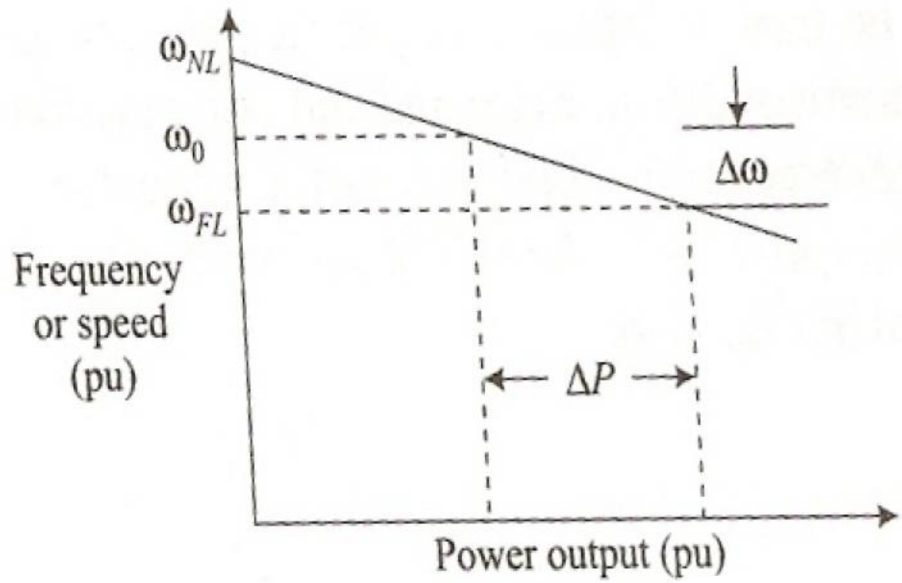
Governor % speed regulation is defined as

$$\%R = \left(\frac{\omega_{NL} - \omega_{FL}}{\omega_o}\right) \times 100$$

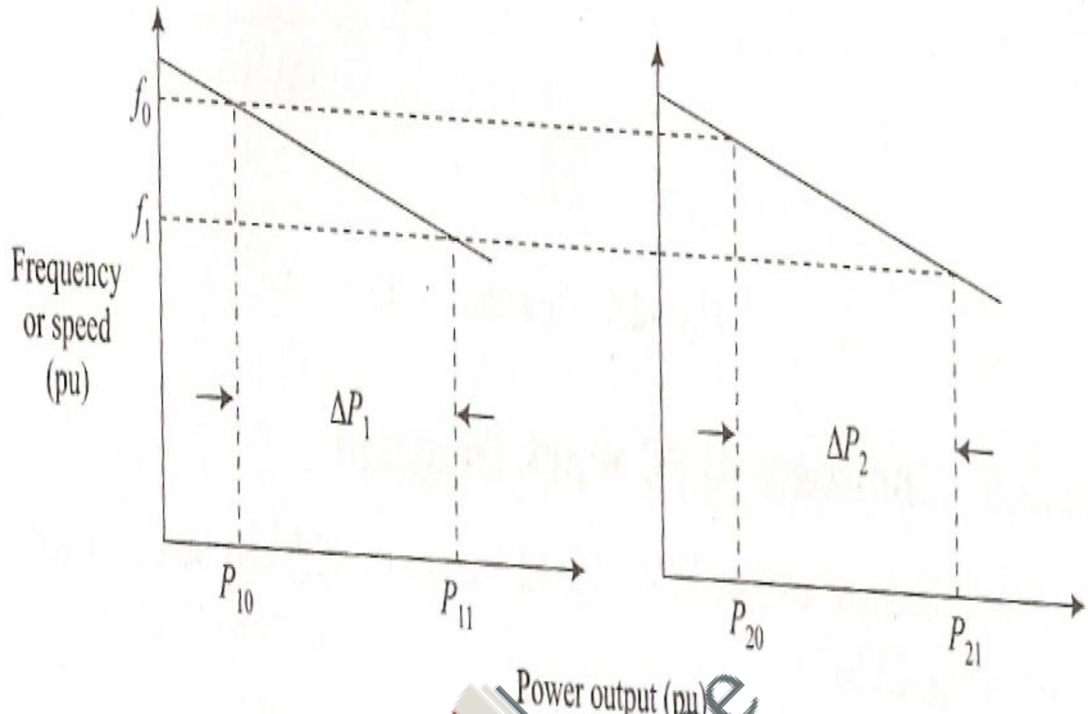
where ω_{NL} = No–load speed

ω_{FL} = Full load speed

ω_O = Nominal speed



To illustrate how load is shared between two generators, consider two generators with droop characteristics as shown in Fig below



Let the initial frequency be f_o and the outputs of the two generators be P_{10} and P_{20} respectively. If now the load increases by an amount ΔP_L, the units slow down and the governors increase the output until a common operating frequency f_1 is reached. The amount of load picked up by each generator to meet the increased demand ΔP_L depends on the value of the regulation.

$$\Delta P_1 = \frac{\Delta f}{R_1}$$

$$\Delta P_2 = \frac{\Delta f}{R_2}$$

$$\frac{\Delta P_1}{\Delta P_2} = \frac{R_2}{R_1}$$

The output is shared in the inverse ratio of their speed regulation. The output of the speed governor is ΔP_g , which is the difference between the set power ΔP_{ref} and the power $\frac{\Delta \mu}{R}$ which is given by the governor speed characteristic.

$$\Delta P_g = \Delta P_{ref} - \frac{\Delta\mu}{R}$$

$$\Delta P_g(s) = \Delta P_{ref}(s) - \frac{\Delta\mu(s)}{R}$$

The hydraulic amplifier transforms the command into valve/gate position ΔP_V. Assuming a time constant τ_g for the governor,

$$\Delta P_V(s) = \frac{1}{1+\tau_g s}\Delta P_g(s)$$

Complete ALFC block diagram:

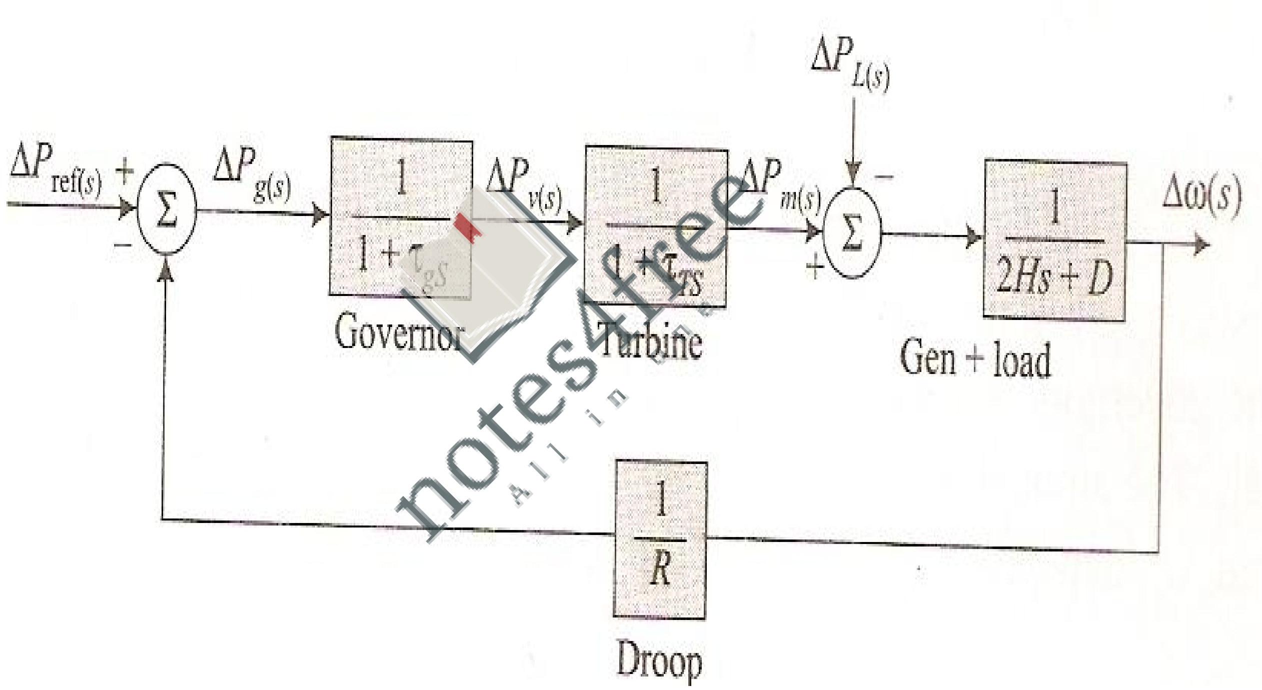
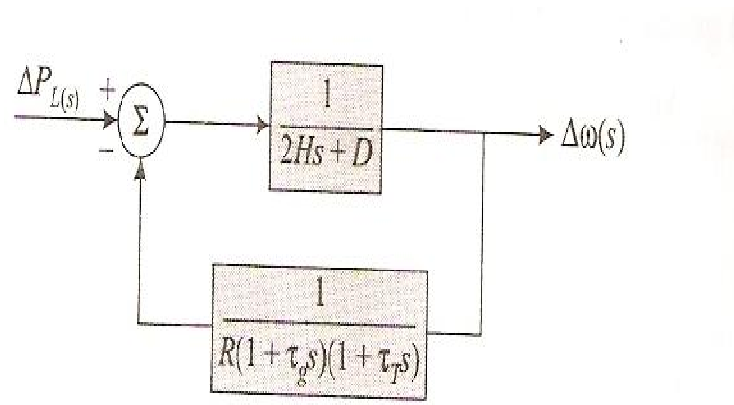


Fig Block diagram of complete governor system

The Figure shows the complete load frequency control for an isolated generator supplying a load. Since we are interested in the change in speed for change in load, we can obtain the transfer function $\frac{\Delta\mu(s)}{-\Delta P_L(s)}$ from Fig below which is a reduced order model.



Reduced block diagram

The closed loop transfer function is obtained from Fig

$$\frac{\Delta\mu(s)}{-\Delta P_L(s)} = \frac{\left(\frac{1}{2Hs+D}\right)}{1+\left(\frac{1}{2Hs+D}\right)\left(\frac{1}{R(1+\tau_g s)(1+\tau_T s)}\right)}$$

$$= \frac{(1+\tau_g s)\,(1+\tau_T s)}{(2Hs+D)\,(1+\tau_g s)\,(1+\tau_T s) + \frac{1}{R}}$$

We can write

$$\Delta\omega(s) = \Delta P_L(s)\, T(s)$$

If we consider a step change ΔP_L in the load,

$$\Delta P_L(s) = \frac{\Delta P_L}{s}$$

The steady state frequency deviation $\Delta\omega_{ss}$ is given by the limit of $\Delta\omega$ as $t \to \infty$. This can be obtained by application of the final value thereon

$$\Delta\omega_{ss} = \lim_{t\to\infty} \Delta\omega = \lim_{s\to 0} s\Delta\omega(s)$$

$$= \lim_{s\to 0} s\left(\frac{-\Delta P_L}{s} T(s)\right)$$

Now $$\lim_{s \to 0} T(s) = \frac{1}{D + \frac{1}{R}}$$

$$\Delta\omega_{ss} = (\ \ \Delta P_L)\ \frac{1}{D + \frac{1}{R}}$$

If there are no frequency sensitive loads, D = 0; in which case

$$\Delta\omega_{ss} = \frac{-\Delta P_L}{\frac{1}{R}}$$

The steady state speed deviation thus, depends on the governor speed regulation. If several generators are connected to the system, the composite frequency – power characteristics depends on the combined effect of the droops of all the generator speed governors. If we consider *n* generators with a composite load damping coefficient D, the steady state speed deviation after a load change ΔP_L is given by

$$\Delta\omega_{ss} = \frac{-\Delta P_L}{D + \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots \frac{1}{R_n}\right)} = \frac{-\Delta P_L}{D + \frac{1}{R_{eq}}}$$

where $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots \frac{1}{R_n}$$

The *stiffness* of the system, β is given by

$$\beta = \frac{-\Delta P_L}{\Delta\mu_{ss}} = \frac{1}{R_{eq}} + D \ \text{MW/Hz}$$

β is also called the *frequency bias factor* and is indicative of the change in frequency which would occur for a change in the load.

(* The pu speed deviation Δω is same as pu frequency deviation Δf)

Example 1: A system consists of 4 identical 250 MVA generators feeding a load of 510 MW. The inertia constant H of each unit is 2.5 on the machine base. The total load varies by 1.4% for a 1% change in frequency. If there is a drop in load of 10MW, determine the system block diagram expressing H and D on a base of 1000MVA. Give expression for the speed deviation, assuming there is no speed governor.

Solution:

H for 4 units on 1000MVA base = $4 \times 2.5 \times \frac{250}{1000} = 2.5$

Load after drop of 10 MW = 510 10 = 500 MW

D for load on base of 1000 MVA is given by

$$D = 1.4 \times \frac{500}{1000} = 0.7\%$$

[note that a change of load of 1.4% on base 500 MW corresponds to 0.7% on base of 1000MVA]

The standard first order transfer function form is given by $\frac{K}{1 + sT}$. In the reduced order model, the feedback loop is zero, since no governor is modeled. Substituting the values, and expressing in standard form we get

The gain = 1.428 and time constant = 7.14 secs.

$$\Delta P_L = \quad 10 \text{ MW} = \frac{-10}{1000} = \quad 0.01 \text{ pu.}$$

$$\Delta P_L(s) = \frac{-0.01}{s} \text{pu}$$

From block diagram

$$\Delta\omega(s) = -\left(\frac{-0.01}{s}\right)\left(\frac{1.428}{1+7.14s}\right)$$

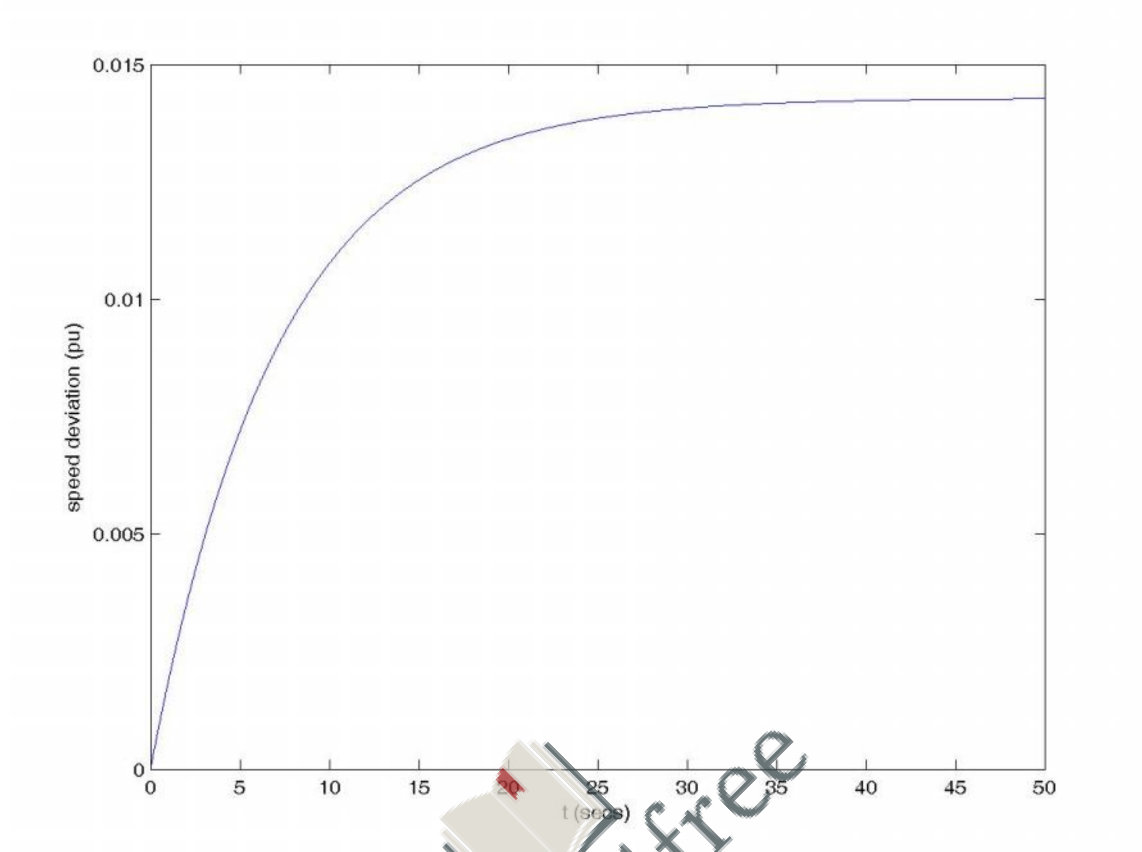
$$= \left(\frac{0.01}{s}\right)\left(\frac{1.428}{1+7.14s}\right) = \frac{0.01428}{s} - \frac{0.10196}{1+7.14s}$$

$$= \frac{0.01428}{s} - \frac{0.01428}{s + 0.14}$$

Taking inverse laplace transform

$$\Delta\omega(t) = 0.01428\ (1 \quad e^{\ 0.14t})$$

The pu speed deviation as function of time is shown in Fig below.



The steady state speed deviation is 0.01428 pu. If frequency is 50 Hz, steady state frequency deviation = 50 × 0.01428 = 0.714 Hz. The frequency deviation is positive since a decrease in load leads to increase frequency.

sent.

Example 2.

An isolated generator and its control have following parameters

Generator inertia constant = 5sec

Governor time constant τ_g = 0.25sec

Turbine time constant τ_T = 0.6sec

Governor speed regulation = 0.05 pu

D = 0.8

The turbine rated output is 200 MW at 50 Hz. The load suddenly increases by 50 MW. Find the steady state frequency deviation. Plot the frequency deviation as a function of time.

Solution:

The transfer function is given by

$$\frac{\Delta\mu(s)}{-\Delta P_L(s)} = T(s) = \frac{(1+0.25s)(1+0.6s)}{(10s+0.8)(1+0.25s)(1+0.6s)+\frac{1}{0.05}}$$

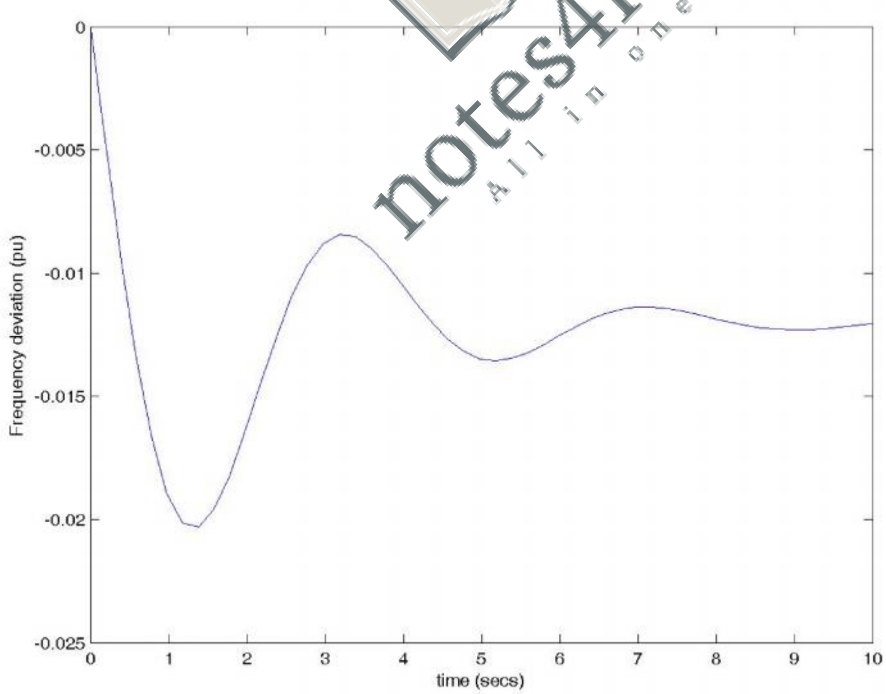
$$= \frac{0.15s^2 + 0.85s + 1}{1.5s^3 + 8.62s^2 + 10.68s + 20.8}$$

$$\Delta P_L = \frac{50}{200} = 0.25 \text{ pu}$$

$$\Delta\omega_{ss} = \frac{-\Delta P_L}{D + \frac{1}{R}} = \frac{-0.25}{0.8 + \frac{1}{0.05}} = \quad 0.01202 \text{ pu.}$$

Steady state frequency deviation Δf_{ss} = 0.01202 × 50 = 0.601 Hz.

The frequency decreases since the load has increased. The time response to a step deviation of 0.25 is shown Fig



Frequency deviation for step response for example 2

CONCEPT OF AUTOMATIC GENERATION CONTROL (AGC):

With the primary ALFC, it was seen that a change in the system load results in a steady state frequency deviation, depending on the regulation and frequency sensitivity (as indicated by D) of the load. All the connected generator units of the system contribute to the overall change in generation, irrespective of the location of the load change. Thus, restoration to nominal system frequency requires additional control action which changes the load reference set point to match the variations in the system load. This control scheme is called the Automatic Generation Control (AGC). The main objectives of the AGC are to regulate the system frequency and maintain the scheduled power inter changes, between the interconnected areas, via the tie–lines. A secondary objective of the AGC is to distribute the required change in generation among the various units to obtain least operating costs. During large transient disturbances and emergencies, AGC is bypassed and other emergency controls act.

AGC in a single area:

In a single area system since there is no tie–line schedule to be maintained, the function of AGC is only to bring the frequency to the nominal value. This is achieved by introducing an integral controller to change the load reference setting so as to change the speed set point. The integral controller forces the steady state speed deviation to zero.

The gain K_I of the integral controller needs to be adjusted for satisfactory response in terms of over shoot, setting time etc.

The closed loop transfer function with integral controller is given by

$$T(s) = \frac{\Delta \mu(s)}{-\Delta P_L(s)} = \frac{s(1+\tau_g s)(1+\tau_T s)}{s(2Hs+D)(1+\tau_g s)(1+\tau_T s) + K_I + \frac{s}{R}}$$

Example 4:

In example 2, an integral controller with gain $K_I = 6$ is added .Obtain the dynamic response, if all other conditions are same.

Solution:

From the transfer function

$$\frac{\Delta \mu(s)}{-\Delta P_L(s)} = \frac{0.15s^3 + 0.85s^2 + s}{1.5s^4 + 8.62s^3 + 10.68s^2 + 20.8s + 6}$$

The change in P_L, $\Delta P_L = 0.25$ pu.

The response of the above function for a step change of 0.25 pu is plotted using Matlab. The response is shown if Fig below.

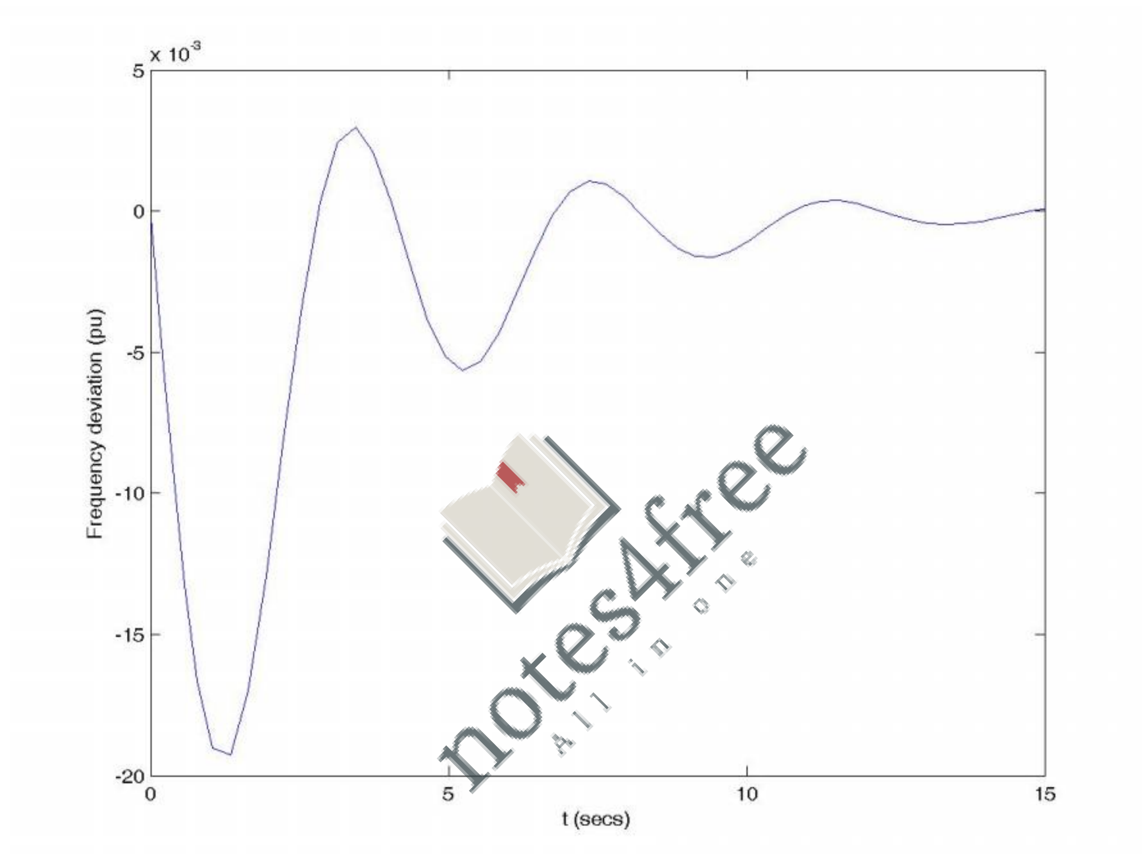


Fig: Dynamic response of example 4

It can be seen that the steady state frequency deviation is now zero. However, the overshoot and setting time are more.

AGC in multi area systems:

In inter connected systems, a group of generators are closely coupled internally and swing in phase. Such a group is called coherent group. The ALFC loop can be represented for the whole area, referred to as control area.

Consider two areas interconnected by lossless tie–line of reactance X_{tie}, with a power flow P_{12} from area 1 to area 2 as shown in Fig. a. Let the generators be represented by a single equivalent generator for area 1 and area 2. The generators are modeled simply as constant voltage sources behind reactance as shown in Fig. b. We first consider only primary ALFC loop as shown in Fig. c.

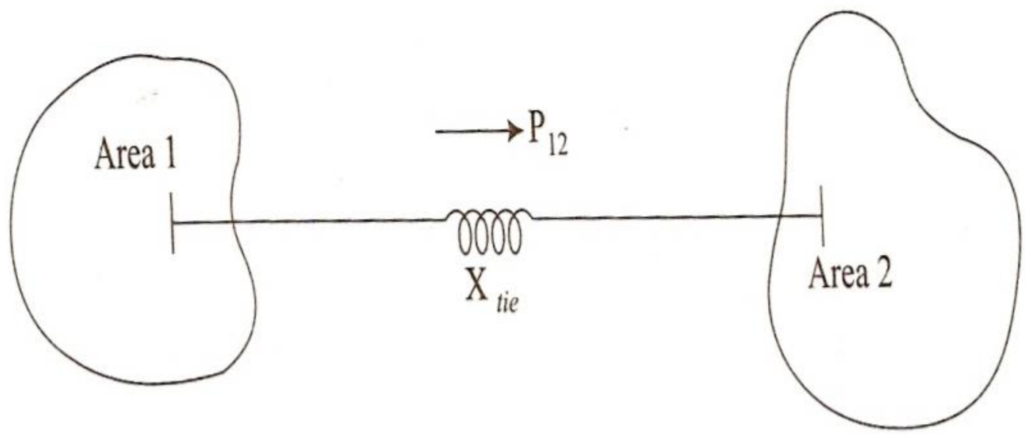


Fig a : Two area system

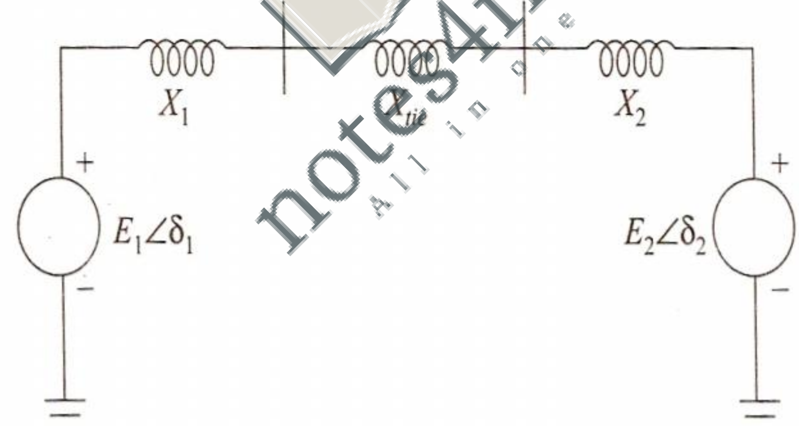


Fig 12.19b: Electrical equivalent

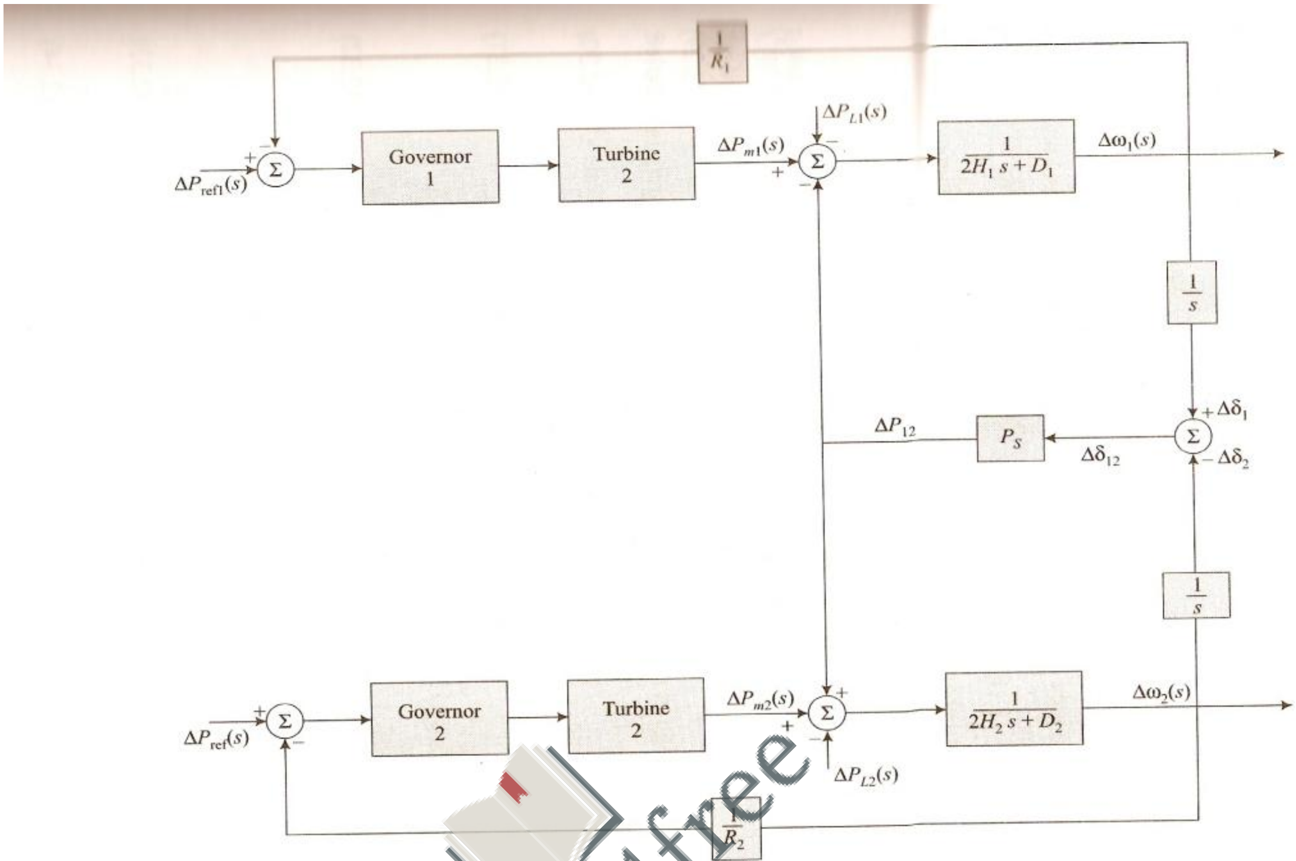


Fig c: Two area system with primary ALFC

H is the equivalent inertia constant of each area. The turbines are represented by the effective speed droop R and load damping constant D.

Under steady state the power transferred over the tie–line is given by

$$P_{12} = \frac{|E_1|\,|E_2|\,\sin ¢_{12}}{X_{12}}$$

where $X_{12} = X_1 + X_{tie} + X_2$ and $\delta_{12} = \delta_1 - \delta_2$. For a small deviation ΔP_{12} of the tie line power flow, we can write

$$\Delta P_{12} = \left.\frac{\partial P_{12}}{\partial ¢_{12}}\right|_{¢120} \Delta ¢_{12} = P_S\,\Delta\delta_{12}$$

where $\left.\frac{\partial P_{12}}{\partial \text{¢}_{12}}\right|_{\text{¢}120}$ = $P_{S,}$ is the slope of the power angle curve evaluated at the initial operating point ($\delta_{120} = \delta_{10} \quad \delta_{20}$) and is the synchronizing power coefficient..

$$P_S = \left.\frac{\partial P_{12}}{\partial \text{¢}_{12}}\right|_{\text{¢}120} = \frac{|E_1|\,|E_2|}{X_{12}} \cos \text{¢}_{120}$$

$$\Delta P_{12} = P_S\,(\Delta\delta_1 \quad \Delta\delta_2)$$

A positive ΔP_{12} occurs when $\Delta\delta_1 > \Delta\delta_2$ and indicates a flow of real power from area 1 to area 2. This has the effect of increasing load on area 1 and decreasing load on area 2. Hence ΔP_{12} has negative sign for area 1 and positive sign for area 2 in Fig c.

Consider a change in load ΔP_{L1} in area 1. The steady state frequency deviation Δf is same for both the areas. Hence

$\Delta f = \Delta f_1 = \Delta f_2$.

For area 1,

$$\Delta P_{m1} \quad \Delta P_{12} \quad \Delta P_{L1} = \Delta f D_1$$

For area 2,

$$\Delta P_{m2} + \Delta P_{12} = \Delta f D_2$$

The change in mechanical powers depends on the respective regulations.

$$\Delta P_{m1} = \frac{-\Delta f}{R_1}$$

$$\Delta P_{m2} = \frac{-\Delta f}{R_2}$$

Substituting we get

$$\Delta f\left(\frac{1}{R_1} + D_1\right) = -\Delta P_{12} - \Delta P_{L1}$$

$\frac{1}{R_1} + D_1 = \beta_1$, the frequency bias factor for area 1.

$\Delta f\, \beta_1 = -\Delta P_{12} - \Delta P_{L1}$

Similarly

$\Delta f\, \beta_2 = \Delta P_{12}$

Solving for Δf we get

$$\Delta f = \frac{-\Delta P_{L1}}{_1 + \ _2}$$

$$\Delta P_{12} = \frac{-\Delta P_{L1}\ _2}{_1 + \ _2}$$

Thus an increase of load in area 1 reduces frequency in both areas. Similarly for a change in load ΔP_{L2} in area 2,

$$\Delta f = \frac{-\Delta P_{L2}}{_1 + \ _2}$$

and $\Delta P_{12} = \quad \Delta P_{21} = \frac{-\Delta P_{L2}\ _1}{_1 + \ _2}$

Example 5

A two area system connected by a tie–line has following parameters on 1000 MVA base;

$R_1 = 4.5\%$; $D_1 = 0.6$; $H_1 = 4.5$;

$R_2 = 6\%$; $D_2 = 0.85$; $H_2 = 5.0$;

The units are running in parallel at a frequency of 50 Hz. The synchronizing power coefficient is 1.9 pu at the initial operating angle. A load change of 150 MW occurs in area 1. Determine the new steady state frequency and the change in tie–line power flow.

Solution:

$$\Delta P_{L1} = \frac{150}{1000} = 0.15 \text{ pu.}$$

Steady state frequency deviation is

$$\Delta f = \frac{-\Delta P_{L1}}{_1 + \ _2}$$

$$\beta_1 = \frac{1}{R_1} + D_1 = \frac{1}{0.045} + 0.6 = 22.822$$

$$\beta_2 = \frac{1}{R_2} + D_2 = \frac{1}{0.06} + 0.85 = 17.516$$

$$\Delta f = \frac{-0.15}{22.822 + 17.516} = \quad 0.0037 \text{ pu}$$

Steady state frequency = 50 (0.0037 × 50) = 49.815 Hz.

$\Delta P_{12} = \Delta f\ \beta_2 =$ 0.0037 × 17.516 = 0.0648 pu = 64.8 MW.

Since ΔP_{12} is negative, it implies that 64.8 MW flows from area 2 to area 1.

Change in mechanical powers is given by

$$\Delta P_{m1} = \frac{-\Delta f}{R_1} = \left(\frac{-0.0037}{0.045}\right) = 0.082 \text{ pu} = 82 \text{ MW}$$

$$\Delta P_{m2} = \frac{-\Delta f}{R_2} = \left(\frac{-0.0037}{0.06}\right) = 0.0617 \text{ pu} = 61.7 \text{ MW}$$

Change in load in area 1 due to frequency sensitive loads is $\Delta f D_1$ = (0.0037) (0.6) = 0.0022 pu = 2.2MW. Similarly for area 2 $\Delta f D_2$ = (0.0037) (0.85) = 0.0031 pu = 3.1MW. Total change in load is 5.3 MW. The power flow of 64.8 MW from area 2 to area 1 is contributed by an increase in generation of area 2 by 61.7 MW and reduction in load of area 2 by 3.1 MW.

Tie–line bias control:

From discussion it can be seen that, if the areas are equipped only with primary control of the ALFC, a change in load in one area met is with change in generation in both areas, change in tie–line power and a change in the frequency. Hence, a supplementary control is necessary to maintain

- Frequency at the nominal value
- Maintain net interchange power with other areas at the scheduled values
- Let each area absorb its own load

Hence, the supplementary control should act only for the areas where there is a change in load. To achieve this, the control signal should be made up of the tie–line flow deviation plus a signal proportional to the frequency deviation. A suitable proportional weight for the frequency deviation is the frequency – response characteristic β. This is the reason why β is also called the frequency bias factor. This control signal is called the area control error (ACE). In a two area system

$ACE_1 = \Delta P_{12} + B_1\ \Delta f$; $B_1 = \beta_1$

$ACE_2 = \Delta P_{21} + B_2\ \Delta f$; $B_2 = \beta_2$

The ACE represents the required change in area generation and its unit is MW. ACEs are used as control signals to activate changes in the reference set points. Under steady state ΔP_{12} and Δf will be zero. The block diagram with the supplementary control is shown below. It is applied to selected units in each area.

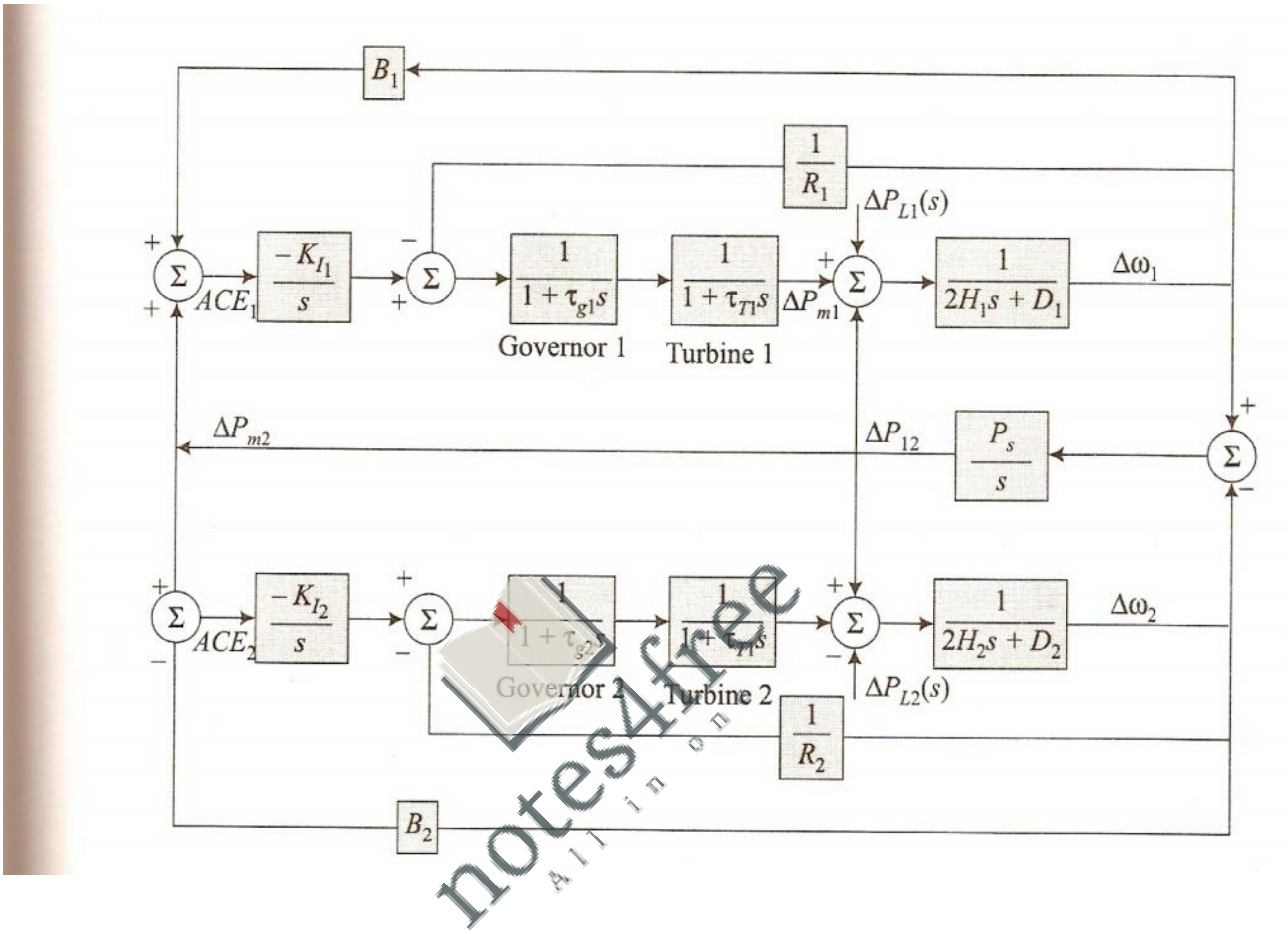


Fig : Block diagram with supplementary control

The operation of the ACE can be explained as follows. Consider an increase in load of area 1, which leads to a decrease in system frequency. The primary ALFC loop limits the frequency deviation to

$$\Delta f = \frac{-\Delta P_{L1}}{{}_1 + {}_2}$$

The tie–line power has a deviation $\Delta P_{12} = \beta_2 \Delta f$. The slower acting supplementary control, starts responding now.

$$ACE_1 = \quad \Delta P_{12} + B_1\, \Delta f = \frac{\left({}_1 + \quad {}_2 \right)}{\left({}_1 + \quad {}_2 \right)} \left(-\Delta P_{L1} \right) = \quad \Delta P_{L1}$$

and $$ACE_2 = \quad \Delta P_{12} + B_2\, \Delta f = \frac{\left(-\Delta P_{L1} \right)}{\left({}_1 + \quad {}_2 \right)} \left(- \quad {}_2 + \quad {}_2 \right) = 0$$

Thus only supplementary control of area 1 responds to ΔP_{L1} and the generation changed so that ACE_1 becomes zero.

<u>Example 6</u>:

Two areas are connected via an inter tie. The load at 50Hz, is 15,000 MW in area 1and 35,000 MW in area 2. Area 1 is importing 1500 MW from area 2. The load damping constant in each area is D = 1.0 and the regulation R = 6% for all units. Area 1 has a spinning reserve of 800 MW spread over 4000 MW of generation capacity and area 2 has a spinning reserve of 1000 MW spread over 10,000 MW generation. Determine the steady state frequency, generation and load of each area and tie–line power for

(a) Loss of 1000 MW in area 1, with no supplementary control

(b) Loss of 1000 MW in area 1, with supplementary controls provided on generators with reserve.

β_1 = 250 MW/0.1 Hz and β_2 = 400 MW/0.1 Hz

<u>Solution</u>:

(a) Assume a lossless system

<u>Area 1</u>: Load = 15,000 MW

Power import from area 2 = 1,500 MW

Generation = 15,000 – 1500 = 13,500 MW

Reserve = 800 MW

Total generation capacity = 13,500 + 800 = 14,300 MW

<u>Area 2</u>: Load = 35,000 MW

Export to area 1 = 1,500 MW

Generation = 35,000 + 1,500 = 36,500 MW

Reserve = 1000 MW

Total generation capacity = 36,500 + 1000 = 37,500 MW

Regulation of 6% on a generation capacity of 14,300 MW (including reserve) corresponds to

$$R_1 = \frac{0.06 \times 50}{14300}$$

$$\frac{1}{R_1} = \frac{1}{0.06} \times \frac{14300}{50} = 4{,}766.67 \text{ MW/Hz}$$

Similarly

$$\frac{1}{R_2} = \frac{1}{0.06} \times \frac{37500}{50} = 12{,}500 \text{ MW/Hz}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = 17{,}266.6 \text{ MW/Hz.}$$

Load damping D = 1.0, which means a 1% change in load occurs for a 1% change in frequency. D_1 is computed for area 1 on a total load of 14000 MW, considering the loss of 1000 MW.

$$D_1 = 1 \times \frac{14000}{100} \times \frac{100}{50} = 280 \text{ MW/Hz.}$$

Similarly D_2 is calculated on a load of 35,000 MW since there is no change in load of area 2.

$$D_2 = 1 \times \frac{35000}{100} \times \frac{100}{50} = 700 \text{ MW/Hz.}$$

$$D_{eq} = D_1 + D_2 = 980 \text{ MW/Hz.}$$

The change in system frequency is given by

$$\Delta f = \frac{-\Delta P_L}{\dfrac{1}{R_{eq}} + D_{eq}} = \frac{-(-1000)}{17{,}266.6 + 980} = 0.0548 \text{ Hz.}$$

Load changes due to load damping are,

$$\Delta P_{D1} = D_1\,\Delta f = 280 \times 0.0548 = 15.344 \text{ MW}$$

$$\Delta P_{D2} = D_2\,\Delta f = 700 \times 0.0548 = 38.36 \text{ MW}$$

Change in generations are

$$\Delta P_{G1} = \frac{-\Delta f}{R_1} = \quad 4766.6 \times 0.0548 = \quad 261.2 \text{ MW}$$

$$\Delta P_{G2} = \frac{-\Delta f}{R_2} = \quad 12500 \times 0.0548 = \quad 685 \text{ MW}$$

We recalculate powers as follows

<u>Area 1</u>: New load = 15,000 1,000.00 + 15.344 = 14,015.344 MW

New generation = 13,500 – 261.2 = 13,238.8 MW

Deficit = 14,015.34 13,238.8 = 776 MW

<u>Area 2</u>: New load = 35,000 + 38.36 = 35,038.36 MW

New generation = 36,500 – 685 = 35,815 MW

Excess = 35,815 35,038.36 = 776 MW

Thus tie–line power is 776 MW and flows from area 2 to area 1.

Steady state frequency = 50 + 0.0548 = 50.0548 Hz

(b) *With supplementary control and β_1 = 250 MW/0.1 Hz and β_2 = 400 MW/0.1 Hz*

Generating capacity with supplementary control in area 1 is 4000 MW (on reserve) and in area 2 it is 1000 MW. These supplementary controls will keep ACE_1 and ACE_2 at zero.

$ACE_1 = \beta_1 \Delta f + \Delta P_{12} = 0$

$ACE_2 = \beta_2 \Delta f + \Delta P_{21} = \beta_2 \Delta f \quad \Delta P_{12} = 0$

This means $\Delta P_{12} = 0$ and $\Delta f = 0$

Thus the load and generation in area 1 are reduced by 1000 MW. There is no steady state deviation of tie–line power flow and frequency. The generation and load of area 2 also do not change.

2.1 What is a fault?

Any undesired / unwanted condition is known as a fault.

2.1.1 Reasons for fault occurrence?

- Lightning strokes / thunderstorms / switching surges
- Tree falling
- Kites
- Birds can cause faults
- Aircraft / vehicle
- Earthquake
- Wind / ice / rain
- Deterioration of insulation / ageing
- Monkey / reptiles / snakes

Sr. No.	Causes	% of Total
1	Lightning	12
2	Wind / mechanical consideration	20
3	Appliance failure	20
4	Switching to a fault	20
5	Misc. (Trees, birds, etc.)	28

Sr. No.	Equipment	% of Total
1	O.H. lines	50
2	Wind / mechanical consideration	20
3	Appliance failure	20
4	Switching to a fault	20
5	Misc. (Trees, birds, etc.)	28

2.1.2 Type of faults

Symmetrical Faults

- LLL
- LLLG

Unsymmetrical Faults

- LG
- LL
- LLG
- LL and 3rd Ground

Sr. No.	Faults	% of Total
1	LG	70
2	LL	15
3	LLG	10
4	LL or LG	2-3
5	LLLG	2-3
6	LLL	2-3

2.2 Introduction

- Symmetrical faults are caused in power system accidentally through insulation failure of equipment or flashover of lines initiated by lighting stroke or through accidental faulty operation.
- Disconnecting the faulty part of the system by means of circuit breakers operated by protective relaying is necessary to protect against flow of heavy short circuit current. For proper choice of circuit breakers, we should study this chapter.
- Most of the system faults are not three phase faults but faults involving one line to ground or occasionally two lines to ground. Though the symmetrical faults are rare, the fault leads to most severe fault current.
- The synchronous generator during short circuit has a characteristic time-varying behavior. In the event of a short circuit, the flux per pole undergoes dynamic change with associated transients in damper and field windings.

2.3 Transients on a transmission line

- Certain simplifying assumptions are made
  1. The line fed from a constant voltage source.
  2. Short circuit takes place when the line unloaded.
  3. Line capacitance is negligible and the line can be represented by a lumped RL series circuit
- As shown in fig. 2.1 short circuit is assumed to take place at $t=0$. The parameter α controls the instant on the voltage wave when short circuit occurs. It is known form circuit theory that current after short circuit is composed of two parts i.e.

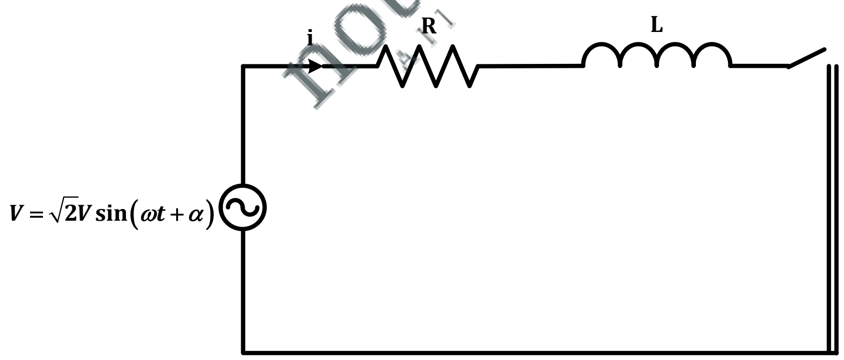


Figure 2.1 Short Circuit Model of Line

$$i=i_s+i_t$$

$i_s =$ steady state current

$$=\frac{\sqrt{2}V}{|Z|}\sin(\omega t+\alpha-\theta)$$

$$Z=\sqrt{R^2+\omega^2L^2}\quad \angle\theta=\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$i_t =$ transient current

(it is such that $i(0) = i_s(0) + i_t(0) = 0$ being an inductive circuit; it decays corresponding to the time constant L/R)

- A plot of i_s, i_t, and $i = i_s + i_t$ is shown in fig. 2.2.

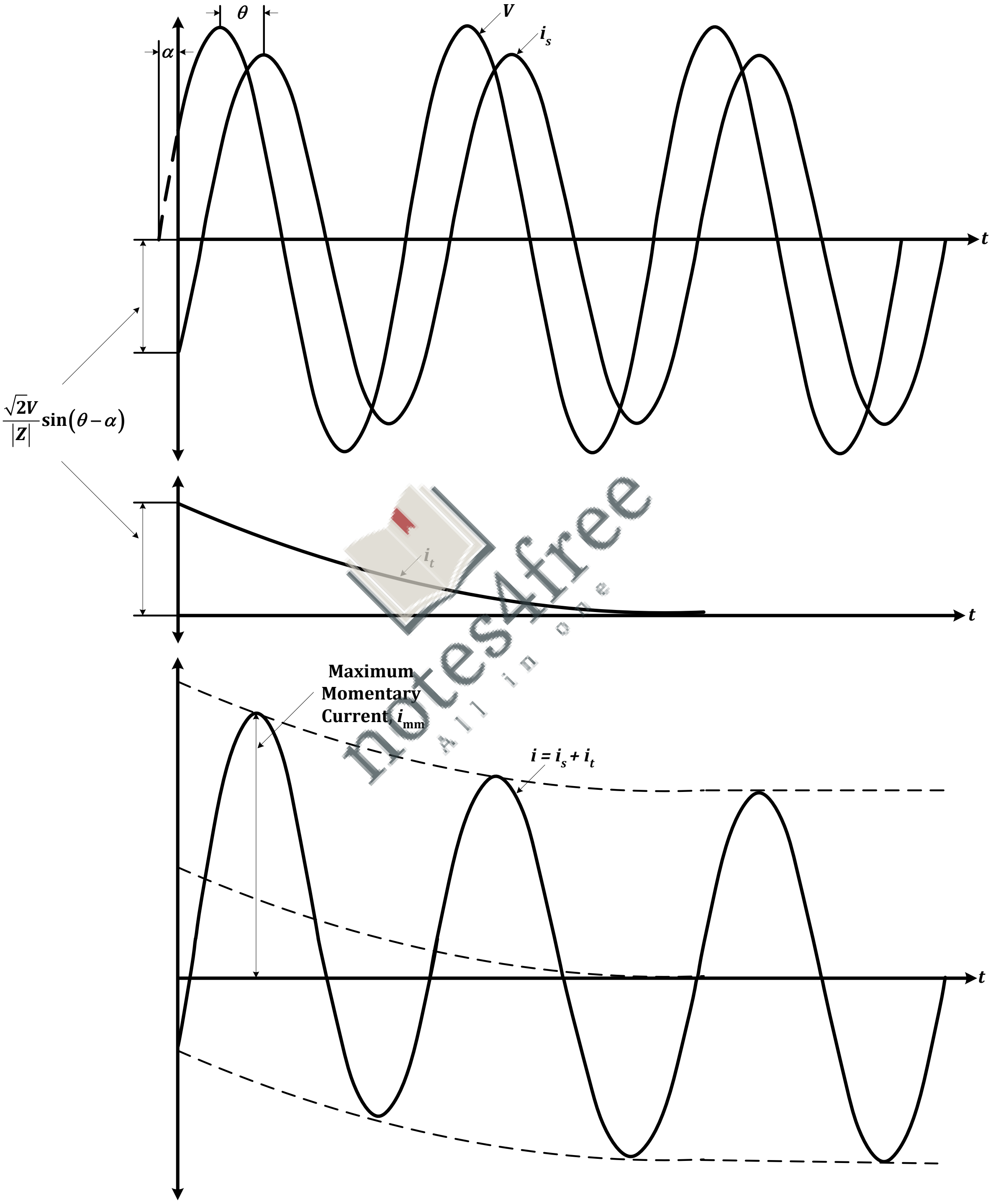


Figure 2.2 Waveform of Short Circuit Current on a Transmission Line

$$i_t = -i_s(0)e^{-(R/L)t}$$
$$= -\frac{\sqrt{2}V}{|Z|}\sin(\alpha-\theta)e^{-(R/L)t}$$
$$= \frac{\sqrt{2}V}{|Z|}\sin(\theta-\alpha)e^{-(R/L)t}$$

Thus, short circuit current is given by

$$i = \frac{\sqrt{2}V}{|Z|}\sin(\omega t+\alpha-\theta) + \frac{\sqrt{2}V}{|Z|}\sin(\theta-\alpha)e^{-(R/L)t}$$

= symmetrical SC current + DC-offset current

- In power system terminology, the sinusoidal steady state current is called the symmetrical short circuit current and the unidirectional transient component is called the DC-offset current, which causes the total short circuit to be unsymmetrical till the transient decays
- The maximum momentary current, i_{mm} corresponds to first peak. If the decay of transient in short time is neglected,

$$i_{mm} = \frac{\sqrt{2}V}{|Z|}\sin(\theta-\alpha) + \frac{\sqrt{2}V}{|Z|}$$

- Since transmission line resistance is small $\theta \cong 90°$

$$i_{mm} = \frac{\sqrt{2}V}{|Z|}\cos\alpha + \frac{\sqrt{2}V}{|Z|}$$

- This has the maximum possible value for $\alpha = 0°$, short circuit occurring when the voltage wave is going through zero.

$$i_{mm}(\text{max possible}) = \frac{2\sqrt{2}V}{|Z|}$$

= twice the maximum of symmetrical SC current (doubling effect)

- For the selection of circuit breakers, momentary short circuit current is taken corresponding to its maximum possible value (a safe choice)
- It means that when the current is interrupted, the DC offset i_t has not yet died out and so the computing the value of DC offset at the time of interruption (this would be highly complex in a small network), the symmetrical short circuit current alone is calculated. This figure is then increased by multiplying factor to take in account of the DC offset.

2.4 Short circuit of a synchronous machine (On NO Load)

- Under steady state short circuit condition, the armature reaction of a synchronous generator produces a demagnetizing flux.
- In terms of a circuit, this effect is modelled as a reactance X_a in series with the induced emf.

- This reactance when combined with the leakage reactance X_l of the machine is called synchronous reactance X_d (direct axis synchronous reactance in case of salient pole machines) as shown in fig. 2.3.

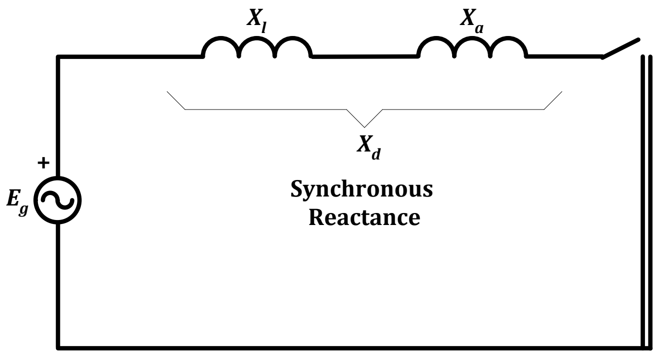


Figure 2.3 Steady State Short Circuit Model of a Synchronous Machine

- Consider now the sudden short circuit (3ϕ) of a synchronous machine initially operating under open circuit conditions.
- The circuit breaker must, of course interrupt the current much before steady conditions are reached.
- Immediately upon short circuit, the DC offset currents appear in all the three phases, each with different magnitude since the point on the voltage wave at which short circuit occurs is different for each phase.

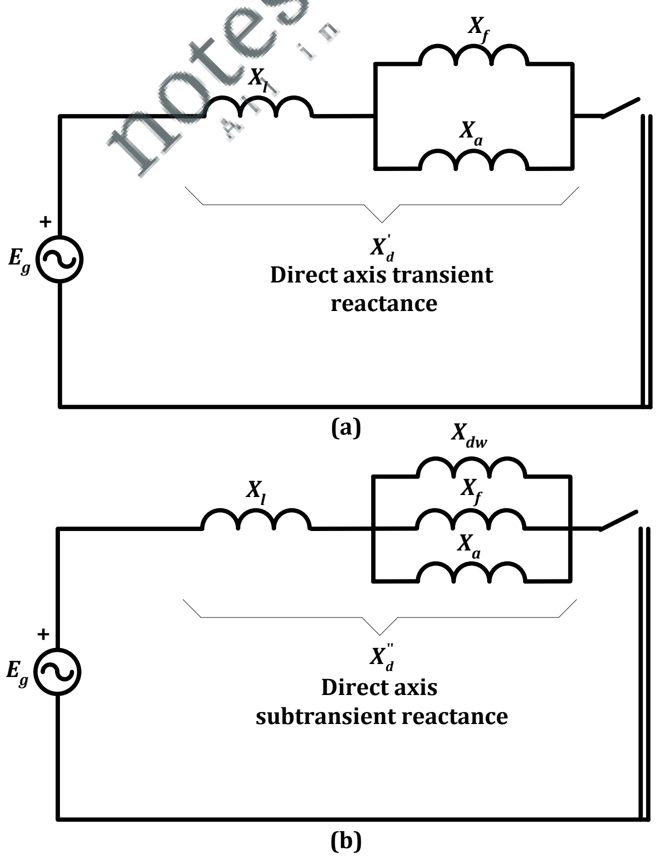


Figure 2.4 Approximate Circuit Model During (a) Subtransient Period of Short Circuit (b) Transient Period of Short Circuit

- Immediately in the event of a short circuit, the symmetrical short circuit current is limited only by leakage reactance of machine.
- Since the air gap flux cannot change instantaneously, to counter the demagnetization of the armature circuit current, current appears in field winding as well as in the damper winding in a direction to help the main flux.
- The current decays in accordance with the winding time constants. The time constant of damper winding which has low leakage inductance is much less than that of field winding with high leakage inductance.
- The machine reactance, thus changes from parallel combination of X_a, X_f and X_{dw} during the initial period (fig. 2.4 (b)) of short circuit to X_a and X_f in parallel in the middle period (fig. 2.4 (a)) of the short circuit and finally to X_a in steady state.

$$X_d'' = X_l + \frac{1}{\frac{1}{X_a} + \frac{1}{X_f} + \frac{1}{X_{dw}}}$$

$$X_d' = X_l + \frac{1}{\frac{1}{X_a} + \frac{1}{X_f}}$$

$$X_d'' < X_d' < X_d$$

$$I'' = \frac{E_g}{X_d''}$$

$$I' = \frac{E_g}{X_d'}$$

$$I = \frac{E_g}{X_d}$$

2.5 Short circuit of a loaded synchronous machine

- Fig. 2.5 shows the circuit model of a synchronous generator operating under steady conditions supplying a load current I^0 to the bus at a terminal voltage of V^0. E_g is the induced emf under loaded condition and X_d is the direct axis synchronous reactance of the machine.

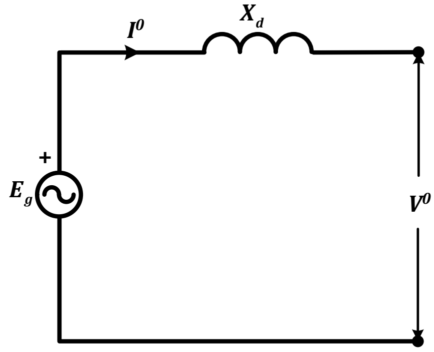


Figure 2.5 Circuit Model of a Loaded Synchronous Machine

- When short circuit occurs at the terminals of this machine, the circuit model to be used for computing short circuit current is shown in the fig. 2.6 for subtransient current and transient current.

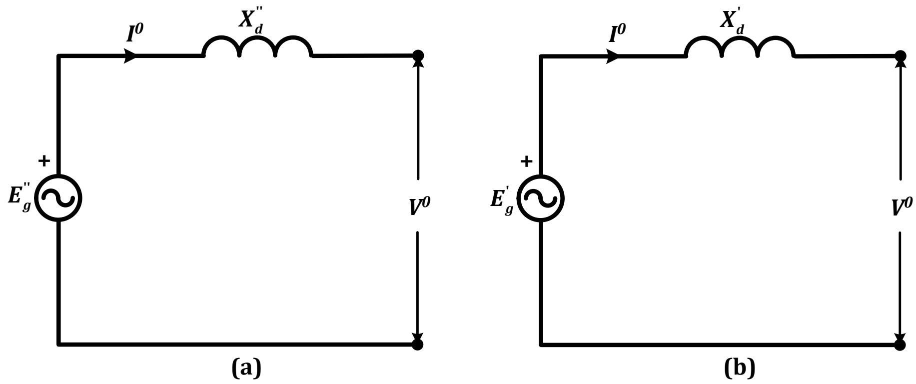


Figure 2.6 Circuit Model for Computing (a) Subtransient Current (b) Transient Current

$I^0 = I_L =$ Load current of the bus before fault

$V^0 =$ Teminal voltage

$E_g =$ Induced emf under loaded conditon

$E_g^{''} =$ Voltage behind subtransient reactance

$E_g^{'} =$ Voltage behind transient reactance

$$E_g^{''} = V^0 + jX_d^{''} I^0$$
$$E_g^{'} = V^0 + jX_d^{'} I^0$$

- Synchronous motors have internal emfs and reactances similar to that of a generator except that the current direction is reversed. During short circuit conditions, these can be replaced by similar circuit models except that the voltage behind subtransient/transient reactance is given by

$$E_m^{''} = V^0 - jX_d^{''} I^0$$
$$E_m^{'} = V^0 - jX_d^{'} I^0$$

- In case of short circuit of an interconnected system, the synchronous machines (generators and motors) are replaced by their corresponding circuit models having voltage behind subtransient (transient) reactance in series with subtransient (transient) reactance. The rest of the network being passive remains unchanged.

2.6 Short circuit current by Thevenin Theorem

- An alternate method of computing short circuit current is through the application of thevenin theorem.
- Consider a synchronous generator feeding a synchronous motor over a line. Fig. 2.7 (a) shows the circuit model of the system under conditions of steady load. Fault computations are to be made for a fault at *F*, at the motor terminals.

- As a result the circuit model is replaced by the one shown in fig. 2.7 (b), wherein the synchronous machines are represented by their subtransient reactances (or transient reactances if transient currents are of interest) in series with voltages behind subtransient reactances. This change does not disturb the prefault current I^0 and prefault voltage V^0 (at F).

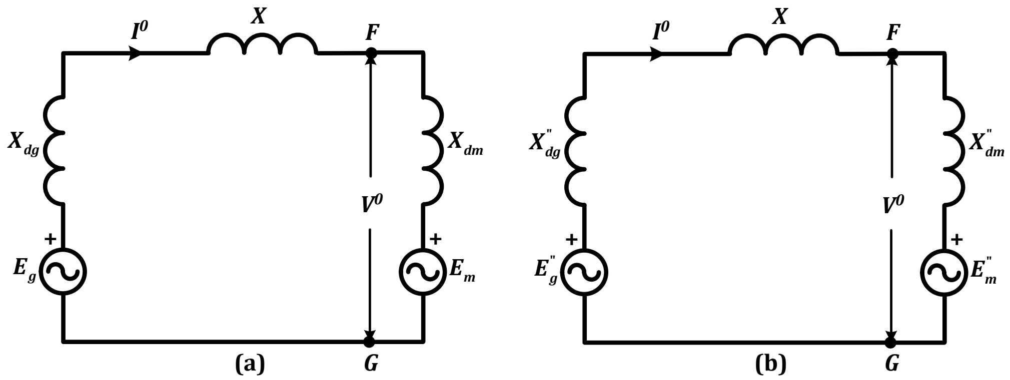


Figure 2.7 Circuit Model under (a) Steady State (b) Subtransient State

- As seen from FG the thevenin equivalent circuit of fig. is drawn.

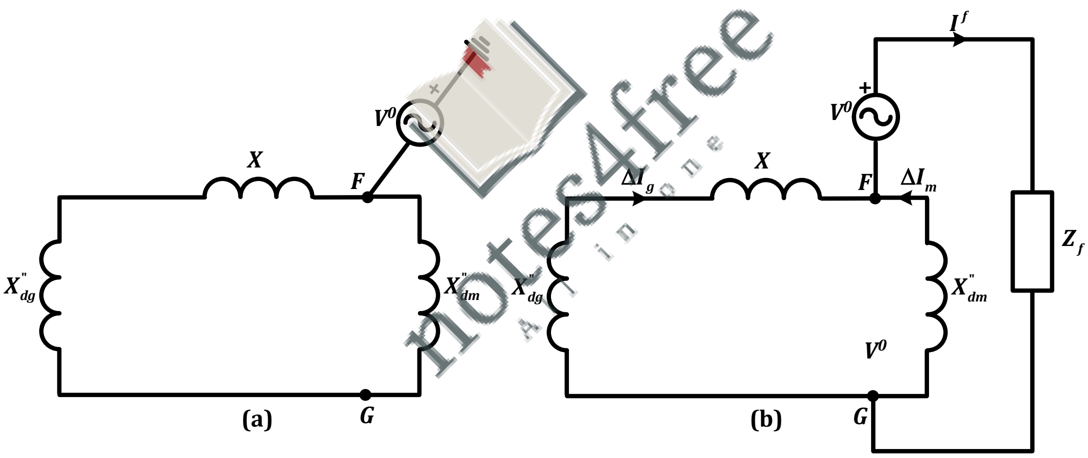


Figure 2.8 (a) Computation of SC by Thevenin Equivalent Circuit (b) Thevenin Equivalent System Feeding Fault Impedance

- Consider now a fault a F through an impedance Z^f. Fig. 2.8 (b) shows the thevenin equivalent of the system feeding the fault impedance.

$$X_{TH} = \left(X_{dg}^{''} + X\right) \| X_{dm}^{''}$$

$$X_{TH} = \frac{1}{\dfrac{1}{\left(X_{dg}^{''} + X\right)} + \dfrac{1}{X_{dm}^{''}}}$$

$$X_{TH} = \frac{X_{dm}^{''}\left(X_{dg}^{''} + X\right)}{X_{dg}^{''} + X + X_{dm}^{''}}$$

$$I^f = \frac{V^0}{jX_{TH} + Z^f}$$

- Current caused by fault current in generator circuit

$$\Delta I_g = \frac{X''_{dm}}{X''_{dg} + X + X''_{dm}} I^f$$

- Current caused by fault current in generator circuit

$$\Delta I_m = \frac{X''_{dg} + X}{X''_{dg} + X + X''_{dm}} I^f$$

- Postfault currents and voltages are obtained as follows by superposition:

$$I''_g = I^0 + \Delta I_g$$

$$I''_m = -I^0 + \Delta I_m (\text{in the direction of } \Delta I_m)$$

- Postfault voltage

$$V^f = V^0 + \left(-jX_{TH} I^f\right)$$

$$= V^0 + \Delta V$$

- So, the prefault current flowing out of the fault point *F* is always zero, the postfault current out of *F* is independent of load for a given prefault voltage at *F*.
- Steps for solving short circuit current by thevenin theorem approach
  1. Obtain the steady state solution using load flow. Formulate the circuit model.
  2. Replace reactance of synchronous machine by their transient or subtransient values.
  3. SC all emf sources and find the value of Z_{TH} or X_{TH}.
  4. Compute the required currents using thevenin's and superposition theorem.
- Assumptions
  1. All prefault voltage equal to 1pu.
  2. Load current neglected as it is very less as compared to SC current.

2.7 Selection of circuit breakers

- Two of the circuit breaker ratings which require the computation of SC current are: rated momentary current and rated symmetrical interrupting current.
- Symmetrical SC current is obtained by using subtransient reactances for synchronous machines. Momentary current (rms) is then calculated by multiplying the symmetrical current by a factor of 1.6 to account the presence of DC-offset current.
- The DC-offset value to be added to obtain the current to be interrupted is accounted for by multiplying the symmetrical SC current by a factor as tabulated below:

Table 2.1 Circuit Breaker Multiplying Factor

Sr. No.	Circuit breaker speed	Multiplying factor
1.	8 cycles or slower	1.0
2.	5 cycles	1.1
3.	3 cycles	1.2
4.	2 cycles	1.4

- The current that a circuit breaker can interrupt is inversely proportional to the operating voltage over a certain range, i.e.

Amperes at operating voltage = amperes at rated voltage x $\frac{\text{rated voltage}}{\text{operating voltage}}$

- Of course, operating voltage cannot exceed the maximum design value. Also, no matter how low the voltage is, the rated interrupting current cannot exceed the rated maximum interrupting current.
- It is therefore logical as well as convenient to express the circuit breaker rating in terms of SC MVA that can be interrupted, defined as

  Rated interrupting MVA (three phase) capacity = $\sqrt{3}x|V_{line}|x|I_{line}|_{\text{rated interrupting current}}$

  Where $|V_{line}|$ is in kV and $|I_{line}|$ is in kA

- Thus, instead of computing the SC current to be interrupted, we compute three-phase SC MVA to be interrupted, where

  SC MVA (three phase) = $\sqrt{3}$ x prefault line voltage in kV x SC current in kA

  SC MVA (three phase) = $|V|_{\text{prefault}}$ x $|I|_{\text{SC}}$ x $(\text{MVA})_{\text{Base}}$ (if in P.U.)

- Obviously, the rated MVA interrupting capacity of a circuit breaker is to be more that (or equal to) the SC MVA required to be interrupted.
- For the selection of a circuit breaker for a particular location, we must find the maximum possible SC MVA to be interrupted with respect to type and location of fault and generating capacity (also synchronous motor load) connected to the system. A three phase fault though rare is generally the one which gives the highest SC MVA and a circuit breaker must be capable of interrupting it.
- In a large system various possible location must be tried out to obtain the highest SC MVA, requires repeated SC computations.

2.8 Algorithm for SC studies

- Consider an n bus system shown schematically in fig. 2.9 operating at steady load. Consider r as faulted bus.

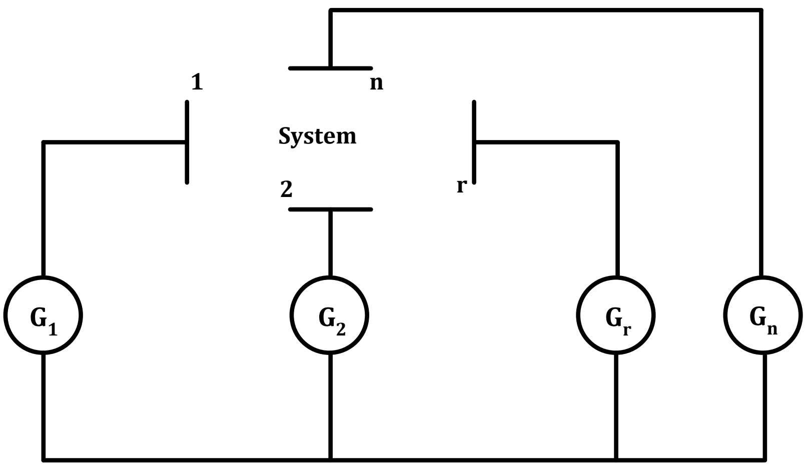


Figure 2.9 n-bus System under Steady Load

- The first step towards short circuit computation is to obtain the prefault voltages at all buses and currents in all lines through load flow studies.

- Step-1 To find out prefault voltages at all buses.

$$V_{BUS}^{0} = \begin{bmatrix} V_1^0 \\ V_2^0 \\ \vdots \\ V_n^0 \end{bmatrix}$$

- Step-2 Bus no. r is to be faulted through fault impedance Z^f

$$V_{BUS}^{f} = V_{BUS}^{0} + \Delta V \text{ (change)}$$

- Step-3 Form Thevenin's equivalent circuit. Short circuit all emf sources and replace all reactances by their respective reactances transient / sub-transient as shown in fig. 2.10

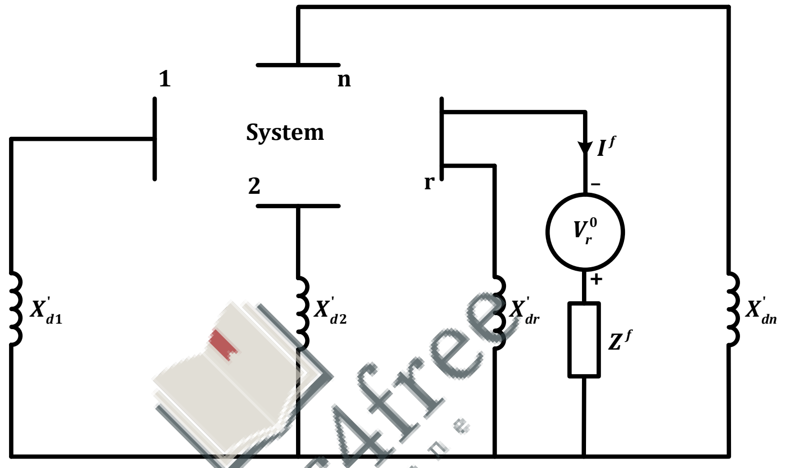


Figure 2.10 Network of System for Computing Post Fault Voltages

- Step-4 Compute the value of ΔV

$$\Delta V = Z_{BUS} J^f$$

Where,

$$Z_{BUS} = \begin{bmatrix} Z_{11} \cdots Z_{1n} \\ \vdots \quad \ddots \quad \vdots \\ Z_{n1} \cdots Z_{nn} \end{bmatrix} = \text{bus impedance matrix of the passive Thevenin network}$$

$$J^f = \begin{bmatrix} 0 \\ 0 \\ -I^f \\ \vdots \\ 0 \end{bmatrix} = \text{bus current injection vector}$$

- Step-5 Find the post fault voltage for the rth bus

$$\Delta V_r = -Z_{rr} I^f$$

$$V_r^f = V_r^0 + \Delta V_r$$

$$V_r^f = V_r^0 - Z_{rr} I^f$$

$$V_r^0 = Z^f I^f + Z_{rr} I^f$$
$$I^f = \frac{V_r^0}{\left(Z^f + Z_{rr}\right)}$$

- Step-6 Find the post fault voltage for any i[th] bus

$$\Delta V_i = -Z_{ir} I^f$$
$$V_i^f = V_i^0 + \Delta V_i$$
$$V_i^f = V_i^0 - Z_{ir} I^f$$
$$V_f^0 = V_i^0 - Z_{ir} \frac{V_r^0}{\left(Z^f + Z_{rr}\right)} \quad \left(\because I^f = \frac{V_r^0}{\left(Z^f + Z_{rr}\right)}\right)$$

- Step-7 Find the post fault current in lines

$$I_{ij}^f = Y_{ij}\left(V_i^f - V_j^f\right)$$

2.9 Z_{BUS} formation by step by step method

Notation: i, j – old buses; r – reference bus; k – new bus.

i) Type-1 Modification: - Branch Z_b is added between new bus and reference bus
ii) Type-2 Modification: - Branch Z_b is added between new bus and old bus
iii) Type-3 Modification: - Branch Z_b is added between old bus to reference bus
iv) Type-4 Modification: - Branch Z_b is added between two old buses

- Type-1 Modification: - Adding a branch Z_b between new bus k and reference bus r as shown in fig 2.11.

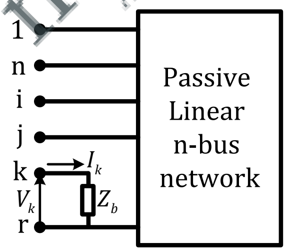


Figure 2.11 Type-1 Modification

$$V_k = Z_b I_k$$
$$Z_{ik} = Z_{ki} = 0;\ i = 1,2,\ldots,n$$
$$Z_{kk} = Z_b$$

$$Z_{BUS}(\text{new}) = \left[\begin{array}{ccc|c} & Z_{BUS}(\text{old}) & & 0 \\ & & & \vdots \\ & & & \vdots \\ & & & \vdots \\ \hline 0 & \cdots\ \cdots & 0 & Z_b \end{array}\right]$$

- Type-2 Modification: - Adding a branch Z_b between old bus j and new bus k as shown in fig. 2.12.

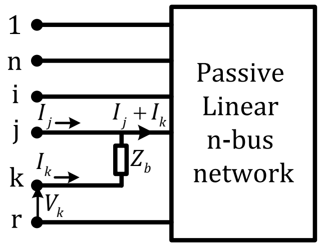


Figure 2.12 Type-2 Modification

$$V_k = Z_b I_k + V_j$$
$$= Z_b I_k + Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{jj}\left(I_j + I_k\right) + \dots + Z_{jn} I_n$$

Rearranging,

$$V_k = Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{jj} I_j + \dots + Z_{jn} I_n + \left(Z_{jj} + Z_b\right) I_k$$

$$Z_{BUS}(\text{new}) = \left[\begin{array}{ccc|c} & Z_{BUS}(\text{old}) & & \begin{matrix} Z_{1j} \\ Z_{2j} \\ \vdots \\ Z_{nj} \end{matrix} \\ \hline Z_{j1} & Z_{j2} \; \cdots & Z_{jn} & Z_{jj} + Z_b \end{array}\right]$$

- Type-3 Modification: - Adding a branch Z_b between old bus j and reference bus r as shown in fig. 2.13. This case follows by connecting bus k to the reference bus r, i.e., by setting $V_k = 0$.

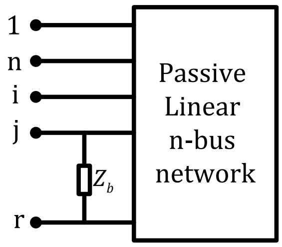


Figure 2.13 Type-3 Modification

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline 0 \end{bmatrix} = \left[\begin{array}{ccc|c} & Z_{BUS}(\text{old}) & & \begin{matrix} Z_{1j} \\ Z_{2j} \\ \vdots \\ Z_{nj} \end{matrix} \\ \hline Z_{j1} & Z_{j2} \; \cdots & Z_{jn} & Z_{jj} + Z_b \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_k \end{bmatrix}$$

Eliminate I_k in the set of equations contained in the matrix,

$$0 = Z_{j1}I_1 + Z_{j2}I_2 + ... + Z_{jn}I_n + \left(Z_{jj} + Z_b\right)I_k$$

$$I_k = -\frac{1}{Z_{jj} + Z_b}\left(Z_{j1}I_1 + Z_{j2}I_2 + ... + Z_{jn}I_n\right)$$

Now,

$$V_i = Z_{i1}I_1 + Z_{i2}I_2 + ... + Z_{in}I_n + Z_{ij}I_k$$

$$V_i = \left[Z_{i1} - \frac{1}{Z_{jj} + Z_b}\left(Z_{ij}Z_{j1}\right)\right]I_1 + \left[Z_{i2} - \frac{1}{Z_{jj} + Z_b}\left(Z_{ij}Z_{j2}\right)\right]I_2 + ... + \left[Z_{in} - \frac{1}{Z_{jj} + Z_b}\left(Z_{ij}Z_{jn}\right)\right]I_n$$

In matrix form,

$$Z_{BUS}(\text{new}) = Z_{BUS}(\text{old}) - \frac{1}{Z_{jj} + Z_b}\begin{bmatrix} Z_{1j} \\ Z_{2j} \\ \vdots \\ Z_{nj} \end{bmatrix}\begin{bmatrix} Z_{j1} & Z_{j2} & \cdots & Z_{jn} \end{bmatrix}$$

- Type-4 Modification: - Adding a branch Z_b between old bus i and old bus j in fig. 2.14.

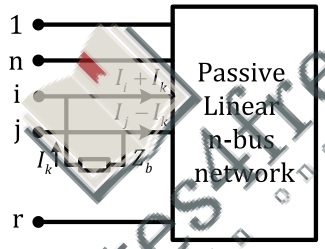


Figure 2.14 Type-4 Modification

$$V_i = Z_{i1}I_1 + Z_{i2}I_2 + ... + Z_{ii}\left(I_i + I_k\right) + Z_{ij}\left(I_j - I_k\right) + ... + Z_{in}I_n$$

Similar equations follow for other buses. The voltages of the buses i and j are, however, constrained by the equation

$$V_j = Z_bI_k + V_i$$

$$Z_{j1}I_1 + Z_{j2}I_2 + ... + Z_{ji}\left(I_i + I_k\right) + Z_{jj}\left(I_j - I_k\right) + ... + Z_{jn}I_n =$$

$$Z_bI_k + Z_{i1}I_1 + Z_{i2}I_2 + ... + Z_{ii}\left(I_i + I_k\right) + Z_{ij}\left(I_j - I_k\right) + ... + Z_{in}I_n$$

Rearranging,

$$0 = \left(Z_{i1} - Z_{j1}\right)I_1 + ... + \left(Z_{ii} - Z_{ji}\right)I_i + \left(Z_{ij} - Z_{jj}\right)I_j + ... + \left(Z_{in} - Z_{jn}\right)I_n + \left(Z_b + Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji}\right)I_k$$

In matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline 0 \end{bmatrix} = \left[\begin{array}{cccc|c} & & Z_{BUS}(\text{old}) & & (Z_{1i}-Z_{1j}) \\ & & & & (Z_{2i}-Z_{2j}) \\ & & & & \vdots \\ & & & & (Z_{ni}-Z_{nj}) \\ \hline (Z_{i1}-Z_{j1}) & (Z_{i2}-Z_{j2}) & \cdots & (Z_{in}-Z_{jn}) & Z_b+Z_{ii}+Z_{jj}-2Z_{ij} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_j \end{bmatrix}$$

Eliminate I_k on lines similar to what was done in type-2 modification, it follows that

$$Z_{BUS}(\text{new}) = Z_{BUS}(\text{old}) - \frac{1}{Z_{ii}+Z_{jj}+Z_b-2Z_{ij}} \begin{bmatrix} Z_{1i}-Z_{1j} \\ Z_{2i}-Z_{2j} \\ \vdots \\ Z_{ni}-Z_{nj} \end{bmatrix} \begin{bmatrix} (Z_{i1}-Z_{j1}) & (Z_{i2}-Z_{j2}) & \cdots & (Z_{in}-Z_{jn}) \end{bmatrix}$$